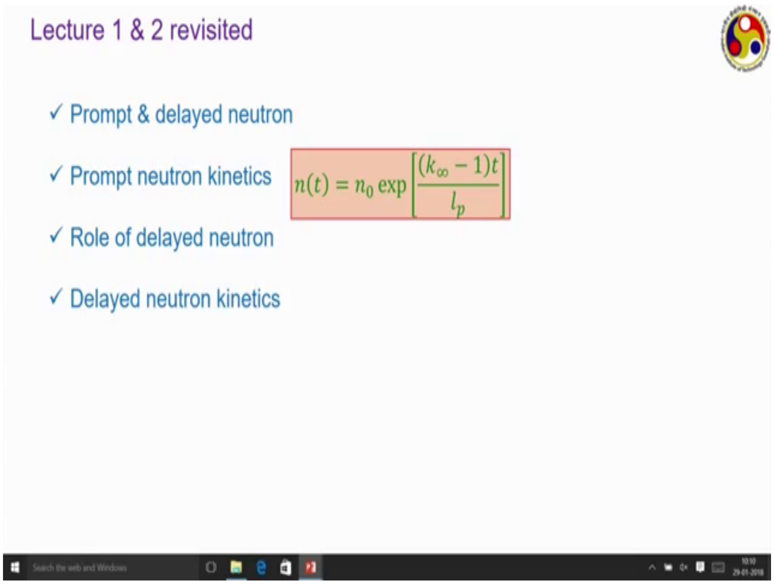
Fundamentals of Nuclear Power Generation
Dr. Dipankar N. Basu
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 06
Reactor Control
Lecture – 03
Different control mechanisms & various effects

Hello friends, welcome back to our 6th module where you are discussing about the topic of reactor control. While someone has already started to step up in different parts of India, but we are in Guwahati or in northeast India, we are quite a bit blest in terms of temperature, and it is quite chilly morning at Guwahati. But let us go back to our topic.

(Refer Slide Time: 00:55)



Lecture 1 & 2 revisited

- ✓ Prompt & delayed neutron
- ✓ Prompt neutron kinetics $n(t) = n_0 \exp\left[\frac{(k_\infty - 1)t}{l_p}\right]$
- ✓ Role of delayed neutron
- ✓ Delayed neutron kinetics

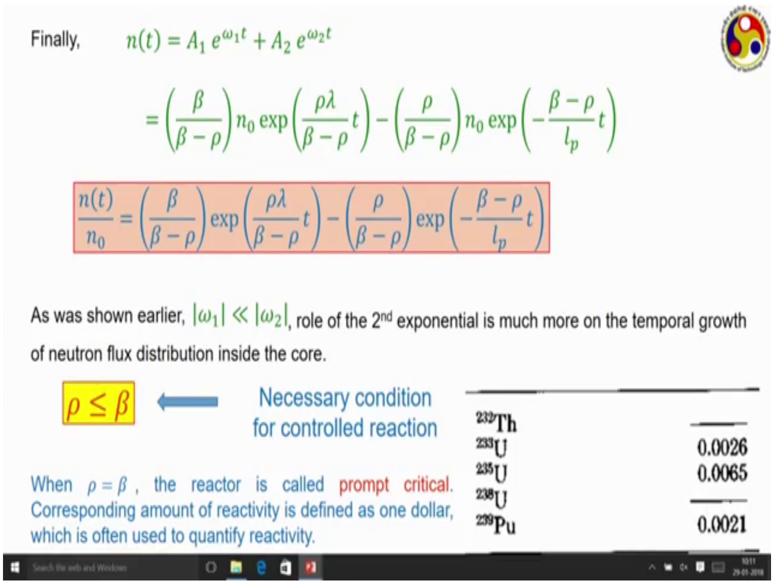
Here on this particular module. We have already discussed about the role of prompt and delayed neutrons played in reactor control or on deciding the neutron flux profile.

There first we discuss about the from neutron kinetics to get a equation like this, and by putting the terms like multiplication factor, and the prompt neutron like time etcetera, they are magnitudes. We have seen that it is quite difficult to control the reactor if it is operating fully based upon from neutrons. Rather the time that is available for us to put any kind of controlling mechanism or activity kind of controlling mechanism that is extremely small.

And then we discussed about the role of delayed neutrons, and in the last lecture that is in the second lecture of this particular module we have discussed the delayed neutron kinetics. Where you have called develop this particular equation here instead of considering 6 or 8 different groups of delayed neutron precursors, we have consider just a single group and beta represents corresponding delayed neutron fraction.

We have analyzed this equation and from there we have identified the condition of prompt criticality to be something like rho equal to beta; that is, as long as the reactivity is less than the delayed neutron fraction or at the limiting case it is equal to delayed neutrons fraction, then we have sufficient time to activity or control mechanism and disposable to control the reactor. Basically because of these presence of the delayed neutron or the activity of the delayed neutrons only we are able to control the a practically nuclei reactor.

(Refer Slide Time: 02:30)



Finally, $n(t) = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t}$

$$= \left(\frac{\beta}{\beta-\rho}\right) n_0 \exp\left(\frac{\rho\lambda}{\beta-\rho} t\right) - \left(\frac{\rho}{\beta-\rho}\right) n_0 \exp\left(-\frac{\beta-\rho}{l_p} t\right)$$

$$\boxed{\frac{n(t)}{n_0} = \left(\frac{\beta}{\beta-\rho}\right) \exp\left(\frac{\rho\lambda}{\beta-\rho} t\right) - \left(\frac{\rho}{\beta-\rho}\right) \exp\left(-\frac{\beta-\rho}{l_p} t\right)}$$

As was shown earlier, $|\omega_1| \ll |\omega_2|$, role of the 2nd exponential is much more on the temporal growth of neutron flux distribution inside the core.

$\boxed{\rho \leq \beta}$ ⟵ Necessary condition for controlled reaction

When $\rho = \beta$, the reactor is called prompt critical. Corresponding amount of reactivity is defined as one dollar, which is often used to quantify reactivity.

^{232}Th	—
^{233}U	0.0026
^{235}U	0.0065
^{238}U	—
^{239}Pu	0.0021

Now, towards the end of our last lecture that is in my last slide I used some technical features for which some part of the slides going to missing. And I had to write on the screen hopefully that was clear, but still for the purpose of completeness, and this is a slide that I had there I should have shown you there, here nt represents the neutron density or number of neutrons per unit volume and this variation with respect to time N naught is the initial one and this is the expression which dominates the neutron kinetics; where we have taken into consideration both delayed and prompt neutrons, and beta is the delayed neutron precursor or concentration on delayed neutron fraction rather where we are considering just the single group of delayed neutron precursor. And here this is is omega one this is omega 2 and is a spooked earlier omega one is extremely small compared to omega 2 therefore, it is a second exponential term which dominate the temporal growth of this neutron flux distribution. And from there we have identified rho is equal to beta is the condition for a control reaction.

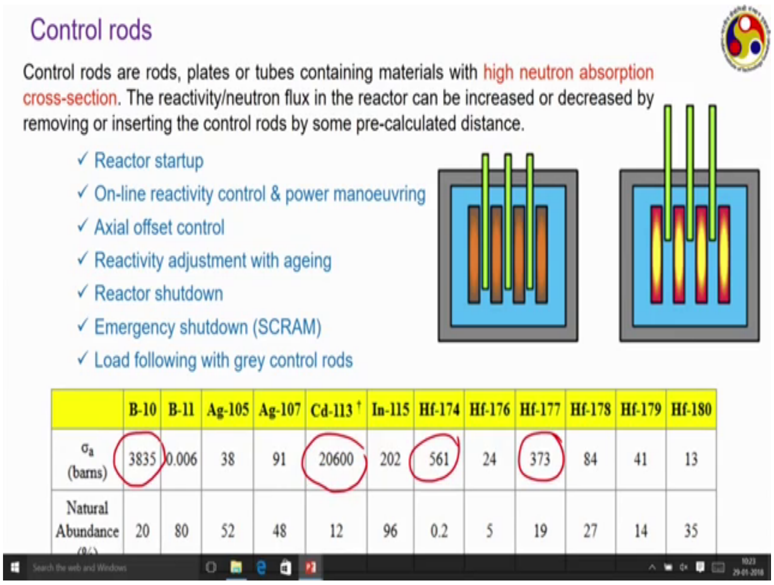
When rho equal to beta we call the reactor to be prompt critical and corresponding amount of here reactivity is called one dollar. It is a unit sometimes quite often used in nuclear industry to denote the condition or to I signify the condition corresponds to this prompt criticality. One-dollar risk for refers to the amount of critically the amount of reactivity requires to achieve this prompt critical condition. Of course, the value of beta depends upon the fuel and as the condition for prompt criticality is rho equal to beta; so, the magnitude of dollar that also keeps on varying depends on depending on fuel therefore, it is not an absolutely unit.

Like for uranium 233, it is 0.00 to 6 is the value of beta whereas, for uranium 235 it is 0.0065. So, correspondingly the value of one dollar for a reactor operating a uranium 235 will be 0.0065; whereas, if it is operating on plutonium 239 corresponding magnitude of one dollar will be 0.0021, but once we know which fuel we are using in that reactor, then we of course, know what is the magnitude of one dollar, and then the value of reactivity is often referred in terms of this dollar.

In fact, 100 of a dollar is also called the cent, just equivalent to the currents in inertia notations, and it is quite common you will find in a nuclear reactor that it is mentioned it is presently it is reactivity at the moment is 10 cents; that means, it is reactivity is 10 percent of beta. Like; it is working on uranium 235. So, 1 dollar is supposed to 0.0065 or 0.65 percent; therefore, 10 cent will refer to 0.065 percent, and hence this reactivity will be 0.065 in that case 0.065 percent I should say.

With this we have discussed about the role of prompt and delayed neutrons on determining the criticality or the reactivity of a thermal reactor. And let us now move to the mechanisms which are used in practical reactors to achieve this particular conduct control.

(Refer Slide Time: 05:54)



Control rods

Control rods are rods, plates or tubes containing materials with high neutron absorption cross-section. The reactivity/neutron flux in the reactor can be increased or decreased by removing or inserting the control rods by some pre-calculated distance.

- ✓ Reactor startup
- ✓ On-line reactivity control & power manoeuvring
- ✓ Axial offset control
- ✓ Reactivity adjustment with ageing
- ✓ Reactor shutdown
- ✓ Emergency shutdown (SCRAM)
- ✓ Load following with grey control rods

	B-10	B-11	Ag-105	Ag-107	Cd-113 †	In-115	Hf-174	Hf-176	Hf-177	Hf-178	Hf-179	Hf-180
σ_a (barns)	3835	0.006	38	91	20600	202	561	24	373	84	41	13
Natural Abundance (%)	20	80	52	48	12	96	0.2	5	19	27	14	35

And the most common one of them is the control rods. Control rods are some elements which can be of the shape of rods or plates or tubes which contains materials which has very high absorption cross section.

Therefore, whenever this kind of elements or these kinds of structures are present inside the reactor, they can eat up a significant portion of the neutrons in the neighborhood area, and accordingly they can reduce the reactivity or you can reduce the neutron flux distribution there neutron flux concentration there. Therefore, just by this is a schematic diagram, on the first hand on the left-hand side the diagram shows, this particular situation, where the all the control rods are completely inserted into the reactor. And at the moment there may be some value of reactivity that is present.

Now, if we remove these reactors or by a certain distance, then the corresponding absorption cross section effective absorption cross section corresponding this controller that reduces, and hence the neutron flux concentration that a neutron concentration you say the reactor that has to increase. Their accordingly by changing the position of these control rods, we can control the amount of neutrons that are available to cause fission reaction.

Generally, the position of these rods are properly calibrated that; that means, depending upon what level of reactivity you wants, we can always say that this is the this should be the position of the control rods and accordingly they are adjusted. Their control rods serves in serve several purposes first in reactor startup means at the when you are starting a reactor from the cold condition, then we need very high neutron flux at the moment generally control rods are completely outside they completely remove from the reactors so that whatever amount of neutrons that are produced or that are supplied from some external source, all of them are available to induce fission reaction.

So, we get a very high rate of reaction which keeps on increasing with time. And as the neutron concentration keeps on increasing then we may slowly get these control rods into the reactor, thereby putting some sort of control and once we attain the desired level of equilibrium that is for instant number of neutrons produced, because of fission and number of neutrons consumed by these control rods that comes in equilibrium then we can attain the proper chain reaction.

And next is online reactivity control and power maneuvering as I mentioned, once we need to change the multiplication factor reactivity we can change the position of the control rods, like if we want to increase the reactivity; that means, if all need is to increase the power production from the reactor, then we can remove the control rod by certain distance so that the neutron concentration increases and accordingly we have an increase in reactivity. The opposite is true when you want to reduce the power production, then we need to introduce the control rods a bit more we need to insert them by a further distance.

Then axial offset control, as we have seen in our discussions in the previous modules, that the neutron flux while you want the neutral flux to be uniform throughout the reactor it is practically not possible, rather we generally have the highest neutron flux close to the center line, and a much lower at the at the edges, if the reactor is an reflect to do on that is there is no reflected, then theoretically the neutron flux at the edges should be equal to 0.

Now, using the control rods, we can somehow offset this axial distribution or we can ah; that means, we can at the center line where the neutron flux is more then we can insert the control rods by a larger amount where as close to the edges. We can keep the control rods removed by a bigger amount so that we can maintain somewhat uniform neutron flux distribution; that means, what I am referring to if this is your reactor, at the center line that may be position of the control rod which is inserted with a big amount. Whereas, at the edges it can be inserted by a much smaller amount

And by adjusting the position of these two rods or whatever axial distribution that we have of these control rods, we can somewhat mental maintain a more uniform distribution. Then reactivity adjustment with aging; we have discussed this one also earlier. As time goes on once we have loaded a reactor with fresh fuel the reactivity generally is very high. Because there are more number of fuel isotopes are available.

But as a time goes on the number of fuel isotopes keeps on reducing, and accordingly the net fission cross section and net macroscopic fission cross section that keeps on reducing because that is really proportional to the nuclear density.

Now, that aging effect can be somewhat compensated by the use of control rods. Like, and the beginning when the fuel concentration or concentration of fuel nuclei is very high, we can have control rods inserted by a larger amount, but as the concentration of nuclei keeps on reducing, then we can gradually take these control rods out the.

Next is the reactor shutdown, when there is a plant shutdown required then we can gradually get these control rods into larger insertion position so that the reactivity keeps on reducing gradually, and once these control rods are completely inserted, then they should be able to eat up all the neutrons available inside the reactor thereby completely stopping the reaction.

So, the control rods are used both during the startup and during reactor shutdown. And then emergency shutdown which is also called scram, I do not want to expand this term just equate this acronym at the moment this is related to an emergency shutdown. When there is an emergency need of shutting down the reactors, then this control rods are inserted very, very rapidly that is from whatever position they were in at the moment, they go into the completely insertion position within a very short amount of time so that they are able to consume all the available neutrons thereby completely stopping the reaction.

So, control rods have important roles to play, both during the normal operation to not both and also during emergency operation. Normal during normal operation they can adjust for the axial distribution they can adjust for the aging and also, they can be used for a short time power change; that is, by changing the reactivity to the desired level. And also during a emergency condition like emergency shutdown situation the control rods are generate the most important mechanism to stop the reaction inside the reactor.

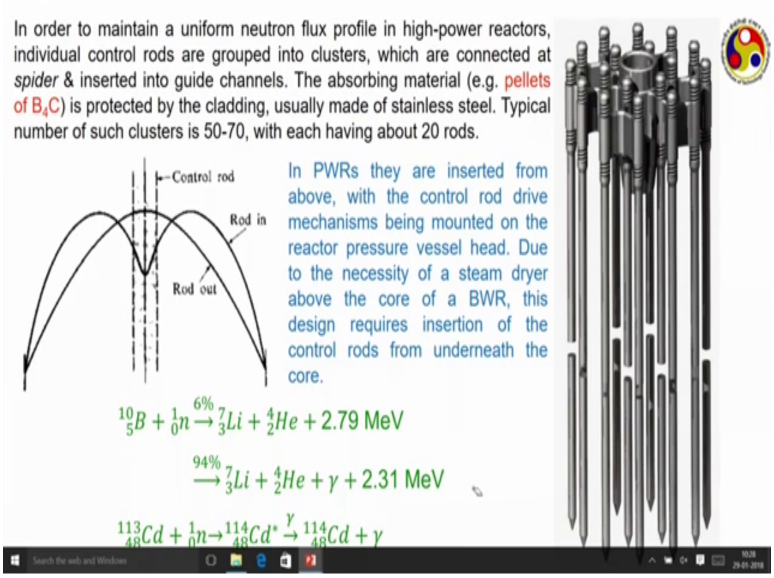
Now, the material of the control rod should be something that has very high neutron absorption cross section ok. Actually I have missed one the final performance or final function of the control rods is load following it grey control rods. Grey control rods are elements which has somewhat a laser absorption cross section compared to the normal cross control rods, which are also called the black control rods. And therefore, when there is a smaller change in the load on the reactor using these grey control rods also you can change the reactivity.

Next these are a table of some materials and their cross sections. And if of course, only few selected materials I am showing here, namely boron silver cadmium indium and hafnium. And all their isotopes or most of the isotopes are shown here. You can see out of this the boron 10 as an extremely high absorption cross section, and cadmium 113. So, again it is much much higher than boron 10. Hafnium 174 or 177 also has reasonably high absorption cross section. And therefore, they can act as quite good material for this neutron or for these control rods. It is also important here I am mentioning about absorption cross section where actually all these cross sections are non-fission capture kind because their fission cross section is essentially 0.

Now, boron has 2 naturally occurring isotopes boron 10 and boron 11 boron in common or you will find boron 10 to of 20 per to consume 20 percent by volume. Whereas, boron 11 is 80 percent, but boron 11 has extremely small absorption cross section, and therefore, boron 10 is a material it is primarily used for control rods or any kind of control related machinery. Cadmium can also be used, but there can be other reasons with cadmium which I shall be showing shortly.

Hafnium one big advantage of hafnium is that there are 2 isotopes hafnium 560 hafnium 174 or 177 both have high fission of cross sections. And also, other isotopes hafnia hafnium like 178 that is also if it reasonable amount of absorption cross section. And therefore, hafnium can also be used for control rod.

(Refer Slide Time: 15:16)



In order to maintain a uniform neutron flux profile in high-power reactors, individual control rods are grouped into clusters, which are connected at *spider* & inserted into guide channels. The absorbing material (e.g. pellets of B_4C) is protected by the cladding, usually made of stainless steel. Typical number of such clusters is 50-70, with each having about 20 rods.

In PWRs they are inserted from above, with the control rod drive mechanisms being mounted on the reactor pressure vessel head. Due to the necessity of a steam dryer above the core of a BWR, this design requires insertion of the control rods from underneath the core.

$${}^{10}_{5}B + {}^{1}_{0}n \xrightarrow{6\%} {}^{7}_{3}Li + {}^{4}_{2}He + 2.79\text{ MeV}$$

$$\xrightarrow{94\%} {}^{7}_{3}Li + {}^{4}_{2}He + \gamma + 2.31\text{ MeV}$$

$${}^{113}_{48}Cd + {}^{1}_{0}n \rightarrow {}^{114}_{48}Cd^{*} \xrightarrow{\gamma} {}^{114}_{48}Cd + \gamma$$

This is a typical profile for neutron flux, the this particular curve corresponds to when there are no control rods. So, because there are no control rods present, this is an unreflected reactor. So, at the center line the concentration is very high whereas, close to the edges the concentration is 0; however, once you have inserted one central control rod, then you can see there is an immediate reduction, this is the profile, you can see immediate reduction in the neutron flux profile at the center line. Of course, that increases as we move away from the center and then decreases again to become 0 at the edges.

So, just by using this using this a single control rod, we can have we can make the difference between the center line value, and this edges value to be smaller, and then if we can use some more control rods with a proper orientation, then we can get a much more uniform profile which I have mentioned earlier.

Now, just a single control rod therefore, is not sufficient, rather we need to have several control rods. And therefore, individual control rods are grouped into clusters, and this something like this. There are several control rods they are all grouped into grouped into clusters, and they are connected at the top by a structure which is commonly called a spider, and then they are inserted to the gride guide channels.

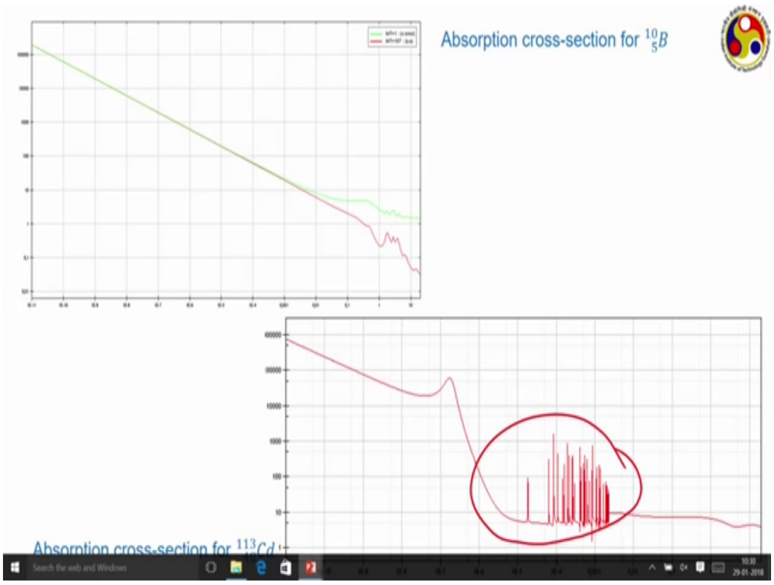
These because of the structural stability point of view just plain boron powder can not be used. So, use some kind of compounds of boron something like a boron carbide can be a very interesting one. So, pellets of boron carbides are generally used in control rods, and they look quite similar to the fuel rods, and they also are protected by cladding made of stainless steel. Typically we can have 50 to 70 number of such clusters in a modern-day reactors with each clusters having about 20 rods.

Now, in a PWR's this control rod clusters are inserted form the top. The driving mechanisms and all other accessories are mounted at the top of the pressure vessel, and then they can enter from the top. But in case of a boiling water reactor, when we allow the liquid to change it is phase, and then the vapor phase it is a steam that is separated in a steam drum the it is generally required to mount that stream drum to the highest possial possible location, and therefore, this control rod driving mechanism is generally mounted from in the bottom, and control rods also entered the reactor from the lower side.

As I have mentioned boron on cadmium are the two most important materials used in control rods particularly boron. So, this is the reaction that boron can go through, when a boron absorbs neutron it generally goes through an alpha decay process; that is, it is produces a lithium 7, and 1 helium isotopes. And in 6 percent of cases if reduce it corresponding mass defect, it emits 2.79 MeV of energy it will. But in rest 94 percent cases, you will generally find one gamma photon and 2.3 on a maybe of energy, and the energy carried by this gamma photon or the kinetic energy the gamma photon actually is the difference between these 2.79 and 2.31 thereby conserving the energy.

Cadmium eats also absorbs a neutron and generally cadmium 113 which has the highest absorption cross section. That gets converted to cadmium 114, it is generally at the excited state. So, it releases one it goes through a gamma decay process that is releases the gamma photon, and then come back to the ground state.

(Refer Slide Time: 19:13)



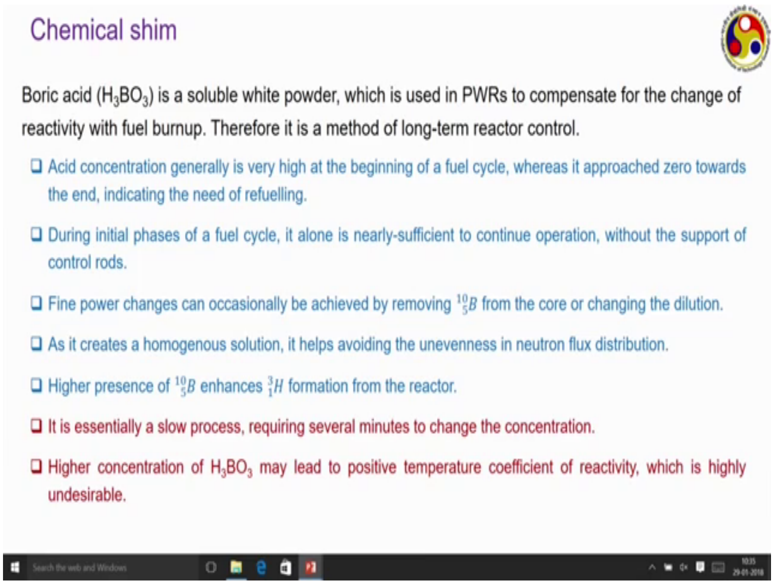
Absorption cross-section for $^{10}_{5}B$

Absorption cross-section for ^{113}Cd

These are the absorption cross sections for these 2 isotopes; that is, a boron 10 and cadmium 113. In case of boron 10 it is very interesting to observe that there is hardly in a resonance. That is as we are moving from a very low energy level like that thermal energy level corresponds to 0.025 electron volt of energy that is a, 2.5 into 10 to the power minus 8 electron volt, which will be somewhere here. So, from the thermal range to quite high energy levels like here it is showing something like 10 MeV at the highest one. It is more or less a straight-line distribution.

However, in case of cadmium there is a significant absorption zone, resonance absorption that is. And that is one reason that we should be careful about using cadmium 113 as the control material. Boron is quite easy to use, and we can see that does not have any kind of a resonance absorption. And hence there it maybe it is a primary reason of going for boron or boron 10 I should say as the common material for absorption control rods rather.

(Refer Slide Time: 20:27)



Chemical shim

Boric acid (H_3BO_3) is a soluble white powder, which is used in PWRs to compensate for the change of reactivity with fuel burnup. Therefore it is a method of long-term reactor control.

- ❑ Acid concentration generally is very high at the beginning of a fuel cycle, whereas it approached zero towards the end, indicating the need of refuelling.
- ❑ During initial phases of a fuel cycle, it alone is nearly-sufficient to continue operation, without the support of control rods.
- ❑ Fine power changes can occasionally be achieved by removing $^{10}_{5}B$ from the core or changing the dilution.
- ❑ As it creates a homogenous solution, it helps avoiding the unevenness in neutron flux distribution.
- ❑ Higher presence of $^{10}_{5}B$ enhances $^{3}_{1}H$ formation from the reactor.
- ❑ It is essentially a slow process, requiring several minutes to change the concentration.
- ❑ Higher concentration of H_3BO_3 may lead to positive temperature coefficient of reactivity, which is highly undesirable.

This is another technology along this chemical shim is something that is used along with the control rods. In means all practical reactors generally use control rods, but chemical shim is an additional technology that is used generally with PWR that is pressurized water reactors. In chemical shim, we use some kind of soluble powder commonly a boric acid, h 3 bo 3 which is a soluble white powder, and it is mixed with the water which is present inside the reactor pressure vessel, and it creates because it is soluble it creates a homogenous mixture and thereby occurs the entire portion of this reactor.

So, it is able to compensate for reactivity by or the change in reactivity with the change in the fuel concentration. Therefore, it is in a way method of long term reactor control not a short term one or it is also in a way it is not a measured which can be used under in case of emergency.

Now, chemical shim also similar to boric acid sorry similar to the control rods it all as a import several characteristics. Like the acid concentration the concentration of boric acid generally is very high at the beginning of the fuel cycle. And as time goes on these boron 10 isotopes that absorbs neutron gets converted to boron 11, and as we have seen just 2 slides back boron 11 has a negligible absorption cross section, and therefore, it becomes a dormant means it does not take any further part this.

So, as time goes on the concentration of this acid, or concentration of boron 10 isotopes that keeps on reducing, but the you should also take into account with time the number of fuel isotope that also keeps on reducing, in a properly designed reactor the rate at which reactivity decreases because of the depletion of fuel should be nearly equal to the increase in reactivity because of the reduction in boron 10 concentration. And if we can achieve that kind of situation we do not need control rods at all.

The basically in a properly designed pressurized water reactors, chemical shim alone is able to take care of all the regular operations or day to day operations. Control rods are generally you are almost removed condition. Only for there is a rapid changing reactivity required, we use the control rods and of course, in under emergency situations.

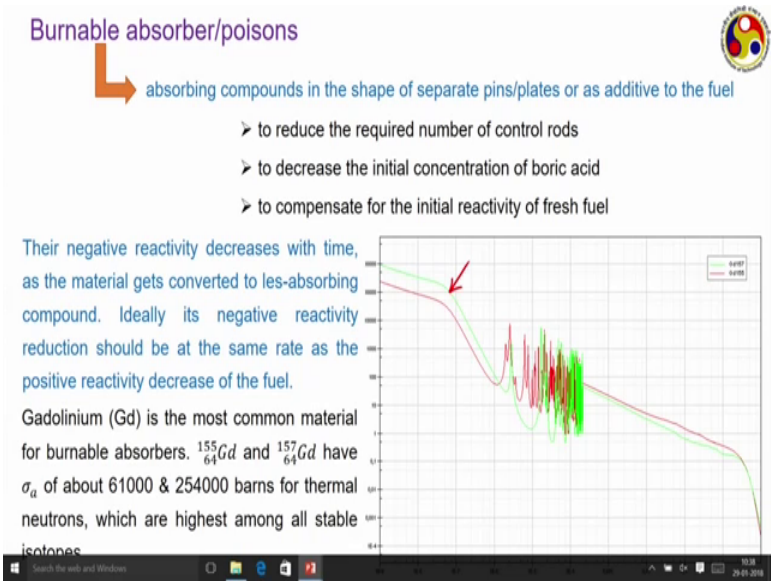
During the initial phases of the fuel circle, it alone is nearly sufficient to continue the operation without the support of control rods like I have just mentioned. But as time goes on the concentration of boron 10 reduces and when we approach the end of the fuel cycle it is concentration virtually become 0, which is also an indicative that we need to refuel the reactor again.

Small power changes can also be achieved with by changing the concentration of this boric acid, that is by adding some further acid powder or by adding some further coolant we can change the concentration of the solution. And so, smaller change or fine tuning of this reactivity can be done. Yeah, also it creates a homogenous solution therefore; it helps in avoiding the unevenness in the neutron flux distribution. And another important issue is that because of the appearance of this boron 10 the decay of the boron 10 often is found to lead to the formation of tritium. In fact, the decay of boron 10 or neutron a capture by boron 10 is the primary mechanism of tritium formation.

And also like it is also true for the control rods whenever a boron absorbs a neutron it can undergo a alpha decay process, and the helium that are produced because of that alpha decay, that increase the pressure inside the reactor; which also help somewhat helps in this entire procedure. But chemical shim also has it is own limitations like it is essentially slow process, it for any change in concentration several minutes are required.

Higher concentration of boric acid may lead to positive temperature coefficient of reactivity which is highly undesirable we shall be discussing the shortly. Because a temperature coefficient of reactivity you just keep in mind and also what the moment you assume that the positive value of this temperature coefficient of reactivity is undesirable; which may happen if the concentration of boric acid is too high. And also, it is not associate not suitable for boiling water reactors. Because whenever there is a phase change present inside the reactor the concentration of boric acid that keeps on increasing. So, that can significantly affect this reactivity.

(Refer Slide Time: 25:39)



Burnable absorber/poisons

absorbing compounds in the shape of separate pins/plates or as additive to the fuel

- to reduce the required number of control rods
- to decrease the initial concentration of boric acid
- to compensate for the initial reactivity of fresh fuel

Their negative reactivity decreases with time, as the material gets converted to les-absorbing compound. Ideally its negative reactivity reduction should be at the same rate as the positive reactivity decrease of the fuel.

Gadolinium (Gd) is the most common material for burnable absorbers. $^{155}_{64}Gd$ and $^{157}_{64}Gd$ have σ_a of about 61000 & 254000 barns for thermal neutrons, which are highest among all stable isotopes

There can be a third mechanism which is called burnable absorbers. Burnable absorbers, again mechanism is always the same that is to use a material with high neutron absorption cross section, just the mode of introducing that material that is different. Like in case of control rods we are making these materials in the shape of rods or plates, and inserting those into the reactor or removing as for our need. In case of chemical shim, we are making this material in the shape of soluble powder in the form of a soluble powder, and then and distributing that inside the reactor uniformly distributing there solution.

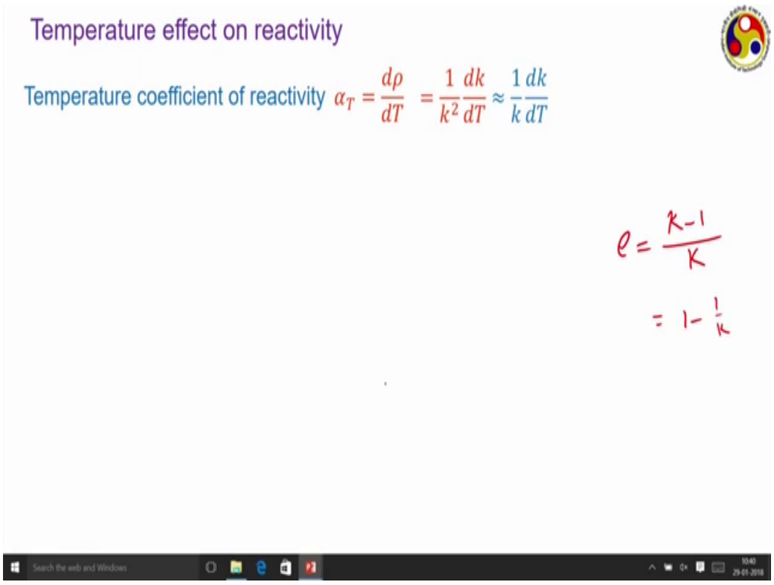
In case of burnable absorbers, we have separate pins or plates which are placed permanently inside the reactors. In certain situations, they are also added to the fuel itself; that is this burnable absorbers sometimes may become a part of the fuel pin itself. This also their activities quite similar to the chemical shim, but they are located at some their fixed at a particular locations and therefore, they can control the neutron flux profiles around that location.

Thereby they can reduce the required number of control rods. They can also reduce the initial concentration of boric acid because of a chemical shim process. And then similar to the chemical shim they can also compensate for the initial activity of fresh well. As the fuel becomes older fuel starts to edge, then this burnable absorbers also the after absorbing neutrons they also keeps on becoming inactive inside the reactor quite similar to chemical shims therefore, the reduction in reactivity because of the decaying fuel generally gets compensated, because of the reduction in this boron concentration or this absorber concentration.

The negative reactivity decreases with time, and as the material gets converted to low absorbing material, and as I mentioned this negative reactivity or rective reactivity reduction should be the same as this positive reactivity decrease of the fuel. Along with boron, gadolinium is another material which is very commonly used as burnable absorbers. Gadoliniums 155 and 157's are 2 isotopes which probably has the highest absorption cross section among all-natural isotopes.

Like for Gd 155 has an absorption cross section of something like 61,000. And that is significantly larger for 157 which is 12 2 54000. And these are the corresponding their variation of their cross section with energy, and you can see in the thermal energy level it is extremely high. You can compare this with the figure shown for boron 10 and cadmium earlier, and you will find this is at least 2 order is higher compared to them. That is why gadolinium can also be used that is particularly used as burnable absorbers.

(Refer Slide Time: 28:26)



Temperature effect on reactivity

Temperature coefficient of reactivity $\alpha_T = \frac{d\rho}{dT} = \frac{1}{k^2}\frac{dk}{dT} \approx \frac{1}{k}\frac{dk}{dT}$

$\rho = \frac{k-1}{k}$

$= 1 - \frac{1}{k}$

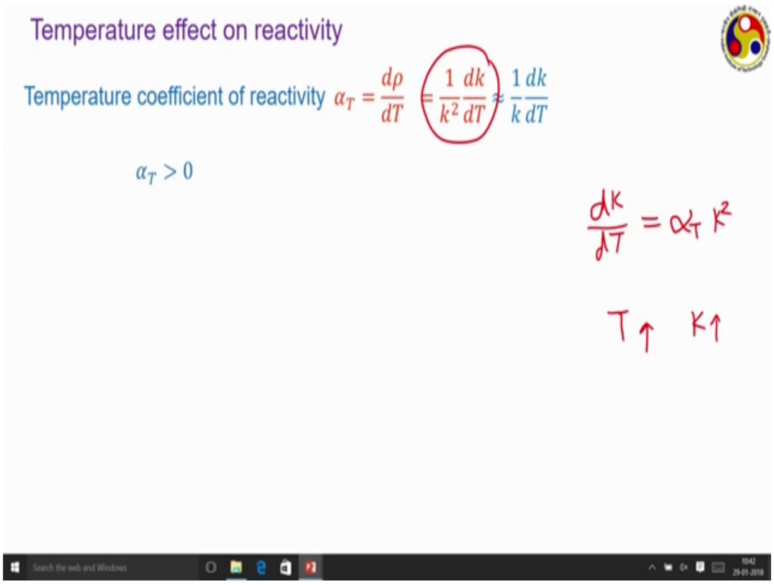
So, these are the common modes of the activity control. There can be a few others also some innovative methods like reflectors sometimes can also act in contriving the reactivity somewhat. Because we know that reflectors are used to prevent the leakage. It is their objective is to the get this neutrons is a leaking out of reactor back into the core. And by changing the orientation of these reflectors we can control this neutron leakage, and accordingly we can control the neutron flux inside the reactor. In certain innovative designs some removable fuel rods are also used.

Like when we need higher reactivity instead of removing the control rods, this removal fuel rods are inserted. And accordingly that can increase the reactivity inside the reactor. And there can be achieve other mechanisms also, but generally control rods and chemical shim are the most control rod is universal mechanism and chemical shim is also you will find particular in pressurized water reactors, and these 2 at the most common of reactivity control.

Now, we come to the term which may was mentioned shortly before temperature if it coefficient of reactivity. Reactivity we now know the how the delayed and prompt neutrons modify the reactivity, and we have also just studied the practical mechanisms by which you can control the reactivity. Now we are going to see the effect of different parameters on this reactivity like the temperature like the fuel burn up etcetera.

So, reactivity temperature the first term is temperature coefficient of reactivity defined as the rate of change of reactivity with temperature. And we also know that the definition of reactivity is equal to k minus 1 by k where k is a effective multiplication factor. And that is equal to 1 minus 1 upon k. If we introduce the definition here then it comes to be 1 by k square d k dT. And for most of the reactors, the initial condition k being equal to 1 this is sometimes written by this form shown by blue color as well. But we can stick to the original form there is 1 upon k square into dk dT.

(Refer Slide Time: 30:43)



Temperature effect on reactivity

Temperature coefficient of reactivity $\alpha_T = \frac{d\rho}{dT} = \frac{1}{k^2}\frac{dk}{dT} \approx \frac{1}{k}\frac{dk}{dT}$

$\alpha_T > 0$

$\frac{dK}{dT} = \alpha_T K^2$

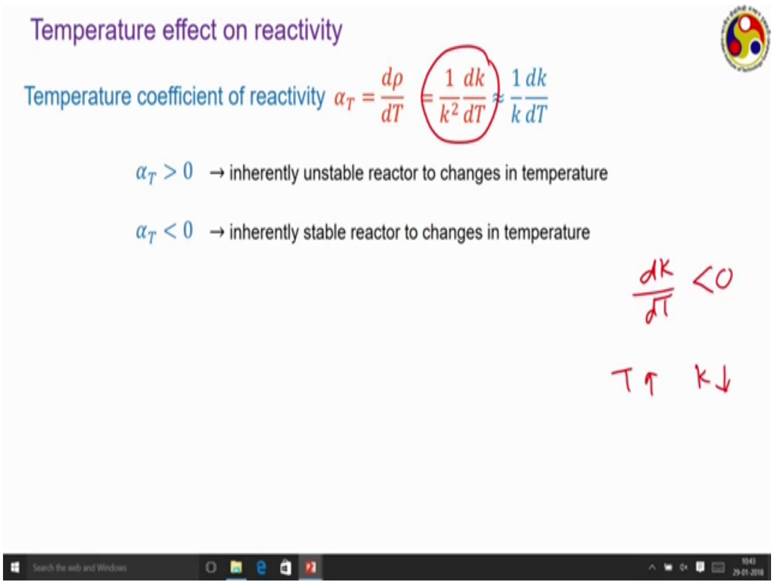
$T\uparrow \quad K\uparrow$

Now, when alpha T is positive; that is when we are having a reactor which is having a positive value of this temperature coefficient of reactivity. Then what may happen? You can just look at this expression here. K square is always a positive quantity if fact k itself is always a positive quantity. Here alpha is also a positive quantity. Let us write this as dk dT is equal to alpha p into k square. Now we are taking alpha T to be positive k square itself is always positive. So, for dk dT is also positive.

So, whenever there is a rise in temperature there will be a rise in the reactivity. And as the reactivity rises; that means, the power produced by the reactor that will also rise. So, energy released by the fuel rods that will for the rise further, and hence that will cause further increase in the furniture of the reactor code the fuel pin the moderator and all the materials in world.

So, that will caused even further increase in the reactivity; that means, once the temperature starts increasing, for a reactor with positive value of this alpha t, the reactivity keeps on increasing continuously. And unless we put some kind of intervention through some kind of controlling mechanism, the reactivity will keep on increasing infinitely with even a small change in it is temperature.

(Refer Slide Time: 32:08)



Temperature effect on reactivity

Temperature coefficient of reactivity $\alpha_T = \frac{d\rho}{dT} = \frac{1}{k^2}\frac{dk}{dT} \approx \frac{1}{k}\frac{dk}{dT}$

$\alpha_T > 0$ → inherently unstable reactor to changes in temperature

$\alpha_T < 0$ → inherently stable reactor to changes in temperature

$\frac{dk}{dT} < 0$

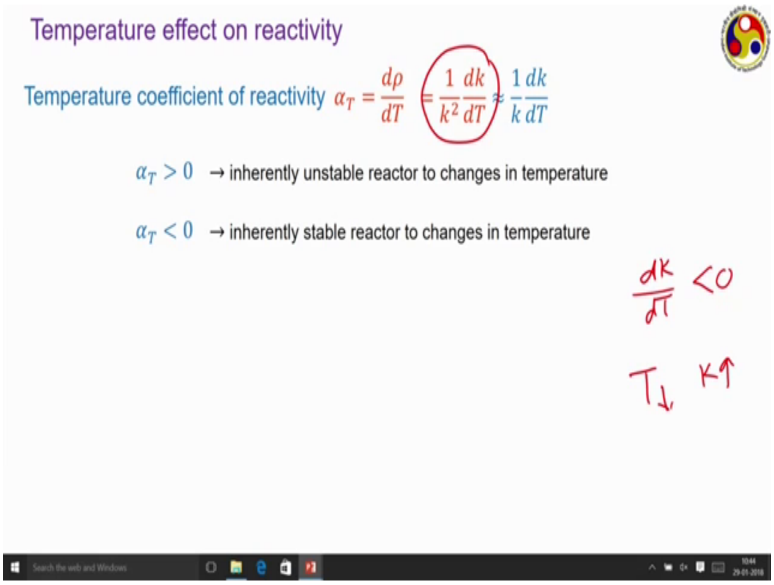
$T\uparrow \quad k\downarrow$

Which is a very much undesirable situation; so, the reactor is inherently unstable with changing temperature.

When alpha T is negative, then what may happen? In that case our situation is dk dT is negative. So, if the temperature increases, then k has to decrease. If reactivity decreases, then the power produced by the reactor that also decreases. And hence the temperature of the fuel pin the moderator that will come down, and once that starts to come down, that will affect the reactivity again.

Therefore, it is in a way is acting like feedback mechanism in and it is trying to get the temperature back to it is original thing original value. That is any change in temperature any rising temperature here will cause a reduction in reactivity which itself will try to reduce the temperature. And thereby it will lead to a very stable situation. And therefore, a reactor which is having a alpha T less than 0, or a negative value of this temperature coefficient of reactivity, it is a very much stable reactor.

(Refer Slide Time: 33:21)



Temperature effect on reactivity

Temperature coefficient of reactivity $\alpha_T = \frac{d\rho}{dT} = \boxed{\frac{1}{k^2}\frac{dk}{dT}} \approx \frac{1}{k}\frac{dk}{dT}$

$\alpha_T > 0$ → inherently unstable reactor to changes in temperature

$\alpha_T < 0$ → inherently stable reactor to changes in temperature

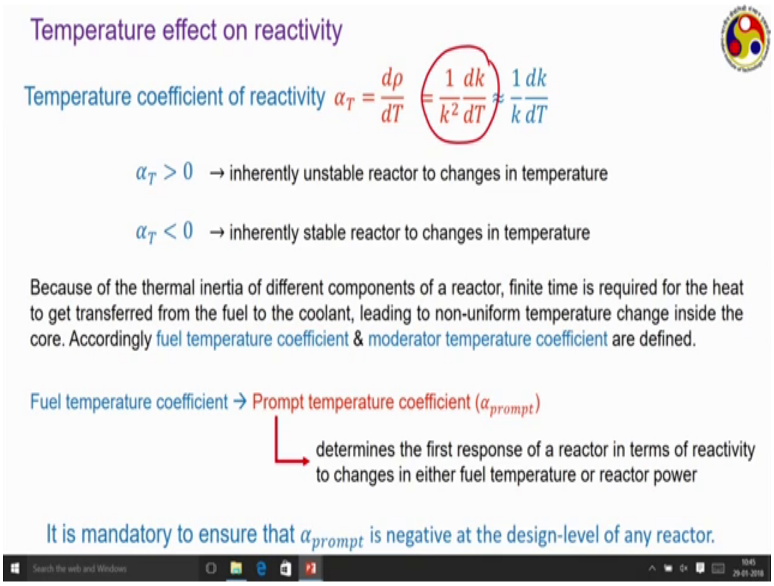
$\frac{dk}{dT} < 0$

$T\downarrow \quad K\uparrow$

We can we will study the other case also. Like, if there is a reduction in the temperature. Any temperature reduction will cause on increase in the reactivity, and once the reactivity increases. So, the power produced by the fuel pins that will increase. So, that will try to increase the temperature level of the reactor, and here the temperature will try to get back to it is original value thereby restoring the reactor back to it is initial position.

So, whenever this alpha T is negative, it will act like a proper feedback mechanism proper controlling mechanism to get the reactivity back to it is original value.

(Refer Slide Time: 33:54)



Temperature effect on reactivity

Temperature coefficient of reactivity $\alpha_T = \frac{d\rho}{dT} = \boxed{\frac{1}{k^2}\frac{dk}{dT}} \approx \frac{1}{k}\frac{dk}{dT}$

$\alpha_T > 0$ → inherently unstable reactor to changes in temperature

$\alpha_T < 0$ → inherently stable reactor to changes in temperature

Because of the thermal inertia of different components of a reactor, finite time is required for the heat to get transferred from the fuel to the coolant, leading to non-uniform temperature change inside the core. Accordingly fuel temperature coefficient & moderator temperature coefficient are defined.

Fuel temperature coefficient → Prompt temperature coefficient (α_{prompt})

→ determines the first response of a reactor in terms of reactivity to changes in either fuel temperature or reactor power

It is mandatory to ensure that α_{prompt} is negative at the design-level of any reactor.

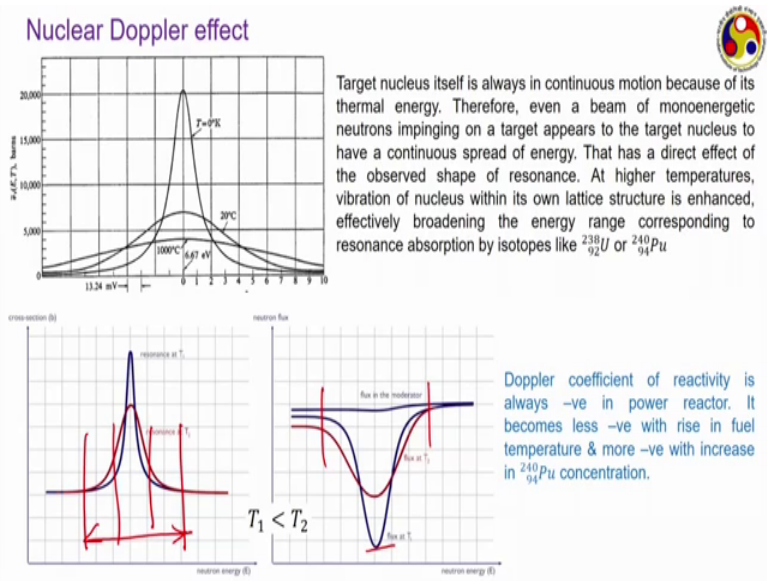
Generally, because of the thermal inertia of the different components of reactor the temperature inside is not uniform. Like, whenever there is a change in the reactivity is the fuel pin whose temperature increase first. Then that temperature will be transferred to the cladding that energy will get trans transfer to the cladding and from cladding to the coolant or to the moderator. And this entire energy transmission process it may require some finite amount of time.

And that is why the fuel rod and the coolant may not be at the same temperature. At the at a particular instant. Accordingly, we sometimes defined a fuel temperature coefficient and moderator temperature coefficients. Fuel temperature coefficient refers to the change in reactivity with unit change in the temperature of the fuel, similarly moderator temperature coefficient of reactivity defines, the change in reactivity with units change in the temperature of the moderator.

The fuel temperature coefficient is of practically importance that is also sometimes called the prompt temperature coefficient. Because it determines the first response of the reactor in terms of a reactivity change when there is a change. In either the fuel temperature or reactor temperature; this prompt temperature coefficient is probably the most important definition of temperature coefficient of reactivity is commonly used. And as per our discussion that is shortly here, we can simply say that it is mandatory that the prompt temperature coefficient has to be negative, or we have to ensure that is negative at the design level itself.

In fact, in several countries while licensing a nuclear reactor, it is one of the most important criteria like in United States no reactor is going to get the license; unless, it can show that the prompt temperature coefficient is a negative one.

(Refer Slide Time: 35:40)



Nuclear Doppler effect

Target nucleus itself is always in continuous motion because of its thermal energy. Therefore, even a beam of monoenergetic neutrons impinging on a target appears to the target nucleus to have a continuous spread of energy. That has a direct effect of the observed shape of resonance. At higher temperatures, vibration of nucleus within its own lattice structure is enhanced, effectively broadening the energy range corresponding to resonance absorption by isotopes like $^{238}_{92}U$ or $^{240}_{94}Pu$

Doppler coefficient of reactivity is always –ve in power reactor. It becomes less –ve with rise in fuel temperature & more –ve with increase in $^{240}_{94}Pu$ concentration.

An associated effect is the nuclear Doppler effect. It is actually an another effect of this temperature itself.

Now, in all the discussions related to the neutron nuclease interaction that we had so far. We have always considered the inherently assume that the neutron is moving and the nucleus is stationary. But practically speaking the nucleus itself is also not stationary. Because of it is own thermal energy level, evenly we are talking about this solid fuel the nucleus there is vibrating always. And therefore, even when their particular vibrating nucleus is subjected to a beam of strictly mono energetic neutron, it is interaction with different neutrons will be different, and accordingly this nucleus actually sees this mono energetic beam of neutron to have a distribution of energy. And this has a direct effect on the observed shape of resonance.

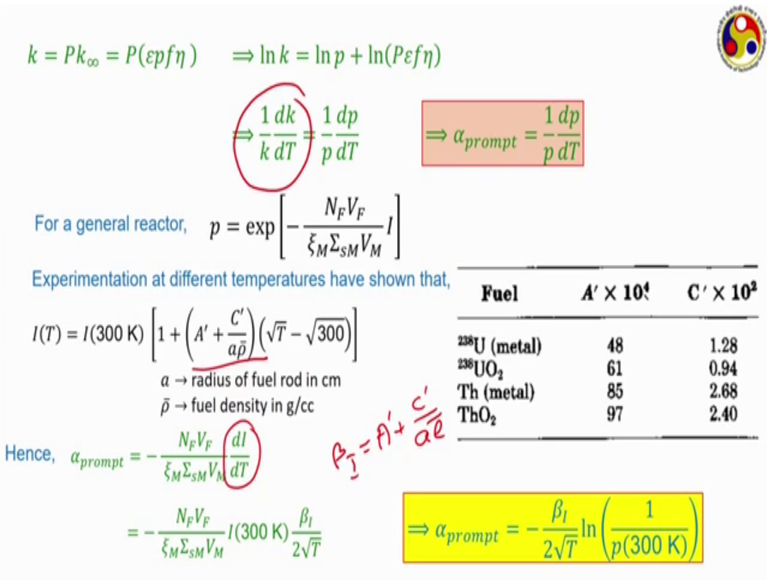
Like, look at this figure that is shown there. At when temperature is 0 Kelvin, then that is that is at the absolute 0 condition the fuel nucleus they are perfectly stationary. What are the temperature keeps on increasing? Like in this situation because of it is own motion it will find this neutron flux distribution to be much wider.

So, you can see the peak has come down to this, but the spread earlier when for 0 Kelvin it has restricted to this, when it has gone to 20-degree Celsius in the spread is this much. So, this is a much wider level of energy this is a much wider level of neutron energy for to for which this resonance absorption or resonance capture can take place. There thereby whenever and isotopes like uranium 238 or plutonium 240 are present inside the reactor which has very high resonance absorption cross section, this rising temperature can enhance the probability of resonance absorption.

This is the another picture of the same thing, like in the we are having 2 temperatures T 1 which is lower and the T 2 which is higher. Because of this Doppler effect, in this is for T 1, we can see the resonance absorption is restricted to this zone, but for T 2 because of this new Doppler effect it is a much wider zone, and but the peak is lower.

And correspondingly while the neutron flux has a very sharp deep here, close to the close to that resonance zone close to that particular energy level, here there deep is smaller, but this drop takes place over much wider energy spend. So, this Doppler coefficient of reactivity is also negative in power reactor, as a fuel temperature rises it becomes less negative, but it still always stays negative. And as the concentration of isotopes like plutonium 240 keeps on increasing, it becomes a bit more negative.

(Refer Slide Time: 38:42)



$$k = Pk_\infty = P(\varepsilon p f \eta) \quad \Rightarrow \ln k = \ln p + \ln(P\varepsilon f\eta)$$

$$\Rightarrow \frac{1}{k}\frac{dk}{dT} = \frac{1}{p}\frac{dp}{dT} \qquad \boxed{\Rightarrow \alpha_{prompt} = \frac{1}{p}\frac{dp}{dT}}$$

For a general reactor, $p = \exp\left[-\frac{N_F V_F}{\xi_M \Sigma_{sM} V_M} I\right]$

Experimentation at different temperatures have shown that,

$$I(T) = I(300\text{ K})\left[1 + \left(A' + \frac{C'}{a\bar{\rho}}\right)\left(\sqrt{T} - \sqrt{300}\right)\right]$$

$a \rightarrow$ radius of fuel rod in cm

$\bar{\rho} \rightarrow$ fuel density in g/cc

Fuel	$A' \times 10^4$	$C' \times 10^2$
^{238}U (metal)	48	1.28
^{238}UO$_2$	61	0.94
Th (metal)	85	2.68
ThO$_2$	97	2.40

$\beta_I = A' + \frac{C'}{a\rho}$

Hence, $\alpha_{prompt} = -\frac{N_F V_F}{\xi_M \Sigma_{sM} V_M}\frac{dI}{dT}$

$$= -\frac{N_F V_F}{\xi_M \Sigma_{sM} V_M} I(300\text{ K})\frac{\beta_I}{2\sqrt{T}} \qquad \boxed{\Rightarrow \alpha_{prompt} = -\frac{\beta_I}{2\sqrt{T}}\ln\left(\frac{1}{p(300\text{ K})}\right)}$$

Let us see mathematically the effect of this and Doppler reactivity, we know from the definition of multiplication factor it is the product of p into k infinity. The k infinity is the infinite multiplication factor and p is the total non-leakage probability, which combines both first and thermal non-neutral non-leakage probability normally.

So, k infinity can be represented by the 4-factor formula, or if we assume all other parameters apart from this resonance to be constant, we can write in a form like this and now differentiating both side with respect to the absolute temperature we get this. From your alpha prompt can we remember alpha prompt can roughly be equated to this quantity as a kd means close to one in reactors. So, it is equal to 1 upon p d p dT, but p is a resonance absorption or resonance escape probability it should same.

This was the expression for p that we have showed earlier. We have not derived it rather this was some kind of empirical relation corresponding to a reactor, which uses uranium as a fuel. We got a more specific form of that where I is the integral where the integral just concept integral is performed what is entire energy level of resonance that is from the first thermal neutron level.

This is a very general expressions actually. Earlier the expression that we have seen that was only for uranium 238 this is a very general expression where in the numerator we have the subscript f refers to fuel, here NF is the nuclear density of the fuel and V is the volume. In the denominator M refers to moderator. VM is the volume of the moderator sigma sM is the scattering cross section or average scattering cross section of the moderator. And zeta M is of course, the logarithmic energy decrement of the moderating isotope.

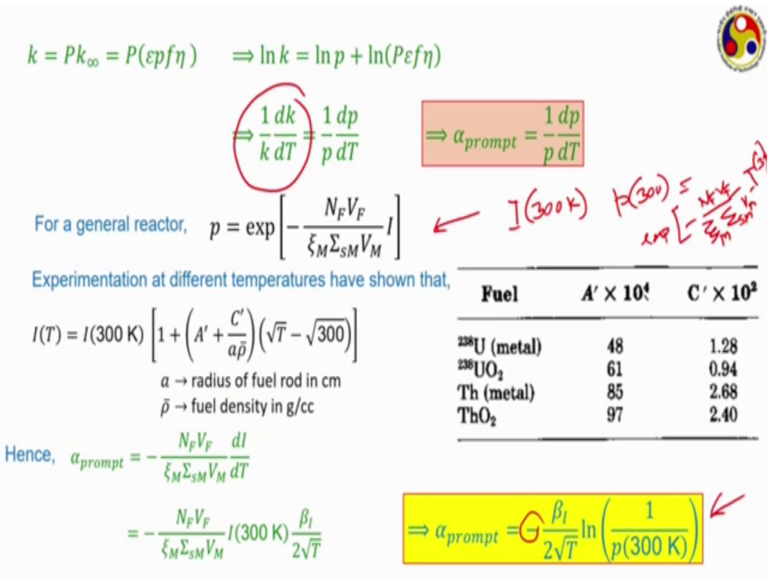
And during experiments it has been shown that this integral, can various temperature following a relation like this. Here T is the absolute temperature. 300 Kelvin has been taken as a reference temperature. This is a purely experimental correlation, which generally involves several kinds of isotopes, but primarily uranium 238, and also involves over a large range of temperature.

Here A prime and C prime are 2 coefficients. Small a, this one is actually the radius of the fuel rod, but it is given a centimeter simulated rho bar is the density of the fuel given in gram per centimeter cube. And A prime and C prime can vary with the fuel these are the values that are given like for uranium 238 in the form of uranium oxide the A prime values 61 in to 10 to the power minus 4.

You please take a note of this point here. This refers to actually the value of A prime for uranium 238 for this one is 61 into 10 to the power minus 4. And C prime is point 9 4 into 10 to the power minus 2, with these values we can put the expressions for this p into the expression for alpha prompt and perform the differentiation just with temperature to we so, we get this.

And now by integrating, sorry by differentiating I which has to the temperature, we get this here beta I is a nothing but this particular constant. That is here, this beta I is A prime plus C prime divided by a into rho bar. And then by rearranging the term, here another replacement that we can do; that is, if we put the expression; for I at 300 Kelvin into this expression, then p at 300 Kelvin will be equal to exponential of minus NF vf by zeta M capital sigma sM VM into I at 300.

(Refer Slide Time: 42:42)



$k = Pk_\infty = P(\varepsilon p f \eta) \quad \Rightarrow \ln k = \ln p + \ln(P\varepsilon f \eta)$

$$\Rightarrow \frac{1}{k}\frac{dk}{dT} = \frac{1}{p}\frac{dp}{dT} \qquad \boxed{\Rightarrow \alpha_{prompt} = \frac{1}{p}\frac{dp}{dT}}$$

For a general reactor, $p = \exp\left[-\frac{N_F V_F}{\xi_M \Sigma_{sM} V_M} I\right]$

$I(300\,K) \quad p(300) = \exp\left[-\frac{N_F V_F}{\xi_M \Sigma_{sM} V_M} I(3\right.$

Experimentation at different temperatures have shown that,

$$I(T) = I(300\text{ K})\left[1 + \left(A' + \frac{C'}{a\bar{\rho}}\right)\left(\sqrt{T} - \sqrt{300}\right)\right]$$

$a \rightarrow$ radius of fuel rod in cm

$\bar{\rho} \rightarrow$ fuel density in g/cc

Fuel	$A' \times 10^4$	$C' \times 10^2$
^{238}U (metal)	48	1.28
^{238}UO$_2$	61	0.94
Th (metal)	85	2.68
ThO$_2$	97	2.40

Hence, $\alpha_{prompt} = -\frac{N_F V_F}{\xi_M \Sigma_{sM} V_M}\frac{dI}{dT}$

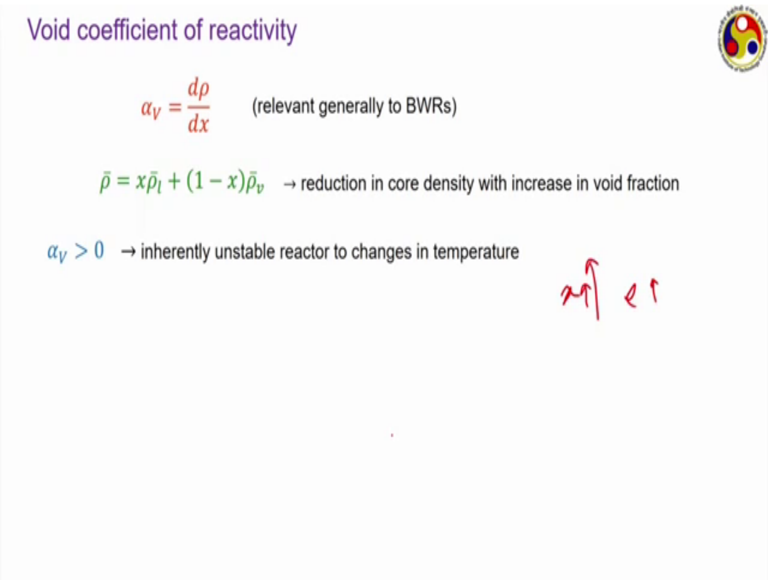
$$= -\frac{N_F V_F}{\xi_M \Sigma_{sM} V_M} I(300\text{ K})\frac{\beta_I}{2\sqrt{T}}$$

$$\boxed{\Rightarrow \alpha_{prompt} = -\frac{\beta_I}{2\sqrt{T}}\ln\left(\frac{1}{p(300\text{ K})}\right)}$$

And once we put it back here then we get this expression for this alpha prompt

So, this way by using whatever kind of fuel that we were using we can calculate the value of this prompt reactivity. Note there this log of 1 by p at 300 Kelvin, can be an can also be written as log of minus log of p at 300 Kelvin, but we have kept this minus sign outside because we always want this alpha prompt to be negative.

(Refer Slide Time: 43:50)



Void coefficient of reactivity

$\alpha_V = \frac{d\rho}{dx}$ (relevant generally to BWRs)

$\bar{\rho} = x\bar{\rho}_l + (1-x)\bar{\rho}_v$ → reduction in core density with increase in void fraction

$\alpha_V > 0$ → inherently unstable reactor to changes in temperature

$x\uparrow \uparrow \rho\uparrow$

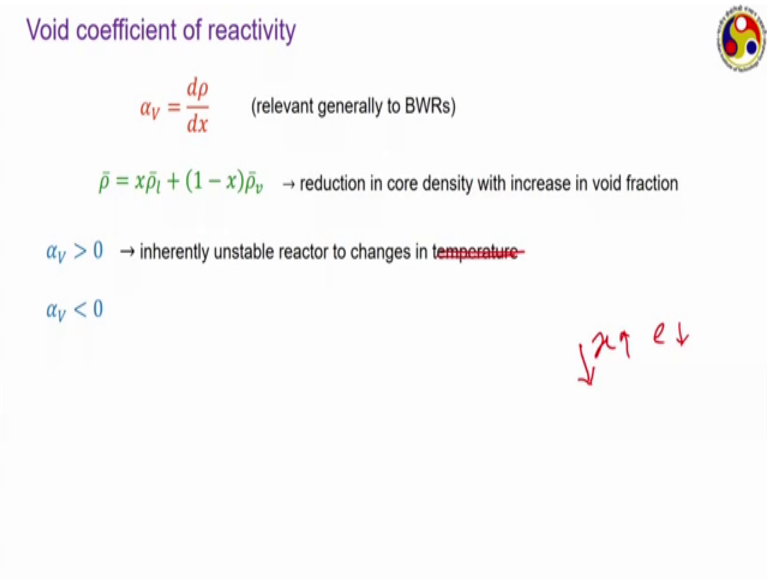
There is another effect the void coefficient of reactivity, which actually refers to the rate of change of reactivity with x, x is called void fraction, void fraction can be defined as the volume fraction of the vapor phase in a mixture of liquid and vapor.

Now, when you are dealing with a mixture of 2 phases a liquid and vapor phase, then the density of mixture can be given by this. Remember here x is the volume fraction or void fraction, but in order the mass fraction. This particular definition is particularly relevant to the boiling water reactors. In a boiling water reactor, water is used both as a moderator and coolant.

Now with energy supply this water can get converted to the wafer phase and once it become gets convert to the wafer phase they starts to leave towards the steam drum, thereby changing the total nuclear density of other moderator inside the reactor which can effect affect the scattering process, and accordingly it can affect the total number of thermal neutrons availability in fission; Thereby, reducing the reactivity of the reactor.

And this void coefficient of reactivity is a way of relating the reactivity, and also the void generation during this phase change process. And so, we can say when alpha v is greater than 0. Just similar to this temperature coefficient of reactivity, when alpha v is positive, then whenever there is a change in x, x increases rho also should increase. And as the reactivity increases it will lead to further power production or increasing the power production, as the power production increases that will transferred much larger amount of heat to the moderator, and hence there will be a further wafer generation. So, x will keep on increasing even further. And quite similar to the previous case, therefore, alpha the greater than 0, signifies an inherently unstable reactor, with I should not write with changes in temperature we changes in void fraction.

(Refer Slide Time: 46:03)



Void coefficient of reactivity

$\alpha_V = \frac{d\rho}{dx}$ (relevant generally to BWRs)

$\bar{\rho} = x\bar{\rho}_l + (1-x)\bar{\rho}_v$ → reduction in core density with increase in void fraction

$\alpha_V > 0$ → inherently unstable reactor to changes in t~~emperature~~

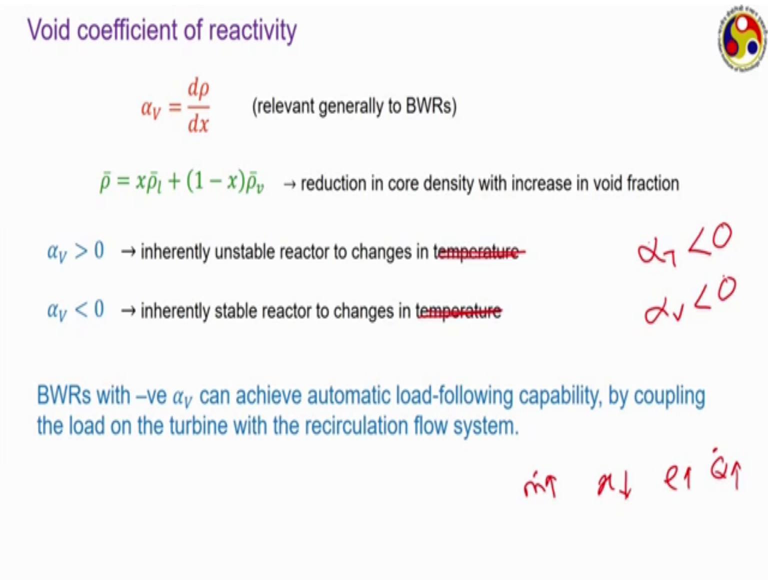
$\alpha_V < 0$

$\downarrow x\uparrow \rho\downarrow$

Similarly, when alpha v is less than 0, in that case, when x increases rho, reduces as the rho is reducing, then corresponding power production from the reactor also reduces. Hence lesser amount of energy is transferred to the moderator. They were reducing the total void generation, and that therefore, tries to reduce this x itself acting like a feedback mechanism.

So, alpha v less than 0 leads to an inherently stable reactor to changes in void fraction. It should not be temperature again changes in void fraction.

(Refer Slide Time: 46:33)



Void coefficient of reactivity

$\alpha_V = \frac{d\rho}{dx}$ (relevant generally to BWRs)

$\bar{\rho} = x\bar{\rho}_l + (1-x)\bar{\rho}_v$ → reduction in core density with increase in void fraction

$\alpha_V > 0$ → inherently unstable reactor to changes in ~~temperature~~ $\alpha_T < 0$

$\alpha_V < 0$ → inherently stable reactor to changes in ~~temperature~~ $\alpha_V < 0$

BWRs with –ve α_V can achieve automatic load-following capability, by coupling the load on the turbine with the recirculation flow system.

$\dot{m}\uparrow \quad x\downarrow \quad \rho\uparrow \quad \dot{Q}\uparrow$

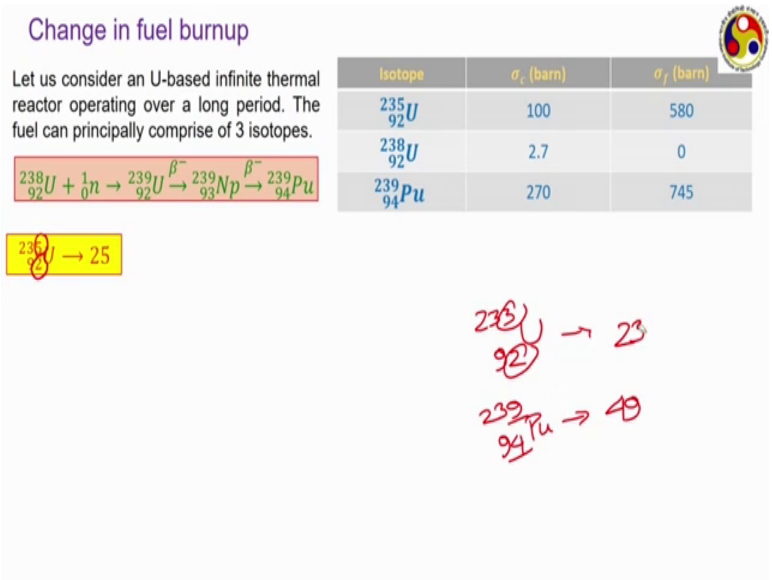
Therefore we have seen the temperature coefficient of reactivity should be negative for a stable operation. Similarities void coefficient of reactivity should also be negative. The boiling water reactor is negative alpha v we can achieve automatic load following incapable, because just the way I have mentioned. They couple the load on the turbine with the recirculation flow systems.

And so, whenever there is an increase in the load on the turbine, the flow through the reactor also increases, and as flow through the recirculation channel that also increases, and as the flow through the recirculation channel increases, the power produced or if the power production remains constant, then the void production inside the reactor that should reduce because total mass of coolant has increased, but power is the same.

And as the void fraction reduces, because of the negative value of alpha v, that will cause an increase in the value of reactivity; that means, what you are saying when the load increases, total mass flow rate of coolant that increases, and if power remains constant then increasing mass flow rate will reduce the x, and because of the negative value of alpha v that will cause a increasing the value of rho, and once rho increases total q dot increases, total power production that will also increase. Which of course, we will try to get the x back to it is original value. But that will achieve something called the automatic load following capability.

So, in a nutshell, we want both temperature coefficient of negativity temperature coefficient of reactivity to be negative, and void coefficient of reactivity also to be negative for stable operation of our reactor.

(Refer Slide Time: 48:25)



Change in fuel burnup

Let us consider an U-based infinite thermal reactor operating over a long period. The fuel can principally comprise of 3 isotopes.

$$^{238}_{92}U + ^{1}_{0}n \rightarrow ^{239}_{92}U \xrightarrow{\beta^-} ^{239}_{93}Np \xrightarrow{\beta^-} ^{239}_{94}Pu$$

Isotope	σ_c (barn)	σ_f (barn)
$^{235}_{92}U$	100	580
$^{238}_{92}U$	2.7	0
$^{239}_{94}Pu$	270	745

$^{235}_{92}U \rightarrow 25$

$^{238}_{92}U \rightarrow 23$

$^{239}_{94}Pu \rightarrow 49$

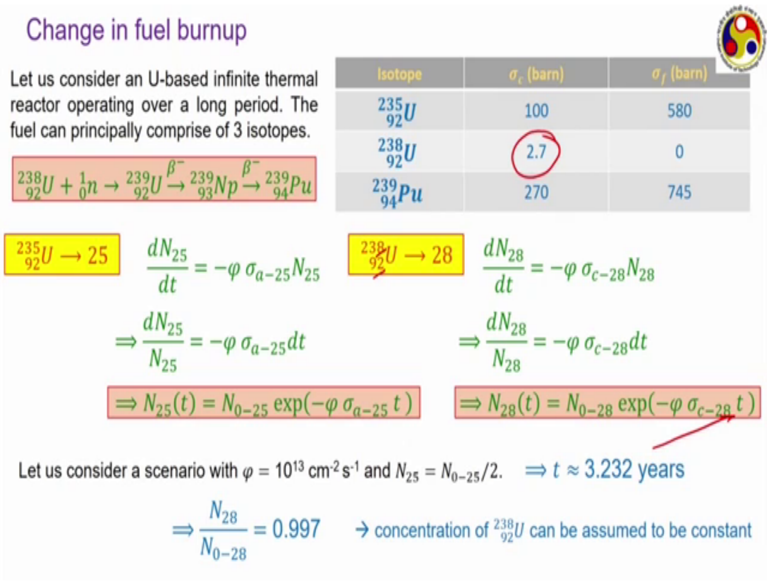
Next let us check the effect of fuel burn up, we have seen the effect of temperature now fuel burn up or the change in the fuel fraction inside the reactor. Let us consider an infinite thermal reactor which is operating based upon uranium. Now, we remember that you natural uranium generally comprises of three different isotopes; Uranium 235 about 0.7 percent, uranium 238 99.3 percent and very stress quantity of uranium 234.

Here we are neglecting uranium 234, because they are hardly has any kind of nuclear characteristics. Then there should be 2 different isotopes present inside the reactor, you 2 different fuel isotopes, I should say uranium 235 and 238. But here we are also mentioning plutonium 239. Now from where; the plutonium may come into picture? Because of this the non-fission capture reaction of uranium. Uranium 238 absorbs one neutron and to produce uranium 239 which goes through 2 different steps of beta dk to produce plutonium 239. This particular reaction is called breeder reaction, and this particular reaction is a basis of first beta reactors which is will be seeing studying in a later module.

So, let us check all these 3 isotopes of their constitution separately. Here we are using one shorthand notation. Uranium 235 is represented as 25. Actually, to denote this kind of notation, we will first use the last number of this, and the last number of this. Accordingly say uranium 233, that will take come taking this 2, and taking this 3 will be called 3.

Similarly, 94 Pu 239, this is a shorthand notation it should use this 4 and should this 9, 49. This way we sometimes use a shorthand notation to denote isotopes during such mathematical analysis.

(Refer Slide Time: 50:30)



Change in fuel burnup

Let us consider an U-based infinite thermal reactor operating over a long period. The fuel can principally comprise of 3 isotopes.

$^{238}_{92}U + ^{1}_{0}n \rightarrow ^{239}_{92}U \xrightarrow{\beta^-} ^{239}_{93}Np \xrightarrow{\beta^-} ^{239}_{94}Pu$

Isotope	σ_c (barn)	σ_f (barn)
$^{235}_{92}U$	100	580
$^{238}_{92}U$	2.7	0
$^{239}_{94}Pu$	270	745

$^{235}_{92}U \rightarrow 25$

$$\frac{dN_{25}}{dt} = -\varphi\,\sigma_{a-25}N_{25}$$

$$\Rightarrow \frac{dN_{25}}{N_{25}} = -\varphi\,\sigma_{a-25}dt$$

$$\Rightarrow N_{25}(t) = N_{0-25}\exp(-\varphi\,\sigma_{a-25}\,t)$$

$^{238}_{92}U \rightarrow 28$

$$\frac{dN_{28}}{dt} = -\varphi\,\sigma_{c-28}N_{28}$$

$$\Rightarrow \frac{dN_{28}}{N_{28}} = -\varphi\,\sigma_{c-28}dt$$

$$\Rightarrow N_{28}(t) = N_{0-28}\exp(-\varphi\,\sigma_{c-28}\,t)$$

Let us consider a scenario with $\varphi = 10^{13}$ cm^{-2} s^{-1} and $N_{25} = N_{0-25}/2$. $\Rightarrow t \approx 3.232$ years

$$\Rightarrow \frac{N_{28}}{N_{0-28}} = 0.997$$ → concentration of $^{238}_{92}U$ can be assumed to be constant

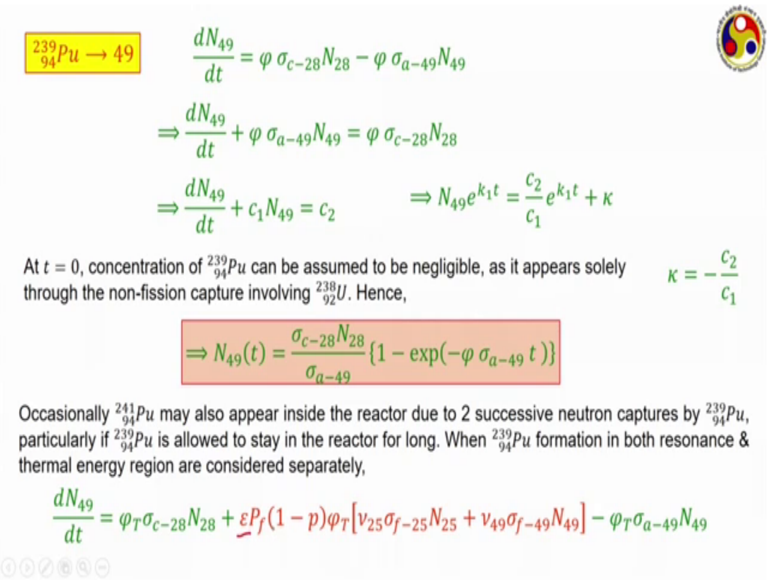
So, this is the corresponding equation uranium 235 there is no source of production, but it decays by absorption of neutron. And then we rearrange the equation, and we get this exponential solution.

Similarly, uranium 238 what should be this notation? We are taking this 2 and then we are taking this 8. So, it should be denoted as 28. So, it also goes through the decay process, there is no source of producing uranium 238 as well. So, we get is very similar exponential reaction. Here N naught 25 and N naught 28 represents their initial concentrations.

Now, let us considered scenario, where phi is 10 to the power 13 and in we want to calculate the time required to reduce the number of uranium 235 isotopes to half of the initial value. Then calculations means putting in this number here, and also this phi here, and also the using the value of sigma a for 25. Sigma a for 25 will be this 100 plus 580, that is 680, we get this time to be 3.23 years. That is to reduce the number of 230 you 235 isotopes to half of this original value, we need more than 3 years. And then how much will be the change in the concentration uranium 238 over that period?

Total absorption cross section for uranium 238 is extremely small, because this fission cross section is 0 and capture cross section is a 2.7, and now putting T equal to 3.232 years in this expression and also corresponding value of phi and sigma c, we get N 28 by N naught over that same period to be 0.997; that means, there is a hardly any change in the concentration of uranium 238, and hence uranium 238 concentration can be assumed to be constant.

(Refer Slide Time: 52:29)



$^{239}_{94}Pu \rightarrow 49$

$$\frac{dN_{49}}{dt} = \varphi\,\sigma_{c-28}N_{28} - \varphi\,\sigma_{a-49}N_{49}$$

$$\Rightarrow \frac{dN_{49}}{dt} + \varphi\,\sigma_{a-49}N_{49} = \varphi\,\sigma_{c-28}N_{28}$$

$$\Rightarrow \frac{dN_{49}}{dt} + c_1 N_{49} = c_2 \qquad \Rightarrow N_{49}e^{k_1 t} = \frac{c_2}{c_1}e^{k_1 t} + \kappa$$

At $t = 0$, concentration of $^{239}_{94}Pu$ can be assumed to be negligible, as it appears solely through the non-fission capture involving $^{238}_{92}U$. Hence,

$$\kappa = -\frac{c_2}{c_1}$$

$$\boxed{\Rightarrow N_{49}(t) = \frac{\sigma_{c-28}N_{28}}{\sigma_{a-49}}\{1 - \exp(-\varphi\,\sigma_{a-49}\,t)\}}$$

Occasionally $^{241}_{94}Pu$ may also appear inside the reactor due to 2 successive neutron captures by $^{239}_{94}Pu$, particularly if $^{239}_{94}Pu$ is allowed to stay in the reactor for long. When $^{239}_{94}Pu$ formation in both resonance & thermal energy region are considered separately,

$$\frac{dN_{49}}{dt} = \varphi_T\sigma_{c-28}N_{28} + \varepsilon P_f(1-p)\varphi_T\left[\nu_{25}\sigma_{f-25}N_{25} + \nu_{49}\sigma_{f-49}N_{49}\right] - \varphi_T\sigma_{a-49}N_{49}$$

Next, we move to plutonium, plutonium as a short for notation is 49, this is a corresponding conservation equation there are 2 terms. This is the decay rate of uranium 238, because of which is plutonium is getting formed, and this is the dk rate of plutonium itself.

So, we are in the terms, and as already mentioned this we can be assumed to be constant because the N 23 is a constant, and this also a constants we are having a form like this. So, we rearranging we can write like this. We are multiplying both equation to e to the power k 1 into T, sorry by performing the integration, we are getting a form like this because here it is your k 1 T is a integrating factor; and now at T equal to 0, concentration of plutonium can be neglected, because it will start forming in the reactor only because of the decay also uranium 238.

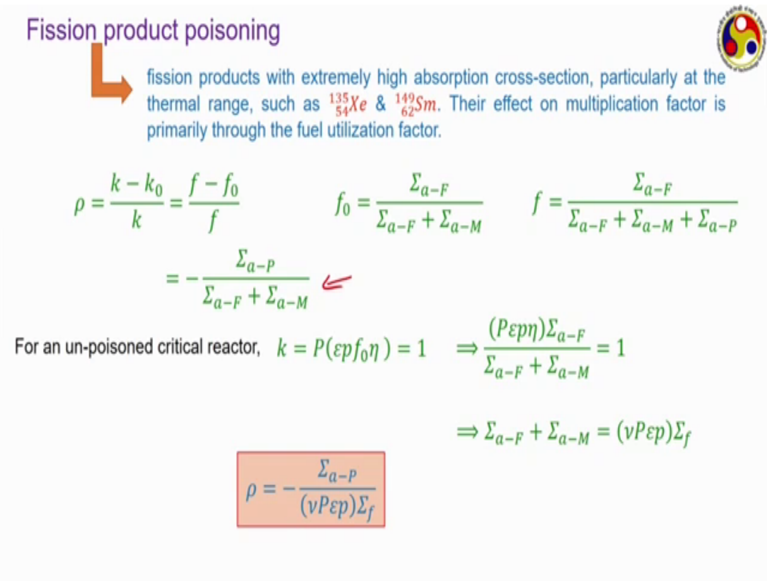
So, taking a concentration of the N 49 to be 0 at T equal to 0, we get this constant of integration this kappa to be equal to minus c 2 upon c 1 and 2 putting this is the final expressions of plutonium. So, when we are analyzing over a long period of time, such changes in the concentration of all these isotopes should also be need to be taken into consideration, because that can affect the reactivity as well.

In this particular analysis, we have consider only plutonium isotopes to be 239, but certain situation plutonium 239 may also go through to steps of neutron capture to form plutonium 241 or sometimes may plutonium 240 also, but that I have been neglected here. And also, plutonium 239 can get formed in both resonance capture and thermal energy region and when that is considered we get an expression like this.

Here if this is the first fission factor Pf is the first fission factor then Pf is the first fission non-leakage probability. And small p is the resonances (Refer Time: 54:34) probability. And this is the neutron production because of the decay of first neutron production, because of the decay of uranium 235. And this is the first neutron production because of the decay of plutonium 239.

Please go through these reactions properly and you will be able to understand.

(Refer Slide Time: 54:54)



Fission product poisoning

fission products with extremely high absorption cross-section, particularly at the thermal range, such as ${}^{135}_{54}Xe$ & ${}^{149}_{62}Sm$. Their effect on multiplication factor is primarily through the fuel utilization factor.

$$\rho = \frac{k-k_0}{k} = \frac{f-f_0}{f} \qquad f_0 = \frac{\Sigma_{a-F}}{\Sigma_{a-F}+\Sigma_{a-M}} \qquad f = \frac{\Sigma_{a-F}}{\Sigma_{a-F}+\Sigma_{a-M}+\Sigma_{a-P}}$$

$$= -\frac{\Sigma_{a-P}}{\Sigma_{a-F}+\Sigma_{a-M}} \leftarrow$$

For an un-poisoned critical reactor, $k = P(\varepsilon p f_0 \eta) = 1 \quad \Rightarrow \frac{(P\varepsilon p\eta)\Sigma_{a-F}}{\Sigma_{a-F}+\Sigma_{a-M}} = 1$

$$\Rightarrow \Sigma_{a-F}+\Sigma_{a-M} = (\nu P\varepsilon p)\Sigma_f$$

$$\boxed{\rho = -\frac{\Sigma_{a-P}}{(\nu P\varepsilon p)\Sigma_f}}$$

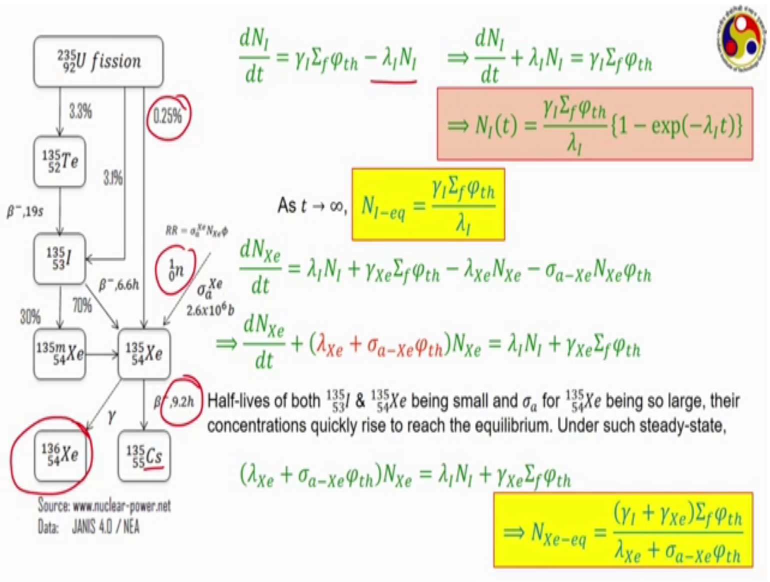
Let me go through quickly to the last part of this particular module, where we shall be discuss about fission product poisoning. Poisons refers to certain isotopes which may appear following radioactive decay of the fission products, which is extremely high absorption cross section. Like, xenon 135 and samarium 149.

They have extremely high neutron absorption cross section, and therefore, they can significantly affect the neutron flux distribution. Now their effect on the multiplication factor will come (Refer Time: 55:26) with the fuel utilization factor. And therefore, reactivity can also be in somewhat like this. Here f or k refers to the general situation this is and f naught and k naught refers to the situation without the presence of any poison.

Now, f naught is like this the fuel utilization factor without the presence of poisons will be the macroscopic absorption cross section of the fuel divided by the same for fuel and moderator together. Whereas, f will be where we also need to consider the effect of this poison and if we put the terms back into original expressions then rho comes to be something like this.

Now, for an un poisoned critical reactor, then we know k can be represented like this. For a critical reactor of course so, we are rearranging the terms, and then taking this sigma af plus sigma am into back into this particular equation we get such an expression for this reactivity in terms of the macroscopic absorption cross section of this poison.

(Refer Slide Time: 56:31)



Source: www.nuclear-power.net
Data: JANIS 4.0 / NEA

$$\frac{dN_I}{dt} = \gamma_I \Sigma_f \varphi_{th} - \underline{\lambda_I N_I} \quad \Rightarrow \frac{dN_I}{dt} + \lambda_I N_I = \gamma_I \Sigma_f \varphi_{th}$$

$$\boxed{\Rightarrow N_I(t) = \frac{\gamma_I \Sigma_f \varphi_{th}}{\lambda_I}\{1 - \exp(-\lambda_I t)\}}$$

As $t \to \infty$, $\boxed{N_{I-eq} = \frac{\gamma_I \Sigma_f \varphi_{th}}{\lambda_I}}$

$$\frac{dN_{Xe}}{dt} = \lambda_I N_I + \gamma_{Xe} \Sigma_f \varphi_{th} - \lambda_{Xe} N_{Xe} - \sigma_{a-Xe} N_{Xe} \varphi_{th}$$

$$\Rightarrow \frac{dN_{Xe}}{dt} + (\lambda_{Xe} + \sigma_{a-Xe}\varphi_{th}) N_{Xe} = \lambda_I N_I + \gamma_{Xe} \Sigma_f \varphi_{th}$$

Half-lives of both $^{135}_{53}I$ & $^{135}_{54}Xe$ being small and σ_a for $^{135}_{54}Xe$ being so large, their concentrations quickly rise to reach the equilibrium. Under such steady-state,

$$(\lambda_{Xe} + \sigma_{a-Xe}\varphi_{th}) N_{Xe} = \lambda_I N_I + \gamma_{Xe} \Sigma_f \varphi_{th}$$

$$\boxed{\Rightarrow N_{Xe-eq} = \frac{(\gamma_I + \gamma_{Xe}) \Sigma_f \varphi_{th}}{\lambda_{Xe} + \sigma_{a-Xe}\varphi_{th}}}$$

Now, this xenon 135, that can appear by 2 ways. One is there is 0.25 percent 0.25 percent probability of appearing directly from fission. Other is this T 135, and iodine 135 both can be produced in fission actually T 135 as an extremely small half-life only 19 seconds. So, it is presence can completely be neglected. And it can be assumed that iodine 135 has a fission yield of about 6.4 percent. And this iodine has a 70 percent probability of going through a beta decay to produce this xenon 135, but that is basically the primary reaction iodine 135 it is half life is also very small only 6.6 hours. So, it gets converted to this xenon 135.

So, if we write the expression for iodine, then rate of change of this iodine concentration has 2 parts, this is the production here gamma refers to it is fission yield, which is this 3.3 plus 3.1 percent, and macroscopic fission cross section multiplied by the thermal neutron flask, and then this is it is reduction that is it is decay rate into the number of neutrons, we rearrange the terms to get a distribution like iodine this way.

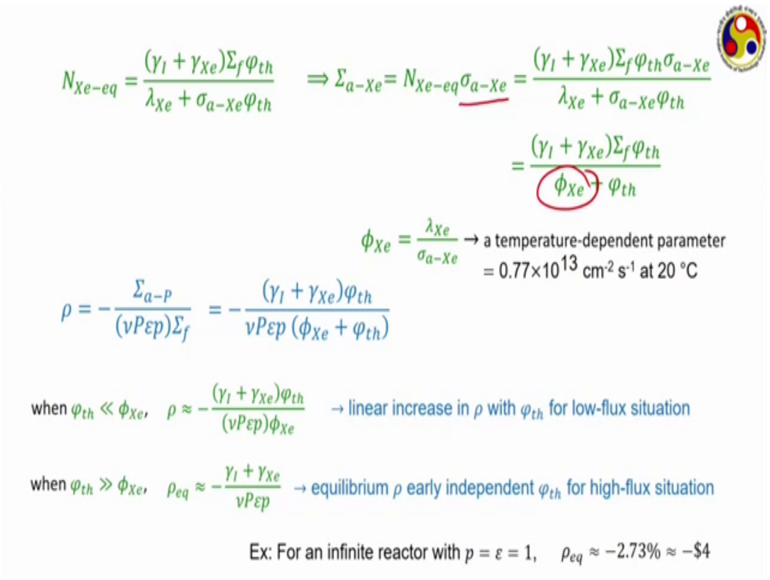
But xenon now there are two routes we have found, that can get produced decay from fission or because of the decay of iodine, and also there are two ways it may disappear. One is it can it is actually radioactive in nature. So, it can go through a beta decay to produce cesium. Other is it can absorbs a neutron to produce xenon 136, which is a ISO which is stable isotope, and it have a very small absorption cross section.

So, this is the expression for xenon. Before that putting in this previous expression at T tends to infinity we get the equilibrium concentration of iodine. But rate of change of xenon has this is it is production because of the decay of iodine, and this is production because directly from the fission, this is it is decay to through the beta decay procedure, and this is the neutron capture product.

So, we rearrange the term, and here this part written in red can be assumed to be an effective decay constant for this xenon 135, and half-lives for both iodine and xenon being quite small, xenon has a half-life it is shown here. This xenon has a half-life of just 9.2 hours.

So, both being quite small, and also the sigma a 4 xenon being such a large amount value, their concentrations quickly rise to is the equilibrium. We have already seen the equilibrium concentration of iodine, and putting these values here, we can get the equilibrium concentration of xenon to be like this.

(Refer Slide Time: 59:37)



$$N_{Xe-eq} = \frac{(\gamma_I + \gamma_{Xe})\Sigma_f \varphi_{th}}{\lambda_{Xe} + \sigma_{a-Xe}\varphi_{th}} \quad \Rightarrow \Sigma_{a-Xe} = N_{Xe-eq}\underline{\sigma_{a-Xe}} = \frac{(\gamma_I + \gamma_{Xe})\Sigma_f \varphi_{th}\sigma_{a-Xe}}{\lambda_{Xe} + \sigma_{a-Xe}\varphi_{th}}$$

$$= \frac{(\gamma_I + \gamma_{Xe})\Sigma_f \varphi_{th}}{\boxed{\phi_{Xe}} + \varphi_{th}}$$

$$\phi_{Xe} = \frac{\lambda_{Xe}}{\sigma_{a-Xe}}$$ → a temperature-dependent parameter = 0.77×10^{13} cm^{-2} s^{-1} at 20 °C

$$\rho = -\frac{\Sigma_{a-P}}{(\nu P\varepsilon p)\Sigma_f} = -\frac{(\gamma_I + \gamma_{Xe})\varphi_{th}}{\nu P\varepsilon p\,(\phi_{Xe} + \varphi_{th})}$$

when $\varphi_{th} \ll \phi_{Xe}$, $\rho \approx -\frac{(\gamma_I + \gamma_{Xe})\varphi_{th}}{(\nu P\varepsilon p)\phi_{Xe}}$ → linear increase in ρ with φ_{th} for low-flux situation

when $\varphi_{th} \gg \phi_{Xe}$, $\rho_{eq} \approx -\frac{\gamma_I + \gamma_{Xe}}{\nu P\varepsilon p}$ → equilibrium ρ early independent φ_{th} for high-flux situation

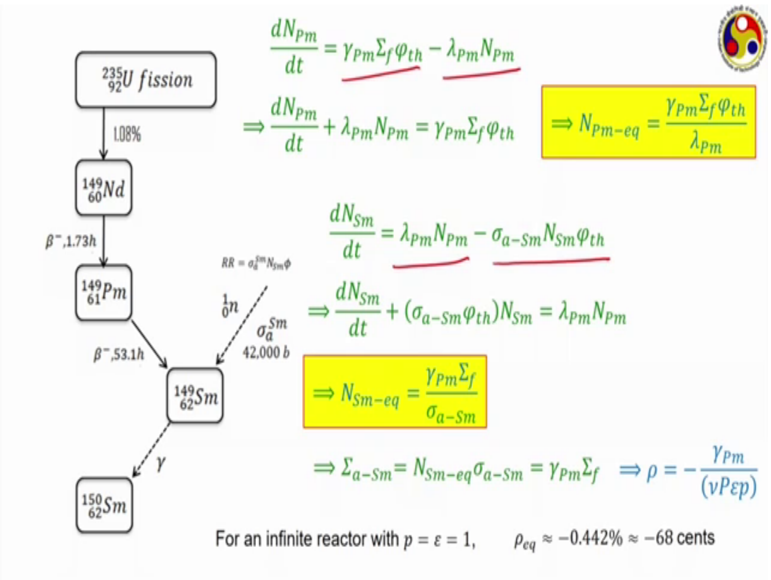
Ex: For an infinite reactor with $p = \varepsilon = 1$, $\rho_{eq} \approx -2.73\% \approx -\$4$

And proceeding further in this equilibrium concentration, we are using to calculate the macroscopic cross section means you consider multiply this with the, sorry, once we multiplying this equilibrium concentration to the corresponding absorption, microscopic absorption cross section we get the corresponding microscopic absorption cross section, this here this phi x c is actually the lambda by sigma a for xenon it is a temperature dependent parameters. It is value is this for 20-degree Celsius in changes quite rapidly temperature.

Now, this was the expression for rho that we have obtained earlier. So, once we put this expression for this sigma a xenon, which is a poison in this case we get an expression like this. Once this phi thermal is extremely small compared to the phi xenon, then rho is found to be a linear function of this phi thermal or low flux situations. On the contrary, when phi thermal itself is extremely high compared to this phi xenon, then rho equilibrium remains nearly a constant number; that is, it is independent of the phi thermal at high flux situations.

And if we assume, an infinite reactor with rho and epsilon to sorry epsilon and p to be equal to 1; this equilibrium value of rho comes to the minus 2.73 percent, which is roughly; minus 4-dollar taking the value of uranium 235.

(Refer Slide Time: 61:00)



$$\frac{dN_{Pm}}{dt} = \underline{\gamma_{Pm}\Sigma_f\varphi_{th}} - \underline{\lambda_{Pm}N_{Pm}}$$

$$\Rightarrow \frac{dN_{Pm}}{dt} + \lambda_{Pm}N_{Pm} = \gamma_{Pm}\Sigma_f\varphi_{th} \qquad \boxed{\Rightarrow N_{Pm-eq} = \frac{\gamma_{Pm}\Sigma_f\varphi_{th}}{\lambda_{Pm}}}$$

$$\frac{dN_{Sm}}{dt} = \underline{\lambda_{Pm}N_{Pm}} - \underline{\sigma_{a-Sm}N_{Sm}\varphi_{th}}$$

$$\Rightarrow \frac{dN_{Sm}}{dt} + (\sigma_{a-Sm}\varphi_{th})N_{Sm} = \lambda_{Pm}N_{Pm}$$

$$\boxed{\Rightarrow N_{Sm-eq} = \frac{\gamma_{Pm}\Sigma_f}{\sigma_{a-Sm}}}$$

$$\Rightarrow \Sigma_{a-Sm} = N_{Sm-eq}\sigma_{a-Sm} = \gamma_{Pm}\Sigma_f \quad \Rightarrow \rho = -\frac{\gamma_{Pm}}{(\nu P\varepsilon p)}$$

For an infinite reactor with $p = \varepsilon = 1$, $\rho_{eq} \approx -0.442\% \approx -68$ cents

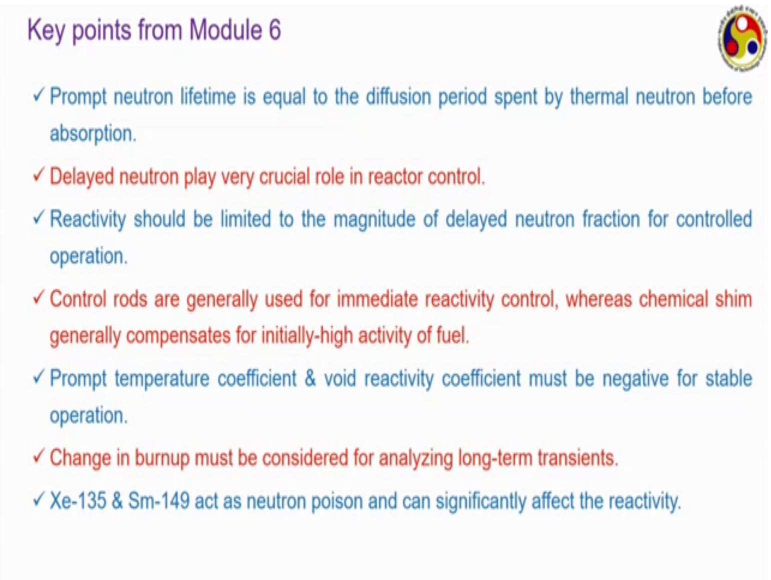
The other important poison is summary on 149, it actually can produce only because of the beta decay 2 successive between decay process Nd 149 is directly produced from fission, that goes to a beta decay to produce Pm 149, and then further beta decay produce sM 149. SM 149 use a stable isotopes there is no radioactive decay possible for this, but it is it has a extremely high absorption cross sections, 42000 bonds you probably have noticed the absorption cross section for xenon that was given earlier xenon 135 is an absorption cross section of the order of 10 to the power 6 bonds.

So, this is a 149-absorbing neutron becomes sM 150. This is the equation for VM in a number of concentration for Pm this is the fission yield. And this is the decay rate correspondingly we get it is an equilibrium concentration like this. And now we write the equation for sM. It again has just one way of producing that is by decay of Pm and one way of consuming because of the neutron capture. Accordingly, we gets this equilibrium concentration to be like this. And then we calculate corresponding microscopic cross section to get the value for an infinite reactor with p and epsilon equal to 1 to be just point minus 0.442 percent about minus 68 cents.

So, it is much smaller compared to the xenon, but still quite significant from reactivity control. You please go through these slides, this calculation deserve being just a repetition of earlier calculations I have gone through them quickly, but please try to do on (Refer Time: 62:36) So, these beings out towards the end of this particular module where you have discussed different effects of different parameters on reactivity.

And the keep on that we have learnt from module 6 is that the prompt neutron lifetime is virtually equal to the diffusion period by thermal neutron because scattering fluid is extremely small delay neutron play a very crucial role in reactor control. Particularly, the reactivity should be limited to the magnitude of delayed neutron fraction to have a control reaction.

(Refer Slide Time: 63:11)



Key points from Module 6

- ✓ Prompt neutron lifetime is equal to the diffusion period spent by thermal neutron before absorption.
- ✓ Delayed neutron play very crucial role in reactor control.
- ✓ Reactivity should be limited to the magnitude of delayed neutron fraction for controlled operation.
- ✓ Control rods are generally used for immediate reactivity control, whereas chemical shim generally compensates for initially-high activity of fuel.
- ✓ Prompt temperature coefficient & void reactivity coefficient must be negative for stable operation.
- ✓ Change in burnup must be considered for analyzing long-term transients.
- ✓ Xe-135 & Sm-149 act as neutron poison and can significantly affect the reactivity.

We have discussed about the role of control rods and chemical shim etcetera; control rods are generally used for immediate reactivity control. Whereas, chemical shims generally compensate for initially high activity of fuel, or for slow control chemical shim can be used. Then we have discussed about the effect of fuel burn up, the effect of temperature we have discussion the prompt new temperature coefficient and void reactivity coefficient both should be negative for stable operation.

The burn up also must be considered for analyzing long term transient particularly the appearance of different isotopes of plutonium is very important, even in a uranium rich reactor. And finally, we have discussed about the role of 2 neutron poisons xenon 135 even sM 149. Both of them has been found to effect the reactivity quite significantly.

So, that is it for the 6th module, when I have discussed about the topic of reactivity control. And in this particular lecture we have discussed about the effects of different parameters like fuel burn up temperature and neutron poisons on reactivity. In a practical reactor all these factors need to be taken into consideration, while deciding the fine the role of the controlling mechanisms or the activity of the controlling mechanisms.

So, thanks for your attention, in the next week we shall be moving to the 7th module we shall be talking about the thermal reactors; so, bye for now.