Fundamentals of Nuclear Power Generation
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Module - 06
Lecture – 02
Reactor Control

Hello, friends. We are back with the second lecture, where we are discussing about the reactor control or basically the way of controlling a nuclear reactor where we have started with discussing the role of neutrons and particularly the role of prompt and delayed neutrons separately and also the reactor kinetics. Now, anything related to the nuclear reactor always counts down to how the neutron behave inside the reactor or how can we control the behavior of the neutron inside the reactor.

And, accordingly the reactor topic of reactor control is also essentially related to controlling the neutron profile or neutron flux distribution inside the reactor, both these variations respect to pace and also with respect to time.

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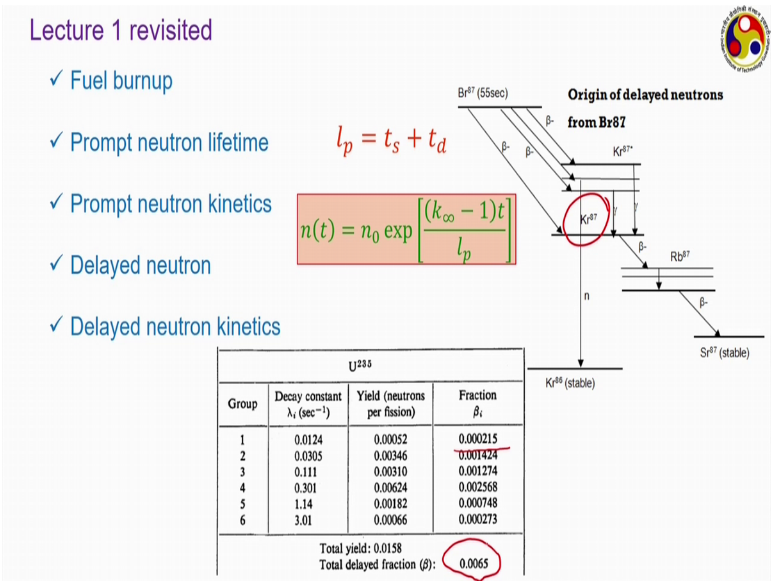

U^{235}			
Group	Decay constant λ_i (sec^{-1})	Yield (neutrons per fission)	Fraction β_i
1	0.0124	0.00052	0.000215
2	0.0305	0.00346	0.001424
3	0.111	0.00310	0.001274
4	0.301	0.00624	0.002568
5	1.14	0.00182	0.000748
6	3.01	0.00066	0.000273
Total yield: 0.0158			
Total delayed fraction (β): 0.0065			

Just a quick recap of the first lecture their initial introduced the concept of fuel burnup where which refers to the amount of fuel consumed by inside the reactor over a period of one day and we have seen that in the fuel burnup for nuclear reactor is significantly smaller compared to thermal reactor and that is why once we have loaded a reactor with a certain quantity of fuel we can keep on running for a very long period which can range from several months to more than a year.

Then, we discussed about prompt and delayed neutrons. It was already introduced in earlier modules that prompt neutron is the one which is the immediate product of efficient reaction and therefore, it appears within 10 to the power of minus 14 seconds of efficient occurrence. However, delayed neutron is a product of the radioactive decay of certain neutron rich fission fragments; like we have seen that while beta decay is the most common type of radioactive decay that fission fragments undergo there are certain situations where some certain neutron rich isotopes appear and they also need to have a certain amount of energy that is the energy level of the concern neutrons need concern nucleus rather should be higher than the binding energy of a single neutron then it can also undergo a neutron decay and thereby contributing that delayed neutron.

Delay neutron can appear significantly after the significant time after the appearance of the actual fission. The time may vary from a few millions to more than a seconds like probably the longest living delayed neutron precursor has a half life of around 55 seconds and that is why there can be wide variety of delay neutron precursors that may be present inside the reactor and accordingly delayed neutrons can become available and different period of time.

But, first we discussed about the prompt neutron lifetime and there we have seen that it is an earlier summation of two components; the scattering period and also the diffusion time. Generally, a scattering time is significantly smaller compared to the diffusion time and hence for particularly for thermal neutrons we can blindly take this l p that is the from neutron lifetime to be similar to the diffusion time and it is generally of the order of 10 to the minus 3. For fast neutrons there is no scattering component. So, it is only the diffusion time and that is the diffusion the first neutron and that is generally of the (Refer Time: 03:43) 10 to the power minus 7.

Then, we discussed about the prompt neutron kinetics considering a single group neutron model, we have developed this particular equation which shows, if n naught is the initial number of neutrons present inside the reactor that is at time equal to t equal to 0, this will be the transient profile of this nuclear reactor or transient profile of neutron inside the reactor where l p is the prompt neutron lifetime and k infinity is the multiplication factor or it can directly related to this reactivity as well.

And, as l p is the order of 10 to the minus 3 commonly, the reactivity when that is of the order of 10 to the minus 3 we have seen that just within a period of a 10 seconds this n number of neutrons can become like when I am saying reactivity is of the order 10 to the minus 3, we can see that this reactor time that is l p divided by rho is of the order of 1, and in that case at just at time t equal to 10 seconds, total number of neutrons can be e to the power 10, that is, it can increase extremely sharply and the rate actually is too high to get controlled by any kind of practically available machineries.

Therefore, if there are the entire kinetics of a nuclear reactor is depending solely upon the prompt neutron then the controller of the reactor becomes extremely critical. Just with a the reactivity of the order 10 to the minus 3 we are seeing such an exponential rate of growth, just think about what may happen in a fast reactor. In case of a fast reactor l p is of the order 10 to the minus 7 and therefore, this even when the reactivity is 10 to the power minus 3 is n t will be n naught into exponential of 10 to the power 4, for just a single second and that is impossible to control.

Therefore, it is a we can conclude that the reactor which is depending solely upon the prompt neutrons the control of that is extremely critical and in most of the cases practically impossible, but thanks to the role played by the delayed neutrons we often find a way actually much easier way of controlling reactors; easier way means basically we have always have much more time to do any kind of operation, to operate our control equipments, because that is because of the role of the delayed neutrons.

And, that role we have roughly discussed and we have seen this is the way the delayed neutron that is produced with a examples of some delayed neutron precursor we have seen. Like it is for barium 87 is given which is the immediate product is krypton 87 and while most of the cases krypton 87 undergoes a bitter decay in certain situation it can also undergo a neutron decay, but only if when this particular isotope has significant amount of energy; energy which is higher than the binding energy of a single neutron.

And, the then we went to the delayed neutron kinetics, which we only we need to started in the previous lecture. As a part of the delayed neutron kinetics we have adopted a 6 group model. Basically, there can be such so many numbers of delayed neutrons that can be present in a single fission reaction or can appear during following the chain of a single fission reaction. We often combine these neutrons into a few groups or we club them into a say few groups and the most preferred approach is to follow 6 group kind of thing. Like, these are the groups for uranium 235 for every group the corresponding parameters like the decay constant the fractional yield for this these are all averaged over this and hence instead of considering the constituents for each group we can just take all of them together.

Like for uranium 235, the first group can have a value of beta which is a fraction corresponding to this group divided by the total number of neutrons prompt plus delayed that may appear following a single fission this extremely small you can see for the first one it is at this 0.000125, that is just 0.0215 and the total delay neutron fraction that we can find is point 0.0065 for uranium 235. It is different for different other fissile isotopes, but, our discussion we are mostly keeping restricted to uranium 235 and so, this combined value of beta is important to remember which is 0.0065. Now, we shall be taking the delayed neutron kinetics forward.

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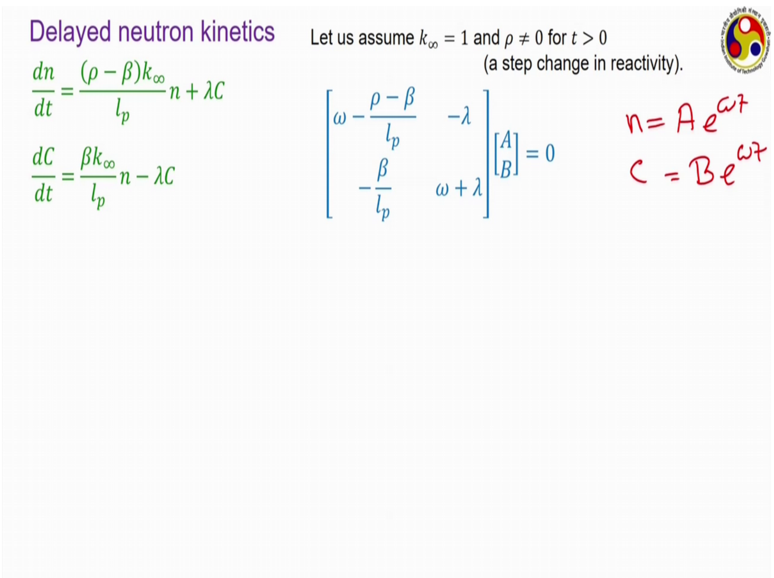

These are the equation just remember these are the equations that we developed during the previous lecture. Here, n refers to the neutron density and the rate of change of the neutron density can have different components such as the neutron produced following the fission that is a prompt neutron which may appear because of the fission reaction.

The delayed neutrons which may appear because of the radioactive decay of the delay into the precursors and also there should be neutron absorption. Combining all, we got this particular equation here instead of considering the 6 or 8 different groups of delayed neutrons we are just considering all of them to be clubbed into a single group and the concentration of that group is given by C and the coefficient, sorry, corresponding parameter holes for the group are taken as single values like lambda is the decay constant, which is representative of the entire group and the same is applicable for beta and k infinity and others.

The second equation dC dt represents the rate at which the concentration of the delayed neutron that changes. At the first term on the right hand side represents its rate of production and second one depend the rate of decay the same. Now, let us assume k infinity to be equal to 1 and rho to be not equal to 0, for t greater than 0, that is, we can consider at the beginning that is when you are starting our analysis the system is working under the steady state and it is critical. So, it is k infinity is equal to 1 and rho is equal to 0.

Now, there is a step change in reactivity that is the reactivity change is very small amount and the change in k infinity is so small that we can neglect it for most of the calculation purpose, but rho is a very small positive quantity for which are going to do this analysis. So, putting these expressions and rearranging the terms we are here where just remember we assumed a solution of this form n equal to A e to the power omega t and C is equal to B to the power omega t, here A and B are 2 coefficients and omega is frequency.

So, this is a single matrix equation that we have got and we have to find a solution for this one in thus particular module.

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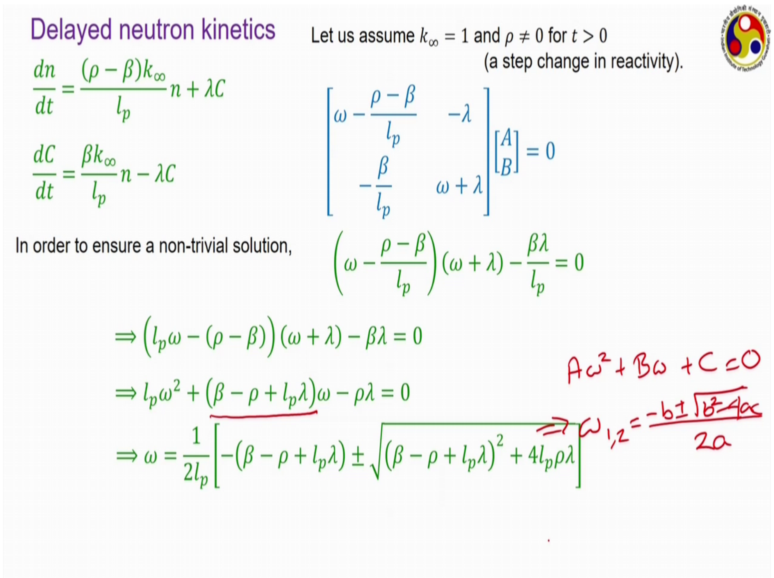

So, this particular set of equations these two equations can have a solution or a non trivial solution only if, the determinant of this one is equal to 0 and from there we are getting this the determinant will be the on this particular component multiplied by this minus this particular one multiplied by this one. So, this has to be equal to 0, in order to have a non trivial solution for this matrix system.

And, then we are simplifying the terms that is we are multiplying the first one first with omega and then with lambda and then combining the terms, this and whatever we are getting now, we are rearranging into a form of a quadratic equation, where omega is the variable and we are having l p into omega square plus this quantity in the bracket into omega minus rho lambda is the third term. This is like any coordinating equation can always be represented as A omega square plus B omega plus C equal to 0.

In that case the solution of this equation, there will be two solutions, should be equal to minus b plus minus root over b square minus 4 a c divided by 2a. So, the same thing is done here if we follow the equation capital A refers to l p.

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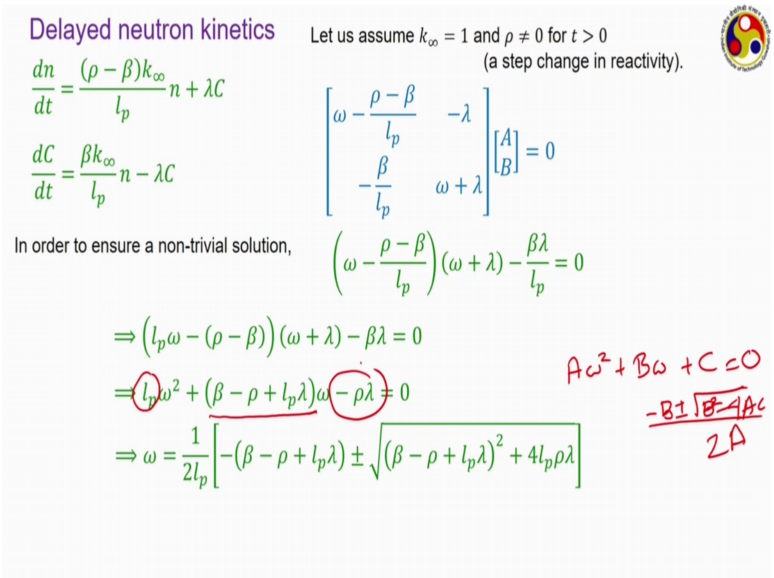

Actually, I have used I should have used capital B here this capital B this is capital A and C and this also is true of capital A. So, in this equation capital A refers to l p, B refers to this underlined quantity and C refers to minus of rho lambda. So, we put them together and then we got the solution for this omega here of course, the plus minus sign appears which represents two different solutions or two different values of omega, one correspond with a positive one and other corresponding to the negative one.

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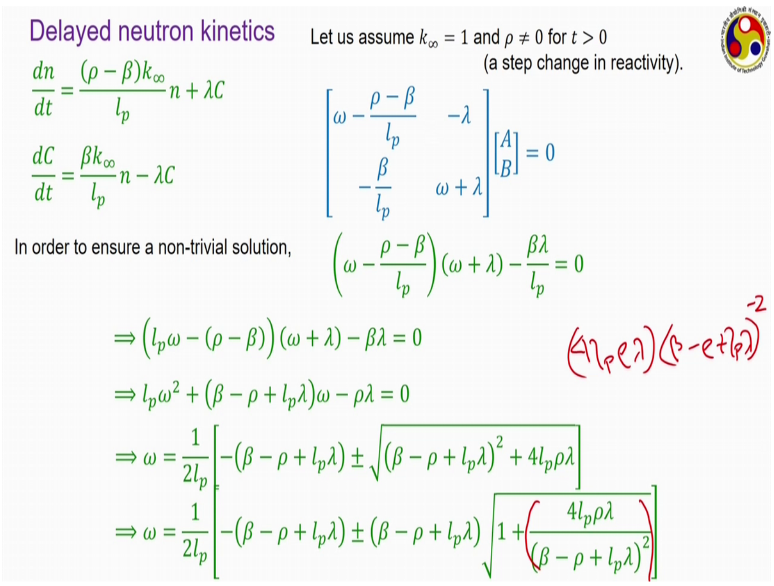

And, a bit of further rearrangement where this squared quantity is coming out of the root and we are having an expression like this for omega.

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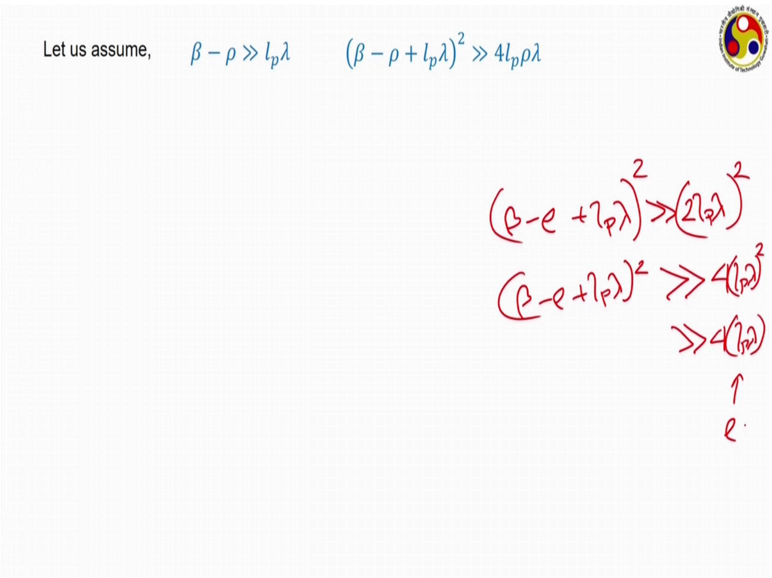

So, now we are taking couple of assumptions. First assumption is beta minus rho is significantly larger compared to this product of l p and lambda and the second one is beta minus rho plus there will be whole square of that is significantly larger than 4 l p into rho into lambda. Actually, the second one can be obtained from the first one, like if you think beta minus rho is greater than l p lambda. So, if we add l p lambda on both side then becomes twice of this.

And, now if I take a whole square of this should also get a whole square of this quantity now it should be greater than this or you know we can say getting a whole square of the first one you should rather than whole square of this or beta minus rho plus l p lambda whole square should be greater than or significantly greater than 4 l p lambda whole square. Now, we can just say it has to be greater than 4 into l p lambda and rho is a quantity which is generally very small positive number at least as per our consideration. So, multiplying this particular term with rho will make it even smaller.

But. of course, the first approximation that is beta minus rho is significantly greater than l p into lambda is something that we shall be proving later on using the magnitudes corresponding to (Refer Time: 15:15) 35. For the moment let us a proceed forward with these two assumptions.

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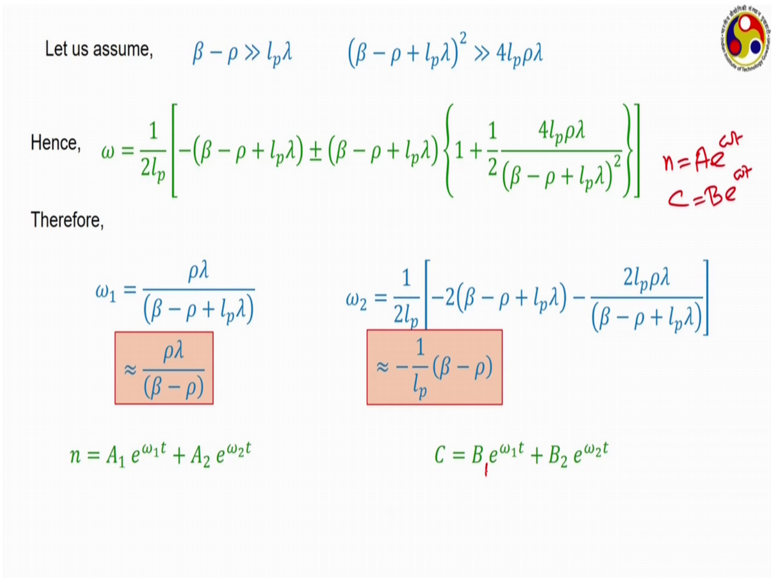

This is as if I go back to the previous slide quickly here, now a this numerator of this quantity is significantly smaller compared to the denominator. So, we can write this one as 4 l p rho into lambda into beta minus rho plus l p lambda whole to the power minus 2 and then if we expand this one following whatever we could like to do as l p lambda is significantly smaller compared to the first term then compare to the l p lambda then we are having an expansion of somewhat this particular form and then we can have two roots from this; one corresponding to the positive sign and one corresponding negative sign. If we take the positive sign then a beta minus rho plus l p lambda this and this one cancels out and what remains that if we simply is of a form like this and to proceed further beta minus rho is significantly greater than l p lambda as per our assumption.

So, now if we put that the first root for omega is coming to be rho lambda divide by beta minus rho. Similarly, if we take the negative sign here then the expression for omega 2 is somewhat like this and there we can use again the first assumption that we had beta minus rho has to significantly larger than l p lambda and if we simply omega 2 is having a solution like this minus 1 upon l p into beta minus rho.

So, we are getting two solutions for omega 1 and omega 2 because of the coordinating nature of the original equation. Now, remember earlier solution that we have assumed is n is equal to e into exponential of omega t and similarly C is equal to B into e to the power omega t, but here we can see that omega can have two values omega 1 and omega

2 and hence the solution of n and C has to be such an additive form it is A 1 into the e power omega 1 t plus A 2 e to the power omega 2 t for n and this should be B 1 e to the power omega 1 t plus B 2 e 2 the power omega 2 t for corresponding to C.

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Let us assume, $\beta - \rho \gg l_p\lambda \qquad (\beta - \rho + l_p\lambda)^2 \gg 4 l_p \rho\lambda$

Hence, $$\omega = \frac{1}{2l_p}\left[-(\beta-\rho+l_p\lambda) \pm (\beta-\rho+l_p\lambda)\left\{1+\frac{1}{2}\frac{4l_p\rho\lambda}{(\beta-\rho+l_p\lambda)^2}\right\}\right]$$

Therefore,

$$\omega_1 = \frac{\rho\lambda}{(\beta-\rho+l_p\lambda)} \qquad \omega_2 = \frac{1}{2l_p}\left[-2(\beta-\rho+l_p\lambda) - \frac{2l_p\rho\lambda}{(\beta-\rho+l_p\lambda)}\right]$$

$$\boxed{\approx \frac{\rho\lambda}{(\beta-\rho)}} \qquad \boxed{\approx -\frac{1}{l_p}(\beta-\rho)}$$

$$n = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} \qquad C = B_1 e^{\omega_1 t} + B_2 e^{\omega_2 t}$$

$$= A_1 \exp\left({}^{t}/_{T_1}\right) + A_2 \exp\left({}^{t}/_{T_2}\right) \qquad = B_1 \exp\left({}^{t}/_{T_1}\right) + B_2 \exp\left({}^{t}/_{T_2}\right)$$

And, we can also write this in this particular form where instead of omega we are writing it 1 by capital T, where capital T is the corresponding time period. Omega is what about the frequency or the angular frequency and capital T is the corresponding time period for this.

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Now, $\dfrac{\omega_1}{\omega_2} = \dfrac{T_2}{T_1}$ $\qquad \omega_1 \approx \dfrac{\rho\lambda}{(\beta-\rho)} \qquad \omega_2 \approx -\dfrac{1}{l_p}(\beta-\rho)$

$$\Rightarrow \left|\frac{T_2}{T_1}\right| = \frac{l_p\rho\lambda}{(\beta-\rho)^2}$$

Assumption: $\beta - \rho \gg l_p\lambda$

$\beta - \rho \gg l_p\lambda\rho$

$$\Rightarrow |T_2| \ll |T_1|$$

Now, omega 1, omega 2 then has to be equal to T 2 upon T 1. These are the expressions for omega 1 and omega 2 that we have derived shortly before and if we put this expressions then T 2 upon T 1, the magnitude of that comes to be l p rho lambda by beta minus rho whole square.

But, in the previous slide only we have considered beta minus rho to be significantly larger than l p into lambda and once you multiply the 2 is the rho being a small number this has to be significantly greater than l p lambda into rho and hence we have we can safely say that this T 2 has to be significantly smaller compared to T 1.

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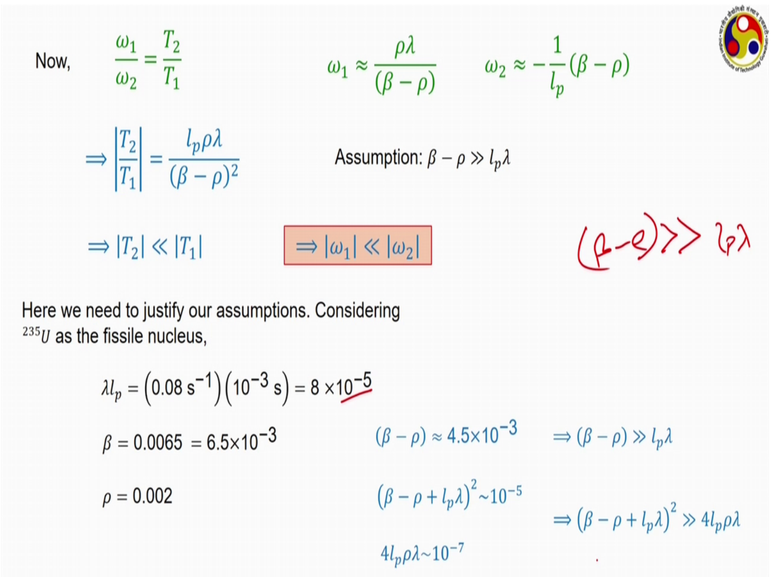

Or omega 1 is significantly smaller than omega 2. Therefore, though we are getting two frequencies and two different characteristic time constants, but one of them is dominant over the other one or it I should say significantly dominate over the other one.

Now, we need to justify those two assumptions that we have considered or particular the first assumption that is beta minus rho is significantly greater than l p into lambda. If we consider uranium 235 has the nucleus then we can say that l p lambda product l p is of the order of 10 to the minus 3 seconds and lambda corresponding into 235 that is the average value of decay constant for all possible delay neutron precursors that may appear with uranium 235 that is just 0.08 second inverse. So, the product of lambda and l p is having a dimension of the order 10 to the minus 5.

Beta now, for uranium 235 beta is 0.065 that was shown earlier also and let us say the rho is 0.002, we are going to consider here. Then, beta minus rho is something like 4.5 into 1o to the power minus 3, that is, beta minus rho is of the order of 10 to the power minus 3. On the contrary, l p lambda is of the order of 10 to power minus 5 and therefore, we can clearly say that beta minus rho is definitely much larger than l p into lambda.

And, going to the second assumption now, beta minus rho plus l p lambda whole square if we a put the magnitudes of beta rho and l p lambda into this then this comes out to be of the order of 10 to the minus 5, but once we multiply this 4 p lambda, sorry, lambda p into lambda into l p with 4 into rho that comes out to of the order of 10 to the power minus 7. So, that shows that the second assumption there is beta minus rho plus l p lambda is much larger than 4 l p rho lambda that can be justified from here.

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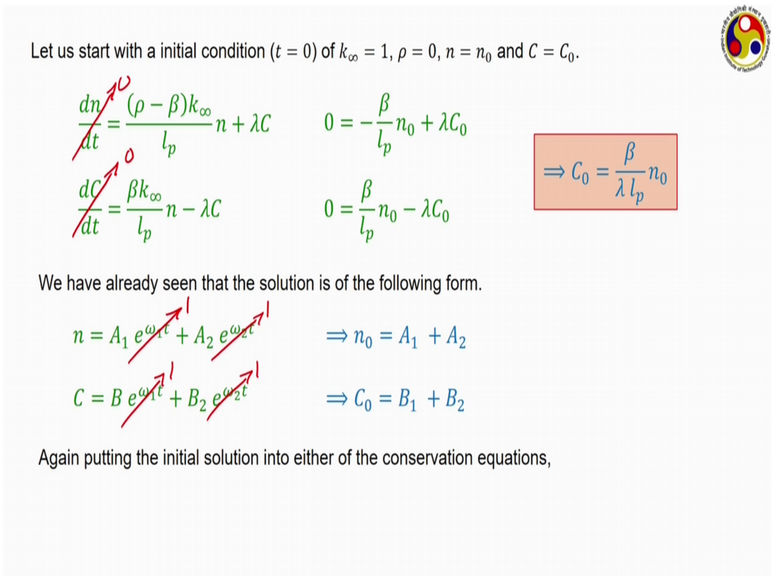

Now, we start with an initial condition and that where at t equal to 0, k infinity is equal to 1, that is, the system was running in the critical condition and it is reactivity there has to equal to 0. Now, because of has to be equal to 0 and the initial concentration of the neutron is given as n equal to n naught and corresponding concentration of the delta to the precursors can be C equal to C naught. So, with these initial conditions let us try to find a solution.

These are the original equations that we already had and once we put k infinity equal to 1 and also all the other steady state values to this then under steady state dn dt or any time variable has to go to 0, these two goes to 0. This is the form that we get from the first equation and this is the one that we are getting this from the second equation here we have of course, put n equal to n naught and C equal to C naught and rho equal to 0. See, both the equations are very much identical and we could use any one of them to get relation between C naught and n naught of this form.

So, editing C naught equal to beta upon lambda l p into n naught there is also you know the value of these three parameters beta, lambda in l p which all of them generally are quite standard; beta equal to 0.0065 for uranium 235, lambda value is also known which the average value for of decay constant for combining all possible decay neutron precursors and l p is generally of the rho into 10 to the power the minus 3. So, we have a relation between C naught and n naught.

Now, you already know that this is the form of solution now we had. So, in that solution now, we can safely put our expressions, that is, at t equal to 0, of course. Once we put t equal to 0 in this equation and put n equal to n naught and C equal to C naught then e to the power omega 1 t goes to 1 and similarly, e to the power omega 2 t also goes to 1, corresponding to t equal to 0. So, we have n naught equal to A 1 plus A 2.

Similarly, this is also 1 and this is also t equal to 0 is equal to 1. So, C naught is also equal to B 1 plus B 2.

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Let us start with a initial condition ($t = 0$) of $k_\infty = 1$, $\rho = 0$, $n = n_0$ and $C = C_0$.

$$\frac{dn}{dt} = \frac{(\rho - \beta)k_\infty}{l_p} n + \lambda C \qquad 0 = -\frac{\beta}{l_p} n_0 + \lambda C_0$$

$$\rightarrow \frac{dC}{dt} = \frac{\beta k_\infty}{l_p} n - \lambda C \qquad 0 = \frac{\beta}{l_p} n_0 - \lambda C_0$$

$$\boxed{\Rightarrow C_0 = \frac{\beta}{\lambda\, l_p} n_0}$$

We have already seen that the solution is of the following form.

$$n = A_1\, e^{\omega_1 t} + A_2\, e^{\omega_2 t} \qquad \Rightarrow n_0 = A_1 + A_2$$

$$C = B\, e^{\omega_1 t} + B_2\, e^{\omega_2 t} \qquad \Rightarrow C_0 = B_1 + B_2$$

$n = Ae^{\omega t}$

$c = Be^{\omega t}$

Again putting the initial solution into either of the conservation equations,

$$\frac{dC}{dt} = \frac{\beta k_\infty}{l_p} n - \lambda C \qquad \Rightarrow B\omega \cancel{e^{\omega t}} = \frac{\beta k_\infty}{l_p} A\, \cancel{e^{\omega t}} - \lambda B\, \cancel{e^{\omega t}}$$

And, again these two conservation equations that we already had like this are the two conservation equations that we had the first one is the conservation of neutron density and second one is the conservation of the delay neutron precursor concentration. There if we put the solutions let us pick up the second equation, that is, let us pick up this particular one and there we put these 2 forms of n and C, this is not a t equal to 0, I am talking about this is the general expression here we are putting n and C. Then, this is the expression or in equation that we had and once we put the expression that is we integrate the expression for C and we are putting it here.

Here, actually we are not put this particular equation rather for simplicity we are putting the original equation that is n equal to A to the power omega t and C is equal to B the power omega t, then this is the form that we are arriving with. From here e to the power omega t can cancels out.

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Let us start with a initial condition ($t = 0$) of $k_\infty = 1, \rho = 0, n = n_0$ and $C = C_0$.

$$\frac{dn}{dt} = \frac{(\rho - \beta)k_\infty}{l_p} n + \lambda C \qquad 0 = -\frac{\beta}{l_p} n_0 + \lambda C_0$$

$$\rightarrow \frac{dC}{dt} = \frac{\beta k_\infty}{l_p} n - \lambda C \qquad 0 = \frac{\beta}{l_p} n_0 - \lambda C_0$$

$$\boxed{\Rightarrow C_0 = \frac{\beta}{\lambda\, l_p} n_0}$$

We have already seen that the solution is of the following form.

$$n = A_1\, e^{\omega_1 t} + A_2\, e^{\omega_2 t} \qquad \Rightarrow n_0 = A_1 + A_2$$

$$C = B\, e^{\omega_1 t} + B_2\, e^{\omega_2 t} \qquad \Rightarrow C_0 = B_1 + B_2$$

Again putting the initial solution into either of the conservation equations,

$$\frac{dC}{dt} = \frac{\beta k_\infty}{l_p} n - \lambda C \qquad \Rightarrow B\omega \cancel{e^{\omega t}} = \frac{\beta k_\infty}{l_p} A\, \cancel{e^{\omega t}} - \lambda B\, \cancel{e^{\omega t}} \qquad \Rightarrow B = \frac{\beta}{l_p(\omega + \lambda)} A$$

And, once we are cancelling out these terms then by rearranging we can say that B is equal to beta upon l p into omega plus lambda into A.

Now, we have to remember that there are two values of omega which we have identified in the previous slide. So, while we have used just a general expression of n equal to A to the power omega t and C equal to B to the power omega t, in this expression to get a relation between A and B we should put omega 1 and omega 2 separately into this which should give the you the relations between A 1, B 1 and A 2, B 2.

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Hence,

$$B_1 = \frac{\beta}{l_p(\omega_1 + \lambda)} A_1 \qquad B_2 = \frac{\beta}{l_p(\omega_2 + \lambda)} A_2$$

$$= \frac{\beta}{l_p\left(\frac{\rho\lambda}{(\beta - \rho)} + \lambda\right)} A_1 \qquad = \frac{\beta}{l_p\left(-\frac{(\beta - \rho)}{l_p} + \lambda\right)} A_2$$

$$\omega_1 = \frac{\rho\lambda}{\beta - \rho} \qquad \omega_2 = -\frac{\beta - \rho}{l_p}$$

$$\boxed{B_1 = \frac{\beta - \rho}{l_p \lambda} A_1}$$

So, that is what we are trying to do now if we put omega equal to omega 1 then there we are having a relation between A 1 and B 1. Similarly, if we put omega equal to omega 2 we are having a relation between A 2 and B 2.

Now, just try to remember what we have developed earlier I hope you are trying deriving this also parallel to me, omega 1 is equal to rho lambda divided by beta minus rho omega 2 is equal to minus of beta minus rho divided by l p. So, let us put this into this expression put omega 1 here, which is rho lambda divided by beta minus rho. So, B 1 simplifies to beta minus rho by l p lambda into A 1.

Similarly, putting the expressions of omega 2 into this let me erase this writings from here as they are overlapping.

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Hence,

$$B_1 = \frac{\beta}{l_p(\omega_1+\lambda)}A_1 \qquad B_2 = \frac{\beta}{l_p(\omega_2+\lambda)}A_2$$

$$= \frac{\beta}{l_p\left(\frac{\rho\lambda}{(\beta-\rho)}+\lambda\right)}A_1 \qquad = \frac{\beta}{l_p\left(-\frac{(\beta-\rho)}{l_p}+\lambda\right)}A_2$$

$$\boxed{B_1 = \frac{\beta-\rho}{l_p\lambda}A_1} \qquad \boxed{B_2 = -\frac{\beta}{\beta-\rho}A_2}$$

$$C_0 = B_1 + B_2 \quad \Rightarrow \frac{\beta}{l_p\lambda}n_0 = \frac{\beta-\rho}{l_p\lambda}A_1 - \cancel{\frac{\beta}{\beta-\rho}A_2}$$

$$n_0 = A_1 + A_2 \quad \Rightarrow \frac{\beta}{\beta-\rho}n_0 = \frac{\beta}{\beta-\rho}A_1 + \cancel{\frac{\beta}{\beta-\rho}A_2}$$

$$\Rightarrow \left[\frac{\beta-\rho}{l_p\lambda} + \frac{\beta}{\beta-\rho}\right]A_1 = \left[\frac{\beta}{l_p\lambda} + \frac{\beta}{\beta-\rho}\right]n_0$$

$$\boxed{A_1 = \left(\frac{\beta}{\beta-\rho}\right)n_0}$$

$$\boxed{A_2 = -\left(\frac{\rho}{\beta-\rho}\right)n_0}$$

So, putting omega 2 here we are having a relation between B 2 and A 2 as B 2 equal to minus beta by beta minus rho into A 2. Now, we have already seen that c naught equal to B 1 plus B 2 and n naught is equal to A 1 plus A 2. So, use the expression for B 1 and B 2 into this and also the relation between C naught n naught that we have developed in the previous slide, if we put all of them this is what we are getting from the first equation as we have already arrived C naught is equal to beta by l p lambda into n naught and these 2 expressions for B one and B 2 you have just developed from the previous just on the above.

And, now, n naught is equal to A 1 plus A 2 try we can just add these two equations together by multiplying the second one with any suitable coefficient so that we can get a solution for n naught in terms of A 1, A 2 or a relation for between A 1, A 2 and n naught. So, let us multiply this equation of n naught is equal to A 1 plus A 2, let us just multiply with this particular factor then, we have beta upon beta minus rho into n naught equal to beta by beta minus rho into A 1 plus beta by beta minus rho into A 2, these two equations

Now, we add them together, let us just this we add them together. So, this cancels out. So, with A 1 we can take this to common we can take these 2 common to get a combined coefficient of A 1 and then simplifies similar we can add these together to get a coefficient for n naught and get a combined expression which comes out to be a form like this and once we simplify A 1 coming to be beta by beta minus rho into n naught and A 2 equal to minus rho divided by beta minus rho into n naught.

So, we are having a solution for A 1 and A 2 in terms of n naught. n naught invariable is a known quantity because just try to think about, how can you calculate the value of n naught. n naught refers to initial number of neutrons that can be present inside the reactor. So, that has to be a known quantity depending upon the initially the generally to initiate the reactor or initially initiate the nuclear reaction. We have to supply some kind of neutron this.

So, some source of neutron which we generally supply to initiate the entire process and once we know that a neutron supplying rate for that source then we always have some information about this n naught and other qualities like beta rho and beta and rho and the other 2 parameters that are appearing here, they are also generally known to us because beta is a standard value for any given nucleus like for uranium 235, beta is equal to 0.0065 and rho is actually the parameter there is reactivity which you are trying to control through the center exercise.

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Finally, $n = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t}$

$$= \left(\frac{\beta}{\beta-\rho}\right) n_0 \exp\left(\frac{\rho\lambda}{\beta-\rho}t\right) - \left(\frac{\rho}{\beta-\rho}\right) n_0 \exp\left(-\frac{\beta-\rho}{l_p}t\right)$$

$$\frac{n(\tau)}{n_0} = \left(\frac{\beta}{\beta-\rho}\right) \underbrace{\exp\left(\frac{\rho\lambda}{\beta-\rho}t\right)}_{e^{\omega_1 \tau}} - \left(\frac{\rho}{\beta-\rho}\right) \underbrace{\exp\left(-\frac{\beta-\rho}{l_p}t\right)}_{e^{\omega_2 \tau}} \qquad |\omega_1| \ll |\omega_2|$$

So, we now have A 1 and A 2 in terms of n naught. So, once we put these two we have we know that n is equal to A 1 into e to the power minus in A 1 e to omega 1 t plus A 2 e to omega 2 t, we are putting this expression of beta by beta minus rho into n naught exponential omega 1 t and the second expression we are putting where we are putting the getting value of omega 2 t and once we are rearranging the terms. This is what we are getting, finally.

Now, this is the final expression or final solution that we were looking to find through this entire exercise, that is, n naught or expression for n as a function of time and it is relation with n naught here it is found to be a combination of 2 exponential quantities this is the first one and this is the second one. The first one actually this e to the power of omega 1 t and this is the e to the power of omega 2 t.

Now, if you remember in the earlier slide we have proved that omega 1 is significantly smaller than omega 2 there is a as per the assumption that we are considering. And, therefore, the contribution for this second quantity that has to be much larger in deciding the neutron flux profile inside the reactor and the first one of course, is a at least when all the terms inside the brackets are positive then the first one will represents a growing exponential the term and second one will represent the decaying exponential term.

But, the role of second one has to be much larger as omega 1 is much smaller than omega 2.

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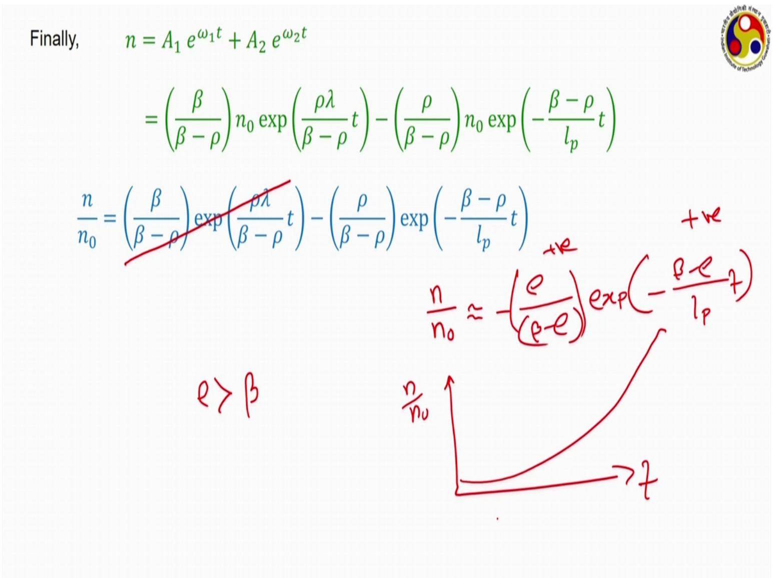

Now, omega 1 being such a small quantity compared to omega 2 as per our assumptions, let us completely neglect this term. In that case n upon n naught is approximately equal to minus rho upon beta minus rho into exponential of minus beta minus rho upon l p into t and let us just analyze this particular term; when beta is greater than rho or I should say let us take the other case when rho is greater than beta then this quantity is positive and this is also positive this coefficient of this entire coefficient and so, your profile that is n is actually a positive exponential function, it will increase exponentially if we plot time on this axis and n upon n naught on the other axis it will exponentially grow, that is what is the situation that we get for prompt neutrons as well.

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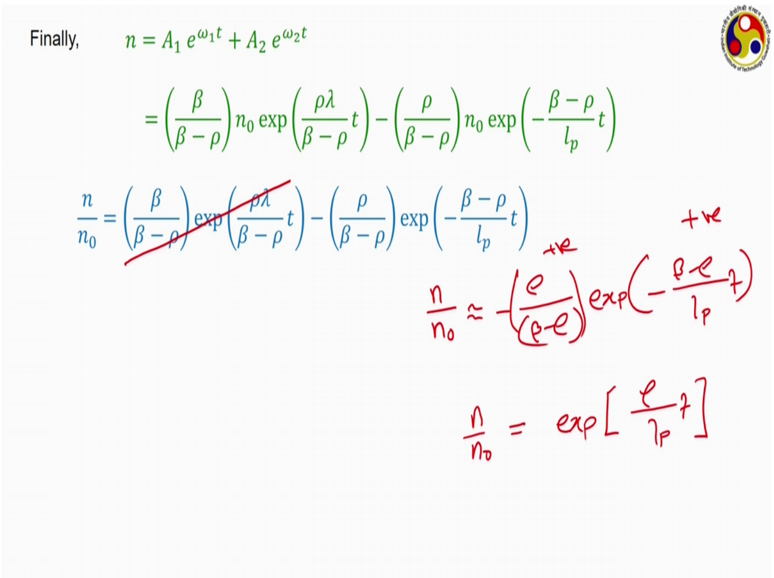

Just in to put into perspective for prompt neutrons we also got a profile of this particular kind of a form upon a exponential is equal to rho upon l p into t is rho and l p both of can when rho is a positive quantity l p being always a positive quantity it was also a positive exponential that is showed a very steep growth.

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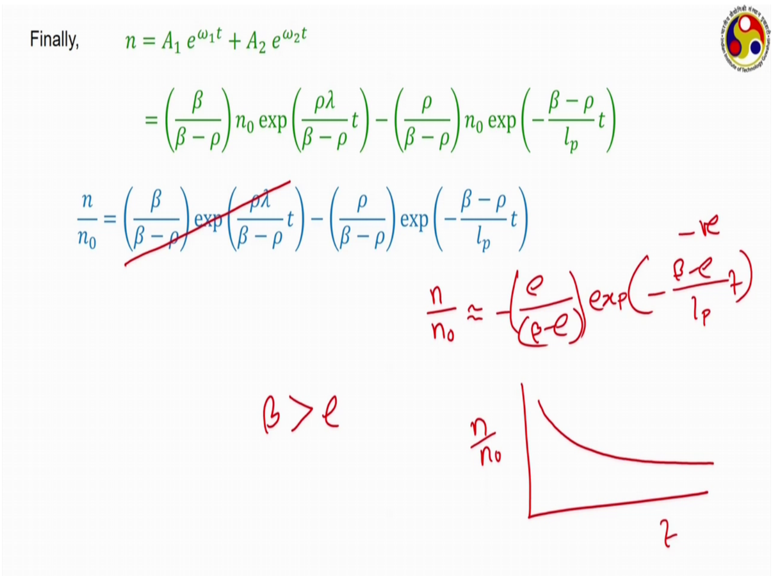

But, now we delay neutrons the beta is coming into picture the other situation when beta is greater than rho, then this quantity is a negative one. So, it will show a negative a profile of this or n upon n naught if we plot with respect to time then that will exponentially decrease and hence, we can have much we can maintain the reactor into subcritical condition as long as the reactivity is less than beta. Again, we would like to put into perspective with what in got him from neutron.

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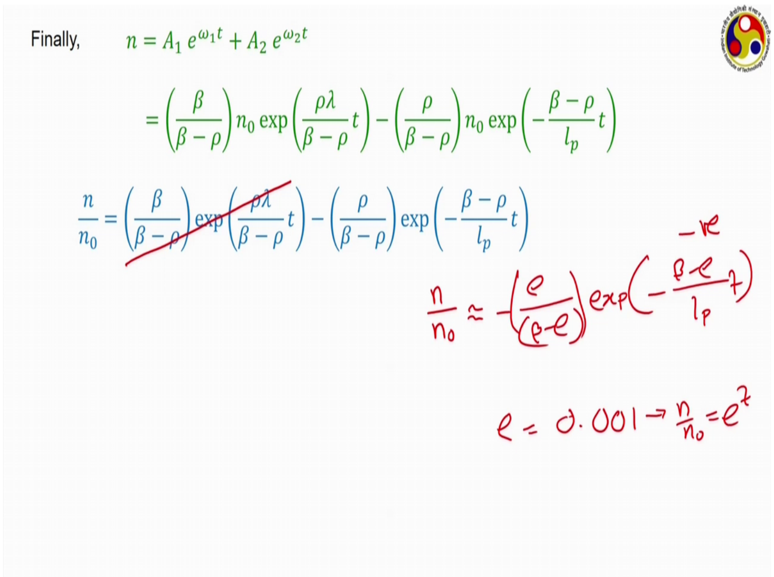

In case of a prompt neutron taking a rho value of one point no sorry taking a rho value of 0.001, we can have n upon n naught the order of t, because l p is typically of the order of 10 to the minus 3 as well and that is why when for a very small value of reactivity rho equal to 0.001, it is a straight forward exponential function of time and it increases very steeply.

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Finally, $n = A_1\, e^{\omega_1 t} + A_2\, e^{\omega_2 t}$

$$= \left(\frac{\beta}{\beta - \rho}\right) n_0 \exp\left(\frac{\rho\lambda}{\beta - \rho} t\right) - \left(\frac{\rho}{\beta - \rho}\right) n_0 \exp\left(-\frac{\beta - \rho}{l_p} t\right)$$

$$\frac{n}{n_0} = \cancel{\left(\frac{\beta}{\beta - \rho}\right) \exp\left(\frac{\rho\lambda}{\beta - \rho} t\right)} - \left(\frac{\rho}{\beta - \rho}\right) \exp\left(-\frac{\beta - \rho}{l_p} t\right)$$

$-ve$

$$\frac{n}{n_0} \approx -\left(\frac{\rho}{(\beta-\rho)}\right) \exp\left(-\frac{\beta-\rho}{l_p} t\right)$$

$\beta = 0.0065 > \rho$

$\beta = \rho \qquad \frac{n}{n_0} = -\left(\frac{\rho}{\beta-\rho}\right) e^0$

But, with delayed neutrons now with uranium 235, beta is equal to 0.0065. So, as long as the value of rho is or this particular value is greater than the reactivity, the reactor growth will or the rather the neutron profile will keep on decaying. Only when this beta and rho becomes equal to each other then we get n upon n naught is equal to 1 or is equal to the coefficient of course, it leads to rho by beta minus rho into e to the power 0, of course, it leads to mathematically an impossible situation because the denominator becomes 0, in this particular situation, but we can seek a solution for this from other means. But, this represents that the rate of neutron growth will remain the same and we can attain a critical situation for this.

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Finally, $n = A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t}$

$$= \left(\frac{\beta}{\beta-\rho}\right) n_0 \exp\left(\frac{\rho\lambda}{\beta-\rho}t\right) - \left(\frac{\rho}{\beta-\rho}\right) n_0 \exp\left(-\frac{\beta-\rho}{l_p}t\right)$$

$$\frac{n}{n_0} = \cancel{\left(\frac{\beta}{\beta-\rho}\right) \exp\left(\frac{\rho\lambda}{\beta-\rho}t\right)} - \left(\frac{\rho}{\beta-\rho}\right) \exp\left(-\frac{\beta-\rho}{l_p}t\right)$$

$-ve$

$$\frac{n}{n_0} \approx -\left(\frac{\rho}{(\beta-\rho)}\right) \exp\left(-\frac{\beta-\rho}{l_p}t\right)$$

$\beta = 0.0065 > \rho$

$\beta = \rho \leftarrow$ Prompt critical

And, this kind of situation the reactor is called prompt critical. So, when beta and rho are equal then only we call it prompt critical and for uranium 235 beta equal to 0.0065. So, reactivity can be significantly higher than 0, significantly higher than that 0.001 or 0.002 which can be disastrous in case of only from neutron kinetics, but still we can have a step operation going on.

So, in a nutshell the entire objective or rather the concept of controlling a nuclear reactor using the delayed neutron kinetics is to maintain the value of rho quite close to beta or maybe less than beta.

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Finally, $n = A_1\, e^{\omega_1 t} + A_2\, e^{\omega_2 t}$

$$= \left(\frac{\beta}{\beta-\rho}\right) n_0 \exp\left(\frac{\rho\lambda}{\beta-\rho} t\right) - \left(\frac{\rho}{\beta-\rho}\right) n_0 \exp\left(-\frac{\beta-\rho}{l_p} t\right)$$

$$\frac{n}{n_0} = \left(\frac{\beta}{\beta-\rho}\right) \exp\left(\frac{\rho\lambda}{\beta-\rho} t\right) - \left(\frac{\rho}{\beta-\rho}\right) \exp\left(-\frac{\beta-\rho}{l_p} t\right)$$

$$\rho \leq \beta$$

Dollar $\rightarrow \rho = \beta$

Or I can summarize it as rho has to be less equal to beta. For different nuclei, beta can have different values and accordingly we generally one new unit of reactivity is often defined which is called dollar, it is the same as the US currency, is dollar. Dollar is defined as that value of reactivity which makes the reactor prompt critical, that is, dollar corresponds to that value of reactivity when rho becomes equal to beta. This is not an absolute unit because a for different isotopes beta has a different numbers it is point 0.00654 uranium 235, but can be anything else for uranium 233 or may be few 239 and hence, the magnitude of dollar also keeps on varying or the definition of dollar I should say keeps on varying.

But, whatever material once we have fixed up one nuclei one for fission reaction then we know what is going to be the value of corresponding beta. And, once we know the value of beta, then corresponding value of rho or actually that particular number should be defined as 1 dollar. One hundredth of a dollar is often called one cent just to be this make the same kind of convention as in currency.

So, that is all about the delayed neutron kinetics that we generally can have. We have, I am keeping this little bit short because we had lots of mathematics. I would urge you to do the entire derivation and if you have any trouble please write to me, we are there to help. But, the conclusion that we can draw from today's lecture is that the role of delayed neutron is extremely important though the quantity of million neutrons that may appear through the field during a fission reaction is extremely small. It is invariably less than point 1 percent, but still that can play extremely important role in reactor control and rather I should say that it is only because of the delayed neutrons, we can develop a proper controlling mechanism.

If there are no delayed neutrons if all the neutrons appearing inside the reactor are of prompt in nature it would have been impossible to control a nuclear reactor. It is only because of the delayed neutrons we can maintain them maintained we can maintain subcritical, critical or supercritical condition inside the reactor as per our choice.

So, that is it for the day. I would like to keep it here itself and in the next class we shall be discussing about other topics related to the nuclear control, where firstly, I should be talking about the two most prominent mechanism by virtue of which we can control the reactivity inside the reactor and hence the power profile inside and also if you associated terminologies will be introduced.

Thank you.