Fundamentals of Nuclear Power Generation
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Module - 06
Reactor Control
Lecture – 15
Prompt & delayed neutrons

Hello friends, we are back with our MOOCS course on the topic of Fundamentals of Nuclear Power Generation. We have already gone through 5 different modules and I hope you are enjoying whatever lectures we had so, far. In the previous lecture, we have covered the topics of neutron flux distribution inside the reactor in and in the recently concluded module 5 which you have covered in the last week there we discussed about the heat transfer aspects of a reactor

Now, today onwards we are going to start sixth module a topic; I am personally very much excited about that is on reactor control and associated kinetics. Now if you remember carefully in the fourth module, you are introduced to the term a critical reactor while in common English is a term critical generally represents some kind of crisis or something.

But here actually critical reactor is the desired condition where for corresponding to each fission reaction only one of the product neutron can induce a subsequent fission. So, that the rate of fission or rate of neutron generation remains constant and accordingly we keep on getting a constant rate of power output from the reactor.

Subsequently, in all those neutron flux profiles that we have derived in your module 5 even also the discussion related to heat transfer in module rather a neutron for distribution module 4. And the heat transfer analysis that we have done in module 5 all them generally corresponds to the critical reactor only because that is the desired condition.

But there may be several situations, there may be several scenarios inside the reactor where we cannot maintain the reactor either intentionally or unintentionally; we cannot maintain it in the critical condition, rather we have to go for subcritical or supercritical

situations. For example, you can think about say your reactor is operating at a critical condition giving constant rate of power, but now there is sudden increase in the demand.

So, you have to increase the power output from the reactor and that is possible only if somehow you can increase the rate of neutron generation or I should say the rate of fission reaction; that means, at least for certain period of time you have to make the reactor supercritical. When the multiplication factor is greater than 1 reactivity is positive and the rate of reaction keeps on increasing with time and once you reach the desired higher level.

Then you can get the reactor back to the critical condition and you can continue operate with that. Similarly, when we have to reduce the power output from a reactor; we must make it subcritical for a certain instant of time. So, that the reaction rate comes to some lower level and then it can continue to be critical again. You can compare this with what we do while driving a car; like in a long smooth highway when you are driving at constant speed, we can maintain a constant pressure on the accelerator so, that the fuel that is getting supplied to the engine that remains constant and so, we get a constant speed of the vehicle a constant power output break power output from the engine.

But if we have to accelerate then we must put some higher pressure on the accelerator so, that the fuel supply to the engine increases and engine keeps on producing larger amount of break power. The opposite is true when we have to increase sorry when you have to decrease the velocity of speed this. So, there are certain situations particularly for controlling the output from the engine; we have to make it subcritical or supercritical.

There may be a further extension of that like suddenly there is a situation where we have to stop the power of production from the reactor because of some emergency or some or maybe just a plane planned maintenance kind of situation. Then you have to make the reactor highly subcritical so, that the rate of reaction quickly decreases and finally, dies down to 0.

So, these are generally the intentional situation, but there can be some long term scenarios as well. Like in a coal base thermal power station we generally supply coal that is the fuel that continuously, there is some kind of coal handling unit which crushes the coal to some small elements; generally of the size of 75 to 100 microns. And those coal particles along with air is continuously fed to the boiler, but there to generally cannot do

in case of a nuclear reactor. Rather we load the reactor with some desired quantity of fuel, you must remember the critical mass; critical mass refers to the mass of fuel required to make a reactor critical.

So, we generally load the reactor with this desired quantity of mass and then keep it running for a long period of time. Long means I really means long which can be 6 to 12 months or even more than a year. And the then only we again recharge the reactor with another set of fuel after that period is over. So, if you consider say at the beginning of this period that is at the time of loading; we have supplied the critical mass to the reactor; that means, the reactor will operate as a critical one initially, but as we end there will be not too much change in the quantity of fuel mass over a period of say 2 days or 5 days or maybe 2 weeks.

But if you take a stock of the situation after 6 months whenever there has to be some kind of reduction in the fuel quantity of fuel. Because whenever a fission reaction happens that corresponds to the destruction or depletion of 1 fuel nuclei or 1 fuel nucleus. So, the quantity of fuel continuously decreases as the reactor keeps on increasing power in producing power. And therefore, after a certain period of time there has to be a significant reduction in the quantity of fuel that is available to cause fission inside the reactor.

Then the challenge is to still maintain the critical condition inside the reactor; if we initially fed the reactor is just critical mass then after 3, 4, 5 months it must get reduced to the subcritical condition. So, we need some kind of controlling mechanism so, that we can maintain the reactor under critical condition both at the beginning of that loading period or when the reactor is freshly loaded and also several months after that.

There is another issue here during the fission reaction; those parent nucleus gets broken into to daughter nucleus. And all those daughter nucleus there also that is those fission fragments they are also radioactive in nature; so, their subsequent degradation produces several kinds of isotopes.

And initially the concentration of these isotopes are virtually negligible inside the reactor, but with trying their concentration keeps on increasing and starting of them unknown as neutron poisons like some of the isotosal xenon which has very high absorption cross section. So, once they starts appearing inside the reactor they can eat up

a significant fraction of the neutron that are available inside the reactor and once that is once they absorb those neutrons then the number of neutrons available to continue the chain reaction that keeps on decreasing.

Now, you consider the scenario one one side you where have the you are having the fuel which is reducing. And the on the other side such kind of neutron absorbing material their concentration is increasing and the net product of these two has to be a reduction in the reactivity reduction in the effective multiplication factor.

So, we need a proper controlling mechanism during by virtue of which despite all the scenarios that is decrease in fuel, increase in neutron poisoned; we can still get a significant amount of power production from the reactor or preferably we can maintain a critical condition inside a reactor. That is what we are going to discuss in this particular chapter that is; firstly, from mathematical point of we shall be sitting in the ground seeing mostly the role of prompt and delayed neutrons on the reactor control. And then we shall be seeing different ways how we can control the reactivity inside the reactor.

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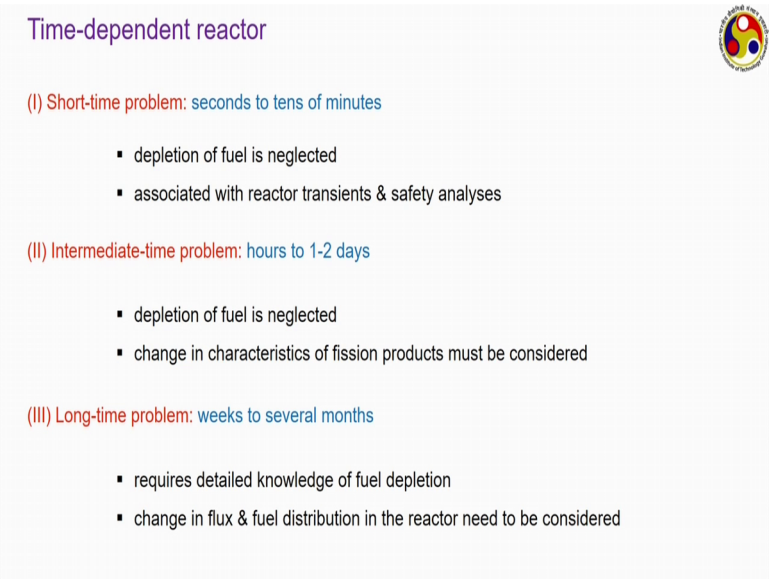

Next is the time dependent reactor; virtually whenever you are dealing with a critical reactor we can deal that has a steady state situation. Because the neutron flux and hence the power output that remains constant with time and so, we can neglect the time derivative of any component. But whenever you dealing with the subcritical or supercritical reactor; definitely the neutron flux and the rate of fission reaction and also

the rate of power generation that keeps on changing with time and so, you have to consider this as a time dependent problem.

But from nuclear kinetics point of view time dependent problems can be divided into 3 categories. The first one is the short time problem, where the time interval under consideration generally ranges from seconds to tens of minutes. And these are the kinds of problems we completely neglect the depletion of fuel that is a mass of fuel is considered to be constant.

And these are the problems that we generally use while solving reactor transients and also the safety analysis. There can be next to the intermediate time problem where the time spend can be a few hours to even 1 or 2 days typically up to 2 days. Here also quite interestingly, we neglect in a depletion of fuel; however, the production of the fission products that needs to be considered because over period of 2 days; the concentration of certain fission products can have significant amount of increase.

So, that changing characteristics of the fission products needs to be considered, but again here also similar to the short time problem; we can consider the fuel mass to be constant. And third is a long time problem where we are going for a very very long period certain weeks to several months; here we have to consider the changing characteristics of fuel, changing stock of fuel and also the change in the flux can fuel distribution inside the reactor.

Because as we have seen in a neutron flux distribution in module 4; the neutron flux while we would like to have that uniform that general is not uniform inside the reactor. Particularly if the reactor is not reflected one then the neutron flux will be extremely low at the edges and very high at the center line. Therefore, the fuel which is present close to the center line or the zone where the neutron flux is high; that will experience larger amount of fission and so, the depletion in fuel there will be much more compared to the depletion in fuel in certain other areas of the reactor that also needs to be considered in this long time problems.

But here our objectives will mostly the surrounding to this short time and intermediate time problems at least in this chapter. Another interesting part of that short and intermediate time problems is the depletion of fuel is neglected, but a question is how can you neglect the depletion of fuel over a period of 2 days; virtually we are considering

the mass of fuel to remain constant, but still you are getting power output. And to justify that we need the concept of fuel burnup.

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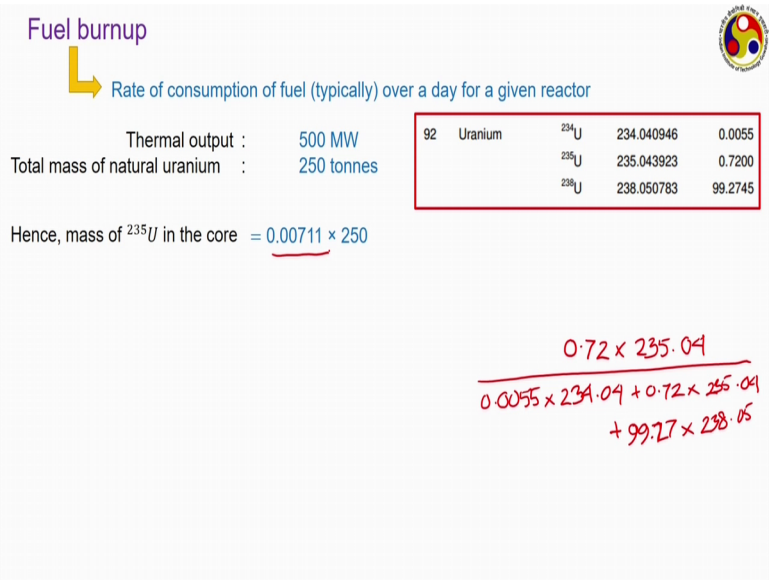

Fuel burnup refers to the rate of consumption of fuel typically over a single day in a given reactor. That is the amount of fuel reactor consumes in one single day that is called fuel burnup and this is the term not exclusively for used for a nuclear reactors, but can be used for any kind of a power generating in a entities.

So, let us take one example we are considering a reactor which is giving 500 megawatt of output and it has initially been loaded with a 250 tonnes of natural uranium. Now if you remember, natural uranium basically comprises of 3 different isotopes; primarily uranium 238 which is have which comprises more than 99 percent of this, but uranium 238 is a fertile isotopes; so, it does not undergo fission at least when it is subjected to thermal neutrons it is a very small fission cross first fission cross section, but that can be neglected generally in thermal reactors.

Here our point of interest is uranium 235, but its fraction is only our 0.7 percent because this is a fissile one. And therefore, it is only this uranium 235 which can produce power by virtue of this fission reaction and small traces of uranium 234 can also be present.

So, to first we need to identify the mass of uranium 235 that is present in the core total mass is 250 tonnes, but that corresponds to an natural uranium. And then we need to

know first the mass fraction of uranium 235 in this reactor and that mass fraction can easily be calculated as we know the molecular weight or atomic weight rather of all these 3 isotopes and also their percentage.

So, the fraction of uranium 235 can be calculated as 0.72 into its atomic weight 235.04; I am not writing the further digits divided by the same for all the a components that is for you 234 this point double naught 5 5 into 234.04 plus 0.72 into 235.04 correspond to uranium 235 plus the last fraction for uranium 238 which is 99 sorry 99.27 into 238.05; now if we do the calculation it will come to be at 0.00711. And once we multiply this mass fraction with the total mass of the fuel that is 250 tonnes.

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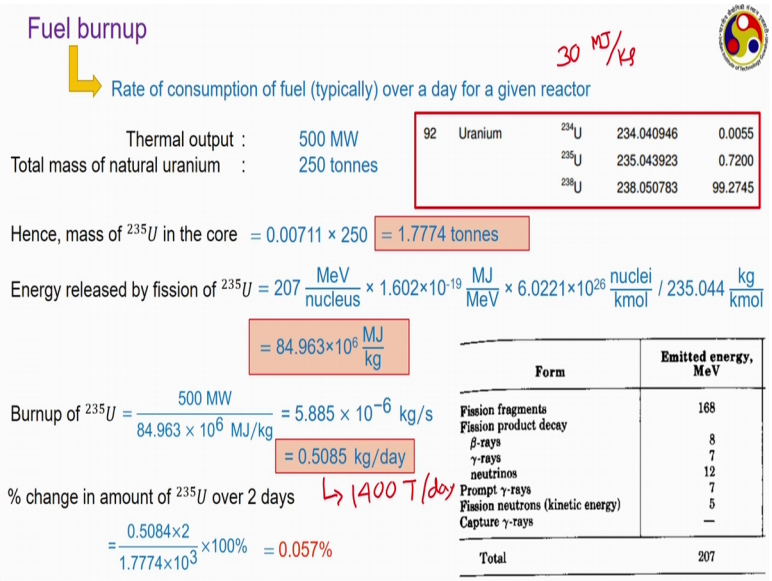

Then we get the amount of uranium 235 available inside the reactor to be only 1.7774. So, this is the mass of actual fuel that is available inside the reactor, but actually the total mass of substance that you are supply this one 250 tonnes, but our calculation we shall be doing which respect to the mass of uranium 235 only.

Now we know that the total amount of energy released during the fission reaction uranium 235 can have several components; this is a chart that was shown in the earlier modules. And it is roughly can we said that we get 207 MeV of energy because of the fission of a single uranium 235 isotope.

So, the total amount of energy that we are getting because of the fission reaction can be this 207 MeV for a single nucleus, this multiplied by 1.602 into 10 to the power minus 19, where we are considering this energy to we are rather converting this energy to mega joule. And then it is a 6.0221 into 10 to the 26 that is the Avogadro number, which converts this from single nucleus to 1 kilo mole.

And so, this 3 quantities is our product of these 3 numbers gives you the amount of energy that is been released by 1 kilo mole of you to rectified and the unit of that is this mega joule. And next if we divided by the molecular mass of uranium 235; then we get the total output as from per kg of fuel. So, we are getting 84.963; 10 to the power 6 mega joule of energy from a single kg of uranium 235.

And total quantity that we have supplied is 1.7774 tonnes. So, the burnup then can be calculated as 500 megawatt is a thermal output that we are getting divided by the amount we are getting from a single kg of this which comes to be something like 5.80 into 10 to minus 6 kg per second or just half of a kg in a single day just triggering number really. That is 500 megawatt of energy we are getting and for to get that amount of energy, we have to spend only half kg of uranium 235 in a single day.

So, if we want to calculate the change in the; percentage change in the amount of uranium 235 inside the reactor over a period of 2 days, then we can calculate this the amount of fuel consumed the single day multiple by two divided by the total mass available and just 0.57 percent or 0.057 percent that is less than 0.06 percent change in fuel mass will happen over a period of 2 days.

And that is why we can neglect this depletion of fuel mass in short time or intermediate time problems. Just to consume is 1 percent of this total mass of fuel that is this one; just to consume 1 percent of this one in a we need something like 34 to 35 days. That is why once we have loaded our reactor with this 250 tonnes of natural uranium or 1.7774 tonnes of uranium 235; we can keep on running it for a long period of time theoretically as 1 percent is consumed in about 35 days.

So, it can go up to 3500s; it can go up to 3500's that is more than 9 years without any further fuel loading. Practically it is of course, not done because of several other reasons, but still once we have loaded the reactor with a certain quantity of fuel; we can keep on using that for a long period of time. Just if you would like to take this one or compare

this one with a conventional coal based power station; in a very good quality bituminous coal that we use nowadays, the typical calorific value is generally of the order of 30 mega joule per kg.

So, this is the typical calorific value of common bituminous coal is a high grade coal. And if we use this particular coal to give the similar thermal power output; then how much energy, you have to how much amount of fuel you have to produce or we have to supply? You can do the calculation you know that the total output is 500 and megawatt and 1 kg of coal is giving about 30 mega joule. So, if you compare this you will find that corresponding this number corresponds to this is will be coming something like 1400 tonnes per day.

Actually it will be greater than 1400 transfer day; that means, while in to get the same amount of power while we are spending just 0.5 kg of uranium 235 we are spending more than 1400 tonnes of coal and that is also a good rate coal. If we compare this with say natural gases; gases may have a higher calorific value something in the range of 15 mega joule still the total amount consumed will be 1000 tonnes for a single day.

And that is the reason the amount of fuel requirement in a nuclear power station is much lesser compared to a conventional coal base power stations. And hence the total volume of the reactor can be much lesser compared to a coal based on. Next we move to the prompt and delayed neutrons the terms these two terms were introduced earlier itself in one of the earlier modules.

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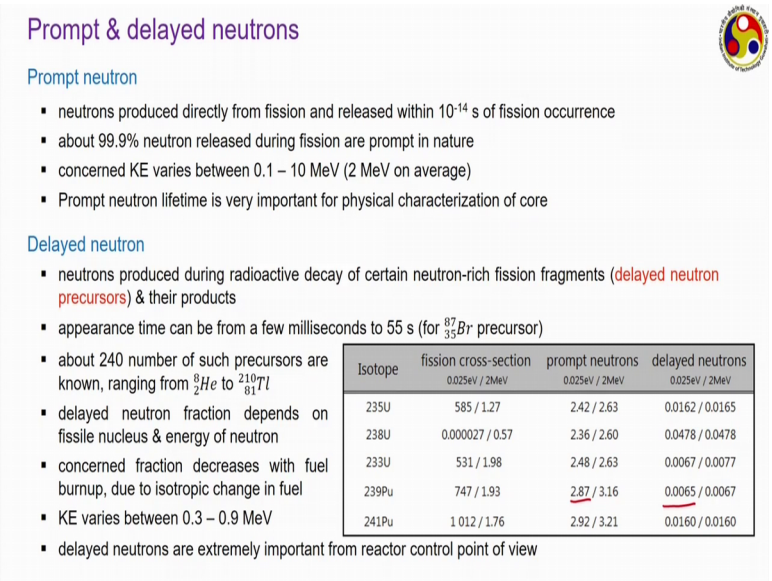

And we clearly know that prompt neutron refers to the neutrons which are produced directly from the fission and therefore, they appear within 10 to the minus 14 seconds of fission occurrence. Whereas delayed neutrons actually are the result of the radioactive decay of certain neutron rich fission fragments. Generally fission fragments undergo beta decay because of which electron comes out of the fission fragments and they get converted to a different isotope.

But in certain fission fragments of during those decay chain of the fission fragments; we may sometimes encountered certain neutron rich elements. And those neutron rich elements can undergo neutron decay and those neutrons are called delayed neutrons. And the isotopes which leads to the formation of these delayed neutrons are known as delayed neutron precursors.

Appearance time for this delayed neutron depends upon the decay rate of this delayed neutron precursors and it can range from a few milliseconds to something of the order of 50 seconds as well. Presently the longest living neutron delayed neutron precursor is bromine 87, which is a decay sorry and half life of about 55 seconds or the mean life of about 55 seconds.

And presently both from nature and also through laboratory control experiments we have identify about 240 numbers of such precursors; it is a very very wide range it can have very low mass numbers such as helium 8 to a high mass number like helium 210 and

several of them actually outside the fission range that is their atomic number is quite small to cause any kind of fission, but still like it is helium 8; this kind of very light isotopes can also act as delayed neutron precursors.

Now, total amount of neutrons that are available from fission more than 99.9 percent are from are prompt in nature. So, the total number of delayed neutrons or total fraction of delayed neutrons that is available is extremely small. And also that keeps on depending on the nucleus itself and also energy of the neutrons. Like this is a chart that I am showing; if you compare the values for uranium 235, then for thermal fission whereas, the number of prompt neutrons average number of prompt neutrons 2.42; corresponding number of a delayed neutron is just 0.016 ;it is an extremely small percentage.

And the same is true for others as well; like you can think about plutonium 239; plutonium 239 has a very high fission cross section and it can produce on an average 2.87 number of prompt neutrons for every thermal fission. But corresponding number of delayed neutral is extremely small 0.0065. So, the total fraction of delayed neutron that may appear during a fission reaction is extremely small compared to the prompt neutrons and also as they appear we after the actual fission occurrence; so, their energy level is also much lesser.

Like the fission reaction sorry the concerned fraction thereof delayed neutron that may appear that can decrease with the fuel burnup because of the change in the isotropic change in the fuel concentration, but from energy point of view, prompt neutron can have very high kinetic energies can range from 0.1 to 10 mega electron volt because they are directly getting a share of the energy released during fission. On an average, prompt neutrons come with an energy of 2 MeV or at least for a calculation purpose generally we consider prompt neutron to have an average 2 MeV of energy.

And if you compare that figure with the delayed neutrons it is just 0.3 to 0.9 MeV. So, much much lower compared to the prompt neutrons for delayed neutrons the average is something like 0.4 MeV. So, 2 MeV for prompt neutrons whereas, only about 0.4 MeV for delayed neutrons; prompt neutrons a very important role in the physical get relation of the core, but delayed neutrons on the question definitely is coming to your mind that where it is a such a small fraction less than 0.1 percent sometimes even lesser like we can always compare this numbers like 2.87 and 0. double naught 65.

So, it is such a small fraction why we at all need to bother about delayed neutrons, but as we shall we saying shortly in this lecture and also in the next lecture that delayed neutron play very extremely very important role in controlling the reactor. In fact, it is only because of the delayed neutrons we can successfully control a nuclear reactor. Otherwise if we are depending only on the prompt neutrons, it would have been it is a an extremely difficult job to control a nuclear reactor is thanks to the delayed neutrons, we can do the coil successfully.

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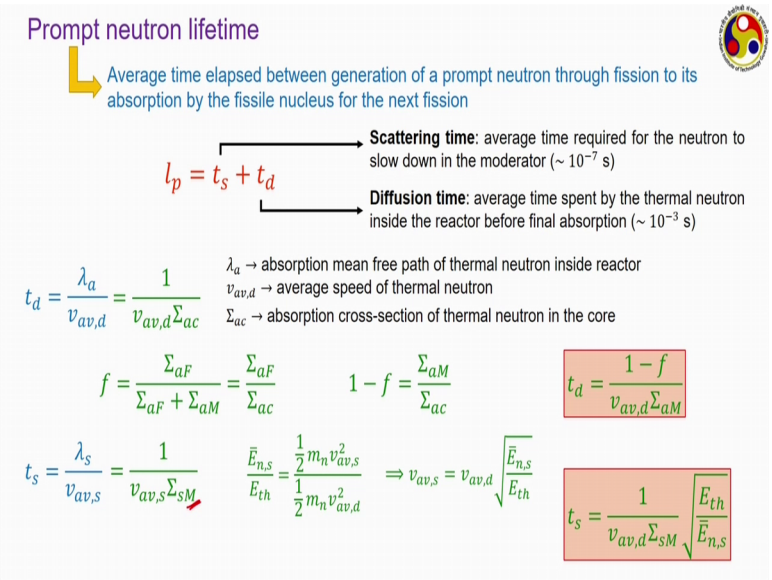

Now, come define the prompt neutron lifetime; prompt neutron lifetime is defined as the average time elapsed between the generation of a prompt neutron through fission to its absorption by the fissile material for the next fission; that is a prompt neutron the moment it appears from a fission reaction, we start calculating this life time; it goes through several levels of phenomenon's like, it can is very very rare cases it can induce a first fission, it can go through the all those scattering and activation through the intermediate energy range.

And finally, once it ones to the thermal neutron level it can get absorbed, it can get diffused to the thermal neutron as well and it can get absorbed in the moderator or in the fuel. And this entire period is the one that we call as a prompt neutron lifetime and mathematically we use the symbol lp to call the prompt neutron lifetime of course, time is denoted by symbol t, but this is the convention that we generally followed.

So, the two typical phenomenon a prompt neutron undergoes is a scattering and also the diffusion. Scattering through the moderator and also the diffusion process that after getting thermalized. Here ts refers to the scattering time which is the average time required for the neutron to slow down in the moderator, td is the diffusion time that is after we can thermalized; is the time it spends inside the reactor. Typically the scattering time is of the order of 10 to the minus 7 seconds and diffusion time is of the order of 10 to the minus 3 seconds; we can shall be calculating them shortly.

Let us try one case that is the typical diffusion time can be defined as lambda a by v average where lambda is the absorption mean free path of the thermal neutron inside the reactor and v average is the average speed for the thermal neutron. And as lambda a is inversely proportional to the absorption cross section of thermal neutron in the core. So, we can always write this here again we are I would like to mention lambda refers to the absorption mean free path and so, sigma ac corresponds only to the absorption macroscopic absorption cross section and v average is the average speed of the thermal neutron.

Next this small f we have defined earlier this is the thermal utilization factor; so, thermal utilization factor is a macroscopic cross section of the fuel divided by macroscopic cross section of all other elements that is present there. Macroscopic absorption cross section of fuel divided by macroscopic absorption cross section of all other elements ; this is after becoming thermalized.

Here we are considering only fuel and moderator; so, f is equal to sigma for absorption for fuel divided by sigma absorption for fuel last moderator. And hence q minus f is equal to sigma of a for moderator divided by sigma a for the core that is a total macroscopic absorption cross section. And hence td can be written as 1 minus f by v average into sigma absorption a for the moderator.

And the same way we can calculate for ts as well it is called the lambda s by v average s; lambda s is the scattering mean free path and v average s is the average velocity of the neutron during the scattering process. One thing this velocity during the scattering process definitely keeps on changing; because the only in energy that neutron can have is the kinetic energy. And during the scattering process its kinetic energy reduces from

something like 2 MeV to 0.025 electron volt and therefore, accordingly this v average can also have a very large range.

So, lambda is can be related to the macroscopic scattering cross section for the moderator. It is a moderator where the scattering process is going on fuel does not have any role in scattering and that is why we are having only the moderator here. And now we use this energy ratios here en thermal the denominator refers to energy of a thermal neutron. And hence it has to be half into mass of a neutron into the average velocity average velocity of the neutron during the diffusion process and the numerator E n s bar that refers to the average energy during the scattering process.

So, it has to be equal to half mnv average square and from there we get a can get an expression of v average scattering and relate that to v average diffusion. So, once we put this final expression; we get this as the final expression for the scattering time. So, now, we have derived both the expressions for the diffusion time and scattering time and let us put them together also add them together to get the prompt neutron lifetime.

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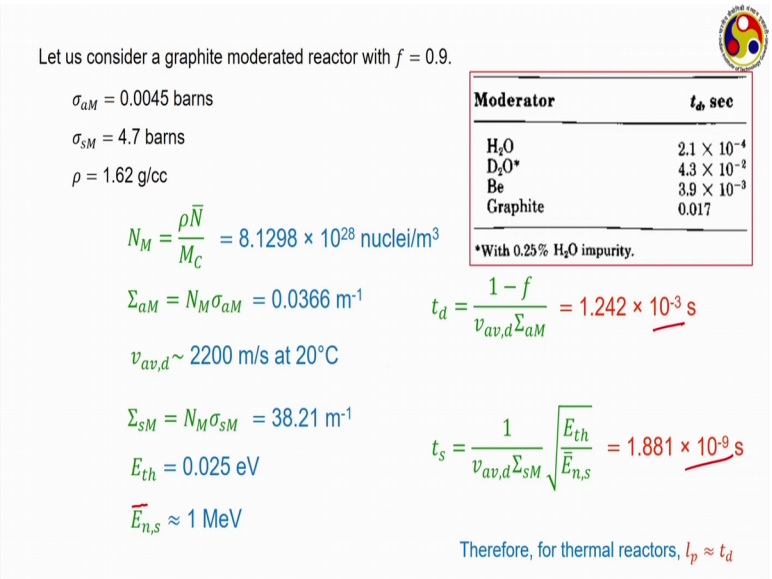

Let us take the example of graphite moderated reactor where the utilization factor is 0.9 and the for graphite we know the microscopic absorption and scattering cross sections and also a density.

Once we know the density, we can use that to calculate the number of such nucleus present inside the moderator per unit or number of moderate a nucleus present per unit volume which can be the density into the Avogadro number divided by the molecular weight of graphite and accordingly it comes to be 8.1298 into 10 to the power 28; number of nuclei per unit volume. And now macroscopic cross section is this number of nuclei per unit volume on nucleus density multiplied by the microscopic cross section. Secondly, this one is the macroscopic absorption cross section for the moderator.

And for a typical thermal neutron; the velocity is generally around 2200 meter per second because this energy is something like 0.025 electron volt. So, the expression for td is already known to you; we are putting these numbers that you have calculated and putting on we are getting it to be 1.2 into 10 to the minus 3 seconds or something like 1 millisecond; that is the diffusion time.

Now, let us calculate these are the typical diffusion times for some common moderator whereas, it is just of the order 10 to the minus 2 for D 2 O with 0.25 percent of impurity. It is only 10 to the minus 4 compared to water; so, the diffusion process in modern the or the rate of diffusion process depends on the nature of the moderator itself.

Now we go to the scattering part; similar to the absorption we can also calculate the macroscopic scattering cross section, which is something like 38.21 per meter; definitely much larger compute the absorption process and the energy for a thermal neutron is 0.025 electron volt. Then energy for a in average energy in the scattering process; it should be average energy bar should be there are several use we can calculate this, but here going for a simple approach; initial energy at the starting of scattering can be written to be 2 electron 2 a mega electron volt and at the end of the scattering process, when it has become thermal is this energy 0.025 electron volt. So, the average of that is 1 mega electron volt.

So, we put that into the mathematical expression for scattering period; scattering time and this coming of the order of 10 to the minus 9 seconds only. So, this diffusion time and scattering time there is a very large difference between them; while diffusion time is of the order of only 10 to the minus 3 second, scattering time is of the order of 10 to the minus 9 second. And therefore, for thermal reactor at least the neutron prompt neutron

lifetime is considered to be equal to the diffusion time only the scattering time can simply be neglected.

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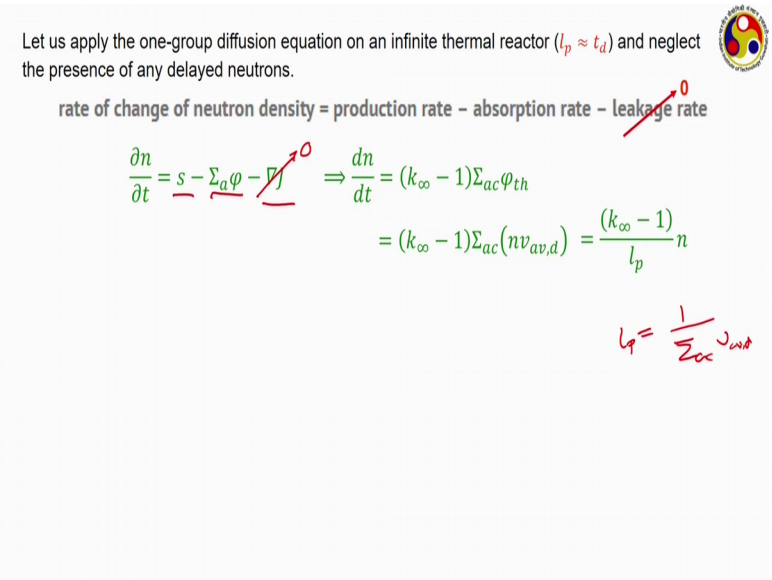

Now, we have to apply the one group diffusion equation on an infinite thermal reactor. Here the reactor is infinite and we are considering from the previous slide whatever we have done; we have we have taking this prompt neutral lifetime to be equal to the diffusion time and also we are neglecting the presence of energy neutrons from any source.

So, this is the equation that we have dealt with; the earlier also the rate of change of neutron is it should be equal to the production rate minus absorption rate minus leakage rate as you are considering an infinite reactor; so there is no leakage then we are having only production and absorption accordingly this is the equation, where s is the production rate and this is the absorption rate and finally, this is the leakage rate, but this leakage rate of course, will be equal to 0.

So, the equation reduces to this particular form; here the neutron density in as we are considering an infinite reactor; infinite thermal reactor. So, we can neglect the variation in neutron density in different parts of the reactor and so, n becomes a sole function of time only. The absorption and the production rate can be related like this and phi is known as n into the average velocity that is the neutral density into the average velocity of the neutron.

So, we again know that just from the previous slide we have seen that lp is equal to one by the absorption cross section into the average diffusion velocity ; accordingly this equation this lp terms comes into play; that is our equation reduces to dn minus dn dt is equal to k infinity minus 1 by lp into n.

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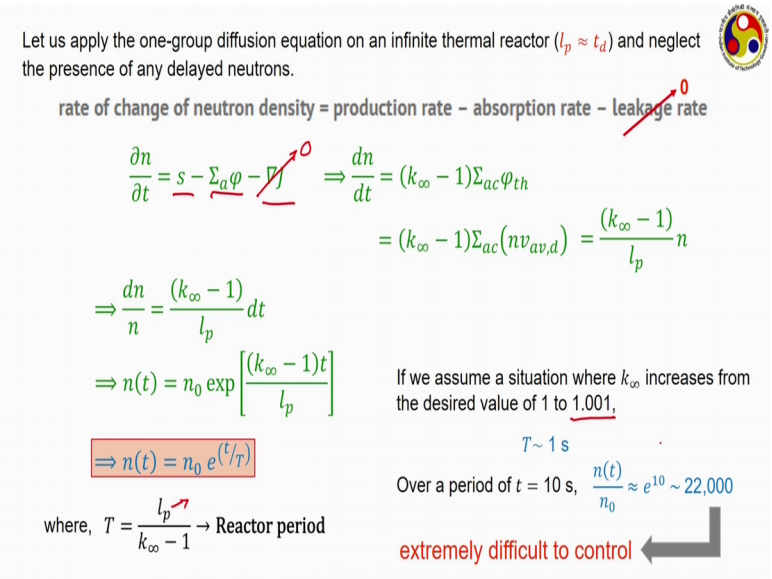

Or putting this into perspective now, we can rearrange the terms and we can perform this integration to get expression of nt equal to n naught into exponential k infinity minus 1 into t by lp, where n naught refers to the neutron density at time t equal to 0 or nt equal to n naught e to the minus T by capital T where T capital T is lp upon k infinity minus 1 that is which is called the reactor period it is called the reactor period.

If we now assume a situation another test case that you are considering, we are considering a reactor which is initially operating as a critical one that is the effective multiplication factor is 1. But because of certain disturbance it has increased by a very small fraction of 0.1 percent and heat has become 0.1001 sorry 1.001; if we put this number into this expression ; first you have to calculate capital T. We know that this from our previous analysis this lp is of the order of 10 to the minus 3 and k infinity after modification is 1.001. So, capital T is of the order of one seconds only and now if we put into the perspective taking small t equal to 10 second; then nt becomes after 10 second because 22000 approximately of the initial neutron density.

That is just over a period of 10 seconds the number of neutrons present inside the reactor will increase by 22000; that is an extremely large number. As the number of neutron keeps on increasing, corresponding reaction rate keeps on increasing and just in 10 second the reactor will produce if it at all can survive is such a rate of such high rate of increase ; then it will produce 22000 larger amount of power compared to the original reactor.

So, that is a situation which is extremely difficult to control and we have to find some other mechanism.

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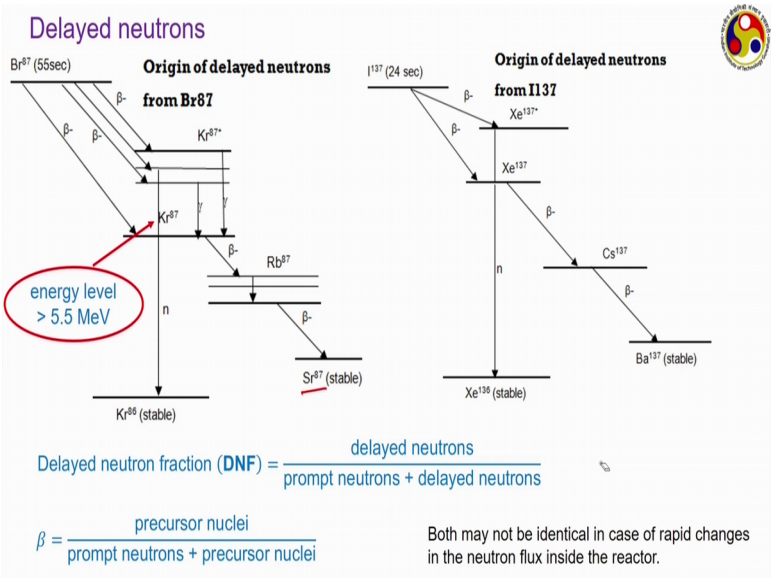

And there where the role of the delayed neutrons comes into play. Delayed neutron as I mentioned are the outcome of the radioactive decay of this fission fragments. During some of the fission fragment during the decay can become can appear to be neutron rich in nature and therefore, they may lead to the emission of neutrons. Like one chain is shown here for bromine 87; it has a half life of about 55 seconds.

Now, during its decay commonly produces krypton; now krypton can again there is can be several kinds of isotopes. Like if we consider this one where krypton 87 star means it is an excited krypton 87; then it can undergo a gamma decay to come back to just normal krypton 87. And then it can go through two levels of beta decay to finally, reach Sr 87, but there can be other rules also by undergoing by undergoing beta decay; it can also produce krypton 87 which are now all is a not any excited states like this.

Now, krypton 87 is something that is neutron rich and therefore, it is very likely to emit a delayed neutron and become a stable krypton 86 isotope. So, while and the while most part of this chain corresponds either beta decay or gamma emission; only one chain or in bromine 87 can lead to the formation of neutron delayed neutrons and krypton 86 the stable isotope.

Here is another example with iodine 137; it has a half life of 24 seconds and again it can go to beta decay through to produce either xenon 137 or xenon 137 star; that is a excited, xenon 137 goes through two levels of beta decay to reduces to Ba 137; barium 137, but it is also possible like here is excited stage its energy content is so, high that it can dislodge one neutral from this dislodge 1 neutron from its nucleus they are very giving this particular delayed neutron.

And final product is xenon 136; there are two transits commonly used while dealing with delayed neutrons; one is the delayed neutron fraction which is defined with a straight forward definition ; delayed neutrons delayed fraction is equal to delayed neutrons divided by total number of neutron that is form plus delayed.

But there is another definition which is used that is beta this called this slightly different, where in a numerator we have the precursor nuclei or delayed neutrons basically has been replaced by precursor nuclei. Beta is equal to precursor nuclei divided by precursor nuclei plus the prompt neutrons um. Generally both of them are same, but when the situations where this is a rapid change in the neutral flux distribution, then both may become identical.

Here also we shall mostly dealing with delayed neutron, but here also we are shall mostly be dealing with the situation where the neutron flux changes not that rapid and so, we shall be considering these two vectors to be equal that is beta and the delayed neutron fraction will be considered to be identical.

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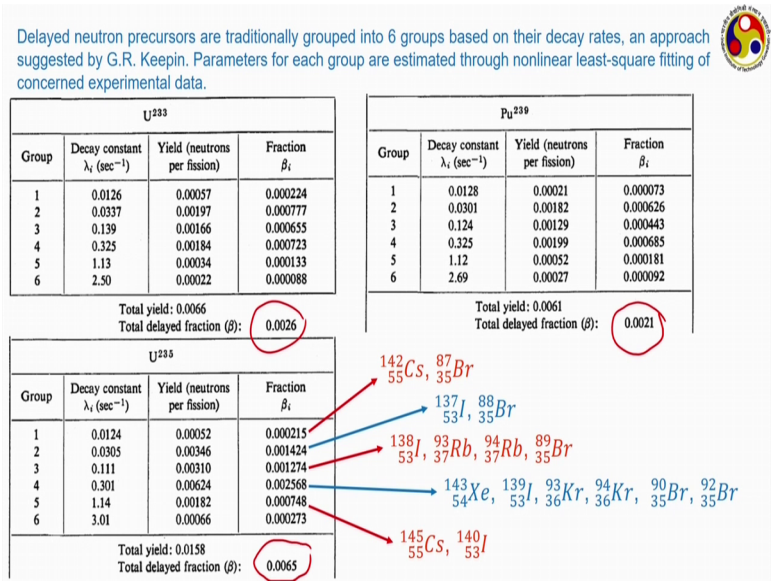

Now, there can be several kinds of delayed neutron precursors which are produced during this nuclear fission. And also subsequent radioactive decays like I have shown number earlier it can be as large as 240 and still increasing because of the newer elements found in the laboratory or newer compounds found in the laboratory.

But it is not possible to consider all those isotope separately; like if I go back to the previous slide one; now just to take a look at this chain for bromine 87. There are how many isotopes involved? We have one bromine 87 this Kr 87, then number 3 Rb 87, then Sr 87 and also Kr 86; there are 5 isotopes which are involved into this. Similar other chain; I137, Xe 137, Cs 137, Ba 137 and Xe 136; so, again 5 here; that means, for successful modeling of that particular kind of chain, we need to consider at least 5 isotopes; only for this kind of chain and there can much much longer chains which is what several kinds of isotopes.

And therefore, it is not possible to treat each of such isotopes separately. Rather G.R. Keepin and his core causes propose to combine the isotopes into 6 groups; these groups are formed based upon the decay rates and half life that is similar in the properties. That is a few precursors which are having similar kind of properties are clubbed together into a single group and then the property for the group is estimated by non-linear least square fitting of the concerned experimental data.

So, the property where is for a single group may not match with any of its constituents, but it will give an overall behavior of that particular constituents. This is the groups that we can get for uranium 233; you can clearly see there are 6 groups and corresponding average decay constant and the fraction of that group there is beta that they are clearly given. Similarly this is a groups for uranium 238; so, in case of uranium 233; you can see that only 0.0026 is beta fraction. So, that it is 0.26 percent of total neutron is this delayed neutron.

Similarly, it is 0.0065 for uranium 235 and this is the chart for plutonium 239; if the delayed fraction is only 0.0021. These are some examples of the isotopes which can appear in different groups like the first group which corresponds to the lowest decay constant and hence the largest half life belongs to cesium 142 bromine 87 etcetera.

The second one also deals with iodine 137 and bromine 88 and in the third case; there can be several kinds of isotopes like iodine 138, rhodium 93, rhodium 93; rhodium 94 rather and bromine 89; you can clearly see there on all these 3 examples you have bromine isotopes involve bromine 87, 88 and 89. We can also bromine 90 and 92 in the fourth group along with xenon 143, iodine 139 and a few others.

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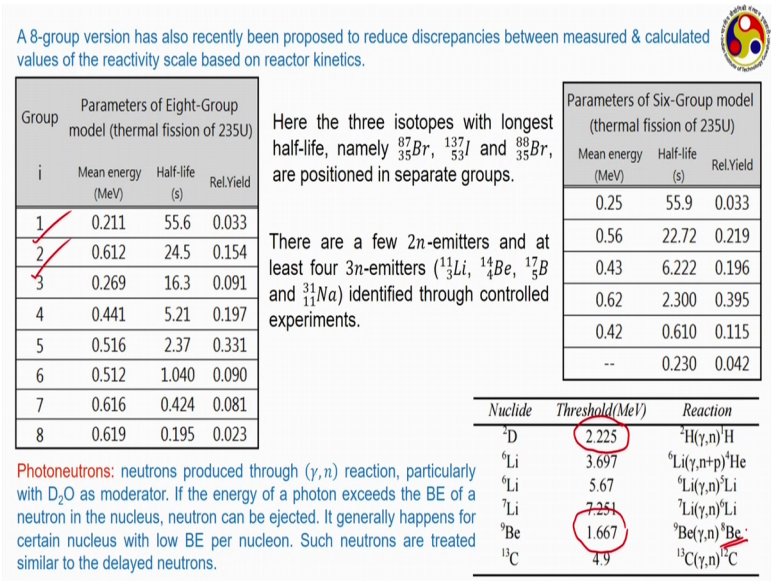

And in the fifth group we have cesium and iodines etcetera; there can also be an 8 group version of this because the adoption of the 6 group model the calculation that we get by

adopting a 6 group model can differ quite a bit compared to the exponential measurements.

So, to make this distribution more uniform certain cases 8 group versions are also used. This is the 8 group version for uranium 235 and this is the corresponding version for uranium 238. You can clearly see the difference between them in case of uranium 235 with 6 groups; the largest half life that you are encountering 55.9; it is 55.6 only in this particular case.

So, they are quite similar, but as we move down this table there is difference like for sixth group case second group has a average half life of 22.72, but that has been broken into two parts into the second and third group for 8 group model. There are 3 isotopes which are having the longest half life; like bromine 87, iodine 137 and bromine 88. And they are the first 3 those will be considered they will be considered into separate groups. Like the first group for each parameter model a is just uses you bromine 87 or it has bromine 87 as the single component.

So, all the half life and other values that are given corresponds to bromine 87 and not average do it some other element. Similarly second one uses the data for you iodine 137 and third one uses for bromine 88. And just for your information in certain situations we may have one delayed neutral precursor which is emitting not one, but two neutrons and even there can be situation when it can emit 3 neutrons also. There are at least 4 elements 4 isotopes which are which can emit 3 neutrons like a check lithium 11 normal isotope for lithium is 3 a like 6 or 7.

And here then its mass number is 11 out of which 3 is proton; then there are 8 neutrons. So, we can clearly see for the others also accordingly they need to emit very large number of neutrons to come back to some kind of ground states and hence we get this 2 n and 3 n emitters. and before I close in on delayed neutrons we I would like to introduce another term called photoneutrons.

Neutrons here these neutrons are not the resultant of the radioactive decay of the fission fragments. Rather they are produced by gamma in kind of reaction; I hope you remember this convention it favors that the parent nucleus has absorbed 1 gamma photon and it has gone through a neutron decay process. That is the daughter is actually another isotope of

the same element, but its atomic number, while it remains the same is mass number will be one less because there will be one less neutron.

This happens particularly when deuterium is a moderator; here actually the gamma photon means whenever a gamma photon strikes a nucleus, this photo neutrons will not be emitted rather the gamma photons should have sufficient energy. So, that it can dislodge one neutron that is the energy of the gamma photon should be larger than the binding energy of a single neutron. There are certain elements or certain nucleus where whose binding energy per nucleon is quite low.

And when that is heat by a photon of sufficient energy, it can lead to the emission of photon neutrons. But these are quite a few examples like you can see for deuterium we have calculated this one earlier into our first module; the deuterium the binding energy per nucleon is a very small number just 2.225 MeV. And whenever it is being heat by a gamma photon of energy something like say 2.23 MeV or 2.3 MeV; then it will lead to the ejection of one nitrogen and the deuterium gets converted a normal hydrogen. Look at this example of beryllium 9; it is having an extremely low binding one of the lowest among all known isotopes this is just 1.667.

And therefore, it is very easy to extract one photo neutron from this beryllium and that leads to the formation of beryllium 8, but from characteristics point of view; this photo neutrons are found to be quite similar to the thermal neutrons and or normal or delayed neutrons I should say. And therefore, they are not distinguished from the delayed neutrons rather during calculations being analysis photoneutrons are considered as a part of the delayed neutrons themselves.

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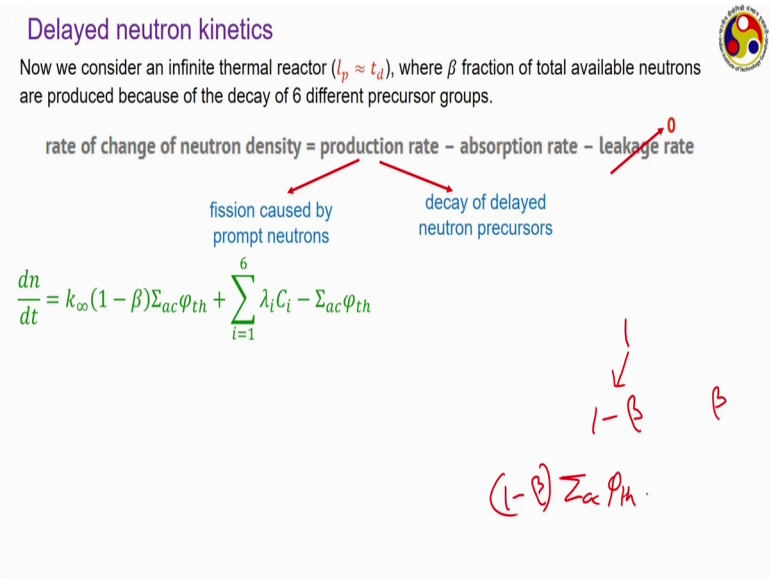

Finally, we shall be starting briefly the delayed neutron kinetics today which a mathematical base we shall be setting up today and we shall finishing off with the next class.

Let us again consider an infinite thermal reactor such that lp is nearly equal to td here are like a earlier case we consider all neutrons to be of prompt in nature, but here beta fraction of the total number of neutrons are coming in the form of; coming from the delayed neutron precursors. There can be 6 different groups of precursors we can consider 8 also that will the analysis will remain the same.

Now, the rate of change of neutron density the same equation it is equal to production and minus absorption minus leakage. Leakage there is no leakage from infinite thermal reactor, but now production while earlier with the considering production only because of the fission caused by prompt neutrons, here we can also a production because of the decay of the delayed neutron precursors. So, there has need to be taken into consideration.

So, this is the equation that we are having here; here actually in this equation there are two parts one part k infinity into sigma into phi thermal is the one that is coming because of the prompt neutrons; fission caused by prompt neutrons. And k infinity into beta into sigma ac into phi thermal is coming from the delayed neutrons.

Actually I should not have written this way this is not the proper way have to writing this sorry for that just strike out what I have told earlier. This is total number of neutrons, if we consider total number of neutron is 1; then out of that 1 minus beta comes from prompt neutron and beta comes from delayed neutron.

So, therefore, total the rate of fission caused by prompt neutron should be the number of fission reaction that is happening because of the prompt neutron is 1 by beta into sigma ac into phi thermal this is a number of prompt neutron that get absorbed and is about to cause a fission reaction. And once we multiply this with the multiplication factor then that gives you the number of prompt neutrons produced because of fission. And the second production part that is a decay of delayed neutron precursor is this here lambda is the decay constant for the i th group of precursor the average decay constant and C is the concentration of that i th group and finally, we have the absorption rate here.

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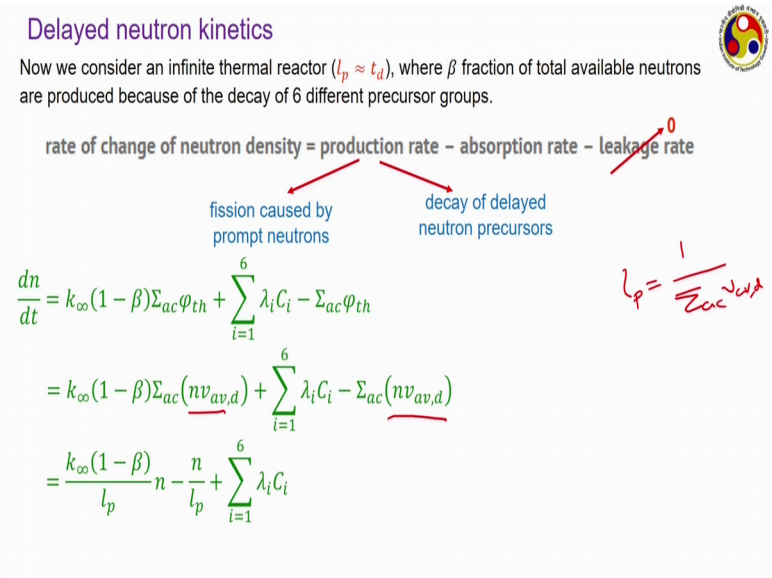

Now, phi we know can be written as the neutron density into the average velocity of neutrons. So, this change has been done here; phi has been replaced by n into v absorption for v average for this diffuse neutron. And now we know that lp is equal to 1 upon sigma ac into v average d. So, that has been introduced here to write or replace the product of sigma ac into v average with 1 upon lp.

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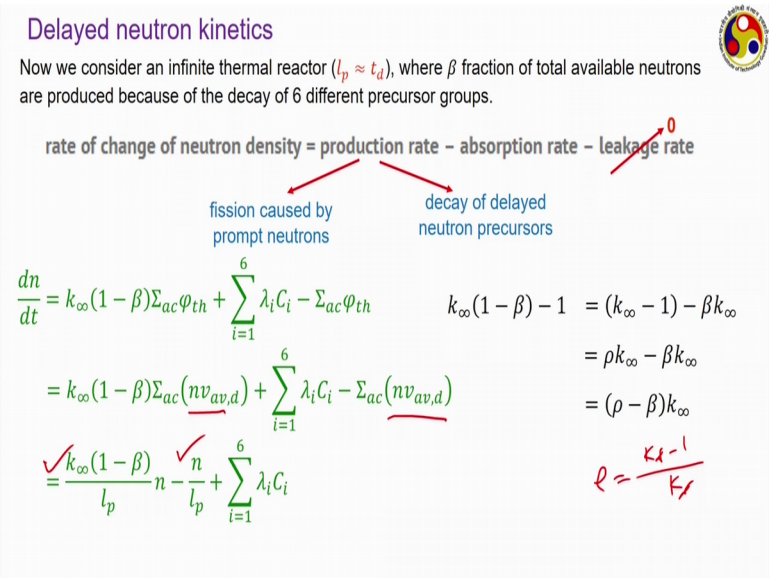

Now, little bit of mathematics we are taking this numerator here and also combining that with this.

So, k infinity minus k infinity to 1 minus beta minus 1 can be written like this and what is the definition of rho reactivity? Rho we know can be is defined as k infinity minus 1 by k infinity. So, that substitution has been done and accordingly we get this to be rho minus beta into k infinity. So, if we substitute that into our equation.

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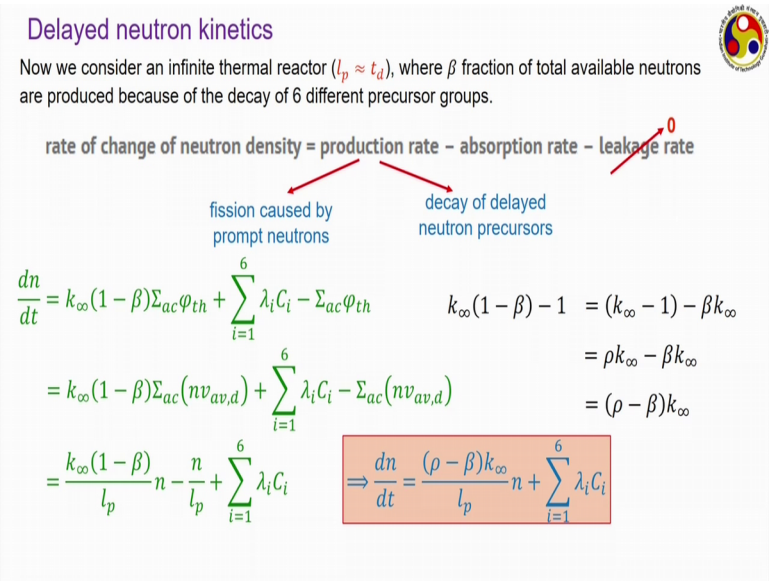

Then this is the final equation for the neutron density that you are getting. Here on the left hand side dn dt represents the rate of change of neutron, in the right hand side first one combines the production from the fission over from neutrons and also the absorption and the second term corresponds to the decay of the delayed neutron precursors.

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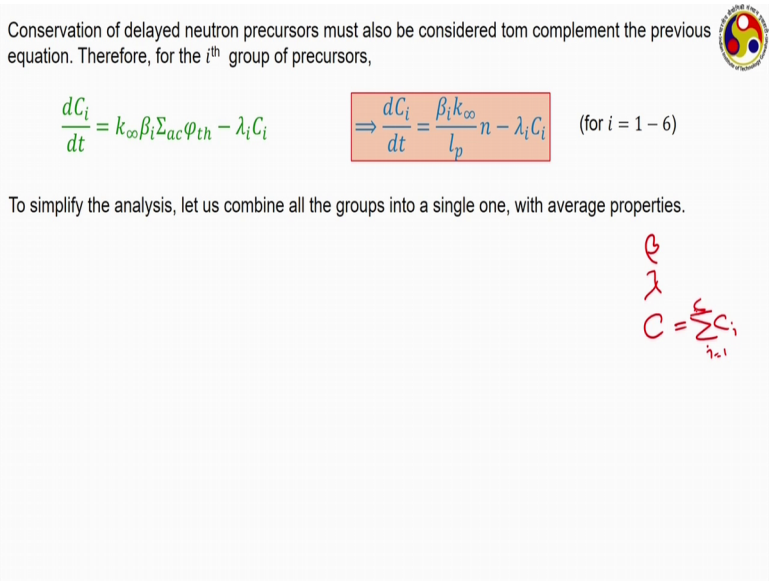

Now, a delayed neutron conservation of the delayed neutron precursors must also be considered. Because here this C term is unknown while others are generally properties we also need to know this C and that is possible only if we consider conservation of this delayed neutron precursors.

So, for the i th group of precursors we can write this conservation equation had the left hand side represent the rate of change in the concentration of the i th group. And on the right hand side the first term represents the rate of production of delayed neutron precursors that I th group because beta is the delayed neutron fraction; for that i th group and lambda i Ci represents the decay rate which we have used in the previous slide also.

And so, we can rewrite this thing into this form where I can vary from 1 to 6. Actually if we consider 8 group model then I will vary from 1 to 8, but generally 8 group models quite complicated in 6 group itself we have to release 6 different precursor groups and their properties in 8 group a complexity is even more. So, we generally are sticking to the sixth group model, but here we are going to do a very simple analysis; if you are dealing with 6 group models then you have to consider that is there was one equation for

neutron density and now there are 6 equation for the concentration of the delayed neutron precursors.

So, there are there was 7 equations in the framework which is are not possible to solve in simple analytical framework of a class. So, we are combining all these 6 group to groups together into a single group. a single group we where the total delayed neutron fraction is beta decay constant is lambda and total concentration is C; remember C refers to the summation of Ci where i is varies from 1 to 6. And similarly the lambda and beta can be calculated as the average quantities for all the 6 groups together.

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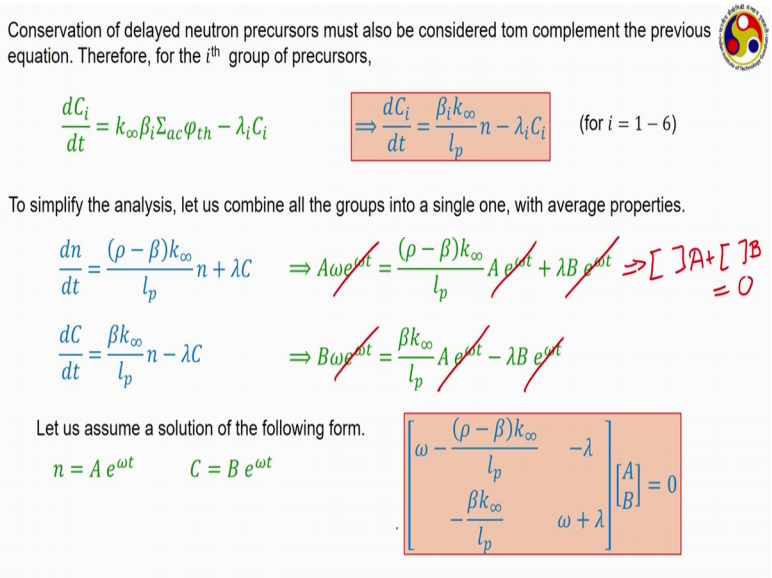

So, these are the two equations no doubt the change; here this is the change where instead of the summation we are writing as lambda into C where lambda is the average decay ratio for all the delayed neutron groups together. And C is their concentration and the second equation is the counterpart of this one. Now we assume a solution of this particular form n equal to A e to the omega t and C is equal to B into e to the power omega t. Here A and B are two coefficients and omega is a parameter which we are looking to identify.

So, once we put it into the first equation then it reduces to form like this; you please try to do it on your own. And the similarly for the second equation now e to the power omega t can be strike out from both sides of both the equations. And now we take everything on one side and we combine it into a form like this into A plus this into B is

equal to 0. Similarly, for the other equation also and we arrive at this particular matrix form this; where we have omega lambda rho and beta and also k infinity involve and definitely you have the prompt neutron lifetime lp.

So, this one will I help us to identify the expressions for A B and also omega the solution of this one is the thing with which we shall be starting our next lecture. So, hopefully you have understood the discussion here, today you have discussed about the concept of prompt neutron lifetime and you have also discussed about the role of delayed neutron.

We have seen that if the neutron kinetics only govern by prompt neutrons, then we have extremely small time to operate any kind of control devices, but as we shall be seeing in the next lecture; it is only because of the delayed neutron we are allowed to have larger amount of time and hence we can control a nuclear reactor you have quite easily. So, that is it for today.

Thanks for your attention.