

Fundamentals of Nuclear Power Generation
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Module – 05
Reactor Thermalhydraulics
Lecture - 02
Axial temperature distribution & heat transfer coefficient

Hello friends, we are back with the 5th module of our MOOC course where we are discussing about the reactor thermal hydraulics and today in this second lecture we are trying to wrap up this particular chapter. As was discussed in the previous module or rather previous lecture, the term thermal hydraulics relates to the fluid flow and heat transfer behaviour and hence in this particular module we are focusing on the heat transfer aspects inside the reactor.

That is the amount of energy which has been released because of fission reaction; how that can get transferred to the coolant which is flowing and it is around the fuel rods or fuel elements that we are discussing here and what are the corresponding temperature profiles? That is in earlier primary importance or primary interest in this particular chapter. So, if we quickly revisit what we have done in the previous lecture?

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Lecture 1 revisited

- ✓ Homogeneous & heterogeneous nuclear reactor
- ✓ Nuclear fuel & cladding
- ✓ General energy consideration

$$\dot{q}''' = \int_0^{\infty} E_R' \Sigma_f(E) \phi(r, E) dE$$

$$\dot{q}''' = \dot{q}_{max}'' J_0 \left(2.405 \frac{r}{R} \right) \cos \left(\frac{\pi z}{H} \right)$$

- ✓ Heat transfer analysis

We discussed about the Homogeneous and heterogeneous nuclear reactor in a homogeneous nuclear reactor; we have a uniform distribution of fuel and moderator basically in most of the homogeneous nuclear reactors are of aqueous solution type where some aqueous solution of nuclear salt like uranium sulphate or uranium nitrate is present and if we take a sample from anywhere in the reactor.

We are going to get the same kind of chemical composition as while homogeneous nuclear reactor has several advantages, but there are quite a few disadvantages also particularly from kinetics and control point of view and that is why the use of homogeneous reactors particularly the aqueous homogeneous reactors are restricted to small scale applications such as a medical isotope separation or hydrogen production by radiolysis and for power generation purpose, we almost universally use heterogeneous reactors where fuels and moderators are placed as discrete elements generally following some kind of regularized pattern.

We have also discussed about different properties of fuel and cladding and now we know what are the desirable properties that we need to have for the fuel like: high thermal conductivity corrosion resistance characteristics good mechanical strength and deliberate temperature cladding has a big role to play in the last point that is cladding size some kind of jacket which covers the fuel. Thereby providing its structural solidity and the cladding material should also have very low absorption cross section and high thermal conductivity.

Sometimes it can also provide some additional or extended heat transfer surfaces particularly when we are using some kind of gas as a coolant then we go to the general energy consideration and there we have seen that while generally more than 200 MeV of energy is produced for every fission reaction we can get only about 75 to 80 percent of that transfer to the coolant. While remaining goes in the form of radiation primarily the volumetric energy generation can be related by relation like; this where ϕ is actually that effective energy that is available from a reaction and ϕ represents the neutron distribution inside the reactor.

Actually this is of course, at a particular location this integration is done. So, you know you can also say this volumetric generation that we are writing this is actually a function of r and then we discussed a specific case for a cylindrical reactor which contains a large

number of a fuel cylindrical fuel rods capital R is the radius of that reactor and capital H is the height of that cylinder and accordingly we got this particular solution for this heat generation and this is functions seem to be a function of both r which is the radial direction coordinate and z which is the axial direction coordinate.

And finally, we have done the heat transfer analysis for two simplify geometries commonly in nuclear reactors fuels are presents either in the form of rectangular plates, very thin rectangular plates and also cylindrical fuel rods. This rods generally are such small diameter they are often referred as fuel pins. So, we have done quite simplified heat transfer analysis for both the cases by assuming constant fuel fluid sorry constant properties for both fuel and cladding.

And also very important assumption by assuming uniform rate of heat generation throughout the reactor and this was the solution that we have obtained for a plate type reactor; where we can we have calculated this temperature difference that we may have across a fuel and also a temperature difference we may have across the cladding. This gap situation we have not analyzed, but practical reactors may have a gap also the air gap between the fuel and the cladding.

And also we have derive the expression for this film temperature difference which is actually the temperature difference across the coolant boundary layer on the surface of the cladding; and we have also got by assuming some temperature value for the fuel centreline we have got expressions for the temperature at the fuel cladding interface at the cladding outer surface and also for the bulk of the fluid and we have seen that primarily.

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Thermal resistance concept

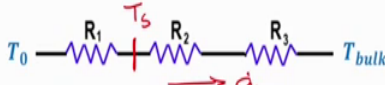


Plate-type fuel: $\frac{a}{2Ak_F}$, $\frac{a}{Ak_{cl}}$, $\frac{1}{Ah}$

Cylindrical fuel rod: $\frac{1}{4\pi Hk_F}$, $\frac{\ln \frac{a+b}{a}}{2\pi Hk_{cl}}$, $\frac{1}{2\pi(a+b)Hh}$

$$\dot{q} = \frac{T_0 - T_{bulk}}{R_1 + R_2 + R_3}$$

Hence, total resistance for a fuel rod $R_{tot} = \frac{\Delta T}{\dot{q}} = \frac{1}{4\pi Hk_F} + \frac{\ln \frac{a+b}{a}}{2\pi Hk_{cl}} + \frac{1}{2\pi(a+b)Hh}$

In practical reactors, invariably $b \ll a$

$$\Rightarrow \ln\left(1 + \frac{b}{a}\right) \approx \frac{b}{a}$$

$$\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} \dots$$

If we neglect the gap between the fuel and cladding we can draw a simple electrical energy for that situation, where T_{naught} is a fuel centreline temperature which can be thought about as a source of energy and T_{bulk} is the bulk fluid temperature and in between there are three resistances through which the heat flux or rate of heat transfer whatever we would like to consider, that is; flowing through the $\dot{q}$ is the rate of heat transfer and R_1 , R_2 , R and R_3 are the three resistances R_1 is the fuel thermal resistance, R_2 corresponds to the cladding thermal resistance both R_1 and R_2 .

Are actually conductive resistance because fuel and cladding generally being solid in nature a liquid fuel can of course, be there as we have discussed in the last lecture, but liquid fuel is primarily used in homogeneous reactors whereas, the heterogeneous reactors primarily use solid fuels only.

So, both fuel and cladding are solid materials and hence R_1 and R_2 both refer to conduction resistances only R_3 rather refers to the convective resistance, because of the convective heat transfer; that is, going from the going between the coolant outer surface and the bulk of the fluid. These are the two geometry that we have considered; the plate type fuel and a cylindrical fuel rod and these are the expressions that we got for the fuel resistance we just derive these expressions without discussing into the details.

We can clearly see that the resistance for both the cases that inversely proportional to the thermal conductivity of the fuel which is a very logical very actually very logical way

observation higher the thermal conductivity. Hence the smaller will be the value of this resistance and therefore, the smaller will be the temperature difference across the fuel or the temperature difference between T_{naught} and T_s which is the surface temperature at the fuel cladding interface.

The difference between T_{naught} and T_s will progressively be smaller as the thermal conductivity of fuel keeps on increasing that is why metallic fuels are highly preferable. So, that we can keep the temperature differential small, but metallic fuel pure metallic fuels or metallic alloys have their own disadvantages like lower melting points etcetera and if we use some oxide fuel or ceramic fuels thermal conductivity will be quite low and we may have significant amount of temperature change across the fuel, then the second group of resistance which is associated with the conduction through the cladding; that is, also proper inversely proportional to the thermal conductivity of cladding.

Now, cladding generally uses some kind of metallic element like: metallic, alloys like: magnox or zinc alloy or maybe stainless steel which have quite reasonable values of the thermal conductivity and hence the temperature will change across the cladding itself is.

Generally, very small and finally, we have the conduction sorry the convection resistance for the associated by heat transfer from a cladding outer surface to the bulk of the fluid and this as expected is inversely proportional to the convey corresponding convective heat transfer coefficient.

So, for any of the elements we can always write that the rate of heat transfer is equal to the effective temperature difference which is T_{naught} minus T_{bulk} divided by the corresponding resistance and the; if we consider the case for a fuel rod, then the total resistance has to be equal to ΔT by q dot is the summation of these three components the first one refers to the conductive resistance to the fuel.

Second one is the same for the cladding and third is of course, the conductive resistance conductive film resistance we should say. So, we can take $2\pi H$ outside the bracket and we get a simplified form for most of the electrical reactors; however, the thickness of the cladding is extremely small compared to the diameter of the fuel itself or radius of the fuel itself fuel things are also having quite small diameter, but size of the cladding can be even significantly smaller than that.

So, whenever we are dealing the situation where b is extremely small compared to a we can perform a simple analysis we know that $\log$ of one plus x can be extended as if x is sufficiently small.

We can always write this as x minus x square by 2 plus x cube by 3 minus dots. So, in this case also when b becomes smaller than a we can write $\log$ one plus b upon a to be nearly equal to b upon a here we are neglecting all the terms from second order onwards and accordingly this equation simplifies to you have simpler form like this.

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Thermal resistance concept

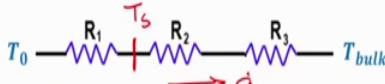


Plate-type fuel: $\frac{a}{2Ak_F}$, $\frac{a}{Ak_{cl}}$, $\frac{1}{Ah}$

Cylindrical fuel rod: $\frac{1}{4\pi Hk_F}$, $\frac{\ln \frac{a+b}{a}}{2\pi Hk_{cl}}$, $\frac{1}{2\pi(a+b)Hh}$

Hence, total resistance for a fuel rod $R_{tot} = \frac{\Delta T}{\dot{q}} = \frac{1}{4\pi Hk_F} + \frac{\ln \frac{a+b}{a}}{2\pi Hk_{cl}} + \frac{1}{2\pi(a+b)Hh}$

In practical reactors, invariably $b \ll a$

$\Rightarrow \ln \left(1 + \frac{b}{a} \right) \approx \frac{b}{a}$

$\dot{q} = \frac{T_0 - T_{bulk}}{R_1 + R_2 + R_3}$

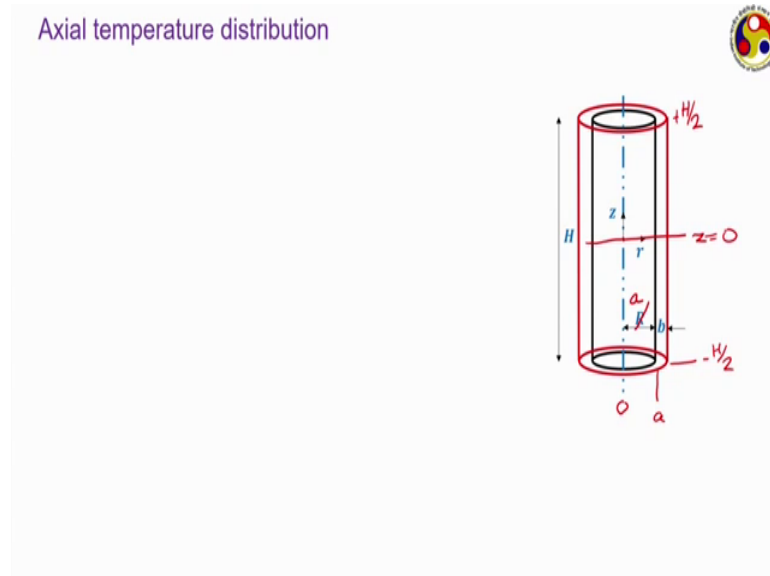
$\approx \frac{1}{2\pi H} \left[\frac{1}{2k_F} + \frac{1}{k_{cl}} \ln \left(1 + \frac{b}{a} \right) + \frac{1}{(a+b)h} \right]$

$\approx \frac{1}{2\pi H} \left[\frac{1}{2k_F} + \frac{b}{ak_{cl}} + \frac{1}{(a+b)h} \right]$

Now, this total resistance is quite important knowledge to have, because of course, we have done this analysis using a very simplified form of the heat generation. We had assume the uniform heat generation throughout the entire fuel pin or fuel plate element.

However, that is not true in practice there is always variation in the heat flux in both radial and axial direction for a fuel rod, but generally the dimension in the radial direction is so small compared to that in the axial direction, we can invariably take the temperature variation in the radial direction to be negligible and consider only the axial directional variation and in that case this consists of thermal resistance is very important we shall be seeing that in this particular case.

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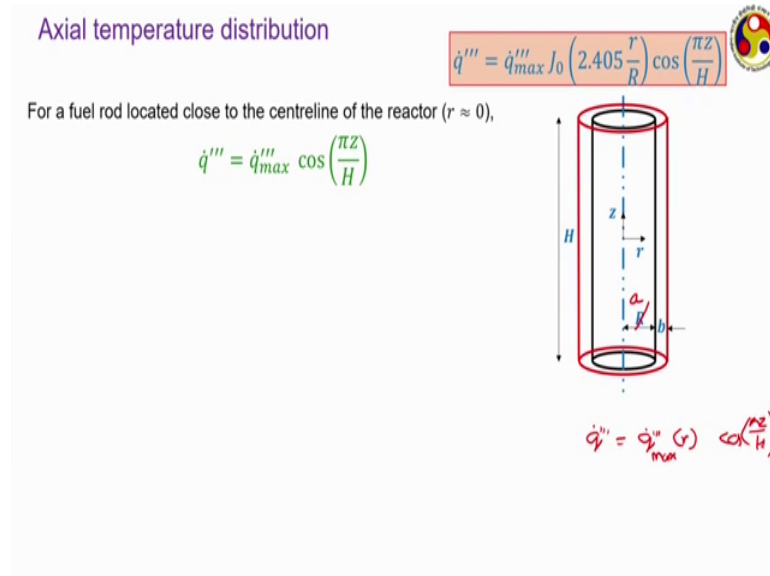


Here our geometry is that of a cylindrical reactor here actually the diagram is not perfectly placed you can say this is a fuel pin that I have shown here. A fuel pin which is having this R is wrong which is having a radius of a and the corresponding thickness of the cladding is b .

And here now instead of one we are taking two dimensions r is the radial direction and z is the axial direction and our centre or the origin of the coordinate system is located at the centroid of this particular fuel pin, that is; we can say that the extent of the fuel pin in the radial direction is from r equal to 0 to R equal to a whereas, that in the axial direction is from some minus H by 2 to plus H by 2 as H is the total height.

And this particular location or sorry let me correct this particular location refers to z equal to 0 and now this is the fuel pin that we have and we are placing this fuel pin inside a large cylindrical reactor of radius capital R and height as the same height of the of each fuel element which is capital H .

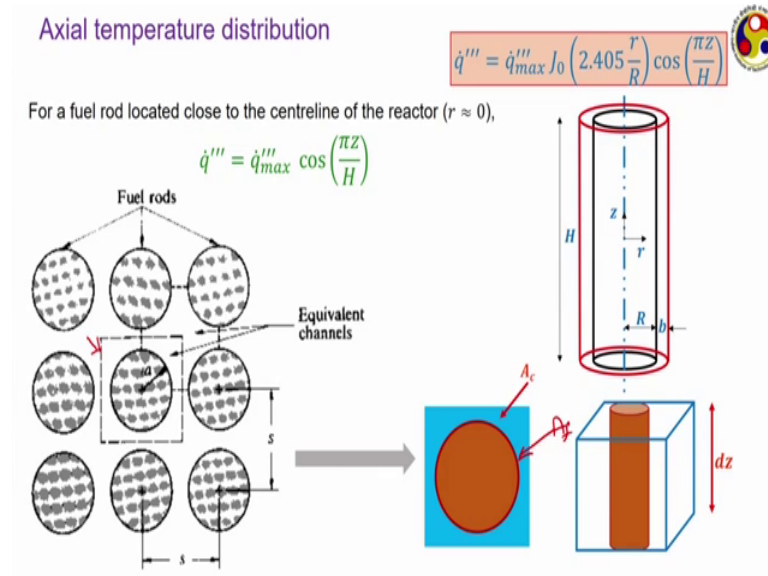
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Now, we have earlier analyzed such a situation and seen that the; in the last lecture we have seen that corresponding distribution of the volumetric heat flux can have a form like this. Now, if we apply this equation to a fuel rod which is located just at the centre of the coordinate system just like this one here; that means, it is located at a position with r equal to 0 and. So, inherently we can reduce this one to a form like this; that means, the heat flux becomes a sole function of z .

But this is applicable only for the fuel rods which are located quite close to the origin that is that r equal to 0 if we are talking about a location which is not very close to the centreline, then q then in that case we can also write q dot triple prime is equal to q dot triple prime max which is a function of r into this $\cos \phi z$ by H where this q dot max will keep on varying in the radial direction and for different radial location we are going to get different value for this maxima, but presently let us restrict ourselves to a fuel rod located close to the centreline very close to a centreline.

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So, that the r dependence can be neglected and hence for that particular fuel element this heat generation becomes a sole function of the axial directional coordinate which spans from minus H by 2 to plus H by 2; now in a practical reactor quite often you will find that the fuels are having an arrangement like this they generate crystalline lattice like this where these are these circles are all fuel rods like 9 circles are shown here each of them is each of them is a fuel rod and all of them are immersed into the large expansion of coolant.

So, there is no excessive coolant channel and the coolant is only flowing to the interval spaces between this fuel pin. Now, we instead of considering all this coolant channel or all this fuel and coolant flow around that let us just consider one and use the corresponding symmetric condition with respect to the other.

So, here each fuel rod is having an the having an radius of a and around each of them we have considered a square of size s where s is the spacing between two neighbouring fuel rod in both directions.

So, and s is also the size or dimension of this particular square block that we have selected. So, we can be visualized to have a form like this where we had our domain reduces to a square block which is being filled up with water and (Refer Time: 15:18) or whatever coolant we are maybe using and also there at the say around the centre end of that we are having a circular fuel pin which is having a radius of a this is the view from

the other direction here, let us consider an inoffensively small portion of this particular fuel rod and surrounding coolant which is having a height of dz .

So, if we apply an energy balance whatever amount of energy that has been produced by this particular fuel element over which is having let us say the fuel element is having cross section area of f and its height is dz . So, over the volume of a into dz whatever amount of energy the fuel releases the intern amount of energy can be visualized to be consumed by the fluid which is restricted in this area A_c .

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Performing energy balance over this differential element of coolant,



$$\begin{aligned} d\dot{q} &= \dot{m}_c c_p dT_b \Rightarrow dq'''(A_f dz) = \left(\rho A_c \frac{dz}{dt} \right) c_p dT_b \\ \Rightarrow dT_b &= \frac{dq'''}{\dot{m}_c c_p} (\pi a^2 dz) \\ \Rightarrow dT_b &= \frac{\dot{q}_{max}''' \cos\left(\frac{\pi z}{H}\right)}{\dot{m}_c c_p} (\pi a^2 dz) \\ \Rightarrow \int_{T_{b0}}^{T_b(z)} dT_b &= \frac{\pi a^2 \dot{q}_{max}'''}{\dot{m}_c c_p} \int_{-H/2}^z \cos\left(\frac{\pi z}{H}\right) dz \end{aligned}$$

$\beta = \frac{\pi z}{H}$

$$\Rightarrow T_b(z) = T_{b0} + \frac{a^2 H \dot{q}_{max}'''}{\dot{m}_c c_p} \left[1 + \sin\left(\frac{\pi z}{H}\right) \right] = T_{b0} + \frac{\Delta T_b}{2} [1 + \sin(\beta z)]$$

Hence total temperature rise of coolant, $T_{b,max} - T_{b0} = \Delta T_b = \frac{2a^2 H \dot{q}_{max}'''}{\dot{m}_c c_p} = \frac{2V_{rod} \dot{q}_{max}'''}{\pi \dot{m}_c c_p}$

Accordingly, we may write this particular energy balance equation, but $d\dot{q}$ is the amount of energy released by the fuel element over the small portion of dz and this is the corresponding temperature rise of this fluid inversely is a corresponding mass flow rate and dT_b is the temperature difference of the temperature difference the fluid experiences. While passing through this inference in a small time of dz .

Now, $d\dot{q}$ can be visualized as the instead of writing power we can consider this one to be a volumetric power multiple of the total area of the heat transfer surface. So, here the heat transfer is taking place from the fuel towards the surrounding coolant and in this case the effective area has to be the fuel surface area which is A_f and now on the right hand side we can expand this mass flow rate as the product of ρ and the corresponding cross section area of the channel in end the velocity.

But it is not necessary also, if we rearrange that using the original form, then the temperature difference of the fluid across the channel comes out to you in this particular form and, then if we put; now the expression for the heat flux; which is of course, as we have neglected any kind of radial variation.

So, the only variation that remains is in the axial direction which is following cosine function and therefore, this temperature difference is also found to be a cosine function; we need to integrate this function from the inlet of the channel till the end of the channel or maybe till some intermediate location.

So, that we can calculate the temperature of the coolant at that particular location so, this way we are performing the integration from some minus $H/2$ which refers to the inlet of the channel to z which can be some arbitrary location inside the channel and corresponding temperatures are T_b refers to the temperature at the location $H/2$ or I should say minus $H/2$. The T_b refers to the temperature of the coolant at this particular location which can be visualized as the inlet to the channel.

So, after performing this integration; this is the expression that we are going to get where we are having T_b the inlet temperature for the coolant and then some constant terms and finally, a sinusoidal component here the total temperature rise of the coolant can be obtained by putting $H/2$ in the above equation. And hence that again brings some sort of this again it is some sort of constant as you can see, if we do not consider the very radial variation in the maximum heat flux other terms all are constant and here this numerator can also be replaced with the term V_{rod} where V_{rod} refers to the volume of each fuel rod. Now, it can be a volume of each fuel rod.

There should be the cross section area of the fuel rod into H and its cross section area is πa^2 into H . So, the term $\pi a^2 H$ in this equation can be replaced by the volume of the rod divided by π which exactly we have done here.

This is the total change in the coolant temperature during phases from the phases to the reactor and, if we use this expression for this total temperature rise, then the temperature profile for the coolant can also be written in a form like this and also the term β that we are using as an abbreviation that is nothing, but πH .

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For a cylindrical fuel element, $\Delta T_{film} = T_c - T_{bulk} = \frac{\dot{q}}{2\pi(a+b)Hh}$

Hence, $T_c(z) = T_{b0} + \frac{\Delta T_b}{2} [1 + \sin(\beta z)] + \frac{\dot{q}}{2\pi(a+b)Hh}$

$\theta_c = \frac{a^2 \dot{q}_{max}'''}{2(a+b)h}$

$= T_{b0} + \frac{\Delta T_b}{2} [1 + \sin(\beta z)] + \frac{\dot{q}_{max}''' \cos\left(\frac{\pi z}{H}\right) (\pi a^2 H)}{2\pi(a+b)Hh}$

Similarly,

$T_s(z) = T_{b0} + \frac{\Delta T_b}{2} [1 + \sin(\beta z)] + \theta_c \cos(\beta z) \left[1 + \frac{(a+b)h}{k_{cl}} \ln\left(1 + \frac{b}{a}\right) \right]$

$T_0(z) = T_{b0} + \frac{\Delta T_b}{2} [1 + \sin(\beta z)] + \theta_c \cos(\beta z) \left[1 + \frac{(a+b)h}{k_{cl}} \ln\left(1 + \frac{b}{a}\right) + \frac{(a+b)h}{2k_F} \right]$

So, for a cylindrical fuel element, now we know that the temperature difference across the coolant film, that is; temperature at the cladding surface minus the wall free temperature is something like this. Now, if we use the concept of the this is the concept of the thermal resistance.

So, if we put the expression for T_c or rather T_{bulk} into this which we have just obtained the previous slide here this, then we can get this particular expression which here which can be simplified by putting the expression for actually not simplified; we can explain this the $\dot{q}$ dot.

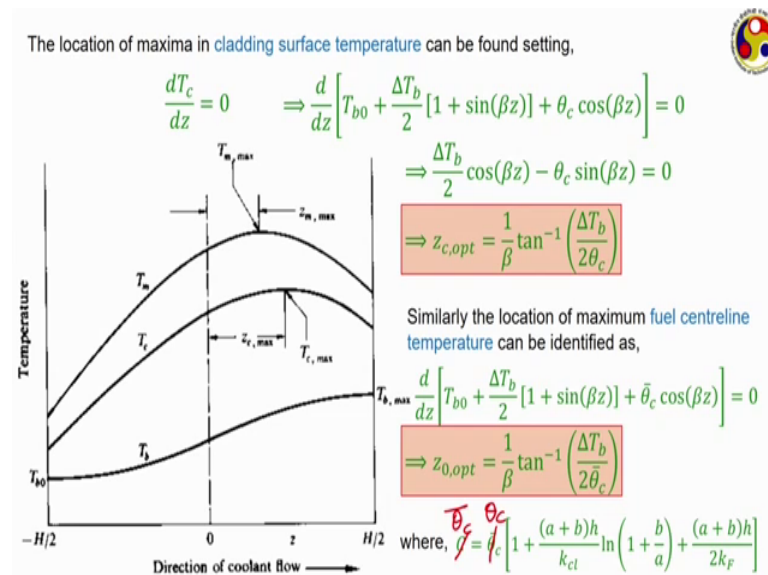
The rate of heat transfer that is there that is been replaced by corresponding cosine term and finally, we arrive at this that is the variation of coolant temperature. In the axial direction can directly related to the coolant inlet temperature and in the sine and cosine term with beta in as one of the coefficients or beta in between the bracket where this theta c is actually a shortened form that we are writing which is equal to a square into q triple prime dot max divide by twice a plus b into h which we are used only to give this equation slightly shorter.

Similarly, we can also calculate the temperature at the interface between the fuel and cladding and also you can calculate the fuel centreline temperature that is exactly that has been attempted to this is the centreline temperature here T_{sz} refers to the temperature. The interface between fuel and cladding and we get a much bigger

expression for this like here we have two different resistances coming in this one corresponds to the film resistance.

And this one corresponds to the cladding resistance and finally, if we write the expression for $T_{naught z}$ which is actually the temperature at the centreline of the solid then we get the same form, but now we have three resistances well this one corresponds to the cladding this one corresponds to the conductive resistance for the fuel and this is the typical temperature variation; that we may find cylindrical reactor generally well this one is for the coolant.

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This is the inlet of the channel this is the exit of the channel and this is the centreline in the axial direction. So, as we move towards the exit that coolant is getting more and more energy increase the amount of energy, because of the fission energy release and hence the cooling temperature continues to increase unless of course, it reaches some threshold value.

However, that is not true for the other two temperatures of interest one is the centre fuel centreline temperature and other is the cladding surface temperature this T_m refers to actually fuel centreline temperature as for our terminology you can see both the cases.

Both are both the profiles show a maxima. So, the location of this maxima for the cladding surface temperature can be found by setting that a d/dz of cladding temperature

to be equal to 0 which reduces to form like this and after performing the differentiation, we are having this and finally, we get the location of this optimum looking the optimum cladding surface temperature.

Similarly, we can also calculate the location of the optimum fuel centreline temperature that is by setting $\frac{dT}{dz}$ equal to 0 and corresponding form is this and we get z centreline optimum value of the centreline equal to $\frac{1}{\beta} \ln \left(\frac{\tan^{-1} \Delta T_b}{2 \theta_{bc}} \right)$. Actually, is a shorthand notation, because as there are quite a few terms involved three different resistances involved and during the calculation for the fuel.

We once you consider them in a short notation of actually; there is a typing error here this is this θ_{bc} that I refer to and this should be θ_c that we are using from the earlier slide onwards. So, this way we can calculate the optima for both the profiles if you want you can also calculate the optimum location for the temperature.

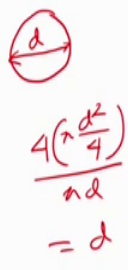
Where the temperature is maximum at the fuel cladding interface, but generally that is not of a great interest or major interest is to know where the fuel centreline temperature is maximum that we can obtain here and by putting these expressions for z_{optima} in the corresponding temperature expression.

We can also get the value of this maximum temperatures like the maximum cladding surface temperature or maximum fuel centreline temperature or maximum temperature value at the interface of the fuel and cladding, but those generally are quite big expressions that is, why? They have been avoided here next we need to know about the heat transfer coefficient.

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Heat transfer coefficient

For most of the common coolant (including water, D₂O, organic liquids & gases), heat transfer coefficient can be correlated to the following form.

$$Nu = \frac{h d_{eq}}{k} = C Re^m Pr^n$$
$$Re = \frac{u d_{eq}}{\nu} \quad d_{eq} = \frac{4 A_c}{\Pi}$$


The diagram shows a circular pipe with diameter d . Below it, a handwritten derivation shows the equivalent diameter d_{eq} for a circular pipe, which is equal to the actual diameter d .

We know that the generally there are three resistances, that we have seen one important thing is that the analysis that we have just done while the concept of fuel terminal the thermal resistances were developed following an uniform; heat transfer distribution or uniform value of the heat generation throughout the reactor, but when we are having some kind of distribution for the heat generation in the radial and axial direction.

Both we can still use the concept of those resistances and two of them being conductive resistances they are very standard, because the thermal once we have selected the material, we know this thermal conductivity and corresponding resistances values as once we fix up the dimensions as well correspondence resistance value are fixed, but the convective resistance depends on the heat.

Transfer coefficient and the heat transfer coefficient may depends on infinite number of factors like it may depend on what is nature of the coolant? What is the nature of the cladding outer surface? That is a surface where the solid and fluid are having some kind of interaction. What is the nature of the flow itself? Whether it is laminar or turbulent and similarly several other factors, but commonly for any general coolant like water or heavy water or organic liquids or gaseous heat transfer coefficient can have a form like this; or Nusselt number can always be expressed as a function of Reynolds and Prandtl numbers.

Here, Reynolds number is defined as form like this you can see here; u is the velocity of the coolant and ν is the kinematic viscosity of that fusion generally corresponding with the average temperatures and other term that you can find in the definition of both Reynolds number, Nusselt number is d_{eq} , which actually is a suitable length scale and it is also called the equivalent diameter d_{eq} or hydraulic diameter also is a much very common name.

Now, equivalent diameter has a definition of $4 A_c$ by π where A_c is the corresponding cross sectional area and π is the weighted perimeter. Now, if we are dealing with a channel with circular cross section say your geometry is that of a circular channel and through which the coolant is flowing.

Then calculation of this d_{eq} is very easy. In this case if d is the diameter of this one, then d_{eq} is 4 into the area of this one area should be πd^2 by 4 and divided by what should be the weighted perimeter for this is π into d .

So, it comes equal to d , that is; when you are having dealing with circular channel this is just reducing to the diameter of that channel, but when you are dealing with non circular dot with channel, then we have to find a way of calculating this d_{eq} like this one is the common representation of fuel and fuel rods and coolants inside the reactor where we are having circular fuel rods each of radius a and we can select a square of size s around each of this here s is the spacing between successive fuel rods.

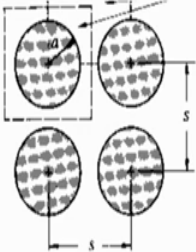
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Heat transfer coefficient

For most of the common coolant (including water, D_2O , organic liquids & gases), heat transfer coefficient can be correlated to the following form.

$$Nu = \frac{h d_{eq}}{k} = C Re^m Pr^n$$

$$Re = \frac{u d_{eq}}{\nu} \quad d_{eq} = \frac{4 A_c}{\pi}$$

$$d_{eq} = \frac{4(s^2 - \pi a^2)}{2\pi a}$$


$\frac{4(s^2 - \pi a^2)}{2\pi a}$

So, this particular portion; this hash portion can be viewed to be occupied by the coolant. So, like we have done we have considered earlier in our earlier analysis.

In this particular case we can calculate equilibrium the dimension to be something like this here; what is the area for each such dot a square is the area of the square and the area occupied by the coolant will be this minus the area of corresponding circular fuel rod.

So, this pi a square this multiplied by the 4 and what will be the corresponding weighted area that has to be the parameter of this particular circle our circular fuel rod which is $2\pi a$, that is; precisely what is written here if your geo this is the most common geometry, but in a geometry something else we can the same way calculate corresponding equilibrium lengths equivalent diameter or the length scale to identify Reynolds number and Nusselt number.

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Heat transfer coefficient

For most of the common coolant (including water, D₂O, organic liquids & gases), heat transfer coefficient can be correlated to the following form.

$$Nu = \frac{h d_{eq}}{k} = C Re^m Pr^n$$

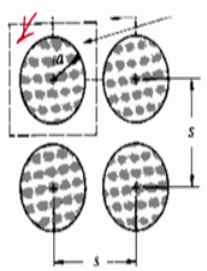
$$Re = \frac{u d_{eq}}{\nu} \quad d_{eq} = \frac{4 A_c}{\Pi}$$

$$d_{eq} = \frac{4(s^2 - \pi a^2)}{2\pi a}$$

Dittus-Boelter correlation: $C = 0.023$
 $m = 0.8 \quad n = 0.4$

For flow of ordinary water through a lattice of rods,

$$C = 0.033 \left(\frac{V_w}{V_w + V_f} \right) + 0.0127$$

$$m = 0.8 \quad n = 1/3$$


There are several ways we can put the values of this coefficient C and this two exponent's m and n. One can be the age old Dittus-Boelter correlation, where C is 0.023 m equal to 0.8 and this is being a heating case n equal to 0.4, but there are several others experimental correlations here we have to depend upon experimental correlations to identify the expression for Nusselt number for ordinary flow for flow of ordinary water through a lattice of rods.

This is one correlation that sometimes is used in nuclear thermal hydraulics where you can see m equal to remains the same, that is; 0.8 and n equal to 1 third, but if C is have been instead of a constant number, it is having some kind of expression here this V w refers to the volume of water, that is present inside.

Such one element that is the square minus circle, whatever is left what is the volume of that? Volume of water that is present there and this is V f, the volume of fuel the fuel rod corresponding to again such each selected module there can be several other collisions as well which are applicable for such kind of coolants.

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With liquid metal as coolant, the correlation proposed by Dwyer is often used for hexagonal lattices.

$$Nu = 6.66 + 3.126 \left(\frac{s}{d} \right) + 1.184 \left(\frac{s}{d} \right)^2 + 0.0155 (\bar{\psi} Pe)^{0.86}$$

$$\bar{\psi} = 1 - \frac{0.942 \left(\frac{s}{d} \right)^{1.4}}{Pr \left(\frac{Re}{10^3} \right)^{1.281}}$$

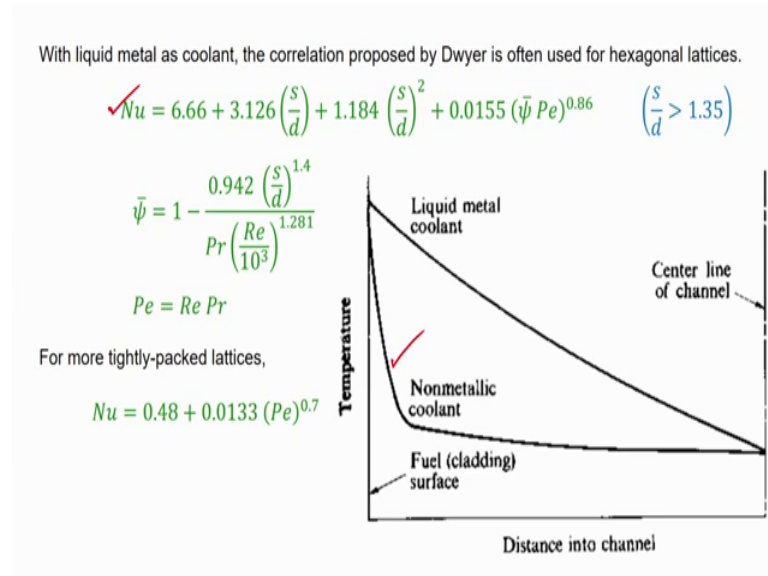
$$Pe = Re Pr$$

$d = 2a$

And if we are using liquid metal as coolant liquid metals have much higher thermal conductivity compared to the other coolants and therefore, those Dittus-Boelter kind of relations are not applicable one correlation; that is, commonly used those proposed by Dwyer.

Here particular for hexagonal lattices is like this here s is the same that is the spacing between neighbouring fuel rods and the d is the diameter of the fuel rods. So, d is actually twice of this a this psi bar is some kind of empirically fitted constant where we can find Prandtl number and Reynolds numbers involved and also you have Peclet number P which is the product of Reynolds and Prandtl number.

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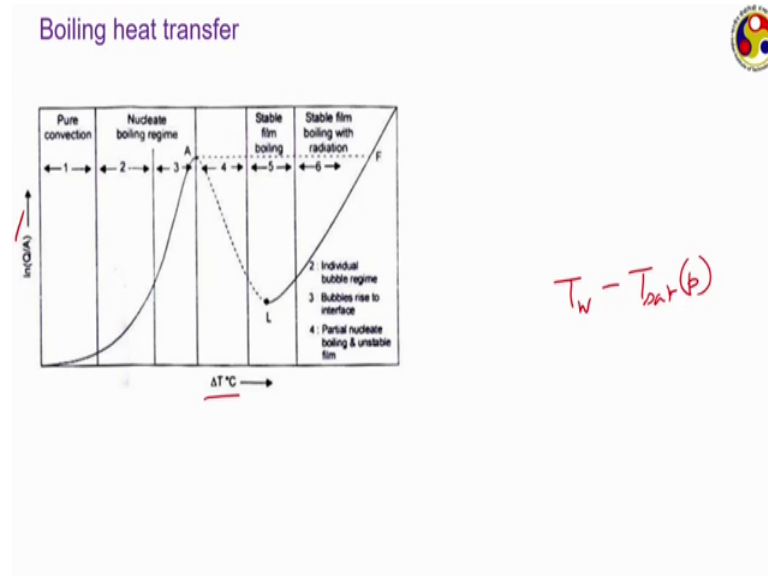
So, this is one correlation that can be used, when we are having liquid metal as the coolant, but this one has a restriction it is applicable only for s/d is greater than 1.35 times the diameter this, and if we are dealing with a channel, where the diameter is much or this s/d ratio is much smaller and much tightly packed lattice this can be one of the relations that can be used; if we are using a lattice which is not hexagonal in nature some kind of say a triangular lattice or a square lattice then this Nusselt number relation can be multiplied with a suitable conversion factor and then the same formula can be used this is a typical nature of the temperature distribution that we get across the liquid film.

When you are having a non-metallic coolant there you can the thermal conductivity being small corresponding resistance is quite large and we can see this is rapid.

Rapid reduction in the coolant temperature quite close to this surface, but if we are using liquid coolant or liquid metal as the coolant, because of high thermal conductivity it increases quite gradually and almost following a straight line kind of relation, but this is applicable these relations are applicable.

When we are dealing with single phase coolant, but there may be similar situations particularly in boiling water reactors where, actually there is a phase change involved inside the reactor that is liquid coolant like liquid water enters the channel. They are after absorbing heat it reaches the corresponding saturation temperature, and then again it can start to get converted to vapour phase; So, in such kind of situation.

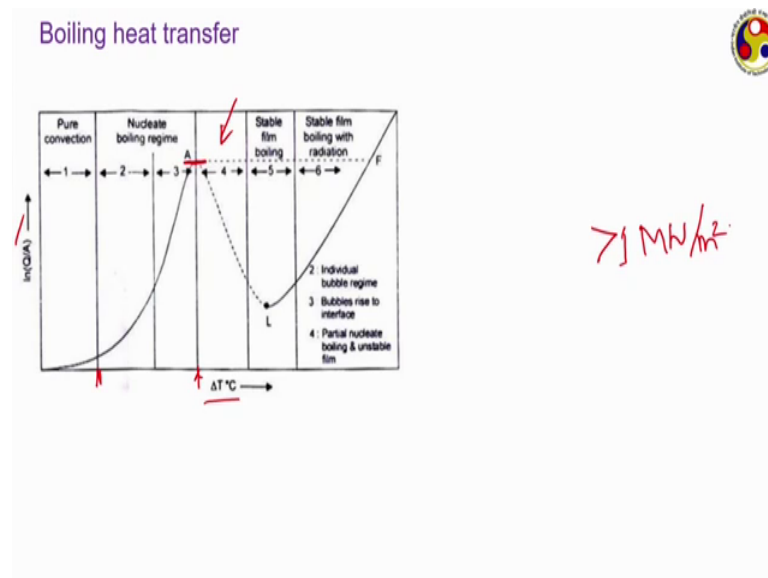
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We need some knowledge on boiling heat transfer. I hope all of you have a idea about this as this is the classical diagram is available in all most every heat transfer book, but just for the purpose of completion. Now, the completion of this one I am mentioning here about the boiling come, that we may get for the pool boiling situation pool boiling refers to where you are heating up a some quantity of fluid in an open container so, that there is no exclusive flow before the initiation of heat transfer. In such kind of case, we get four clear regimes of heat transfer here on the horizontal sorry; the vertical axis.

The heat flux is plot in a lower image scale and in the horizontal axis we have delta T which is actually the wall temperature minus saturation temperature corresponding to the pressure at which this experiment is being conducted. So, initially we get a pure convection zone where liquid is present your single phase liquid and the phase change is yet to started then we enter a nucleate boiling regime; where we have isolated bubble formation at the heating seated surface which after some time may get detached from the surface, and then rise; because of the buoyancy effect then something not marked here.

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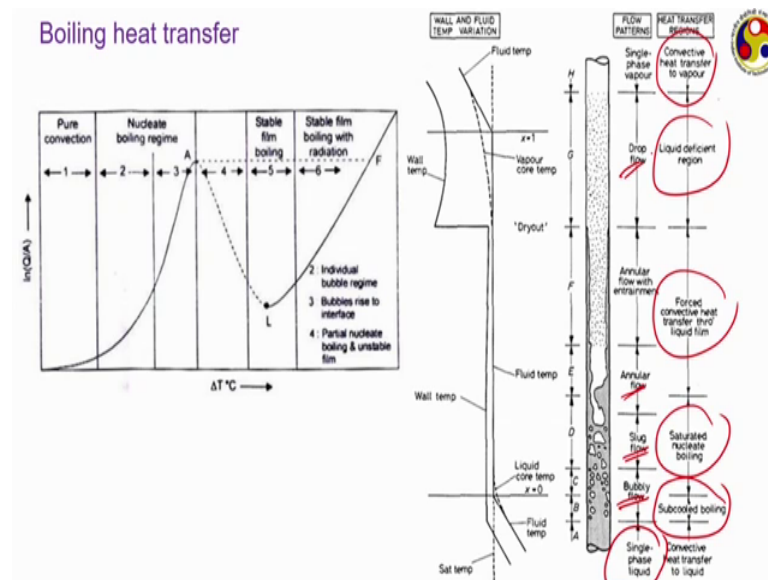
This is an intermediate zone where it is difficult to say exactly; what is happening? And then we have a stable film boiling zone where a film of liquid or I should say a film of vapour is formed on the heated surface. This intermediated zone can have characteristics of both nucleate and film boiling.

This particular point corresponds to the maxima in the maximum heat first corresponds with your nucleate boiling regime and it is hence called a critical heat flux if we are dealing with water and atmospheric condition.

This pure convective zone runs up to 5 to 6 degrees of this delta T whereas, this critical heat flux. Typically, appears around a degree of superheat of 30 degree Celsius and the corresponding heat flux value corresponding to critical heat flux can also be high.

It is invariably greater than 1 megawatt or 10 to the power 6 watt per meter square, because there is a heat flux that we are talking about, but this applies to pool boiling and the situation that we are having here that is not pool boiling rather, that is; flow boiling situation as the liquid is flowing as it is entering the channel; So, in that kind of situation.

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This is again a classical diagram here a liquid is flowing upwards through a heated channel. So, it enters as a single phase liquid here and continues to remain a single phase liquid till location somewhere here where we can see small bubble that may generally gets formed at the surface and as we keep some moving upwards those bubbles keep on increasing in a size. And then they starts getting separated from the walls as we move further these smaller bubbles are generally agglomerate together to form bigger bubbles.

And in case of bigger bubbles we get the slug flow regime and the corresponding bubbles are called slug bubbles or tailor bubbles. These bubbles are generally of a bullet shaped.

That is around having a rounded here and quite flat on the rear side as we move further down this valley flow gives have a to the annular flow regime in the annular flow regime. We have a liquid restricted on into wall and that vapour occupying most of the centre T r part of this of sound of this heated channel and this way it keeps on going in practical reactors.

We generally do not go up to the annular flow regime or maybe it is quite close to that, but we generally do not want to go even beyond, where the entire liquid it is converted to vapour. And we are dealing with something like a single phase vapour zone corresponding heat transfer variation or the wall and fluid temperature variations are shown alongside where the surface temperature that keeps on increasing till some point

here or I should say the surface temperature initially increases, then once the small bubbles starts to appear somewhere here. The surface tension the corresponding wall and fluid temperature variation is shown here.

Initially both the temperature keeps on increasing almost in proportion till small bubbles starts to appear somewhere here the small bubbles starts to appear, but interestingly in this zone the fluid temperature is react to reach the saturation and that is why they are often called the sub cooled boiling zone, sub cooled boiling zone this is a situation where the walls are at a temperature higher than the saturation and that is why small bubbles minsters appearing at the walls.

But, as soon as those bubbles get separated from the walls they come in contact with highly sub cooled liquid and therefore, they gets convinced as the temperature of the bulk fluid keeps on increasing and is about to reach the saturation then some of the sub cooled bubbles may survive in the bulk of the liquid as well.

But once the bulk of the liquid reaches saturation, then we call that saturated boiling. So, once we enters this saturated boiling zone, then the wall temperature remains constant and the fluid temperature also remains constant at the saturation temperature this entire portion is called the nucleate boiling regime and which is analogous to this nucleate boiling regime of pool boiling, but this saturated nucleate boiling continues till the appearance of the annular flow regime, where we have the force convective transfer and finally, the liquid efficient zone where the film on the wall that completely evaporates and when at this particular juncture.

When the liquid film at the wall completely evaporates then the heat that is supplied to the wall it has nowhere to go basically, because there is no liquid stream in contact with the wall there are only vapour which is having a very poor conductivity and maybe some small liquid droplets which you can find here in these which are just floating around in the vapour and they are not likely to come in contact with the wall and therefore, there is a sudden decrease in the communication between the heat transfer communication between the wall and the fluid which is referred as the dry out.

So, in the dry out zone you can clearly see the that the temperature of the wall suddenly jumps to a very high value. And this difference can be quite significant fluid temperature

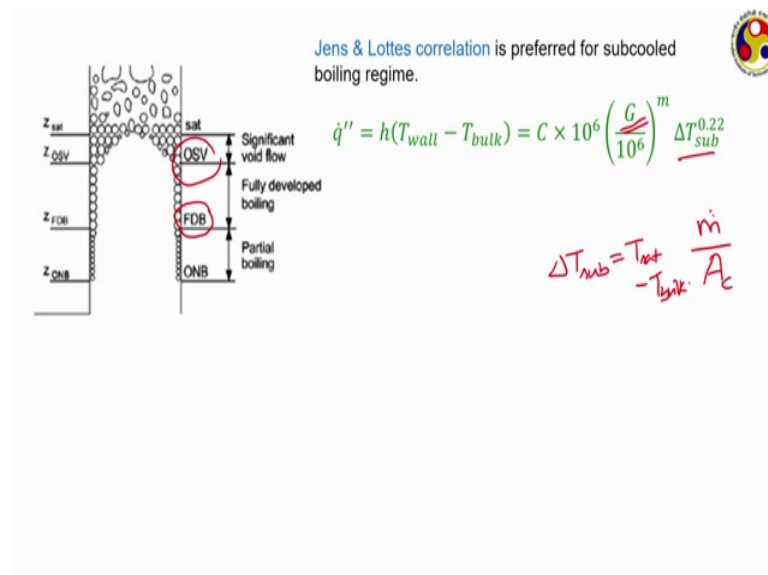
continues to be the saturation temperature till all these liquid droplets completely evaporate.

And then once it reaches a perfect vapour situation, then only it starts to increase this entire topic of boiling heat transfer or this flow and pool boiling is extremely complicated and requires I should say several lectures for this.

But as our purpose is only to get an idea about the nucleate thermal hydraulics; so, just to know the names of all these regimes is sufficient. So, you can see that in a typical coolant flow through a channel in something like in a boiling water reactor we get different regimes like initially a single phase liquid regime, then a sub cooled boiling regime then saturated nucleate boiling regime a force convective heat transfer regime and finally, liquid deficient regime going to the convective heat transfer to single phase vapour and in terms of the flow regimes that is the visual nature of the flow regimes we may have a bubbly flow they we may have a slug flow.

And giving to annular flow and finally, a drop flow regime where we are only having liquid drops floating in vapour phase.

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If we look further into the sub cooled boiling part till some point which is called ONB that is onset of nucleate boiling it is only single phase vapour something like this here,

the flow is considered from somewhere downstream location in the upward direction, but when we do something called FDB the fully developed boiling.

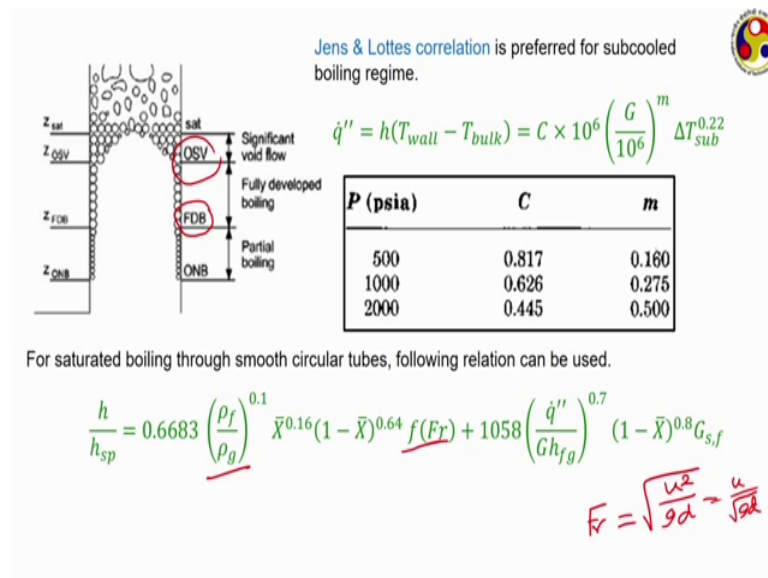
There are only very minor almost microscopical level vapour bubbles can be identified in the walls as we reach this fully developed boiling zone. The temperature difference between wall and situation temperature difference between the saturation temperature and the bulk fluid temperature is much smaller.

So, much bigger bubbles can be seen on the walls, but still there are vapour bubbles in the bulk of the fluid, because in a bulk in thus close to the centreline. The temperature is way below the saturation, then we are is something called OSV onset of significant void. This is the location from where the bubbles can actually get can actually survive even after getting detached from the wall, because where the degree of sub cooling for the liquid is quite small and very soon if the entire fluid reaches the saturation temperature.

So, the mechanism of sub cooled boiling is quite very much different compared to; what we have in case of saturated boiling? In case of saturated boiling the entire fluid is already in the saturation temperature and. So, whatever heat that is supplied that entirely spinned to entirely spinned to vaporize the fluid, that is; the entire heat contributes as latent heat; however, in the sub cooled boiling zone a part is utilized as latent heat to produce vapour in the walls, but a major part is utilized in heating the single phase liquid.

And accordingly, we need to use different kind of heat transfer correlations for sub cooling boiling zone and the saturated boiling zone for sub cooled boiling zone gsn lotus correlation is preferred which is having a form like this here this g is the mass flux which is basically the mass flow rate divided by cross section area of the channel mass flow rate of the coolant by the cross section area and ΔT_{sub} is the degree of sub cooling at that particular location ΔT_{serve} ; as I have mentioned it is just saturation temperature minus bulk fluid temperature m and h is the heat transfer coefficient m and C are two coefficients which depends on the local pressure.

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And these are some representative values; once you reach the saturated boiling zone, particularly in smooth circular tubes this is one relation that can be used you can see it is a quite complicated form. Actually, there are several such correlations exist, because this is an active area of experimental research and it has been found that this heat transfer correlations can depend on infinite number of factors when two phase flow is involved this is just an representative one; to give you a an idea, how complicated it can be?

You can see on the left hand side we have h upon h single phase. Here, h_{sp} refers to the single phase heat transfer coefficient means instead of being a two phase mixture if the entire fluid is flowing as a single phase liquid, then what would have been the corresponding heat transfer coefficient?

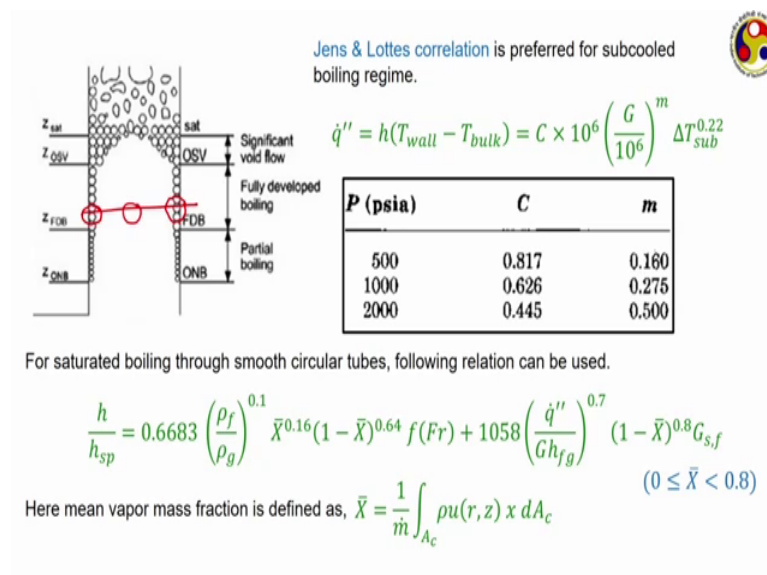
And these h_{sp} is can be calculated using something like Dittus Boelter relation or maybe the one that I have referred earlier on the right hand side first we have the density ratio here ρ_f is a saturated liquid density and ρ_g is the saturated vapour density Fr is the fluid number a fluid number is not a non dimensional number which present a ratio of inertial and gravitation of a forces and fluid number typically is defined as root over u square by g into l , where u is the velocity of the fluid g is the gravitational force and l is some suitable length scale; like if we are talking about a flow through a smooth circular tube. Then l can be the diameter of the tube.

So, it is u by root over g into d in that kind of situation and this function f function of fluid number.

This actually depends on the exact orientation of the flow, that is; where having an upward flow downward flow maybe some idea about the surface roughness etcetera. Then we have this is a heat flux that is applied to the wall h_{fg} is the latent heat of vaporization and g is a mass flux which I have just mentioned.

And finally, there is another g_s which is actually a coefficient which depends on the combination of the metal that is used in the surface and the fluid itself and $\bar{X}$; is the mean vapour mass fraction which is defined like this is the area averaged mean from mass fraction, because in a single cross section there may be significant different in the mass fraction value like if we take a sample somewhere here at this particular cross section.

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At here you will be getting some value of the vapour cross section vapour void fraction or mass fraction here also you will be getting something, but if you take a sample from somewhere you are going to get a mass fraction of 0. So, you need to define an average mass fraction and this $\bar{X}$ value can be put here this relation is applicable for $\bar{X}$.

A wide range of $\bar{X}$ from 0 to 0.8. So, the choice of heat transfer coefficient is a complicated topic in not only in nuclear thermal values, but any heat transfer

applications as we have depending on experimental heat transfer correlations. So, before picking one we should be very very careful; whether that is at all applicable for our case or not like in this the last one that I have mentioned it is applicable for a wide range of X bar that is 0 to 0.8, but say the situation you are dealing with that is having a very high vapour mass fraction somewhere close to the channel outlet of your channel which is something like 0.9, then this is not at all applicable we have to find something suitable.

So, this takes us to the end of this particular module.

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Key points from Module 5



- ✓ Homogeneous nuclear reactors are primarily used in small-scale applications & research.
- ✓ Heterogeneous reactors contain fuel & moderator in a repeated pattern of discrete bodies.
- ✓ Large thermal conductivity & high melting point is preferred for fuel.
- ✓ Cladding provides structural support to the fuel & can also aid heat transfer.
- ✓ Actual amount of energy transferred to coolant is nearly 80% of that released during fission.
- ✓ Thermal resistance of fuel, cladding & coolant film must be considered during thermal appraisal.
- ✓ Both neutron flux & power generation strongly varies along the height of a reactor.
- ✓ While bulk coolant temperature continually increases, temperatures at cladding surface & fuel centreline show a maxima.
- ✓ Proper care must be exercised while selecting the correlation for heat transfer coefficient.

Here we have learned about homogeneous and heterogeneous reactors we have seen that homogeneous nuclear reactors are primarily used for small-scale applications, medical isotope production or hydrogen production kind of the research, but heterogeneous reactors are primarily used in power generation where we have fuel and moderator placed in a repeated pattern of discrete bodies.

We have discussed about the desirable properties of fuel and cladding material large thermal conductivity and high melting point is preferred for a fuel. Cladding provides the structural support to the fuel and all can also add heat transfer particularly in gas cooled reactors, then we have discussed about the energy that can get transferred or that is physically practically available for transfer to the coolant and that is generally 75 to 80 percent of the total, that is; released during fission thermal resistance of fuel cladding and coolant film must be considered in the thermal appraisal and can be some further

instances like; if there is any gap between fuel and cladding surfaces and some further resistances may come into picture both neutron flux and power generation strongly varies along the height of a reactor.

So, during the analysis we need to take that into consideration while bulk fluid temperature continuously increases. During such energy we have found that temperature at the cladding surface and fuel centreline both show a maxima and we have derived mathematical ways of estimating energy corresponding location and also the value of concern temperature and finally, we have discussed a lot about heat transfer coefficients both single phase and two phase versions and we have concluded that proper care must be exercised. While selecting the correlation for heat transfer coefficient so, this is where I would like to finished the 5th module where discussed about thermal hydraulics.

There are quite a few small problems that you can solve on this; please follow all the mathematical parts that whatever derivations that I have done here try to do all those derivations on your own and then check back with the slides whether the they are correct or not and in the assignments mathematical problems will be there please try to solve them if you find any issue do not hesitate to communicate to us we are ready to reply to you.

Thanks a lot.