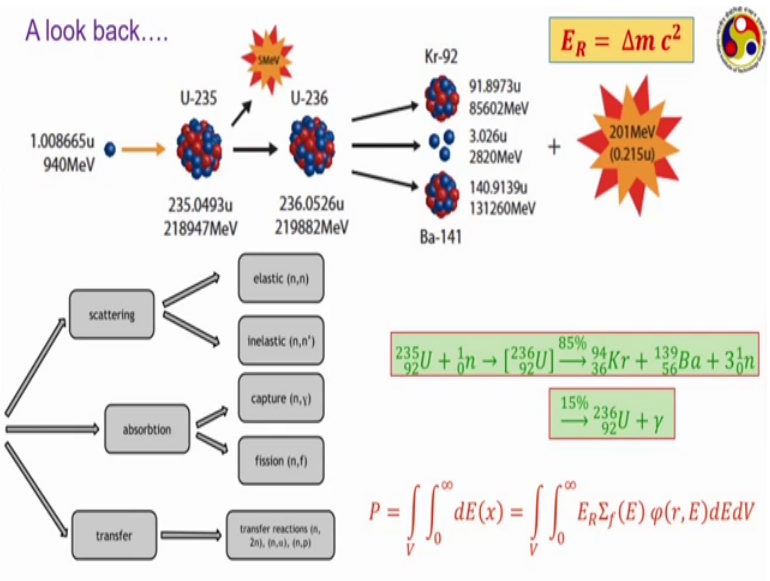
Fundamentals of Nuclear Power Generation
Dr. Dipankar N. Basu
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Lecture – 13
Nuclear Fuel and Simple Energy Consideration

Hello friends. Welcome back to our MOOCS course on the topic of Fundamentals of Nuclear Power Generation. And, today we are going to start the 5th module on reactor thermo hydraulics the term thermo hydraulics some of you may be aware of it some of you may not be. So, just to update on this the term thermal hydraulics involves the term thermal and hydraulics. So, can you guess from here what it may refer to of course, thermal is associated with the analysis of heat transfer and hydraulics is aso is associated with fluid flow?

And therefore, the term thermal hydraulics is a generally used for simultaneous analysis of heat transfer fluid flow problems and hence in this particular module. We are going to look at the heat transfer aspect and also the few aspects whenever necessary of a nuclear reactor to start with just a quick look back at the 4 modules that; we had like a I am taking you back to the first module itself.

(Refer Slide Time: 01:26)



Were this was shown a whenever a neutron or strikes a uranium 235 nucleus a it initially forms a temporary nucleus in the form of uranium 235 and then we the efficient reaction.

When this a intermediate nucleus or isotope brakes into two much lighter isotopes. In this case it is return 92 and barium 141 plus 3 neutrons and it is also associated with a large amount of energy in the module itself.

We have learned whenever such reaction is given without trying to understand, the inherit physics of that we have und learned that; how to calculate this amount of energy release? We know that whenever values of this mass we can calculate the mass defect; that is the mass or combined mass of the products will be generate slightly higher than the combined mass of all this reactants and this difference was termed as the mass defect.

And we know the mass defect multiplied by c square can be a related to the total amount of energy that has been released during this reaction. Whenever we are having any reaction even in common chemical reaction, there is generally small amount of mass defect, but the amount of mass defect that generally we a get in a chemical reaction is negligibly small compare to; what we get in case of chemical reaction and therefore, chemical reactions are a then we get in a nuclear reaction I should say.

And that is why; a nuclear reaction we get huge amount of energy release which can be of the root 10 to the power 6 to 10 to power 7 times greater compare to a equivalent chemical reaction. So, in the first modules itself we have learned; how to calculate this amount of energy? Once we know the mass of all this components involve, but a in the second modules we learned that a while there are several nucleus which can undergo spontaneous radioactive decay.

This particular of fission reaction is a generally in not common or it does not happen naturally, rather we have two induce this kind of reaction by striking the nucleus with the suitably charged particle which comes under the category of artificial radioactivity and event. When we strike a nucleus with a particle commonly neutron is the most suitable particle we can have different kinds of reactions like we can have scattering which is associated in both elastic and inelastic collisions.

That is neutron and nucleus may extent a momentum energy and a can have a perfect well a conservation of both kinetic energy and momentum in case of elastic collision in case of inelastic collision. We have some amount of energy release as well we can also some kind of transfer reactions only in a there is small fraction of total situations. We can have the neutral being absorb in the nucleus and whenever the neutrons gets absorbs in the nucleus it is not guaranteed to fission reaction.

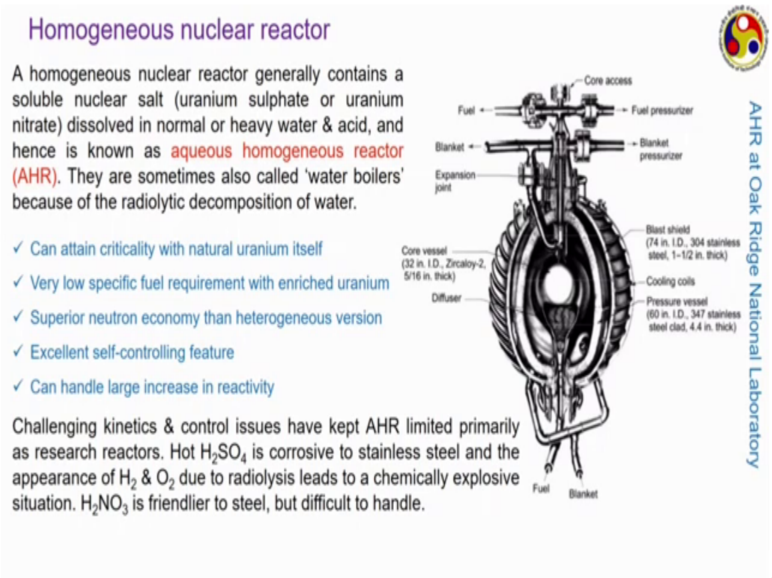
Rather we have learned in the third module that there is only a fraction of this total neutron that gets absorbed can induce fission like a when an uranium 235 absorbs a neutron only in 85 percent case, we can have the fission reaction in remaining situation we may have the non fission capture to produce uranium 235 isotopes and whether this a reaction that all will happen or not that strongly depends upon the energy of the neutron or also the nucleus itself like the equation that I am showing here this particular reaction that is valid only thermal neutron that is a neutron which are having energy of the level of 0.0 to 5 electron volt a high energy neutron generally, will not cause such kind of reaction for uranium 235.

So, in the 4th module we give a much closer look to the neutrons particularly the distribution of neutrons inside the reactors. We are seen the power produced during reaction can be given by such a reaction like this where this particular term is associated with amount of energy released during fission here E R this is one the amount of energy released during a single fission reaction; which is generally of the order of 200 MeV sigma f is the macroscopic cross section and this is the neutron flux generates the thermal neutron flux, but here a as a neutron can have different neutron can pass to different energy level in reactors.

So, we need to integrate this over all possible energy levels at any particular location and then to get the total energy produced by the reactor we need to integrate this over the inter volume. So, know we more or less know about how to calculate the neutron flux distribution like we have studied different cases in the previous module. And once we know the neutron flux distribution the x; the knowledge about E R and sigma f are more or less standard now a days.

And. So, just we knowing the neutron flux distribution we can more or less calculate the total power that has been produced by fission inside a reactor, but once we have the power and then, what to do with that? Of course, we need to transmit the power to some a coolant stream which will we use for subsequent power production and in this particular module we are going to look at that particular power transmission or energy transmission procedure.

(Refer Slide Time: 06:53)



Now, to start with I probably mentioned earlier that commonly we can classify nuclear reactors into two categories: homogeneous and heterogeneous, where homogenous nuclear refers to the one where we have a uniform distribution of all the substance that may be presently inside the reactor; that is from the entire volume of the reactor. Let us see a a this is one reactor. Now, if we pick up sample from somewhere and sample from somewhere here and a third sample from somewhere here the all this samples should have identical chemical composition and a then only we shall be calling it a homogenous nuclear reactor.

Generally, in a homogenous nuclear reactor we have a soluble nuclear salt commonly a sulphate or nitrates salts of uranium which is dissolved in water. Now that water can be the normal water or it can be heavy water also just aa remember that heavy water efforts to the water where hydrogen is actually deuterium. So, here the nuclear salt is generally dissolved in normally heavy water and also acid.

Acid can be sulphuric or nitric acids and a this kind of sig, because of the presence of such an aqueous solution nuclear fuel there are also called aqueous homogenous reactor aviator they are sometimes called as water boilers do not get confused with boiling water reactor which is a common type of nuclear reactor water boiler refers to this aviators, because the water molecules that can be present inside the reactor they can go through a kind of decomposition process. When that is subjected to the radiation such kind of redulate decomposes, because of such kind of redulated decomposition hydrogen and oxygen.

Bubbles may appear inside this reactor; that is why they are called water boilers. This is a classical feature of a very old and popular aviator developed in the oq junction laboratory; we do not want go to the details of this is just for a your idea where we have a common vessel and inside the vessel inside the pressure vessel like a this one inside this pressure vessels we have this aqueous solution of the nuclear fuel and everything explicit inside.

You can see it is a 60 inch diameter which is a very large one. Now homogenous nuclear reactor offers a several advantages like, it can attain criticality with a natural uranium itself if you think about the earlier analysis of a multiplication that we have done a to have a criticality condition we need to have a large value of this thermal fission factor and also significantly large value of the thermal refilaration factor.

Now, a natural uranium has only 0.7 percent of uranium 235 rest is uranium 238. Therefore, it a can have a quit significant value of this resonance absorption probability, but still because of the volume and uniform mix large volume of the shell and uniform mixing of fuel and moderator the homogenous nuclear reactor; generally can attain criticality with natural uranium itself.

And quite frequently there also used with enriched uranium and if there used enriched uranium. They provide an excellent specific fuel requirement which is generally very low compare to other kind of reactors that is heterogeneous nuclear reactors neutron economy is also superior in case of homogenous version because a the neutron are surrounded by neutrons and fuels on all the sides.

And therefore, that is every neutron is surrounded i f e modern return nuclear modern return nuclei or fuel nuclei. And therefore, a there is excellent nutria supi neutron economy compare to heterogeneous version there also very good in self controlling and can handle large increase in radioactivity, because of their large volume of a moderator coolant present said this ; however, homogenous nuclear reactors are not very popular for commercial use because of their challenging kinetics and control issues.

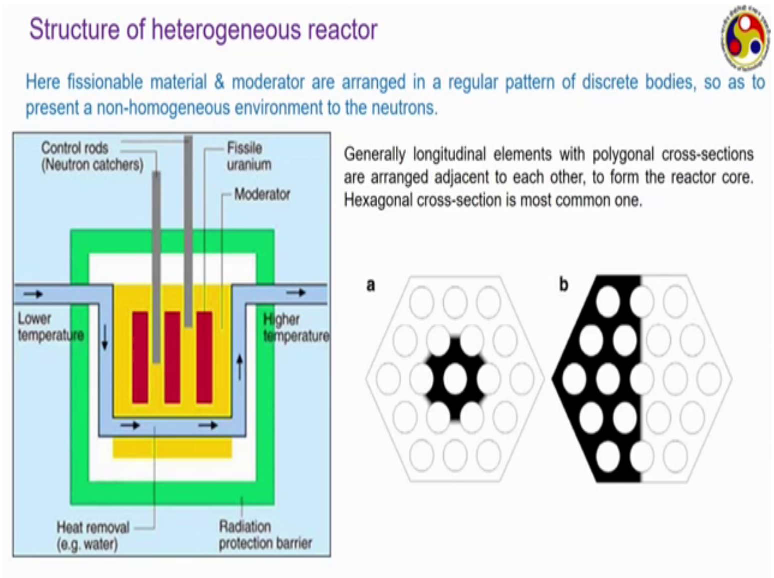
While they have very good self controlling future the external control is quite difficult again because of that homogenous mixing only we cannot intimately control the reactivity yes a because once we get the reactor started. We do not have exclusive control on the mass of fuel and the mass of moderator; that is present here and also we do not have a means it did it control on the on the rea on the reactivity. So, AHR is primarily limited to research reactors left scale small reactors till date a big problem for them is also the use of the sulphate or sulphuric acid bec uranium sulphate in water leads to the production of sulphuric acid and such hot sulphuric acid is extremely corrosive to stainless steel.

Which is one of the common material is used in nuclear reactors and also the appearance of hydrogen and oxygen bubbles, because of the radioactivity radialitic composition is another big issue from control point of view when we are using on the uranium nitrate as the fuel then we do not have sulphuric acid rather we have hot nitric acid nitric acid is much friendlier to stainless steel it does not corrode stainless steel that much, but ev everyone knows how difficult it can be to handle nitric acid particularly at higher temperature.

So, these are the reasons of a not using homogenous nuclear reactors that much effort for commercial power generation rather.

They are more restricted to much small scale use like the medical isotope separation from inline fuel or also; using that radioactive decomposition the production of hydrogen. Now, while homogenous nuclear reactors are tilted mostly restricted to a experimental or a lab scale reactors heterogeneous nuclear reactor prevails almost in all possible applications for new commercial power generation.

(Refer Slide Time: 12:54)



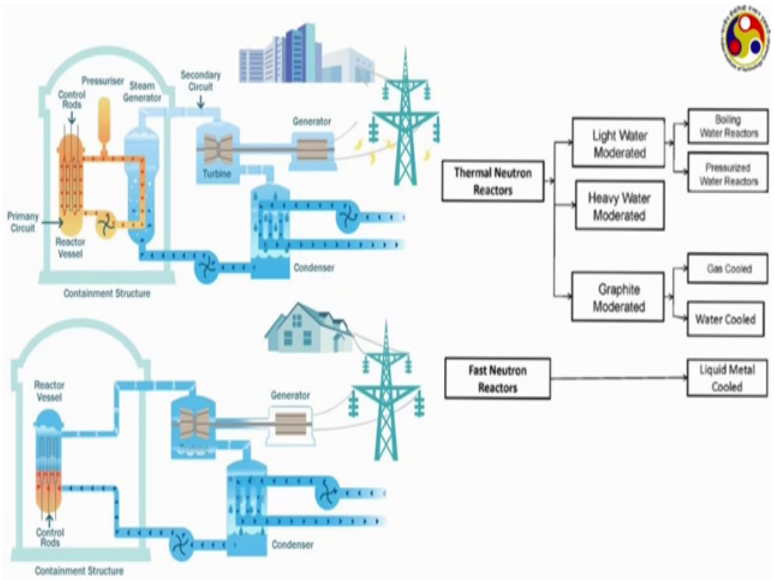
In heterogeneous nuclear reactor we have mod material the fissionable mixer; that is fuel and moderator keep separately, but they generally add in this some kind of regular pattern some kind of repeatable regular pattern and very fuel component is internally separated by a moderator with the adjacent fuel components just a represent picture like this here the red o blocks are actually fuel which are surrounded by moderator this yellow colored one from all possible sides.

The coolant is flowing this a blue line refers to coolant through which is flowing through the reactor coolant that for all the energy that has been generated; because of the fission reaction a that are taken by this coolant stream. We can also have that control rods which are of the controlling events which will discussing them in that next module this is just a representative picture, but this a red blocks of fuel can find in practical cases also they look moralize the same we generally can have two types of designs.

One we can have a plate type of reactor; that is a not reactor I should say plate types of fuel elements, where every fuel element is a shape like rectangular plate very thin rectangular plate. I should say and then what is in a moderator other possible design can be the of circular section cylindrical design again the diameter of cir cylinder generally is kept quite small compare to it is height there are generally longitudinal elements with polygonal cross sections, why? Which are arranged adjacent to each other hexagonal cross section is the most common one just like shown here.

Here all the circular elements are the fuel rods cylindrical fuel elements are called fuel rods. These circles are fuel rods and all the gaps in between are occupied by moderator or the coolant. We can also have rectangular or triangular cross sections in nuclear reactors, but hexagonal is mostly referred.

(Refer Slide Time: 14:58)



Now, nuclear reactors can be classified to different categories as well actually from now onwards we are not going to use the term heterogeneous; because we are going to talk on about heterogeneous nuclear reactors and heterogeneous reactors can have several kinds of classification like one classification common classification is based upon the neutrons that are being used inside.

So, based upon the neutron flux profile we can have fast neutron reactors or fast reactors and thermal neutron reactors or thermal reactors fast neutron reactors of course, utilizes fast neutron for their operation. And hence they must use a fuel which has a some significant value of this fast fission cro cross section similarly thermal neutron reactors utilize thermalize neutrons. So, there is a big role of moderator in case of thermal reactors based upon moderator thermal reactors can be classified as; the light water moderator reactor heavy water moderator reactor and graphite moderator reactor.

Here, I would like to mention again that light water generally is a quite bad term, because it here we talking not about a any water which lighter that the common fluid rather we are talk about water itself, but still just to differentiate them with heavy water this term is universally used in case of the nuclear fluid and therefore, at least in this of names we have to use this light water reactor kind of terminologies.

Now, there can be three kinds of moderators like a normal water, heavy water and graphite and this lighter water. So, called light water reactors again can have two principle classifications boiling water reactor and pressurized water reactor. There can be quite a few others also we shall be talking later on.

Similarly graphite moderator reactors also depending upon nature of the coolant, they can be gas cooled or water cooled and in case of fast neutron reactors what can be the moderator in the fast neutron reactor? What is the purpose of moderator? The purpose of moderator is just to slow down a fast neutron with the thermal neutron level and in a fast reactor we are looking to utilize the fast neutron itself are we and we are not at all trying to slow it down. And therefore, a fast neutron reactor does not have any moderator; rather it a fast neutron itself coi induce fission to the fuel and transfer that energy to the coolant fast neutron reactors are generally liquid metal cooled; that is they use liquid metals as the coolant name.

This is take almost schematic diagram for a pressurized water reactor. In case of pressurized water reactor the water is maintained in single phase by using a pressurizer; the pressurizer maintains a pressure inside the reactor such that that maximum temperature of water is not allowed to cross the saturation temperature and hence we are looking single phase opposition and this is a boiling water reactor.

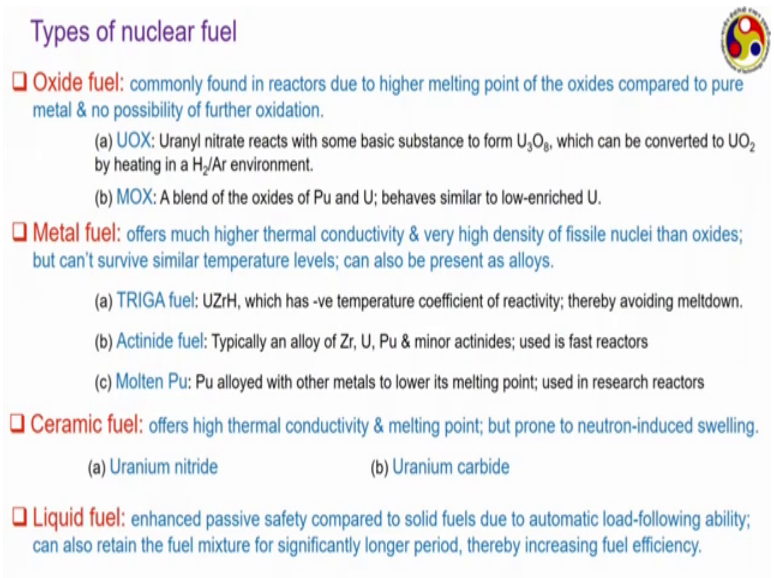
In case of a boiling water reactor the water or the fluid is allowed to boil and gets converted to steam or I should say a part of that water gets converted to steam and that steam is taken through the substitute subsequent a processing in the form of a turbine and a expansion in the turbine and condenser and palm and again back to the core in the form of recirculation. So, we can have wide variety of coolant.

As we can see from here these two both this two figures refers to light water reactors, but in one case we are using single phase water to carry all the heat produced by nuclear fission whereas, in the other case we are using the phase change process to carry the a heat from the nuclear reactor we can also have gas cool reactors which again quite similar to the pressurized water reactor they remain in single phase single phase gaseous medium.

But a they gas is allowed to circulate through the core and transfer the heat to subsequent immersive operation and then come back again same about liquid co metal cool reactor there also similar to p w volts that is the coolant is a liquid one like liquid sodium or liquid potassium and that remains in that same state throughout it is operation. There can be several types of nuclear fuels that we can find one can be oxide fuels it is quite common to find uranium oxide or plutonium oxide or a mixer of that as the fuel in the inside the reactor like a oxide fuels generally offer very high melting point compare to the pure metal itself.

And also as their already in the oxide form so, there is no possibility of any further oxidation; there by expanding the thermal limit of operation oxide fuel commonly can be just uraneul oxi urinuim oxide uranile nitrate generally reacts with some basic substance like ammonia to produce U 3 O 2; which is subsequently converted to uranium oxide by heating it in an a environment of hydrogen or argon generally a environment of inert gas.

(Refer Slide Time: 19:53)



Other can be MOX MOX actually refers to mix oxide mix oxide means.

Here, the fuel comprises of both uranium oxide and plutonium oxide plutonium oxide can comprises about 20 to 25 percent of the total mixer and their found to be behaving like low enriched uranium like a like; that is even in the mixer. When we are using natural uranium, but the overall multiplication factories found to be improved compare to what we get with pure uranium or na you natural uranium.

Other kind of fuel can be the metal fuel where using the pure metal itself or may be some alloy of that their always characterized by much higher thermal conductivity and also very high density of fissile nucleus compare to the oxides, but their temperature level or applicable temperature level is restricted if we go to higher temperature the metal may melt or it may get oxidizes.

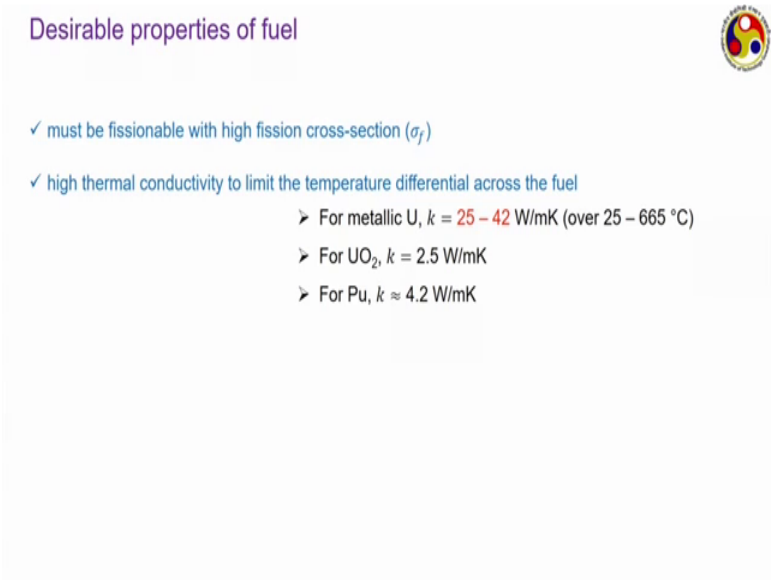
There are several examples for this metal fuel one can be triga fuel triga is very popular type of nuclear reactor which is found which is being used by several countries. And it is actually an alloy comprises of uranium zirconium and hydrogen and it has actually a negative temperature coefficient of reactivity. This is very interesting negative temperature coefficient of reactivity refers to has the temperature increases reactivity starts to decrease and therefore, there is no chance of reactor melt down that is at high temperature or when the reactor temperature keeps on increasing the reactivity keeps on decreasing and in a self sustained way the reactor is able to control it is own temperature.

There can be actinide fuels actinide actually refers to all these elements which can act as nuclear fuel uranium and plutonim uranium and plutonium are the two major actinides other fuels which are having it is atomic number greater than 94; that is after plutonium they are generally called minor actinides. So, actinide fuel generally refers to typical a alloy or zirconium, uranium, plutonium and several minor actinides. they are used a fast reactors third is a molten plutonium plutonium is also alloyed is other metals to lower it is critical point and it is a commonly used in research reactors particularly when experiment with possible reactors.

There can be ceramic fuels as well ceramic offers some very high conductivity and melting point, but they are prone to neutron induced swelling; that is at high temperature there may be a significant change in it is volume there we can have commonly uranium, nitrate and uranium, carbide as the two ceramic fuels and the final category is the that of the liquid fuels liquid fuels are generally used in the homogenous reactors and not that much in heterogeneous reactors for obvious reasons.

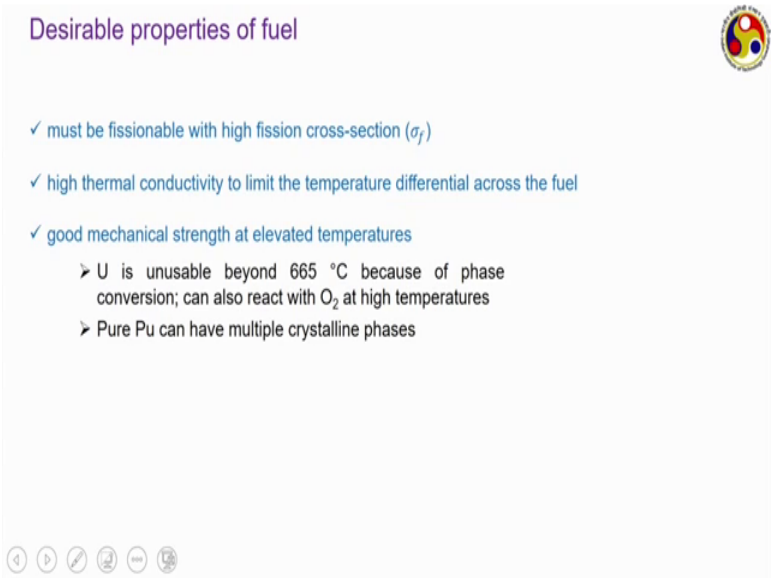
Liquid fuel enhances the passive safely compared to solid fuels; because of automatic load following capability like; we have in case of homogenous reactor they can also retain the fuel mixture for significantly longer period thereby effecting the fuel efficiency.

(Refer Slide Time: 23:20)



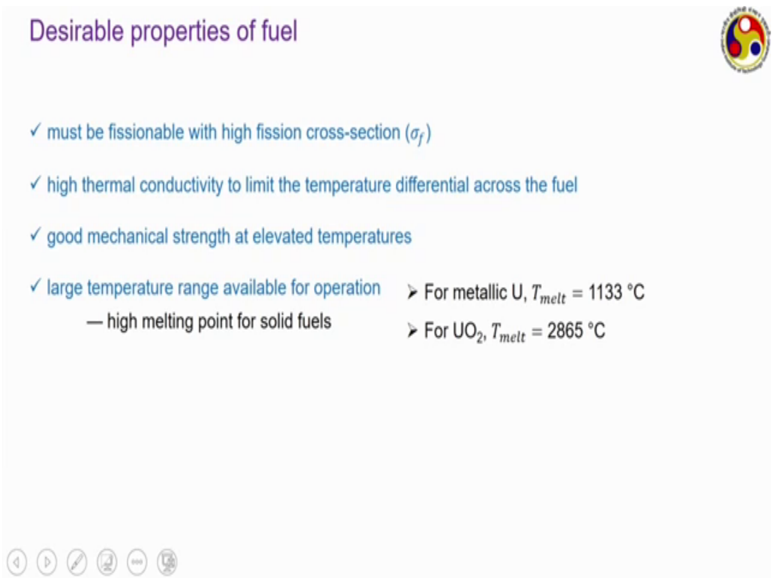
So, with all these kinds of fuels in any reactor while choosing the fuel we can consider several factors like the fuel master with very high fission cross section. Secondly, it should have a very high thermal conductivity the thermal conductivity of the fuel is very high, then the temperature change across the fuel itself who is be qua will be quite small. These are the common values of thermal conductivity like for metallic uranium thermal conductivity may vary between 25 at 20 degree Celsius to 240 voltammeter Kelvin at 665 degree Celsius for uranium oxide it is much lower just of a hmm to 245 voltammeter Kelvin, whereas for plutonium also as a conductivity lower than uranium which is something like four point two voltammeter Kelvin.

(Refer Slide Time: 24:08)



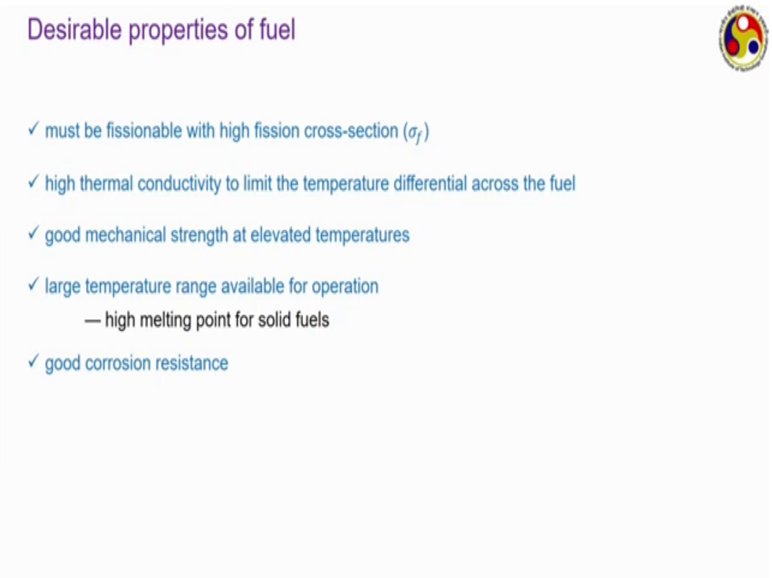
The at higher temperature the fuel should have sufficient mechanical strength should at it can maintain it is own shape and does not get deformed like uranium is unusable beta beyond 665 degree Celsius, because uranium can have several crystalline phases like alpha, beta and gamma phases and beyond 665 degree Celsius. There is a translation from alpha phase to the beta phase uranium can also react with oxygen at high temperatures and get oxides and another problem with plutonium is pure fuel is it also as several crystalline phases and therefore, not suggested to be used at high temperatures are at least high core temperatures.

(Refer Slide Time: 24:50)



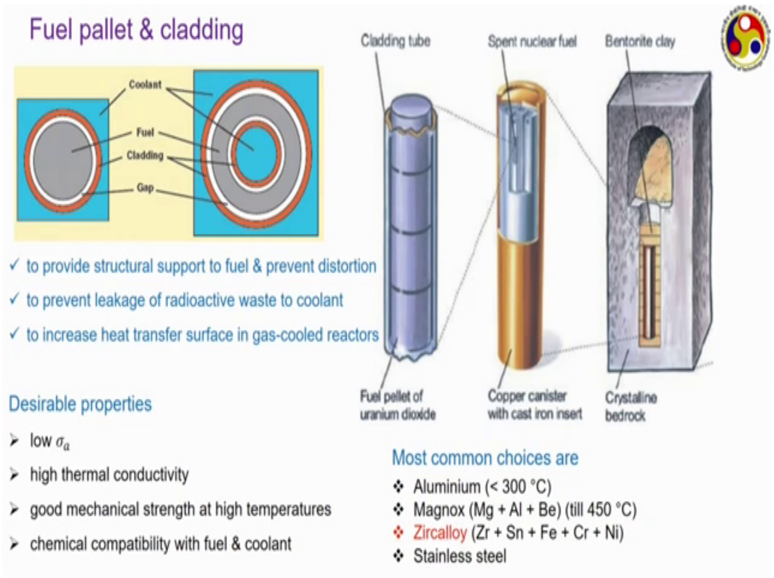
The temperature range available for operation should also be large and that is possible only when the solid fuel as a high melting point like for pure metallic uranium the melting point is something like 1133 degree Celsius whereas, for uranium oxide is significantly higher it is greater than 2800 degree Celsius; that is one the reason of not using metallic uranium rather uranium oxide as the fuel in modern reactors.

(Refer Slide Time: 25:19)



And finally, they should have also good corrosion resistance characteristics; because they generally remain in contact with flowing coolant which a can cause or at least can try to coefficient amount of corrosion.

(Refer Slide Time: 25:32)



Then we come to fuel pallet and fuel cladding pallet refers to each small module of fuel element which can be of the shape of small bullets or may be a just like a small tablet and their covered by a jacket which is called cladding claddings of several important purpose like; a this is a much elaborated diagram where you can clearly see each of the module here like this is the one this one is the fuel module and this module is being covered by the cladding material.

So, some several fluid modules each one is put on top of the other one and their connected from energy point of view and their cladding is also a common material and hence the cladding must have se ma can several important prob important function like; they can provide the structural support to the fuel and there we prevent the distortion ; that means, that higher temperature we need that the fuel a gets often becomes soft, then it will it cannot go just ma directly into the coolant stream rather they remain inside this cladding tube mold tube only.

Another advantage is to prevent leakage of radioactive waste to coolant the radioactive waste that have being produced that generally remains inside the reactor itself it is a later on they can be means once their life time finishes they can be used as this spent fuel and finally, this. So, radioactive leakage of this waste to the coolant can be avoided.

And finally, when in case of gas cool reactors you know that compare to liquid gas generally is associated much lower heat transfer coefficient and hence they need some kind of extended surfaces or fields for heat transfer this cladding by properly designing the cladding tube all we can also enhance the heat transfer in case of a gas cool reactor. So, water should be the desirable properties for the cladding quite similar to the nuclear fuel; that means, a they should have lower relation cross section and a the thermal conductivity it should be high; that means, whatever energy the cladding receives from the fuel that is avail to transmit immediately to the coolant.

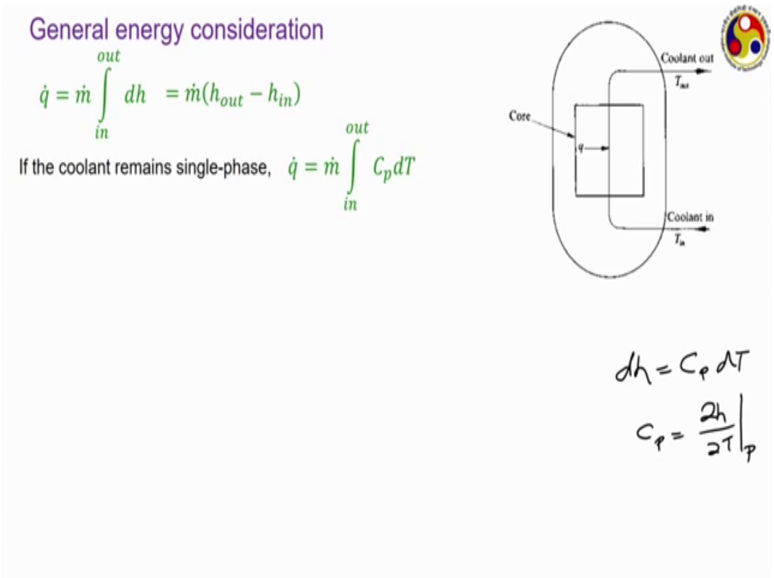
The first point I must go back low absorption cross section we have to ensure; because if the cladding itself is a starts absorbing neutron then that will be a type of loss, then it should have good mechanical strength at high temperature so that you can support the fuel itself and finally, cladding material should have chemical compatibly with the fuel and the coolant particularly the fuel.

This fuel cladding and coolant all this three materials generally are generally must have compatibly with each other and. So, is the moderator these are the common cladding materials like aluminum is can be used only for temperatures less than 300 degree Celsius and a a the second very important cladding material can be magnox which is actually alloy of magnesium and it can have aluminum in barium and along with magnesium in it and it can sustain up to a higher temperature of 450 degree Celsius.

Third one there is a most preferred one zircol zircalloy which is an alloy of zirconium and contains s s small elements like s t n tin iron chromium, nickel, etcetera and stainless steel can also be used, but stainless steel may have it is own issues. So, once we know about structure of the fuel we are now ready to study the energy analysis or perform an energy analysis of a reactor may be just a single fuel element.

We know that every fuel is covered by a jacket called cladding and therefore, the energy; that is produced inside the fuel if we are particularly talking about solid fuel the energy produced inside the solid fuel that will get conducted through the solid from it is center line to the wall and from the wall of the fuel the cladding is there. So, that heat has to be conducted through the cladding material as well and generally it will be able to coming contact to the coolant.

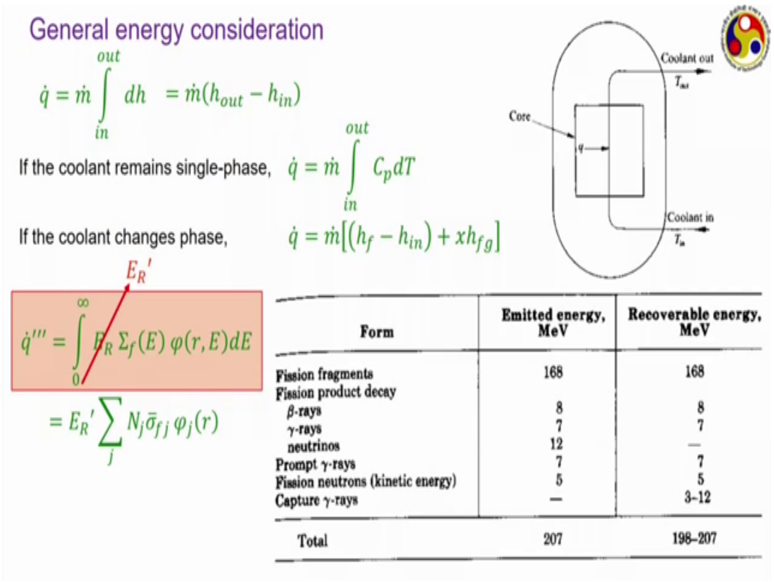
(Refer Slide Time: 30:09)



Now, this is brief schematic representation if q is the amount of energy; that is been produced by the core and if coolant is coming with some temperature and going out at some element temperature, then we write a simple energy balance as q dot equal to m dot into integral of dh, but q dot is the power production rate and m dot is the mass flow rate of the coolant and hence it is mot dot into h out minus h in assuming that m dot remains constant this is of course, under steady state this energy wellness has been retained.

If the coolant re retains the single phase status throw out this passage, then we can always write this thing, because we know as per the definition of specific heat dh equal to Cp d T or Cp is equal to dou h dou t at constant pressure and that is why we can for single phase situation we can write this way if Cp remains constant this is a straight for a integration, but generally Cp is a strong function of time particularly at higher temperature that can show a quadratic kind relationship.

(Refer Slide Time: 31:16)



And if the coolant changes phase, then situation slightly more complicated then this energy balance have two components. When the first one this is the single phase energy right from the inlet temperature to the saturation temperature he h f is the saturated liquid enthalpy and this is the latent heat transferred, where this x is on the quality a or the mass fraction of the supplied fluid coolant which has been converted to the upper phase.

Now, this expression we have found earlier a pa at pa energy released by because of fission found in volume can be given expression like this. So, where E R is the amount of energy released; because of fission sigma f is the macroscopic cross section phi is the flux distribution by the neutron.

Now, let us take a look at this particular chart we have earlier mention that the because of fission reaction there about 200 MeVs slightly greater than 2 MeV. Actually amount of energy is released which corresponds to the mass defect, but how that energy is being transmitted of course, the fission fragments take care of the majority part of that which is about 160 MeV, but there can be several components like fission energy carried by the beta and gamma rays or by the neutrinos the fission and also there can be a significant amount of energy carried by the gamma rays which are released during this reaction.

But out of this all this possible contribution actual this 168 MeV a from the fission fragments and this first 8 MeV from the beta rays combined together are usable or available as thermal energy rest are available either energy through radiation or via some other means something like delayed neutron decay etcetera.

And, hence while we had a total figure of 207 to deal with for this E R practical case we are going to get only about 80 percent of that and hence this E R must be substituted with an effective value of that which come generally is about 80 percent of this theoretically and hence we can perform this integration as an approximate summation where we are considering say j a multiple goods of a neutron flux.

So, ER prime being a constant is outside the integration and here j refers to a here j is the tot a each of the a small neutron groups that will be considered like; if we remember from the previous module that a in a reactor we can have several groups of neutrons and therefore, we often adopt a multi group approach; that is why we divide those neutrons into several groups and some there cat and perform the calculation for each of them then sume that over the entire range of this energy avail and the same has been shown here.

Here, j refer to any particular such group n j is the total number of nuclei present here and sigma f j is the average fission is the microscopic fission cross section under that group and phi of course, is the neutron is to have distribution of that energy level. We already spent lots of time in the previous module discussing the distribution of this pi phi and let us take one result from here.

(Refer Slide Time: 34:52)

For cylindrical reactor, $\boxed{\varphi(r,z) = \dfrac{3.63\,P}{E_R \bar{\Sigma}_f V} J_0\left(2.405\dfrac{r}{R}\right)\cos\left(\dfrac{\pi z}{H}\right)}$

Let us consider a heterogeneous reactor having n number of cylindrical fuel rods, each of radius a and height H. Then average macroscopic cross-section can be calculated as,

$$\bar{\Sigma}_f = \frac{n(\pi a^2 H)\bar{\Sigma}_{f,rod}}{\pi R^2 H} = \frac{n a^2 \bar{\Sigma}_{f,rod}}{R^2}$$

Hence,

$$\varphi(r,z) = \frac{3.63\,PR^2}{n a^2 E_R \bar{\Sigma}_{f,rod}(\pi R^2 H)} J_0\left(2.405\frac{r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$$

$$= \frac{1.16\,P}{n a^2 H E_R \bar{\Sigma}_{f,rod}} J_0\left(2.405\frac{r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$$

Accordingly, $\boxed{\dot{q}'''(r,z) = \dfrac{1.16\,PE_R'}{HR^2E_R} J_0\left(2.405\dfrac{r}{R}\right)\cos\left(\dfrac{\pi z}{H}\right)}$

At the middle ($z = 0$) of the central rod ($r = 0$), $\dot{q}'''_{max} = \dfrac{1.16\,PE_R'}{HR^2E_R}$

This is the one result that was shown there for a cylindrical reactor or a cylindrical fuel element here this J naught is the a here this particular J naught is the basal function a E R is the energy released during fission that 200 MeV in the order of and this one the sigma f bar is the average fission cross section of the reactor. P is the power production or power rating of the reactor and this V is the volume of the entire reactor. So, capital R refers to the radius of the cylinder and capital H refers to the height of the cylinder.

Now, let us consider heterogeneous reactor which has smaller number of cylindrical fuel rods. So, each rod can be viewed like a cylindrical reactor kind a kind of kind and each of this rods are having radius of small a and height of H then the average macroscopic cross section can be calculated like this, where pi a square H is the volume of each fuel rod multiplied by n gives a total volume of all the fuel rods and pi R square H is the volume of the reactor itself.

So, by we can simplify this as a form like this a if we put this expression for sigma f in a whole expression then we get like this which finally, simplifies to a form like this. So, finally, putting this expression for pi in the previous expression we get the volumetric greater generation of form like this here just note that E R E R are both of E Rs where this E R refers to the amount of energy released during a single fission reaction these are theoretical value and E R prime is the effective value of that E R which can be recovered in the form of thermal energy, but still we are using this E R in that denominator, because while calculating the power rating for a reactor generally the E R is used.

As E R prime E R is more less constant, but E R prime that may vary from one situation to another. So, like say this is our reactor the cylindrical reactor this is the center line we are taking this as a core unit system. So, center line refers to r equal to 0 in the vertical direction also let us take the middle of the core cylinder as the coordinate system. So, here z starts from here and hence if we put r equal to 0 and z equal to 0. In this expression that is if we are talking about this particular location then it reduces to a form like this.

Actually cos 0 is equal to 1 z naught 0 is also is equal to 1 it is asymptotical approach is 1.

(Refer Slide Time: 37:58)

For cylindrical reactor, $\boxed{\varphi(r,z) = \frac{3.63\,P}{E_R \bar{\Sigma}_f V} J_0\left(2.405\frac{r}{R}\right)\cos\left(\frac{\pi z}{H}\right)}$

Let us consider a heterogeneous reactor having n number of cylindrical fuel rods, each of radius a and height H. Then average macroscopic cross-section can be calculated as,

$$\bar{\Sigma}_f = \frac{n(\pi a^2 H)\bar{\Sigma}_{f,rod}}{\pi R^2 H} = \frac{n a^2 \bar{\Sigma}_{f,rod}}{R^2}$$

Hence, $$\varphi(r,z) = \frac{3.63\,PR^2}{na^2 E_R \bar{\Sigma}_{f,rod}(\pi R^2 H)} J_0\left(2.405\frac{r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$$

$$= \frac{1.16\,P}{na^2 H E_R \bar{\Sigma}_{f,rod}} J_0\left(2.405\frac{r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$$

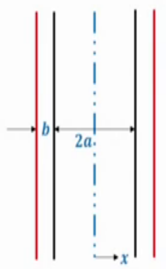
Accordingly, $$\boxed{\dot{q}'''(r,z) = \frac{1.16\,PE_R'}{HR^2E_R} J_0\left(2.405\frac{r}{R}\right)\cos\left(\frac{\pi z}{H}\right)} \boxed{= \dot{q}'''_{max} J_0\left(2.405\frac{r}{R}\right)\cos\left(\frac{\pi z}{H}\right)}$$

At the middle ($z = 0$) of the central rod ($r = 0$), $\dot{q}'''_{max} = \frac{1.16\,PE_R'}{HR^2E_R}$

So, we get this as a expression of a maximum volumetric rate of heat generation and putting that the final expression gets simplified to a form like this. So, this way by knowing the nature of the medium and nature of the heterogeneous medium and also the use in a standard solutions we can always calculate the total amount of energy that is available to be transferred to the coolant.

(Refer Slide Time: 38:21)

Plate-type fuel element



Generally thickness of the fuel element is so small compared to the dimensions in other 2 coordinate direction, it can be viewed as 1-D problem.

For steady-state conduction heat transfer inside the fuel,

$$\frac{d^2T}{dx^2} + \frac{\dot{q}'''}{k_F} = 0$$

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right) + \cancel{\frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right)} + \cancel{\frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right)} + \dot{q}'''_G = \cancel{\rho C_p\frac{\partial T}{\partial t}}$$

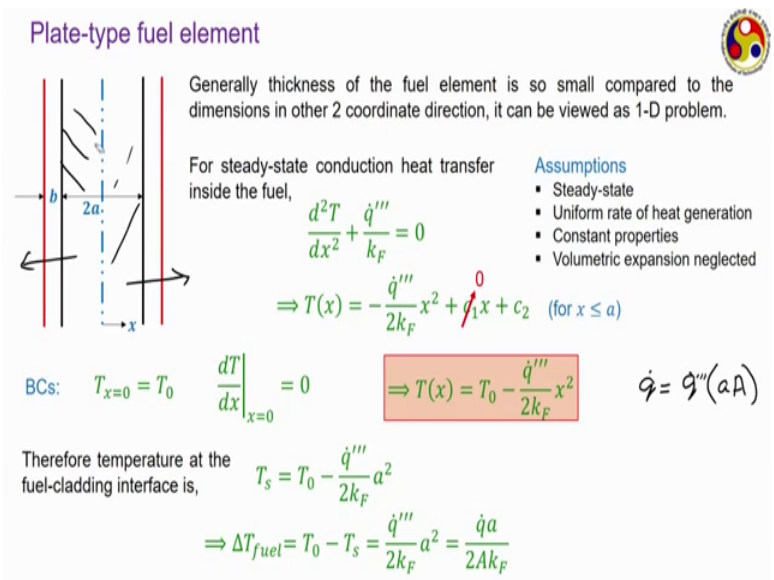
So, let us take our consider couple of simple situations our first case is that of a plate type fuel element as I mentioned commonly reactors have two kinds of fuel pallets one is of a plate kind other is a cylindrical or fuel rod. So, here we have a plate type fuel element of a thickness two a it is which is not possible to showing the diagram is cross section are we are taking as capital a cross section which is perpendicular to this slide and on both sides this element is covered by a cladding of thickness V.

Now, now a generally in general reactors we will find that this dimension two a; is extremely small compare to the dimension in the other two directions and hence it can be considered to be or can be viewed as a one dimensional problem. So, a under steady state the heat transfer in the fuel can le can be retain following the partition collide system can be written like this just a I repeat here conduction is the only mechanism that is happening inside the fuel is being a solid.

And, now the generally heat conduction equation in a three d Cartesian coordinate can be written as these are the three diffusion terms in x, y and z direction with respectively plus the volumetric rate of heat generation is equal to the transient term. Now here putting several assumptions one of course, one assumption is one is already mentioned. So, in case of on d this one goes off and. So, is this one and here being under steady state.

This is the term. So, this one also goes off and leave in our analysis to a one dimensional problem if you other assumptions also we have to consider like one important assumption that we quite often consider is uniform rate of heat generation. Now, truly speaking that is not true and in the next lecture we shall we relaxing this assumption.

(Refer Slide Time: 40:41)



But, here as a as a simplified case let us consider that the rate of fission heat generation is uniform throughout the entire reactor.

We are also considering the properties being constant properties the like this thermal conductivity of fuel and also for cladding that. So, it is come out of the function and also we are neglecting any kind of volumetric expansion that may appear, because of the change in temperature and any such kind of similar effects. So, we get solution of this from x equal to 0 to a a of this second order d.

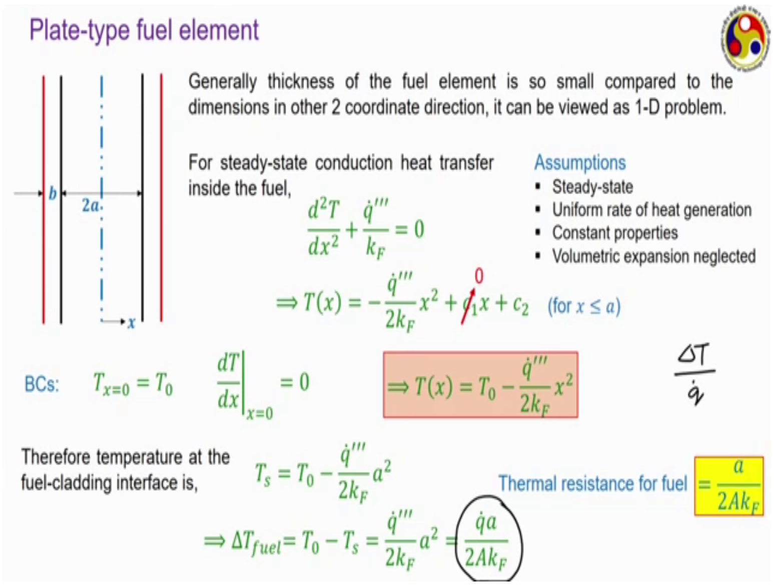
We need two boundary conditions to identify the value of this a two coefficients. So, this can be on boundary condition where T naught refers to the central line temperature and the movement we do not have the too much information about this central line temperature. So, we are just putting a single T naught and then d T is that x equal to 0 is also equal to 0. Now, what does that mean ; that means, that the temperature profile is a mirror image on either side of the center line a with respect to the center line and that is perfectly logical; because the heat generation inside the fuel is uniform and it is dimension in both positive and negative directions are identical and hence temperature distribution should also be identical on both sides or I should say mirror image on either sides.

So, d T d x at x equal to 0, if use the condition then we immediately as e 1 equal to 0, and then putting the second condition we get c 2 equal to T naught. So, this is a final expression for temperature profile inside the fuel therefore, a temperature at the fuel cladding interface that is I am talking about this particular surface here we have to put x equal to a there.

So, if we put x equal to a we are going to get the surface temperature T naught minus q triple triple prime a square by 2 k F and by rearranging this the temperature change across the fuel central line to the cladding surface that is T naught minus T s can be written as q dot triple prime a square by 2 k F and if q dot replace the total power that has been produced by the fuel that should be the a pa a volumetric energy generation multiplied by the volume of this and this volume should be small a into capital A; because small a is the a dimension a of half of the reactor and capital a is the cross section area.

Here, one thing I should mention the energy that has been produced in this half of this reactor that gets transmitted into this side similarly the energy that is getting produced in this side of the reactor that is getting transmitted in this side and therefore, we are only considering half of volume while a doing this energy values or while defining this q dot. So, this q dot you can see as the energy produced or rate of all production of half of the reactor. So, putting this expression here we get this final expression and we can like a; what we are doing in common heat transfer you can draw electrical analogy to this and define a thermal resistance.

(Refer Slide Time: 44:03)



Now, resistance can be defined as the drying force for the heat transfer which is the temperature difference across the fuel divided by a heat transfer rate; that is delta T by q dot and accordingly the thermal resistance for the fuel is a by 2 capital A into k F.

(Refer Slide Time: 44:22)

Neglecting any heat generation inside the cladding,

$$\frac{d^2T}{dx^2} = 0 \qquad \Rightarrow T(x) = c_3x + c_4 \qquad (\text{for } a \le x \le b)$$

BCs: $T_{x=a} = T_s$

$T_{x=a+b} = T_c$

$$\boxed{\Rightarrow T(x) = T_s - \frac{x-a}{b}(T_s - T_c)}$$

Applying interface BC, $-k_{cl}\left.\frac{dT}{dx}\right|_{x=a} = \dot{q}'''a$

$$\dot{q}'' = \frac{\dot{q}''' aA}{A}$$

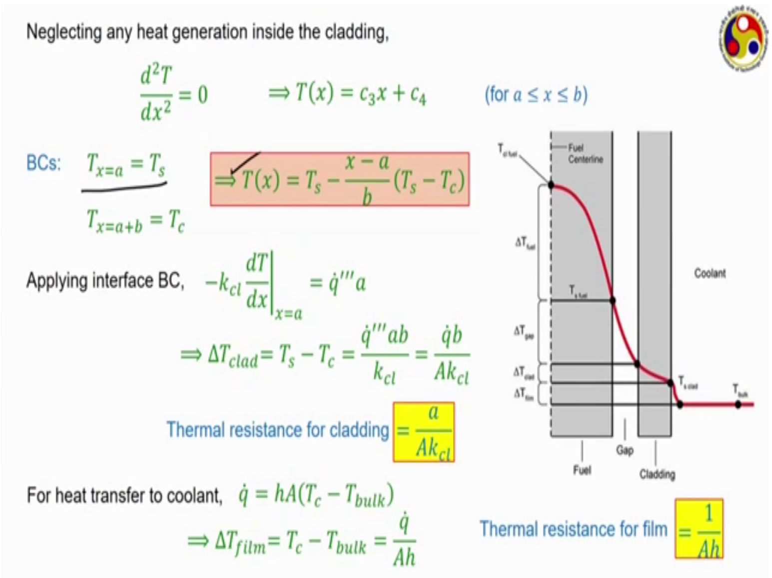
Now, we move to the cladding there is no heat generation inside the cladding and hence a we have a very simple form of homogenous second dot dot d whose solution will be c 3 x plus c 4 for x equal to a to b.

Now, our boundary conditions can be at x equal to a t is equal to T s the surface temperature which we are evolving in the previous slide and at x equal to a plus b; that is at the outer surface of the cladding is equal to Tc. So, if we put both of them this are temperature profile that we are going to get and now we can apply the interface boundary condition interface boundary condition refers the rate of heat transfer a both the temperature and the temperature gradient of the rate of heat transfer should be identical at the interface of the fuel and the cladding; that means, at x equal to a t a temperature should have a continuity and there is same condition we are using here at x equal to a t is equal to t as for both cladding and fuel.

And here we can use the second condition that is the rate of heat transfer conduction is only the mode of heat transfer here because both fuel and cladding are solids. So, minus k d T d x at x is equal to a should be equal to the total amount of heat flux that is getting transmitted through that phase here we have use the fuels of heat conduction and the heat flux just that is coming out through one of the phases.

Let me write it a better way the heat flux coming out of one of the from the phases should be equal to the energy produced in that half of the reactor that is as we have seen in two previous slide to a divided by the cross section area; that is why we are having this term here. Now using this interface boundary condition we can calculate or we can derive a we can derivate get the derivative of this and put the condition x equal to a.

(Refer Slide Time: 46:25)



And subsequently we get the temperature change across the cladding; that is, T s minus r c should be of a form like this q dot b by A into k c l.

So, again drawing the thermal energy we define the thermal resistance for the cladding as small a divide by A into k c l finally, the energy that as reach the cladding must go the coolant stream which is flowing on the surface of the cladding a a along the surface of the cladding and there the mode of heat transfer is convection.

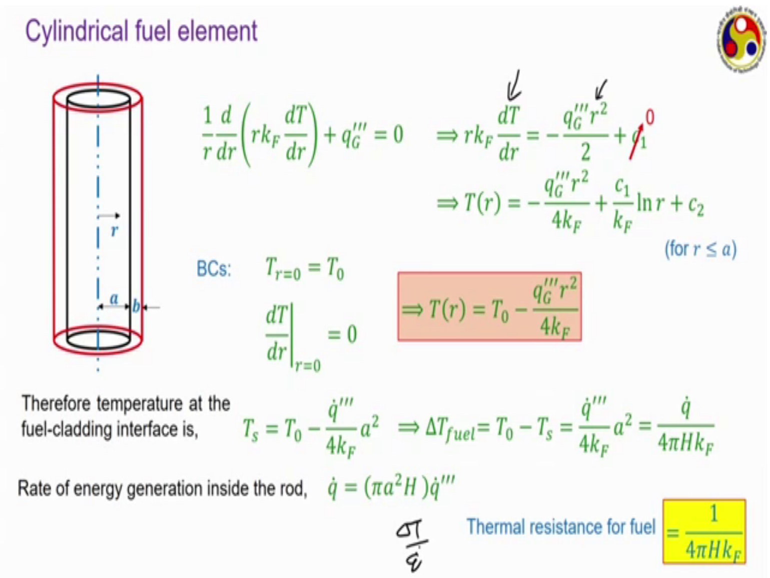
So, we can write a general balance between the cladding at the cladding surface; that is the amount of conduction heat received by the cladding should be equal to the amount of a transfer to the coolant by convection under steady state and hence the total rate of energy generation by half of the reactor should be equal to h A into T c minus T bulk here small h is the corresponding heat transfer coefficient.

And accordingly we can define a fling heat transfer a fling temperature difference delta T film as the temperature difference between T c and T bulk and that is found to be q dot by A h. So, we can define a thermal resistance for fling is 1 of 1 A h for which is the actually the very standard form of thermal resistors for convection in Cartesian coordinate this is a typical temperature profile that we may have actually in reactors you may find that the fuel and cladding are separated by small year gap or gap or by some kind of a inner a honour a some kind of inert gas.

So, inside the fuel we are getting this kind of temperature profile and this is the temperature change across the fuel this is the temperature change across the cladding and if the gap is there we are going to have temperature change across this as well and finally, this is the temperature change across the a liquid film this portion the film of liquid which may get generated on the outer surface of this beam outer surface of this cladding.

So, we shall be doing that in the next lecture again, but as we have already calculated on the three heat transfer resistances. So, we can also combine them to get a combined expression of total temp temperature difference that is the reason for having such a heat transfer which is the difference between the temperature at this particular point and the bulk temperature. So, T naught minus T bulk divided by heat transfer should be equal to the summation of all this resistances. We have already calculated the resistance corresponding to the fuel the cladding and the coolant or heat a coolant film I should say and if there is a gap is involved then also called calculate the resistance for that gap add to this.

(Refer Slide Time: 49:22)



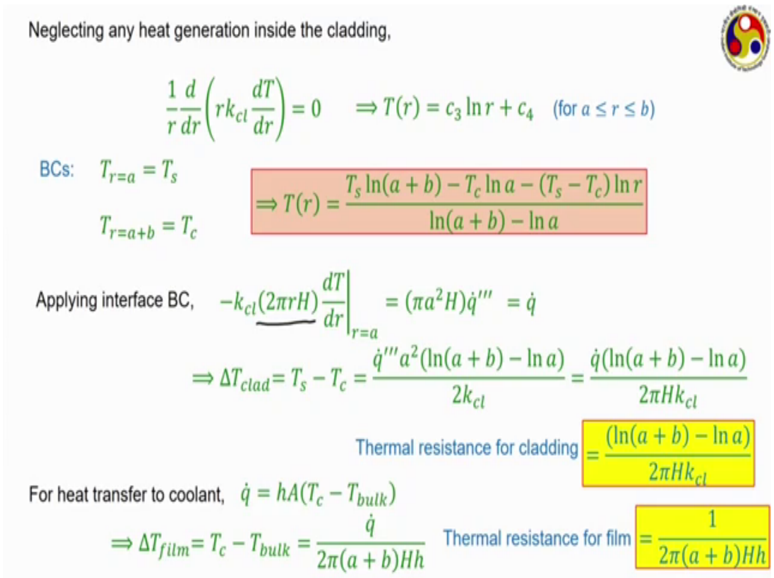
We can perform an analysis of a cylindrical fuel element the same way here we have a cylindrical the same way here we have a cylindrical fuel b in of radius small a and there is a cladding covered in of the a around the cylinder which is having a thickness of small b here we can perform the similar analysis, but following the cylindrical coordinate system. So, this is the one dimensional steady state heat conduction equation in cylindrical coordinate system and by solving this by we are here and then integrating it once more we are getting this particular temperature profile inside this cylinder that is for r less equal to a.

Now, we use the boundary conditions similar boundary conditions at r equal to 0, the temperature is T naught which is center line temperature against the inter line being plane of symmetry the temperature gradient is 0 as well. So, putting the second condition we can straight to as say that c 1 equal to 0, like if we put r equal to 0 here and d T d r equal to 0, here c 1 has to be a 0 and then putting this are the second boundary condition in the remaining this equation we get c 2 equal to T naught and accordingly this is the corresponding temperature profile.

Therefore temperature at the fuel cladding interface where r equal to a we need to put we are getting the corresponding expression and hence the temperature difference across the fuel is q dot triple prime by 4 k F a square rate of energy generation inside the rod can be given by like this by pi a square reaches the volume of this rod and q dot triple prime is the rate of energy generation per unit volume. So, if we use this definition in the previous expression the temperature difference can be written like this.

And now we can again draw the electrical energy to get the thermal resistance for the fuel which actually will be this delta T and divided by q dot. So, is equal to 1 by 4 pi H to k p where h is the height of the cylinder that we are talking about.

(Refer Slide Time: 51:21)



Again neglecting the heat generation inside the cladding we are have having this simplified expression and then we can integrate this way. Now the boundary condition can be to specify temperature on both of the cladding that is T a r is equal to T s and the second condition at the other edge of the cladding that is r equal to a plus b T is equal to T c. So, if we combine them we get this particular expression I would request you to a derive this on your own this looks complicated, but actually it can be quite simple.

And, now again, we apply the interface boundary condition; that is a total amount of energy; that is being produced by the fuel that must through the cladding itself; so k dou t dou h dou r equal to a to the a d r to the corresponding a d r should be equal to the amount of energy produced inside the cladding here 2 pi r H refers to the area of the cylindrical element at any radius r and h is the height and.

Accordingly, we get the temperature difference across the cladding is equal to q dot into log a plus b minus log a by 2 pi a H k c l and hence we can define a thermal resistance for the cladding as this log of a plus b minus log a by 2 pi a H into k c l and finally, again we can define a thermal resistance corresponding to a conductive heat transfer to the coolant by equating the energy transferred from the cladding to the energy received by the coolant small h again is the heat transfer coefficient and hence we can define a film temperature difference as q dot by 2 pi a plus b H into small h.

So, the corresponding thermal resistance for the film is this hence like in case of a planar element for cylindrical element also we have identified the three thermal resistances of course, by assuming the properties to be constant property like the k F and k c l we assume them to be constant; then we know how to calculate the three thermal resistances. And accordingly, we can calculate either the bulk temperature if we have a knowledge about the center line temperature or the central line temperature if we somehow identify all this 3. So, if we a can calculate or measure the bulk fluid temperature which is not very difficult.

One problem of course, will be this small h the conductive transfer coefficient which depends on the properties and also the velocity level of the coolant stream itself a, but the two geometrics that we are discussed here both are quite simple like one important assumption was the uniform heat generation that we are having inside the real fuel element which practically is not true like in the previous module already we have seen that neutron flux can have quite complicated distribution about the reactor and is the neutron flux is varying then the heat generation has to be varying in proportion.

So, in the next lecture we shall be taking it from this point onwards where this thermal resistance value will be utilized, but we shall we considering the neutron flux to be varying. So, thereby we shall be able to do a more complicated and involved heat transfer analysis. So, you I am leaving this up to today please go through the analysis and go through all this derivations I would request you to derive on it is all this steps on your own. And so, when we come for the next lecture you know all the intermediate steps and you know how we are deriving every time this.

Thanks and bye for that.