Fundamentals of Nuclear Power Generation
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Module – 04
Chain Reactions in Reactors
Lecture – 12
Simple reactor theory

Hello friends, we are back with our; the fourth module of our MOOCs course on the topic of fundamentals of nuclear power generation. We already had three lectures on the topic of chain reaction in reactors and today we are going to finish up this particular module by getting some more mathematical analysis about the neutron distribution inside the core and then some idea about how to design a reactor.

Just as a brief recap, I would like to go back primarily to the third lecture here we are discussing or we started discussing in this module about the chain reactions where you are introduced to the topic of neutron life cycle and multiplication factor and. So, now, you have clear idea what is the requirement of having a chain reaction and accordingly how can we control this multiplication factor by controlling several of its components, so that we can ensure a critical reactor critical means, where for every fission reactions only one of the product nucleus is allowed to participate in a subsequent fission.

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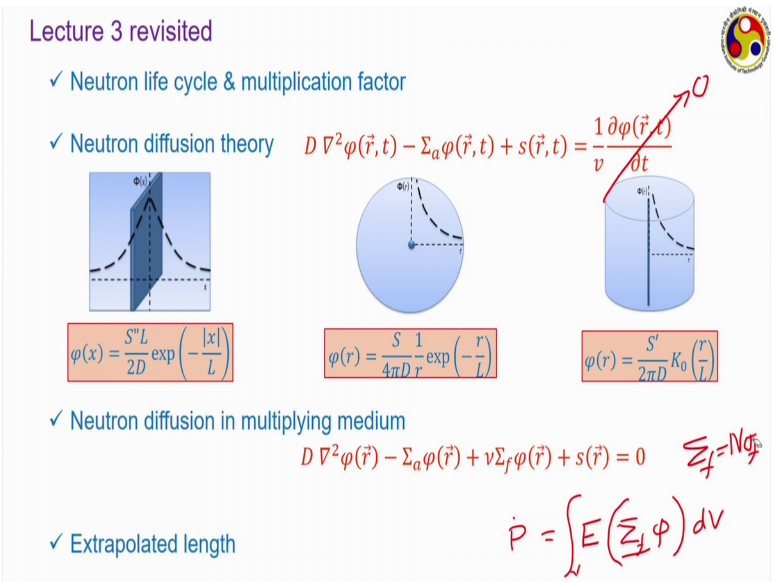

In the previous that is in the third lecture we have discussed in detail about the neutron diffusion theory, where this particular diffusion equation where developed I think it is in the second lecture only were we developed this particular equation, which basically states the balance of neutron inside the reactor or in a particular location. So, right hand side here represent the rate of change of neutron density with this small v being the velocity of the neutron, and on the left hand side we have three terms where this first term is a diffusion of neutron, which is also related to the leakage of neutrons and we generally get this form by rating the current density of neutron with the gradient, where the Ficks law of diffusion then we have the rate of absorption and finally, some source term where source can be of two types it can be an external source of an neutron or it can be neutron production because of the fission commonly thermal fission.

In the last lecture we focused on this particular equation or I should say the steady state version of this equation, because our discussion was limited mostly to the critical reactors, reactor critical means the neutron density is constant and it is producing a constant amount of power. So, the right hand side goes to 0 for a critical reactor and then we solved this particular equation under steady state for three different kind of geometries.

We took three ideal configuration the first one is that of an infinite plate, which is having infinite the dimensions in two coordinate directions, and almost 0 in the third coordinate direction or you can think this one to be an a plate with infinite area, but near 0 thickness and this is a source of neutron, which is immersed into an infinite extent of medium where there is no fission nuclei present. So, the only mechanism a neutron can have are the diffusion and absorptions and accordingly we got this particular dissolution of neutron here is double prime represents the neutron emitted by the source per unit area, d is a diffusion coefficient and l is the diffusion length. If you remember l square was related to d and sigma a.

Then we went to second consequent emission where we had a point source of neutron, this being point source it is capable of emitting neutrons in all possible directions over a sphere. So, we use a spherical coordinate system and got this particular distribution of neutron flask here is represents the source strength that is number of neutrons emitted per unit time by this point source. And the final configuration was again of inline which is infinite in terms of its length and. So, it can emit radiation in all possible directions of having a cylinder and using cylindrical coordinate system accordingly we got this expression, where is prime represents the source strength per unit length of the source and k not comes from the Bessel functions or it is the modified Bessel function or second kind.

So, for all this three distribution one thing is common that, at the which is quite logical as well near the source the strength of neutron flux is the highest, and as we are moving away from this that exponentially decreases. But all this three conditions all being ideal geometry that was done in non-multiplying media; that means, the neutron source was resumed to be inverse into a media, where there was no source of neutron present or no fission going on only mechanism is diffusion or only mechanisms are the diffusion, and absorption, and presence of source was recognised only very close it is close to the centre of the coordinate system or it physically why it is located which provides one of the boundary conditions in calculating these expressions.

But practically in a reactor we can have different kinds of a multiplying media present or I should say it is a not only non-multiplying media rather fuel itself also will be present. So, that the neutrons can participate in fission reaction, and accordingly produce some new neutrons. So, we can have additional sources of neutrons anywhere in the reactor, which accordingly we discussed about the neutron diffusion multiplying medium and we got this expression, where we have only one additional term this is only for a critical reactor.

So, we are taking 0 on the right hand side and we are having this additional term here which represents the generation of neutron because fission here mu is the number of neutrons average number of neutrons produced per fission, sigma f is the macroscopic fission cross section and phi of course, is distribution of a neutron on neutron flux distribution.

Now, and finally, we have discussed about this concept of extrapolate length of course, we solved this diffusion in multiplying media for one consideration of infinite slab, and the condition of extrapolate length was consider a mentioned while we are considering the neutron flux to go to 0, at the edge of the geometry, practical cases generally it does not becomes 0 there rather it can continue to proceed in the downs stream medium. So, by considering extrapolate length refers to if the medium is allow to continue then, where the neutron flux approaches 0. So, for most of the cases extra volatile length can be quite small, but when that is a significant portion then we must add like a if we talk about say.

If we talk about any particular of this reactions, always which we shall be discussing shortly also where this extra volatile length should provide some kind of connection to the physical length scale to consider. But one point I would like to mention before we move any further why we are having so much discussion about this neutron flask. Just think about how can you calculate the amount of energy produced in a reactor because a fission of course, if a e represents the amount of energy released from a single fission reaction, then total amount of energy produced in a reactor should be equal to this e multiply by the number of interactions.

Now how can we get the number of interactions? Number of interactions as per the definition has to be equal to the macroscopic fission cross section into the phi. So, this should be the number of interaction happening per unit volume, because phi represents the neutron flux or it is a number of neutrons per unit area, and sigma f is a microscopic cross section. So, this two together has a dimension of per unit volume, and now we need to multiply this one with the volume of the reactor itself or we may integrate this one over the entire volume of the reactor.

So, that should give you the power produced by the reactor or amount of energy released by the reactor. Now look at this expression here this the first term that this e this e is the amount of energy released from a single fission reaction. Do you remember how to calculate this; we have discussed this one in the second module itself. Once we know the details of the reaction then we can always calculate this very easily.

Because we just need to know the atomic mass of the parent nucleus and also the mass of the fission products, and then we can calculate a mass balance to identify the value of mass defect, and we know that one a m e of mass defect is equivalent to 931 MeV of energy. So, accordingly the mass defect is directly going to give you the value of this e that is the amount of energy released or amount of mass that got converted to energy through the fission reaction. For uranium or plutonium fission this e is the commonly around two hundred MeV or maybe just slightly higher like quite often a value of 212 MeV is considered.

Next the next term is this after we are able to calculate the value of E next term of sigma f. Now sigma f is something that is quite standard means sigma f as for the definition capital sigma or sigma f is equal to the nuclei density into I am sorry I should write correctly the nuclei density into I am repeatedly writing wrong thing sorry. So, it can be as the nuclei density multiplied by the microscopic cross section, now for most of the most of the material which used in a common nuclear reactor, the value of this mu is microscopic cross section microscopic fission cross section or any other microscopic cross section is quite standard. So, we can calculate the value of this macroscopic cross section quite easily once we have idea about this nuclei density. If you think about any reactor later on in one of the later module we should be discussing in details, but commonly reactors can be of two types one is homogeneous other is heterogeneous.

In a homogenous reactor we generally find a homogenous mixture of whatever component that you would like to put. Just you can think about one of the numerical problem we solved in a previous module, say your fuel is something like a compound of uranium like urinary sulphate or urinary carbonate kind of thing, which is dissolved into water, and we put this mixture inside the reactor. Now the uranium 235 that can be present inside the compound acts as the fuel, other components remains as it is and the water that act as a moderator.

So, everywhere as there is this homogenous composition, the value of this cross sections also remains the same and in that kind of situations we know how to calculate the cross section like there the macroscopic cross section or say the average microscopic cross section.

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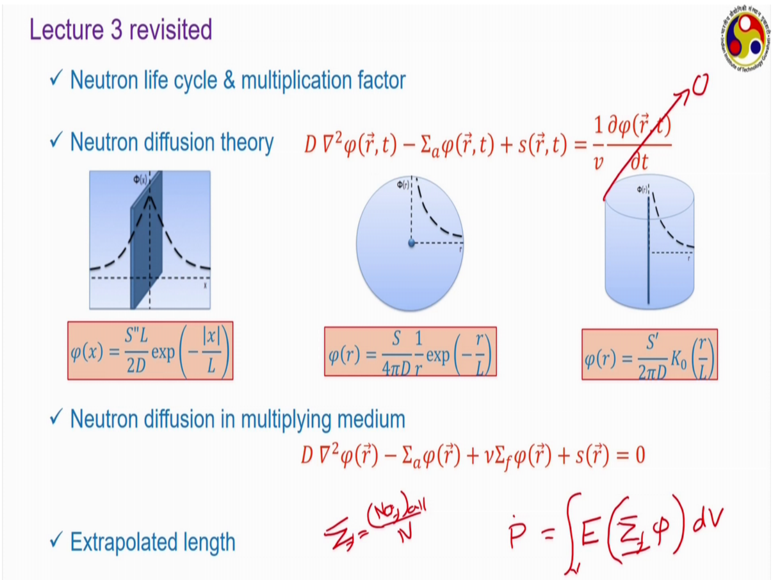

For the entire mixture will be the not microscopic, I should say the average macroscopic cross section for this should be equal to the N sigma f product for all divided by total N or I should say the N sigma I product you have to calculate for all components and then sum may have to get the numerator and then divided by the total number of nuclei that is present inside the reactor that instant of time.

Now, once we have calculated this e n sigma f then we are left with only phi. So, the calculation procedure of both E and sigma f are quite standard in that case the entire evolution of this power that reduces to this phi only, and depending upon the distribution of phi we can have different values of this P in different parts of the reactor, and hence it is at most importance to get a perfect idea about the distribution of this neutron flux.

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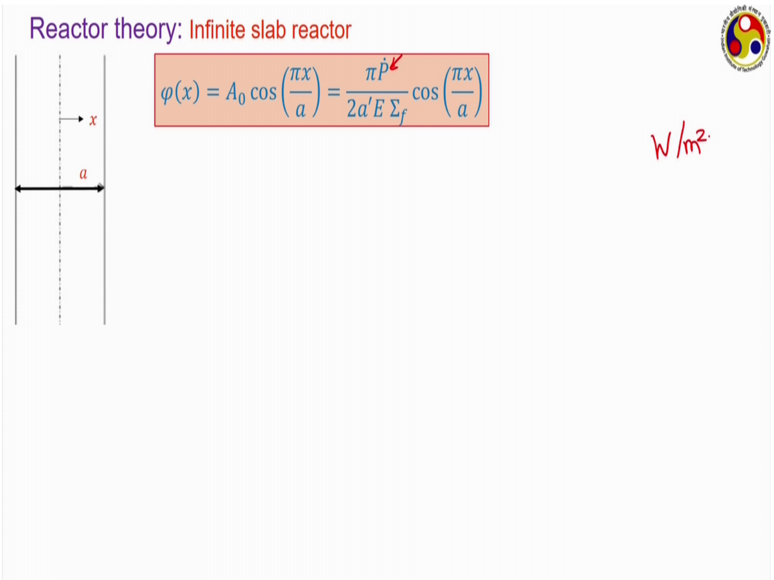

So, now we go back to the problem that we have solved during the previous lecture only. Here geometries of an infinite slab or reactor is of the geometry of an infinite slab, which is having a thickness of a and the coordinate direction was taken to the centre line. So, we have solved this and we got this particular form where A naught is the coefficient, and which can be calculated from the power rating furnace. Here one thing I should mention while P dot I am using here this one I am talking about this P dot refers as power because this is the common symbol that is used, but actually it is more like an energy flux.

Because this P dot has an is I unit of energy released per unit area of the reactor watt or kilo watt whatever. So, this while the symbol P dot it represents or it looks like energy or power I should say, but actually it is energy released per unit area of the slab.

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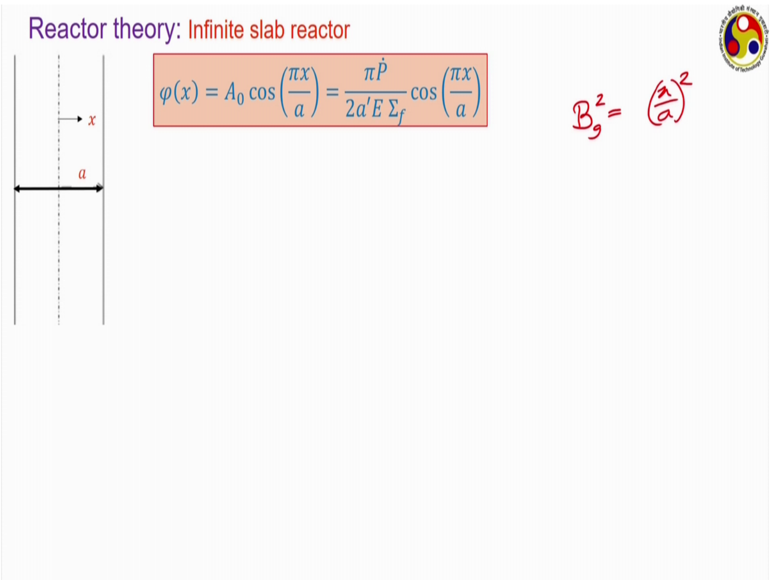

So, we can have already done this calculation and here the buckling parameter that can be calculated as this pi by a whole square, because B was a found to be phi by a. So, this up to the part we have already done in the previous lecture, let us now calculate a few more things from here.

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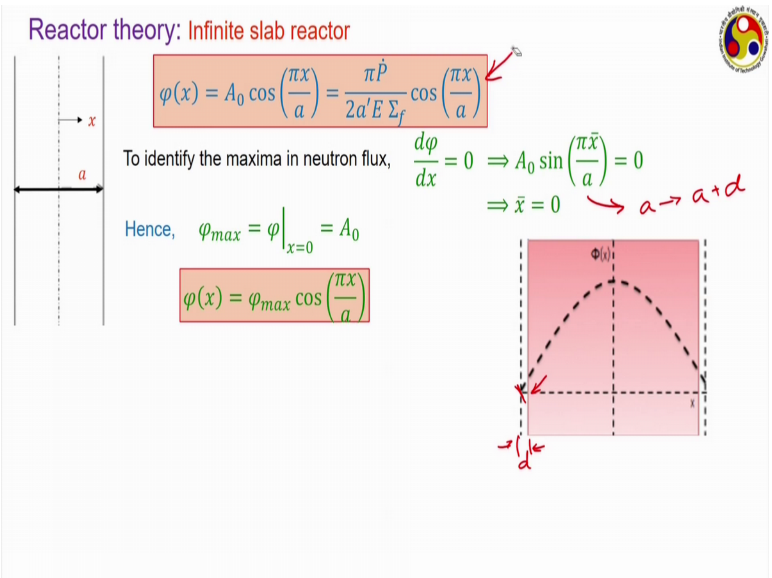

If we this of course, is a cosine distribution and hence we can expect it to vary along the length of the slab reactor and hence to identify the maxima is a quite straight forward procedure we have to get d phi d x and equate that to 0 and then we get this particular form and finally, get x bar equal to 0, where x bar represents the location of this maxima.

So, we can clearly see that the maxima in this neutron flux is located at the centre line itself and hence the phi max by putting x equal to 0 in this particular equation cos 0 becomes equal to 1 and hence we get phi max equal to A naught or whatever expanded form that we have for A naught and therefore, their distribution phi can be written as phi max into cos pi x by a because the entire coefficient is equal to the maximum value of this flask.

But quite this is the distribution that we can have; it is mirror image with respect to the centre line with a maxima at the centre line and gradually decreasing to 0 at the edge. Now this is the edge, but in this drawing it actually is non 0 at the edge and it is expanded to become 0 at somewhere here. So, this particular portion is the extrapolated length that I talked about earlier. Whenever we have this extrapolated length to be significant then this a needs to be corrected or this a net should be replaced by a plus d where d is the this extrapolated length or the thickness of the corresponding layer also along with the maxima quite often an average neutron flux is an important information.

As we can clearly see from this picture that the flask is maximum at the centre line, and sharply reduces to become l is 0 at the edge.

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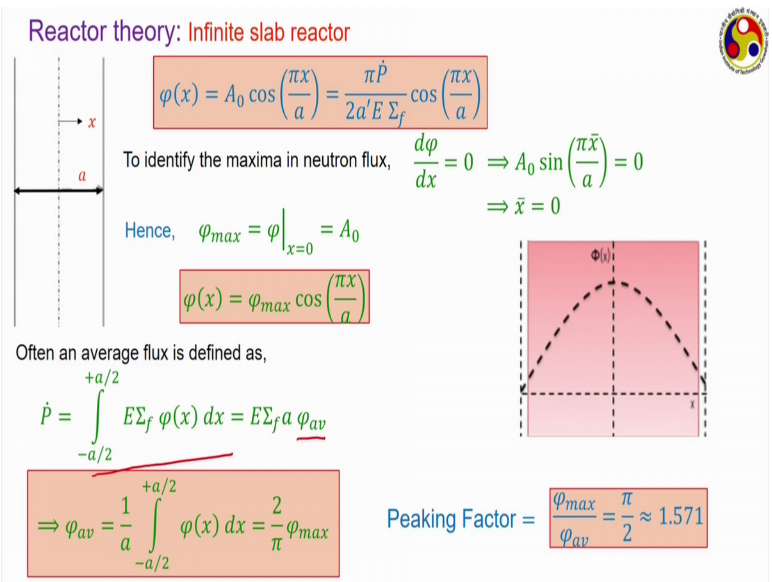

Now, average flask can be defined something like this, which where the this phi average should be such a value that it is able to maintain the same power rating of the reactor. Now the this particular one is the total amount of power produced during the reactor which we have already used earlier and this should be equal to E sigma f a into phi average, where E is the amount of energy released again sigma f is the macroscopic fission cross section and a is the thickness of this slab rector half width of the slab reactor. And hence we finally, get this particular expression this average phi value can be calculated as 1 by a into integral of phi x d x over the entire reactor that is minus a by two to plus a by two it comes out to be 2 by pi f into phi max.

So, quite often to understand the non-uniformity that can be present inside the reactor in terms of neutron flux, we define a peaking factor. This peaking factor is defined as the ratio of the maximum flux and the average flux and this case it is clear the maximum flux is about 1.57 times greater than the average flux. So, that is a strong indicator of the asymmetry that can be present inside the reactor in such configuration of infinite slab.

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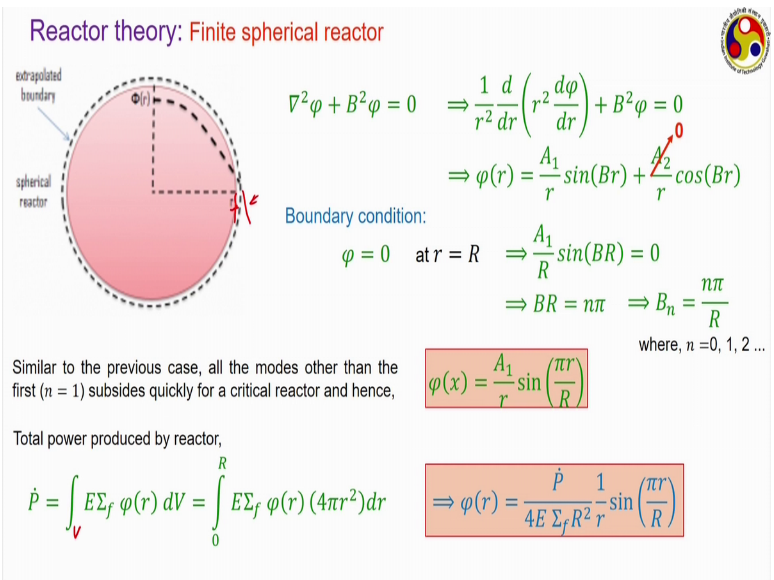

Now, it, but infinite slab reactor is only an idealization because nothing can be infinite in terms of length.

And hence we can we need to go to a more is finite geometry well there are quite a few final geometries, but we have selected the spherical rector as for our consideration. The governing equation remains the same that is Laplacian of phi plus B square phi is equal to 0 the B being the parameter the buckling parameter for critical reactor we know that B g square and B n square as to be equal and which is given by B square. So, this is spherical reactor and hence we have to use a spherical coordinate system, which gives like this if we assume the entire phenomena to the isotropic in nature, then only variation that remains is in the radial direction.

So, according this is the equation that you are having, and after solving this again a very standard ODE. So, after solving we have A 1 by r into sin b r plus A 2 by r into cos B r. Now we have to find the value of these two coefficients A 1 and A 2 for which we need to know the boundary conditions what can be the common boundary conditions? One boundary condition can be the value of this flux at the edge of the reactor that is at r equal to (Refer Time: 19:02) and other condition.

Suppose if we put in this equation r equal to 0 means we are talking about the centre line, centre line r equal to 0 would lead to the if you focus on this term, this gives to a 0 by 0 kind of form in that situation which is completely impracticable and hence only if only if realistic solution that we can get from this state of equation is when this A 2 goes to 0, A 2 has to goes to 0 otherwise the neutral flux becomes infinite at the centre line.

So, we now left with only a single coefficient which is A 1 which we have to finalize shortly. Now this is the other boundary condition that is at the edge of the cylinder not cylinder edge of the spherical reactor is r equal to capital R, we have the condition of phi equal to 0. If we are having some kind of extrapolated length like shown in the diagram this portion is extrapolated length in the diagram this r equal to capital R condition, that we have to impose on the same equation and we can find that at phi equal to 0 or phi equal to 0 r equal to R we have A 1 by R into sin BR and hence as A 1 and R both are non 0 numbers sin 0 has to come out sin 0 we know that that is equal to possible only when the it is equal to n pi. So, accordingly b r equals n pi at n is any integer and hence B n is equal to n pi upon r, n can be any integer from 0 1 2 etcetera

Now, as we have used in a previous exercise also here well they are theoretically several modes of operation, but nothing beyond the two or three that works. So, B n equal to n pi by r where n is equal to any integer starting from 0 going to 1 2 etcetera, but as we have seen in the previous exercise also, here well there are several possible modes, but all the modes for which n is greater than 1, they subsides quite quickly and leaving the final profile to resemble only the profile corresponding to n equal to 1.

Of course n equal to 0 is not possible because if you put n equal to 0 and that you give phi equal to 0 and hence your n equal to 1 is the most realistic solution that we can have of this particular form A 1 by r into sin pi small r by capital R and total power produced in order to get the value of A 1 we have to use the total power produced by the reactor, which is again given by a form like this here E is the amount of energy produced by each fission reaction sigma f is the fission cross section phi is the neutron flux which we have just derived and d v is the volume change it is the volume over which we are doing this entire integration of volume of the reactor. Basically we are doing this integration over the entire volume.

So, it will (Refer Time: 22:14) to a form like this and finally, we get the form of phi as something like this a sinusoidal profile, which is having its maximum at the centre line and it is 0 at the edge that is when small r becomes equal to capital R it is equal to 0, but the following the same pattern we can also calculate the maxima of this flux, and its location and also the average value of this flux.

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Geometry	Dimensions	Buckling	Flux	A	ϕ_{max}/ϕ_{av}
Infinite slab	Thickness a	$\left(\frac{\pi}{a}\right)^2$	$A\cos\left(\frac{\pi x}{a}\right)$	$1.57P/aE_R\Sigma_f$	1.57
Rectangular parallelepiped	$a \times b \times c$	$\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2+\left(\frac{\pi}{c}\right)^2$	$A\cos\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)\cos\left(\frac{\pi z}{c}\right)$	$3.87P/VE_R\Sigma_f$	3.88
Infinite cylinder	Radius R	$\left(\frac{2.405}{R}\right)^2$	$AJ_0\left(\frac{2.405r}{R}\right)$	$0.738P/R^2E_R\Sigma_f$	2.32
Finite cylinder	Radius R Height H	$\left(\frac{2.405}{R}\right)^2+\left(\frac{\pi}{H}\right)^2$	$AJ_0\left(\frac{2.405r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$	$3.63P/VE_R\Sigma_f$	3.64
Sphere	Radius R	$\left(\frac{\pi}{R}\right)^2$	$A\frac{1}{r}\sin\left(\frac{\pi r}{R}\right)$	$P/4R^2E_R\Sigma_f$	3.29

Each of the considered cases shows large variation in neutron flux distribution across the reactor core. Such non-uniformity in neutron distribution is unacceptable in practical reactors, which demands a flat profile, with minimal variation from centreline to the edge.

And for any other geometry also you can keep on repeating the same exercise.

But that will be too much cumbersome thing to do, and hence I have summarised this results of quite a few other geometries in this particular table. For the infinite slab and for the finite cylinder we have already developed is the infinite slab for which we have developed. For finite cylinder we have or not finite cylinder actually for the spherical reactor we have just not asked upon.

And we can have all the geometries like a cylindrical reactor, we can have a rectangular parallel piped type reactor for each of them we can repeat the same procedure by following the proper coordinate system like if you are dealing with the rectangular parallelepiped, then variation is there in all three possible directions and hence we need to adopt a three d method or three dimensional approach to identify the flux profile there same for a infinite cylinder or even a finite cylinder if we know the values of r and H still we need to put the assumption of axis symmetricity and then only we shall be able to analyse this finite cylinder following A 2 dimensional cylindrical coordinate.

But the most interesting factor here is the last column, which is the peaking power. Here we can clearly see all these values are much greater than 1, I am just going back to the previous slide here we can clearly see or in case of infinite slab also that was true the neutron flux is very high at the centre line, and then diminishing very very shortly and approaching 0 close to the edge. That is because of a large difference between the maximum and average value of this neutron flux.

1.57 was a value for infinite slab which we have derived already but the same number for the finite sphere comes to be greater than 3 and for other several geometries as well. Now this is not at all desirable, because inside the reactor we would always like to have a flat profile of the neutron. So, that the rate of reaction remains more or less the same in every part of the reactor. If the neutron flux is high only at the centre line and very low or 0 at the wall then the fluid which is getting energy close to the central line will become super heat very very quickly whereas, the fluid which is closer to the centre may not get that much of energy or rather the power production they are remains 0.

So, the while the coolant goes out of the reactor channel, there will be a large amount of axial variation in temperature and that is that can cause lots of several other issues hence we must ensure that the neutron flux profile remains more of the flat while the infinite slab 1.57 is quite increase in number, but it is still concept only. So, we must ensure that the neutron flux remains flat and so that, we can have a uniform distribution of energy received by the coolant over the entire cross section and one way of achieving that is by the use of reflectors. If you can remember earlier that the reflector we have mentioned in one of the early and discussions that the reflectors are commonly used, to reduce the neutron leakage from the reactor.

Now, neutron going out of the reactor is never desirable, rather because that reduces the neutron density inside the reactor and thereby effects the power generation inside the reactor, but use of reflectors can reflects reflectors are materials which are having higher scattering cross section and. So, they can reflect or diffuse some medium back into the rector and hence they it can provide a reduction in the total mass or the critical mass of the reactor.

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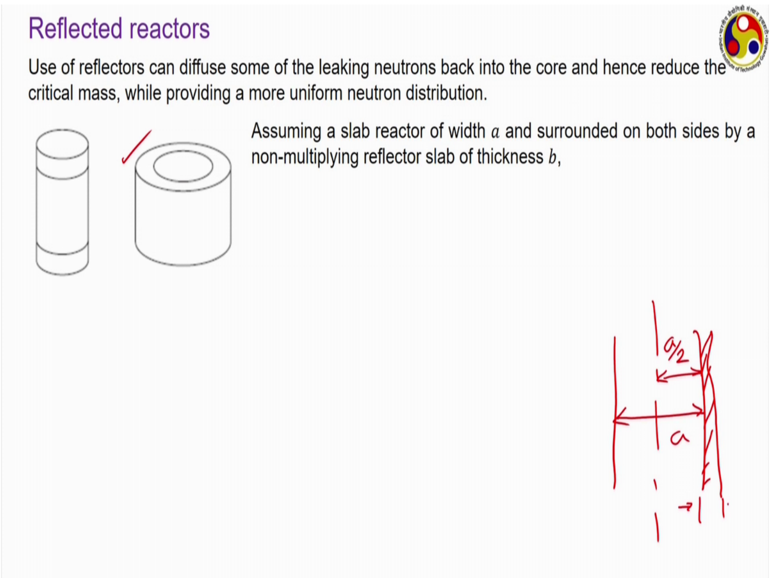

But it was mentioned earlier it can also provide a more uniform neutron distribution, and here we can see how that can be done. So, typical reflectors can have both kinds of designs, it can have actual reflectors or radial reflectors. This is axial reflector where reflectors are used only at the inlet and outlet section of the channel whereas, this is radial reflector or reflector is there all around. Both of them have their own pros and cons, but generally the radial one is found to be more convenient or so much easier to design and therefore, thus radial refractors more commonly used. Particularly in common pressurized water reactor or boiling water reactor the reflector radial kind of reflector is preferred. It is also very common to have the entire cool of a boiling water reactor to get immerse into pool of water because water is a good reflector.

Now, we assume a slab type reactor of width a and is surrounded on both side by a non-multiplying reactor slab of thickness b. Like the geometry of slab which we are taking earlier this is the slab type reactor where thickness is a or you can say at each of the side it is a by 2, but this is the additional part now we are having this reflector of thickness b on both sides of this slab. So, we have to analyse this thing and while the analysis for a such geometry we have already done for infinite slab, but the reflector was not there.

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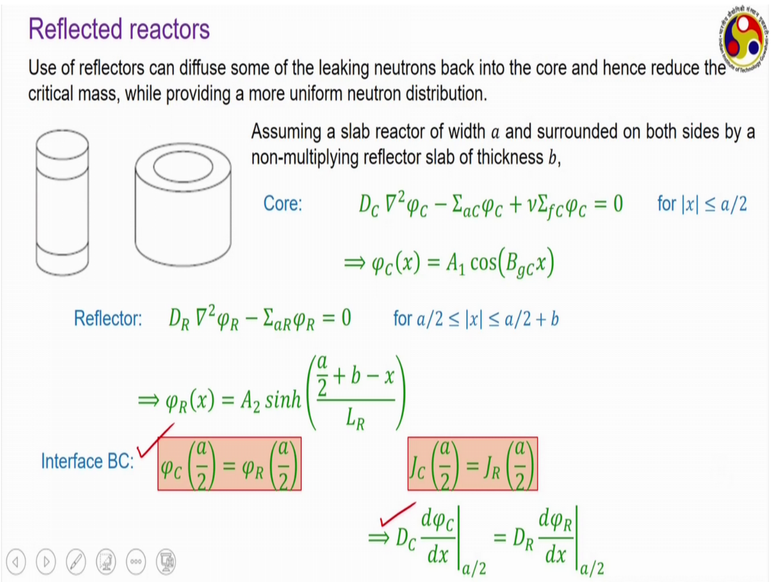

So, for the reactor core we can write the conservation equation to be like this.

Just the same form the diffusion part, the absorption part and the generation part because of fission, but this equation is applicable for x equal to 0 to a by 2, where the mod x is written because diffusion can be equal in both positive and negative coordinate directions and. So, we are taking mod of x. Here the subscript c that has been used that refers to the core. So, phi c refers to the distribution of neutron flux inside the core and. So, this equation being a standard you get this solution. One of the coefficient as was mentioned earlier in order to keep the neutron flux finite at the centre line, one the sinusoidal term goes out we are having only the cos term that is remaining.

Now, for the reflector that is from a by 2 to a by 2 plus b, within that this is the corresponding cross section equation here phi r is the neutron flux inside the reflector, there is no multiplication or no fission going on. So, we do not have a third term, but we definitely have the other two term that is a diffusion of neutron inside the reflector and also the absorption on neutron.

Now, if we solve this one we are going to get this particular form of solution for phi r again boundary condition has been applied and one of the coefficient goes of living only one coefficient. So, this A 1 and A 2 are the two coefficients that we need to solve using the power rating of the reactor and any such information. Now what should we should use as the boundary condition to get this values of this to get a coupling between this two of course, the reflector you have to understand the reflector.

If I draw the diagram again this is the centre line is the edge of the core and this is the reflector. N ow in the reflector there is no source of neutron because there is no fission going on then from where it can get the neutrons. So, that can that is only possible by diffusion from the core itself, and as there is a diffusion going on as there is a strong transfer of moment newton or neutron is going on from the fuel to the reflector this particular interface is of very important large importance, this is corresponding condition is called interface boundary condition.

Interface boundary condition ensures that flux phi does not have any kind of discontinuity or jump at the interface, rather the value of flux neutron flux distribution in the core and the same for the core distribution at inside the reflector, they should be equal at this particular interfacial surface. If and they only we can avoid any kind of jump in the value of the neutron flux and we have a second boundary condition. The equality of flux alone is not sufficient rather we also have to consider the gradient of the flux of neutron current density that also should be equal for both core and reflector at that interface.

So, here j represents the neutron flux density and different current density and by using a Ficks law of diffusion we arrived at this J being d of d phi d x. So, we get a this particular equations. So, accordingly we have two expressions where we are taking the equality of neutron flux between reactor and core and this is another one where we have the gradient of velocity at reactor and also at the reactor core and also at the reflector.

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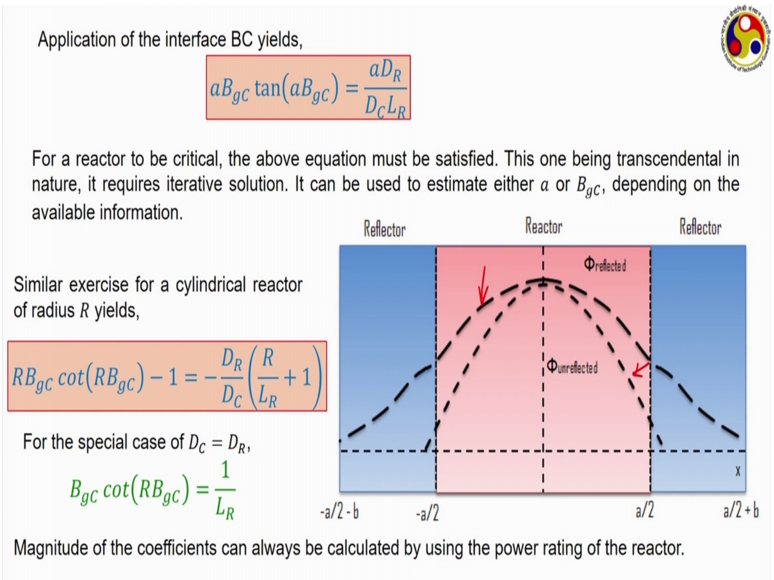

If we combine these two boundary conditions, then this is a solution that we are going to get. Now if combining this two this is the final form of the equation that we are getting. I have skip the intermediate skills just to save some time, here B g c refers to the buckling parameter for the core D C and D R are the diffusion coefficients for the core and reflectors respectively and L R refers to the diffusion length inside a reactor inside the reflector as well. This for this reactor to be critical this above question must be satisfied now this is actually a transcendental equation.

And. So, we cannot solve it directly rather it requires iterative solutions, with some possible starting guess. This can be used to calculate either this buckling parameter B e or any other parameter is a dimension a, that depends on what information is made available to you we can decide which one to calculate from here. Like if your objective is to identify the dimension of this reactor, then you definitely try to solve this for a with a knowledge of this buckling parameter. Similarly if the objective is to identify the buckling parameter then we need to know the information about the a. So, this is a kind of profile that we are going to get the first one that is this one is un reflected, there is no reflector there we can see the rapid decline in the neutron flux distribution, but once we put the reflector in look at what we are having.

Now, this is the corresponding flux distribution and they are definitely much flatter and reaching 0 only at the edge of the moderator. So, use of reflector definitely helps in flattening out the neutron flux distribution and hence it can lead to a higher rate of fission reaction and neutron flux distribution and there by allow a smooth distribution of temperature. Next if we do the same exercise it was done for an infinite slab reactor which is bit of idealization.

If you do it for a cylindrical reactor, then we are going to get this kind of reaction another transcendental equation here r refers to the diameter of the cylinder for the radius of the cylinder and for the special case when both the moderator or the reflector and the core are having identical values of the diffusion coefficient or basically when the same medium is used at the moderator and the in the reflector in the reactor, then this ratio of D C by D R by D C that goes to 1 and it reduces to a even simpler form.

And now this does not seem to be a transcendental anymore, because not special for simplified we get this particular simplified form B g c equal to cot of RB g c equal to one by L R this is not transcendental because if for the value of B g c is given, this can be used to calculate the radius of a cylinder the magnitude of the coefficients I have already mentioned this two coefficients A 1 and a to can be calculated from the power rating of the reactor and finally, we have the two group approach to talk about. So, far whatever analysis we have done there we have considered a single group of neutrons, because we are dealing with one group fission equation or one group newton diffusion equation.

But during the nuclear reactor neutron goes through different levels of energy.

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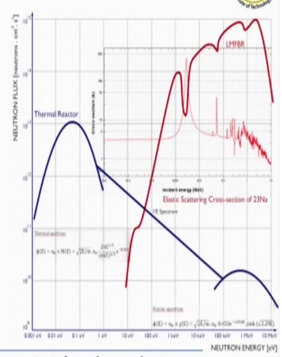
Two-group approach

Fast neutrons produced through fission can diffuse across several energy levels, as they scatters in the moderator and finally get thermalized. In fact, prompt neutrons can have wide energy spectra depending in reactor design.

Therefore it is logical to lump the neutrons in several energy groups during analyses. The simplest approach is to consider 2 groups, namely, fast & thermal neutron groups. The properties for each energy group are averaged over the concerned range of energy.



Thermal-group constants for various moderators				
Material	Σa [1/cm]	λa [cm]	D [cm]	L [cm]
H2O	0.022	45.5	0.142	2.54
D2O	3.3 E-5	30300	0.840	160
Be	1.24 E-3	806	0.416	18.3
C	3.2 E-4	3120	0.916	53.5

Fast-group constants for various moderators			
Material	D1 [cm]	Σ1 [1/cm]	τT [cm2]
H2O	1.13	0.042	27
D2O	1.29	0.0099	131
Be	0.56	0.0055	102
C	1.2	0.0028	369

Like initially the neutrons when they appear because of fission, they are first in nature then they passed to the moderator and becomes intermediate temperature level and when we at the intermediate temperature zone, it can get eaten up or the resonance (Refer Time: 36:25) that happens and. So, it is able to reach the thermal neutron level.

Accordingly we can have a very wide spectrum of neutrons present in the reactor, this is one of the representative diagrams here that this purple colour line represent thermal reactors or thermal neutrons produce because of thermal fission, the red one represents that for fast fission and you can clearly see there is a wide spectrum of energy with which the neutron can appear.

So, it is more logical instead of using a single group assumption, it is more logical to lump the neutrons into several energy groups during analysis. There are several ways they are clumped sometimes people prefer three groups like the fast neutron group the intermediate neutron group and also the thermal neutron group, but here we shall be speaking out cells to the two groups to show an example, where are fast neutron and thermal neutrons. So, once that is in the level of fast neutrons that is kinetic energy is of the order of one MeV, then we shall be taking that into the fast neutron group or we shall be considering that in the fast neutron group and whenever the energy is less than that, we shall be considering that in the thermal neutron group.

And the properties of each group are averaged over the concerned range of energy like these are the these are some numbers which are generally considered for the thermal group neutrons, and they also depends upon the moderator. So, the first one is the absorption cross section second one is the reciprocal of that which is the absorption mean free path, then we have the diffusion coefficient and finally, the diffusion length l, l is again d y sigma s. So, from there also it can be calculated l square is d by sigma s from there l can be calculated.

So, these are the more most common four moderators, and you can clearly see the H 2 O is having a quite small diffusion coefficient, but there are other mediums accordingly they are like D 2 O. D 2 has a much larger diffusion coefficient quite close to graphite and is diffusion length is also much larger and these are corresponding numbers for fast neutron groups again here the graphite or carbon is the one that is showing very large values of these diffusion length.

Now, we have to present a brief analysis. So, we are assuming that the neutrons appear in the fast group as a result of thermal fission only, that is when the thermal neutrons participate in fission reaction the products of those are the one that comprises this or contributes this fast group, but the only source of thermal neutron is when this fast neutrons passes through the resonance absorption zone and because of a scattering it becomes thermalized.

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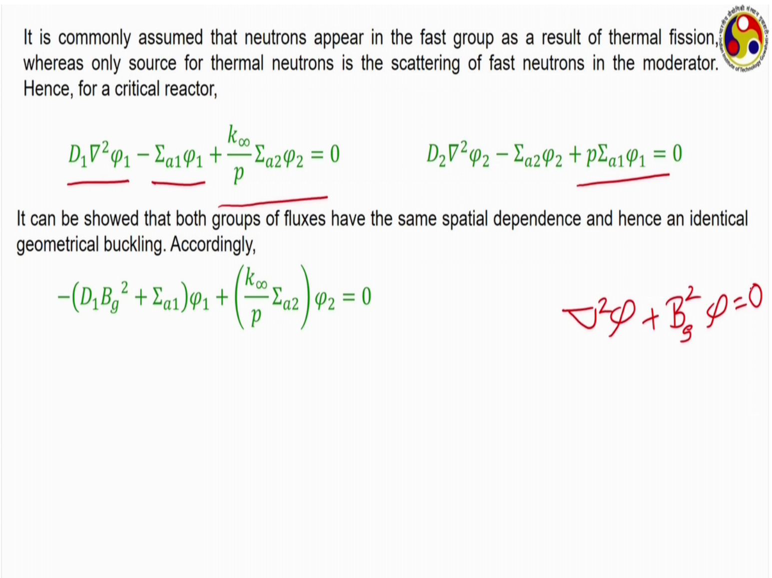

So, for a critical reactor we can write two different equations two different balance equation this is for the fast group of reactor, which you can think about the fast reactor group here we have the first term is the diffusion of fast reactors then absorption of fast reactor because of with the number of neutrons will decrease and third is the production of fast reactors. Of course, k infinity gives you the total multiplication factor the infinite multiplication factor, once you are dividing that with the resonance escape probability this is and the rate we are going to get.

Similarly, for the second group this is the equation, where along with the leakage and absorption term we have this, it represents the amount of neutrons that is the source of neutrons which is the neutrons which are passing through the resonance absorption zone and hence p is the resonance escape probability. So, sigma A 1 multiplied by this p is the amount of neutrons that are able to thermalize, and hence be part of this thermal neutron group.

Now, both kinds of fluxes generally show the same kind of spatial dependence and hence there have an identical geometrical buckling. Accordingly the first (Refer Time: 40:46) takes this particular form here we have only represents that Laplacian of phi as the buckling into phi or minus of that rather or you can also write this as this plus this is equal to 0, and equation which we have repeatedly used. So, with that substitutions we get a form like this and we can do a similar procedure for the second one.

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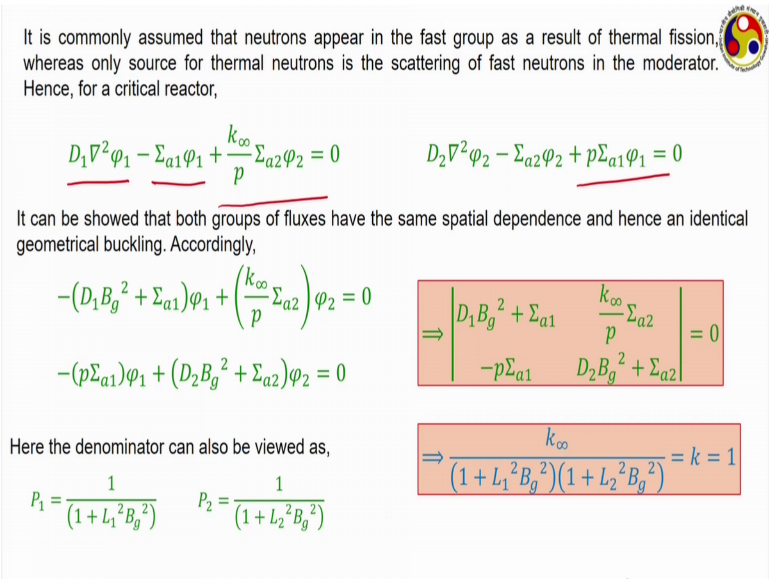

This is the corresponding equations. So, we are having actually two algebraic equations which are coupled with each other, and the solution of non 0 or nontrivial solution for this is possible only one that this particular determine or the coefficients is equal to zero.

So, from there we get this as the final solution. tThis equation has to be satisfied for a reactor to be critical a reactor with reflector to be critical, here the denominator can also be viewed in this way like 1 by L 1 square B g square can be thought about the fast neutron non leakage probability whereas the second term of the reciprocal of 1 plus L 2 square B g square can be viewed as a thermal neutron non leakage probability.

So, instead of P 1 we can also write this as P f and we can also write this as P t h. These are the two non-leakage probabilities because that includes the diffusion length as well diffusion length for the zone 1 and diffusion length for zone 2. So, this way we can analyse this two different groups of neutrons, there can be situations where you have to deal with more than two number of groups and just by putting this equations writing corresponding equations for each of them and applying suitable boundary conditions and also coupling situations you can always solve this kind of systems.

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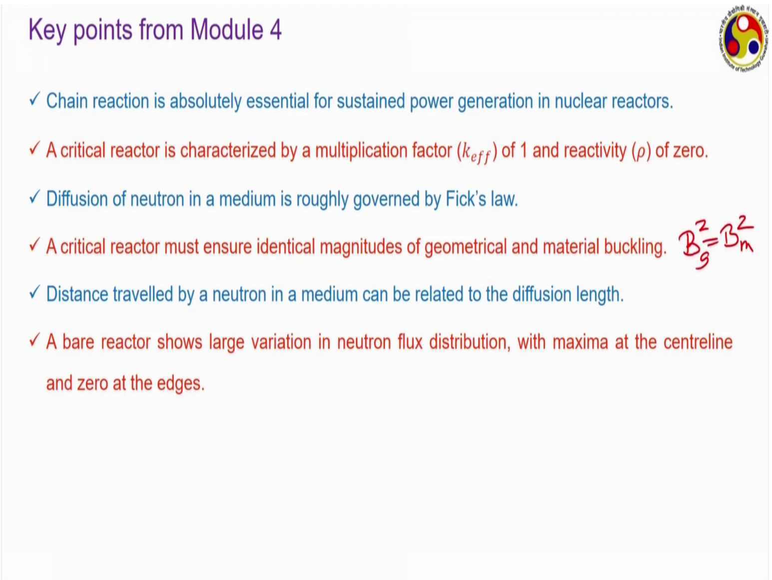

So, that takes us towards the end of this fourth module, where we have discussed in detail about the chain reaction have understood that is absolutely essential for sustained power generation in nuclear reactors, then a critical reactor is characterized by a multiplication factor value of one and reactivity of 0. The diffusion of neutrons in a medium is roughly governed by the Ficks law, using the Ficks law and also neutrons valency I have written the single (Refer Time: 43:19) diffusion equation, which includes the diffusion term the neutron production because of fission or external source term and we can also the absorption term. Then following from there onwards we have identified that the critical reactor must ensure identical magnitudes of both geometrical and material buckling parameters because that is the mandatory condition or we can also write mathematically B g square is equal to B n square.

The distance travelled by their neutron inside a reactor can be related to the diffusion length. There we have also distinguish between the diffusion length and the mean free path mean free path is a total distance travelled by a neutron between two successive interactions where the diffusion length is a straight line distance from the point of its birth till the point where it gets absorbed.

Then we have done the mathematical analysis of the bare reactor, we have seen that a beer reactor shows large deviation in the value of the neutron flux distribution with the maximum at the centre line and the this induce 0 at the edges or in some cases if the extrapolated length is considered is 0 at the extrapolated length.

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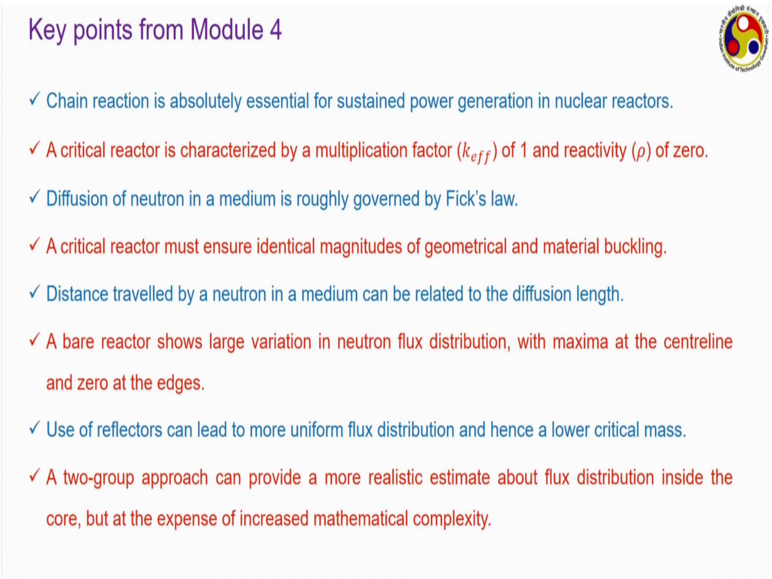

And then we have analysed with the reflectors and it was find that use of reflectors lead to more significant flux distribution and hence a offers a lower critical mass.

So, this is the fourth module where we have discussed about chain reaction and now you have much clearer idea about the possible neutron flux distribution, that we may encounter in a particular reactor and once we know or once we have the complete inversion about this flux distribution, how to utilise this in calculating the power rating of the reactor.

So, this is the end of actually I forgot about mentioning the last one, which is the two group approach which provide a more realistic estimate and, but I did computational or mathematical complexities course. So, we now know how to calculate the total energy that can be produced from a reactor, and in the next module we shall be discussing about how we can harness that energy and transfer that to the coolant, because we shall be talking about the nuclear thermal hydraulics. So, just wait for that and for the moment.

Thank you and bye.