Fundamentals of Nuclear Power Generation
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Lecture – 11
Solution of one group diffusion equation

We are back with the 4th module of our MOOCs course, where we are discussing about chain reactions in reactors. Of course, we already had 2 lectures on this and from there hopefully have understood the importance of chain reaction.

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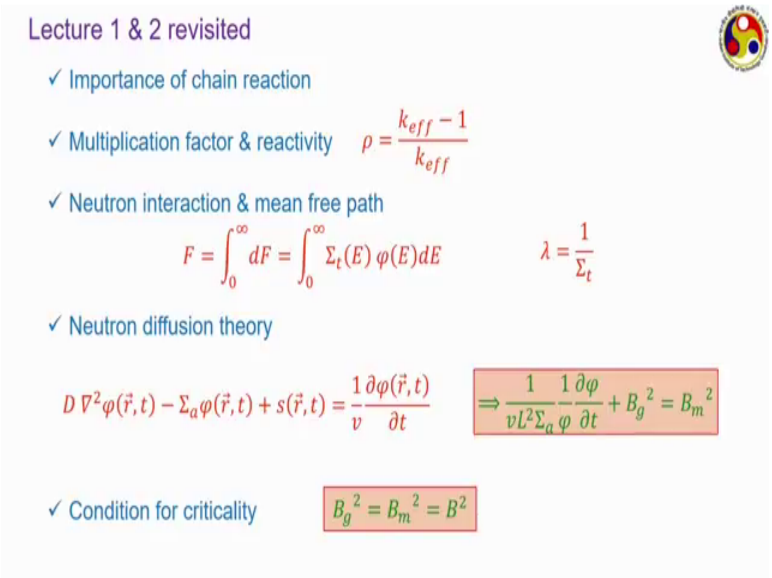

So, just a part of the recap, for whatever we have covered in the earlier 2 lectures, you have so far understood that whenever there is a fission reaction there generally produces neutrons. Now, the number of neutrons produced from each fission may vary between 1 to 7, but commonly it is 2 or 3, but just one fission itself is not sufficient rather we need to have a chain of such fission reactions.

And in this particular context we can connect this particular module with the previous 2 modules as well. Like in module 2, we have understood while discussing about the artificial radioactivity, we have learned that we can induce radioactive decay to any nucleus by striking it with a suitably charged particle and the choice for that particle is generally a neutron.

But, what will be the nature of such kind of neutron nucleus collision, that generally depends upon both the energy level of the neutron as well as the property of the nucleus itself and from our third module we have learned that, out of several possible kinds of interactions fission is just one of them and only very few nucleus like uranium 233 or 235 or plutonium 239 they have some kind of significant fission cross section. So, that they can undergo fission reaction, when being struck by a thermal neutron, uranium 238 has a small fast fission cross section, when it is subjected to first neutrons, but generally most of the common nucleus do not have any fission cross section at all.

Now, so we understand a or we identify particle as a nuclear fuel only when that is having some kind of fission cross section and just inducing a fission by striking the nucleus within neutron itself is not sufficient, where are the neutrons which are being by that fission at least one of them should induce fission to the neighbouring nucleus then only we can have a chain of reactions.

Such kind of chain reaction is quantified in terms of the multiplication factor and reactivity, where this is their relation which we have already learned. A reactor, which is having a multiplication factor of 1 and reactivity of 0, is called a critical reactor, where for every fission reaction only 1 neutron that is produced from the fission is allowed to participate in a subsequent reaction and accordingly the rate of fission reaction remains constant. So, that the reactor is able to produce a constant amount of power over a long period of time.

When, more than 1 neutrons are participating in the subsequent fission reaction we call that a supercritical reactor, here the multiplication factor is greater than 1 and reactivity is positive, whereas for a subcritical reactor multiplication factor is less than 1. You can somehow relate this process of chain reaction to the cell division that happens in our bodies.

Like, in a healthy human body there are always some new cells that are getting produced because of cell division similarly some cells are also dying because of aging issues or maybe some other relevant factors. Now, in a healthy fully grown up body generally the number of such cells produced because of division is balanced by the nominal cells that are dying and accordingly total number of cells is more or less the same; that you can relate to a critical reactor.

Whereas, a supercritical reactor is can somehow resemble the situation with a tumour or with a cancerous tumour there the cell division is uncontrolled, so that the number of cells in that particular tissue that keeps on increasing at a rapid rate and that is not at all being balanced by the date of the cells, accordingly total number of cells keeps on increasing. Similarly, a supercritical reactor also will show sharp increase in the rate of fission reaction, hence a diverging kind of power production rate which we generally relate to nuclear weapons.

But, the most important factor in controlling this multiplication factor like, commonly in power reactors, we would like to have a multiplication factor of 1. So, that we can maintain a critical reactor and hence a constant power generation rate, but in certain situations we may have to increase the power production rates, so we can go for a supercritical situation or sometimes if we like to shut down the reactor or lower the power generation we may go to the subcritical stage.

But controlling this reactivity is very important and the most important factor in that control is the role of neutrons. Therefore, in previous lectures you have discussed a lot about different factors associated neutrons, we have discussed about neutron interaction which is generally given by a relation like this and we the mean free path characterizes the distance; average distance travelled by a neutron between 2 successive interactions or which you can be related directly to the corresponding cross sections and then we went to studying the neutron diffusion theory, the Fick's law was introduced which governs the diffusion of neutrons in a particular medium and using that we can get a balance in the number of neutrons present in a medium, that is the total change in the number of neutrons present in the medium can be related to the rate of growth and the rate of absorption and also the rate of leakage.

Accordingly, we got this one, this single energy group or single group neutron diffusion equation. Here, this D is the diffusion coefficient, sigma is the absorption cross section, s is a source term the source of neutron and v is the velocity of the neutron. So, if we are dealing with a critical reactor that we can come just after some time, this equation can be modified to a form like this, where we have these 2 new terms, these which are called buckling. B g square is called the geometrical buckling as it is generally a function of the geometry, whereas B m square is called the material buckling as it is rated to the material properties.

And when the, we are operating with a critical reactor then the neutron flux that is this phi that remains constant to time. Hence, this term goes to 0 and therefore, for a critical reactor we have this particular condition satisfied that is B g square is equal to B m square. So, up to this part that is till the previous lecture, we have discussed about or we have developed this neutron balance equation and we have discussed about the internal diffusion theory.

So, today we shall be utilizing this equation to study the neutron flux that we may encounter in a particular given kind of reactor in a given geometry and subsequently we would like to use that in design of different reactors. So, let us take 3 different sample problems and our objective is to sample geometries I should say, our objective is to identify the distribution of neutron flux for a critical reactor in each of such configurations.

And I am actually not sure how much time the subsequent slides are going to take place because here we are having lots of mathematics to consider. Had it been a normal class, where I am dealing with a chalk and board, it could have taken much more than a single lecture, but here as everything I am having on the slides it may go through very quickly, but still let us start this.

The first geometry we have is that of an infinite planar source, like our geometry is that of a plane, which is acting as a source of neutron and it is having nearly 0 thickness or extremely small thickness.

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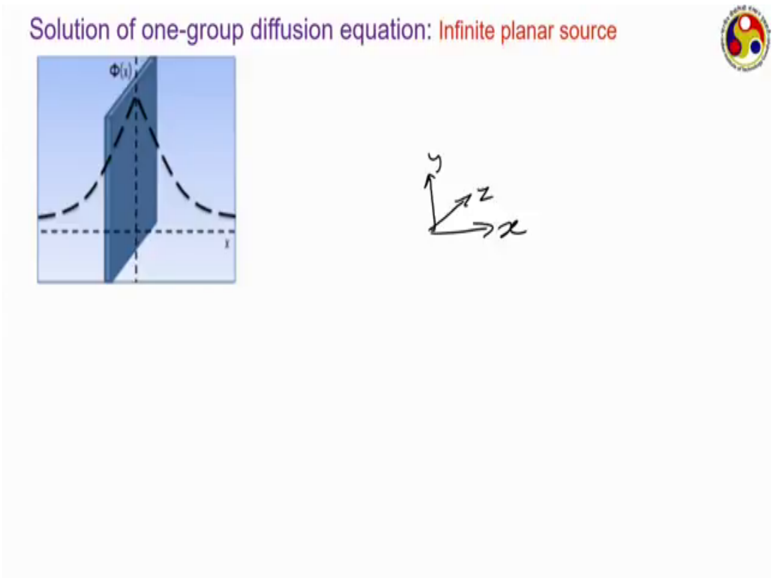

Look at this diagram, here this by this dark colour we have the plate shown, here we can take this particular coordinate system x in this direction that is normal to the plane and these are the other through coordinate systems. Now, here our geometry or our problem definition is the dimension of the plate in the y and z direction is infinitely large, that is the length scale that we have in the y and z direction are much, much larger compared to that in the x direction or we can write, if L x refers to the length scale in the x direction is much, much smaller than length scale in the y direction and length sale in the z direction.

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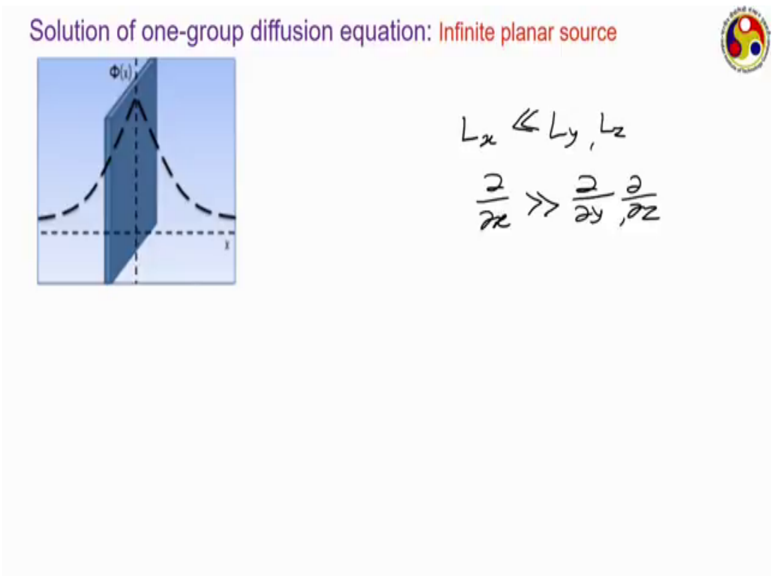

And hence, subsequently we can write the gradient in the x direction, gradient for any quantity that has to be significantly larger compared to the gradient in the y and gradient in the z direction and hence gradient of any quantity in the y and z direction can be neglected reducing this to a 1 dimensional problem, where we can safely consider all variations to be taking place only in the x direction that is perpendicular to this particular plate.

Here, we have taken our coordinate system to be at the centre or line of this plane and the thickness as I have already mentioned the thickness of the sources can be considered to be negligibly small.

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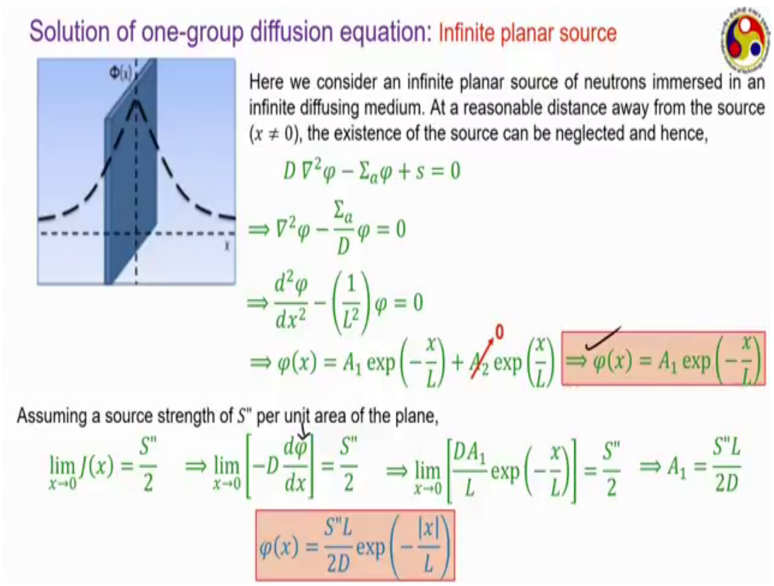

So, this is the standard form of or a for the diffusion equation, which we have already studied in a previous lecture and also mentioned in the previous slide under steady state the transient term goes to 0 and so we have the 3 terms. Where, this is the rate of absorption, this is the generation related term or I should say, I should not say generation rather this is the neutron diffusion or leakage, where we have used the Fick's law of diffusion and this s is the generation from some external source or neutrons supplied to the system from some external source.

Here we are not at all considered any kind of fission reaction, rather you can think about that this plate or the planar source of neutron that is immersed into an infinite stretch of diffusing medium, where we do not have any fission reaction happening.

Therefore only thing that the neutron can encounter is diffusion and also absorption by the medium which is surrounding this plate. So, there is no other source of neutron away from the plate only source is present that is at the centre of this coordinate system that is at x equal to 0. That means, at this particular condition, location we have some kind of neutron source, but which are moving away from the plate there is no neutron source.

Therefore, a small distance away from this neutron source of this plane we can neglect this term s and accordingly this equation simplifies to this. Now, sigma a by D as per our earlier definition is the reciprocal of L square, L will being diffusion length. So, this here we are using the Cartesian coordinate system as that is the most suitable one to this geometry. Accordingly, we have d 2 psi, d x 2 is equal to 1 by L square into phi, which is a very, very standard second order differential equation and we know the solution is going to be of this particular form, that is it is going to be a summation of 2 exponential terms. A 1, A 2 are 2 constants, whose values we have to identify using the boundary conditions.

Now, you see that the neutron flux phi is given as a summation of 2 exponential terms and both of them are functions of x, here L is the diffusion length, here one x, the first exponential term is a decaying one, where is, the second exponential term is a growing one. That is as we are moving away from the plate, while the first term will start to diminish, the second term will keep on growing, but that is not a very feasible situation to have, because you can always expect as we are moving away from the plate, corresponding neutron flux that should decrease we takes and that is possible only when the second term is absent in this and hence this A 2 has to be 0.

So, we are having this single coefficient A 1, that we need to evaluate using the boundary condition. Now, what can be the boundary condition, this is the simple form that we have, but what can be the boundary condition that we need to use. Here, look at the centre line or the dotted vertical line, this particular line that is shown in this problem.

This refers to the, at x equal to 0 we have the source and as we are moving away from this the effect of source gets negated or rather I should say, the effect of source can be eliminated, but at the centre line that is at x equal to 0 location, the source is present which is continuously emitting neutron following some kind of pattern and this neutron flux, our corresponding neutron current density at this centre x equal to 0 should be equal to the strength of the source.

Now, let us consider S double prime as the source strength per unit area of the plate that is the plate is emitting neutrons at a rate of S double prime neutrons per unit area, per meter square say for the plate. So, this source strength S double prime or this neutron emission per unit area should be equal to the neutron current density at the centre line, that is limit x tends to 0, J x should be equal to S double prime by 2, why this by 2 is coming into picture, because you have to consider that S double prime is the a strength of the source, but it is equally emitting in both a positive x and negative x direction and therefore, the positive x direction, which you are considering here is receiving only S double prime by 2 or half of that amount.

So, the neutron current density at x equal to 0 location should be equal to S double prime by 2 and from Fick's law of diffusion, we know that J x is equal to minus D d phi by d x. So, here we can put this particular form of phi that is we can differentiate this form of phi and put it back here. So, we are getting this thing and now putting this limit x tends to 0, we get A 1 to be equal to this particular form S double prime into L by 2 D. Hence, this is the final form of this neutron flux distribution in this infinite stretch of diffuse medium that we are having, which is surrounding this planar source.

Here, we are putting x mod of x because this particular distribution is true on either side of this plate, that is for both positive x and negative x distribution, this distribution is true or you can must say that the distribution is on one side of this dotted line is a mirror image of that on the other side. Corresponding distribution is also shown by the dotted line, also shown by this is the distribution which is already shown here.

Now, this is the situation of a very simple geometry of an infinite planar source.

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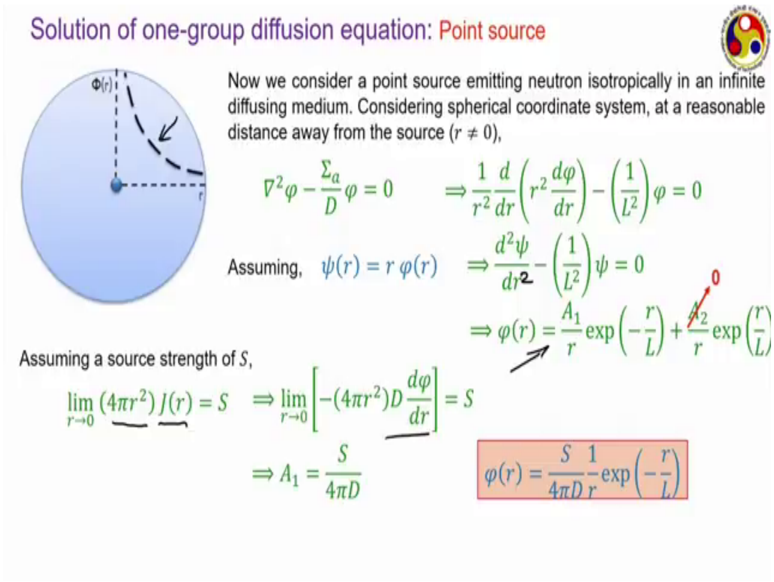

Now, we move to our second problem, where we basically have a point source on, it is a single point, which is emitting neutrons in all possible directions surrounding this and similar to the previous problem here this point is also immersed into an infinite stretch diffusing medium, but there is no fission going on around. This being a point, it can emit in all possible directions of a sphere around this and particularly when this source is an isotropic one, then it is intensity of emission in all possible directions should be equal and here instead of Cartesian coordinate, we have to take the spherical coordinate (Refer Time: 16:45) solving this.

So, again at a reasonable distance away from the source, the presence of the source can be neglected so this is a simple equation, but here while expressing this laplacian you have to use a cylindrical coordinate system and the sphere being or the source being isotropic, we can consider this again to be an 1 dimensional problem because whatever variation in the flux is taking place that is only in the r direction, but the other 2 directions that is theta and z coordinates are not having any influence or rather neutron distribution is symmetric with respect to both of them.

Hence, we are having this particular form. Now, this is a slightly complicated equation compared to the previous one. So, use one substitution, where we define a term psi as the product of r and phi. So, if we put it back into this original equation then we get this particular form d 2 psi d r this should be d 2 psi d r 2 minus psi upon L square. So, this equation again is very similar to the one that we have got in the previous slide say homogeneous equation having a very standard solution like this, A 1 upon r into exponential minus r by L plus A 2 upon r into exponential r by L.

Now, what should be the corresponding boundary condition that we must use here, again as we are moving away from the source we are seeing that the flask phi is a summation of 2 exponential term, 1 decaying with r and other growing with r, but as we are moving away from the source, the neutron flux should decay and hence this A 2 should go to 0 leaving only this negative exponential term. So, we have to identify only a single coefficient that is A 1 here.

How can we identify that? We have to consider that, the neutron current density at the centre that is at r equal to 0 multiplied by the area should be equal to the strength of the source, if the source strength is S, it is not S double prime, it is the strength. That is a source, that point source is emitting S number of neutrons per unit time, then the neutron current density J r multiplied by the area of the sphere of radius r, which is equal to pi r square should be equal to this s because whatever may be the value of this r, but what the number of neutrons emitted by the source that must pass through the entire surface of this particular sphere, which is given by this 4 phi r square and multiplying that with the current density or neutron current density we are getting that to be equal to S, at r equal to 0. So, we put the Fick's law of diffusion, where J becomes D d phi d r. This particular contribution and this phi can be replaced with this particular term.

So, we get A 1 is equal to S upon 4 phi D and this is the distribution of flux in the radial direction. So, for both this 2 problems of an infinite planet source and for a point source, we can just following the standard methodology of solving any differential equation or in differential equation you can calculate the final form of plug distribution. It is quite straight forward, it shows that as we r keeps on increasing, this flux density also keeps on reducing exponentially following this particular trend that is when r tends to infinity this should be equal to 0.

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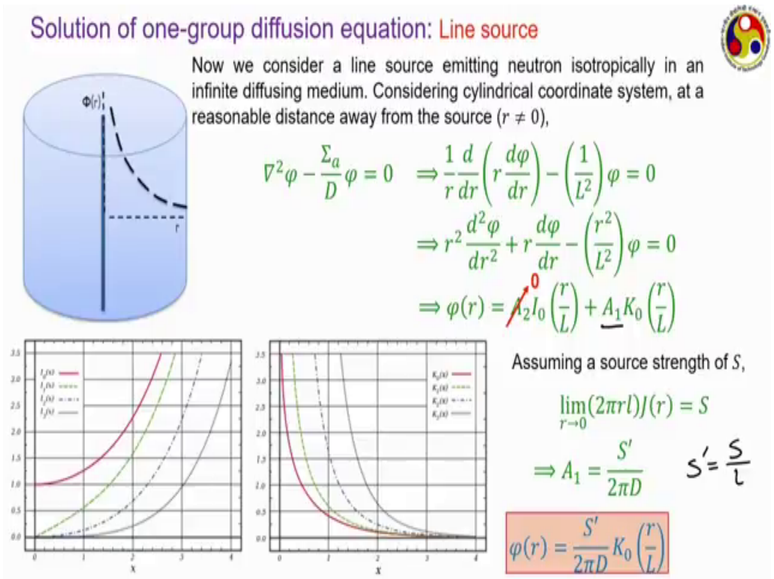

We have a third problem to deal with that is of a line source. Here our source, a neutron source is like a line, which is having 0 thickness, but it is having certain length maybe you can consider this lines strength to be infinite. This being a line, it can emit again neutrons in all possible directions around this and hence the neutrons coming out of this should pass through a cylindrical surface just like that shown here.

So, following the similar methodology this is the governing equation, but here we should use the cylindrical coordinate system. So, this is the form this one upon r d d r of r d phi d r minus phi upon L square is equal to 0 and now we just play around this, that is we break the first differential and then get a formula and multiply everything with r square to get a form like this. It is a quite complicated form actually, but actually the kind of equations that we are having here, that is known as the Bessels equation, which has a very standard solution.

I hope most of you are aware about Bessel functions or modified Bessel functions, if you are not you please go to any standard mathematics book and you will find this form for a Bessel function or Bessel equation, which is a form like this, which also has a very standard solution given by Bessel functions or modified Bessel functions and corresponding solutions will be like this. Here, I naught is the modified Bessel function of first kind and K naught is a modified Bessel function of a second kind. Both of them are a periodic functions and they have a very standard form, which I do not want to repeat here, but the graphical representations are like this.

Here actually I naught and K naught refers to only the 0th mode, they can have several modes like shown in the graph on the left. Here, I naught is having a certain kind of form which you can check from mathematics books, that it keeps on increasing exponentially with x, whereas K naught keeps on decreasing exponentially with x. Now, we know that x or you can say r in this particular coordinate system, now as r keeps on increasing neutron flux the intensity or the current density should reduce with time and hence this I naught should not have any contribution or rather this A 2 should go to 0 leaving only 1 term.

And, now to evaluate this particular constant A 1, we have to use a condition similar to the previous slide where assuming a source strength of S, the neutron current in density multiplied by the area of a cylinder should be equal to the source strength. So, at a distance r from the source or assuming a cylinder of radius r and of height small l, this is the corresponding area of the cylinder is 2 pi r l a surface area multiplied by the current density should be equal to S.

I am not showing the detailed calculation you can follow similar procedure because again J can be related to the gradient of phi, following the Fick's law and this K naught itself is a function of r. So, if you put them you are going to get that A 1 is equal to S prime by 2 pi D, where this S prime is actually source strength per unit length S by small l, so final form of the flux is something like this.

Hence, you can see here that we have always started with the diffusion equation for neutrons, but we have dealt with 3 different kinds of problems, 3 different geometries each of them demands a separate kind of coordinate system adoption, but following the standard procedure we have always, we are always able to get an simplified expression for the final flux distribution form and this exercise also allows us to get a realistic idea or a physical interpretation of the diffusion length.

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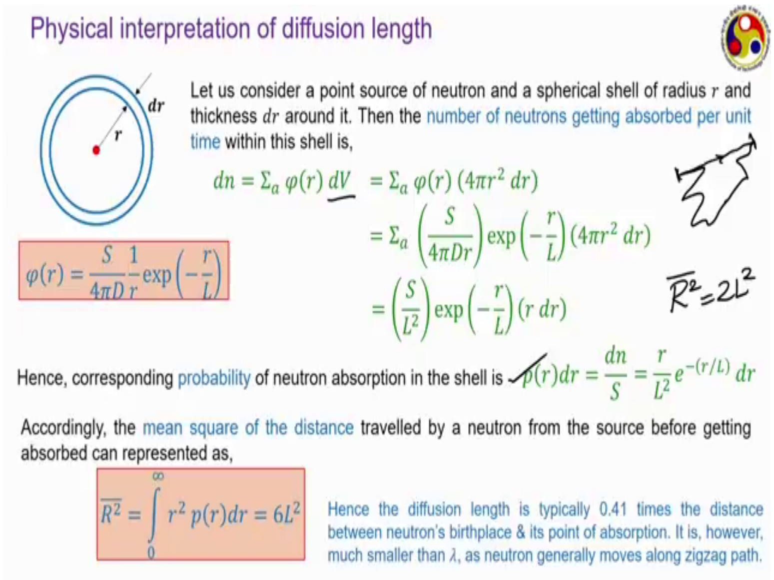

Let us go back to the second problem, where we had a point source here this red dot represent that point source, which is emitting neutrons isotropically in all possible directions over a sphere around this. Now, we consider a spherical shell around this, the inner radius of the shell is r and thickness is d r, then the total number of neutrons that are getting absorbed per unit time within this shell should be equal to the absorption cross section of the silver shield material in to the neutron flux at that particular location into the volume of the shell, here d v refers to the volume of this infinitesimal shell.

And, what is d v? That is 4 pi r square into d r. If d r is extremely small compared to r, we can always represent d v with 4 pi r square into d r. What about phi? Phi you have already solved for in the earlier slides, the solution of phi is already available. So, you can put this directly here and we can now simplify this, that is from these expressions this r and r square cancels out and 4 pi also goes out, leaving us S into r in the numerator and D upon sigma a is equal to L square. So, we are having this d n that is change in the number of neutrons or neutrons getting absorbed is equal to S upon L square into exponential of minus r by L into r d r.

And, now if we want to know the probability of neutron absorption in this shell, the probability is p r d r has to be is equal to d n upon S, because S is the total number of neutrons that are getting emitted from the source and d n is the number of neutrons that is getting absorbed in this. So, it is probability is the number of neutron absorption per unit for every neutron emitted by the source that is d n upon S and putting the expression for d n it is r upon L square into exponential minus r by L d r.

So, now we need to calculate the distance travelled by neutron; the straight line distance travelled by neutron from the source to the point of it is absorption, but I do not know why. In fact, I have not found a proper explanation in the books also, that is a common neutron science does not use the distance, rather they use the square of this distance or I should say the mean square of the distance travelled by the neutron from it is birthplace to the place of absorption.

And, that capital R refers to this straight line distance and R square is the square of that. So, the taking the mean of that squared distance that should be equal to integral 0 to infinity r square p r d r, as r can be of any value from 0 to infinite and here if you put the expression from this particular expression for p r d r you are going to get this to be equal to 6 L square.

That means, the mean square of the distance travelled by a neutron can directly be related to the diffusion length or on other words the diffusion length is a representative of the straight line distance travelled by a neutron; average straight line distance travelled by neutron from the source to the location where it gets absorbed and hence if we have some idea about this diffusion length, we can always calculate this distance travelled by the neutron, but you should not get confused with the earlier defined term mean free path, mean free path is the total distance travelled by a neutron between 2 successive interactions, but here this r talks about only the straight line distance.

Like, generally the movement or passage of neutron in the reactor is quite zigzag, it can move from somewhat like this, but this r is actually the straight line distance only from it is source to the strength, that is it will start from this point and finish up at this point and only comprises straight line between this. So, this distance r is invariably much smaller than the mean free path, but what we can see is that the diffusion length is typically only about 0.4 times of the distance travelled by the neutrons from it is birthplace to the point of absorption on an average.

If we do the same exercise for the infinite planar source, then there you would have got R square bar is equal to 2 L square. I would argue to try this, for just follow the same procedure on a Cartesian coordinate system and you should get R square is equal to twice of L square. So, by knowing the diffusion length, we can get a proper idea about the distance travelled by the neutron.

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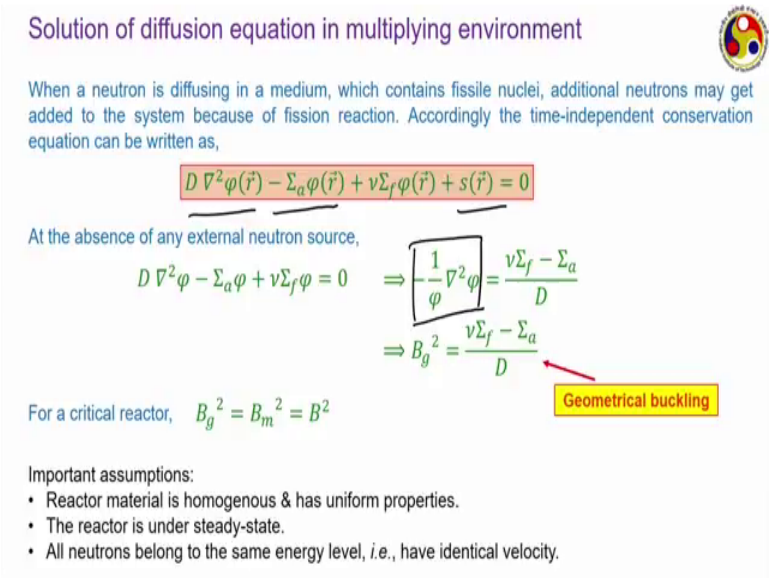

Now, we enter the situation of multiplying environment. Like the 3 problem that we have discussed so far, there we have considered the neighbouring medium or surrounding medium not to have any kind of fissionable nuclei, but when in a typical nuclear reactor we may have fissionable nuclei present as well. Like the neutron may get diffuse through the moderator, but immediately after the moderator or sometimes mixed with the moderate itself we may have well nuclei and whenever there is a fuel nuclei that is being acted upon by neutron we can have additional fission reaction and those fission reactions will produce some further neutrons or will add some further neutrons into the system.

That is what we refer to as a multiplying environment. Corresponding time dependent conservation equation or time independent I should say conservation equation that is for a critical reactor equation is like this. You can see the 3 terms of the same like in a previous case we have the absorption term, we have the leakage term related to the Fick's law of diffusion and we have the source term; external source.

But you also have an additional term in between, which represent the rate of neutron production because of fission. Here, just as per our previous familiarity mu refers to the number of neutrons; average number of neutrons produced per fission. Sigma f refers to the fission cross section and phi is a neutron flux. If the neutron sources absence then this s goes off and then we can rewrite this equation to a form like this minus 1 by phi d 2 phi or the laplacian of phi, should be equal to mu sigma f minus sigma a upon d or if you remember our earlier definition this particular term is referred to as the geometrical buckling B g square. So, that comes out to be nu into sigma f minus sigma a by D. That is by knowing the properties of the medium, we can get some idea about this geometrical; the geometrical buckling as well.

For a critical reactor we know that B g square is equal to B m square is equal to B square. So, we now have to apply this equation; diffusion equation multiplying environment to have more realistic neutron distribution in reactors, but before that you have to remember a few assumptions like reactor material is assumed to be homogeneous and is having uniform properties in all directions and the reactor is working under steady state.

That is, it is a critical reactor and total number of neutrons remains constant with time as are any other parameters and third assumption, all neutrons belong to the same energy level remember what we are doing here is the solution of a single decrease energy diffusion equation, that where all neutrons are assumed to have moderate the same energy level, but the variation in the neutral energy level has been neglected. So, we assume all neutrons to belong to the same energy level that is have identical velocities or identical kinetic energies.

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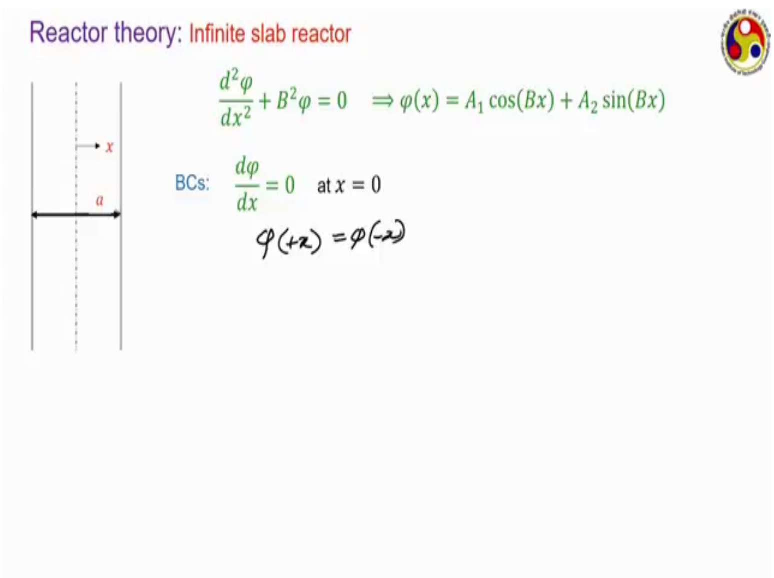

So, with that let us try to study a few reactor designs. The first reactor that we have is for an infinite slab. Here, our reactor you can think this to be an extension of that infinite source problem or infinite plane source that we have discussed shortly back. Here your source is also an infinite slab reactor. Here, your reactor is it is an extension of the previous infinite planner source problem, that is the directories of the shape of an infinite slab or it is extent is infinity in the y and z direction, but it is having some kind of finite width in the x direction.

So, the total reactor is having a width of a, that is from the centre line to the edge the distance is a by 2 on either side of this and again the dimensions in the y and z directions to be much significantly larger than the dimension in the x direction which is a or a by 2 any variation in the y or z direction can be neglected and we are having variations to consider only in the x directions.

So, this is the equation that we have just derived in the previous slide d 2 phi d x 2 plus B square phi is equal to 0, where B is the buckling parameter, which is for as we are assuming a critical reactor. So, B g square and B m square are equal and that is given by this B square and again it is very standard homonymous equation. So, whereas, when a solution will be A 1 cos B x plus A 2 sine B x. A 1 and A 2 are the 2 constants which we have to evaluate using the boundary conditions. So, what can be the possible boundary conditions, at x equal to 0 what can be the boundary condition.

X equal to 0 refers to a centre line and given the geometry we can easily say that the geometry is a mirror image or one side of the geometry also a centre line is a mirror image of the other side, accordingly the centre line can be considered to the plane of symmetry and hence d phi d x has to be equal to 0. Another way of writing the same boundary condition is phi at plus x should be equal to phi at minus x. Solution point of view sometimes this is a much better to write in this particular fashion.

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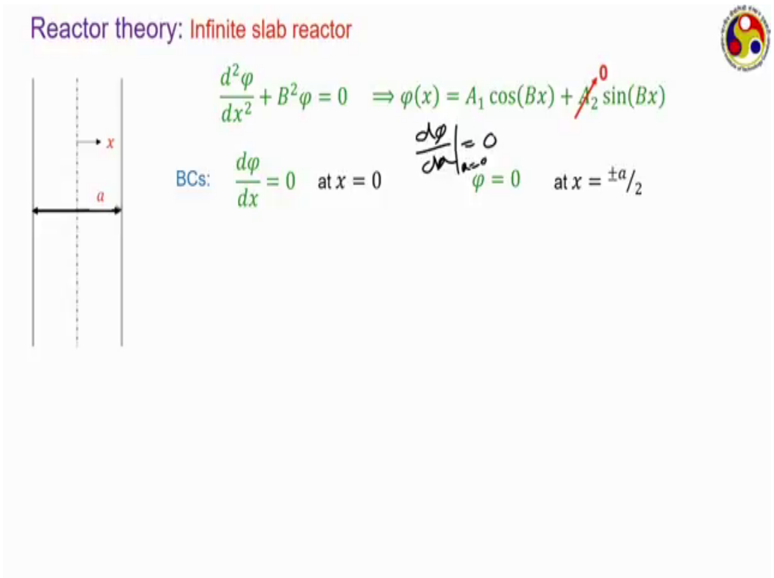

Another boundary condition that we should consider that is at the edge of the reactor, that is x equal to plus a by 2 or minus a by 2. So, here we take phi equal to 0 because as the neutrons are going out of the reactor, so we do not expect any neutrons to exist outside the reactor.

If we put these 2 boundary conditions, particularly if we put the first one, that is we differentiate this equation d phi d x and put the limit x equal to 0 and equate that to 0, then we get this A 2 has to be equal to 0.

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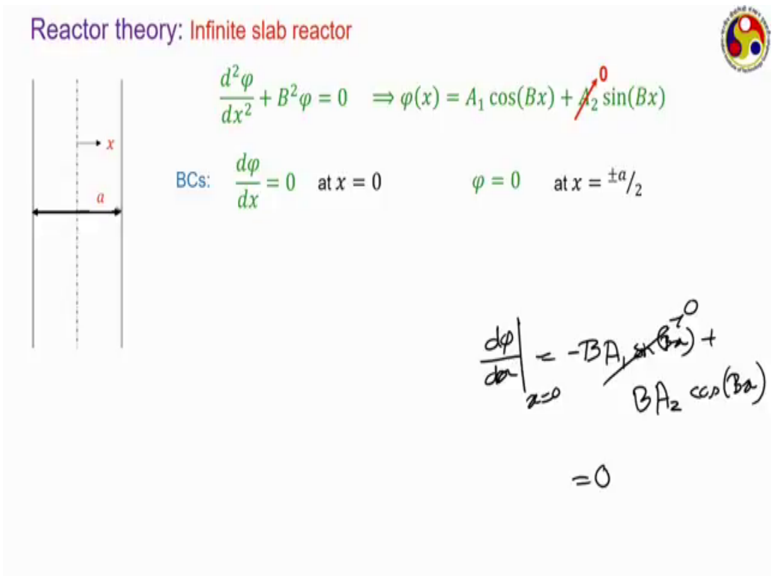

Because the differentiator from what we are going to get by differentiation, d phi d x has to be equal to B A 1 sine B x minus of that plus B A 2 cos B x. Now, if we put the limit x equal to 0, of course sine 0 goes to 0, but as per our definition d phi d x is equal to 0 at x equal to 0, but cos 0 is not 0 and it is possible only if either B or A 2 is equal to 0. B being on the buckling parameter it also is a non 0 number and hence A 2 has to be 0 to get a proper solution.

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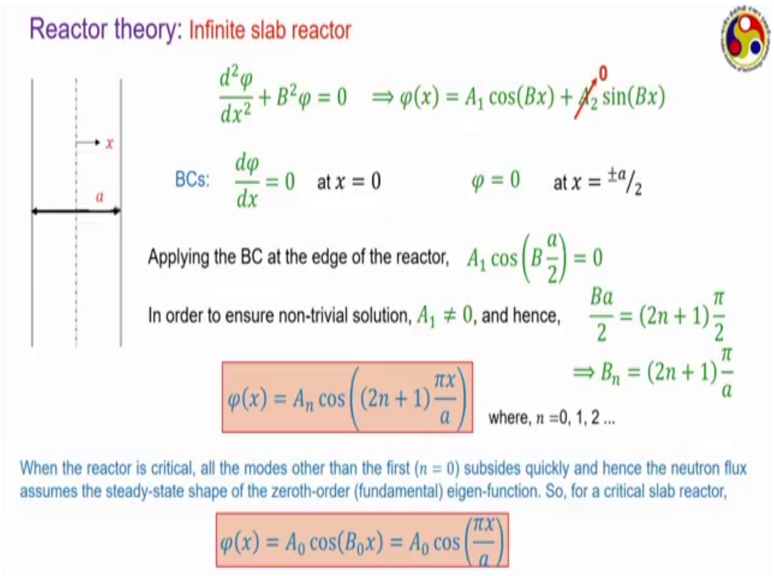

Now, we apply the boundary condition at the edge of the reactor and hence putting x equal to a by 2 plus a by 2, we get A 1 cos B into a by 2 is equal to 0, but in order to ensure a non trivial solution A 1 cannot be equal to 0 and hence cos B by a 2 has to be equal to 0 or in a way B a 2 has to be equal to 2 n plus 1 pi by 2, where n is 0 and any other integer or we can say B n can be equal to 2 n plus 1 into pi upon a, like depending upon the value of n we can have a different values for this B n.

So, this actually is leading to an infinite solution. That is, we are getting phi x equal to A n cos 2 n plus 1 pi x upon a, where A n refers to the value of this coefficient A, corresponds to the nth mode of this function. Practically speaking all the modes other than; when the reactor is critical after certain period of time you will not find the existence of the other modes.

That is all the other harmonics for corresponding to n equal to 1 2 3 etcetera all will die down quite quickly and after some small period of time you will only see n equal to 0.

That is the 0th or a Eigen function of the corresponding fundamental Eigen function, that is existing and the magnitude of the other Eigen functions being negligible and hence for a critical slab reactor our solution is like this phi x is equal to A naught cos B naught x or that is equal to A naught into cos pi x upon a. A being the half width of the reactor, but one important condition that you have to consider here, here you have taken this phi to be equal to 0 the edge of the reactor, but practically that is not true.

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Commonly the diffusion equation is solved in high-density moderator, such as H_2O, D_2O or graphite. Those are typically surrounded by air, which offers much larger mean free path to neutrons. Therefore it is often treated as a vacuum BC.

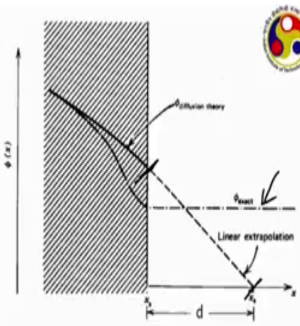
$d \longrightarrow$ extrapolated length

If the magnitude of d is significant, then $a/2 \rightarrow a/2 + d$

Solution of one-group diffusion equation in a weakly-absorbing homogenous media gives,

$$d \approx (0.7104)\lambda_{tr} = (0.7104)(3D)$$

Moderator	D (in cm)	d (in cm)
H_2O	0.142	0.302
D_2O	0.840	1.790
Beryllium	0.416	0.887
Graphite	0.916	1.952



Total power produced by reactor,

$$\dot{P} = \int_{-a/2}^{+a/2} E\Sigma_f \, \varphi(x) \, dx = \frac{2}{\pi} E\Sigma_f A_0 a$$

$$\Rightarrow A_0 = \frac{\pi \dot{P}}{2E \, \Sigma_f \, a}$$

Just look at the diagram, practically we get a profile somewhat like this. That is phi decreases initially as we are moving away from the centre line phi is decreasing, but while eta decreases following model is the same gradient as per the diffusion theory it changes quite sharply in the actual case, but even in constant, once we are crossing at neutron level or once we are coming out of the reactor, but just think about the following the diffusion theory at x equal to 0 at this particular location this is not x equal to 0, I should say x equal to a. at this location there is certain gradient that is being followed by this neutron flux.

So, if we just extend that we are reaching somewhere here that is if the neutron flux is allowed to follow the same trend after coming out of the reactor that would reach 0 at this particular location. This particular distance is often called the extrapolated length, which is d and corresponding boundary conditions generally called the vacuum boundary condition.

So, if for most of the practical reactor problem this extrapolated length is negligible and hence during calculation it can be neglected and the neutron flux can be assumed to approach 0 at the edge of the reactor itself, but if d is not negligible, then in all the previous calculations like in the previous slide this a by 2 has to be replaced by a by 2 plus d and solution of 1 group diffusion equation, solution which is beyond this course.

For this kind of situation in a weakly absorbing homogenous medium, we can find that d is equal to 0.71 into the transport mean free path or it is just 0.7104 into 3 times the diffusion length, because we know that the transport mean free path is equal to 1 upon 3 D. These are certain values for common moderating materials, like you can clearly see the diffusion length is generally quite small starting from 0.1422 in common water to 0.84 and 0.916 in graphite.

Correspondingly the value of this small d or this extra volatile length, that also keeps on increasing. It can be just about 0.3 centimetre in common water, but can be nearly 2 centimetre in graphite, but still even the largest dimension that we are talking about for diffusion length is only about 2 centimetre and when you talk about the reactor; common dimensions of the reactors are generally much larger compared to these values and hence for most of the practical cases this information can be neglected and the boundary condition can be considered to be phi equal to 0 at the edge of the reactor.

But we also have another task click, that is we have to calculate the value of the coefficients, that we have discussed the coefficient A naught, can be calculated by using the power produced by the reactor. Practically speaking, if we do not consider or if you do not impose any other condition we cannot deviate the value of A naught, a rather for every different values of A naught, we can have a different solution.

So, total power produced by the reactor can be represented by this, where this E refers to the energy released by every fission reaction into the absorption cross section into the neutron flask and this is being integrated from minus a by 2 to plus a by 2, that is from 1 edge of the reactor to the other edge of the reactor and corresponding solution is this. So, if we proceed further, we can get A naught to be equal to pi P dot by 2 E sigma f a.

Here, again I repeat sigma f is fission cross section, p dot is the power produced by the reactor and E is the energy emitted by every fission reaction, which is typically of the order of 200 MeV. So, putting this back in the previous expression, we can get the neutron flux distribution in this infinite slab reactor, but actually infinite slab is very much an idealization because practical reactors cannot have infinite stretch, rather they are having only finite size.

So, in the next lecture or in the last lecture for this 4th module, we shall be discussing about different reactors of finite stretch where we shall be taking finite sphere as the finite sphere and finite cylinders as the 2 geometries and we shall be developing corresponding expressions for neutron flux intensity and also we shall be trying to compare different kinds of reactors that are available. So, for today I would like to close it here itself, please revise this lectures and we shall be taking it forward for proper reactor design in the next 1.

Thank you.