Fundamentals of Nuclear Power Generation
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Module – 04
Chain Reaction in Reactors
Lecture – 02
Neutron diffusion theory

Hello friends, welcome back to the fourth module of our MOOCs course on the topic of fundamentals of nuclear power generation, and today we are going to continue our discussion on the topic of chain reactions in reactors.

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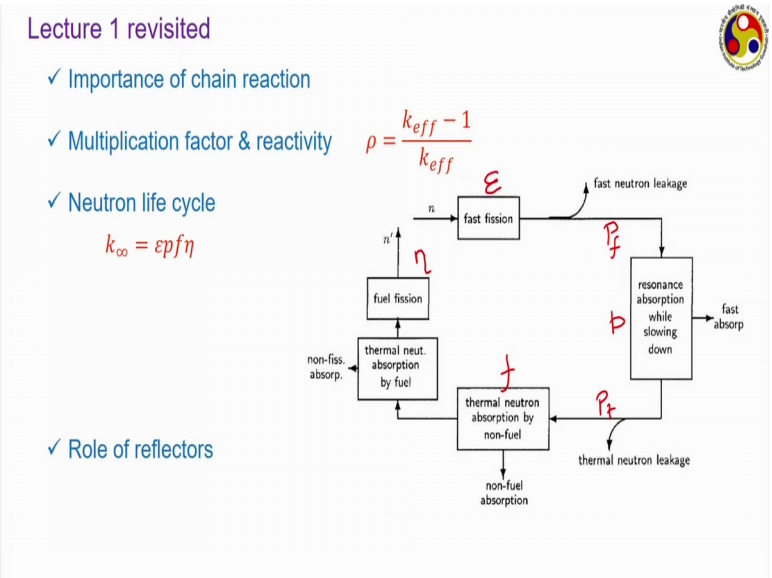

You are introduced to this topic of chain reaction during the previous lecture and if we quickly revise it out the content of the previous lecture, the importance of chain reaction were emphasized there like; it is just one fission will not give you any kind of realistic power production, rather the neutrons which are produced from the fission reaction must induce fission to some neighbouring nucleus as well, and then only we shall be able to sustain a continuous power production procedure and that chain reaction therefore, is extremely essential. However, it is as we have already seen it is not a straight forward that just the neutrons coming out or getting produced from a fission reaction all of them will be able to produce fission a subsequent fission, rather they will go through several

kinds of phenomenon and accordingly there is very less chance that a neutron can cause a fission. In fact, that is just one of the several possibilities.

So, to characterize that we have you are introduced to the topic of multiplication factor and reactivity; multiplication factor basically gives you the idea about the number of neutrons that are produced per unit neutrons absorbed in a reactor, and reactivity is defined by this relation which is multiplication factor minus 1 divided by multiplication factor or effective multiplication factor.

There are several factors on which this multiplication factor depends on to understand that we discussed about the neutron life cycle like this picture was used for that, a freshly produced or introduced fast neutron goes through several stages inside a nuclear reactor like first is goes through fast fission, which we can characterize using the fast fission factor, then it can undergo through a fast neutron leakage.

So, corresponding non leakage probability can be identified with something like this, then it goes through the moderation process giving which the fast fission or rather fast neutron gets converted to thermal neutron level, but several elements present inside a reactor particularly uranium 238 has very high resonance absorption capability, that is the absorption peaks are extremely high in the resonance captured zone, accordingly some of the neutrons may get lost in the form of resonance absorption.

And corresponding escape probability was identified (Refer Time: 03:07) known as small p; the newly produced thermal neutrons then some of them can get leaked outside, which we shall be discussing very shortly this leakage procedure or a movement of neutrons, but corresponding non leakage probability was identified is P t or P t i sometimes to make it thermal. Then neutron will get absorbed this thermal neutrons whatever remains whatever survives leakage they will get absorbed in the reactor, but the there are not only fuel, but there can be several other elements also which are absorbing this thermal neutrons like the moderator, the coolant itself and several other elements which can be there like the control elements etcetera. So, our interest is to identify how much fraction of whatever getting absorbed is actually getting absorbed by the fuel, that was characterized by a thermal utilization factor small f, and then whatever factor that gets whatever fraction that gets absorbed by the fuel that goes through or induces a thermal fission procedure. So, all of them again may not induce a thermal fission because

some of them may go through a non-fission capture process in the fuel itself and finally, whatever we get back that is characterized in terms of a thermal fission factor eta.

So, this all the 6 factors are introduced out of this 2 of them are leakage characteristics this p f and p t and therefore, they depends upon the configuration of the reactor, but others they also depends upon the configuration reactor plus fuel plus whatever other assemblies are there inside etcetera.

So, this particular very popular and very important relation was introduced, where k infinity on the left hand side represents the infinite multiplication factor it is basically the multiplication factor for an infinite reactor; that means, when there is no leakage. So, corresponding multiplication factor is given as a product of this 4 factors, such that it is also called a 4 factor formula. Then we have discussed about the process of calculating all these 4 factors, and in between the role of reflectors are also mentioned to reduce or capture the leakage and reflect the neutrons back to the reactor.

So, to start with today we shall be first seeing on numerical example to see how we can calculate all these 4 factors in a real reactor situation and also how can we use them to calculate the reactivity.

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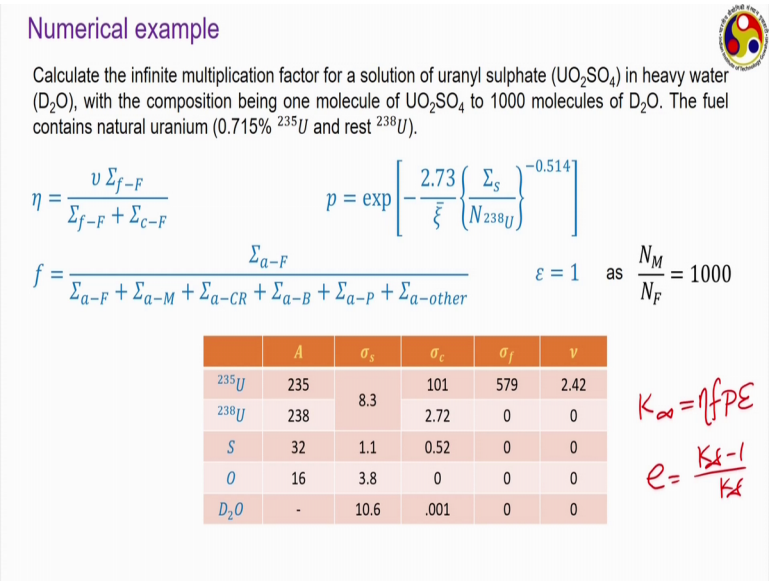

	A	σ_s	σ_c	σ_f	ν
^{235}U	235	8.3	101	579	2.42
^{238}U	238		2.72	0	0
S	32	1.1	0.52	0	0
O	16	3.8	0	0	0
D_2O	-	10.6	.001	0	0

So, this is the example problem that I have selected please read the problem carefully. Here the problem is to calculate the infinite multiplication factor and also reactivity for a

reactor, where we have a solution of urinary sulphate. Now remember uranium is of course, a fuel uranium 235 particularly is a fissile isotope which you would like to use for fission reaction, but uranium is also quite active chemical from chemical point of view and therefore, in the uranium that we introduce in the reactor that is not purely uranium that is rather even more in form of some kind of compound like some oxides or carbonates something like that.

And here this problem uses a sulphate which is urinary sulphate UO 2 SO 4 and it is also immersed in heavy water. Heavy water I have mentioned earlier refers to the water where the hydrogen actually is deuterium that is deuterium isotope. So, here you are having urinary sulphate as 1 element, inside urinary sulphate molecule you have uranium isotopes or even nucleus, which is actually your fuel and also you have heavy water heavy water is a very very strong moderator.

So, that will act as a moderator the composition is also given for every molecule of urinary sulphate, we have 1000 molecules of heavy water, and the fuel uranium can have different kinds of isotopes actually. So, here the fuel contains natural uranium that is out of all the uranium isotopes present only 0.715 percent is uranium 235 rest are uranium 238. So, you have to calculate the multiplication factor the infinite multiplication factor.

So, let us just revise it the mathematical relation that we have derived during last lecture; this is the thermal fission factor of course, it is defined as a ratio numerator having the macroscopic fission cross section of the fuel, you have to remember there are several elements associated in this particular problem like what are the elements how many elements you can identify? One is the uranium, then second is oxygen, third is sulphur and fourth is this heavy water. There is no need of separating D and O here we can treat them the as the same element or just a single element, but. So, we have 1 2 3 and 4, but is it correct probably something is still missing because uranium is a not a single element rather we can have 2 kinds of isotopes of uranium, one is uranium 235 and other is uranium 238.

So, basically there are 5 different kinds of elements or isotopes that are involved in this particular problem. So, we need to know the characteristics of all this 5, but out of the 5 only 1 has fission cross section, at the corresponding to whatever neutron level that we are dealing with. This problem actually concerns thermal neutrons. So, corresponding to

thermal neutrons only uranium 235 is the one that is having fission cross section. So, to calculate the fission thermal fission factor in the numerator we need to use the macroscopic fission cross section of uranium 235, multiplied by a mu is the number of average number of neutrons produced for fission for this uranium 235, divided by the total absorption cross section of the fuel.

Now, they have fuel refers to both uranium 235 and uranium 238. So, while calculating the denominator we have to consider the absorption cross section for both these 2 isotopes 235 and 238; however, while calculating the numerator it has to be only uranium 235 because uranium 238 does not have any kind of fission cross section for thermal neutrons.

Next is the thermal utilization factor. Now in the numerator we have that total absorption cross section of the fuel basically the denominator which we have used here the same denominator is coming to this as the numerator, but what we have in as the denominator for this thermal utilization factor, here the total absorption cross section for all elements that can be present. So, here we have the fuel absorption cross section for the fuel or absorption macroscopic absorption cross section for the fuel same as the numerator. E have the absorption cross section for the moderator, which is the D 2 O, here this control rod this boron this poison all can be eliminated as know information are given about them, but we have also this others. Now what comprises the other in this problem it is the oxygen and the sulphur which are also present in this actual problem.

Next is the resonance absorption cross section this mathematical relation was given which is based upon empirical formula here this sigma s in the numerator refers to the total scattering cross section of all the elements present inside the reactor, and zeta bar is the weighted average logarithmic energy decrement for all the elements again. We have seen how to calculate these 2 factors.

Finally is the fast fission factor, now what should be the value of fast fission factor it was mentioned earlier, that when the moderate and moderator nuclei bi fuel nuclei that ratio is more than 50, we can neglect any kind of fast fission and can take this fast fission factor to be equal to 1. Here this ratio is 1000 because for every fuel isotopes or fuel molecule we have thousand molecules of D 2 O. So, we can take this fast fission factor to be equal to 1.

Now, we need to have several properties that is all this cross section values that we need to know this. So, these are the property information that are given to us, atomic weight for all these elements are given, scattering cross section is given for here both U 235 and 238 are having the same scattering cross section for the others they are given to a definitely have a much higher scattering cross section. Sigma c that is the capture or non fission absorption cross sections are given, uranium 235 has a quite high, but very interesting that oxygen is value is 0 for oxygen and for D 2 O it is also extremely small almost negligible. So, deuterium is an is an excellent moderator, where are it is scattering cross section is quite high it is absorption cross section can be is a negligible.

Then we have the fission cross section of course, all these elements are non-fissionable for thermal neutrons. So, only you uranium 235 is having the fission cross section, and fine correspondingly the average number of neutrons produced per fission is given for the same element. So, with this property values now we have to solve this problem that is we have to basically calculate the values of all these 4 factors, because as we have derived in the last class k infinity is equal to the product of this all 4 factors; the thermal fission factor,' thermal utilization factor, resonances escape probability and the fast fission factor. So, we have to calculate all these 4 factors and accordingly we shall be able to calculate this k infinity, and also if our interest is to know the reactivity rho that is in this case k infinity minus 1 by k infinity of course, the actual definition of reactivity uses in effective multiplication factor, but has no information is given about the leakage characteristic. So, we can take k effective to be equal to k infinity or we can consider this problem to relate to some kind of infinite factor.

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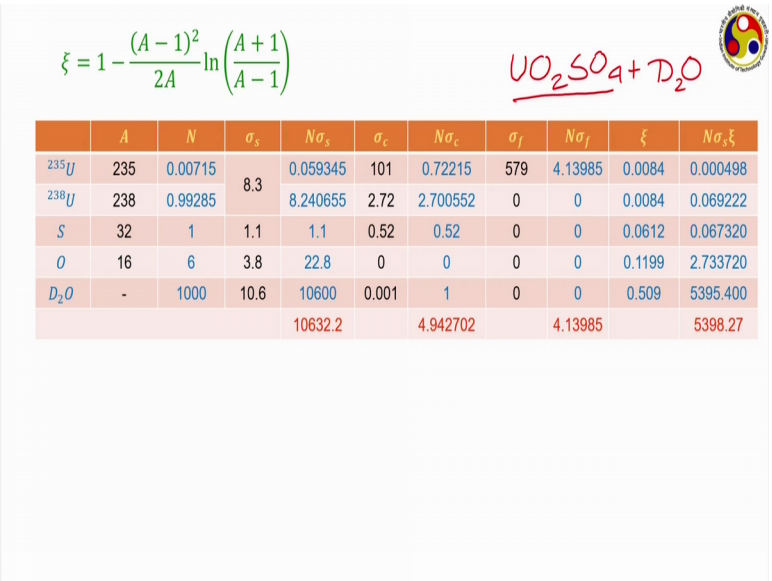

Now, another relation that we need to use that is a zeta, this relation we have derived as a part of our third module once we know the atomic weight, for any isotope we can calculate or mass number for any isotope we can calculate the zeta for this.

So, now expand the table, here this the same table from the previous slide I have taken those written in black where the information given to us, but the those in blue that we have newly calculated I have calculated this for you and I have filled up the table, but let us explain 1 by 1. First is n; n refers to the number of isotopes for that particular element or particular compound present for every every molecule of the fuel. So, for every molecule of the fuel we know our fuel is UO 2 SO 4 or we have UO 2 SO 4 and D 2 O.

So, for every molecule of UO 2 SO 4 we have how many molecules of sulphur? just 1 and for oxygen that is 2 plus 4 that is 6 and it is also given for every molecules of UO 2 SO 4 we have thousand molecules of heavy water. So, that is 1000, but every molecule of oxygen also sorry every molecule of uranium also contains just 0.715 percent of uranium 235 and rest uranium 238. So, uranium 235 corresponding N is 0.00715 and for U 238 is 0.99285 then sigma s is given. So, we compute n into sigma s just multiply all the sigma s values it corresponding n and then take their summation or if I write properly this is the summation for n sigma s. Just same way we get n sigma c that is here n sigma s represents what this represents.

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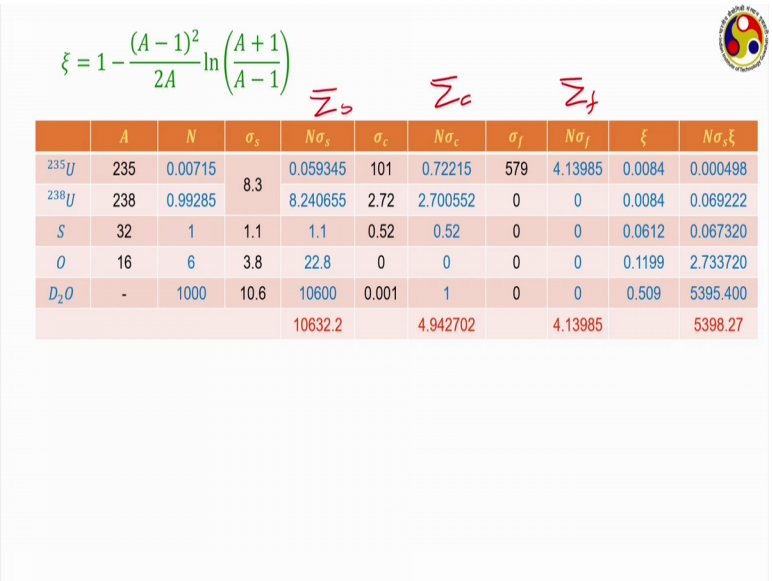

	A	N	σ_s	$N\sigma_s$	σ_c	$N\sigma_c$	σ_f	$N\sigma_f$	ξ	$N\sigma_s\xi$
^{235}U	235	0.00715	8.3	0.059345	101	0.72215	579	4.13985	0.0084	0.000498
^{238}U	238	0.99285		8.240655	2.72	2.700552	0	0	0.0084	0.069222
S	32	1	1.1	1.1	0.52	0.52	0	0	0.0612	0.067320
O	16	6	3.8	22.8	0	0	0	0	0.1199	2.733720
D_2O	-	1000	10.6	10600	0.001	1	0	0	0.509	5395.400
				10632.2		4.942702		4.13985		5398.27

The macroscopic cross scattering cross section for every element and this is the total summation.

Now, this represents the macroscopic capture or non-fission absorption cross section, and this 1 n sigma f is a macroscopic fission cross section. So, we have calculated using the microscopic cross section values and value of n we have calculated the macroscopic cross sections for scattering capture and fission next is zeta. The relation is already given for this the value of zeta we can compute for all these elements, apart from D 2 O the value of zeta for D 2 O actually is given in the problem statement, because this particular relation here A refers to the mass number of any isotope, and hence we can calculate it only for a single element, but not for a compound like heavy water.

So, this value of point 5 O9 that has to be given in the problem here we can assume it to be given and once we have evaluated the zeta value for all the elements, we calculate n sigma s into zeta for all of them and again we take a summation of that. So, once we have all of them now we have to calculate the 4 factors first is the eta the thermal fission factor. So, how to calculate thermal fission factor? We know it is a ratio, in a numerator what we have average number of neutrons produced for fission.

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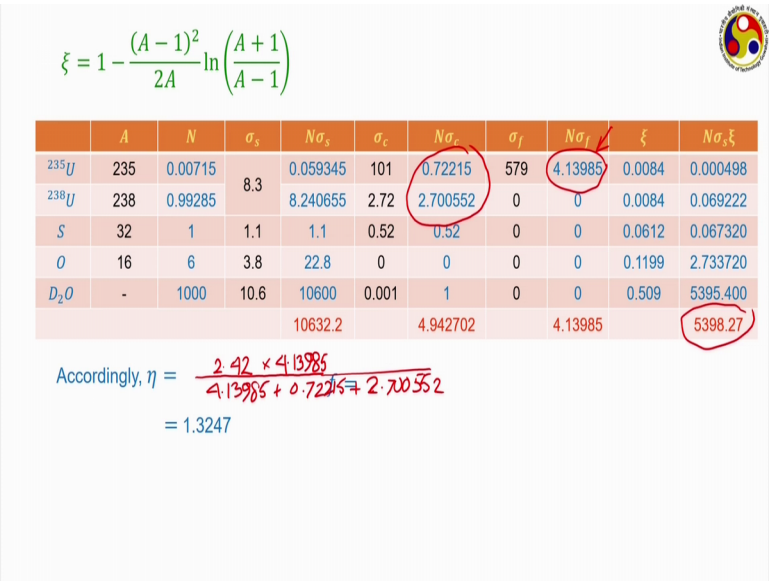

	A	N	σ_s	$N\sigma_s$	σ_c	$N\sigma_c$	σ_f	$N\sigma_f$	ξ	$N\sigma_s\xi$
^{235}U	235	0.00715	8.3	0.059345	101	0.72215	579	4.13985	0.0084	0.000498
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S	32	1	1.1	1.1	0.52	0.52	0	0	0.0612	0.067320
O	16	6	3.8	22.8	0	0	0	0	0.1199	2.733720
D_2O	-	1000	10.6	10600	0.001	1	0	0	0.509	5395.400
				10632.2		4.942702		4.13985		5398.27

Now, it was given in the previous slide it is 2.42 for uranium 235 multiplied by total macroscopic fission cross section for that fuel which is this particular thing. So, that will come here, in the denominator what we have? It should be the macroscopic absorption cross section for the fuel.

Now, fuel refers to uranium 235 and 238 together. So, in the denominator we shall be having the total non-fission capture cross section for these 2 elements that is this plus this particular one the absorption cross section. So, once we put all these back or if we write carefully now in a numerator then we are going to have again 4.13985, in a denominator corresponding to fission part 4.13985 plus 0.72215 plus sorry 2.700552.

So, once you put all these numbers then corresponding value of eta comes out to be 1.3247. We have to calculate thermal fission factor let us just erase this clutter from here. So, to calculate the thermal fission factor now, we have to take the ratio again in the numerator what we are going to have, that is going to be the macroscopic absorption cross section for the fuel, that is macroscopic absorption cross section for the fuel refers to the denominator that we have actually used for eta that is this 2, plus this this is the numerator and what about the denominator? Denominator refers to the total absorption cross section for all elements involved. So, denominator is this quantity plus this one.

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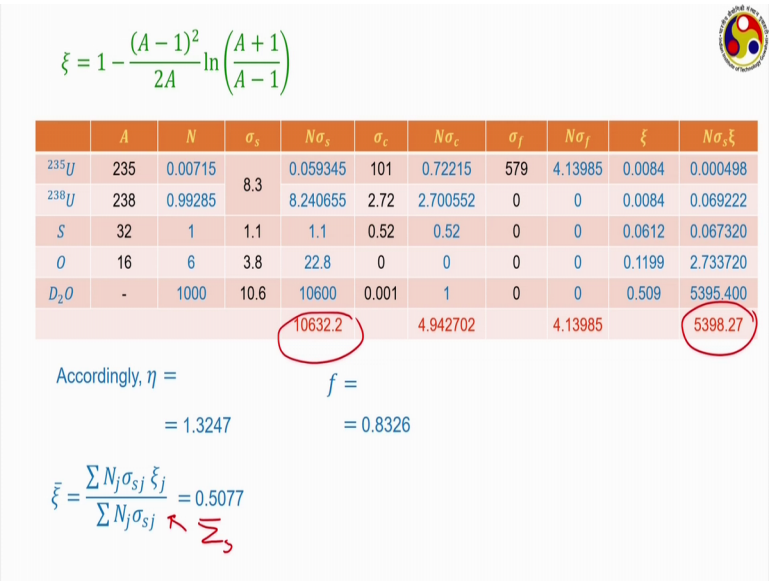

So, and once we put it back then we get f equal to 0.8236. So, we have now 2 quantities n and eta and f. Now we come to the complicated 1 that is the resonance escape probability. Now to calculate resonance escape probability first we need to know the weighted average value of zeta. So, zeta weighted average value that was given earlier as summation of n sigma is zeta for all elements divided by summation of n sigma s where this denominator actually is the total scattering cross section, which is this quantity and a numerator this one that we have already evaluated. So, if we get the ratio it comes to a 0.5077.

Ah and now mathematical relation for p was given earlier. So, which is exponential of minus 2.73 by zeta bar into sigma s by n number of U 238 isotopes, whole to the power minus 0.514.

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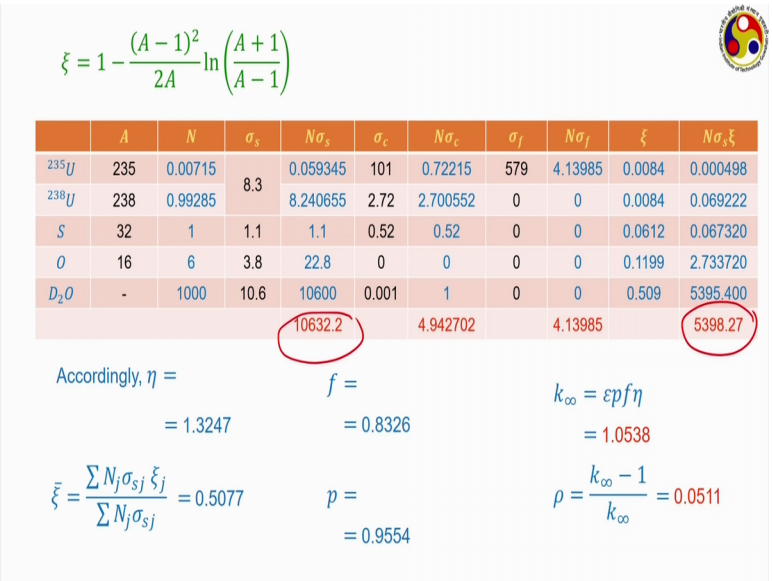

So, if we put it back then we are going to get p equal to 0.9554. So, and epsilon already you know it is equal to one. So, we combine all of them and finally, the value of effective multiplication factor is coming to be 1.0538. Then what should be the reactivity? Reactivity is 0.0511 that is it is a what kind of reactor this is critical subcritical or supercritical. When k effective is equal to 1 that is a critical reactor, when it is less than 1 that is subcritical, but in this case it is a supercritical reactor because reactivity is positive and k effective greater than 1; that means, the rate of reaction will keep on increasing with time, and we shall be having a diverging chain reaction.

So, this way we can calculate the values for the multiplication factor and reactivity for any kind of situation. I would request you to try different variations of this problem like if you hear this problem concerns natural uranium; you can try the same for enriched uranium. Let us say the fuel contains 5 percent uranium 235 and rest uranium 238 and see how it varies.

And also you can try to see the effect of that such enrichment on this eta this is natural uranium, which is points 7 percent uranium 235 and corresponding value of eta is 1.32. Please try to calculate the value of eta for different enrichment factor as I have shown in a graph you will find that beyond 10 percent, it becomes almost saturated and the corresponding value is around 2.06. Correspondingly you can try to calculate the reactivity values for different enrichment factor and also by taking other kind of (Refer

Time: 21:54) like instead of D 2 O, if you are taking normal water H 2 O which has a slightly lower scattering cross section and much more significant absorption cross section non fission absorption that is.

So, a correspondingly what value you are getting you can try to do the calculation. But procedure always remains the same, when a whatever data are given to you, you always try to compute the microscopic values and take their summations; like the macroscopic scattering cross section it is summation, the capture cross section, fission cross section generally there will be only a single element for fission, but you may encounter problem where you have both uranium 235 and uranium 239 then not one, but there will be 2 fissile isotopes and. So, the fission cross section will be having contributions from both. You also need to compute this summation of n sigma s zeta and of course, you need to compute the values of all these zetas. Using this we can finally, can compute all the values of effective multiplication factor and the reactivity.

Now, this is a problem that was evaluated as neglecting any variation in the neutron flux, that is while we evaluated the expressions for this f suppose or say eta, we have considered say let us take the example for eta. While writing this we have neglected any kind of effect of the beam intensity of neutron, that is we have assumed the beam intensity of neutron that is incident on uranium 235 and uranium 238 both are identical and much more strikingly for calculating this thermal utilization factor, we have considered this intensity to be equal for all elements like moderator coolant etcetera. But practical reactors may not be having that scenario rather neutron flux can vary a significantly over a reactor even in case of a homogeneous reactor.

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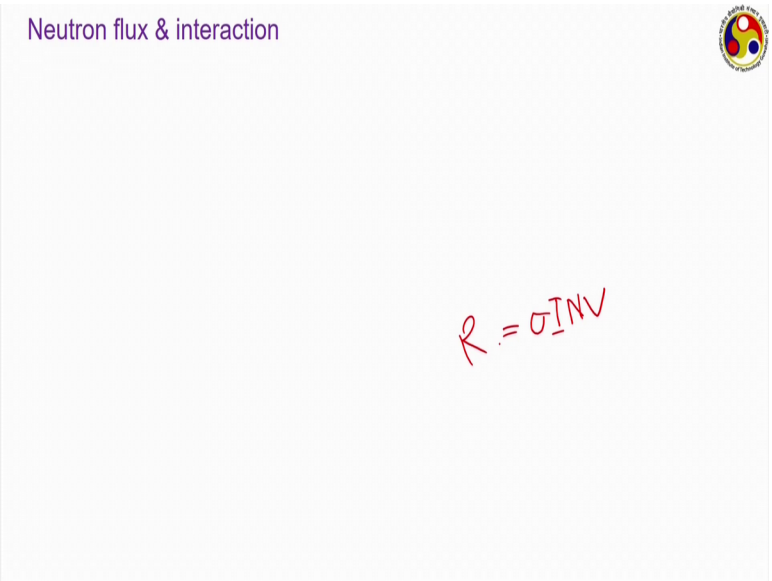

So, let us discuss about neutron flux and corresponding interaction.

If we just if you can just go back to the second module, that we discussed there we have defined the reaction rate to be proportional to several components, and accordingly we have found the reaction rate was to be equal to sigma into the neutron beam intensity, into the number of neutrons into volume, if say R represents the reaction rate. And if our interest is to know the reaction rate per unit volume, then it was something like this.

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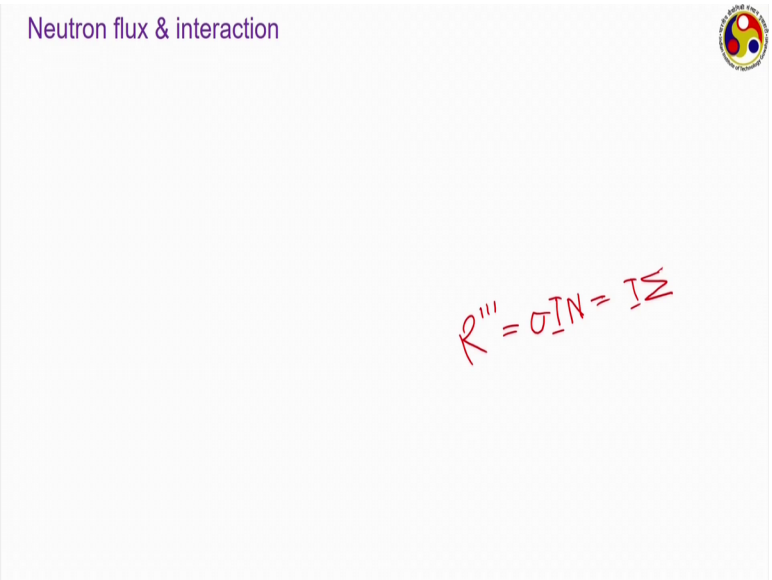

Correspondingly it was also mentioned it was I into capital sigma, because nuclei density and microscopic cross section their product is the macroscopic cross section.

Here this I refers to the intensity of neutron beam, but as we are talking about the variation of neutron flux inside this reactor, instead of I quite often another symbol is used which we shall be seeing shortly. But the issue is that while writing that simplistic expression, the variation in the beam intensity was neglected, but let us try to get a more involved expression now.

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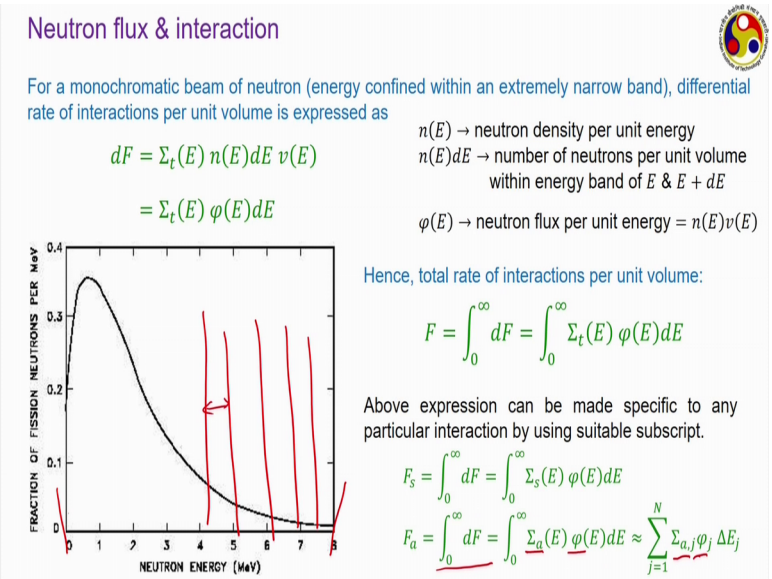

Let us assume a monochromatic beam of neutron; monochromatic refers to all the neutrons are having more or less the same energy or the energy of the neutrons are confined in a very very narrow band so that they can be viewed to be almost a single energy.

Then in correspondingly n E refers to the neutron density per unit energy, and n E d E this product then represent the total number of neutrons per unit volume which are having energy within the small band of E to E plus d E. So, any n E d E represent the neutron density at this particular energy level, sigma t E is the macroscopic total cross section at that energy level, because you have to remember that cross section varies with energy. So, it will also it is also dependent on the energy and finally, the velocity of this neutron, velocity being directly related to the kinetic energy.

So, it is also is going to vary with the energy level. Then once we have specified one energy level E or maybe we have considered a very small energy band across this E of thickness d E, then the total number of interactions that we are expected to have we are expecting to have per unit volume is given by this expression or this n into v can be combined into something called neutron flux per unit energy, this is actually identical to that symbol I that we are using earlier, but generally phi is a more common symbol and I is more used to characterize the intensity of neutron beam that is incident on some kind of surface. Like we have an artificial source of neutron and we are producing neutron layer that neutrons are striking some targets, but here we are talking about the movement of neutron inside a medium. So, it is more convenient to use a classical symbol phi.

So, phi E refers to the neutron flux at that particular energy level, but neutron energy or the varies strongly with or the fraction of the neutron, which are appearing at the starting energy will very strongly with energy. Like you can see from this diagram at this particular energy band the neutron density is quite high whereas, at levels somewhere here the neutron density is quite low.

So, with the energy level both neutron density and accordingly the cross section and velocity also varies, and hence to and get an idea about the total cross section that is happening over this entire band starting from this 0 energy level to some extremely high energy level up to infinite, then we need to integrate this equation from 0 to infinity and this is what is the total rate of interaction per unit volume we are going to have. So, here this sigma t refers to the total macroscopic cross section corresponding to a particular energy level, similarly this phi is also the neutron flux at a particular energy level, and both are strong functions of energy neutron energy that is, and if we know their nature then we can of course, evaluate this integral.

Quite often and instead of calculating the total interactions you are our interest is to know about any particular interaction. Like say if you are dealing with a moderator our primary interest is to know how many collisions will be required for a fast neutron to slow down to the thermal neutron level, and hence our major interest is to know the rate of scattering collisions or scattering interactions.

So, we can just replace this subscript of this replace the subscript of this total cross section and use the corresponding subscript, and we can get the corresponding rate of

interaction. Like for scattering we need to consider the scattering cross section, similarly our interest is to know the absorption total absorption that is, we need to consider the absorption cross section.

And quite often now this with the advent of computers, we always like to go for digital techniques. So, instead of going for a continuous integration like if we want to evaluate this particular integral we need to know the energy relation or we need to know how sigma E and phi both are depending on energy. But quite often numerical references are used, numerical approach refers to this total domain of energy starting from here to this is being divided into several small elements; like we take several small elements each of some constant of varying thickness. So, this is the thickness of 1 element, and to know the value of this interaction at that, we can always multiply the thickness of the element with the cross section at that location and phi at that location, and then we can sum that product up for all such elements present in the (Refer Time: 29:49) to get the total rate of interaction.

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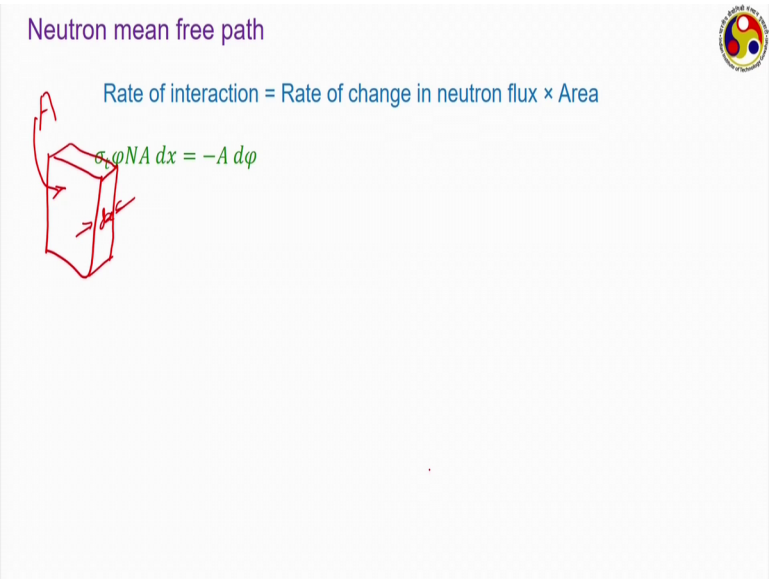

So, once we know the rate of interaction, then we need to define something called the mean free path. But now rate of interaction whenever there is an interaction of course, a neutron goes out of the package and therefore, there will be a change in the neutron flux as well, we can equate these 2 factors to get some kind of balance. That is rate of

interaction should be equal to the rate of change in the neutron flux, multiplied by the cross section area.

Let us consider some elemental area something like this, this area is A and this thickness is d x, then the rate of interaction as for our classical relation is sigma t into phi N A d x, where sigma t again is the total cross section, phi is the corresponding neutral flux across which is going through this area N is the nuclei density and A d x refers to the volume. That should be equal to the rate of change in the neutron flux into the area, here this minus sign is used to indicate that as we are moving forward in the x direction the neutron flux is reducing.

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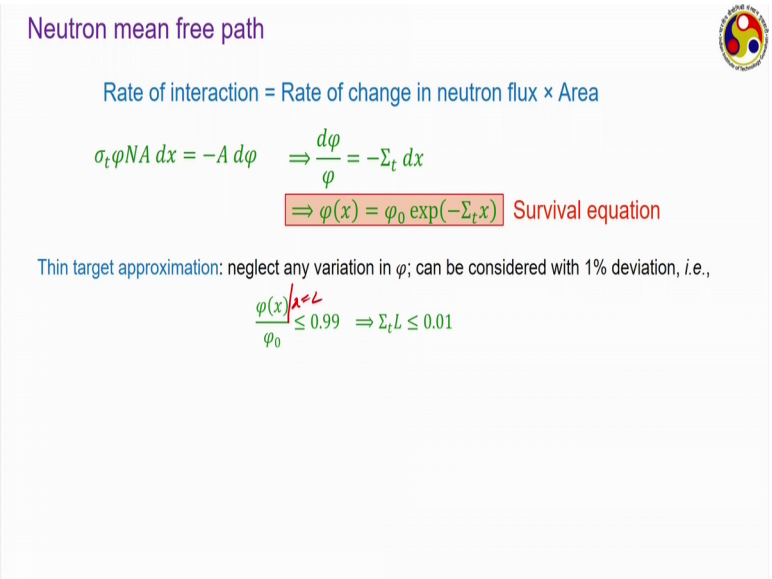

So, if we rearrange we are going to get this and finally, we arrive at this integrating it from x equal to 0 to means we are integrating this equation from some x equal to 0 to some x. So, phi at x equal to 0 is referred to as phi 0, and correspondingly we get an exponential relationship that is this neutron flux in such a geometry will follow an exponential relationship. This is called a survival equation because these characterize the rate at which neutrons are survived while flowing through such a medium and here sigma t we are using.

So, it considers all possible interactions that can be there; and me the there is often something called thin target approximation that we take, like thin target approximation refers to we use the earlier approach; that means, we neglect any variation in neutron

flux rather we assume the neutron flux to be constant over the entire domain of our interest. If we generally that kind of approach can be used when the variation in this phi is less than 1 percent of it is initial level.

So, we can use this to integrate approximation when this phi x at any location, they are basically we are talking about the x equal to L the end of the target at this location the value of phi is within 99 percent of the initial value and. So, if we rearrange the term that gives a criteria of this sigma t into L should be less than 0.01, that is when this criteria is satisfied we can go for this thin target approximation; that means, we can neglect any kind of variation in the value of phi, and we can assume it to be constant everywhere in our target nuclei.

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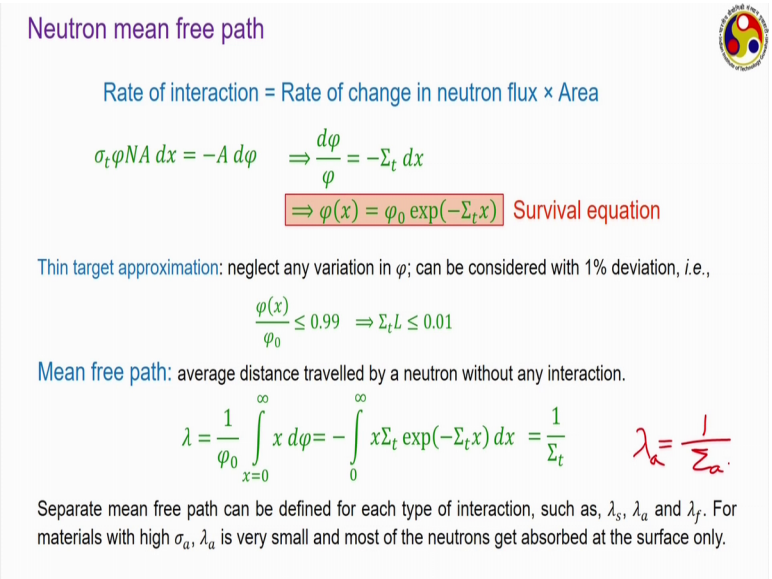

Now, mean free path is a defined as the average distance travelled by neutron without any kind of interaction that is once a neutron enters a medium of course, different neutrons will interact differently with the neighbouring molecules or neighbouring nuclei, and accordingly the travel distance will be different for different neutrons. But mean free path differs the average distance travelled by a neutron, without any such kind of interaction. And hence this is defined as lambda is the symbol that we commonly use, and using the symbol lambda we can define it as 1 by phi not into integration x equal to 0 to infinity x d phi.

If we put the expression for phi that like this particular survival equation, then we can simplify to this and finally, lambda coming to be the reciprocal of sigma t; that is the mean free path is found to be reciprocal of the total microscopic cross section. Instead of going for the total approach or all kind of interaction we can make it specific to all any particular interaction as well accordingly we can define a lambda s, that is scattering mean free path lambda a absorption mean free path and all even a fission mean free path as well. All of them will be reciprocal to the corresponding macroscopic cross section like if our interest is to know the lambda a that is absorption mean free path, that is the average distance travelled by neutron before it will get absorbed in the medium, that will be 1 reciprocal to absorption cross section macroscopic absorption cross section.

Now, when the value of this absorption cross section is extremely high or I should say when the value of this absorption cross section is extremely high, the value of this mean free path corresponding lambda a is definitely much lower, and accordingly most of the neutrons may get absorbed in the surface only before going into the interior of the medium. But this concept of mean free path is a very important 1 and once we have the data available about the cross section we can all calculate all kinds of mean free path.

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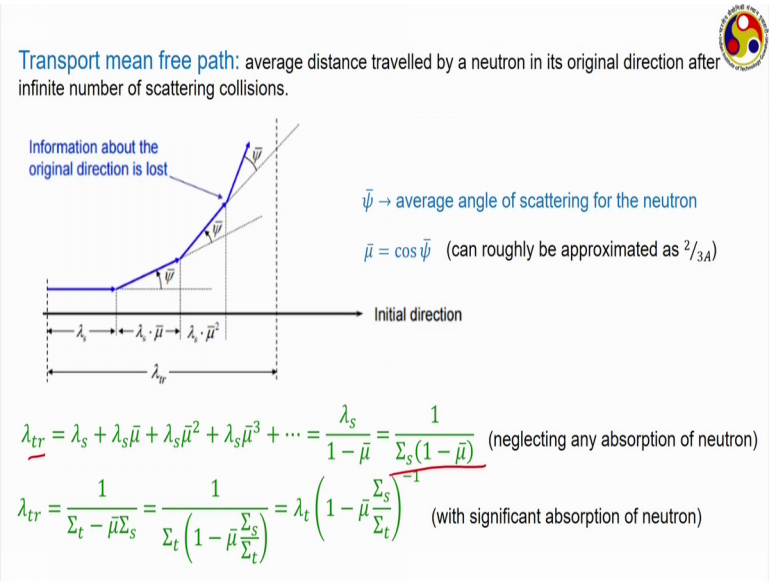

But there is another mean free path which is often defined that is called the transport mean free path. Transport mean free path refers to the average distance travelled by neutron in it is original direction after infinite number of scattering collisions. Let us look

at this diagram, say this is the point from where the neutron is entering the medium. So, before it encounters any kind of scattering interruption, it is likely to travel lambda s distance lambda s being the scattering mean free path.

So, here it will be encountering on an average this is the location where it will be including the first scattering location scattering interruption, and it will hence it will get scattered by some angle, then the if this angle is psi then it will encounter again following this particular line, it is likely to travel lambda s amount of distance and hence after it has travelled lambda s into mu bar distance it will encounter another scattering collision here and this will it will keep on continuing here, this psi bar refers to the average rate of angle of scattering for the neutron of course, different scattering may lead to different angle of deflection, and psi bar is the average of that, and this mu bar is the cosine of that psi bar.

So, while the neutron will first experience any kind of scattering after travelling this lambda s distance, it will be able to travel only lambda s into mu bar before in this original direction of motion, it is important to note that we are doing everything in terms of the initial direction of motion. So, lambda s mu bar will be the distance it will travel before the second scattering collision, and mu bar square before the third one and this it will continue till infinity. And this mu bar something that is beyond us the statistical it can mu bar is actually 2 by 3 into a d sorry 3 into a where a is the mass number of the target nucleus, which or the moderator nucleus which is inducing this scattering.

So, if we want to know the value of this transport mean free path t s transport we need to sum up all these distances that is lambda s for the first one, then lambda s into mu bar for the second one, lambda s mu bar square for the third one till infinity, and mu bar being a cosine function. So, it is always less than 1. So, this infinite series can be combined into lambda s into 1 upon mu bar or lambda s being the reciprocal of sigma s there is microscopic scattering cross section, we can write this particular thing the transporter mean free path is equal to the reciprocal of means scattering macroscopic scattering cross section into 1 minus mu bar.

So, why do you need to know the value of this macroscopic cross section, that is microscopic scattering cross section and molecular density, nuclei density that is we also need to have an idea about average angle at which this scattering is happening that is the

psi bar. Of course, this formula is valid neglecting any kind of absorption in the medium that is we are considering the neutron to go through only scattering cross section, but if neutron is present this distance will be not the same rather this distance will be slightly lesser depending upon the value of other cross sections. So, with significant absorption is present this is another relation that can be derived, where lambda transpose becomes equal lambda transport becomes equal to the reciprocal of total microscopic cross section minus mu bar into scattering cross section, and we can do this calculation we can found a relationship between the 2 as well.

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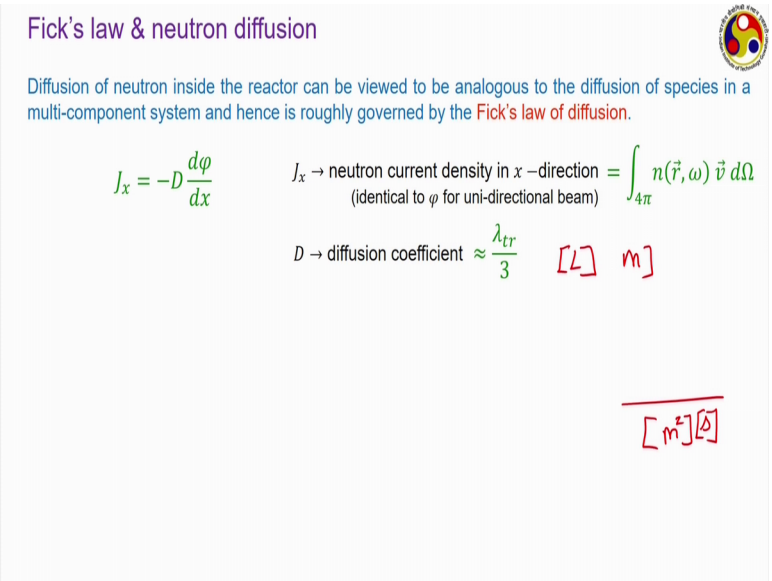

Next we have the diffusion of neutron; this is related to the leakage of neutron that we are talking about. Leakage means neutron is not able to undergo any kind of interaction rather whatever medium through which it is existing it is travelling through the molecules of the medium, and going from a place of higher concentration to a place of lower concentration quite analogous to the movement of heat or diffusion of heat or diffusion of mass.

Like the mass diffusion is often expressed in terms of the Ficks law this diffusion phenomenon of neutron can also be expressed in terms of the Ficks law, which is this here this J refers to here phi of course, is the neutron flux that we have mentioned earlier and J is neutron current density in the x direction see is given by this. It looks very similar to the phi, but there is one difference. Like the expression for phi is where it

looks similar, but here this v is actually speed that is a scalar quantity accordingly neutron flux phi is also a scalar quantity, but the v here in the definition of current density is a vector quantity because it considers the direction of that velocity vector.

You can think this about this way like it is also possible when there is no need motion of the medium still significant amount of interaction is taking place; that means, the total speed of the medium is 0, but there are may be individual components which are participating in interactions and hence the they are interacting or oriented such that their net is coming to be equal to 0.

So, neutron current density is a vector and phi neutron flux is a scalar, if you are talking about an unidirectional flow then they have to be identical this both in terms of magnitude and direction phi does not have a direction, but still the magnitude point of view J and phi will be equal only for unidirectional flow. D is the diffusion coefficient here, which can often be related to the one-third into the transport mean path.

Now, what it should be the unit of this diffusion coefficient can you tell from this Ficks law expression? Here phi is the neutron flux that is so, what should be the unit for phi? Iis the number of neutrons divided by the area through which it is flowing through. So, it is meter square and per unit time and it is also same for J, in case of J only one thing here it is a this being a vector J x basically refers to an area, which is having it is perpendicular in the x direction and J refers to the neutron current density which is flowing in that same direction or perpendicular to the area yes.

So, the unit for phi and J are equal and hence what should be the unit for d? The unit for d should be or dimension should be that of length like it is s i unit should be meter then. So, diffusion coefficient is something that characterizes a diffusion process of neutron through this, and diffusion coefficient we can calculate using the knowledge of the transport mean free path and we can do the calculation of transport mean free path in terms of the scattering cross sections.

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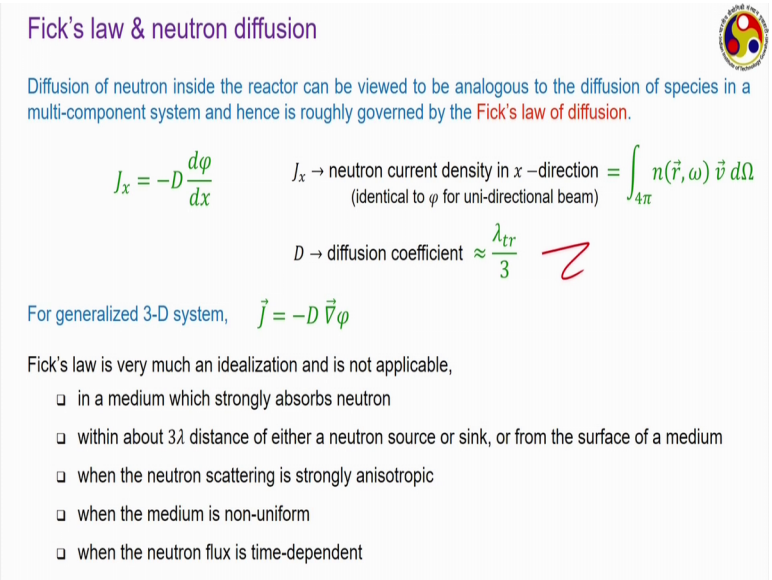

This being a vector we can always write all the 3 components for this neutron current density and if we combine all of them, then we can write for a generalized 3 D system that is J to be equal to minus D into grad of phi, but Ficks law before we apply Ficks law any further, Ficks law is very much an idealization and there are several limitations for this like it is limited only to the mediums or it is where the absorption is extremely low, that is when the absorption is significant absorption of neutrons, then we should not use Ficks law.

Similarly the we have neglected the presence of any kind of source therefore, when the neutron is or the location that we are analyzing is very close to some kind of source of sink as a (Refer Time: 43:24) guideline about 3 mean distance away from the source or sink or from the surface of a medium we should not use this. The neutron scattering if it is strongly anisotropic then also we cannot use it, because in the neutron intensity will keep on varying in different directions.

If the medium is non-uniform then the value of the diffusion coefficient and other scattering cross sections etcetera that will keep on changing with the location. So, we should not use fission the Ficks law and finally, when the neutron flux becomes time dependent, this very much a steady state equation. So, either neutron flux is time dependent we cannot use this.

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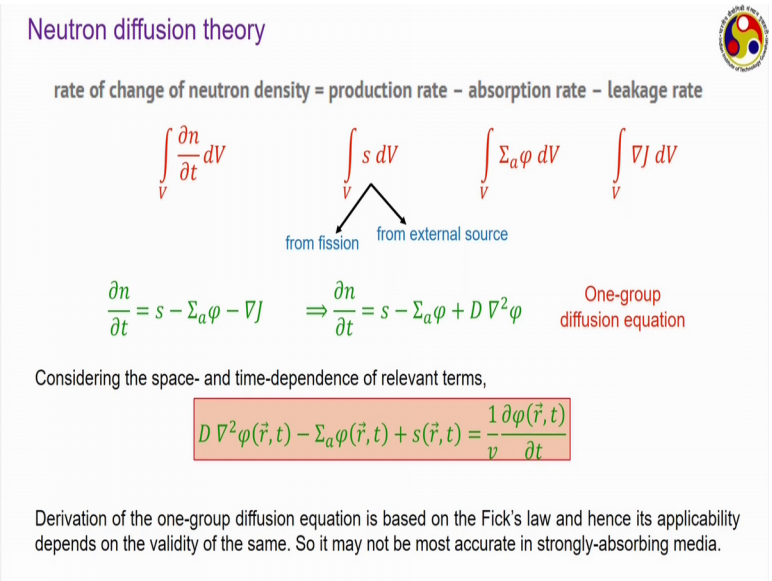

Using this idea of Ficks law let us write a neutron diffusion theory. Neutron diffusion theory governs the movement of neutron in a medium. So, a rate of change of neutrons it is similar to any kind of say mass diffusion theory or mass diffusion equation or maybe energy diffusion equation, thus the same way you can write rate of change neutron density should be equal to the production rate minus absorption rate minus leakage rate. Rate of change of neutron density we assume a small volume d V infinitesimally small volume, then the rate of change of neutron within that volume can be given as dou and dou t of d V integrated over the volume.

Production rate is can be someone to production rate accordingly we get a simple expression, production can be of can from 2 possible sources, one is the fission reaction other is if there is an external source of neutron the absorption rate and the absorption rate; we can find the total absorption cross section multiplied by the phi integrated over the volume that you are dealing with and finally, leakage rate it represent the grade of the current density and g refers to the current density expression, which we have derived it can be related to the Ficks law of diffusion. So, we combine all these terms together here and representing J in terms of the Ficks law, we arrive at this because J is equal to minus d grade psi. So, we get this here this grade square represents the laplation operator that is the second derivative of phi, and this particular equation is called the one group diffusion equation.

Also we know that in is phi the flux is equal to n into v where v is just the speed. So, we can considering the space and time dependence of relevant terms, and using this relation of phi and n, we can express this one group diffusion equation completely in terms of phi which is d into Laplacian of phi minus sigma a into phi plus s is equal to 1 upon v into dou phi by dou t. This equations the equation of this one is very much based on Ficks laws.

So, whatever limitations of Ficks law that we have mentioned in the previous slide all of them are applicable here. So, there are situation like when there is a strongly absorbing medium or a non-uniform medium or a medium where there is an isotropic scattering is going on there this is not applicable. Still one group diffusion equation is quite simple and offers us a reasonable solution in several situations, particularly to get a rough idea about the optimal design of a reactor to attain the critical condition.

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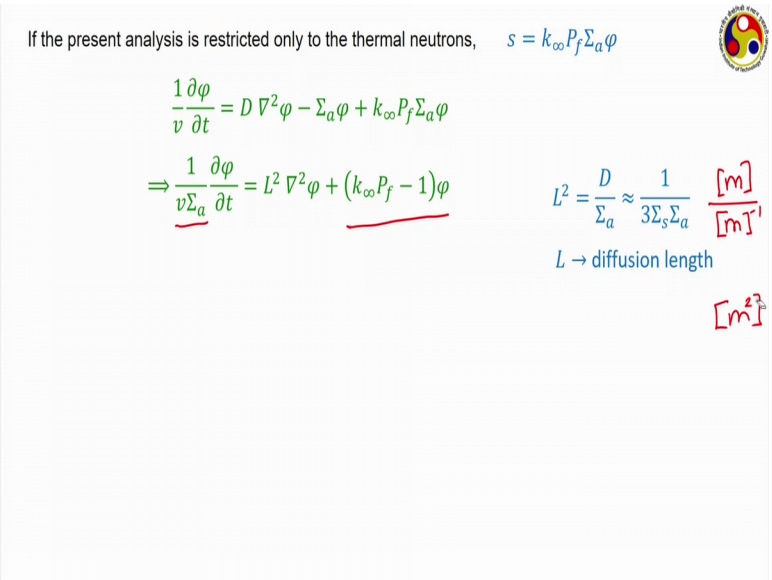

If we restrict this analysis only to the thermal neutrons, then we are talking about this source term here. We are talking only about thermal neutron that is all neutrons present in the system or thermal neutrons then the s can be really written as something like this, k infinity being the infinite multiplication factor into the fast fission leakage probability, this is into sigma a into phi sigma is a absorption cross section corresponding to thermal neutrons and phi is the neutron flux.

So, these are the total number of neutrons which are available or which are getting absorbed during the in the medium as the form of thermal neutrons. So, putting this back in the previous expression we are we get this, and now we divide this entire equation with sigma a on both side. Then here we are having v into sigma in the denominator and the epsilon related terms they are combined into this, and we are writing L square as D upon sigma a or d nearly being the 1 by 3 of into sigma s.

So, this is L square this L is called a diffusion length a very very important parameter in reactor design point of view, important some reason we shall be discussing in the next lecture L square being because of the similarity is why often called a area like what should be the unit of this L or L square, what is a unit of d? Unit of d is meter or length. So, a s I unit of d is meter as we have seen in the previous slide, what is the unit of macroscopic cross section? S i unit is meter inverse accordingly unit for L square is meter square and L is meter. So, that is why you use that term L we use L as length or we defined as a diffusion length, and L square quite often called a diffusion area and as this a dimension is of area.

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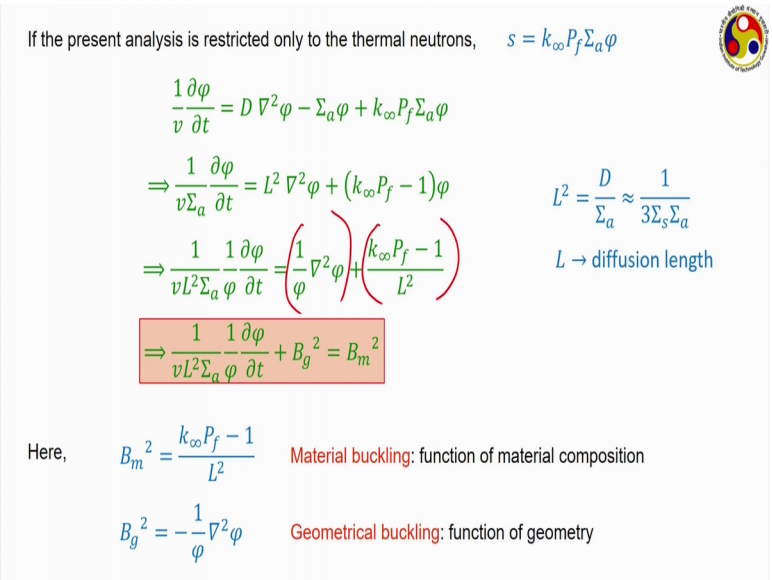

Now, we move forward with this and we have divided this entire equation by L square now, to get reduce to this form and finally, we replace the 2 right hand terms and with B g square and B m square. Here B m square is equal to this particular quantity and B g square is minus of this particular quantity, they are called buckling B m square is called

material buckling as it is a function of material composition only, and B g square is called geometrical buckling because it is a function of geometry, values of both this buckling parameters are very important while designing any reactor shape.

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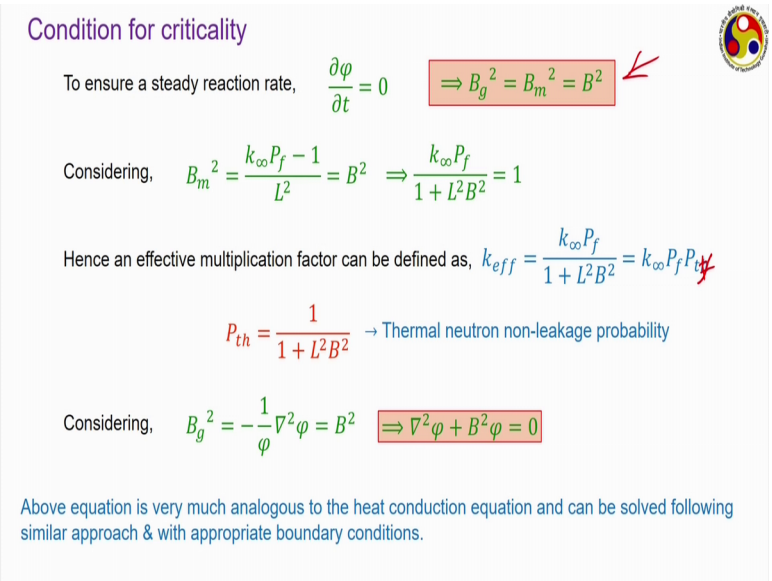

If we consider steady state if we want to attain a critical condition; that means, k effective should be equal to 1 or reactivity should be equal to 0, and the rate of reaction should remain constant over a long period of time. Now to ensure a stable reaction dou phi dou t is equal to 0. If we put it in the previous expression that is here then what we are going to get is B g square is equal to B m square is equal to say B square is called the buckling parameter.

Considering B m square equal to B square we get k infinity into P f y 1 plus L square B square is equal to 1. Accordingly we can define an effective multiplication factor effective thermal multiplication factor as, we can define an effective multiplication factor by separating out this k infinity and P a from this expression and whatever remaining we can call it P t h or just P t because I think earlier we are used a term P t h. So, neglect this earlier we have used P t. So, neglect this h here, and let us continue with P t only.

So, here this P t h or P t is a thermal neutron non leakage coefficient is 1 upon 1 plus L square plus B square. And if we take the second part of this particular equation then we get B g square is equal to B square and rearranging Laplacian of phi plus B square phi equal to 0.

This equation seems familiar have you seen it anywhere else? It looks very similar to the heat diffusion equation, and hence it is solution procedure is also quite similar to that. But there are boundary conditions, separate kind of boundary conditions which we must take into consideration while solving this. So, once we have it is properly defined boundary condition you can solve for this particular equation and get an idea about the nuclear reactor behaviour or how to design a nuclear reactor.

So, in the next lecture we shall be trying to see or trying to subject this particular equation to different kind of situations and see the corresponding solutions, and we shall be defining the importance of this diffusion length properly. So, thanks for your attention today and we shall be seeing in the next lecture.