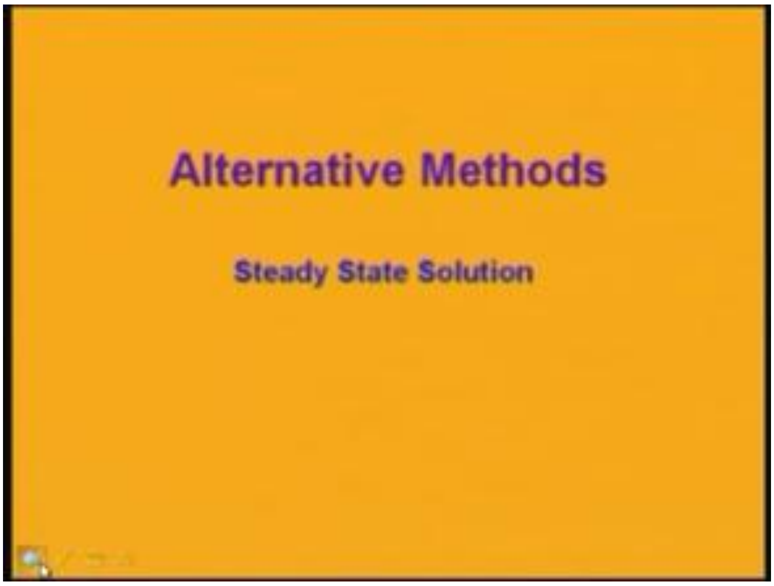
Mechanical Vibrations
Prof. S. K. Dwivedy
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

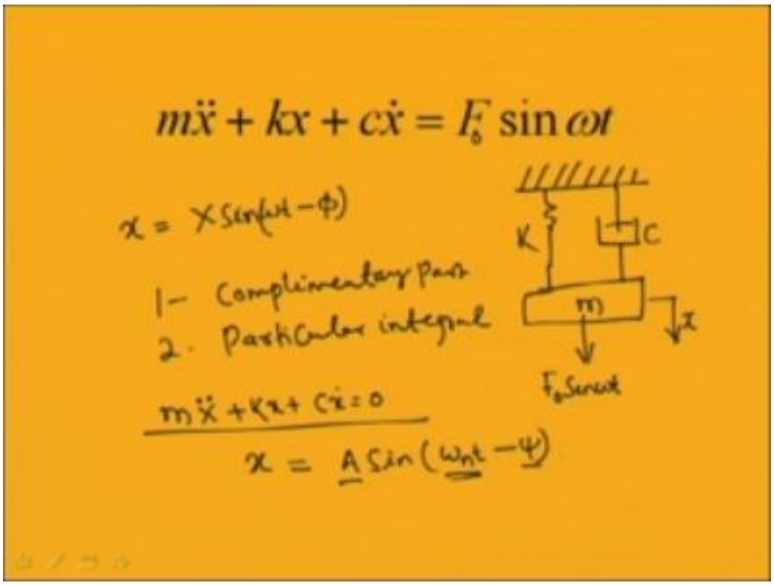
Module - 4
Single DOF Forced Vibrations
Lecture - 2
Laplace Transform, Superposition Theorem

(Refer Slide Time: 00:59)



In the last class, we have studied about the steady state response of a single degree of freedom systems subjected to harmonically exited response. So, there we have found the magnification factor by using a force polygon method we have obtained the response of the system. So, today we are going to study about different alternative methods to find the response of the system. So, these methods include the Laplace transform method in finding the particular solution of the differential equation or by using some numerical methods.

(Refer Slide Time: 01:39)



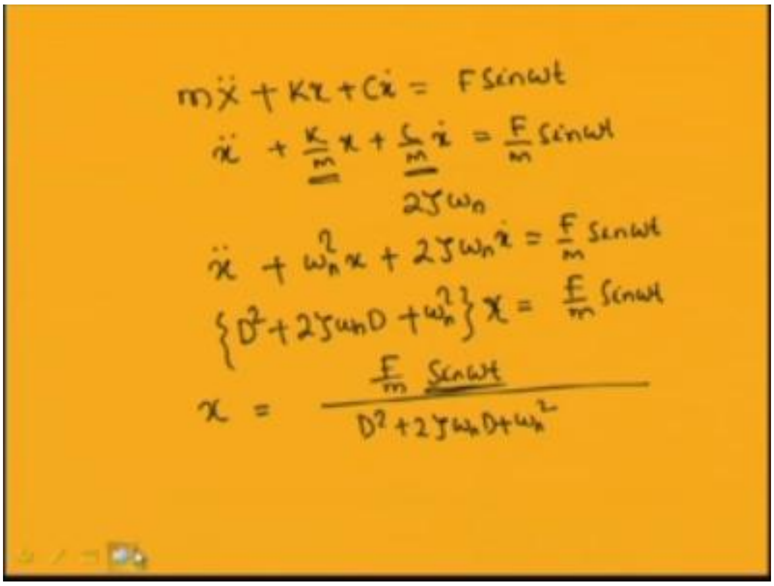
So, for a single degree of freedom system already we know the equation motion can be written in the form of m x double dot plus k x plus c x dot equal to F sin omega t; that can be represented from this spring mass damper system. So, a spring and damper system is shown in this. So, here k is the spring constant, c is the damping, and m is the mass, x is the displacement of this mass, and it is subjected to a force of F 0 sin omega t. So, due to this harmonic force already we have seen the steady state response of the system can be written as x equal to X sin omega t minus phi, where X is the amplitude of the response and phi is the phase of the response.

In the previous classes, you have study about the free vibration response of the system. So, for a system with equation m x double dot plus k x plus c x dot is equal to F sin omega t. It is for finding its solution, it will contain two parts. One is the complementary part; for this differential equation the solution will contain two parts. First is the complementary part. And second one is the particular integral.

So, we are interested to find the particular integral of the system as the complementary part is same as that of the free vibration response of the system or the response when you are taking this m x double dot plus k x plus c x dot equal to zero. So, this the free vibration response; this complementary part will contain the free vibration response of the system and the particular integral; that is when you are taking considering the force into account, the solution will be the particular integral.

So, this complementary part you know. So, this is equal to this x you have already found that it is equal to A sin omega n t minus psi, where omega n is the natural frequency of the system. And A and psi are the constant depending on the initial condition of the system; the initial conditions will be the displacement and velocity are t equal to 0. So, when we find the particular solution from this equation. So, to find the particular integral I can write this auxiliary equation of the system. So, the auxiliary equation of the system I can write.

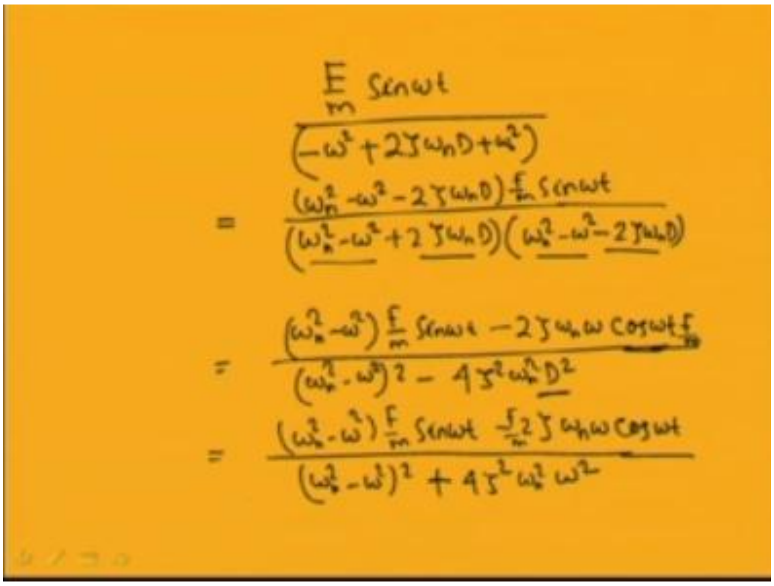
(Refer Slide Time: 04:38)



Before that I can modify this equation. So, m x double dot plus k x plus c x dot equal to F sin omega t; so that equation I can write equal to x double dot plus k by m x plus c by m x dot equal to F by m sin omega t. So, this k by m equal to omega n square or square of the natural frequency of the system and c by m equal to 2 zeta omega n, where zeta is the damping factor of the system. So, this equation can be written as x double dot plus omega n square x plus 2 zeta omega n x dot equal to F by m sin omega t.

So, the auxiliary equation I can write will be equal to D square. So, it is equal to D square plus 2 zeta omega n D plus omega n square x equal to F by m sin omega t. So, this x or particular integral can be obtained from this equation by this way. So, F by m sin omega t by this. So, this is D square plus 2 zeta omega n D plus omega n square. So, for a system with sin omega t in the numerator, the solution can be obtained by substituting this D square equal to minus omega square. So, by substituting D square equal to minus omega square, this equation will reduce to this form.

(Refer Slide Time: 06:42)



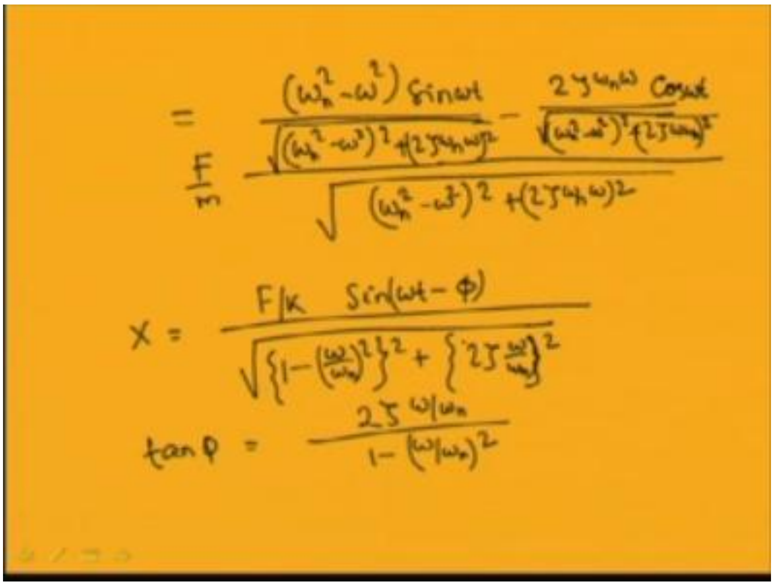
So, this will be equal to F by m sin omega t; the D square I have replaced by omega square. So, this is equal to omega minus omega square plus 2 zeta omega n D plus omega n square. So, I can multiply in the numerator and denominator the terms minus omega square plus 2 zeta omega n D. So, this term I can multiply in the numerator and denominator. So, this thing I can write it equal to this is omega n square minus omega square plus 2 zeta omega n D.

So, I will multiply with this omega n square minus omega square minus 2 zeta omega n D. So, D is the differential operator. So, this is equal to omega n square minus omega square minus 2 zeta omega n D into F by m sin omega t. So, this is in the form of a plus b into a minus b. So, the denominator can be written as omega n square minus omega square whole square minus 2 zeta omega n D whole square.

So, it can be written 4 zeta square omega n square D square 4 zeta square omega n square D square. And in the numerator part it will be equal to omega n square minus omega square into F by m sin omega t minus 2 zeta omega n. This D operator is nothing but the differential operator. So, one can differentiate that thing. So, one can obtain 2 zeta omega into omega. So, this sin will give differentiation of sin is cos. So, this is cos omega t into F by m.

So, this equation can be simplified further and one can write this is equal to. So, this can be written in the form. Here one can observe that this D square operator n can be replaced by this minus omega square. So, if it is replaced by this minus omega square, then this thing can be written as omega n square minus omega square F by m sin omega t minus 2 zeta omega n omega F by m into 2 zeta omega n omega cos omega t by omega n square minus omega square whole square minus 4 zeta square omega n square. And this thing can be replaced by minus omega square. So, minus minus this becomes plus. So, this becomes this. So, one can take this F by m common. So, this equation can be written as F by m.

(Refer Slide Time: 10:21)

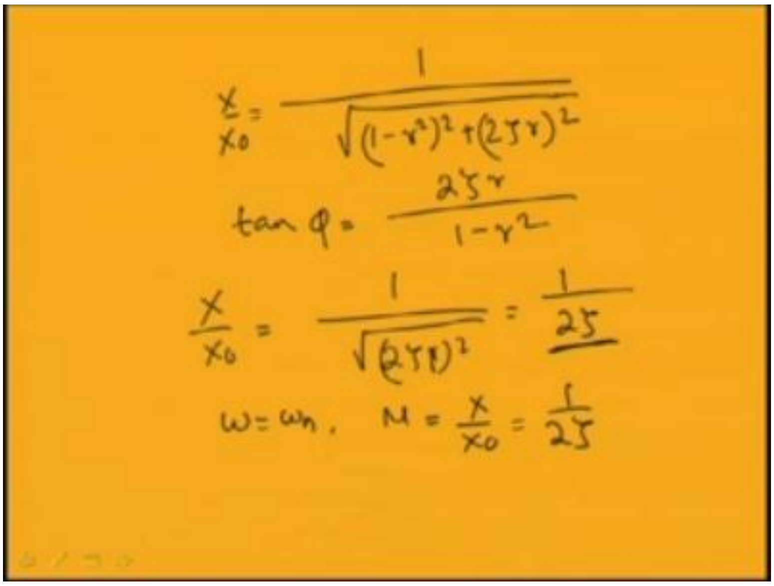


So, this is equal to F by m into omega n square minus omega square by root over minus omega n square minus omega square. One can simplify this thing, and easily it can be written in this form. So, this is equal to F by m omega n square minus omega square by root over omega n square minus omega square whole square plus 2 zeta omega n into omega whole square into sin omega t minus 2 zeta omega n omega root over omega n square minus omega square whole square plus 2 zeta omega into omega n whole square into cos omega t whole by root over omega n square minus omega square whole square plus 2 zeta omega n into omega whole square.

So, from this one can observe that this can be written in terms of sin a into cos b minus cos a into sin b by this term. So, this can be written. So, this thing can be taken as cos phi and this thing as sin phi. So, sin omega t into cos phi minus cos omega t into sin phi from this can be written. So, this x can be written as. So, this thing can be written by F by k; finally, it can written by F by k into sin omega t minus phi by root over 1 minus omega by omega n whole square plus 2 zeta omega by omega n whole square.

So, this is same as that by equation we have observed before, and here tan phi also written you can write in terms of 2 zeta omega by omega n by 1 minus omega by omega n whole square. So, from this expression this expression can be obtained by taking this omega n square common out, and one can write this expression by using this omega n square. This omega n square term if you take it out, it will cancel, and it will reduce to this form. So, by using this particular integral method one can find the particular integral of the system.

(Refer Slide Time: 13:45)



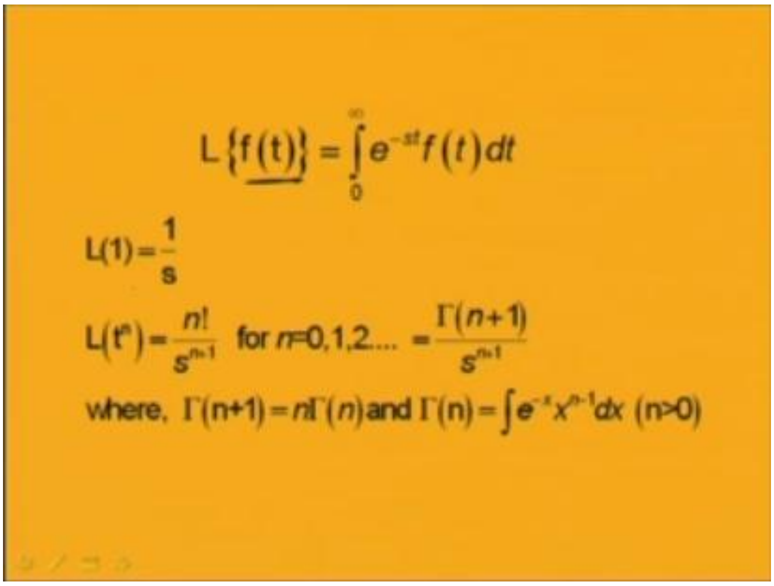
Particular integral of this equation can be given by x equal to F 0 by k or this F 0 by k one can write this x 0. So, one can write this x by x 0 equal to 1 by root over 1 minus r square whole square plus 2 zeta r whole square, and tan phi equal to 1 tan phi equal to 2 zeta r by 1 minus r square. So, from this expression you can find when r equal to 0. So, when r equal to 0, this x by x 0 equal to 1 by root over 2 zeta r square or this is equal to r equal to 1.

So, when r equal to 1 you can find this part equal to zero. So, then this part will be equal to 1 by. So, this is equal to 1. So, this is equal to 2 zeta square root over. So, this becomes 2 zeta. So, when omega equal to omega n, then the response of the system is dominated by or is reduced by damping only. So, at omega equal to omega n, this magnification factor M equal to x by x 0 equal to 1 by 2 zeta.

(Refer Slide Time: 15:29)

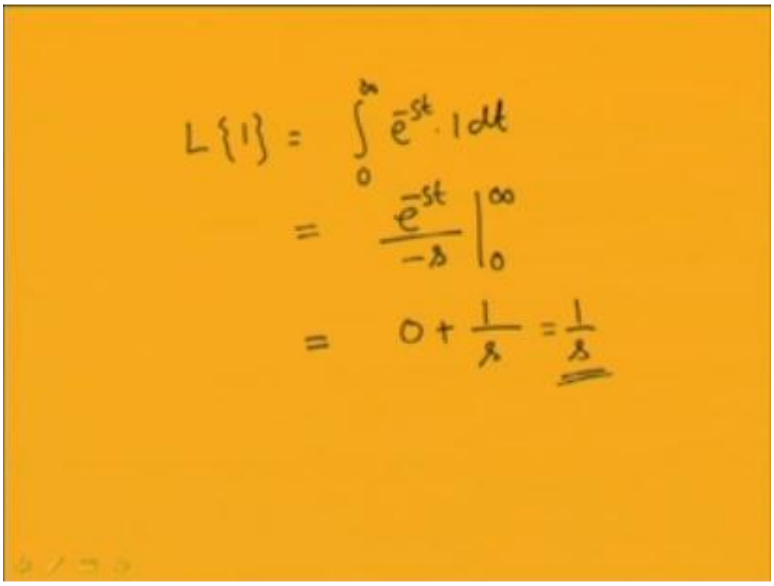
Now, one can use some other method that is the Laplace transform method to find the response of the system. So, before proceeding for the Laplace transform method, we should review briefly about the Laplace transform method.

(Refer Slide Time: 00:15:46)



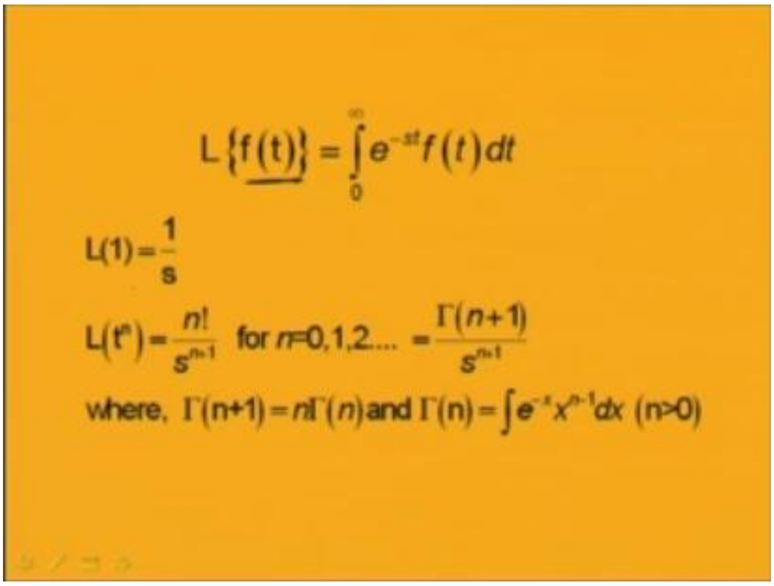
So, the Laplace transform of a function can be given by L f t. If f t is a function, then the Laplace transform of this function can be given by integral 0 to infinite e e to the power minus s t f t d t. So, for some commonly used terms that is 1 t to the power n and sin cosine terms, you should remember the Laplace transform let us find for one.

(Refer Slide Time: 16:13)



So, Laplace for finding Laplace transform for one. So, it will be equal to minus. So, 0 to infinity e to the power minus s t into e to the power minus s t into 1 dt. So, this is equal to. So, this becomes e to the power s t. So, this is e to the power minus s t by minus s 0 to infinity. So, when one substitute the value of infinity. So, this becomes 1 by e to the power minus infinity. So, this is e to the power infinity. So, this becomes 1 by. So, one can obtain this is equal to 0 minus 1 by s. So, when one substitute t equal to 0, this becomes 1. So, minus minus plus; so this becomes 1 by s. So, Laplace transform of 1 equal to 1 by s.

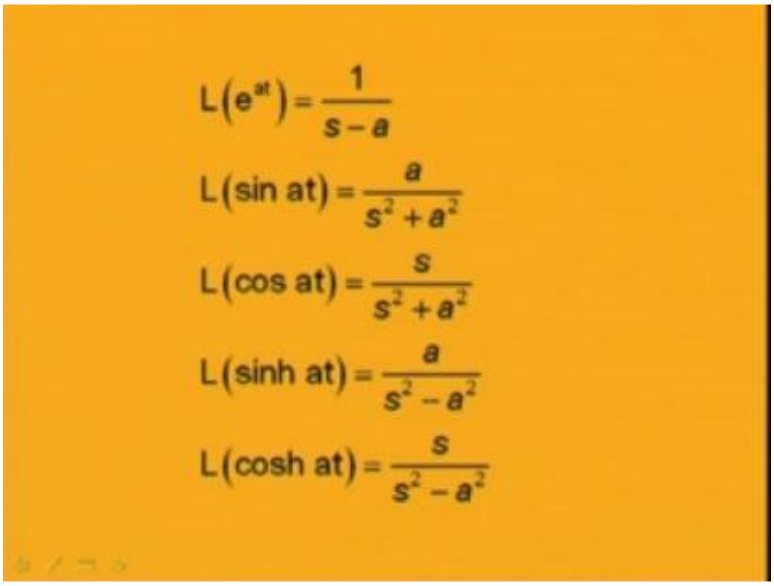
(Refer Slide Time: 17:21)



Similarly, one can find the Laplace transform for other terms. So, Laplace transform of t to the power n equal to n factorial by s to the power n plus 1. So, for n equal to 0, 1, 2, you can find t to the power 0 that is 1. So, it reduces to 1 by s. Similarly, t for Laplace transform of t will be equal to 1 by s square and so on. So, this will be equal to Laplace transform of t equal to n factorial by s to the power n plus 1, which can be represented by using the gamma function also.

So, this is equal to gamma n plus 1 by s to the n plus 1 and you know the special function gamma can be written. So, this gamma n plus 1 can be written equal to n gamma n, and gamma n function is written by this integral minus e to the power minus x x to the power n minus n d x.

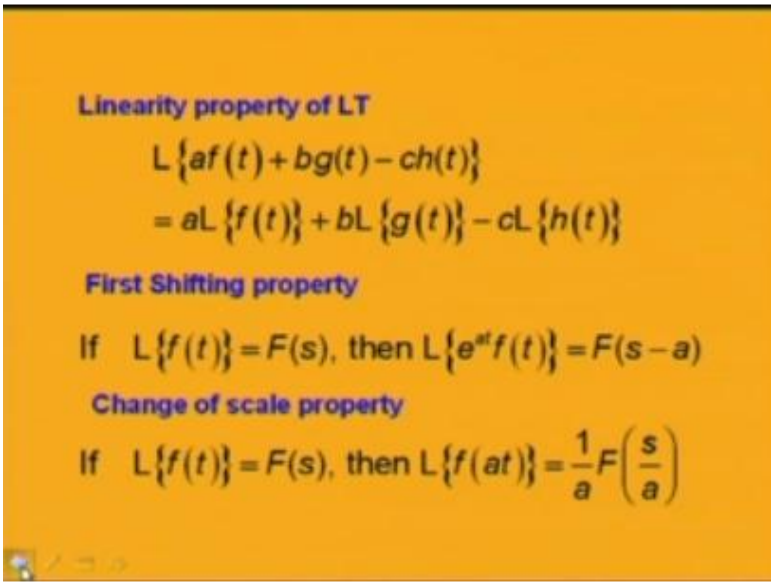
(Refer Slide Time: 18:22)



So, for some commonly used terms, the Laplace transform are given here like Laplace transform of e to the power a t equal to 1 by s minus a. Laplace transform of sin a t equal to a by s square plus a square. Laplace transform of cos a t equal to s by s square plus a square. Laplace transform sin hyperbolic a t equal to a by s square minus a square. And Laplace of cos hyperbolic a t equal to s by s square minus a square.

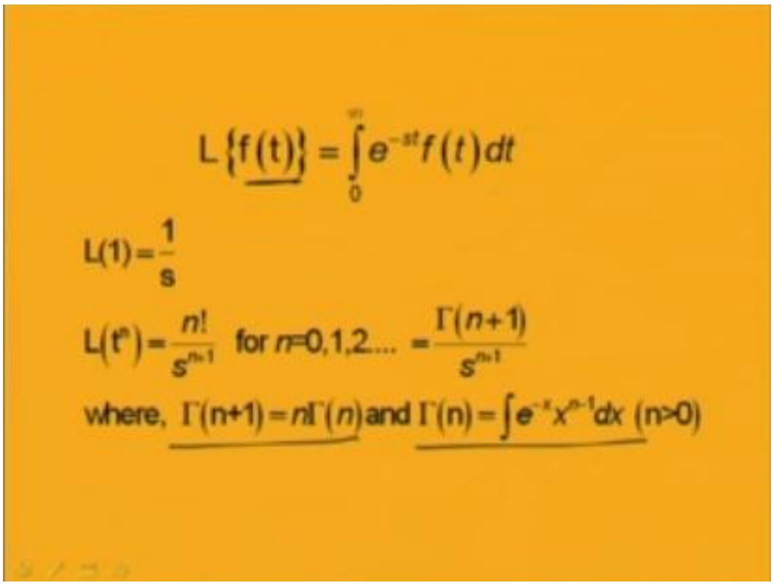
So, you may remember this thing for sin and cos for this two trigonometric function, it is the denominator is s square plus a square. And when you are using a hyperbolic function, then the denominator containing minus sign; otherwise, sin a t equal to a by s square plus a square/ And sin hyperbolic a t equal to s by s square minus a square. Similarly for cos in the numerator in both the cases it is s, what in the denominator the difference is the sin of this terms. So, this is plus, and this is minus.

(Refer Slide Time: 19:40)



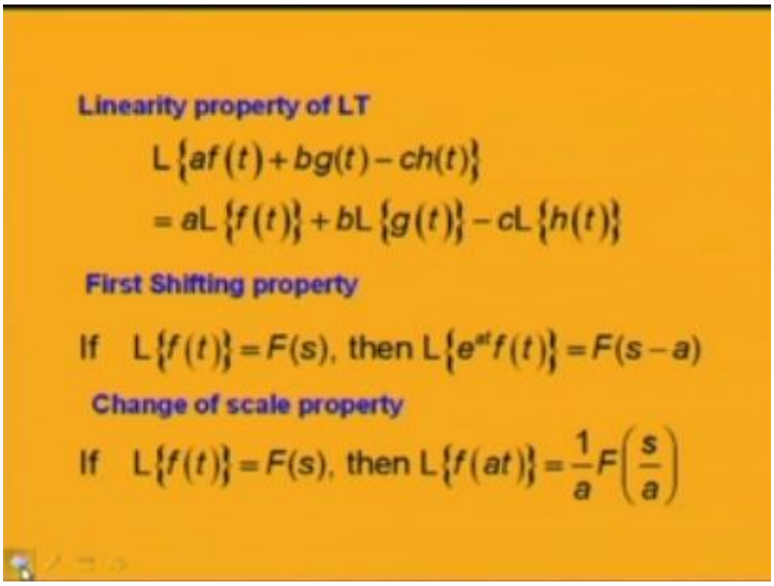
Similarly for other terms, you can find the Laplace transform by using the first principle.

(Refer Slide Time: 19:48)



That is by integrating or by using this formula e to the power minus s t f t d t.

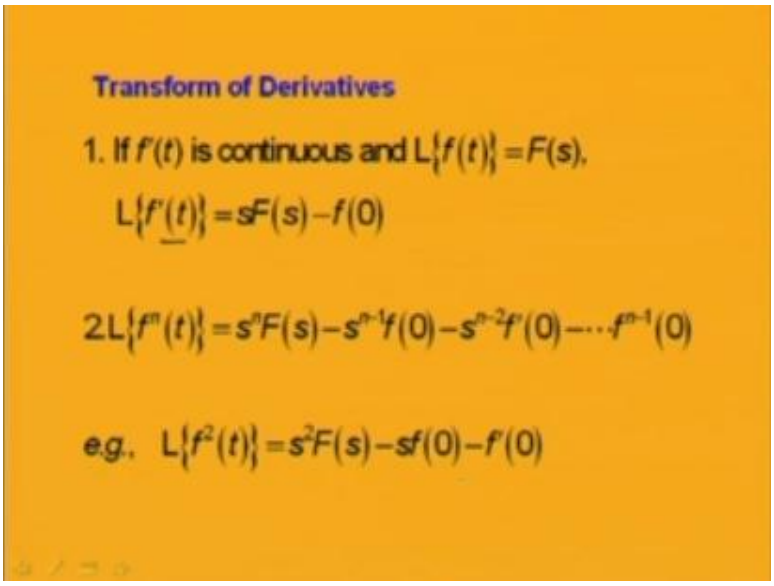
(Refer Slide Time: 19:56)



So, some of the properties of the Laplace transform are given here. So, first the linearity property of the Laplace transform if let you have different function like f t g t and h t. And if they are related and we want to find the Laplace transform of a f t plus b g t minus c h t. So, in that case it will be equal to the Laplace transform of the individual term. So, it will be equal to a into Laplace transform of f t plus d into Laplace transform of g t and minus c into Laplace transform of h t. So, this is the linearity property of this Laplace transform.

Similarly, one can view with this first shifting property. So, in case of first shifting property if L f t equal to F s, then Laplace transform of e to the power f t equal to F s minus a. This is a very important property, and you should remember this property. So, change of scale property. So, if the Laplace transform of f t equal to F s, then if you are multiplying a constant term with t; that is I am taking the Laplace transform, then Laplace transform of F a t will be equal to 1 by a into F s by a. So, this is the change in scale property. So, the scale is changed. So, you just see when a is multiplied here. So, it will divided by a 1 by a and s will be divided by a. So, it will be 1 by a F s by a.

(Refer Slide Time: 21:34)



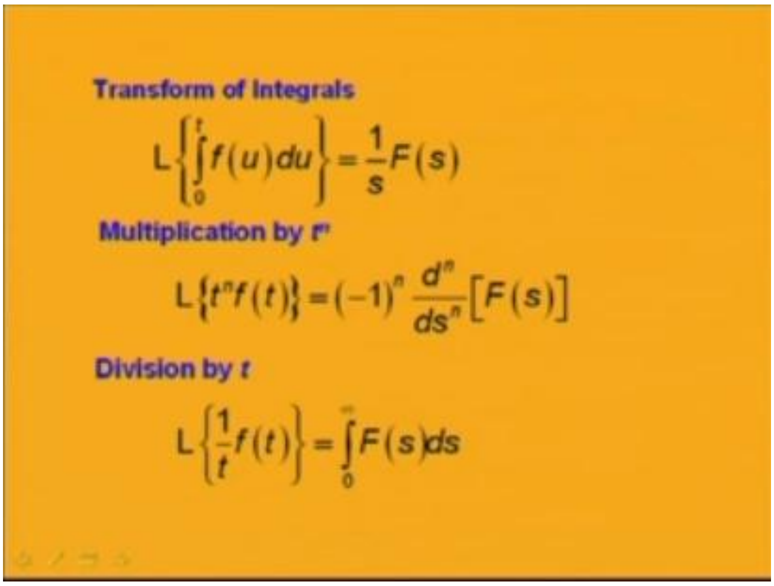
Some other property like transform of derivatives, if we are taking the Laplace of the derivative of a function, then that thing can be given by this. This thing can be directly found from the first principle of the definition of this Laplace transform. So, Laplace transform of f dash t, f dash t is d by d t of ft. So, this Laplace transform of f dash t equal to s into F s minus f 0. So, in general if we are finding the Laplace transform of nth derivative of a function.

So, Laplace transform of a nth derivative if of a function will be s to the power n into F s minus s to the power n minus 1 F 0 minus s to the power n minus 2 F dash 0 and it goes on. And the last term will be s to the power n minus 1 that is s to the power 0 into F n minus 1 0. So, you can remember this thing easily in this way. So, the power of s is decreasing. So, this is s to the power n. Then next term with a negative with s to the power n minus 1, next term s to the power n minus 2.

So, in this way the power will decrease, but the power of this function that is f 0. So, first term is F 0; second term is the derivative of 0; third will be the second derivative of f 0. So, in that way it will go on increasing, and the last term be n minus 1 derivative of F with the initial condition t equal to 0. So, if you want to find for example, if you want to find the Laplace transform of d square f by d t square. So, it will be equal to s square a square f s minus s, here n equal to 2; so 2 minus 1 equal to 1.

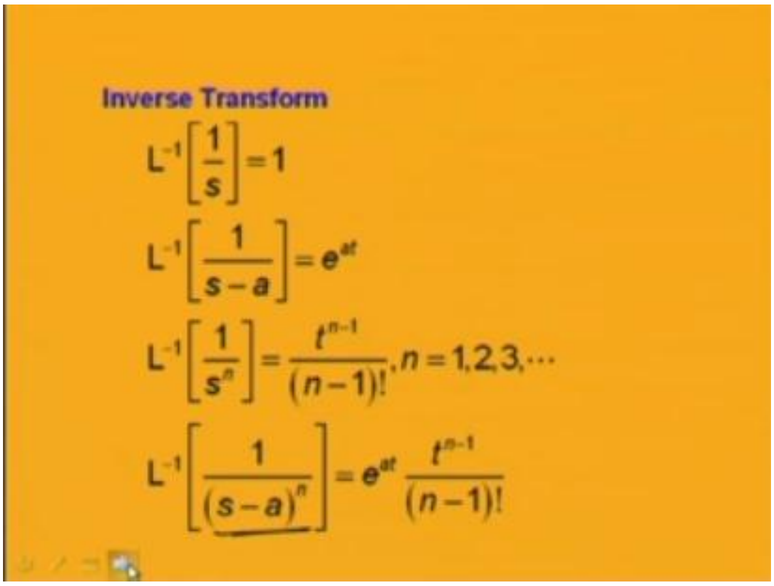
So, this is equal to s s f 0 minus this n equal to 2. So, 2 minus 2 equal to 0. So, s to the power 0 equal to 1 into F dash 0. So, Laplace transform of F square t equal to s square f s minus s F 0 minus F dash 0.

(Refer Slide Time: 23:53)



Similarly, you can find the transform of integrals. So, if you have integral term that is 0 to t f u d u if you want to take the Laplace of that. So, it will be simply 1 by s F s, and if this term is multiplied with a t n. So, if the Laplace transform of F t equal to F s, then t n into f t will be equal to minus 1 to the power n d n by d s n F s. So, if you are defining a term by t. So, Laplace transform of 1 by t F t will be equal to integral 0 to infinity F s d s.

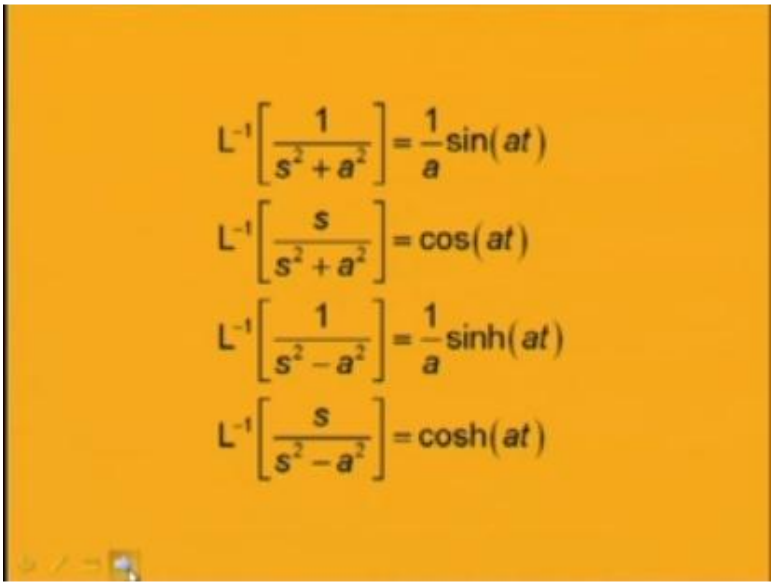
(Refer Slide Time: 24:34)



So, now let us see the inverse Laplace transform; so already we known that Laplace transform of 1 equal to 1 by s. So, inverse Laplace transform of 1 by s equal to 1. Similarly, e to the power a t, Laplace transform of e to the power a t equal to 1 by s minus s. So, inverse Laplace of 1 by s minus a equal to e to the power a t. Similarly, inverse Laplace of 1 by s to the power n equal to t to the power n minus 1 by n minus 1 factorial; already you know the Laplace transform of t to the power n.

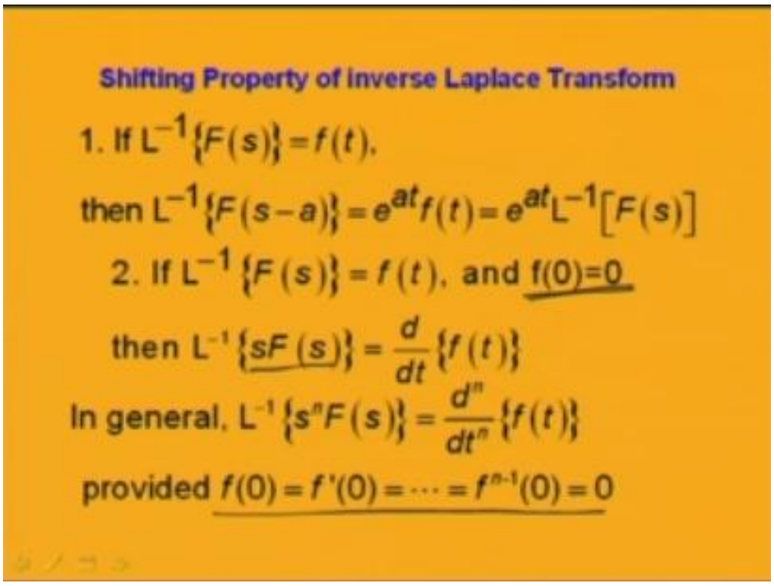
So, in a reverse way you can find the inverse Laplace of 1 by s to the power n. Similarly, you can see this is the shifting theorem is applied here 1 by s minus a to the power n. So, you know the Laplace inverse of 1 by s to the power n. So, inverse Laplace of 1 by s to the power n is equal to t to the power n minus 1 by n minus 1 factorial. So, now here a is subtracted here to the power n. So, it will be multiplied by e to the power a t; so e to the power a t. So, inverse Laplace of 1 by s minus a to the power n is equal to e to the power a t t n minus 1 by n minus 1 factorial.

(Refer Slide Time: 25:56)



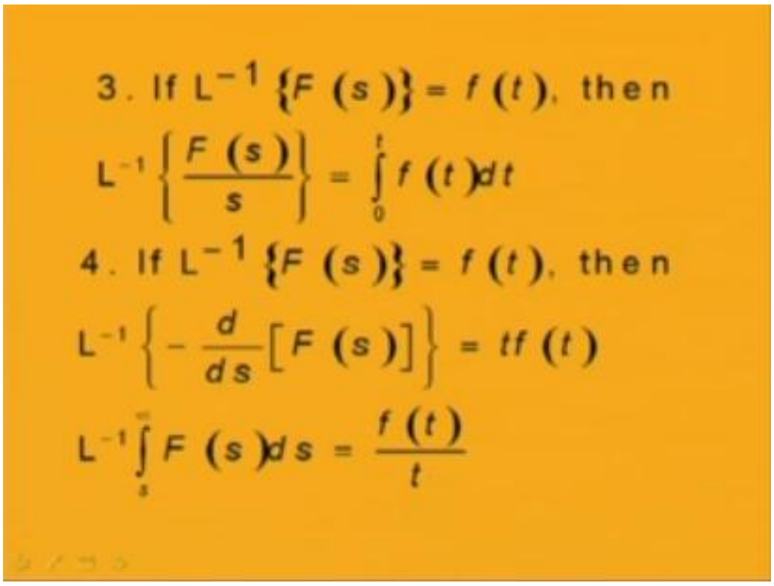
Similarly, you can find the inverse Laplace of 1 by s square plus a square; already you know the Laplace transform of sin a t equal to a by s square plus a square. So, inverse Laplace transform of 1 by s square plus a square will be equal to 1 by a sin a t. And inverse Laplace transform of s by s square plus a square will be equal to cos a t. And inverse Laplace of 1 by s square minus a square equal to 1 by a sin hyperbolic a t. And inverse Laplace of s by s square minus a square equal to cos hyperbolic a t. So, if this s is replaced by s minus omega, then you can multiply a to the power omega t here and by applying the shifting theorem, you can write the inverse Laplace.

(Refer Slide Time: 26:46)



So, this is the shifting property of the inverse Laplace already I told if L inverse F s equal f t, then L inverse F s minus a will be equal to f t multiplied by e to the power a t. So, it will be e to the power a t into f t. Similarly, if L inverse f s equal to f t and the initial condition is 0 that is f 0 equal to 0, then this f s is multiplied by s, then it will be the inverse Laplace will be the derivative of this. So, d by d t of t, and in general you can write inverse Laplace of s to the power n f s equal to d n by d t n f t. If all the initial conditions are 0; all the initial conditions f 0 equal to 0 and its derivatives are 0, F dash 0 equal to 0 and fn minus 1 0 equal to 0.

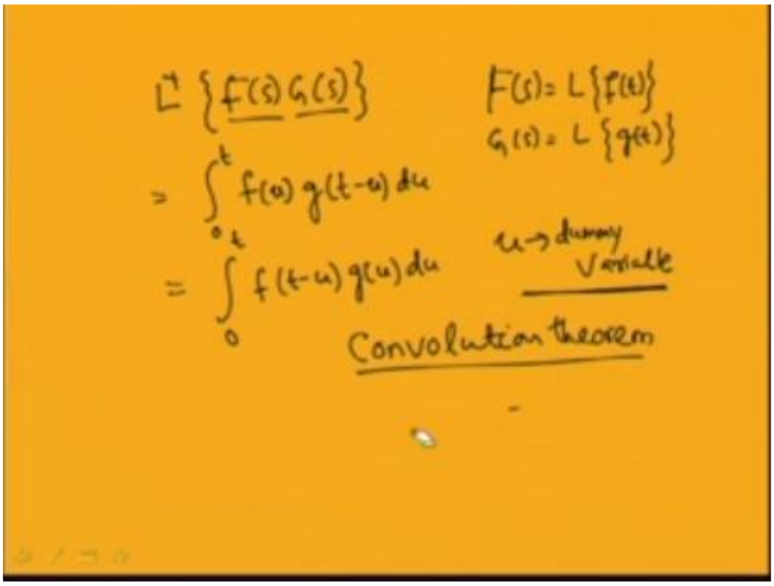
(Refer Slide Time: 27:46)



Also you can find the inverse Laplace of F s by s. So, that will be equal to integral 0 to t F t d t and inverse Laplace of minus d by d s F s. So, it is derivative and it is divided by s. So, if it is divided by s you just take the integral, and when it is derivative then you just multiply by t. So, minus d by d s F s will be equal to t into f t and inverse Laplace of F s d s will be equal to f t by t. So, in this way you can find the Laplace transform and inverse Laplace transform of different terms.

Now you can apply the convolution integral also; by applying convolution integral you can find the inverse Laplace of the terms when they are more than one terms. Let you have two terms; you want to find the inverse Laplace of…

(Refer Slide Time: 28:55)

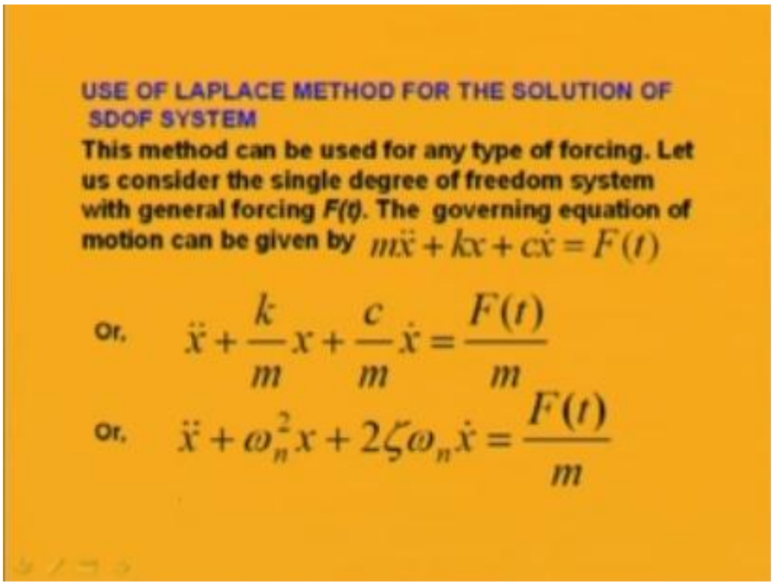


So, even to find the inverse Laplace of F s and G s; so let Fs is the Laplace transform of f t, where F s equal to Laplace transform of f t and G s equal to Laplace transform of g t. So, if we want to find the inverse Laplace of F s G s that is multiplication of these two terms. So, you can apply the convolution integral or convolution theorem. So, we state that inverse Laplace of F s G s, this will be equal to integral 0 to t f u g t minus u d u, or same thing you can write it as 0 to t f t minus u g u d u.

So, when you have product of two terms. So, you can use this convolution theorem. So, using this convolution theorem you can find the inverse Laplace of terms when it is multiplied. So, inverse Laplace o F s G s will be equal to integral 0 to t f u g t minus u d u which is equal to f t minus u. So, this is tau not t. So, this is tau; tau is the dummy variable. So, this is equal to this is t g t minus u, and u is the dummy variable; here u is the dummy variable.

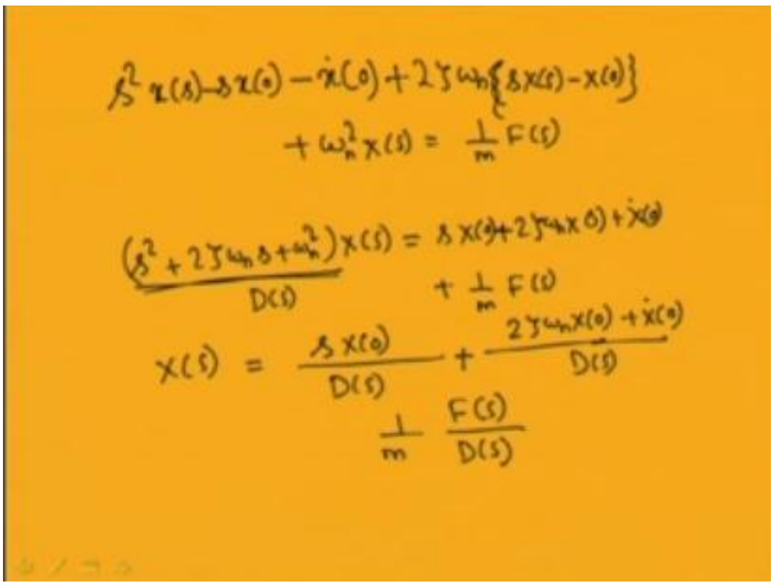
So, this convolution theorem is an important theorem which you can find applications for finding the steady state response of the system. So, when you have a forcing function. So, if you know the forcing function. So, you can find. So, let a force F sin omega t acts on a system, and you know the Laplace transform of this, and you want to find the response. So, that time you can use this convolution theorem to find the response; that thing we will see after few minutes.

(Refer Slide Time: 31:34)



So, let us use this Laplace transform method to find the solution of a single degree of freedom system. So, for the single degree of freedom system the governing equation already you know can be written in this form m x double dot plus k x plus c x dot equal to F t, or I can write this equation in this form x double dot plus k by m x plus c by m x dot equal to F t by m or this thing can be written in this form x double dot plus omega n square x plus 2 zeta omega n x dot equal to F t by m. Now to find the response I can take the Laplace transform in both sides; so x double dot that is equal to d square x by d t square.

(Refer Slide Time: 32:24)

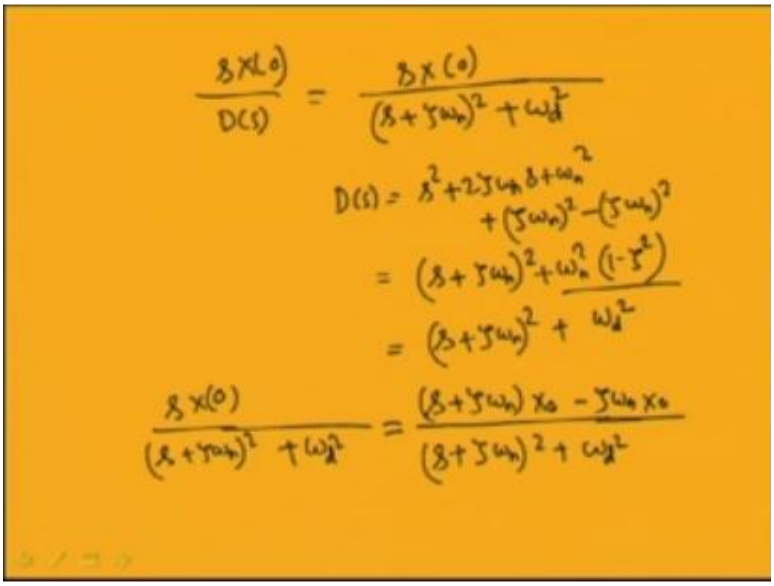


So, its Laplace transform can be written in this form s square X s minus s x 0 and the Laplace transform of 2 zeta omega n x dot equal to. So, this Laplace transform of x double dot equal to s square x s minus s x 0 minus x dot zero. And the Laplace transform of 2 zeta omega n x dot equal to 2 zeta omega n into s x s minus x 0, and the Laplace transform of omega n square x is simply omega n square x s. So, this will be equal to 1 by m F s.

So, I can rearrange these terms, and I can write this equal to s square plus 2 zeta omega n s plus omega n square x s will be equal to s x 0 plus 2 zeta omega n x 0 plus x dot 0 plus 1 by m F s. So, we have three terms. So, I can write this X S equal to. So, X S will be equal to three terms. So, the first term will be equal to s x 0 by I can write let this term I am writing as D s. So, let this is equal to D s. So, I can write this first term equal to s x 0 by D s plus the second term I can write equal to 2 zeta omega n x 0 plus x dot 0 by D s.

And the third term I can write it equal to 1 by m F s by D s. So, to find x t, I can find the inverse Laplace of these 3 terms. So, I have to find the inverse Laplace of this s x 0 by D s, then inverse Laplace of the second term and the third term, and I can add these things to find the total response of the system. So, let us take the first term. So, the first term is.

(Refer Slide Time: 35:03)



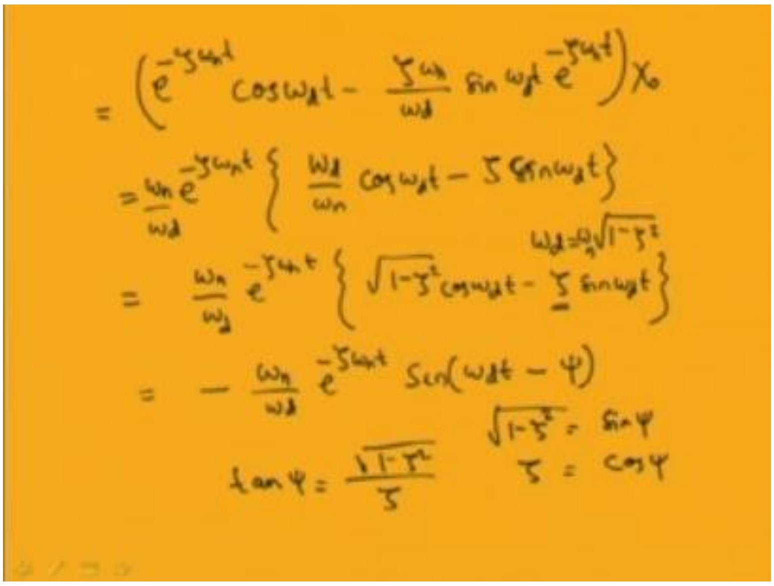
So, the first term I can write equal to X 0 by D s. So, this D s is nothing but. So, this is equal to s x 0 by s plus. So, this D s I can write. So, D s is s square plus 2 zeta omega n s plus omega n square. So, I can add plus. So, let me add zeta omega n whole square and subtract zeta omega n whole square. So, this will be equal to s square plus 2 zeta omega n s plus zeta square omega n square. So, this will give rise to s plus zeta omega n whole square. And these 2 terms omega n square minus zeta square omega n square, I can write this equal to omega n square into 1 minus zeta square.

So, this omega n square into 1 minus zeta square is nothing but the damped natural frequency that is omega d square. So, it becomes s plus zeta omega n square plus omega d square. So, I can substitute this here. So, s X 0 by D s I can write equal to s plus zeta omega n whole square plus omega d square. So, I can take the inverse Laplace of this. So, inverse Laplace of this will become. So, we have to find the inverse Laplace of this term. So, to find this thing I can write this equal to.

So, the first term is x 0 by s plus zeta omega n whole square plus omega d square. So, I have to find inverse Laplace. So, I can write this will be equal to. So, as this is plus zeta omega n. So, it will contain a term e to the power zeta minus zeta omega n term and as there is s in the numerator; so you can think of that there is a cos term is there and to find the inverse Laplace. So, I can add and subtract zeta omega n term. So, s will be replaced as s plus; in the denominator you have s plus zeta omega n. So, in the numerator also you should have s plus zeta omega n.

So, you can add s plus zeta omega n in the numerator and denominator. So, this becomes this and minus zeta omega n x 0 by s plus. So, this becomes s plus zeta omega n. So, this becomes s plus zeta omega n whole square plus omega d whole square. So, it contains two parts. The first part equal to s plus zeta omega n into x 0 by s plus zeta omega n whole square plus omega d square, and second term equal to minus zeta omega n x 0 by s plus zeta omega n whole square plus omega d square. So, the first part will be the inverse Laplace will contain e to the power minus zeta omega n into cos term. And the second part will contain the sin term.

(Refer Slide Time: 38:45)

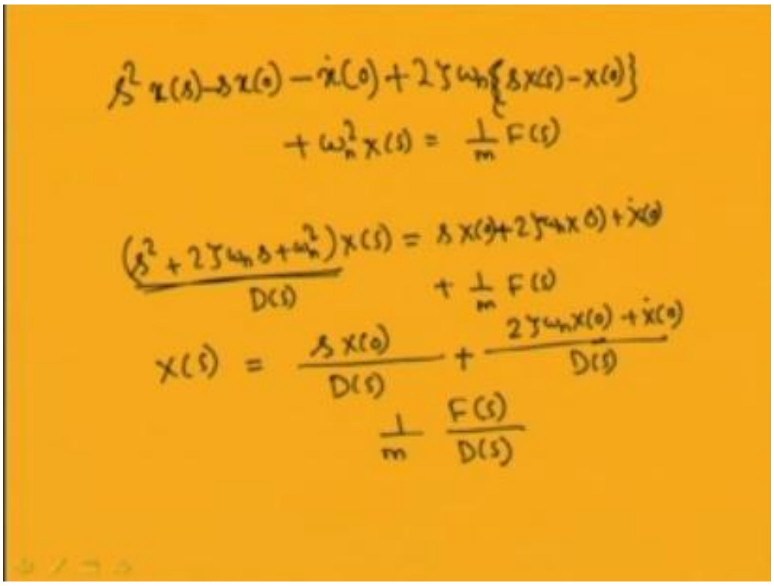


So, first term can be written as this is equal to e to the power minus zeta omega n t cos omega d t minus zeta omega n by omega d sin omega d t into e to the power minus zeta omega n t into x 0. So, I can simplify this term. So, to simplify this let me take this e to the power minus zeta omega n t common. So, this becomes e to the power minus zeta omega n t into. So, I can take this omega n by omega d also common, omega n by omega d if you take common. So, this becomes omega d by omega n into cos omega d t minus here it becomes zeta into sin omega d t.

And you know this omega d by omega n is nothing but root over 1 minus zeta square as omega d is equal to 1 minus zeta square into omega n. So, this omega n omega n cancels. So, this becomes root over. So, this becomes omega n by omega d e to the power minus zeta omega n t into root over 1 minus zeta square cos omega d t minus zeta sin omega d t. So, you can think of this zeta equal to cos phi and this becomes sin phi. So, if zeta is cos phi for a under damped system, zeta is less than 1.

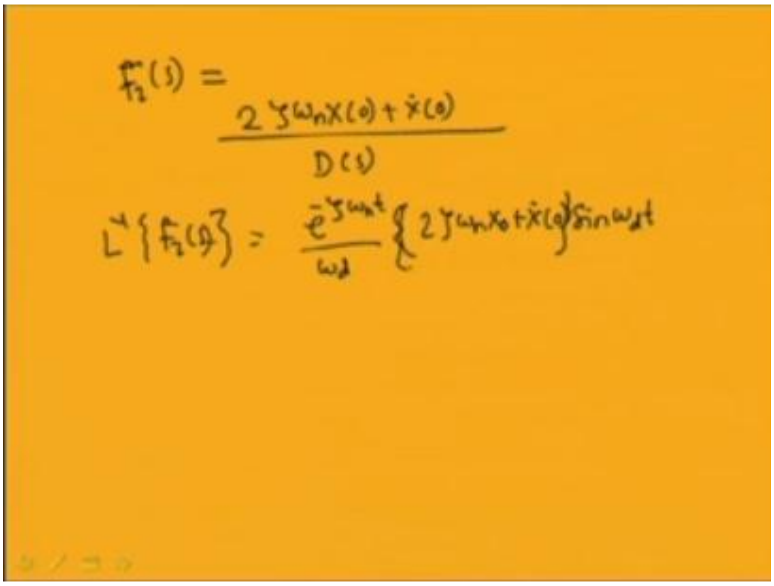
So, zeta can be taken as cos phi, and this can be taken as 1 minus zeta square as sin phi. So, this equation becomes sin phi into cos omega d t minus cos phi into sin omega d t. So, this thing can be written as. So, if I take a negative sign or negative common. So, this is omega n by omega d e to the power minus zeta omega n t sin omega d omega d t minus psi. I have taken this 1 minus zeta here 1 minus zeta square root over I have taken as sin psi and zeta as cos psi or tan psi I have taken equal to root over 1 minus zeta square by zeta. So, the first term is reduced to this minus omega n by omega d e to the power minus zeta omega n t sin omega d t minus psi.

(Refer Slide Time: 41:45)



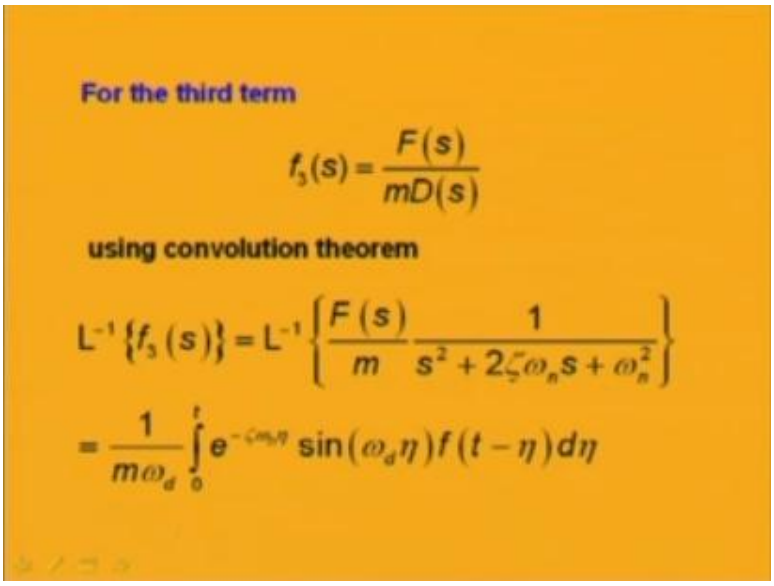
So, you can see the second term is nothing but 2 zeta omega n x 0 plus x dot 0 by D s. So, the second term can be written like this.

(Refer Slide Time: 42:01)



So, the second term f 2 s equal to 2 zeta omega n x 0, so 2 zeta omega n. So, the second term equal to 2 zeta omega n x 0 plus x dot 0 by D s; already you know the inverse Laplace of D s, and this is a constant. So, the inverse Laplace will be equal to. So, inverse Laplace of F 2 S. So, this is capital F 2 S. So, this will become e to the power minus zeta omega n t by omega d into 2 zeta omega n x 0 plus x dot 0 into sin omega d t. So, you got the inverse Laplace of the second term.

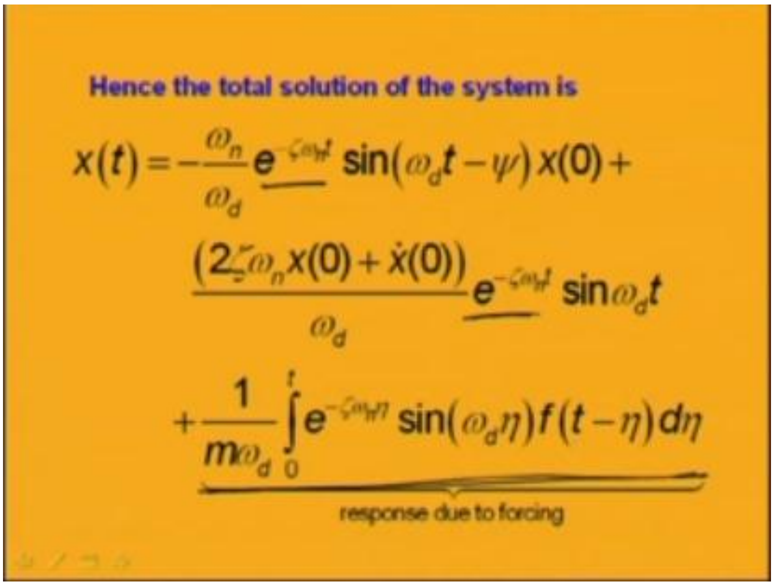
(Refer Slide Time: 43:10)



Third term can be given by F s by m D s and using convolution theorem. So, you know the inverse Laplace of f s equal to F sin omega t or the forcing function applied to the system f t, and the inverse Laplace of D s already you know. So, 1 by D s, so it is equal to 1 by s square plus 2 zeta omega n s plus omega n square. And you have already known the inverse Laplace of this term. So, the inverse Laplace of this can be obtained by using the convolution theorem.

And that can be written as 1 by m omega d integral 0 to t e to the power minus zeta omega n eta sin omega d t sin omega d eta into f t minus eta d eta. So, here eta is the dummy variable, and you can find this integral from these. So, the total solution will be.

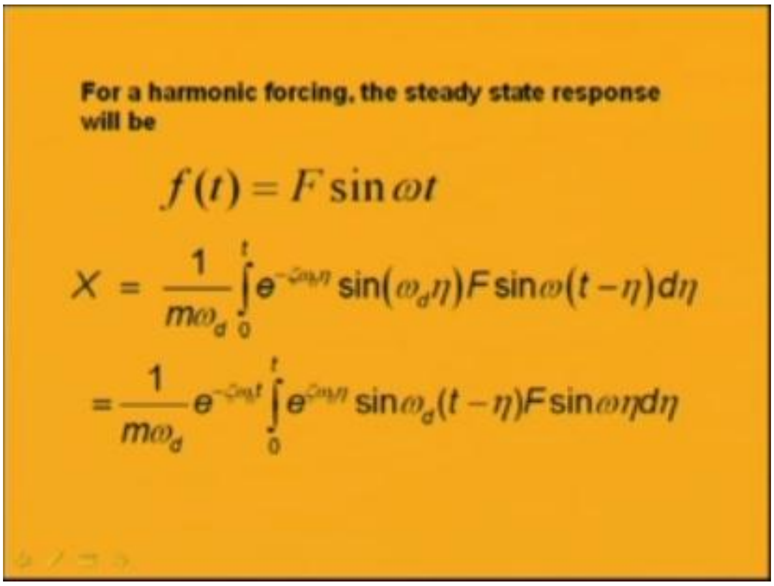
(Refer Slide Time: 44:09)



So, the total solution can be written as x t equal to first term, second term and the third term you can add. So, from the first term you have seen that x t equal to minus omega n by omega d e to the power minus zeta omega n t sin omega d t minus psi into x 0. For the second term you have let us seen that this is equal to 2 zeta omega n x 0 plus x dot 0 by omega d e to the power minus zeta omega n t sin omega d t. And for the third term by using the convolution theorem, you can find this is equal to 1 by m omega d into integral 0 to t e to the power minus zeta omega n eta sin omega d eta f t minus eta d eta.

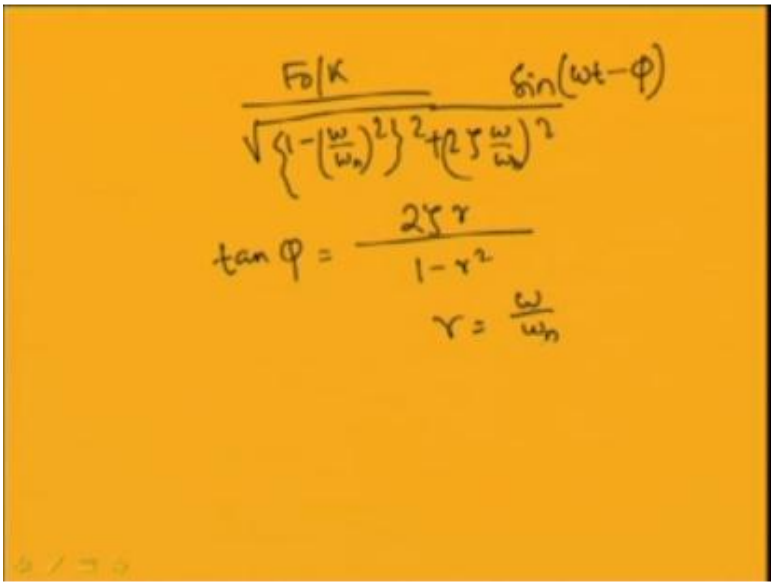
So, these two parts you can observe that it contains a term e to the power minus zeta omega n t. So, here you have a term e to the power minus zeta omega n t, and here you have a term e to the power minus omega n t. So, when t tends to infinity these two terms will die down; it will reduce to 0. So, these two are the transient response of the system and when t tends to infinite, the remaining response will be nothing but this part. So, this part is the steady state solution of the system and for a particular forcing.

(Refer Slide Time: 45:37)



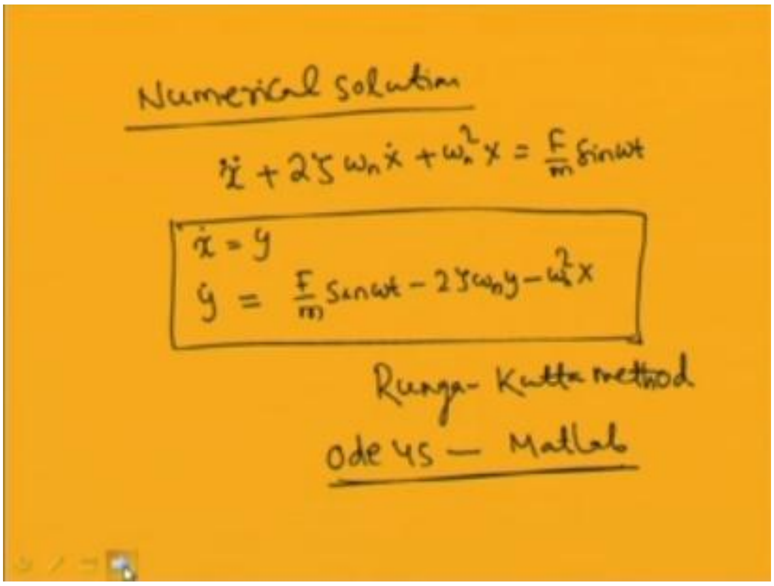
So, if it is harmonically excited system. So, if f t equal to F sin omega t, you can find this x equal to. So, by using this convolution theorem you can find.

(Refer Slide Time: 45:51)



So, this will be equal to F 0 by k root over 1 minus omega by omega n whole square whole square plus 2 zeta omega by omega n whole square sin omega t minus phi, where this tan phi equal to 2 zeta r by 1 minus r square. So, here r equal to omega by omega n. So, in this way you can find the response of a single degree of freedom system by finding the particular integral, and by applying the Laplace transform method also you can find the same thing by using some numerical technique. So, in case of numerical technique you can use Runge-Kutta method to find this solution, you can use MATLAB or you can use any language C or FORTRAN, c plus plus to find the response of the system.

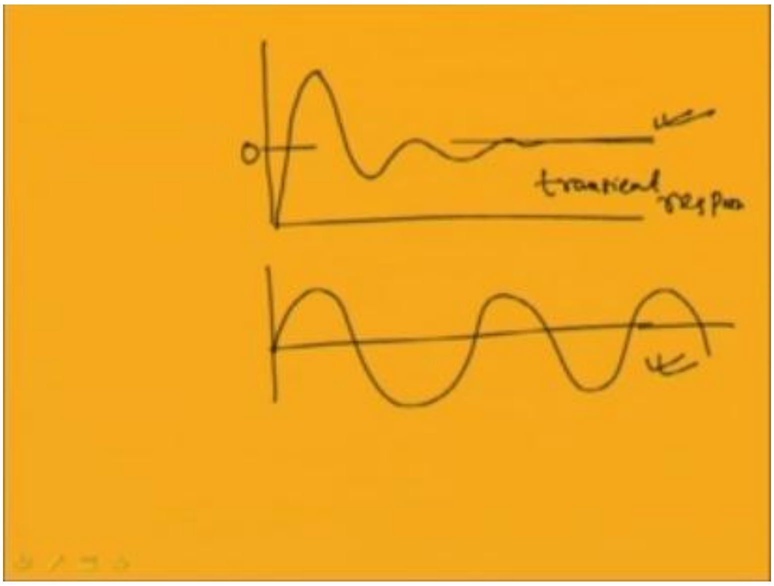
(Refer Slide Time: 46:58)



So, in case of numerical solution you should reduce the second order differential equation to a set of first order system, or you can write this equation in this form. So, your equation is this x double dot plus 2 zeta omega n x plus omega n square equal to F by m sin omega t. So, you can write this equation in this form; you just take x dot equal to y. So, this is the first equation, and the other equation you can write y dot y dot which is equal to x double dot. So, that will be equal to F by m sin omega t minus 2 zeta omega n y minus omega n square x.

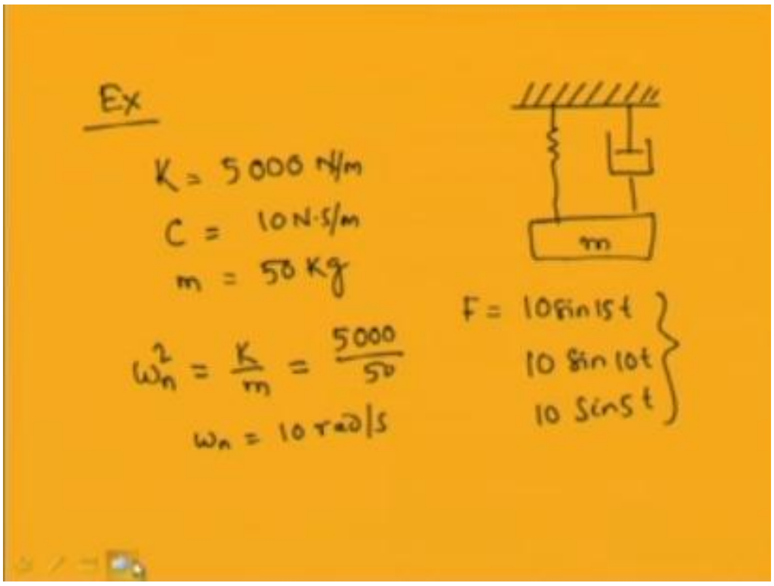
So, the second order system is reduced to a set of first order differential equation or two first order differential equation. So, this two first order differential equation can be solved by using the Runge-Kutta method. In MATLAB you can use the ode function ode 4 5; ode 4 5 in MATLAB you can used to find the response of the system. So, you have seen that this function or the solution will contain two parts; one is the transient response. So, already you know that this transient response.

(Refer Slide Time: 48:46)



So, you can plot this transient response. So, the transient response you know. So, if this is 0 line. So, this transient response will die down with time, and this is the transient response. So, this is your 0 line and the steady state response will be. So, it is sin omega t. So, when you are applying a force of sin omega t, the response will be sin omega t minus phi. So, the total response of the system will be this transient part because this is steady state part of the response. So, in this way you can find the total response of the system and you can find the total system response. So, let us take one example.

(Refer Slide Time: 49:51)

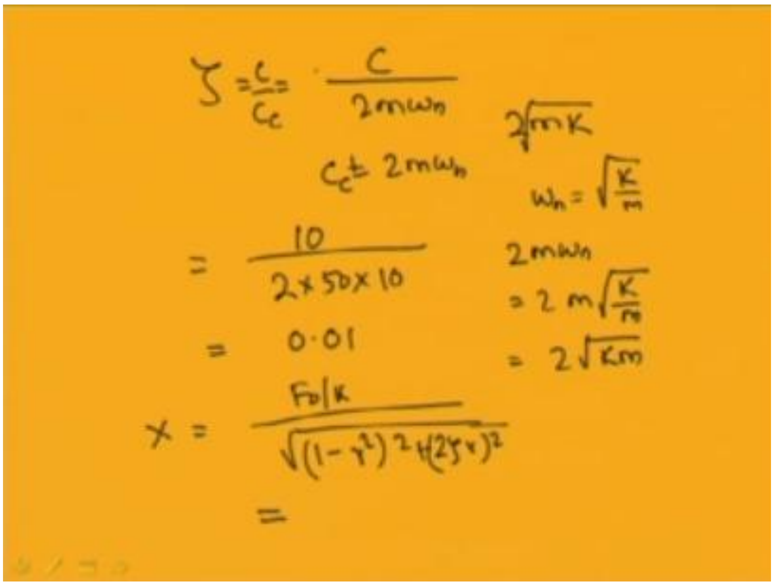


So, previously in the last class we have taken two examples. And in the first example we have found the maximum response; we have found the frequency at which the response is maximum. And we have obtained that expression r equal to 1 minus 2 zeta square, and we have found the maximum amplitude of the response also. And now, let us take another example. Let us take a system, which is modeled as a single degree of freedom system with spring mass damper.

And in this case let us take this k equal to 5000 Newton per meter; let us take c equal to 10 Newton second per meter and mass equal to 50 kg. So, we will find the response of the system for a different value of the forcing parameter. In the first case we will take F equal to 10 sin 15 t, in the second case we will take it is equal to 10 sin 10 t and we will take a third case also. So, here we will take force will be equal to sin 5 t. So, we will find the response for this three different forcing function.

So, already you know this k m c value of the system. So, the natural frequency of the system omega n square you can write equal to k by m natural frequency square equal to k by m. So, this is equal to 5000 by 50. So, this is equal to 100, and this omega n will be equal to 10 radian per second. Also you know your zeta; zeta is nothing but zeta equal to c by c c.

(Refer Slide Time: 51:58)

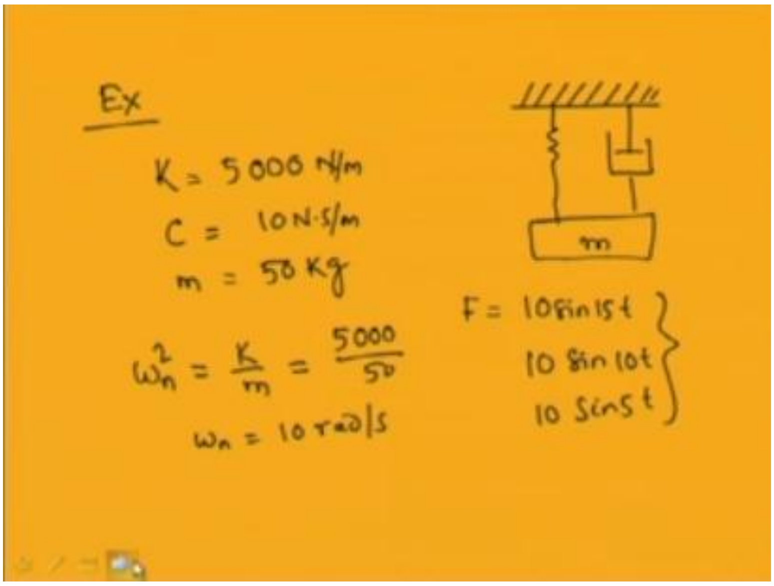


So, this is equal to c by C c that is damping by critical damping which you can write in terms of. So, you can write in terms of mass and omega n of the system, mass and damping mass and spring one stand of the system. So, this is 2 m omega n. So, you have written in terms of mass and natural frequency of the system, or you can write the same thing as 2 m root over k m as this omega n equal to root over k by m. So, 2 m omega n you can write equal to 2 m into root over k by m. So, this is equal to 2 root over k m.

So, in this way also you can write zeta. So, given the system parameter m k and c, you can find zeta. So, that is equal to now 2 into m, m equal to 50 kg. And omega n already you have found. So, omega n equal to 2 m omega n. So, in this way you can find c by c c, zeta equal to c by cc, and this c c equal to 2 m omega n. So, c by cc equal to c by 2 m omega n. So, your zeta becomes c equal to c already given this is equal to 10 Newton second per meter, and 2 into 50 into omega n equal to 10.

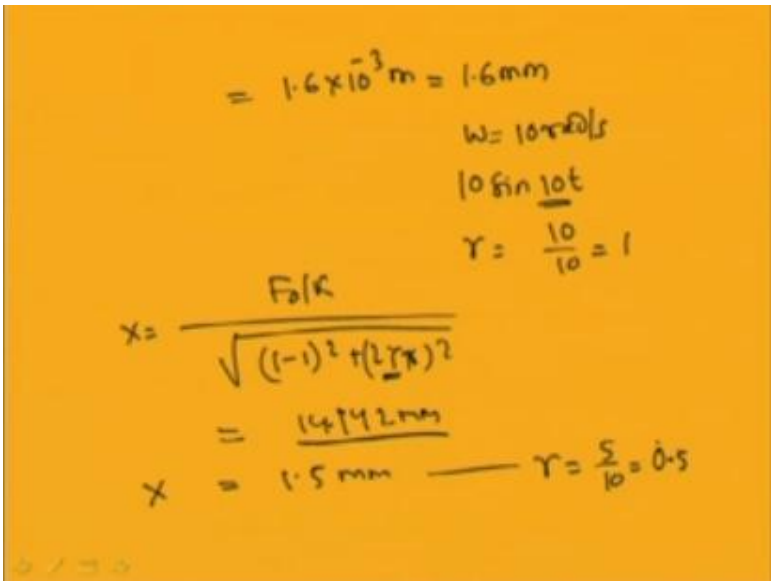
So, this gives rise to 0.01. So, the damping ratio equal to 0.01. And natural frequency of the system equal to 10 radian per second. So, the system response you can find x will be equal to F 0 by k. So, F 0 by k 1 minus r square whole square plus 2 zeta r whole square. So, this will become equal to we will substitute all these values you can see.

(Refer Slide Time: 54:25)



So, in the first case you can substitute the value F equal to 10 and omega equal to 15. So, this r in the first case r will be equal to omega by omega n. So, this will be equal to 15 by 10. So, this is equal to 1.5. So, by substituting this F 0 equal to 10; so this is 10 by 5000, and this becomes root over 1 minus 1.5 square whole square plus 2 zeta equal to 0.01, and r equal to 1.5.

(Refer Slide Time: 55:07)



So, in this way you can find the response of the system which is equal to 1.6 into 10 to the power minus 3 meter, which is equal to 1.6 mm and in the second case when omega equal to 10 radian per second. So, you have applied a force of 10 sin 10 t. So, in this case omega equal to 10; so your r becomes 10 by 10. So, this is equal to 1. So, you can observe that in this case your magnification factor x by x 0 can be given by or you can give your x equal to F 0 by k it is equal to root over 1 minus r square.

So, r square equal to 1 square plus 2 zeta r whole square r equal to 1. So, this damping factor only will limit this thing. So, you can find this is coming to 14.142 mm. In the third case, you can find x will be equal to 1.5 mm. So, you can see that at resonance condition the response is coming to be 14 point very high in comparison to the response when it is away from the omega equal to omega n. So, in the third case your r equal to 5 by 10; this is equal to 0.5.

So, today's class we have studied about different methods. Three different methods I have told you to find the response of the system. First, I told you about the particular integral method; by taking the particular integral of the differential equation you can find the steady state response of the system. Second, I told you the Laplace transform method. This Laplace transform method is very useful particularly when you have a forcing function. For any forcing function, you can use this Laplace transform method.

Third I told you the numerical method. So, in case of numerical method you can convert this second order differential equation to a set of first order differential equation and you can solve it numerically by using Runge-Kutta method. And in this way you can find the response of the system. So, we have obtained the steady state response of the system, and the total response of the system is the steady state response and the transient part. And you know for a system with damping, this transient part die down with damping, and the response of the system becomes the steady state response. So, in the last two classes, we have studied about the steady state response of harmonically excited system. Next class we will study about the rotating unbalance and wheeling of shaft.