Mechanical Vibrations

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Module - 4
Single DOF Forced Vibrations
Lecture -1
Forced Harmonic Vibration, Magnification Factor

In this module, we will study about the harmonically excited single degree of freedom systems, and in this module there will be four lectures. In the first lecture, we will study about the forced harmonic vibration, magnification factor using graphical method. In the second lecture, we will study about the steady state response using alternative approach like solving the differential equation and Laplace transform method. And we will study about the superposition principles also.

In third lecture, we will study about the rotating unbalance and wheeling of shaft. In the fourth lecture, we will study about the vibration, isolation, and transmissibility. In the previous lectures, you know about the free vibration of single degree of freedom systems; also you have studied about how to find the equation motion of a single degree of freedom system.

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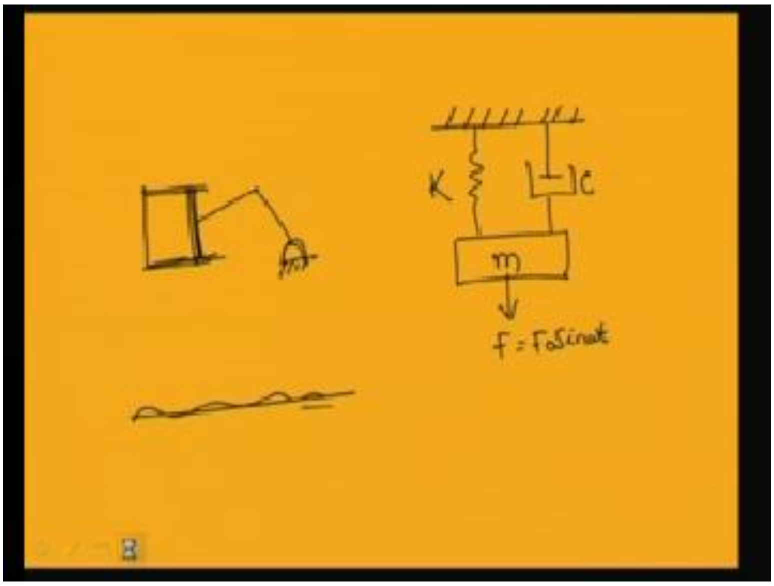

So, let us take a simple spring mass damper system. In the simple spring mass damper system which is the model of most of the systems, you can write the mass equal to m, spring stiffness equal to k, and c is the damping. It is subjected to a force F equal to F 0 sin omega t. So, this simple model can be a model of an IC engine or reciprocating engine. In case of a reciprocating engine you know that the unbalance force always there is the presence of an unbalance force.

So, this is a reciprocating engine; this is the crank, and this is the connecting rod, and this is the piston. And when due to this reciprocating part, always they are presence unbalance force in this. And due to that unbalance force, it will be subjected to this base, or the support will be subjected to some motion. Also you may consider a vehicle moving on a rough surface or moving on the road. So, the tier of the vehicle will be subjected to a force due to the road undulation. Also you may consider a robotic arm in case of a robotic arm.

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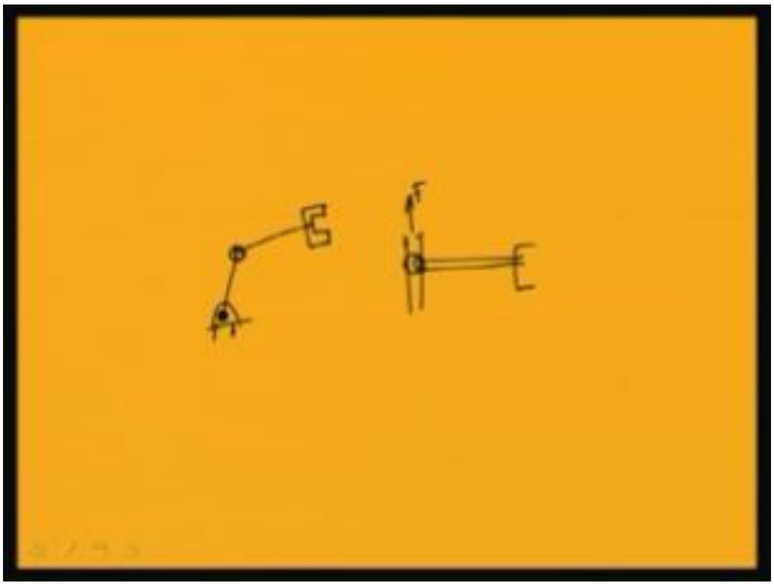

So, this is a simple robotic arm one-link or two-link robotic arm. So, this is the end defector of the robotic arm, and this is a single link or the joints are supplied with motors which rotate these links. So, we are applying some torque or force to these joints; in case of a revalue type of robotic manipulator you are supplying the torque, and in case of a Cartesian type of manipulator we are giving the force. So, in case of a Cartesian type of manipulator; so this is a simple schematic diagram of a Cartesian manipulator. So, it is translating in this direction, and this is a revalue of type of manipulator in which you are applying torque. So, in case of Cartesian manipulator you have applied force F. So, in some cases we are applying the force to the system.

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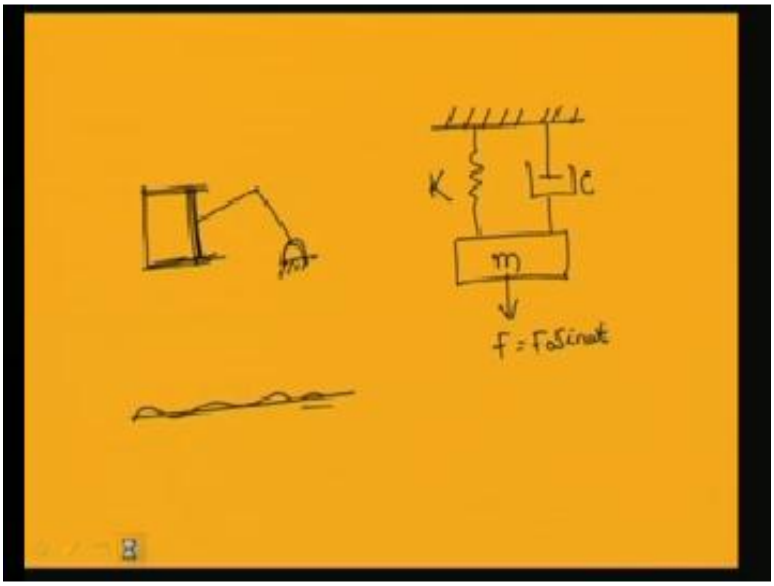

And in some cases like in case of the reciprocating engine or in a vehicle moving on the road; so some force is transmitted to the system itself. So, due to the transmission of this force or due to these forces, the system is subjected to forcing. So, this force may be of different types.

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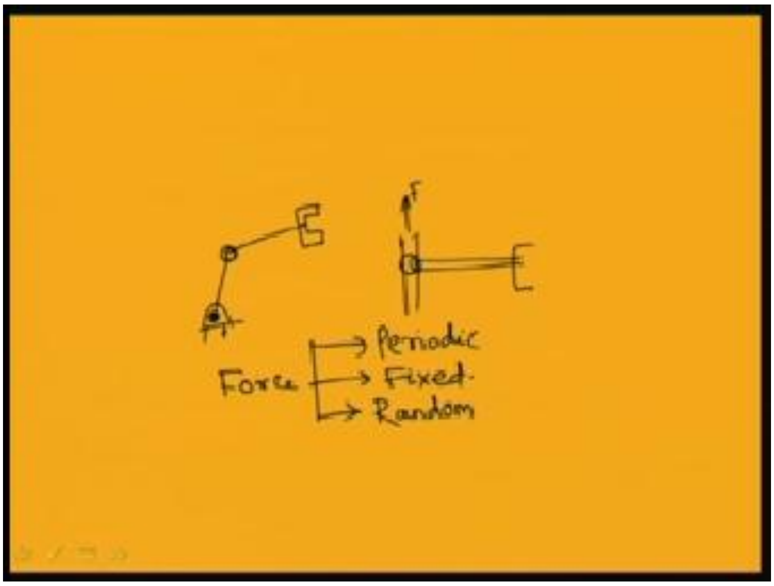

So, this force can be of many different types. So, it may be periodic, or it may not be periodic. So, periodic or it may not be periodic. So, when it is not periodic, it may be a fixed force you are applying, fixed amplitude of force you are applying or you are applying some force which is random force. So, in case of random force that is in case of an earthquake, the buildings will be subjected to some force, which is equivalent to the random force.

And in case of fixed force you are applying a constant force to the system, and in case of periodic force you are applying a force which may be harmonic or may not be harmonic. So, this periodic force may be harmonic.

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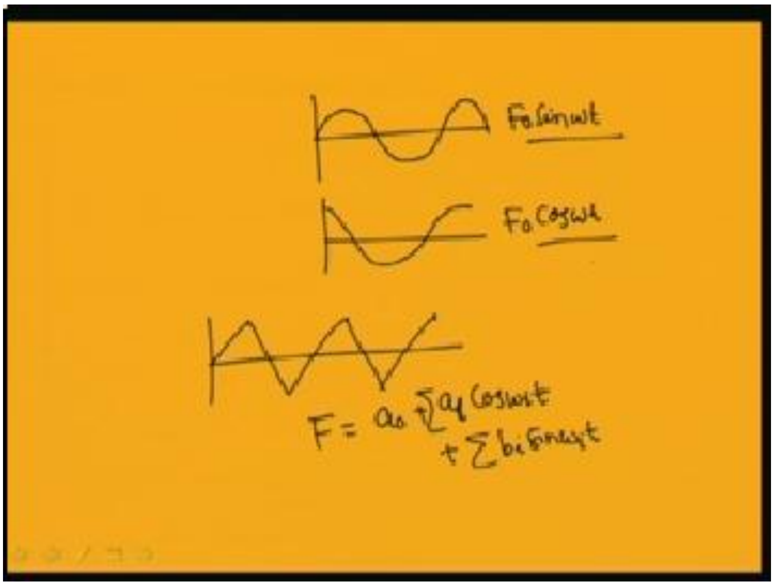

So, in case of harmonic it will be either similar to a sin curve or a cosine curve, and in case of non-harmonic periodic, it may be triangular or it may be rectangular, or it may be of any shape. So, when the system is subjected to harmonically excited force; that is this force equal to F 0 sin omega t or the force equal to F 0 cross omega t. So, we can find the response of the system and when it is periodic but not harmonic like in this case of this triangular wave, we can apply the Fourier series to convert that thing to these harmonically excited one.

So, we can convert this triangular wave by applying Fourier series to some terms containing sin and some terms containing cos and we can write the resulting force equal to summation. So, it will be a 0 plus a 1 summation a i cos I omega t omega I t plus summation b i sin cos omega I t cos omega I t plus summation I equal to b i sin omega I t. So, we can convert this periodic force, but when they are not harmonic we can convert them to the harmonic force by use of Fourier series.

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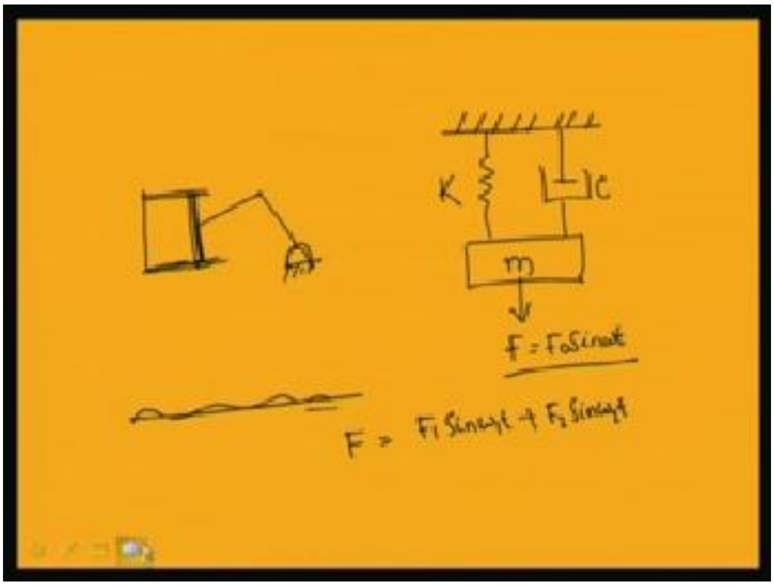

So, in case of the spring mass damper system; so this force we are writing this force equal to F 0 sin omega t. For simplicity we are taking a single term F 0 sin omega t; one may take a force which may be equal to F 1 sin omega 1 t plus F 2 sin omega 2 t or summation of multiple terms, or one may take some other type of forces also. So, in most of the cases or the most of the systems for simplicity in analysis we are converting that thing to single degree of freedom system, or we are refraining that thing by a single spring and spring mass damper system.

So, most of the systems can be reduced to this form. So, we are studying for the simplicity this simple system, the simple spring mass damper system. So, today I will tell you about the graphical method to solve this equation motion. So, previously you have studied how to derive this equation motion.

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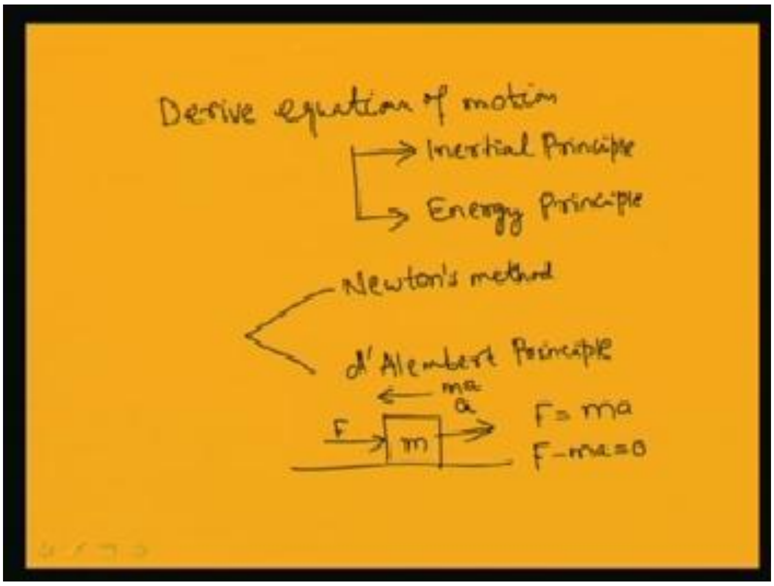

To derive the equation motion; so you have equation motion. So, either you have used energy principle or you have used inertial principle. In case of inertial principle, you have used either Newton's method or you have used d'Alembert principle. Both the methods are same. This d'Alembert principle converts a dynamic system to an equivalent static system by introducing the inertia force on the system. So, if you have a system let us consider this mass; it is subjected to a force F. So, when this force is applied to this mass; so it will have an acceleration.

So, according to Newton's second law you can write F equal to ma but according to d'Alembert principle. So, you are converting the system to an equivalent static system by applying an inertia force, which this inertia force is acting in the direction opposite to the direction of acceleration, and its magnitude is mass into acceleration. So, this is the inertia force acting on the system. So, according to d'Alembert principle this F minus m a equal to zero, so summation. So, this minus ma is considered as the inertia force.

So, summation of all the forces equal to 0 which gives the equilibrium conditions of a static system. So, by applying d'Alembert principle, you can convert this dynamic system to an equivalent static system, and by using Newton's method also you can derive the equation motion. And also you can derive this equation motion by applying this energy principle.

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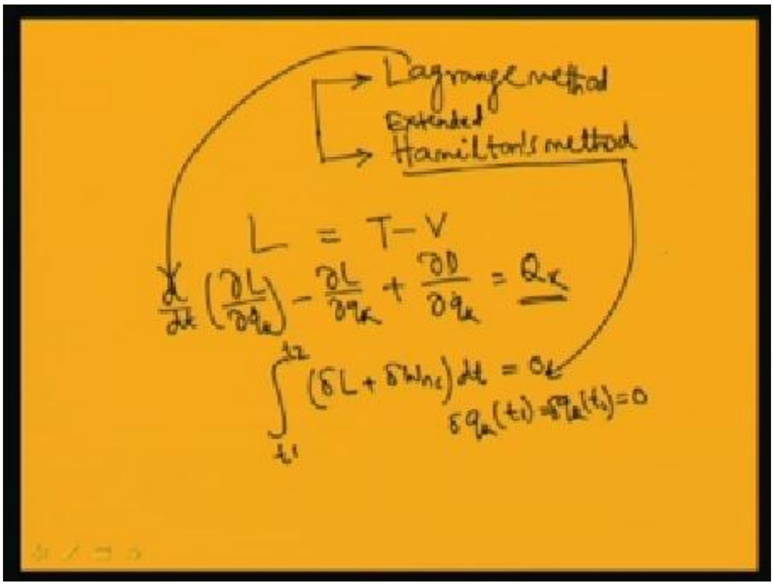

In case of energy principle you know the Lagrange method or you also know about the Hamilton's principle. Generally Hamilton's principle is used for the continuous system, and this Lagrange method is used for multi degree of freedom systems. In case of Lagrange method, we are defining a Lagrangian L that is equal to T minus V, where T is the kinetic energy of the system, and V is the potential energy of the system.

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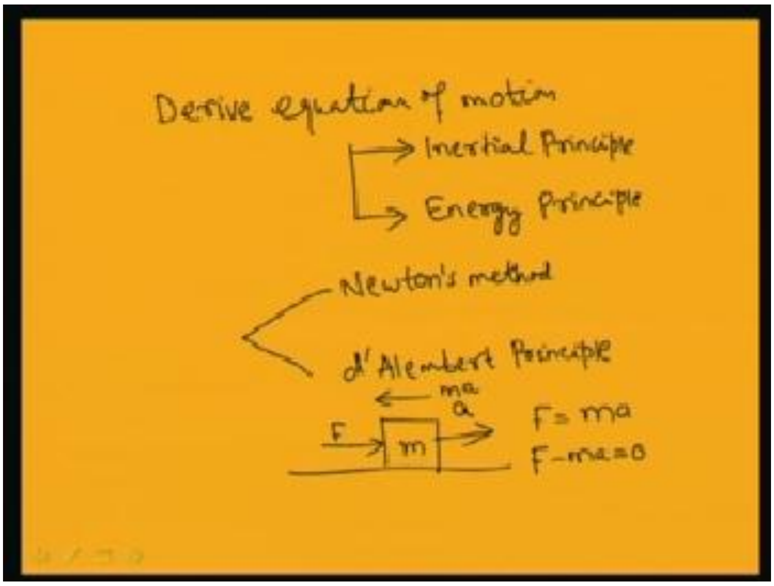

And we can write the Lagrange's equation by these. So, d by d t of del L by del q k dot q k dot minus del L by del q k plus del D by del q k dot equal to Q k. So, here this D is the dissipation energy and L equal to T minus V; that is difference between the kinetic energy minus potential energy. So, by knowing the kinetic energy, potential energy and dissipation energy of a system, one can find the equation motion by using this Lagrange method. Here Q k is the generalized coordinate, and this capital Q k is the generalized force, and the small q k is the generalized coordinates of the system.

Similarly, using Hamilton principle generally Hamilton principle is applied to a conservative system. So, for some non-conservative system or when some forces are acting on the system, one may use extended Hamilton principle. So, one may use extended Hamilton principle. So, extended Hamilton principle can be given by it. So, it is t 1 to t 2 del L plus del w n c. So, it can be given by del L plus del w n c d t equal to 0 and q k at t 1 equal to q k at t 2 or del q k at t 1 equal to del q k at t 2 equal to 0.

So, using extended Hamilton principles. So, this is extended Hamilton principle, and this is the Lagrange principle. So, using extended Hamilton principle also you can derive this equation motion. So, already you know how to derive this equations motion.

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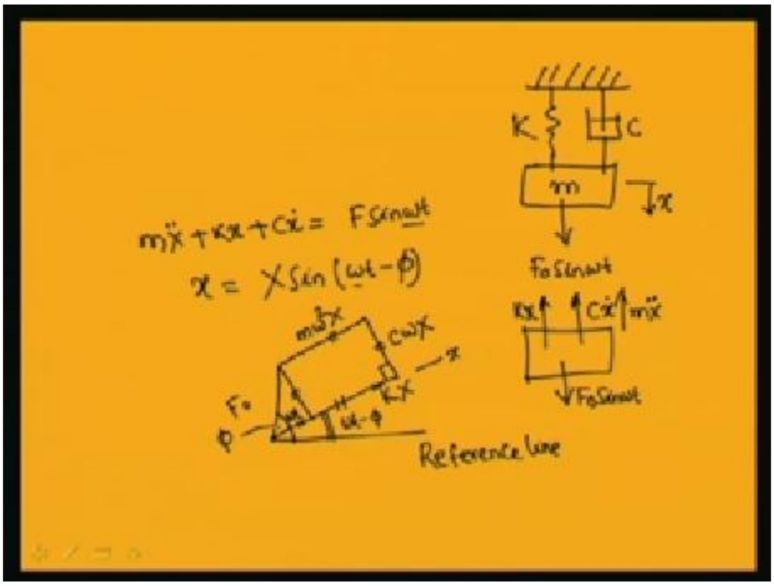

So, for a single degree of freedom system and for this simple spring mass damper system you have. So, for this simple spring mass damper system already you have derived the equation motion and that equation motion can be written as. So, let these are the displacement x. So, when it is subjected to a force F 0 sin omega t. So, the equation motion; so this is k and this is c. So, the equation motion is given by m x double dot plus k x plus c x dot equal to F sin omega t. So, here by drawing the free body diagram one can find this equation motion.

So, when this body is moving downward with x. So, this spring will be stress downward, and so it will be exert a force in upward direction, and that force is proportional to the displacement. So, that force is given by k x, and this damper also will be subjected to a force that is equal to c x dot, and that force is also opposite to the direction of motion. So, this is equal to c x dot, and the inertia force is opposite to the direction of acceleration. So, this inertia force can be given by mass into acceleration of the system, but it acts in the direction opposite to the direction of acceleration.

So, it is given by m x double dot mass into acceleration. X double dot is the acceleration, x dot is the velocity, and x is the displacement from the equilibrium position. So, all these forces will be equal to F 0 sin omega t; this is the externally applied force. So, the externally applied force equal to this spring force k x and damping force c x dot and the inertia force m x double dot. So, by equating these forces, one can write the equation motion of the system equal to m x double dot plus k x plus c x dot equal to F sin omega t.

Now I will tell you a very simple graphical method to solve this equation motion. Already you know for a similar differential equation motion, the solution will be in the form of x will be equal to X sin. So, you know x will be in the form of X sin omega t minus phi. So, here X is the amplitude of the response, and phi is the phase angle. So, you may note that the frequency of this external forcing and the frequency of the response are same. So, I can draw a very simple force polygon to find the amplitude and phase.

So, let me draw this force polygon; let me take a reference line. So, this is the reference line. The external force you may note that this external force is at an angle omega t from this reference line. So, I am just plotting only the magnitude of all the forces. So, the spring force I can represent by. So, spring force is k x small x. So, this magnitude will be k capital X and I can draw that force. So, this is k into x. So, this is k x force, and this damping force will be c omega x cos omega t; so cos omega t minus phi. So, it will be at an angle 90 degree to the spring force. So, this angle is 90 and its magnitude equal to c omega x.

Similarly, the inertia force m x double dot. So, this will be equal to minus m omega square x. So, this is opposite to the direction of the spring force, and it will act in this direction. So, it will be parallel to this line and its magnitude equal to m omega square x, and this closing side will be the applied force. So, this is the applied force that is equal to F 0. So, F 0 will be equal to the vector sum of k x c omega x and m omega square x. Now to find this x I can draw a line parallel to this here. So, I have drawn a line parallel to this.

So, now, this F 0 is acting at an angle omega 2. So, this is the reference line. So, from this reference line F 0 is at an angle omega t. So, this angle is omega t and so this angle I can take this inside angle between this force and this k x line equal to phi. So, this angle equal to omega t minus phi. So, as this angle is omega t minus phi. So, this x response a x is at an angle omega t minus phi with the reference line. So, this force is at an angle omega t with the reference line the displacement.

So, this is the displacement line along this displacement. So, it is at an angle omega t minus phi. So, now, I have to find the magnitude of this x. So, from this triangle one may find that F 0 will be equal to this square plus this square, and this part equal to k x minus m omega square x, and this is c omega x. So, this square plus this square will be equal to this F 0 square. So, I can write this equation in this form.

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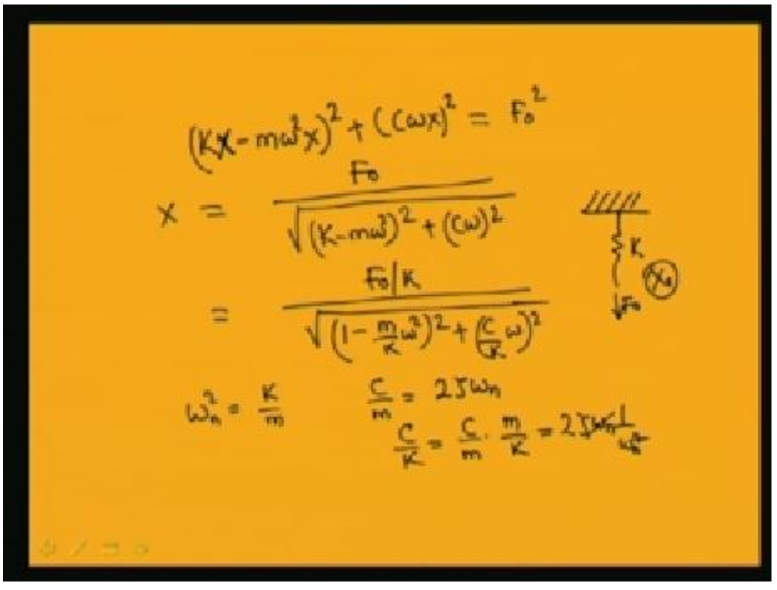

So, k X minus m omega square X whole square plus c omega X whole square equal to F 0 square. So, from these one can find this X. So, this is capital X. So, one can take X common. So, it is equal to X square I can write, or X can be written as X will be equal to F 0 by square root of k minus m omega square whole square plus c omega whole square. So, no one can divide k in both numerator and denominator, and write this equation in this form. So, this will be equal to F 0 by k. So, this will be equal to 1 minus m by k. So, this m by k omega square whole square plus c by k omega whole square.

So, already you know that m by k root over k by m. So, omega n square equal to k by m, and also you know that c by m equal to 2 zeta omega n. So, this c by k you can find. So, c by k will be equal to c by m into m by k. So, this is equal to c by m I can write equal to 2 zeta omega n and this m by k equal to already m by k will be equal to 1 by omega n square. So, it is equal to omega n square.

So, this omega n omega n cancels. So, this is equal to 2 zeta by omega n. So, this c by k term I can write equal to 2 zeta by omega n. So, this term become 2 zeta omega by omega n whole square, and this becomes omega by omega n whole square, now this F 0 by k. So, this is the spring has its stiffness k, and it is subjected to a force F 0. So, it will have a displacement X 0. So, this is the static displacement of the spring, or this is static displacement of the system with a spring constant k. So, when a force of magnitude F 0 acts on the system. So, it will have a displacement of X 0. So, this F 0 by k can be called as the static displacement of the system.

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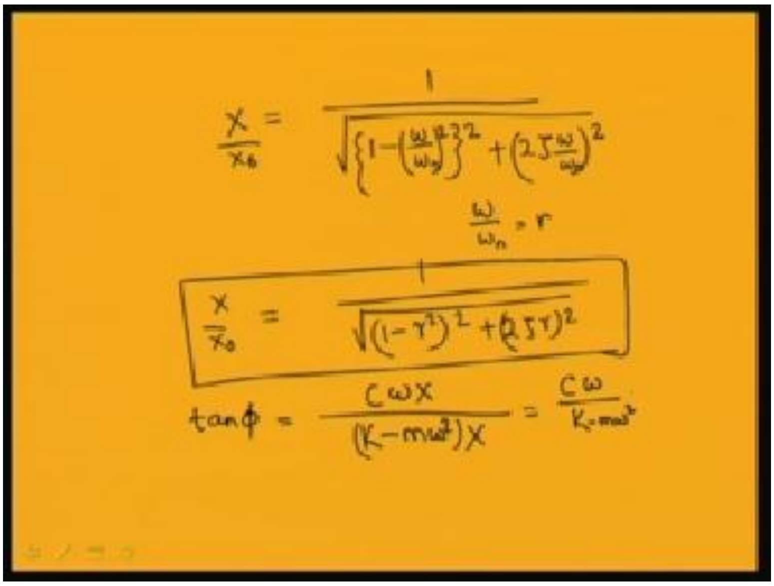

So, this displacement X can be given by. So, this X can be given by this or X by X 0 can be given by 1 by root over 1 minus omega by omega n whole square plus 2 zeta omega by omega n whole square. So, I can write this omega by omega n equal to r, frequency ratio. So, this equation I can write equal to. So, this will be equal to 1 by root over 1 minus r square whole square plus 2 zeta r whole square.

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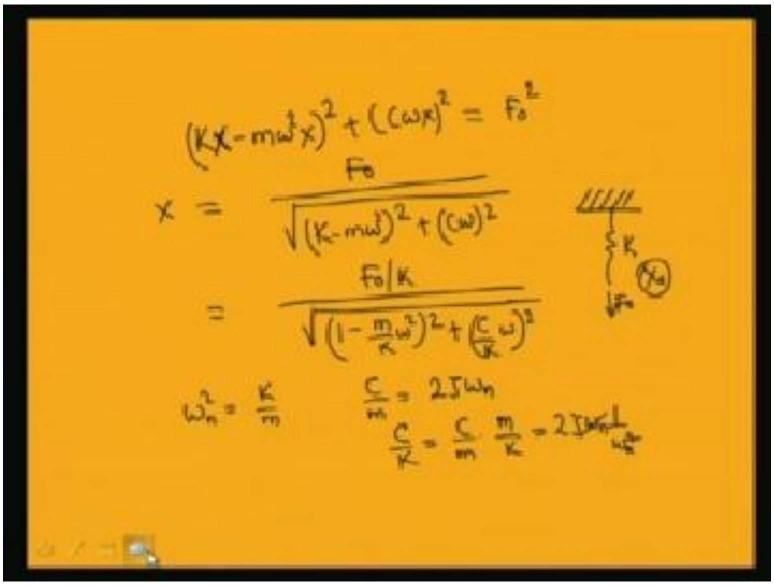

So, now I have to find. So, from this you know that this dynamic amplitude or this amplitude of response equal to F 0 by k or X 0 by root over 1 minus r square whole square plus 2 zeta r whole square. So, this is an important equation which you should remember.

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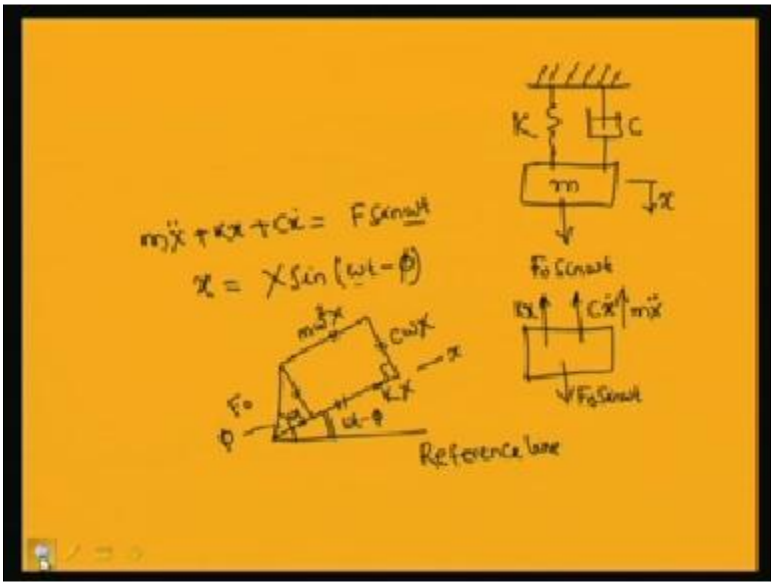

Now I have to find the phase angle. So, this phase angle phi also can be obtained from this triangle. So, in this triangle this tan phi will be equal to c omega X by k X minus m omega square X. So, I can write tan phi equal to c omega X by k minus m omega square X. So, cancelling this X X, it is equal to c omega by k minus m omega square. So, again now dividing this k in the numerator and denominator I can write this equal to.

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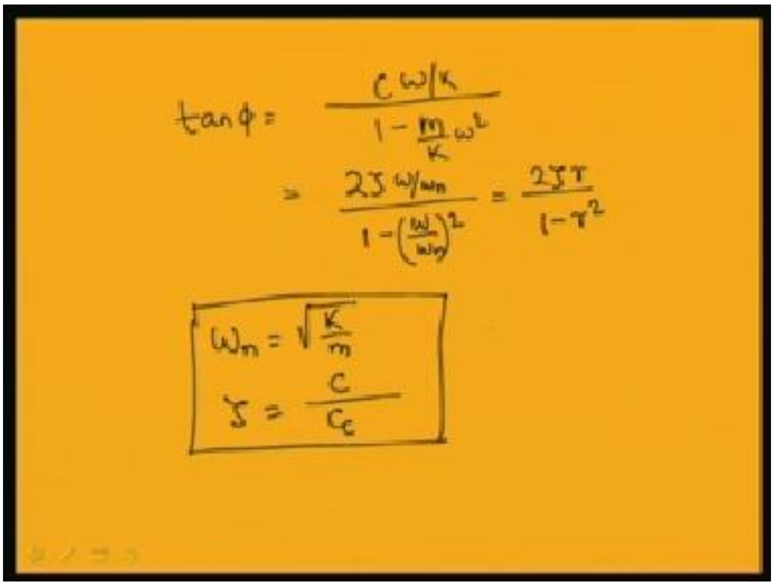

So, this tan phi I can write equal to C omega by 1 minus. So, this is equal to C omega by k minus m omega square 1 minus. So, this is K minus m omega square. So, I am dividing k. So, this is c by k omega by 1 minus m by k omega square. So, this is c omega by k and 1 minus m by k omega square, and already you know that c by k equal to already we have derived the that c by k.

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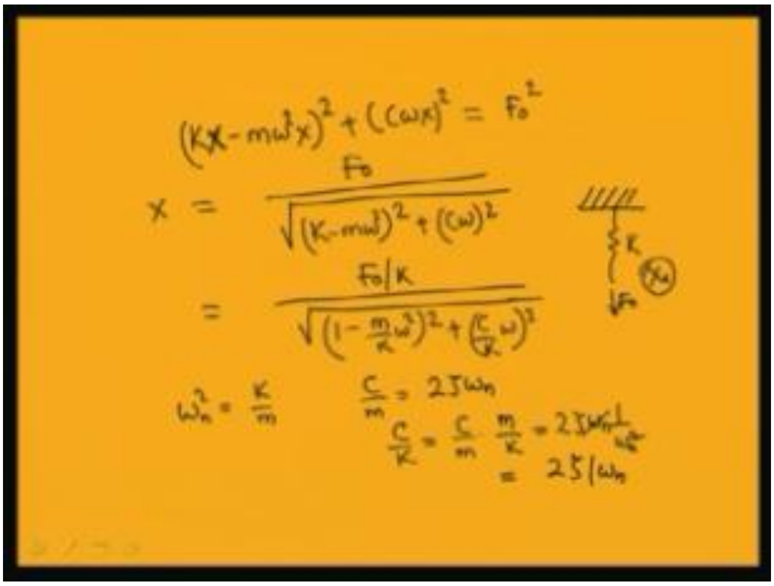

C by k equal to 2 zeta. So, this is equal to 2 zeta by omega n. So, this is equal to 2 zeta omega by omega n by 1 minus this m by k equal to k by m equal to omega n square. So, this is equal to omega by omega n whole square or this thing can be written as 2 zeta r by 1 minus r square. So, for the given system parameter m k and c, we can find the response of the system when it is subjected to a force F 0 sin omega t. So, here zeta is the damping factor, and omega n is the natural frequency of the system; natural frequency of the system you know equal to root over k by m.

And also you know that your zeta that is damping ratio that is equal to damping factor c by critical damping c c. So, you know this thing from the free vibration analysis of the system. So, using all the system parameters you can find the response of the system. So, this factor X by X 0 is known as the magnification factor of the system.

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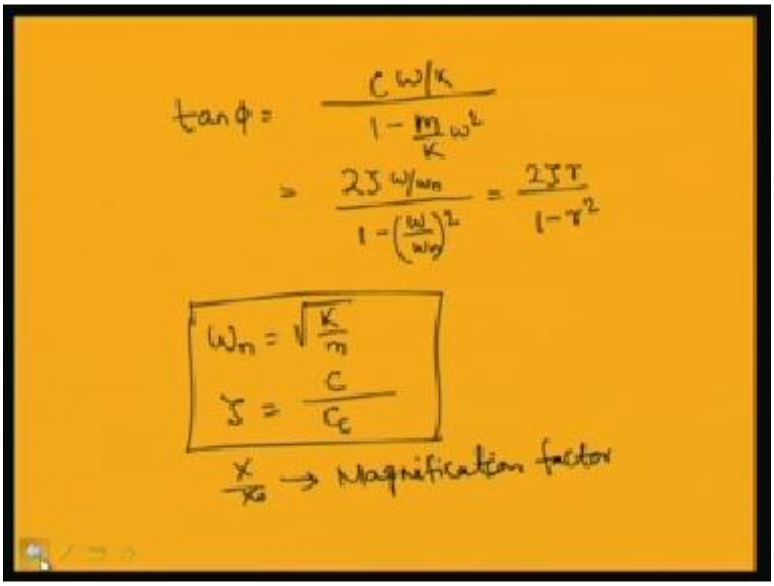

X by X 0 is known as the magnification factor of the system, and we can plot. So, this is magnification factor. So, one can plot this magnification factor versus the frequency ratio of the system. So, from this equation you can see that when r equal to 0 that is when omega equal to 0. So, this becomes 1. So, r equal to 1. So, this becomes 2 zeta. So, this is when r equal to 0. So, this becomes 1 and this part equal to 0. So, this is 1 by 1. So, this becomes 1.

And for an undamped system, undamped system that is a system without damping zeta equal to 0. So, this becomes 1 by root over 1 minus r square whole square. So, when zeta equal to 0, this X by X 0 tends to infinity when r equal to 1. So, when r equal to 1, this part equal to 0. So, r equal to 1 if you substitute in this equation, 1 minus r square will be equal to 0. So, this X by X 0 tends to infinity. So, we can plot this magnification factor versus omega by omega n.

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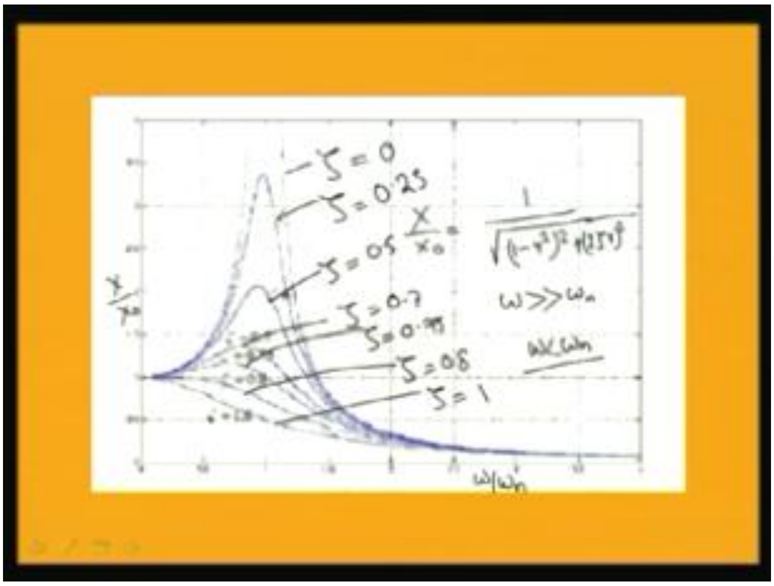

So, you can see this curve. So, already I have plotted this. So, this is X by X 0 versus omega by omega n. So, for a undamped system that is zeta equal to 0 at omega equal to omega n or this receiver equal to 1, the response tends to infinity. Also you have seen that irrespective of damping when r equal to 0, this X by X 0 equal to 1. So, this X by X 0 expression you already know. So, this is equal to X by X 0 equal to 1 by root over 1 minus r square whole square plus 2 zeta r whole square. So, from this you just see that when r equal to 0 this becomes 1 and for a undamped system that is zeta equal to 0. So, this tends to infinity.

So, when omega is very very greater than omega n or when the system is operating at a frequency which is very away from the natural frequency you can observe that. So, this one term can be neglected with respect to this r square term. So, at that time it will be equal to. So, this will be equal to 1 by. So, this will be very large term. This denominator will be very large term, and this will tends to 0, or this will be a number very small number you will get, or this X by X 0 will be very, very less.

That means when the system is operating at a frequency away from the natural frequency, this response amplitude is very, very less, and it is less than the static amplitude of the system x 0. Also you can observe this if for different value of zeta, this magnification factor is plotted. So, this is zeta equal to 0.5, okay. So, for different value of zeta the response is being plotted. So, this is zeta equal to 0.25, and this is zeta equal to 0.5. This is zeta equal to 0.7, and this curve is zeta equal to 0.75; this is zeta equal to 0.8, and this curve is plotted for zeta equal to 1.

So, you can see that with increase in zeta that is increasing the value of damping ratio, the maximum amplitude of the system decreases. So, by using this damping when the response is near this omega 1. So, you can decrease the resonant amplitude of the system. And you can observe that for a value that is more than 0.75, the magnification factor is less than 1. So, you can observe that with increase in damping the response of the system decreases, and also you can observe the maximum peak is occurring towards the left of the value omega equal to omega n.

For an undamped system, you are absorbing that the maximum amplitude occurs at a frequency omega equal to omega n, and for the system with damping it occurs at a value slightly to the left of the omega n; that is omega slightly less than omega n. You are getting this maximum amplitude, and when omega by omega n is very large, then the response amplitude is very small.

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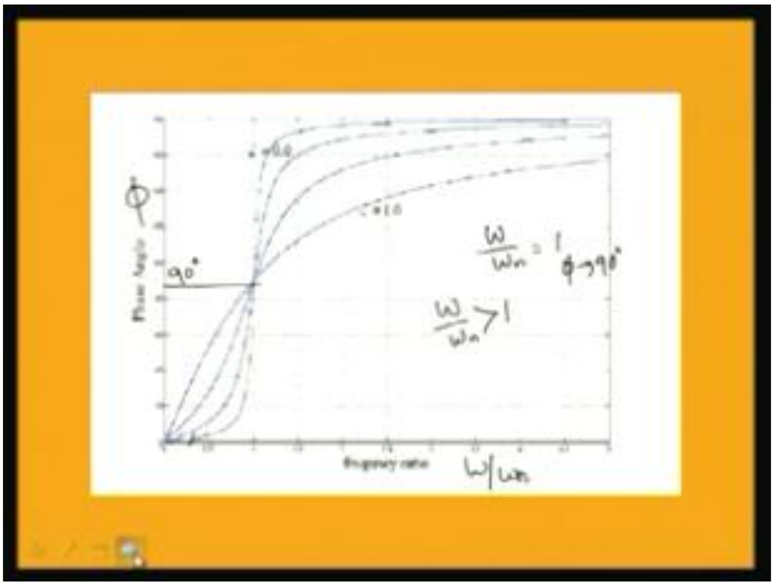

Also you can plot the phase angle versus phase angle phi versus this omega by omega n. So, in this case you can observe that for low damping or for an undamped system, the angle phi is almost 180 degree for frequency omega by omega n greater than 1. And for all this systems you can see that for very low value of omega by omega n, this phase angle is near 0. Also irrespective of damping you can observe that it has a phase angle of phi at omega equal to omega n. So, at omega equal to omega n, this angle is 90 degree.

So, irrespective of value of zeta, all the systems will have a phase angle of 90 degree when omega equal to omega n or omega by omega n equal to 1, phi tends to 90 degree. And when omega by omega n is very, very greater than 1, you may observe that for low damping it tends to 180 degree or with increase in damping this value decreases. So, in this way one can find the magnification factor of the system and also one can find the phase angle of the system. So, now we should find what is the peak amplitude or peak value of this response and what is the frequency at which this peak quantity value is obtained.

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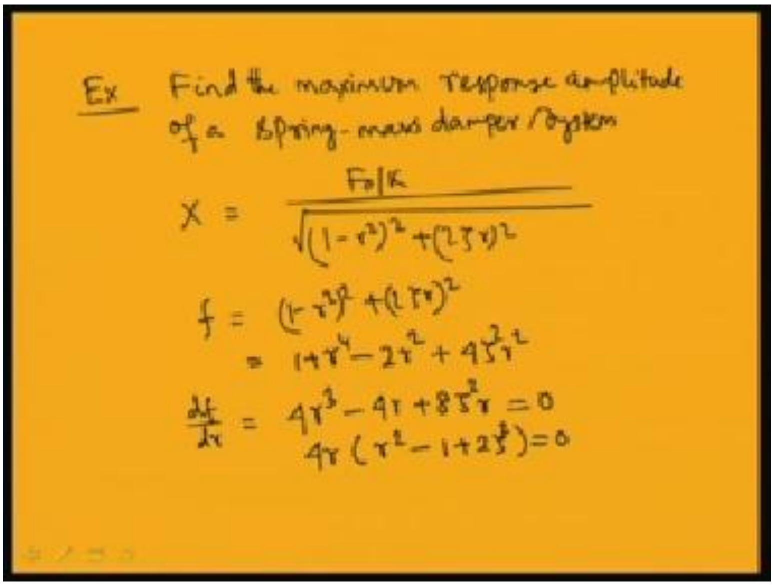

Let us take this example. So, in this example we have to find the maximum amplitude. So, we have to find the maximum response amplitude of a spring mass damper system. So, in this case already we know that the response can be written as X equal to F 0 by k. So, this is F 0 by k root over 1 minus r square to the power r square plus 2 zeta r whole square. So, this will have a maximum amplitude when this denominator is minimum. So, we have to find. So, for what value of r this denominator is minimum.

So, for that I can write this let me take a function f. So, this function f 1 minus r square whole square plus 2 zeta r whole square. So, this would be minimum. To find this minimum value, I can differentiate this d f with respect to r, and equate it to 0, and I will find the value of r. So, in this way I can find. So, this 1 by r square whole square I can write this equal to 1 plus. So, this becomes r fourth, and this becomes minus 2 r square plus 4 zeta square r square. So, now, df by dr. So, I can differentiate this thing. So, this becomes 3. So, if I differentiate this. So, I can have r fourth.

So, this becomes four r cube minus. So, this becomes 4 r and differentiating this; it will become plus 8 zeta square r. So, this will be equal to 0. So, I can take r common here. So, if I take 4 r I can take common. So, if I take 4 r common, so 4 r into. So, this becomes r square minus. So, this becomes 1 plus 2 zeta square equal to 0, or I can see that at r equal to 0 or at r square equal to 1 minus 2 zeta square, the response amplitude will be maximum. So, in r equal to 0 you have seen that response amplitude equal to 1 and when this becomes.

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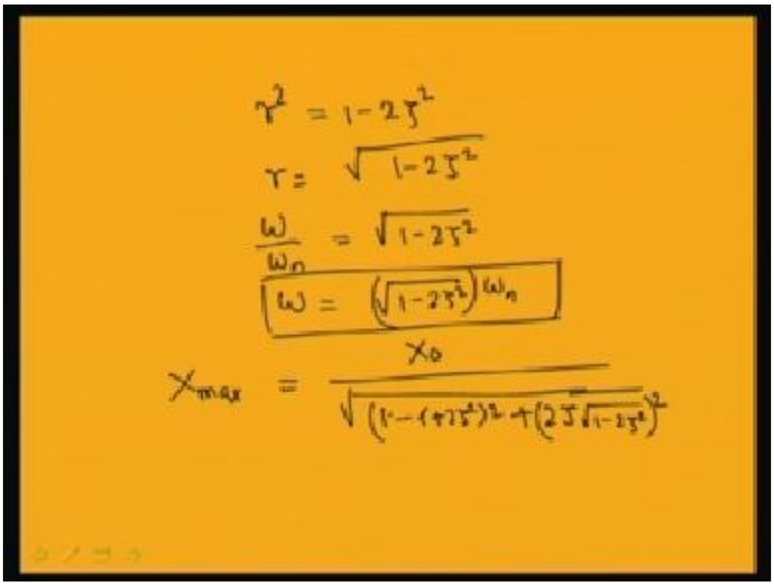

So, you can find that when r square equal to 1 minus 2 zeta square or r equal to root over 1 minus 2 zeta square, the response amplitude is maximum. So, this is the frequency ratio. So, for this frequency ratio you can find the response to be maximum or when omega the frequency of the external excitation equal to 1 minus 2 zeta square time the natural frequency of the system, the system will have a maximum amplitude. We have to find this maximum response amplitude; I can substitute it in that equation.

So, this will be equal to F 0 by k or x 0 by root over 1 minus r square; for this r, I can substitute this value. So, 1 minus square of this; so this becomes 1 minus 1 plus 2 zeta square whole square plus 2 zeta r. So, for this r, I can write root over 1 minus 2 zeta square this whole square. So, in this way I can find. So, this is equal to. So, this part one one cancels.

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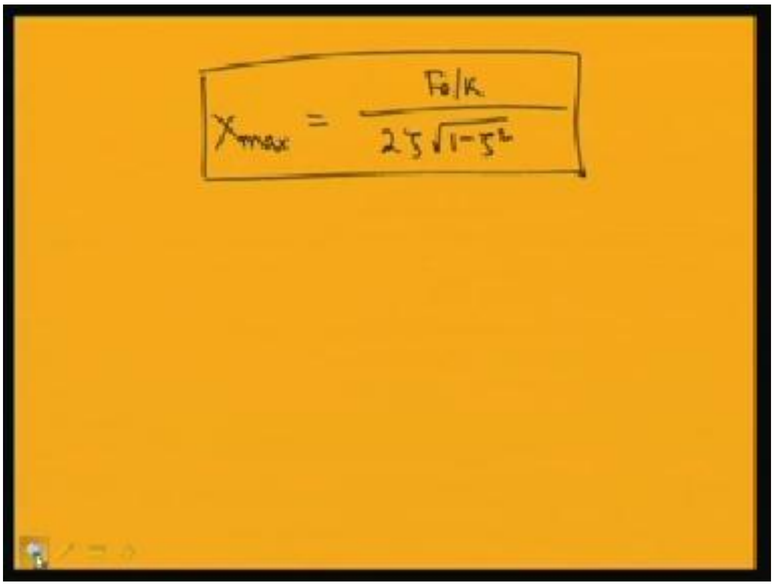

So, this X max I can write equal to X 0 or this F 0 I can write equal to F 0 by k into 2 zeta root over 1 minus zeta square. So, the maximum response amplitude becomes F 0 by k by 2 zeta root over 1 minus zeta square. So, when zeta equal to zero. So, this response amplitude tends to infinity. So, now let us solve another problem.

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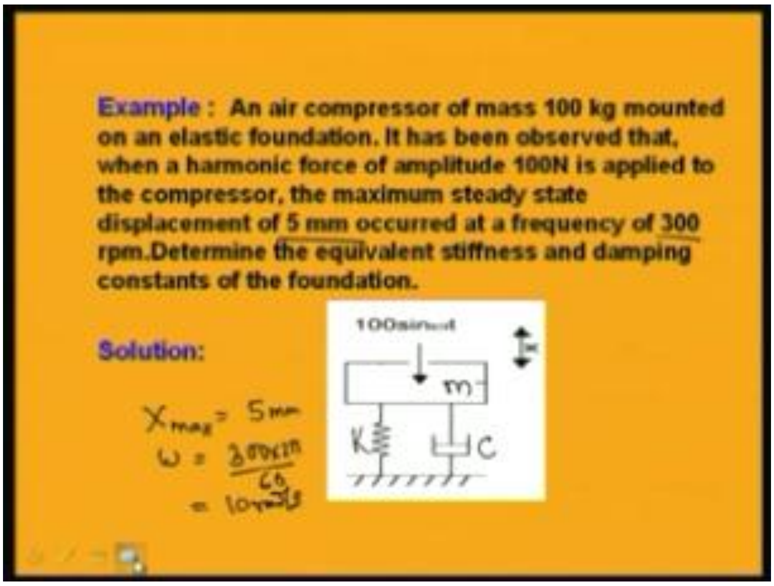

So, in this problem we will take an air compressor. So, this air compressor has a mass of 100 kg. It is mounted on an elastic foundation. So, this elastic foundation can be modeled as a spring of constant k and a dumping c. So, it has been observed that when the harmonic force of amplitude 100 Newton is applied to the compressor, the maximum steady state displacement is found to be 5 mm which is occurring at a frequency of 300 rpm. So, we have to determine the equivalent stiffness and damping constant of this foundation.

So, this compressor we can model it as a single degree of freedom system with mass m equal to 100 kg k. So, you have to find this k and c of this system. So, in this case it is given that the maximum steady state displacement is 5 mm. So, this X max is given to be 5 mm, and it is occurring at a frequency omega also it is given. So, omega equal to 300 rpm. So, this rpm I can convert it to radian per second by multiplying 2 pi. So, this is 2 pi by 60 I can multiply.

So, this becomes 10 radian per second. So, this is equal to 10 radian per second. So, I know omega, I know X max. Now I have to find the equivalent stiffness and damping of the system. So, in this case I can find it like this.

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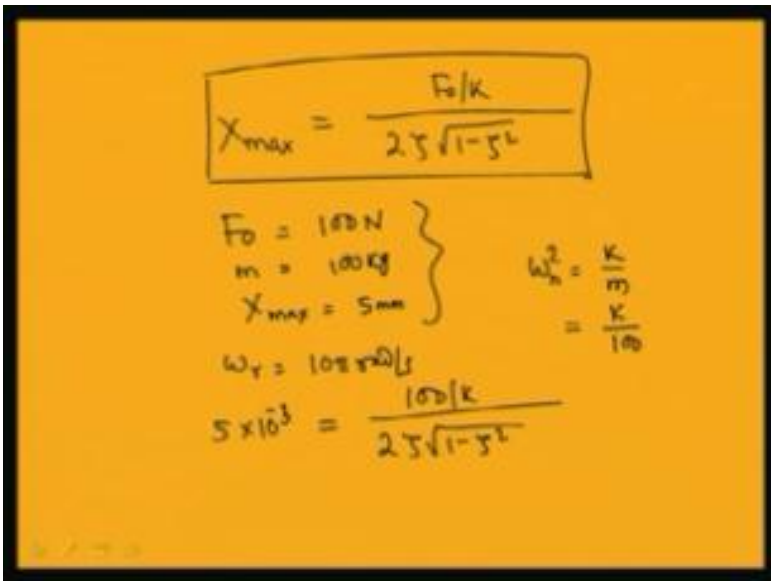

So, I know this formula X max equal to F 0 by k by 2 zeta root over omega 1 minus zeta square. So, X max equal to F 0 by k 2 zeta root over 1 minus zeta square, and already F 0 is given to be 100 Newton. And m is given to be 100 kg, and steady state maximum X max is given to be 5 mm. So, the resonant frequency also is given omega r equal to already we have found that is equal to 10 pi radian per second. So, from this equation I can write that this 5 into 10 to the power minus 3. So, I have to convert this thing to meter.

So, 5 into ten to the power minus 3 equal to F 0 that is 100 by k root 2 zeta into root over 1 minus zeta square. So, this is the thing. Also I know this omega n square equal to I can use this formula omega n square equal to k by m, and m is known to me. So, this is equal to k by 100 and this r is I know the relation between r and zeta for maximum amplitude.

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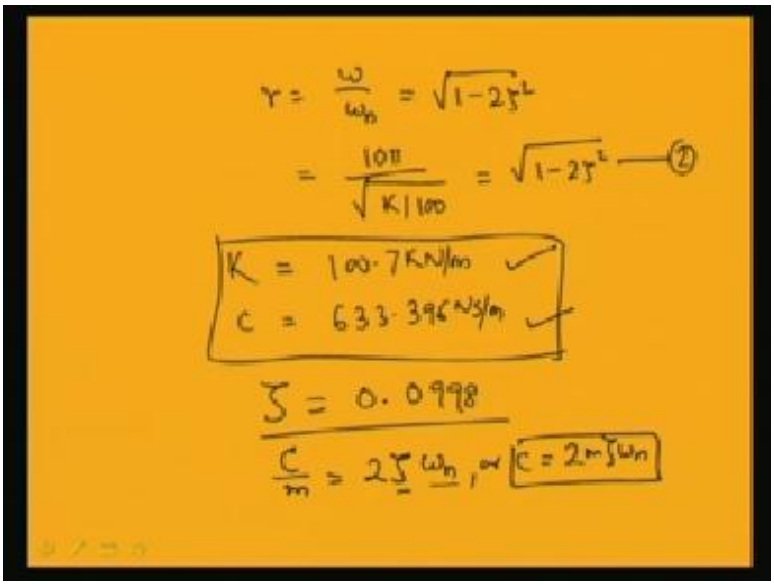

So, I know r equal to omega by omega n. So, this is equal to root over 1 minus 2 zeta square. Already you know that maximum amplitude will occur at a frequency of root over 1 minus 2 zeta square. So, this omega n is root over k by m, and this omega is known to you. So, this is equal to 10 pi by root over k by 100. It is equal to root over 1 minus 2 zeta square. So, you have two equations.

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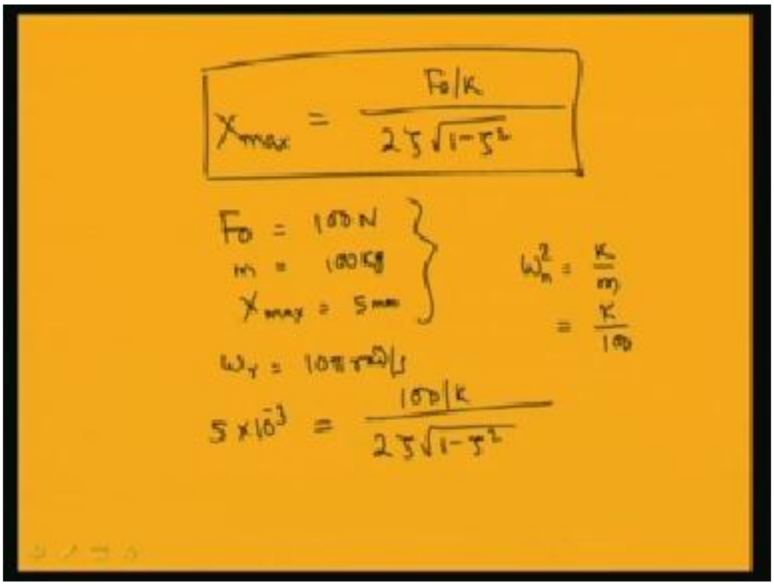

So, from these equation and the previous equations also you know this. So, by solving these two equations you can find k the value of k and zeta. So, the value of k and zeta you can find. So, they are obtained to be. So, this k you can find equal to 100.7 kilo Newton per meter, and the equivalent damping c equal to 633.396 Newton second per meter. So, in this case you are using the formula zeta. So, first you can get the value of k and zeta from these two equations. So, from this equation this is your equation two and, this is equation one.

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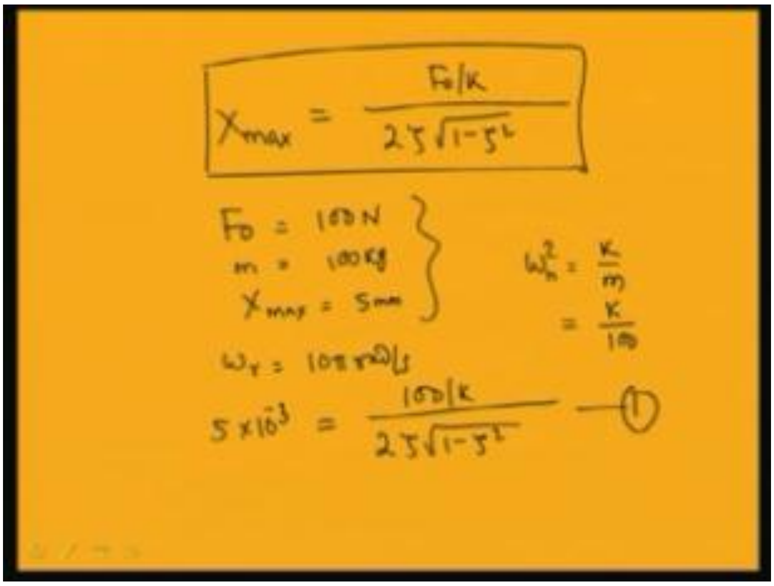

So, from these two equations you can find. So, this is 1 minus zeta square. So, this 1 minus zeta square term you can substitute it from this. And you can find this zeta value first, and after finding this zeta value you can note that zeta you are getting equal to 0.0998. So, this thing you can obtain; now first you square these things. So, you will get a relation between k and zeta.

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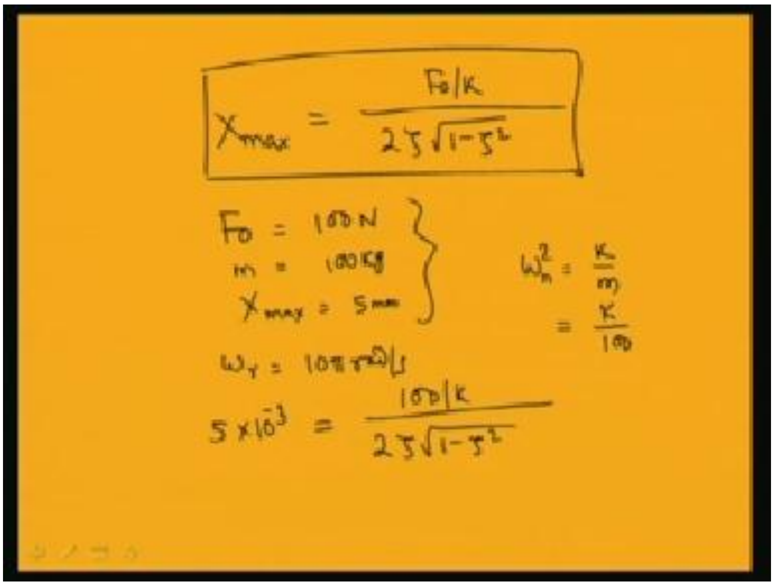

And substitute this K and zeta relation in this expression. So, you will get the relation for zeta, and that zeta equal to 0.998. And you know that zeta equal to c by m equal to you know this relation c by m equal to 2 zeta omega n. So, after knowing k and already you know the value of m. So, root over k by m will give you omega n. And after getting omega n you can find zeta; already you found zeta. So, omega n zeta is known to you. So, c by m equal to 2 zeta omega n, or you can find C equal to 2 m zeta omega n.

So, using this relation c equal to 2 m zeta omega n you can find the value of c, or this is the equivalent damping of the system, and k is the equivalent stiffness of the system. So, in this case you have seen that we have taken a compressor, and that compressor we have modeled it as a single degree of freedom spring mass damper system; we have taken the mass of the compressor. And we have found the equivalent damping c and equivalent spring constant of the system.

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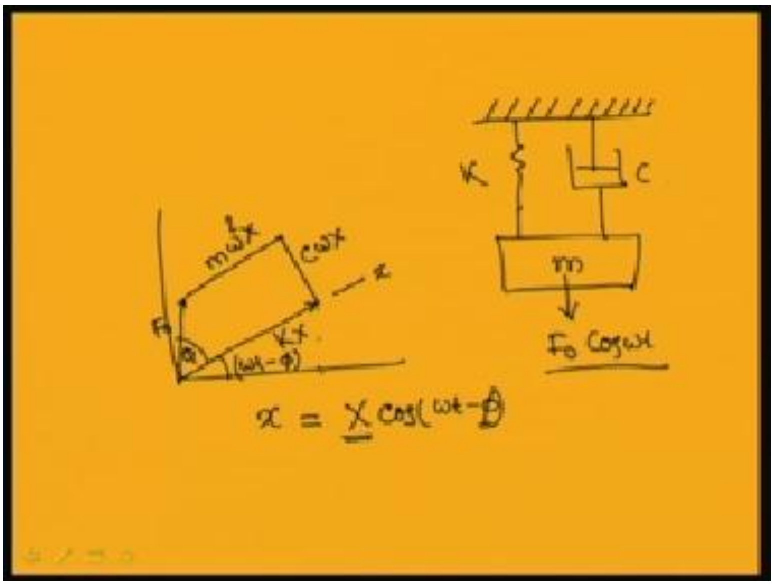

So, when the system is subjected to a force other than this F 0 sin omega t; let us take a system when it is subjected to some other type of force. So, I am applying a force of F 0 cos omega t. So, this is the mass, and this is k, and this is c. Let me apply a force of F 0 cos omega t. So, when I have applied a force of F 0 sin omega t if you have observed that force polygon, you can note or you just see these things. So, this is the spring force k X I have taken. So, this is the damping force c omega X. And this is the m omega square X; this is the inertia force taken for the system, and this is the applied force for the system.

So, applied force F 0 of the system. So, in this case you can observe that if I draw a vertical and horizontal line. So, this reference line, the horizontal line and I will draw a vertical line also. So, this is the x, and you can see this angle is phi, and this angle is omega t minus phi. So, you can see that k x; so the vertical component of this k x equal to k x sin omega t minus phi. Similarly, the vertical component of the damping force will be equal to c x dot and the inertial force.

So, you can see that the vertical component. So, when you are applying a force of sin omega t. So, we are taking the vertical component, or the forces are represented by the vertical component of these forces shown in this graphical method. So, when a force of F 0 cos omega t will act instead of a force of sin omega t, you can take the horizontal component of this force. So, the equation will remain unchanged and the same expression you will get for the response and the response will be written like this.

So, your response will be x will be equal to X; instead of sin you can write, it is equal to X cos omega t minus phi. So, you can get same amplitude of response and same phase angle when you are applying a force of F cos omega t instead of a force of F sin omega t. So, as we are taking a linear system. So, as this is a linear system when instead of applying a single force.

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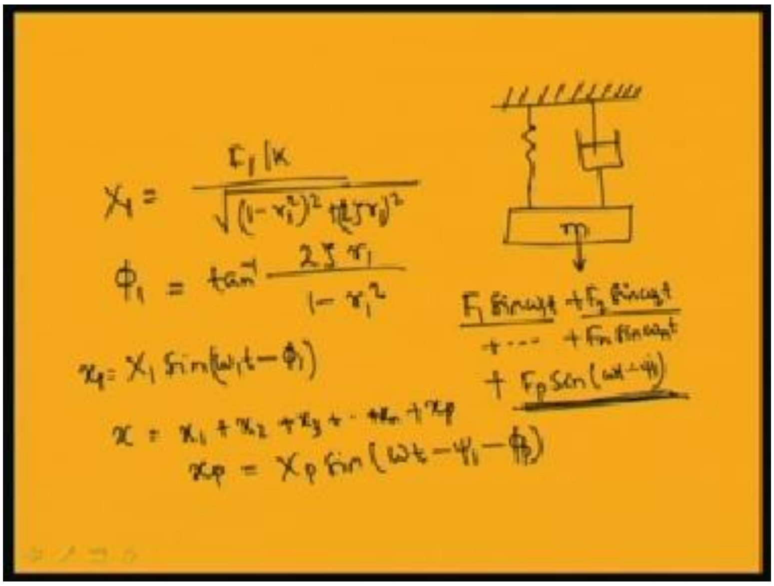

So, if we are applying a number of forces on this system. So, let a force. So, this is the system, and I am applying a force. So, let a number of force are this force which I can write as F 1 sin omega 1 t plus F 2 sin omega 2 t plus F n sin omega n t, or some force which I can write some force F p which is acting at a time sin, let it is omega t plus or minus phi minus some other angle psi 1. So, I can find the response by finding the response due to these individual forces.

So, I can first find what is the response due to the force F 1 sin omega 1 t, then I can find what is the response due to this force F 2 sin omega 2 t. So, in this case this X 1 can be written as F 1 by k root over 1 minus r 1 square whole square plus 2 zeta r 1 square. And this phi 1 I can write that will be equal to tan inverse this is 2 zeta r 1 by 1 minus r 1 square. So, for the first force I can write the response equal to X 1 sin. So, for the first force the response is X 1 sin omega 1 t minus phi 1.

Similarly, for the second force it will be X 2 sin omega 2 t minus phi 2 and for the nth force. So, I can write this X as different component. So, I can write this x equal to X 1 plus. So, I can write this x equal to x 1 plus x 2 x 3 plus x n and for this I can write. So, this x 1 you can write this x 1 equal to X 1 sin omega 1 t minus phi 1. Similarly, x 2 will b equal to capital X 2 sin omega 2 minus phi 2 and x n equal to capital X n sin omega n t minus phi n.

So, for this, what really this represents? This omega t minus psi 1 represent a force is acting on this system with a phase angle of psi 1. So, the response also will be then the response will be late by an angle phi another angle psi 2 I can write. So, due to this the response will be let I am writing this response equal to x p. So, this x p will be equal to capital X p sin omega t minus phi minus psi 1 minus psi 1 minus phi p minus psi 1 minus phi p, where this X p is given by.

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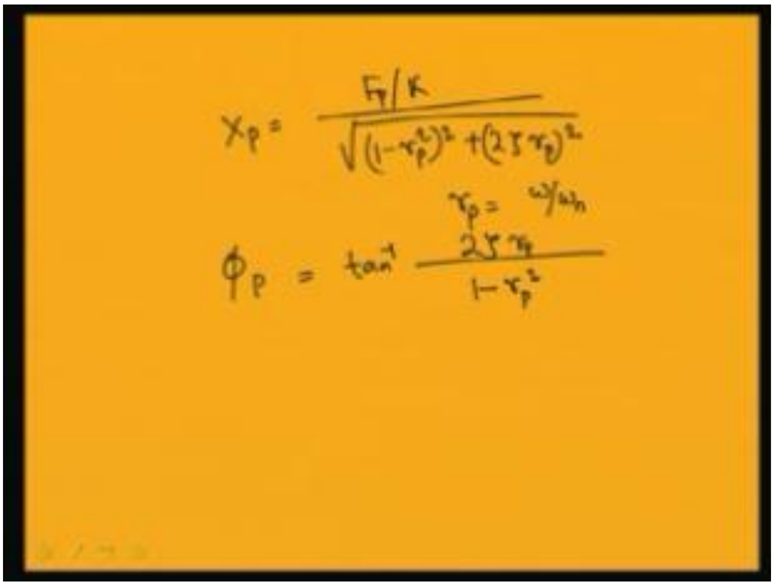

So, I can write this expression for X p. This is equal to F by F 0 by k or F p by k root over 1 minus. So, root over 1 minus r p square whole square plus 2 zeta r p whole square, where r p equal to omega by omega n. And this phi p equal to tan inverse 2 zeta r p by 1 minus r p square.

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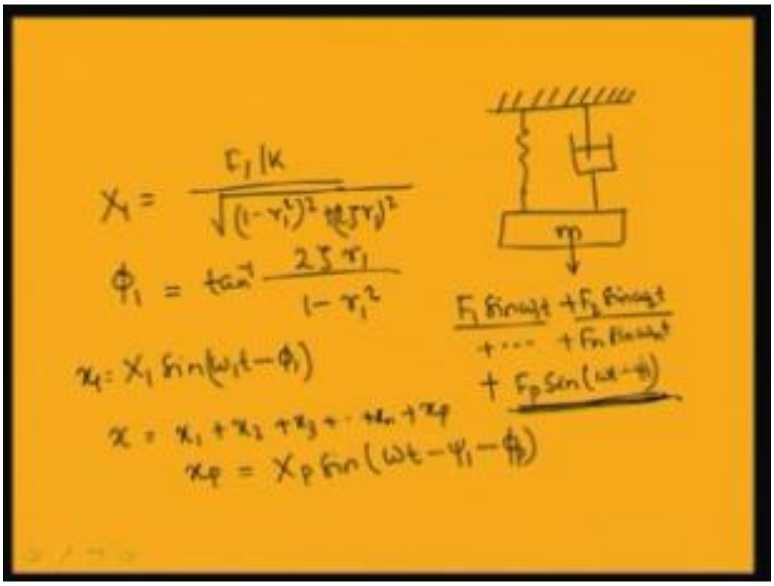

So, in this way when we have a number of forces acting on the system, then we can find the response of the system by finding the response of individual forces and then adding these forces.

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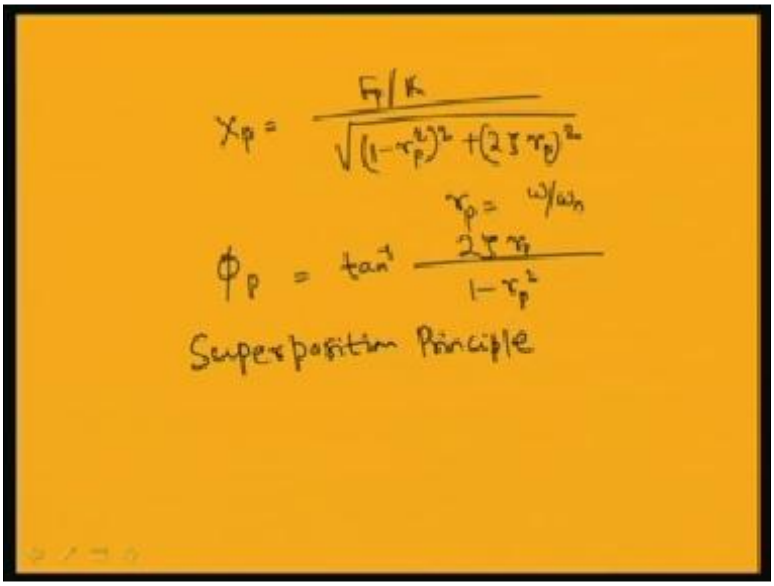

So, this is known as superposition principle. So, using superposition principle we can find the response when a number of forces are acting on the system. So, today class we have studied about a graphical method using, which we can get the response of the system, also we have studied the magnification factor.

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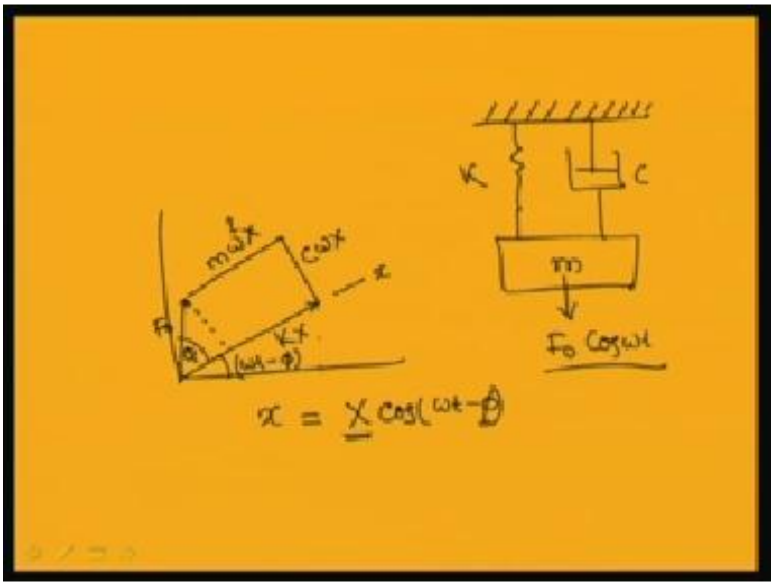

And in this graphical method I have taken the force polygon by drawing the forces, the spring forces, damping forces and the inertia force. And I have found the response of the system by drawing a line parallel to this damping force and forming a right hand triangle. So, in this right hand triangle I got this square plus this square equal to four square. So, I got the response of the system.

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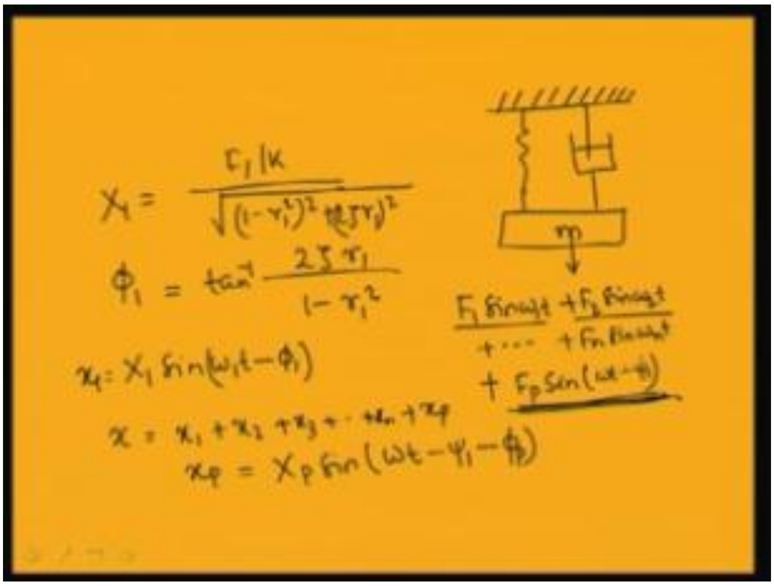

So, I got the expression for the response of the system like X equal to F 0 by k root over 1 minus r square whole square plus 2 zeta r square, also I got the phase angle. So, in this class you learned about the magnification factor, and also you learned about the superposition theory. By applying the superposition theory, you can obtain the response of the system for a number of forces when acting on the system. Also we have solved two examples. And in the next class I will tell you some alternative method to solve this differential equation of motion.