Mechanical Vibrations

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Module - 12
Signature Analysis and Preventive Maintenance
Lecture - 4
Condition Monitoring

Today we will study regarding the transverse or bending vibration of shaft. You have already some idea about what is the transverse or bending vibration. Here mainly we will be focusing on the development of the equation of motion specifically for the finite element analysis of the transverse vibration. In this particular case specially we will be using the Galerkin method for development of the elemental equation of the shaft element for mass and stiffness and forces; internal and the external forces and through examples we will see the applicability of the method for 1 case. Especially in that the assembly procedure of the matrices and application of the boundary conditions will be explained.

And so we will be starting with let us say a beam ah which is getting into the bending vibration due to the transverse loads. Loads can be either distributed load or concentrated load and in this particular case we are assuming the Euler-Bernoulli hypothesis or the theory of beam, in which we assume that a particular plane of a beam remains plane after bending. Also that particular plane which is perpendicular to the neutral axis of the beam, after bending also it remains perpendicular to the neutral axis.

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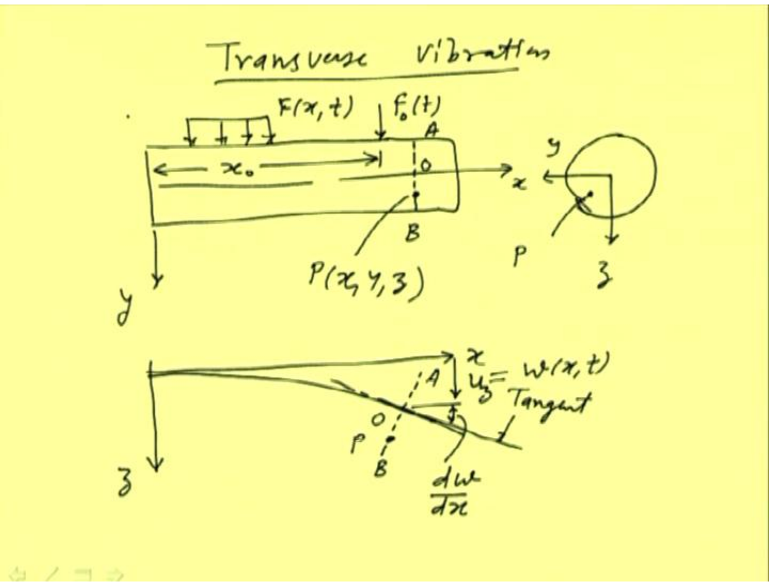

So, let us see this through the diagram. So, we are dealing with the transverse vibration and in this say there is a beam, this is axis of the beam. This is the vertical axis and on this beam there are various kind of loads acting. Let us say this is the distributed load which is function of x and t. There is concentrated load also, which is function of time and the location of the concentrated load is let us say x 0, and if we consider 1 plane.

Let us say A B and a point on this P; this particular point P the coordinate of that is x, y and z and in the side view this is the cross section of the beam and their circular in nature and this is the axis, point P is somewhere here. Now, let us draw another diagram in which we want to show the elastic line of the shaft in the bend configuration. So, this is the elastic line of the shaft which is bent. Now, this particular plane AB is here, because it remains plane and even it is perpendicular to the neutral axis after bending also.

And if we draw a tangent to the neutral axis, this is the tangent to the neutral axis at point O; this is the point O. So, you can see that there are 2 displacements which is taking place 1 is the linear displacement of the point O. That is let us say, we are representing because that displacement is in the z direction uz is equal to w which is x t; it function of x and t. And this particular slope which is dw by dx, and because of this slope you can see that particular point P which was originally here has occupied this position P here, let us say P prime.

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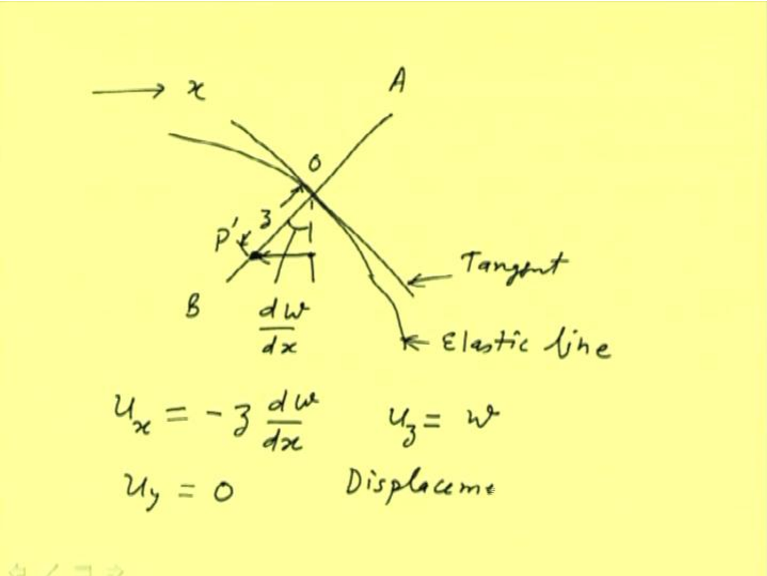

I will draw this particular diagram separately for more clarity. So, let us say this is the elastic line of the shaft. This is the line AB and this is the tangent at the elastic line at O this is the tangent. This is the elastic line of the shaft point P is somewhere here; A P prime let us say and the location of that was z, because initially the coordinate of P toward the downward direction was z and this tilting of the plane AB is dw by dx. So, you can see that the displacement of P has taken place from here to this point.

So, the displacement of point P in the x direction; a x direction is this direction is minus z this, because minus because it is opposite to the displacement positive x direction and this much displacement is this angle into the length of this radial distance. So, it is the displacement of the point P in x direction according to hyper Euler-Bernoulli hypothesis. The displacement in the y direction that is a plane which is perpendicular to the plane of the board or the screen is 0. So, when we are applying a transverse vertical load there is no displacement in the horizontal direction and uz is w.

So, this is the displacement field this is the displacement field. And once we have the displacement field we can able to obtain the strain and stress, and then we can obtain and strain energy kinetic energy and work done onto the beam. And that will be useful for developing the equation of motion of the beam. So, with this the strain field is given as only 1 strain will be nonzero the others will be 0 and rest are 0.

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$$\epsilon_{xx} = -z \frac{\partial^2 w}{\partial x^2} \text{ and rest are zero}$$

$$\sigma_{xx} = -E z \frac{\partial^2 w}{\partial x^2} \text{ and rest are zero.}$$

$$U = \int_V \left[\frac{1}{2} \sigma_{xx} \epsilon_{xx} \right] dV$$

$$= \frac{1}{2} \int_0^l E \, I_{yy} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 dx$$

$$I_{yy} = \int_A z^2 \, dA$$

Similarly, the stress now we can able to obtain stress also only 1 components component will be nonzero or this will be 0. Using this we can get the potential energy of the system which is summation of the stress and strain because other components are 0. So, we will not be having those components. So, if we substitute this 2 expressions here and simplify we will get these terms where Iyy is given as z square d A. So, this is the potential energy of the system.

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$$T = \int_0^l \int_0^A \frac{1}{2} \rho \left(\dot{u}_x^2 + \dot{u}_y^2 + \dot{u}_z^2 \right) dA \, dx$$

$$\dot{u}_x \approx 0 \qquad \dot{u}_y \approx 0 \qquad \dot{u}_z = \dot{w}(x,t)$$

$$= \frac{1}{2} \int_0^l \rho A \left(\frac{\partial w}{\partial t} \right)^2 dx$$

$$W = \int_0^l \left[F(x,t) \, w(x,t) + F_0(t) \, \delta^*(x - x_0) \, w(x,t) \right] dx$$

Direc delta fn.

$$\delta^*(x - x_0) = 1 \text{ for } x = x_0$$
$$= 0 \text{ other}$$

Now, let us see the kinetic energy of the system. So, kinetic energy will be given as various velocity components and only this velocity component is there. The in this velocity or this is approximately 0 uy is already 0 and uz will be wxt. So, if we substitute this we will get the kinetic energy of the system like this. Now, work done by the distributed force and the concentrated force can be written as; this is the distributed force, this is the displacement plus this is the concentrated force and we will using a direct delta function for this. So, this is the work done regarding direct delta function, it has certain properties let us see these properties. It is equal to 0 x minus x naught is 1 for when x is equal to x naught and it is 0.

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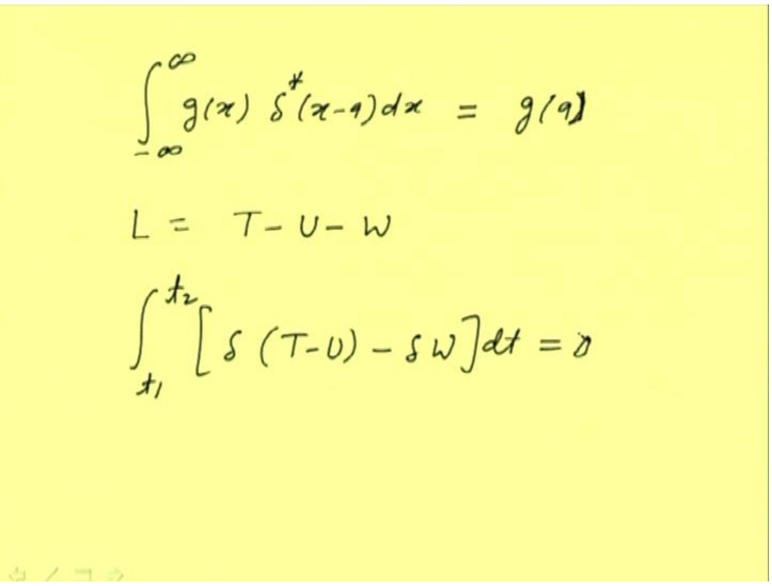

Also it has integration property which will be using it. If there is a function gx, let us say and we are integrating a direct delta function along with it. So, this will give us this,, so this property we will be using it for this purpose. So, once we have got the expression of the kinetic energy potential energy and work done, we can able to use the Lagrangian and with the help of Hamilton principle we can able to obtain the equation of motion.

Let us see what is the expression for the Lagrangian T minus U T minus U W and the the Hamilton principle stated like this. T minus U delta Q dt is equal to 0. So, using a Hamilton principle in the earlier lecture we already explained how to get the equation of motion. So, I will be skipping these steps and finally, I will be giving the final equation of motion, which we will be getting from these energies.

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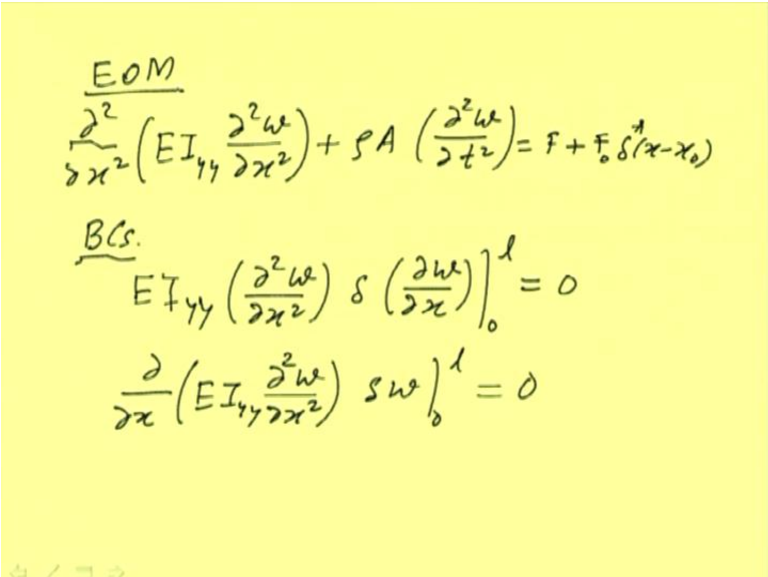

So, the equation of motion which we will be getting;, so this is the term corresponding to the elasticity of the system. Then this is the term which is corresponding to the inertia and this is the external force continuous and the concentrated force with the direct delta function. And boundary conditions and the boundary conditions are a first term is the bending moment and this is the variational operator; this is the slope these are the limits. And second boundary condition this is the shear force and this is the displacement.

Now, we will be analyzing these equations through finite element method as you know this finite element method is an approximate method, in which we assume the solution in the form of polynomial. We substitute these solutions in the equation of motion and obviously because these solutions are approximate some residual will be left out all the equations will not be satisfied. Using a Galerkin method we will minimize this residual to get the elemental equation for the finite element of the transverse vibration of the shaft.

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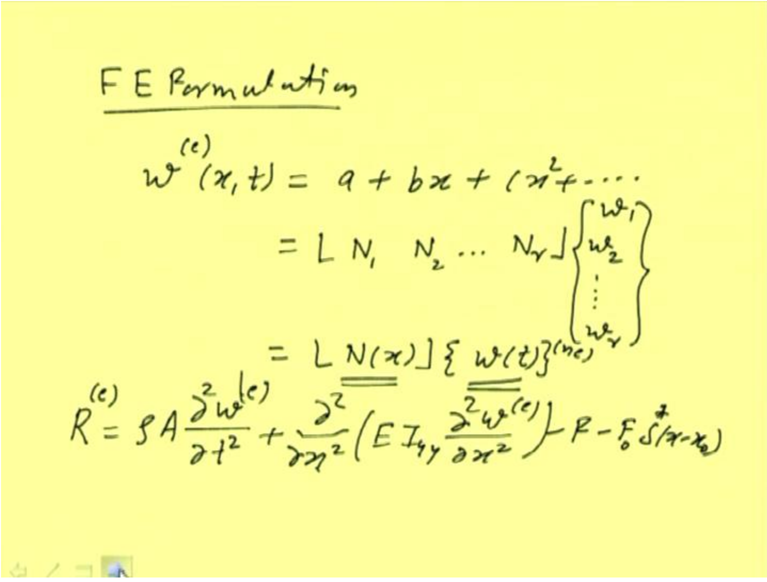

So, for this first we assume the solution the displacement for a element in a polynomial form. Let us say a plus bx plus cx square can be a several terms number of terms finally, it depends upon the type of problem we are handling. At present we are keeping in the general form and this can be expressed as various functions: let us say there are a number and corresponding displacements at various nodes we have and in more general form it can be written as, so here, n is a function of the x only and w is function of time. So, once we substitute this approximate solution into the equation of motion the residual will be; this is the residual rho A. This is the inertia term now in all displacement terms we are writing for the element, this the elastic term then force is.

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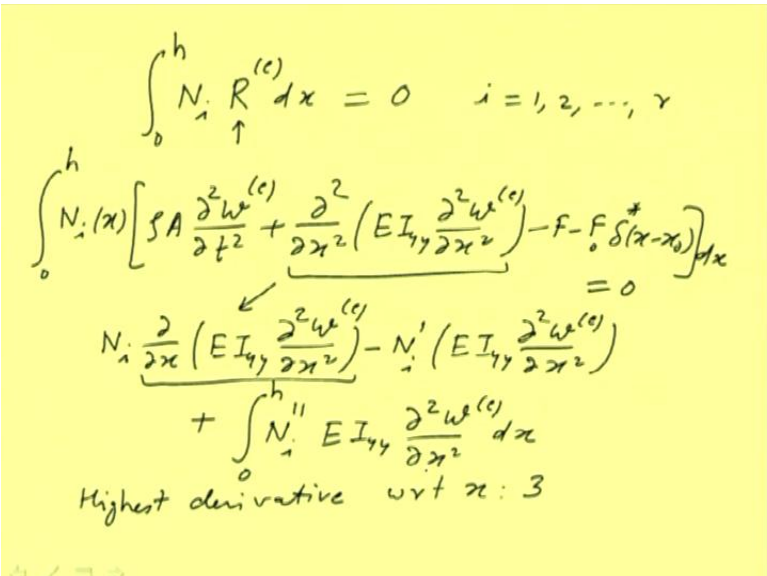

Now, we will be minimizing this particular residue over the element by multiplying by a shape function and we will equate this to 0 and where i is from 1 2 r. Now, we are substituting the residual here. So, this is the residual terms. In this I am giving all the steps; intermediate steps so that all the steps are clear is equal to 0. Now, in this particular term this particular term will be doing integration by part so that, we can have less tangent compatibility and completeness condition for the assumed solution, which we are assuming in a polynomial form.

So, let us do the integration by part of this particular term. So, this will give us rest of the terms will remain same. First term integration of the second term, then differentiation of the first term and integration of the second term, plus differentiation again of the first term and second term as it is and in this, limit is from 0 to h, h is the element length. So, this particular second term we have expanded little like this and so from here, you can see that now the highest derivative in the expressions are now or here that is 3. Highest derivative with respect to x is 3 with respect to x is 3. So, we require the compatibility the completeness up to the I can go to next slide.

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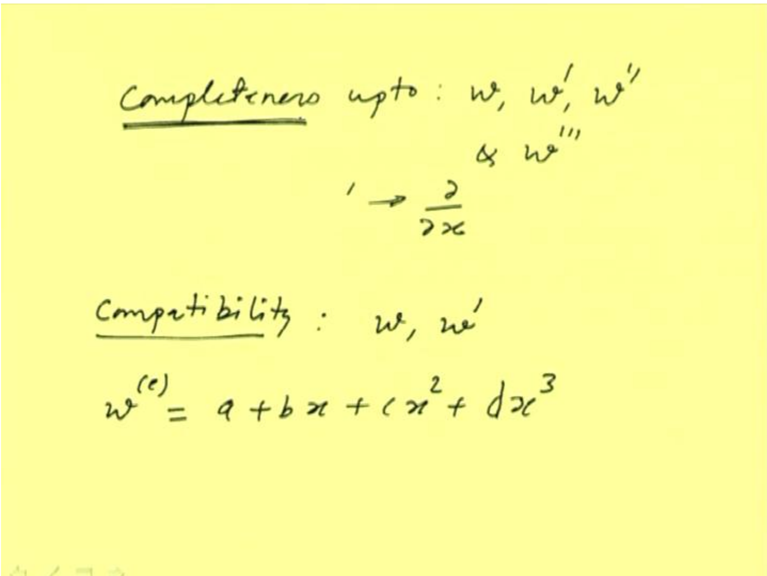

So, completeness up to w w prime w double prime and w 3 primes these primes represent derivative with respect to x. So, you can see that a cubic polynomial we will able to satisfy these completeness condition. Also in the previous term if we see the inside the integral the maximum derivative is second; inside the integral after taking the integration by part. So that means, the compatibility condition we require of w and w prime; that means, 1 order less because it is having derivative up to certain.

So, 1 order less compatibility that w and w prime compatibility is required for the shear function to satisfy. So, as we have seen that for completeness the cubic polynomial is sufficient, because it will not vanish if we take the third derivative of w. A constant term d will be remaining, so it satisfy the completeness condition. Now, let us because these abc are the constants that need to be obtained and that will be obtaining through the boundary condition of the element.

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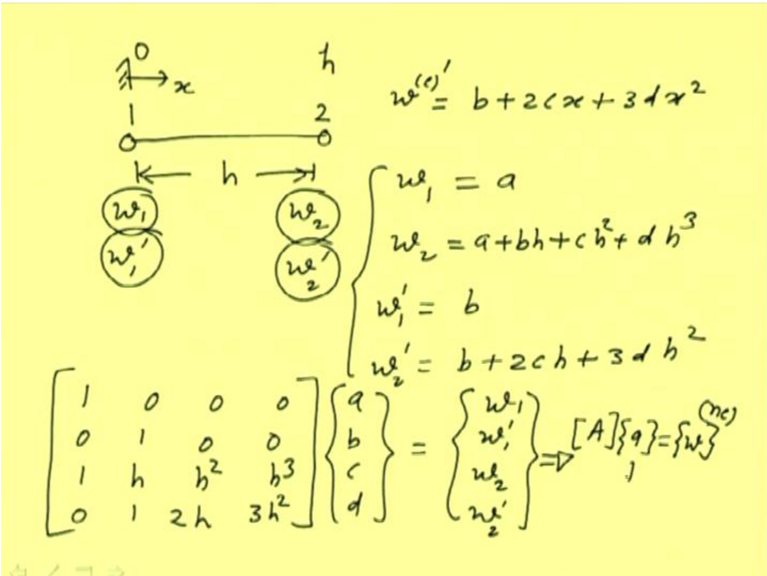

So, let us say this is a element, the length of the element is h this is the beam element and nodes are 1 and 2 and each nodes is having these are the conditions. Now, here the local axis we can able to assign x. So, this is x is equal to 0 and this is x is equal to h. So, we have these 4 conditions which we need to satisfy into the polynomial a polynomial we had of the w. But the prime of that; that means, the derivative with respect to x of that can also be obtained because it is there in the boundary condition.

Now, we are satisfying the 4 boundary conditions and so that, these 4 constants can be obtained in terms of the boundary conditions. Let us see, how the expressions will be through a first boundary condition this 1 will give us w 1 as a. Then second boundary condition will be give us w 2 as a plus bh, because we have to substitute for x is equal to

h. Then the third boundary condition of this will give us w 1 prime equal to b and the fourth 1 give us b plus 2 ch plus 3 d h square.

So, we have now 4 equations and 4 unknowns are there this expression can be put in a matrix form like this: w 1, w 1 prime; w 2, w 2 prime. The ordering has to be taken care ordinarily it is not important thing, because we can arrange these equations in a NAD ne form. So, here we will be having terms like this: 0 0 0 0 1 0 01 h h square h cube 0 1 2 h 3 h square. Now, to get abcd we have to need to invert this matrix. Let us say, this particular matrix we are writing in more compact form as A small a is equal to w. So, we need to obtain this. So, we need to invert this A capital a matrix.

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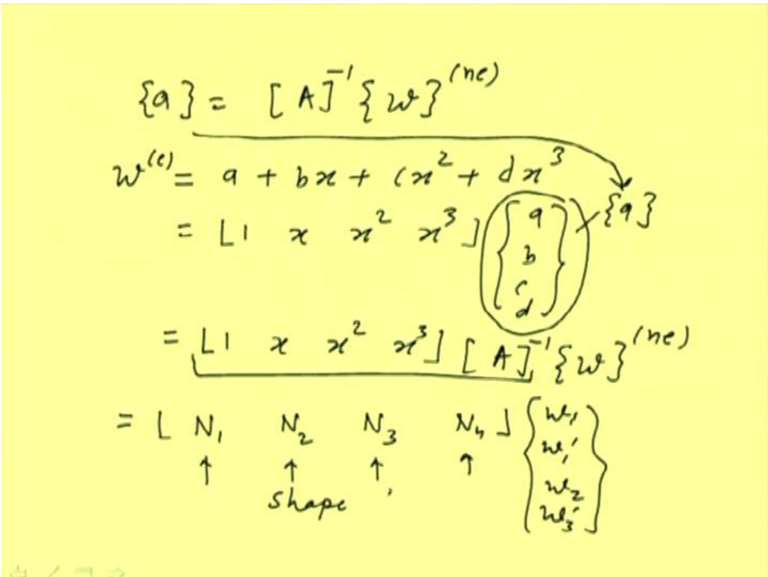

So, a vector we can able to if we invert the A matrix and multiply by the w vector. And this polynomial which we had earlier that is a plus bx plus cx square plus dx cube. This can be written as: 1, x, x square, x cube and abcd, which can be written as: 1, x, x square, x cube. Because this quantity is nothing but a small a vector, so this we can able to substitute here. So, once we substitute that there we will get A inverse capital A inverse w ne. Now, we will combine these 2. So, we will be getting a 4 terms: I am writing N 1, N 2, N 3, N 4 and these are w 1, w 1 prime w 2, w 3 prime. Now, I will be giving expression for these shape function they are called shape function they are function of x.

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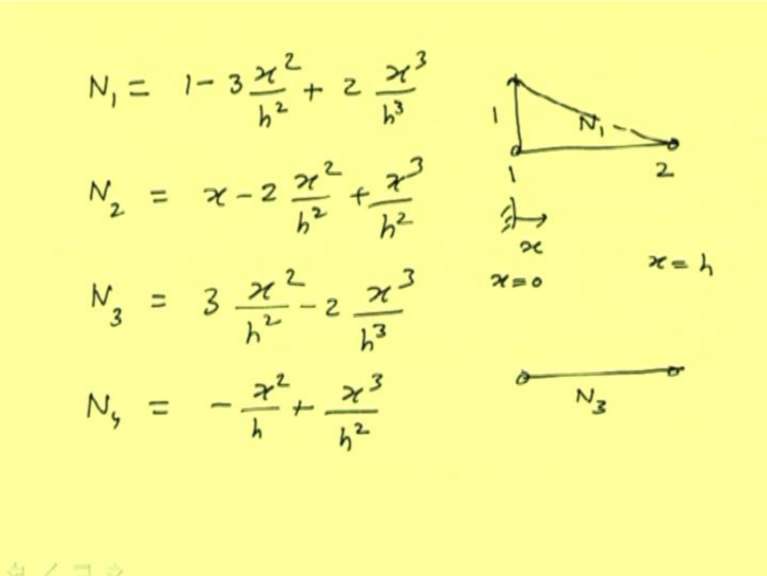

So, these functions can be expressed as if we multiply those vectors. They are in terms of polynomial, these shape functions are having unique property which we will see subsequently. So, it is the third shape function and fourth shape function is x square by plus x cube by h square. Now, these shape functions have unique property let us say this is a particular element this is node and 2. So, if we want to plot the N 1 over this you can see that, when x is equal to 0 this is the x axis and here x is equal to 0 here x is equal to h.

So, when x is equal to 0 N 1 is having value 1 and when x is equal to h if we substitute here it becomes 0 and in between it varies cubic variation. Similarly, N N 2 this is N 3 let us say N 3; N 3 is having opposite property when x is equal to 0 it becomes 0 and when x is equal to h it becomes 1, in between it is having this property. Similarly, N 2 and N 3 their derivatives are having similar property that we will be using it in subsequent analysis. So, once we obtain the displacement, now we should say substitute those into the equation of motion and that will give us the elemental equation.

So, we will be using these equations which we have already earlier obtained. Here you can see that the i subscript is there which is varying now from 1 2 3 to 4, because now we have 4 a shape functions so; it will be varying up to 4. So, these are the 4 such equations. So, let us combine them first and then we will be substitute the displacement here.

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$$\int_0^h \rho A \{N\} \frac{\partial^2 w^{(e)}}{\partial t^2} dx + \int_0^h E I_{yy} \{N''\} \frac{\partial^2 w^{(e)}}{\partial x^2} dx$$

$$= -\begin{Bmatrix} N_1 \left(E I_{yy} w^{(e)''}\right)' \Big|_0^h \\ N_2 \left(E I_{yy} w^{(e)''}\right)' \Big|_0^h \\ N_3 \left(E I_{yy} w^{(e)''}\right)' \Big|_0^h \\ N_4 \left(E I_{yy} w^{(e)''}\right)' \Big|_0^h \end{Bmatrix} + \begin{Bmatrix} N_1' \left(E I w^{(e)''}\right) \Big|_0^h \\ N_2' \left(E I w^{(e)''}\right) \Big|_0^h \\ N_3' \left(E I w^{(e)''}\right) \Big|_0^h \\ N_4' \left(E I w^{(e)''}\right) \Big|_0^h \end{Bmatrix}$$

$$+ \int_0^h \{N\} \left[F + F_0 \, \delta^*(x - x_0) \right] dx$$

So, now after combining those 3 a 4 equations it will take this form. So, now, in place of N it is having a vector form. Here we have double derivative of N and the internal forces I am writing in the expanded form, double prime inside and outside another prime 0 to h. N 2 Eyy w e double prime single prime outside the bracket 0 to h. So, this can be, so these are basically shear forces and fourth term or similar terms are there for the bending moment 0 to h this for bending moment.

These terms will get simplified once we start substituting the boundary condition 0 and h condition because we know the shape functions are having some inherent property rather they become 1 or 0 at these ends. And apart from this the external force components it is the distributed force and this is the concentrated force; that this is the direct delta function. Now, as we know the property of N it is 1 for x is equal to 0, but it is 0 x is equal to h. So, those property we with be using here in all other places.

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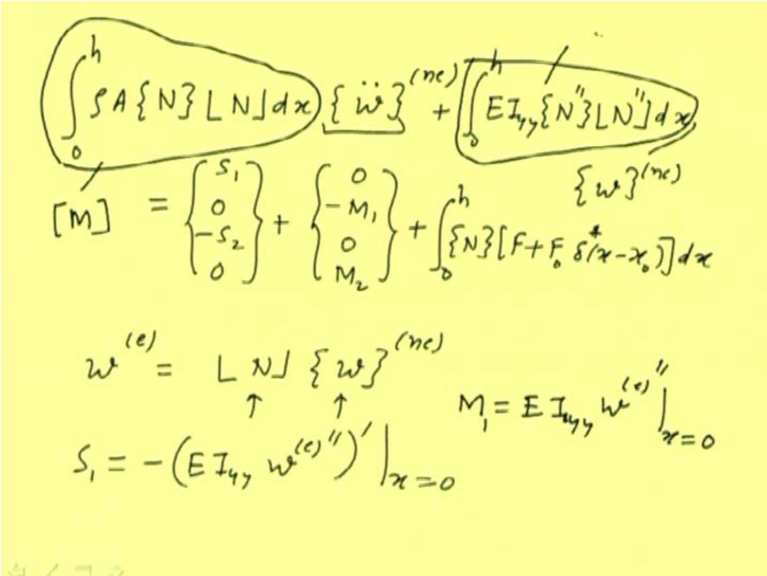

So, these equations will get simplified. So, these equations will get simplified and apart from this we will be substituting the displacement which we obtained earlier here in all other places. So, let us substitute that. So, we will get 0 to h rho A N which is vector then a rho vector of N dx w will take the time derivative. Actually, what we are doing we are substituting for w which is like this. So, this is function of x and this is function of time.

So, time derivative is taken by the w ne and this is inside because now there is not function of x. So, it can be taken out similar terms are therefore the inertia for the similar terms are there for the elastic force a elastic term. Here N is having double derivative this is equal to and here we have w ne. So, earlier the expressions after substituting the value of N 1 and N 2 and other terms for x is equal to h and x is equal to 0. They will take this simple form these are nothing but, shear force and bending moment at node 1 and 2.

And this is the external force term which remains as it is as previous 1. Regarding the shear force s 1 is given as E Iyy w e double prime whole single prime at x is equal to 0, because x is equal to h N 1 was 0. So, that term will not contribute in the first term this is coming from (Refer Slide Time: 32:01) the previous expression here. So, N 1 is 0 for x is equal to h and it is 1 for x is equal to 0. So, that gives us the s 1 and this is the s 2 term. Similarly, this is M 1 M 2 term other terms are getting vanished. So, M 1 is Eyy w e double prime at x is equal to 0. So, now you can see that this particular expression we will be calling as a M matrix; which will call as mass matrix, this term we will call it as a k matrix stiffness matrix.

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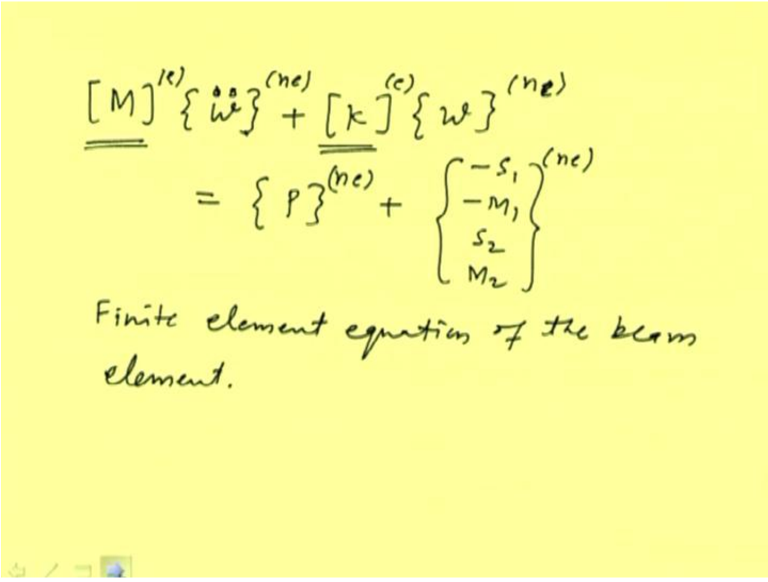

And let us write those expressions separately. This is the element matrix this is the an acceleration term. Then stiffness matrix it is the displacement term that in various nodes is equal to the external force plus the internal force and moments, because this is the elemental equation for element 1 having node 1 and 2. So, these 2 nodes are appearing here. So, this is the elemental finite element equation of the beam element. Now, we will see these term in the expanded form their all elemental equation these are the node.

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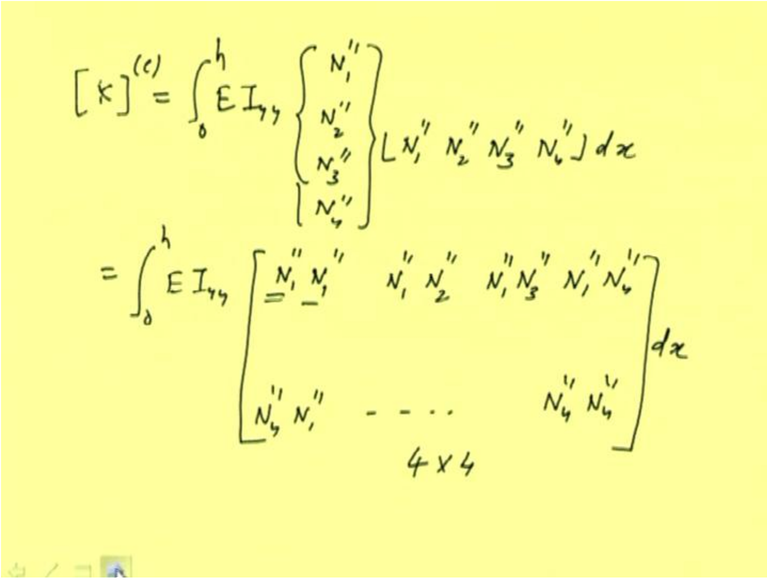

So, let us see the stiffness matrix first. Which is given as, now I am expanding this N this is row vector of N dx. So, if we multiply this we will get N 1 double prime N 2 double prime N 1 double prime. This will be N 1 itself; this will be N 2 double prime, N 1 double prime, N 3 double prime, N 1 double prime N 4 double prime. So, it will go like this here it will be N 4 double prime N 4 double prime. So, other terms similarly we can able to obtain, this is a basically 4 by 4 matrix. So, because we know N 1 N other terms their derivatives can be obtained and it can be integrated.

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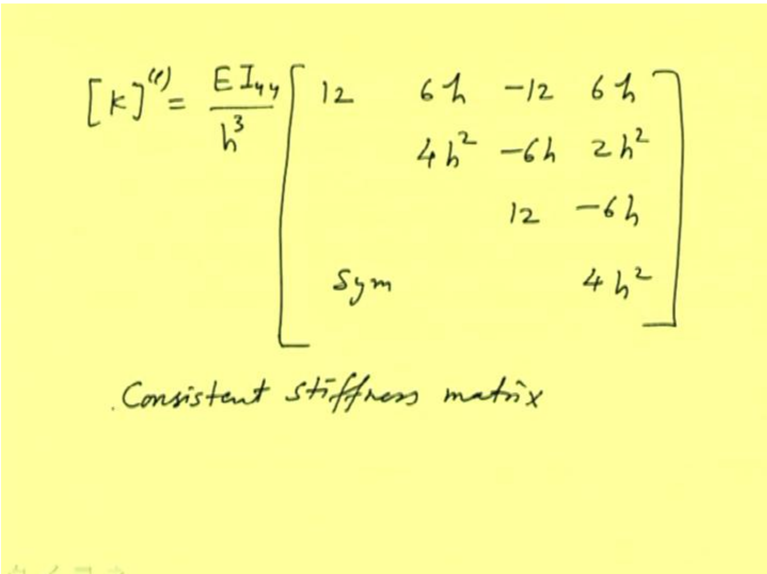

So, if we integrate this the k matrix finally, we will be of this form 12 6 h minus 12 6 h 4 h square minus 6 h 2 h square twelve minus 6 h 4 h square and this is symmetric matrix. So, this is the consistent stiffness matrix.

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$$[M]^{(e)} = \int_0^h \rho A \{N\} \lfloor N \rfloor dx$$

$$= \int_0^h \rho A \begin{bmatrix} N_1^2 & N_1 N_2 & N_1 N_3 & N_{14} \\ & & & \\ N_4 N_1 & - - - & & N_4^2 \end{bmatrix} dx$$

$$= \frac{\rho A h}{420} \begin{bmatrix} 156 & 22h & 54 & -13h \\ & 4h^2 & 13h & -3h^2 \\ \text{Sym} & & 156 & -22h \\ & & & 4h^2 \end{bmatrix}$$

And on the same time we can able to obtain the mass matrix. Which is given by this, so if we expand this as we did before this will be N 1 square, N 1, N 2; primes will not be there in this case. And if we integrate this we will get rho A h by 420 156 22 h 54 minus 13 h. And this also it is symmetric. So, I am writing 1 of the matrix this is h 4 h square a symmetric matrix. So, this is the elemental mass matrix.

Once we have seen the expended form of the mass matrix and stiffness matrix also the internal force vector that contain shear force and bending moment. Let us see, how the force a matrix the external force matrix takes the form. Specially, when we are having different kind of load distribution either it is linearly varying or it is that is uniformly distributed load or concentrated load. How the elemental equations for that can be developed.

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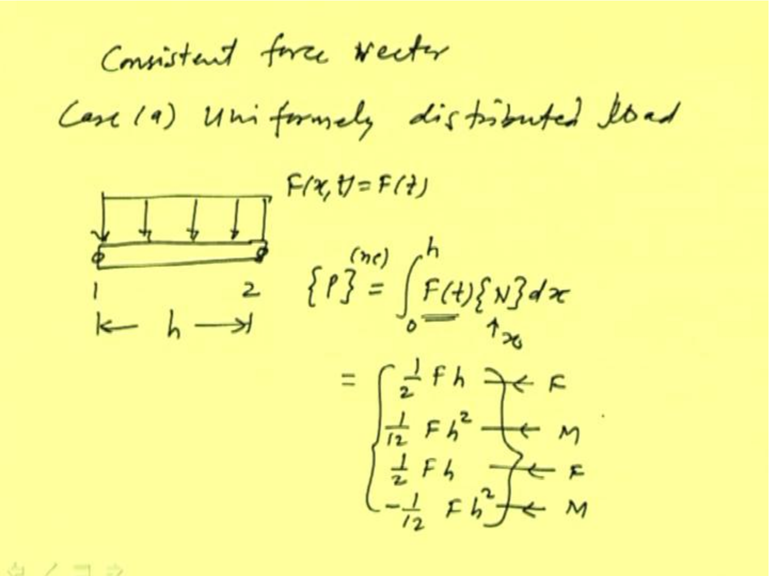

Let us say first case we are considering which is uniformly distributed load, uniformly distributed load. So, in this case this is the element 1 and 2 nodes are there this is the element length load is acting uniformly from here to here. And this is let us say, f some constant value. Now, the external load vector can be written as 0 to h integration over the element Ft, because now it is constant N dx. This is coming from the previous equation of motion of the element and if we substitute because this is a constant it will go out if we because this is function of x.

So, if we integrate it we will get half Fh 1 by 12 F h square again half Fh and minus half Fh square. So, this is the consistent force vector when distributed load is there. You can see that, these are corresponding to the force and they are corresponding to the moments. These are forces and these are moments as we had in the internal load also these were the shear forces and these were the moments.

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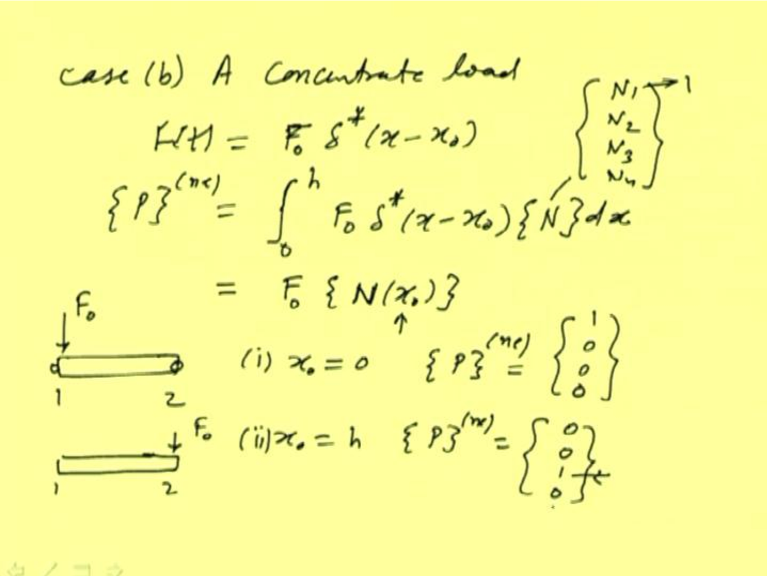

Now, let us see case b in which a a concentrated load is acting. So, for concentrated load Ft is given as: F star x minus x naught and P vector is given as integration 0 to h F naught; this is the F naught direct delta function and N. So, basically as we see the property of the direct delta function, by which we can able to say that this we will get. We have to just substitute in all N's x is equal to x naught. So, we will get the integration of this quantity. So, let us see this particular thing for different cases of concentrated load.

So, let us say force is acting here at node 1. So, for this case x naught is 0 because this is the position of the concentrated load. So, if we substitute x is equal to x naught is equal to 0, the P vector will be 1 0 00. Because N is nothing, but it contain N 1, N 2, N 3, N 4. And you can able to check that at x is equal to 0 only N 1 is having 1 value all others are having 0 value. Similarly, if F naught is acting at node 2 then x naught is equal to h and P will take the form of 0 01 0. So, because only N 3 will be 1 others will be 0.

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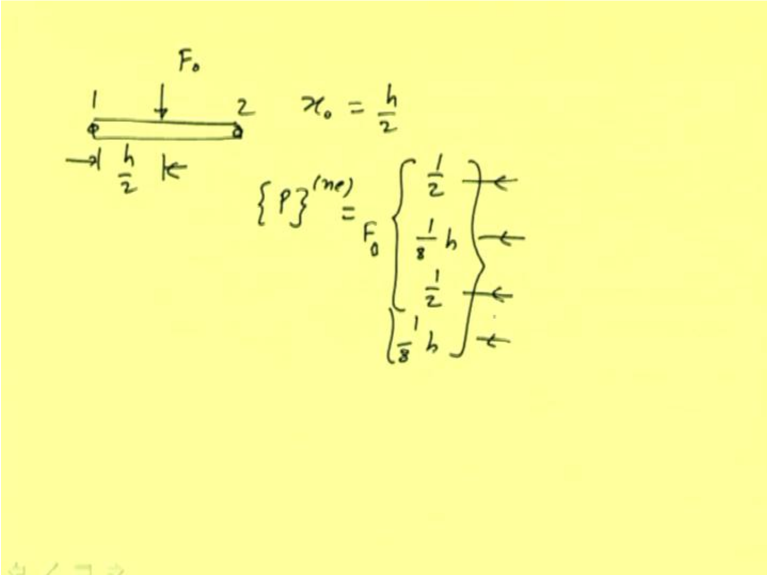

And in third case when the load is acting in between the node let us say, at middle of the element h by 2. So, in this case x naught is h by 2, and if we substitute this in the previous expression we will get this as half 1 by h again half 1 by 8 h. So, you can see that when load is acting the middle not only the forces, also the moments will also be appearing at the nodes. So, we have seen the how the load a load vector takes the form when uniformly distributed load is there or when a concentrated load is there.

Here we are talking about consistent force vector and consistent mass and consistent stiffness matrices. These are, there are another version of these matrices that is called lump mass or lump stiffness matrix. Or generally lump mass and the lump force matrices are there. They can they simplify the analysis, because of this specially when we are doing the lump mass analysis, various forces various inertia forces get decoupled and we have more simplified version of the equation of motion. So, especially now let us see another form of the force which is: distributed force and linearly varying. How the we can able to get the load is a load matrix load vector how we can get load vector.

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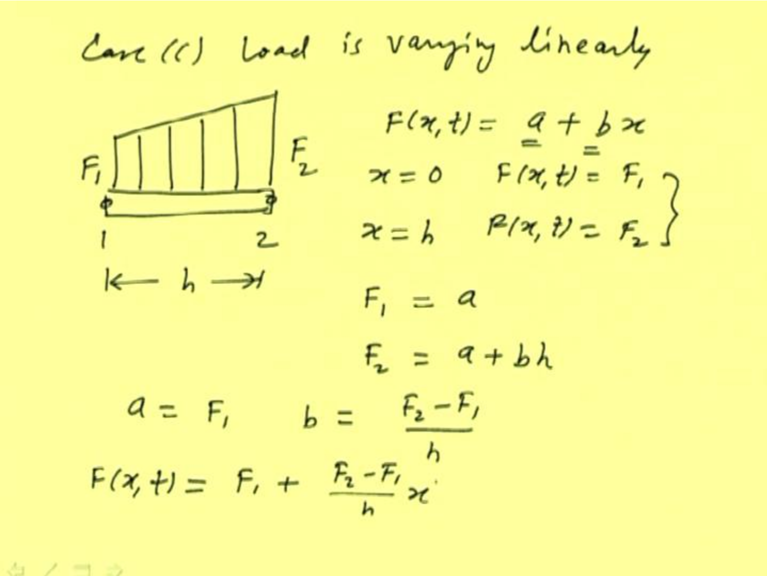

It is the case c in which load is varying linearly. So, for this case, this is the element and node is varying from 1 end to another end. Let us say amplitude which is here is F 1 here it is F 2 and in between it is varying linearly. So, because force is varying linearly with the element length, so we can able to write this as a linear function, where a and b is some other constant, they are not related with the previous a and b.

And we have boundary condition that x is equal to 0 we have Fxt as F 1. And we have at x is equal to h Fxt as F 2. So, these 2 conditions can be used to obtain the a and b. So, let us substitute 1 by 1. So, first condition give F 1 is equal to a second condition will give F 2 is equal to a plus b h and these 2 can be solved for a and b. So, we will get a as F 1 and b as F 2 minus F 1 by h. So, the force which is varying linearly can be written as F 1 plus F 2 minus F 1 by h x.

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$$F^{(e)}(x,t) = F_1 + \frac{F_2 - F_1}{h} x$$

$$= \left(1 - \frac{x}{h}\right) F_1 + \left(\frac{x}{h}\right) F_2$$

$$= \left\lfloor \left(1 - \frac{x}{h}\right) \quad \frac{x}{h} \right\rfloor \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$= \lfloor N_f \rfloor \{F\}^{(ne)}$$

$$\{P\}^{(ne)} = \int_0^h \underbrace{\{N\} \lfloor N_f \rfloor} dx \, \{F\}^{(ne)}$$

Or it can be simplified or it can be simplified as Fxt for the element as F 1 plus F 2 minus F 1 by hx. So, the F 1 and F 2 can be separated. So, we will be getting expression 1 minus x by h F 1 plus x by h F 2, which can be written as: 1 minus F x by h and x by h F 1 F 2. Now, this can be written as the shape function for the force and this is the nodal forces. So, if we substitute this in the force vector, N was already there. Now, another shape function from the force which is varying linearly will come and there will be a force ne term.

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$$\{P\}^{(ne)} = \begin{bmatrix} \frac{7}{20} h & \frac{3}{20} h \\ \frac{1}{20} h^2 & \frac{1}{30} h^2 \\ \frac{3}{20} h & \frac{7}{20} h \\ -\frac{1}{30} h^2 & -\frac{1}{20} h^2 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$= \begin{Bmatrix} \frac{7}{20} h F_1 + \frac{3}{20} h F_2 \\ \left(\frac{1}{20} F_1 + \frac{1}{30} F_2\right) h^2 \\ \left(\frac{3}{20} F_1 + \frac{7}{20} F_2\right) h \\ \left(-\frac{1}{30} F_1 - \frac{1}{20} F_2\right) h^2 \end{Bmatrix}$$

Now, if we substitute these 2 and integrate we will get a term of P like this. Even this is coming from the integration 3 by 20 h, 7 by 20 h and 1 30 h square, 1 by 20 h square. Here force vector is also there at nodes F 1 and F 2 if we multiply them; we will get 7 by 20 h F 1 plus 3 by 20 h F 2. Other terms can be obtained on the same lines even we can able to simply them in more compact form. So, this you can see that now they contain terms in all places.

Even at the force and moment place they contain the terms and contribution of the F 1 and F 2 amplitude set, either end of the linearly varying force are appearing at every place. So, on the same lines if the force is varying with some other function like parabola then, we can able to take the the polynomial as quadratic in nature. Their 3 constants will be there and shape function for same can be obtained. Only thing another node will be requiring in the middle of the element so that, the 3 constants can be obtained.

In the subsequent lecture, we will see with some examples a application of these element matrix which we have developed. So, today we have already seen that for the transverse vibration or the bending vibration. Especially for the Euler-Bernoulli hypothesis how the elemental matrices of the mass stiffness and forces takes the form. And in the subsequent lecture we will be explaining through example, the assembly procedure of the application of this method even, the application of the boundary condition and how we can use this for obtaining the natural frequency or the force vibration of a beam.