Mechanical Vibrations
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Module - 2
Single DOF undamped free vibrations
Lecture - 2
Energy method, principle of virtual work

Previous two classes we have already seen what is the importance of free vibration analysis, especially when we are talking about. Because we just started with the very simple system in which single spring mass system we considered, and for obtaining the natural frequency of the system, we used Newton's second law. I mean in that we were balancing the forces of the mass once we are drawing the free body diagram of that particular mass. There are other methods, which are quite powerful especially those are based on scalar quantity like energy.

And specially when we have conservative systems in which there is no dissipation of the energy, then we can able to use energy methods quite effectively to obtain the equation of motion, because for a conservative system we know that the sum of the potential energy of the system at a particular time that remains constant, and this can be used to obtain the equation of motion. So, today we will see how the energy method can be used to obtain the natural frequency of some simple systems. And in subsequent lectures we will be elaborating more on this how we can extend these methods for more complex systems.

So, in this Energy method we will be using the principle of conservation of energy; that is the basic principle based on which will be developing the equation of motions. And according to this principle we have the total energy of the system; that is partly it will be the kinetic energy, and partly it will be potential energy. The kinetic energy will be stored in the system in the form of this velocity. And the potential energy will be stored in the form of the strain energy or may be work done against the some field like magnetic field or gravity field.

So, we can able to write this expression let us say kinetic energy we are representing as T and potential. So, these terms we can able to write in terms of variable let us say T is the kinetic energy, and U is the potential energy. Kinetic energy will be stored in the system as say velocity terms, and potential energy will be stored in the form of strain energy or may be work done again some field like gravity field or magnetic field, and this quantity is a constant for a conservative system. So, the change of this with respect to time is zero, because this quantity is not changing. So, let us see this how we can able to use for obtaining the equation of motion.
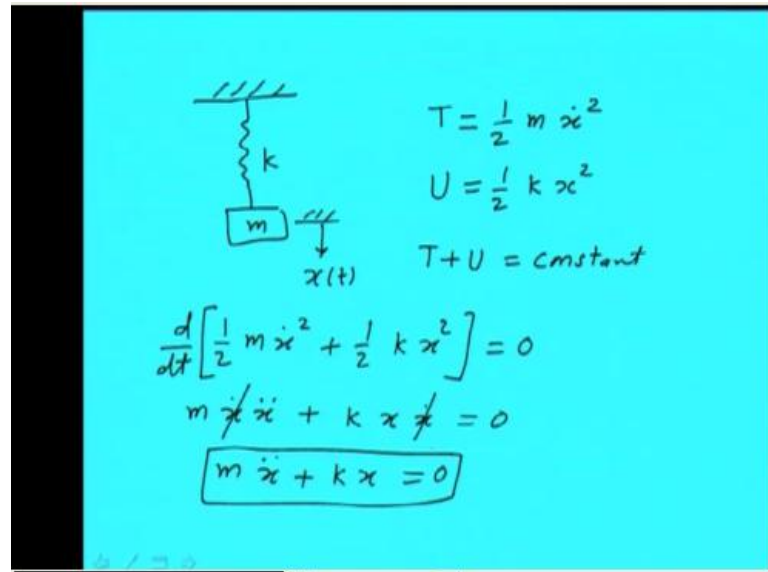
So, let us take a simple example of a spring mass system, and we have taken a reference from the static equilibrium position, and we are disturbing the mass from there; that is there are displacement of the mass from its equilibrium position. Now the kinetic energy and the potential energy terms we can able to write. The kinetic energy will be mass into velocity into square of that, and potential energy will store the springs that will be half k x square. So, total energy T plus U is constant for any particular instant.

So, we can able to sum them up, and because this is a constant quantity if we differentiate this, we should get zero. So, let us differentiate this. So, this will be x double dot because two will cancel this. Then k x x dot is equal to 0. You can see that x dot is common, it will go out. So, we will be left with equation of motion of the spring mass system which is we already derived earlier. Now to elaborate more on the energy method, let us take some more simple examples.
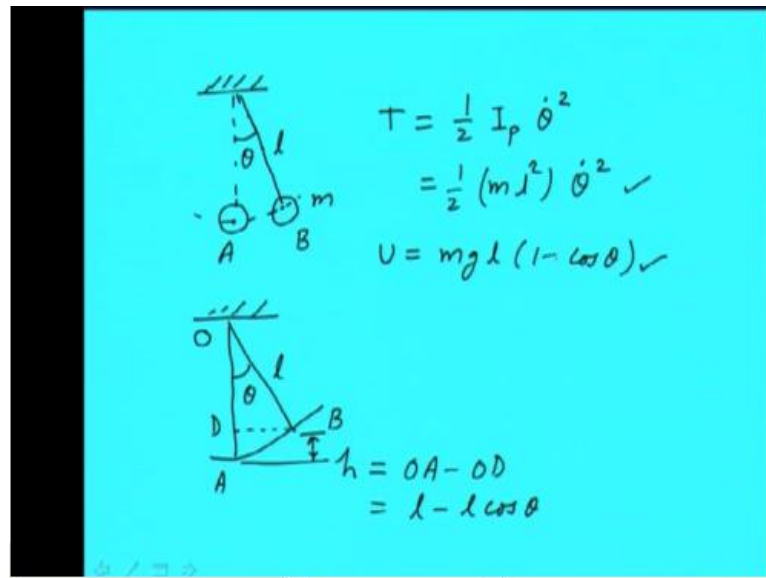
First we will take example of a simple pendulum which will give us more clear-cut idea about the method. Then we will apply this to slightly complicated systems. So, this pendulum is oscillating. And this is the position of the pendulum at time t, and angle is theta. Length of the chord is l, and mass of the mass is m. And from here we can see that kinetic energy can be written directly that will be half I p theta dot square, where I p is the mass moment of inertia of this particular ball about the centre of rotation. So, that is m l square theta dot square.

Now when this particular mass is going from let us say position A to position B, it is working against the gravity field. So, let us see how much potential energy it is gaining because of this. So, here I can draw a separate figure to elaborate this point more clearly. So, it has come from A to B, and it has gained this much height. This angle is theta, length of this is l. So, this height we can able to calculate. This will be if we project this here, let us say this is point o, and this is D.

So, h is nothing but OA minus OD, and OA is equal to the length of the chord, and OD is l cos theta. So, total height it has reached here is l minus l cos theta. So, the potential energy at the system will be mg l 1 minus cos theta. Now we got the kinetic energy term and the potential energy term.

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We can sum them up and if we differentiate, we will get, I am writing all the terms first. So, this is the kinetic energy, then potential energy, and this should be equal to 0. So, if you differentiate we will get m l square theta dot theta double dot plus m g l. This will be sin theta, because minus will get cancelled is equal to 0. So, if we take out the common term from here, we can write this is theta double dot plus g by l sin theta is equal to 0, because this mass term will cancel, length will go. So, we will be having this.

Now previously we obtained this expression earlier also that this is the non-linear function; for small angle of theta sin theta can be written as theta. So, this is the familiar equation as we obtained using the Newton method, Newton's second law. Let us take another example of energy method how we can able to obtain the equation of motion and the natural frequency of the system. This particular system, which we are now considering is slightly complicated; that is interconnected system is there, but still it is single degree of freedom system. Let us see how this system looks like.

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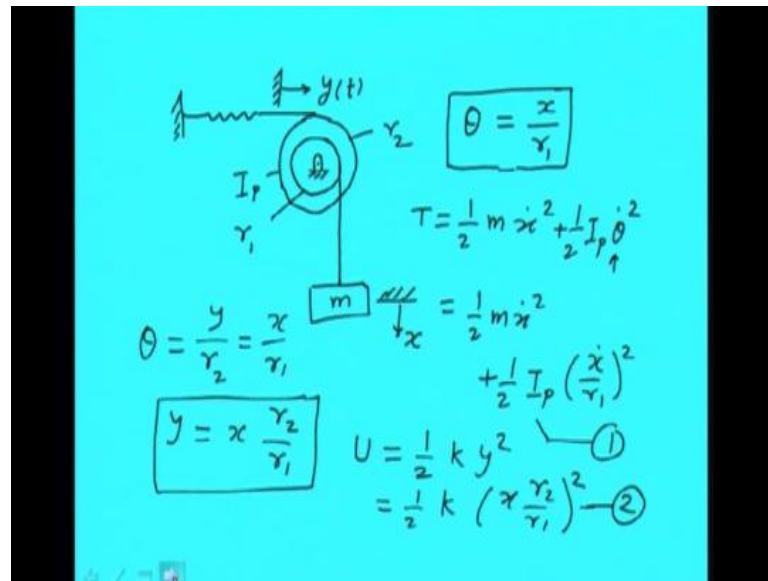
So, we have a spring here which is through chord that is wrapped around a cylinder, and this cylinder is pivoted here. And there is another circular cylinder is here which is attached to the first one there single member. And through this another chord is attached and then there is a mass. So, these particular cylinders are arranged. The first cylinder is having radius r 2 and the inner one is having r 1, and the polar mass moment of inertia of this is I p. So, if we want to disturb this particular mass form its static equilibrium position by x t and let us say the extension of the spring because of this disturbances y t. Then we have even if we see this particular disk or the cylinder will be having oscillation; so how they are related this oscillation and this displacement that we will see now.

So, as we know that if the fly wheel is getting rotated by theta angle; if this is the x displacement, and because of that this is got rotating by theta angle. So, we will be having theta is equal to x by radius of that particular circle. So, you can see that theta and x now we have relationship. They are not independent variable, this x and the rotation of the disk is not independent. They are related with this expression with the radius of this. So, if you want to write the kinetic energy of this whole system, we can able to write the kinetic energy of their mass as half m x dot x square plus kinetic energy of this particular cylinder which is having polar mass moment of inertia I p.

So, that will be given as half I p and theta dot square, because it is oscillating with theta and velocity is theta dot. So, we can see that both the linear and rotational kinetic energy is there in this. And now we are having a displacement or the extension of the spring because of this displacement. Displacement is y, but this also we can able to relate with the theta and subsequently with the x. So, how y and thetas are related? On the same means we can write theta is equal to y by r 2; that is the radius of the outer circle, because this chord which is coming from this spring is as attached to the outer circle. And as we have already seen this is equal to r 1 also.

So, we have relationship between the x and r 1. So, this is also related. So, basically this system is single degree of freedom system in which we have taken let us say x as the variable. So, here we can able to replace this quantity with x. So, kinetic energy will be half m x dot plus half I p; theta can be replaced like this. So, this is the kinetic energy of this system. Now we will see the potential energy of the system that is, because spring is getting extension of y. So, we can able to write this as this is the potential energy of the system, and y can be replaced in terms of x. So, this is the second equation. First one is the kinetic energy, second is the potential energy. Now we got both the energies; we can sum them up.

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And if we take differentiation of this we will get let us say first we are writing this which is constant. So, I am writing all the terms, which we obtained in the previous slide. This

is kinetic energy and another term of potential energy and this is constant. If we differentiate this we will get I p r 1 square x dot x double dot plus k r 2 by r 1 whole square x x dot is equal to zero. So, half terms are getting cancel during the differentiation. So, here we have differentiated with respect to time. So, we add these terms.

If we simplify this or we can take the common terms like x dot is common throughout. So, we can take out x double dots, we can take common of that also. So, m plus I p r 1 square x double dot plus k r 2 by r 1 whole square x is equal to 0. So, this is the equation of motion of this system. And if we write this in standard form we can able to easily see that what is the natural frequency of the system. So, this is nothing but square of the natural frequency of the system, because we have standard equation as this; this is the square of the natural frequency.

So, here you can see that basically in the denominator, we are getting the effective mass, and in the numerator this is the effective stiffness or square of those quantities, and this gives us the natural frequency of the system; this is the square of natural frequency.

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$$\omega_n = \sqrt{\frac{k\, r_2^2}{m\, r_1^2 + I_p}}$$

In simplified form this natural frequency will take this form k r 2 square by m r 1 square plus I p. So, this is the natural frequency of the previous system. From the previous method there is a very general method in which we have that summation of the cut total energy of the system, kinetic and potential energy is a constant. So, this is valid for all

the positions of the dynamic system. So, either it is we are talking about at equilibrium position or it is one of the extreme position; it is valid for all positions. So, these particular energies when we are talking about at the equilibrium position, as we know at equilibrium position most of the energy will be in the form of kinetic energy.

And when this particular system is going to one of the extreme positions, then all these kinetic energy will convert in to the potential energy and there the kinetic energy will be 0. And whereas at the equilibrium position, the whole energy is at the kinetic energy and potential energy is 0. So, let us see that how this can be used to obtain the equation of motion of system very quickly.

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$$T_1 + U_1 = T_2 + U_2 = \text{Constant}$$
1 $\rightarrow$ static equilibrium position
$$U_1 = 0 \qquad T_1 = T_{max}$$
2 $\rightarrow$ maxi. displacement of the mass
$$T_2 = 0 \qquad U_2 = U_{max}$$
$$\boxed{T_{max} = U_{max}}$$

The kinetic energy at one position plus kinetic energy at another potential energy at the same position is equal to their value at second position, and they remain constant. So, first position is the static equilibrium position, and if so when this is the condition then you will find the U one will be 0. And T 1 will be its maximum value. And if two is corresponding to the maximum displacement of the mass or the extreme position, there we will find that the kinetic energy will be 0. We will see this particular thing with the help of one example also how they become 0 and the potential energy will be its maximum.

So, basically through this we can able to say that if we equate the maximum kinetic energy of the system, and the maximum potential energy of the system or this will give

us the natural frequency of the system directly. So, this particular method we will see through one small example of spring mass system in which we will calculate the maximum kinetic energy of the system and maximum potential energy of the system. And we will equate that to get the natural frequency of the system.

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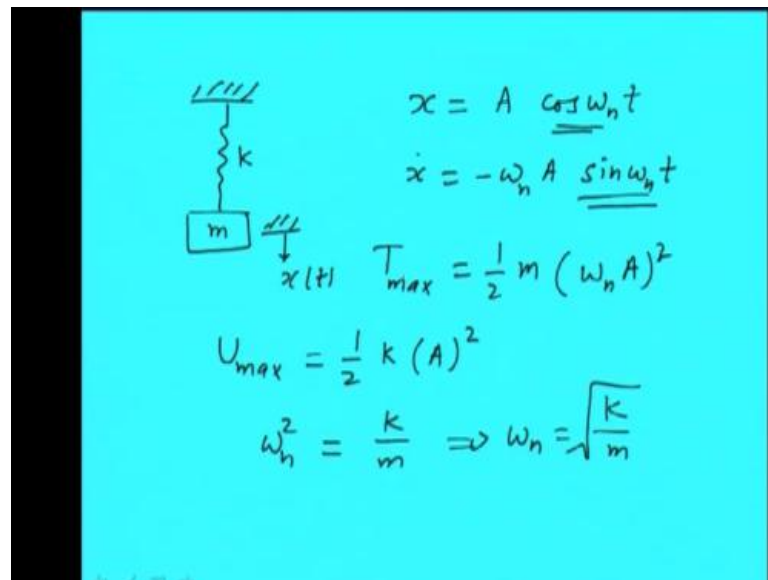
So, in this case when we are considering the spring mass system we are always taking this small spring mass system as very representative problem; it is very easy to understand the concept of this. So, all the basic concepts we are trying to understand using this simple problem. So, in this particular case we know that the displacement is given as amplitude and the harmonic function. So, velocity can be obtained by differentiating this quantity, which is given as this. So, maximum kinetic energy of the system can be obtained as the mass into maximum velocity. Maximum velocity will be when this quantity is one, unity.

So, we can able to write this as this. So, this is the maximum kinetic energy of the system. And this particular kinetic energy will be at its static equilibrium position, and maximum potential energy will be at its extreme position when the spring is having maximum extension; maximum extension maximum displacement will be when this quantity is unity; that means capital A square. So, we got the maximum potential energy and maximum kinetic energy; we can equate them. And if we equate them we will get

omega n square is equal to k by m which is this very familiar equation, because this is the natural frequency of the system.

So, you can see that by equating this two quantity which are maximum, we can get the natural frequency directly. For energy method we will take another interesting example in which one particular roller which is attached through a spring is trying to roll over its surface without any slip, how we can able to get the natural frequency of such system.

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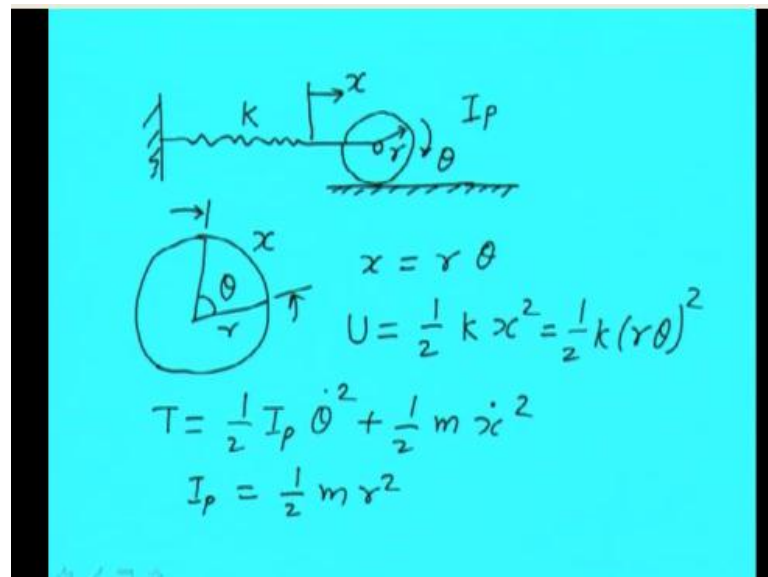
So, in this particular case we have one spring which is attached to a cylinder. So, there is no friction between the springs which is attached to the centre of the roller. And this roller is this particular cylinder is rolling over a surface without any slip. So, surface in friction is there between the roller and the surface ground. So, that it is not skidding, and pure rolling is taking place between them. And in this particular case the stiffness of this spring is k. This roller is having polar mass moment of inertia I p and the radius of this roller is small r.

Now we are giving a rotation to this particular roller, so that the springs are getting stretched. Let us say the spring is getting x extension because of this theta. Now because these two quantities we can able to relate, how they are related? If we have this circle because this is the radius; this is the theta angle. So, when we are giving a theta angle rotation to this cylinder the circumference of this particular cylinder will be having displacement of x. So, x is a related with r as this.

Now we can write the potential energy of this system for a given displacement that is given as k x square; that is the extension of that spring which can be written as k r theta square. Now the kinetic energy of this system can be written; that is kinetic energy will be in two forms. One is because this cylinder is rotating. So, it will be having the kinetic energy because of its rotation, also because it is going toward this direction in the x direction. So, it will be having linear motion, and because of that also it will be having kinetic energy.

So, let us say first due to the rotation this is the kinetic energy if rotation and plus due to the linear motion, this. So, this is the total kinetic energy, and for cylinder that I p is given as half m r square. So, you will be using this, also will be replacing X with r theta. So, that all the quantity of the energy will be expressing in terms of theta.

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$$U + T = const.$$

$$\frac{1}{2}k r^2 \theta^2 + \frac{1}{4} m r^2 \dot{\theta}^2 + \frac{1}{2} m r^2 \dot{\theta}^2 = const.$$

$$k r^2 \theta \dot{\theta} + \frac{1}{2} m r^2 \dot{\theta} \ddot{\theta} + m r^2 \dot{\theta} \ddot{\theta} = 0$$

$$\frac{3}{2} m r^2 \ddot{\theta} + k r^2 \theta = 0$$

$$\ddot{\theta} + \frac{2k}{3m} \theta = 0 \qquad \omega_n = \sqrt{\frac{2k}{3m}}$$

So, if you do that the total potential energy and kinetic energy which is constant can be written as half k r square theta square plus 1 by 4 m r square theta dot square plus due to the linear term kinetic energy linear motion, this kinetic energy is coming as a constant. If you differentiate this that quantity will be 0. So, let us differentiate this. So, this will be theta. Then theta dot plus half m r square theta dot. Then theta double dot plus m r square theta dot theta double dot is equal to 0.

So, you can see this quantity is common; we can eliminate that. Also these two quantities can be coupled. So, this will give us 3 by 2 m r square theta double dot plus k r square

theta is equal to 0. So, from here we can put this in more standard form by dividing this quantity throughout. This r square is also common. So, that will also get cancelled; theta is equal to zero. So, you can see that the natural frequency of the system will be given as root of 2 k by 3 m. So, this particular example to clarify the concept more clearly let us see how we can able to solve this natural frequency of the system using the Newton's second law. So, it will give us some more confidence in clearing the concept.

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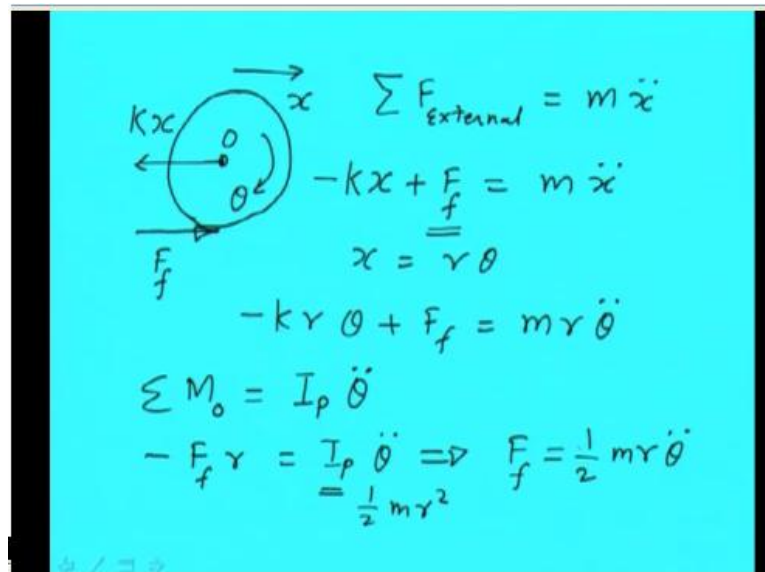
So, for this particular case when we are interested in solving this using Newton's second law; so let us take the free body diagram of the cylinder. So, when you are given a displacement X, the spring will exert a force in this direction at the roller center and the ground, because this pure rolling is taking place. So, sufficient friction is there between the roller and the ground. So, there has to be some friction force which is allowing the motion to take place, the pure rolling motion. So, this is the friction force, and we know that this particular cylinder is having theta rotation, but linear displacement of this is taking place in this direction is x.

So, we can apply the Newton's second law, and we can sum all the external force, and we can equate that to mass into acceleration of the cylinder. So, what are the external forces? The spring force and the friction force, and you can see that displacement direction x in this direction is there toward the right side, but the spring force is in the negative direction. So, this will be negative quantity. Friction force is acting in the same

direction as x, and this should be equal to the inertia of the system. So, we can even replace x with r theta as we did earlier, because x is equal to r theta.

So, this we can able to use it and here also we can able to use r theta. So, this will be r theta double dot. Now we will see the rotational. Now we will consider the moment about the centre of the cylinder, because it is rotating with theta. So, it will be having rotary inertia also. So, we can take the summation of all the moments about centre of the roller equal to that will be equal to polar mass moment of inertia of the cylinder into the angular acceleration.

So, we can see that when we are taking moment about the centre, this spring force will not give any moment. But friction force will give moment, and that is the only external moment, and that acts opposite to the theta direction. So, this moment which is coming from the friction will be negative, and that is the only external moment acting on the cylinder. So, you can see that the friction force we have got from this which is even we can able to write I p as half m r square. So, we will get half m r theta double dot as the friction force, and this friction force we can able to substitute here. So, let us write this equation after substituting this friction force.

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$$-k\,r\,\theta - \frac{1}{2}m\,r\,\ddot{\theta} = m\,r\,\ddot{\theta}$$

$$\frac{3}{2}m\,\ddot{\theta} + k\,\theta = 0$$

$$\ddot{\theta} + \left(\frac{2k}{3m}\right)\theta = 0$$

$$\omega_n^2$$

So, this will take the form, and this is the friction force is equal to m r theta double dot. So, this two quantities can be coupled. So, that this two quantity will give us this 3 by 2 m into theta double dot plus k r theta is equal to 0. And this can be written as k r; r is not

here, because it is common throughout. It will get cancelled; r is not there. So, 2 by 3 m theta is equal to 0. And this is the square of the natural frequency of the system. So, now we can see how these methods the Newton's second law of motion or the energy method can be used effectively to obtain the natural frequency of the system.

Now we will be complementing this energy method by another important method that is called virtual work method. This particular method has been developed by Bernoulli, and this is very useful method especially for large system where system is interconnected, and because at present we are concentrating on the very simple systems single degree of freedom system. So, we will try to elaborate this particular method of a virtual work through simple examples. So, that the method is clear, and in subsequent lectures we will be elaborating more detail, because this is very powerful method, and we will be explaining more and more using this method.

Because the most advantage of the energy method is that these are the scalar quantities. So, there is no need to bother about the science direction of the force generally which is required in the Newton's second law method. So, this particular virtual work principle is based on is this particular virtual work method can be stated as if we have a dynamic system or static system which is in equilibrium. Then if we give some virtual displacement to this particular system keeping the constraints of the system same, then what were the virtual works done during this process is 0.

So, this is the basic concept or the concept of the virtual work that a system which is in equilibrium which is subjected to various forces and constraint. And if we give some virtual displacement to this particular system, keeping in mind that all the constraints are not violating whatever the support conditions are there of the system, they are not violating. Then the work done due to these virtual small displacements which are very small infinite decimal displacements, because of this total work done will be 0. This virtual work is basically we calculate based on all the active forces which are acting on the system. We do not consider the reaction for the support forces when we will be calculating the virtual work.

So, we will only be considering the active forces which are acting on the system for calculating the virtual work; during the process of calculation of the virtual work because whatever the virtual displacement we give to the system that is very small. So, because

of that there is not much change in the geometry of the dynamic system take place. So, whatever the active forces are acting under the system, we assume that when we give this virtual displacement to the system, they remain same. They remain constant; they do not change during the displacement.

The principle of virtual work was originally formatted by Bernoulli for the static system. And then later on by d'Alembert's he extended the same principle for the dynamic system also by introducing the concept of the inertia force. This inertia forces d'Alembert's he considered as an active force apart from the other external force. And during the calculation of the virtual work, these forces also contribute to the work done. So, based on the virtual work principle let us take simple examples how we can able to get the system equation and the natural frequency of the system. So, again we will come back to the spring mass system first.

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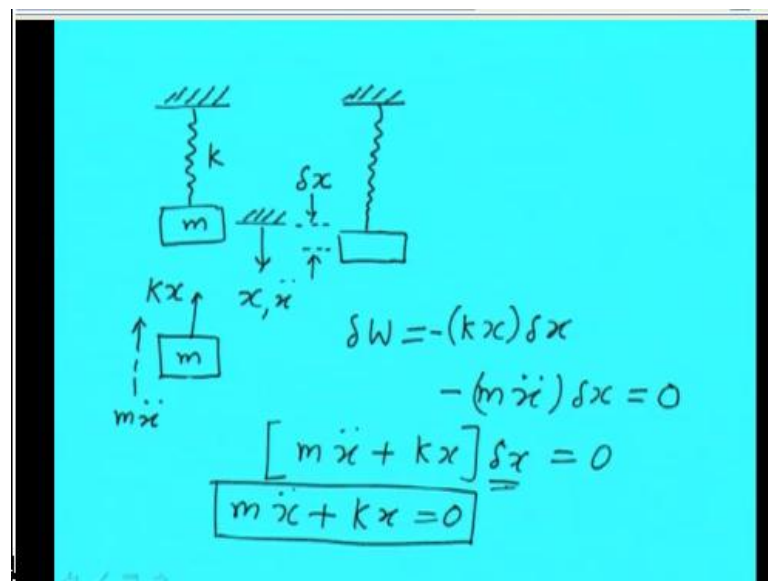
So, in this particular case let us say this is a spring in which at the end mass is attached. Spring is having stiffness k and, from its static equilibrium position we are giving a displacement x. And we know that from this static equilibrium we have given x. Now we are giving a virtual displacement to the mass after this, and that virtual displacement is very small, and we are calling as delta x. And if we see the forces acting on this particular mass when it is in x displacement, if we declare that or let us consider the

gravity we are neglecting, because now we are considering the equilibrium position as the static equilibrium position.

So, spring force will act upward, and acceleration is downward. So, inertia force will be acting like this. So, these are the two active forces acting on to the mass. And the virtual work done from this can be calculated as k x into delta x, and because it is doing work in the opposite direction of the motion it will be negative. So, this force is getting displaced by delta x. So, this is the work done by the spring force plus the inertia force. So, this is also getting displaced by the same amount delta x, and according to the virtual energy principle this virtual work done is zero.

So, we can see that we can simplify this term like this, or this is x. This virtual displacement which we have given to the system there is an arbitrary; that is we can give any value to that. So, obviously, this cannot be 0. So, we need to equate the terms within the bracket zero, and that gives us the equation of motion of the system. Let us take an example of the simple pendulum how we can able to get the equation of motion of a simple pendulum using the virtual work method.

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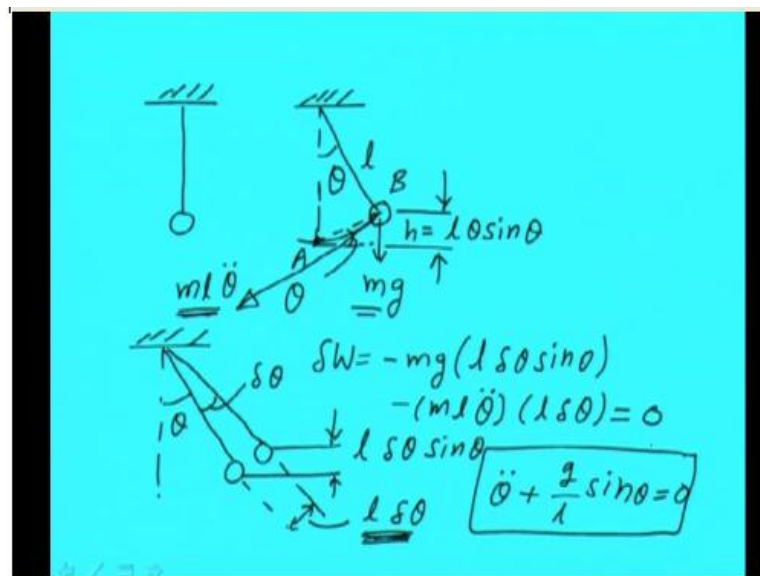
So, in this we have one pendulum. This is in the static equilibrium position, and when we are displacing this, it occupies this position. At this position it has displacement theta; this is the l length of the chord. Weight of this ball is acting downward, and the total lift of the pendulum from its equilibrium position can be obtained; that will be because this

is length of the chord, and this is theta. So, from here to here this chord length will be l theta, and we can able to show that this particular length will be l theta into sin theta. Because if we draw this line, this angle is theta, and let us say this is A, this is B. So, AB chord length is l theta.

So, this particular height h will be AB into l sin theta l. So, h is l theta sin theta. Now we are giving a virtual displacement to this particular pendulum. So, this particular pendulum we are giving virtual displacement. Now we are giving a virtual displacement to this pendulum, and I am drawing that diagram here. This is the theta angle, and further we are giving a virtual displacement of delta theta. This particular distance would be l into delta theta because this is the length and this is the angle, and this vertical moment of the pendulum on the same line as the previous one can be written as l delta theta sin theta.

Now we know various positions of the ball with the help of these dimensions. Now we can write the virtual work. So, virtual work will be that is first term is m g; mg is getting displacement of l delta theta sin theta. So, from this position it is getting lifted to this particular position. So, this is the virtual work done in that particular mass. Another term is because this particular ball is having a rotational motion. So, we have another component of that force that is tangential to the path here; that is tangent to the path at B, and that is m l theta double dot. So, it is the tangential direction force. And this we know how much displacement we are giving when we are giving the virtual displacement.

So, this will contribute because here also there this force is opposite to the motion of l delta theta. So, this quantity is the virtual work done against this force. There are two forces; one is gravity and another is this inertia force. So, this is the total virtual work done. Now according to this principle this should be 0, and if you simplify this we can see that this gives theta double dot plus there is g by l sin theta which is familiar equation. For us to elaborate more the virtual work done method let us take another example in which there is interconnected system, but the degree of freedom remains the same as one.
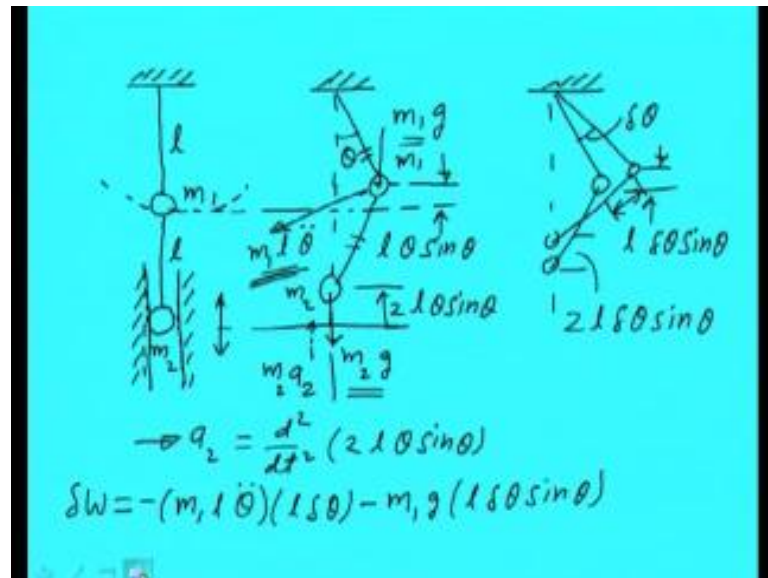
So, let us see how this particular system can be solved, how this particular system can be analyzed. So, this is a pendulum, the length of the chord and the mass of this pendulum is m 1. There is another pendulum, which is attached below the first pendulum mass, and mass of this pendulum is m 2. The only constraint, which we are giving to the second mass that it is constrained in this small cavity and this particular ball can have up and down motion only. And we are assuming that there is no friction between the surface and the ball.

So, it is smoothly it is going up and down on this particular slot, but this mass can have this oscillation; length of the this chord is also l, both chord lengths are l, masses are m 1 and m 2. So, we need to obtain the equation of motion of this system also the natural frequency. So, let us give a displacement theta to the upper pendulum, so that it occupies this position, and this is the lower pendulum position. Now you can see that the lower position lower pendulum has moved here, because this particular mass m 2 can go on this line only, but this particular mass m 1 is free to revolve in a circular path.

So, as compared to the previous position, the position of this ball can be obtained; that is as we did in the previous case that will be l theta sin theta. Similarly, this will be because this length and this length of the chords are same. So, the moment of the lower ball will be double the upper one. It will be exactly doubled. So, now we will be applying various

forces which are acting on these mass. So, we know that here mass m 2 g is acting, and here we have m 1 g in downward direction.

Then in the tangential direction to this path of the m 1, we have m 1 l theta double dot. And this particular mass will be having linear acceleration; that is let us say m 2 a 2, a 2 is the linear acceleration of the mass. This a 2 can be obtained by because this particular ball is having this much displacement. So, double derivative of that will give us the acceleration of that. And actually this particular acceleration is having very insignificant contribution towards the virtual work done. So, we will be neglecting this particular inertia force. So, we will only be considering the gravity forces and inertia force due to this term for calculation of the virtual work done.

So, if we give virtual displacement to these masses. So, these are the virtual displacements. So, you can see that this particular mass is getting lifted by l delta theta sin theta as we did earlier, and this will be twice of l delta theta sin theta. Now we can able to write the virtual work because of this quantity. So, I am writing here. Virtual work done from this first term there is minus m 1 l theta double dot and the l del theta this much is this particular distance is l delta theta. Then m 1 g upper ball is having this much virtual displacement, and we will go to the next slide.

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$$-m_2 g \left(2\, l\, \delta\theta \sin\theta \right) = 0$$

$$\delta\theta \left[\frac{m_1\, l\, \ddot{\theta} + (m_1 + 2 m_2)\, g \sin\theta}{} \right] = 0$$

$$\ddot{\theta} + \left(1 + \frac{2 m_2}{m_1} \right) \frac{g}{l}\, \theta = 0$$

$$\omega_n = \sqrt{\left(1 + \frac{2 m_2}{m_1} \right) \frac{g}{l}}$$

The second mass is having this much virtual work done, and this should be 0. Then you will see that if we simplify this, we will get this quantity. And it will be multiplied by the

delta theta, because this virtual work done this is a virtual displacement. So, obviously, the terms containing in this particular bracket will be equated to 0. And that will give us theta double dot 1 plus 2 m 2 m 1 g by l; theta is equal to 0. So, from here because this is standard equation of motion of single degree of freedom; so natural frequency will be given as 1 plus 2 m 2 m 1 g by l.

Today we have seen the energy method, which is a scalar quantity, how it can be used for finding the equation of motion and natural frequency of its simple systems. These methods are quite powerful as well progressed in these particular lectures. We will see that as the systems becomes very complicated, then the method of the Newton's second law of motion, it become very cumbersome. And these energy methods because they are scalar quantity, there very much helpful for the complex systems; apart from the energy method we have seen the virtual work principle how it can used for finding the natural frequency as well as the equation of motion of the system.

This particular method also we will be extending for large system in subsequent lectures, or in the next lecture, we will be covering the free vibration of the damped system especially of the single degree of freedom system. We will see various property of the damping how it is affects the response of a dynamic system.