Mechanical Vibrations
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Module - 12
Signature Analysis and Preventive Maintenance
Lecture - 2
Vibration Testing Equipment: Signal Analysis

So, in the previous lecture we have already studied different kind of measurements which will be requiring, what type of display we will be doing with those measurements. Specially, we need to capture the data and time domain and then we have to display them in various forms, in specially in the frequency domain form. And right from capturing the data through sensors there are errors involved in digitizing the data.

And once we digitize that data then for once we do further processing of that then also errors comes in the data. And those errors some of them I have already pointed out in the previous lecture. Now, even the some of the processing techniques like Fourier series and Fourier transform we have studied in the previous lecture. Now, the same concept we will be extending for discrete data when we have a measure data which is neither a periodic in nature or nor that is a infinite in length.

Then, because we cannot apply either Fourier series or Fourier transform to these data. So, what we do we captured a vibration signal of sufficient length and we assume that that particular signal repeats after that time interval. So; that means, that particular data we have captured we assume that is a complete period of that vibration which is taking place in in to the system. And we obtain the Fourier series of that particular periodic virtual periodic signal which is which we obtain. And then what are changes that we need to do in the Fourier series now we will see.

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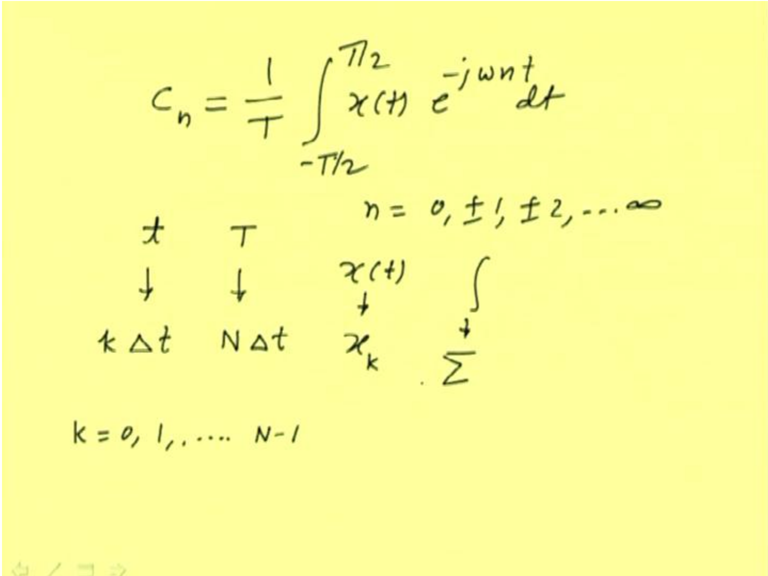

So, in the previous case we had the Fourier series of this form and this is the signal xt that was continuous and n is up to infinity. Now, to for this particular signal in which we have finite length we need to replace some of the terms like time we have to replace to a term like k delta t. Where k is number which varies from 0 1 up to N minus 1 where, N is the total number of data digitize that we have.

And time period is given as capital N into delta t where N is the total number of data. delta t is the time sampling interval and text t which is there in the continuous form that we need to replace with a x subscript k where, k again is given from 0 to N minus one. And the integration we have to replace by a summation.

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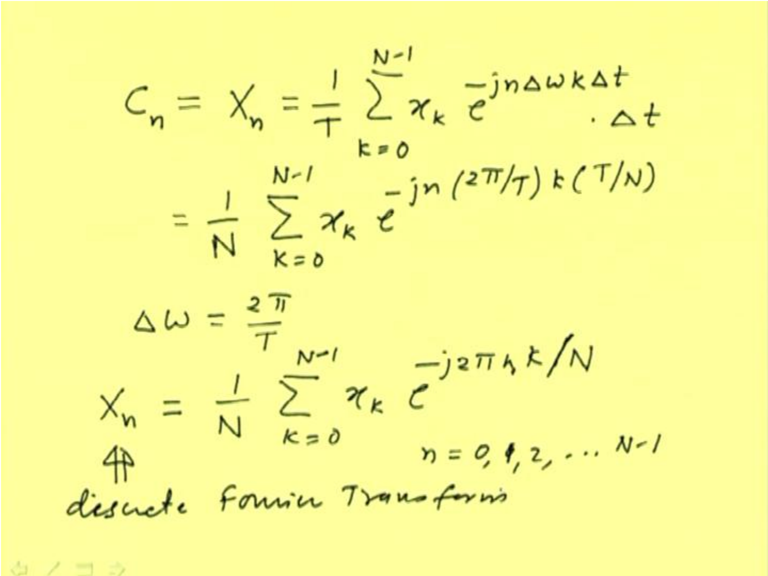

So, if we use this transformations the Fourier coefficient now which we will be calling it has Xn to distinguish it from the Fourier series as: 1 by T summation k 0 to N minus 1 xk then exponential term. So, I am replacing all the terms which can be simplified as 1 by N. Because T can be written has capital N into delta t summation k is equal to 0 N minus 1 xk exponential raise to these terms where again I am replacing some of the terms.

Like omega; delta omega is nothing, but 2 pi T that I have replaced. So, finally, we will be getting Xn as N 1 by N summation k is equal to 0 N minus 1 xk 2 pi n k by capital N. Where, small n is varying from 1 2 up to capital N minus one. So, this particular term is called Discrete Fourier transform.

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$$\underline{x_k} = \sum_{\underline{n=0}}^{N-1} X_n \, e^{j2\pi nk/N}$$

$$k = 0, 1, 2 \ldots (N-1)$$

Inverse DFT (IDFT)

$$A_n = \frac{1}{N} \sum_{k=0}^{N-1} x_k \cos \frac{2\pi nk}{N} \qquad n = 0, 1, \ldots N-1)$$

$$B_n = \frac{1}{N} \sum_{k=0}^{N-1} x_k \sin \frac{2\pi nk}{N}$$

And on the same way we can able to define the inverse discrete Fourier transform which will be given as where k is varying from 0 1 up to N minus one. So, it is the inverse discrete Fourier transform or IDFT. So, this particular transformation converts the time domain data which are infinite in number to this frequency domain data which are again infinite in number, but they are in complex they are complex in characteristics. And this by the inverse DFT we get back the time domain data from the frequency in domain data.

These transformations as you can see they are complex transformations we have real transformations also. And the relation especially the coefficient of those real Fourier discrete Fourier transform are given like this. This is the An and another term is Bn similar to the Fourier series, in which sin term is there where these N are varying from 1 0 1 to up to N minus 1.

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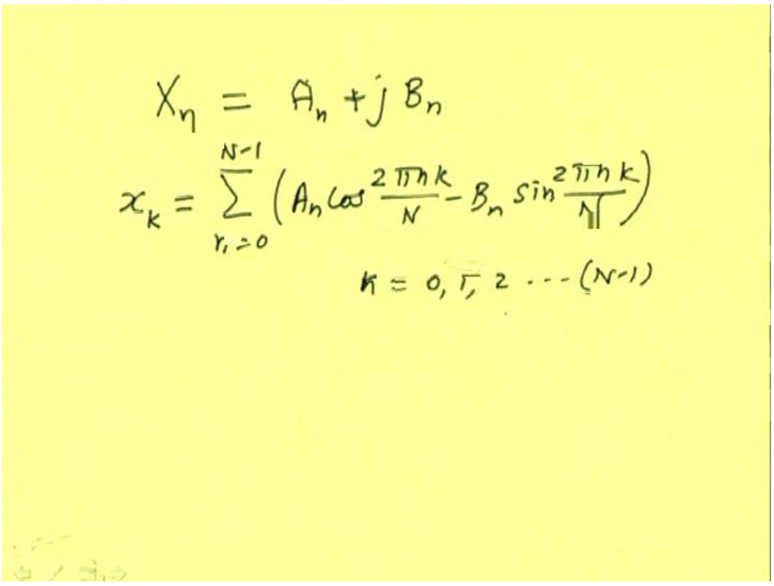

And the relationship between the complex Fourier discrete Fourier transform and the real Fourier transforms are like this. And the inverse real discrete Fourier transform can be written on the same lines in terms of the real coefficients like this. Here again the k is varying from 0 1 to up to N minus one. So, same number of data will be there in the frequency domain and the time domain. But, in frequency domain the data will be the data will be complex in nature.

Now we will take up 1 example in which we will be using a a pulse a square a square wave. And we will discretize that wave and we will do the TFT of that. So, how the spectrum looks after discretizing we will see.

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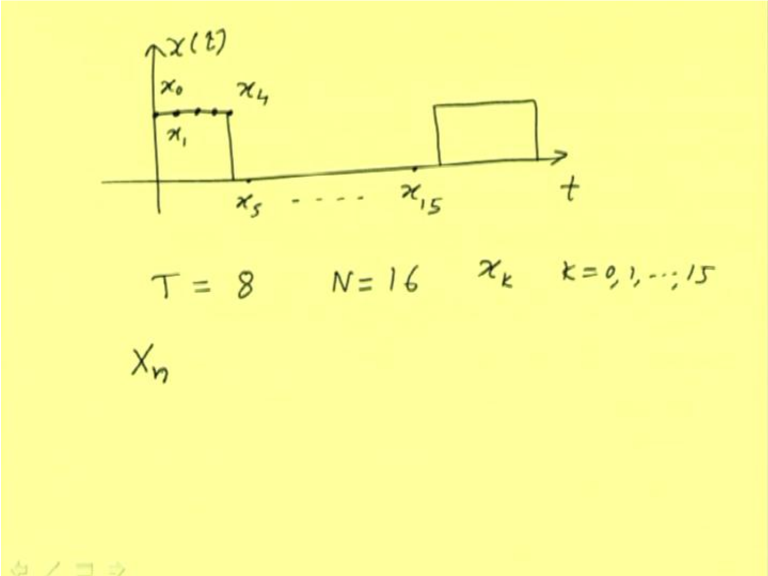

So, the square wave is something like this; this is the time and this is the x t. So, square data is, so total period of the this particular signal is 8 units and we are discretizing in into 16 number of data N is sixteen. So, I will start with this x naught, x 1, x 2, x 3 this corner is x 4. Likewise, we will be doing this will be x 5 go up to this, this is x fifteen. After that signal repeats because corresponding to x 16 x naught is there, so signal repeats after that. So, we have total xk where is k is from 0 to 15; total 16 data are there

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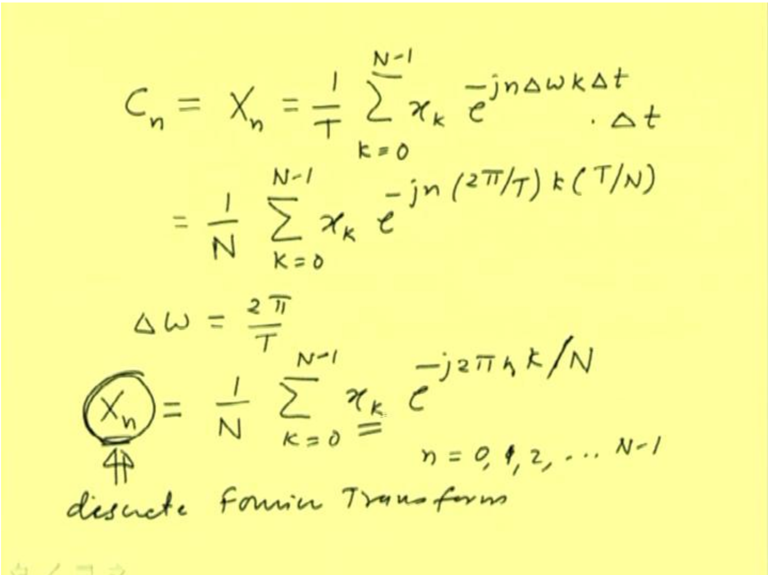

So, using the previous relations or these simple relations we can get the discrete Fourier transform directly after substituting various values because all values we know here. And as I told this will be complex Xn is complex. So, let us plot the real part of the discrete Fourier transform with respect to frequency also the imaginary part.

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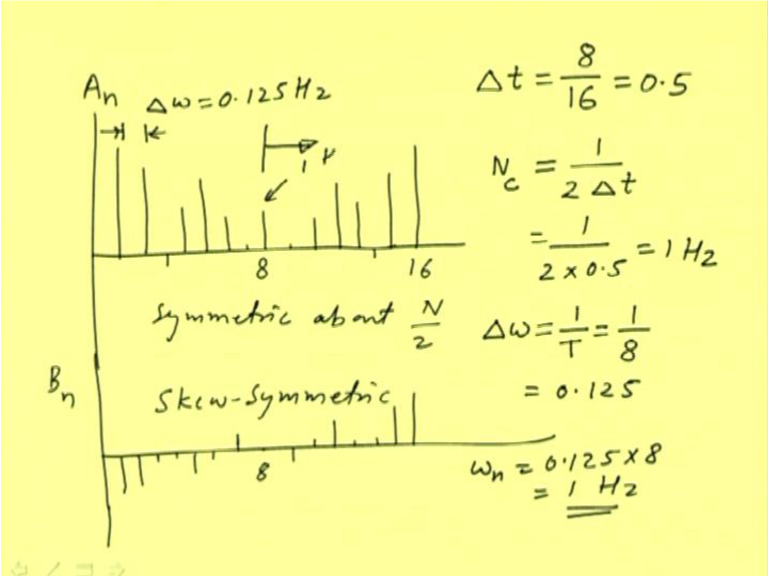

So, it is the let us say real part An of the signal. So, this will look like something like this, these are the amplitude of various An when we change the n of A. So, this is 1 2 3 4 5 6 8 here 8 is there. Then after this it is symmetric. So, whatever the signal about 8 it will symmetric like this, this is symmetric about N by 2 because total 16 data is there. So, at 8 it will be symmetric about 8 axis vertical axis. Similarly, if we want to draw the Bn we will be having this as something like this 4 5 6 7 eight.

So, about 8 this becomes skew symmetric so; that means, this positive this side. So, this side it becomes negative. And when this is negative this becomes positive about eight. So, just this will be skew symmetric this, so is the spectrum of the signal which we digitize it. So, you can see that. So, many spectrum are visible.

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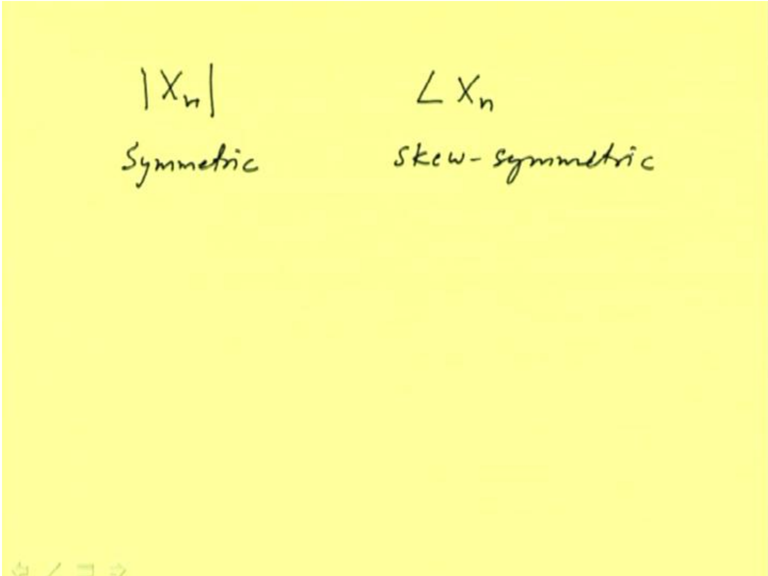

On the same lines we can able to plot the complex discrete Fourier transform as an magnitude of that or the phase of that. This also will be symmetric about the midpoint and this will be skew symmetric about the midpoint. Now, in the previous plot whatever the signal is there after it actually all these spectrum are virtual. Because, whatever the frequencies are there above it they are very fast frequencies, we which we cannot able to measure it the due to the aliasing effect.

Whatever the spectrum I have shown half of that is virtual let us see, how they are virtual. Actually, if we recall the time interval will be given as because 8 unit is the total duration of the time period. And total number of data we have sampled is sixteen. So, time interval is point five; that means, between 2 data points the time interval is point five.

And the frequency which we can able to measure accurately is given has the Nyquist frequency which is given has 1 by 2 delta t. So, delta t is point five. So, this will become 1 hertz. So, maximum we can able to measure accurately the frequency up to 1 hertz. Now, in the frequency spectra the interval between 2 spectra is given has 1 by T that is, there is a fundamental frequency that is 1 by 8; 8 is the unit of the time period.

So, it becomes 0.125, so that means, between any 2 spectra the frequency is equivalent to 1 2 5 hertz. And up to 8 what is the frequency up to 8 we will be having 0.125 into 8 is equal to 1 hertz. That means at 8 the frequency is corresponding to total 1 hertz after that it becomes more. And according to the Nyquist frequency calculation we have to 1 hertz as the limitation. So, beyond the 1 hertz we cannot able to measure for this particular sampling period. Now, let us refine the sampling period instead of sampling 16 data parts, let us sample 32 data points.

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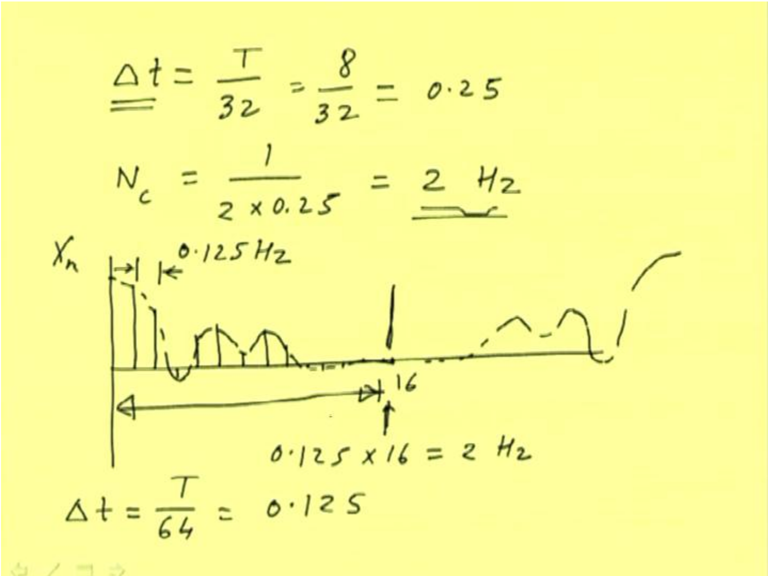

So, when we are sampling with 32 data points this sampling interval will be by 32; that means, 8 by 32 that will 0.25. So, instead of 0.5 now it has become 0.25. So, the Nyquist frequency will be 1 by 2.25, so this will become 2 hertz. So, you can see that when we refined the sampling interval the the frequency which we can able to measure accurately increases. And for this particular case when we have 32 number of data points the spectrum will look like this.

This will come from the formulae directly when we substitute the values of various n in that. Here, it will very small up to 16 and then again best will be if you can have the envelope of this. And after 16 it will be symmetric like this. So, here you can see that this particular is 0.125 hertz this remains same. So, up to 16 points point 1 to 5 into 16 that becomes 2 hertz. So, now up to 16 data we are having a good signal, but after that these are all virtual signal which is not to be analyze anything.

So, our useful range is this only, this where no aliasing effect is taking place when we are processing the data. If we increase further the sampling rate; that means, if we let us say 64 points we are taking. So, over sampling it will be refined further. So, we will be getting more and more data points or an more number of frequencies which we can able to capture accurately And if we compare this particular plot with the previous plot which we obtained for a square signal or a pulse with infinite number of period.

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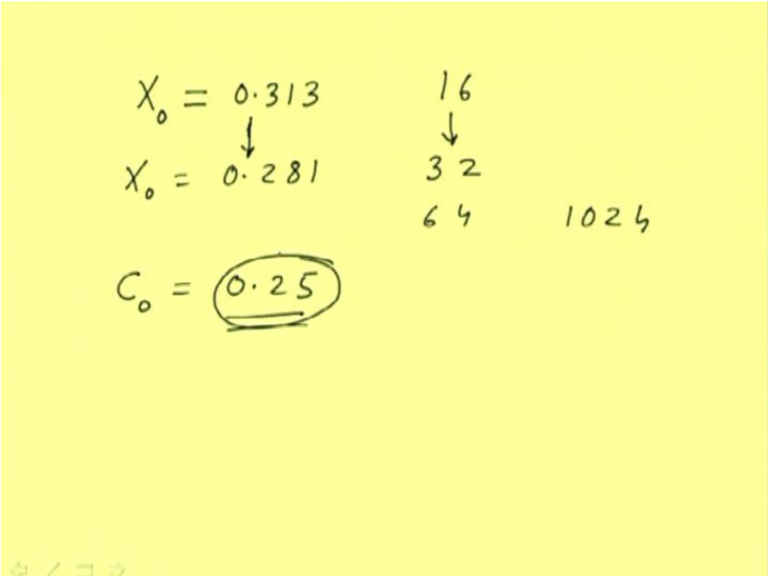

If we compare that we will find that currently that X is n is equal to 0 the amplitude is point 1 3 or with 16 number of data. If we increase the number of data it comes to 0.281 with 32 number of data. And using the Fourier series we got the value of that amplitude has 0.25. So, you can see that when we are increasing the number of data these values are getting closer to 0.25.

So, if we take 64 or higher number of data 1024 then we may reach up to this at that particular peak. And again if we recall this particular spectrum is similar as we obtain in the Fourier series case only thing here, we have shown half of that. Their left half left hand side half we have not shown here, but is symmetric about the vertical axis. So, this curve is symmetrical to the Fourier series transform of a pulse wave which we have obtained. But, this is for the discritized data and the Fourier series was for the continuous wave.

So, there is a direct correlation between them as we increase the number of sampling data, we can get exactly same coefficient as of the Fourier series. In the present case shown 1 type of discrete Fourier transform, but various forms of the DFT that is discrete Fourier transform are available .2 another kind of discrete Fourier form now, I would like to show. Only question is that you must know which form of the discrete Fourier transform we are using it. And according you should you should interpret the spectrum.

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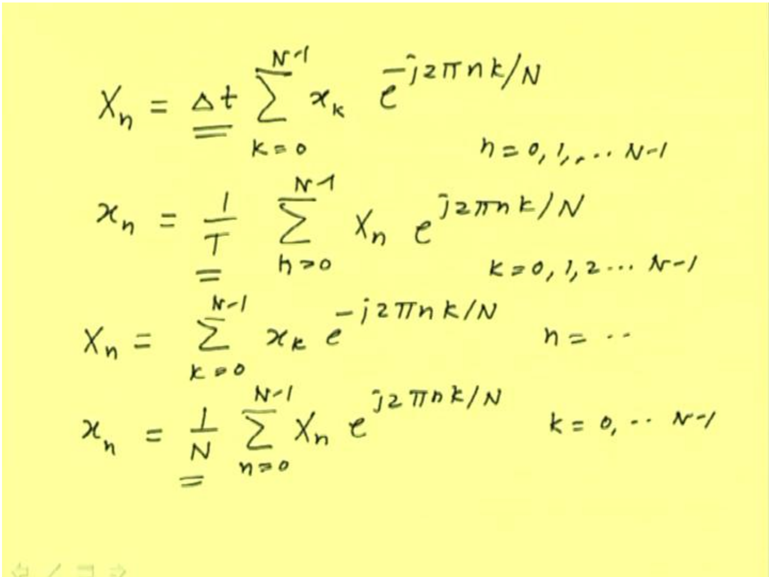

So, another form of the spectrum that is discrete Fourier transform is given as where n is varying from 0 to N minus 1. And this is the inverse discrete Fourier transform where, k is varying from 0 up to N minus one. So, here the difference is here another form of the discrete Fourier transform which MATLAB generally uses is this one. And the inverse discrete form transform is here also there coefficients are getting change otherwise the expressions remain the same. Here, n is varying as usual from 0 to N minus 1; k is also varying from 0 to N minus one.

So, now you have seen the discrete Fourier transform the 1 limitation of the discrete Fourier transform is that it requires extensive computation. So, to that that is a limitation by for which we cannot able to get the real time spectrum of the data which we are measuring it. So, generally we have to offline because it takes lot of time to compute, but in 1964 a Cooley Tukey they developed an algorithm that is which was faster as compared to the discrete Fourier transform. So, that is why that algorithm is called Fast Fourier transform what they do in that the data in that that they found that because of the cyclic nature of the discrete Fourier transform.

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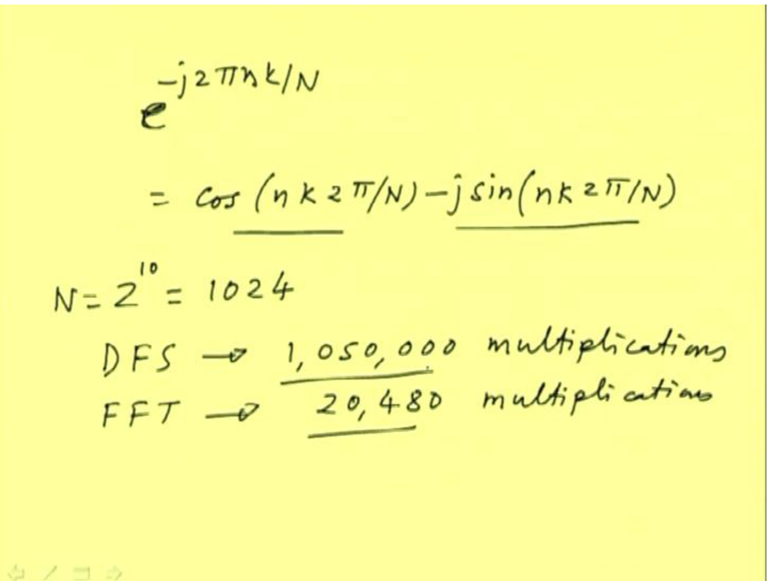

I just want to show some terms these kind of exponential terms are there in the discrete Fourier transform. And that can be expanded as and the sin term of that. So, during calculation of the discrete Fourier transform these kinds of terms which is cyclic in nature are they repeat repetitive multiplication of the similar terms take place.

So, Tukey Cooley they found that repetitive multiplications. And they avoided those calculation in there algorithm to calculate the Fourier Transform and, because of that they could able to get very high speed of calculation of the Fourier Transform just to give a some idea I will give an example that when they used around number of data 2 raise to 10. That is the 1024 data when you they used the ordinary Discrete Fourier Transform takes around 1 million 50,000 multiplication it requires that many multiplications, but using Fast Fourier Transform they required only 20,480 multiplications.

So, we can see that how much difference is there in the number of multiplications and computer takes that much time in calculation of this. And if we increase these data further the difference between these two increases drastically between the DFT. DFT I think this is a DFT and FFT. So, the difference in the calculation time is changes drastically and because of this fastness of this algorithm. Now, it is practical to get the frequency domain data of the real signal directly as and when you are capturing you can able to online in the frequency domain.

So, after saying the FFT the concept behind the FFT let us see more closely the error involved in the processing of the signal like leakage error of as we have already discuss that, when we are doing the Discrete Fourier Transform of a signal measured signal we assume that, that particular signal which we are captured it repeats after that interval. So, that is the assumption and based on that we get some errors.

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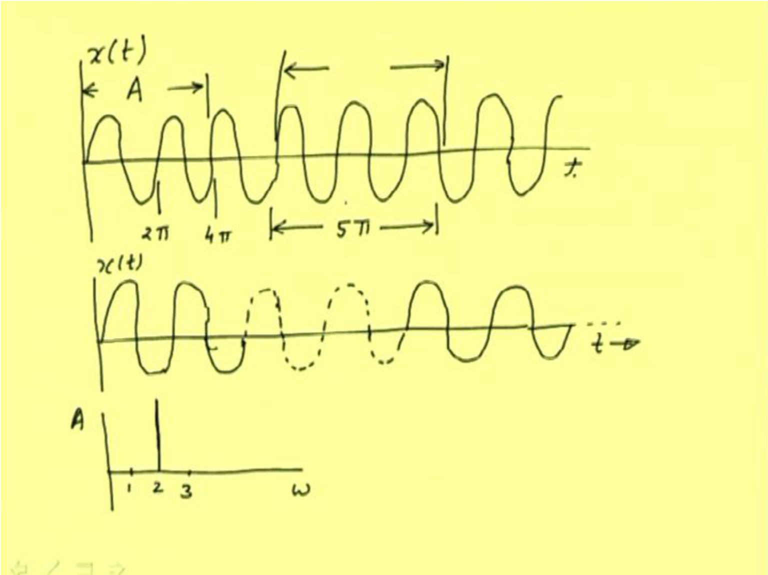

So, let us see what is the leakage error more closely on the slide here we have 1 sinusoidal signal like this and we are captured 1 particular signal. let say this is 2 pi it is 4 pi. So, we have a captured a signal which is this is signal A which is exactly coinciding with the 4 pi period that is you should say this is time and this is the amplitude, so this particular signal if you want to do the DFT of this we I am drawing here that captured signal.

Now, for doing DFT of this particular signal we have to cut and past the same signal subsequently like this again the same signal we are substituting. So, it is going on continuously that is the assumption behind doing FFT at that signal. If you see closely this particular signal which we have made after cutting and pasting it looks like similar to the pervious signal there is as such difference between them. So, if we want to do the DFT of this we will get a, this is the frequency this the amplitude.

So, a single peak will get because there is a sinusoidal curve. So, frequency domain we will get a single peak Let us say in particular original signal now, we are capturing another signal which is up to here that means you can see that from here to here now total duration is 5 pi and if you have to do the DFT of this.

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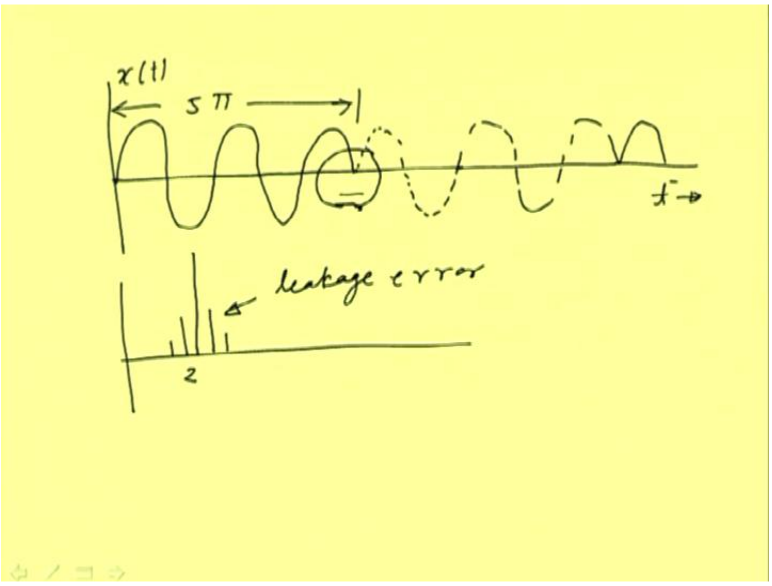

Let us go to the second slide this the signal which we have captured which is something like this which is a 5 pi period. Now, we are cutting and pasting the same signal here this and subsequently again will be cutting and pasting that will go like continuously up to infinite there is a assumption behind DFT. Now, you can see that this particular signal which we have got now there is some discontinuity specially here and because of this we will be having problem that we will not able to get the DFT of this properly.

So, what we will get we will get a same frequency as of the original signal, but apart from that we will getting some more spikes on this they are nothing but, due to the leakage error. Leakage error is coming due to the discontinuity which we have got here on to the signal. So, how to avoid this leakage error that now we will see, so we from the previous example we have seen that if we capture a signal if that signal is having the time period equal to that is multiple of the time period of the original signal then we will get the correct frequency spectra.

If we are having there is a difference between the time period of the captured signal and the original signal then we get the leakage error. So, the best way to avoid the leakage error is if we can match the time period perfectly or with the integer multiples of the time period if we are capturing the signal. Now, let us see some other way of a doing reducing the noise level because what happens we may not knowing the exact period of the original signal.

So, how we can avoid this situation let us see. So, to counter it this leakage error first thing is first way of doing is windowing and what are the various correct type of window available that we will discussing windowing is nothing but, we multiply the signal with some weights specially at the middle of the signal we multiply with the higher value at and as we are going toward the ends of the signal we multiply with the smaller value of the factor. So, that when we join 2 such signals at the boundary there joining is smooth and because of that a we will getting less leakage effect.

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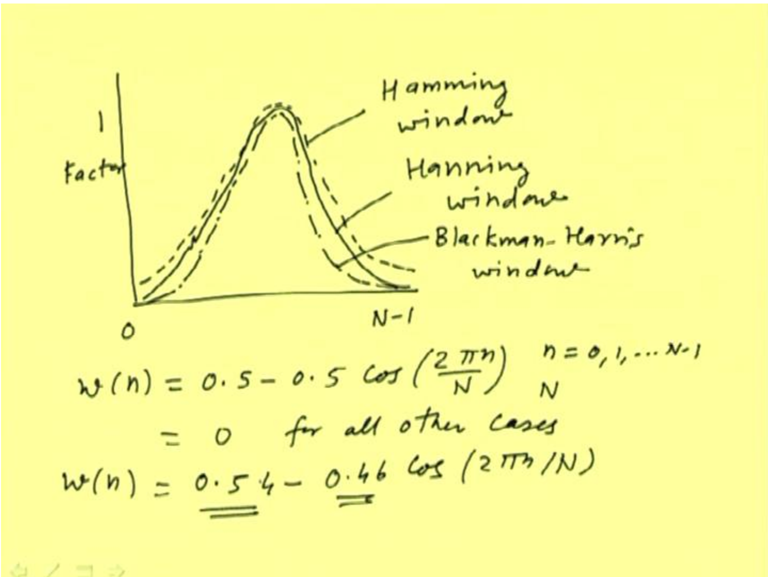

So, let us see this what are the type of window we have. So, window is something like this which is a time windowing. So, maximum value of the time factor is 1 and this is 1 kind of a window in which we have these are the factors which we have to multiply of the signal you can see here we have 0 to N minus one. So, various data which are there any numbers will be multiplying by these factors.

So, at the ends the factors are small. So, the smoothing the joining of the 2 signals will be smooth this particular window is having particular name is called is called Hamming window. There is expression for this is also available that is 2 pi n by N where n is the time instant from 0 to N minus 1 and N is the total number of data and this is 0 I can delete this, this is 0 for all other cases there are other versions of the windows there is another kind of window which is called Hanning window.

So, we can see that at low portion the values are becoming still lower as compare to the hamming window and there is a another improved version of these window that is more steep and values of the factors at ends are still lesser. So, that the smooth of the joining of the 2 signals take place this is Blackman Harris window expression for these windows are also available like for Hanning window we have w n is equal to 0.45 minus 0.46 cos 2 pi n by N. So, minor modification in these coefficients are there.

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$$w(n) = 0.423 - 0.498\cos\left(\frac{2n\pi}{N}\right) + 0.0792\cos\left(\frac{4\pi n}{N}\right)$$
$$= 0 \quad \text{for all other cases.}$$

Similarly, for the Blackman Harris window we have the expression like this. So, it is having 1 extra term and they are 0 for rest of the for all other cases prevention of the leakage error can be reduced by another way that is coinciding the period of the captured signal with the original signal. And there are specially when we are doing this numerical simulation of a particular phenomena, then it is easier to match this particular period because may be you can have first trial simulation of the response and for there you can get the actual period of the signal.

And then you rerun the your simulation to match the period which you which you are going to capture by changing the time steps of the simulated data. But, the problem comes when we are doing the experiment there whatever, the instrument we which we have there the limitations is there in tuning the the sampling frequency so that whatever, the data we are capturing the period of that matches with the actual signal there is limitation in the how much the fine tuning we can be able to do on to the time captured.

So, to avoid that sometimes specially in the rotating machinery there is a flexibility of changing the speed of the rotor is itself near by a small point, so that we can able to match the period of the actual signal and the captured signal. So, that we get the better spectra of the signal. Apart from the errors which comes from the signal processing during measurements also we have to take care of the noise should not enter into the signal which we are trying to measure like in actual experiment when we perform.

So, error can come from various sources right for the instrument the senses which we are measuring the cables or may be the other measuring instruments like oscilloscope or spectrum analyzer. So, proper care has to be taken while measuring the signal because sometimes it may happen that the original signals will be totally suppressed behind the noise. So, there are 2 basic type of problems may come 1 is the electrical noise which comes through various sources which we will see 1 by 1 another is the run out is another specially is the rotating machinery you can have run out due to the geometrical imperfection of the rotor. or they can be some kind of the magnetic run out that we will discuss subsequently.

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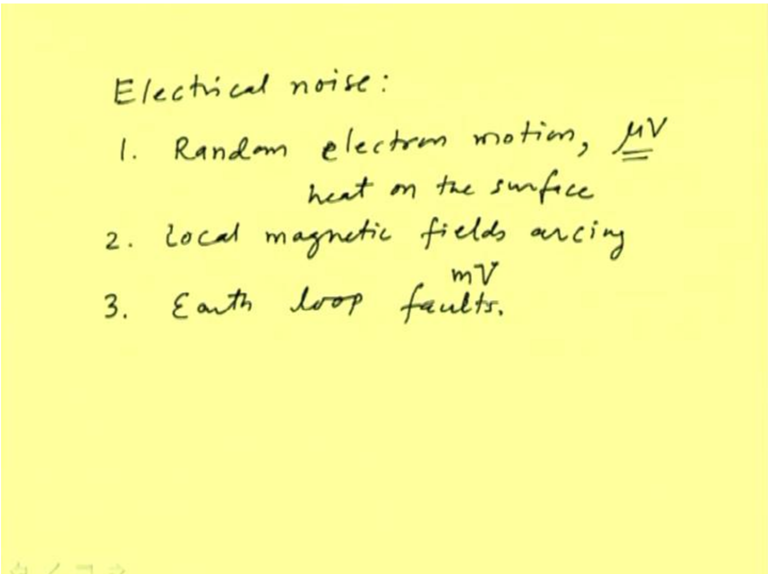

So, let us see first the electrical noise. So, first is the random electron motion. In this case generally the order of the voltage which it generates is in the micro microvolt, so because this voltage is very less. So, it is not of much concern and physically it comes out has a heat on to the surface where we are doing the measurement second kind of a electrical noise is the local magnetic field arcing and this is order of generally millivolt.

So, there appreciable amount of noise generates and in this particular case the magnetic field of an instrument are of which is very close to the either the electrical wiring or to the sensor then a effect the signal then a noise can enter into that this kind of magnetic noise may come when 2 instrument which are generating magnetic field are close by then also they can have the the noise electrical noise in into the measured signal.

So, generally to prevent these kind of electrical noise we have to shield the wiring properly or those instruments which are generating the magnetic field they should be kept on a some metallic cabins third type of the electrical noise is the earth loop faults and generally it occurs in electrical arcing in the switches and other similar components And here also proper care of the shielding is required. So, that the arching does not take place.

So, that the measurement which we are taking is free from the electrical noise. So, another kind of noise which comes into the measurement is run out. The run out generally it depends upon the transducers or the proximity probes which are using to measure the measure the vibration the reactance of that effects due to 2 reasons 1 is 1 kind of run out is the mechanical run out another is the electrical run out. In mechanical run out due to the when the shaft is not perfectly circular may be it is eccentric or electrical in shape or may be when the surface dilution of the rotor surface is there.

Then whatever, the measurements we will getting that will be contaminated with the this kind of run out, because that is not the vibration which that particular sensor is sensing. It is the fault of the surface of the rotor which is giving the some kind of noise into the measured signal. Another kind of noise comes run out comes due to the may be if the surface of the rotor is magnetize may be due to the manufacturing process or may be some kind of non destructive testing procedure which has been followed to check the compactness of the shaft.

So, because of this there may be interference of the field around the the sensor which generally it will introduce some kind of noise into the signal. So, we must remove this kind of magnet field if it is there on to the surface like may be by degaussing or other kind of techniques and for removing the run out mechanical run out there we have to machine the rotor properly. So, that it becomes circular or if it is bend and because of that mechanical run out is coming may be you have to straighten the rotor.

So, these precautions we have to take before taking the actually measurement. The electrical and mechanical run out can be measured using proximity probe which depends upon the gap between the probe and the shaft and the magnetic field if is there on to the surface it effects that also. So, what we will doing we will rotating the rotor at slow speed, and we will measure the whatever the signal we are getting from that that will contain the run out due to the mechanical run out and electrical run out.

So, the mechanical run out we have already measured using the dial gauge. So, vectorially we have to substrate the total run out due to the electrical and mechanical with the mechanical run out to get the run out from the electrical run out and when we measured the actual vibration signal, then we have to nullify these run out from the measured signal. So, that and the this nullify will be vectorially we have to do it and even there is a nullifying of the response due to various modes also we have to do it like if we are measuring the vibration near the second mode then we have to nullify the effect of the first mode, because that first mode effect will also be there at the second mode vibrations. So, that that also we have to nullify now, let us see this subtract vectorial subtraction in more close form.

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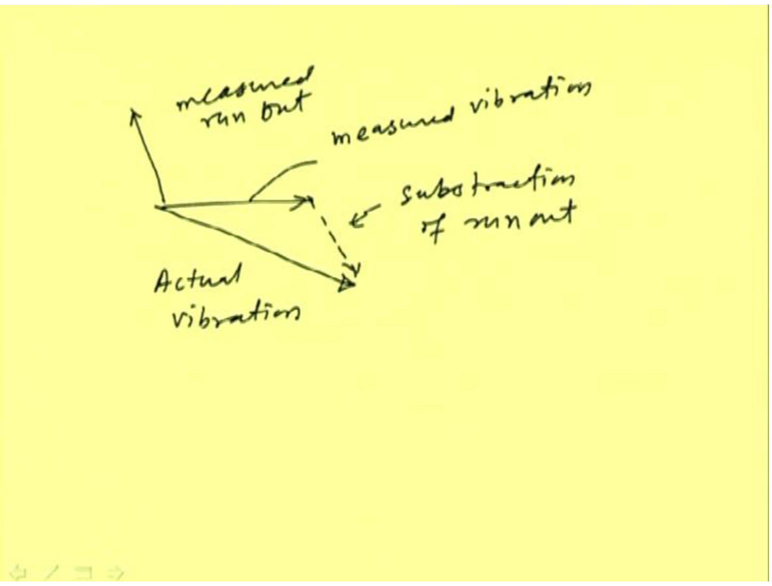

So, this is the actual vibration which we have measure and let us say, this is the measured run out. So, we have to subtract this measured run out this the substraction of run out from the measurement. So, this will be the measured vibration this is the actual measured vibration another kind of error involves when we integrate the signal or differentiate the signal like from accelerometer we get the acceleration measurement. And if we want to the get velocity and displacement signal we have to integrate them to get that quantity. And the error is introduced in this when we do the integration or differentiation. So, let us see what generally we do when we do the integration. When we do integration of let us say acceleration then basically, we divided the signal by its frequency. This is acceleration we have we want to get the velocity from this.

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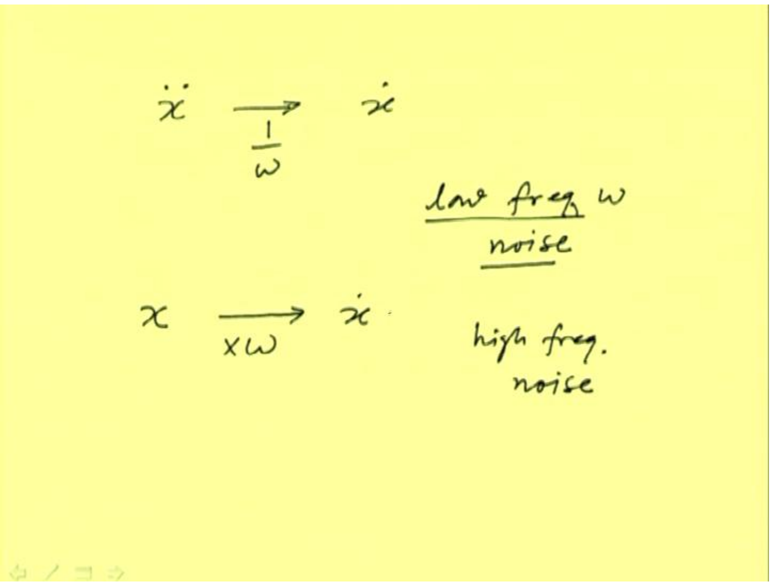

So, basically we have to divide this by omega the frequency which it has to get the velocity. Now, if this particular signal contains very low frequency components then you can see that when a low frequency component of specially noise is there low frequency noise is there. So, error involved will be more because here it is coming in the denominator frequency. So, once it will magnify these noise which are of low frequency.

So, similar when we want let us say we have measured displacement and we want the velocity by a differentiating this. What we have to do basically we have to multiply by that frequency. And if this, particular signal contain high frequency noise then these noise will get magnified and specially at high frequency whatever, the signal we are getting will be having more noise. Generally when we want to differentiate or integrate there are 2 kind of devices are available 1 is passive type of devices they give more error into the signal

So, there are some active devices are also there they give less error into the signal but, they are more expensive because inbuilt amplifier as to be there in that. So, in this particular lecture series we have see the various background and the fundaments of the signaled processing right from the Fourier Series, Fourier Transform, Discrete Fourier Transform and Fast Fourier Transform and while we measure the signal various kind of noise comes into the signal.

So, we should know what is this noise may come and how we can able to avoid them. So, that whatever the signal which we measured for analysis purpose it should be pure signal which should not contain any kind of noise. Otherwise whatever the information's are there in that particular pure signal will be biased, because of noise. And you cannot able to pinpoint the cause of the vibrations.