

Mechanical Vibrations
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Module - 12
Signature analysis and preventive maintenance
Lecture - 1
Vibration Testing Equipments: Signal Measurements

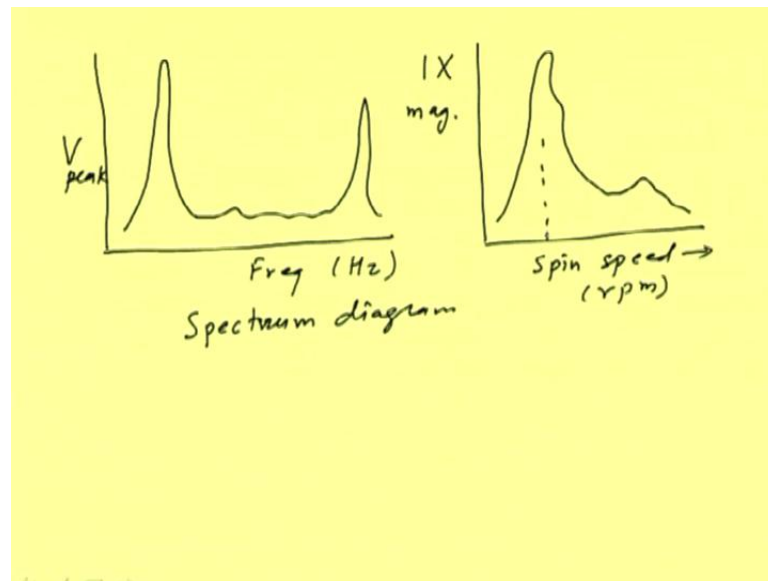
Today, we will be studying regarding measurement and signal processing, till now we have studied lot of analysis how to find dynamic system natural frequency or its force response and there instability analysis in any machinery generally, when we design the machinery, when we manufacture it and when we commission the machine then because of inherent dynamics inside the machine it is having vibrations which is nothing but, after effect of whatever, is the dynamics going on inside the machine comes out as a vibration and noise.

Measurement of these vibrations is very important specially in terms of when we are looking it to the machinery condition monitoring, and generally we measure vibration in terms of other displacement velocity or acceleration and measurement of the vibration is very important, and if whatever we are measuring if is not correct or whatever the instrument which we are using is not proper. Then we may conclude wrong decisions regarding the condition of the machine another aspect is there when we measure the vibration then we depict those vibration in various forms.

So, that we can understand what kind of our dynamics is going on inside the machinery which element of the machine is having certain problems, those problems will be reflecting in the measurements. So, when we take the measurement we should have enough background behind the signal processing So, that we should not make any we should not make any mistake in measuring them also if we, have research program in by which we are trying to understand the condition of the machine.

Then, the theoretical background behind the signal processing is also important. How it works? Because, there are various equipments which has all the built in function in by which we can able to measure the response, but you should know how correct they are and whether you are using them properly.

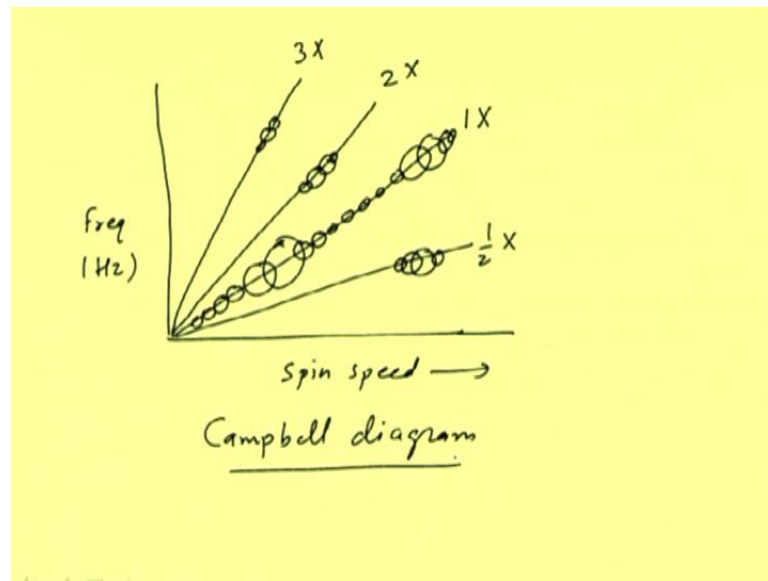
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So, first let us see various kinds of plots which we will be making it after the measurement. So, this is a spectrum diagram in which we have frequency here may be it is in hertz and here peak amplitude of either velocity or displacements. So, if we take the measurements the signal may of this form. Basically, this is the time domain data has been captured and we have done the Fast Fourier Transform of the signal, so it looks like this.

Now, another form of the signal is in which we have spin speed here that is in rpm which is continuous variation of the or let us see, a rotor speed and here we have 1 times the rotational speed magnitude of vibration may be velocity or displacement. So, it is having some variation like this. So, here you can see this is corresponding to the critically speed of the shaft. Other form of vibration measurements

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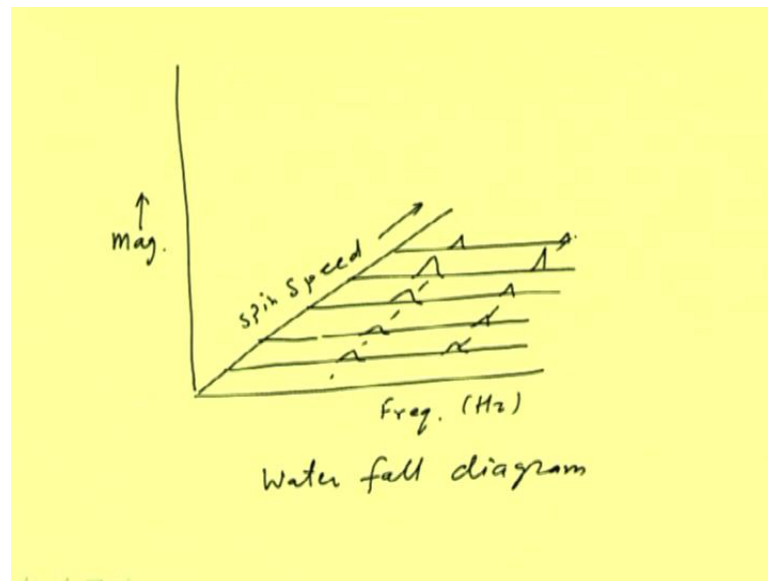


We can have like, this is a Campbell diagram in which this axis is spin speed and there is the frequency of the vibration which is in Hz. Now, we have this line which is corresponding to the rotational frequency component and some more components are there which are twice the rotational frequency or may be 3 times the rotational frequency or even we can have half the rotational frequency components.

Now, when we achieving the speed of the rotor. We were find there will be corresponding to 1 X frequency some amplitudes as it will reach the critical speed amplitudes will increase, the circle radius represent the amplitude and as will go away from the critical speed it will decrease. Again may be if it, has second critical speed we can have amplitudes increasing. And certain faults they give 1 half the rotational frequency components like this, but in the rest of the locations they will be not there.

Similarly, here may be they can have second X rotational frequency or some the small peaks may be in the third rotational frequency. So, this is a Campbell diagram of this particular diagram we have, information of the spin speed also the frequency of excitation also the amplitude 3 information's are there another kind of a diagram which is a 3 dimensional diagram.

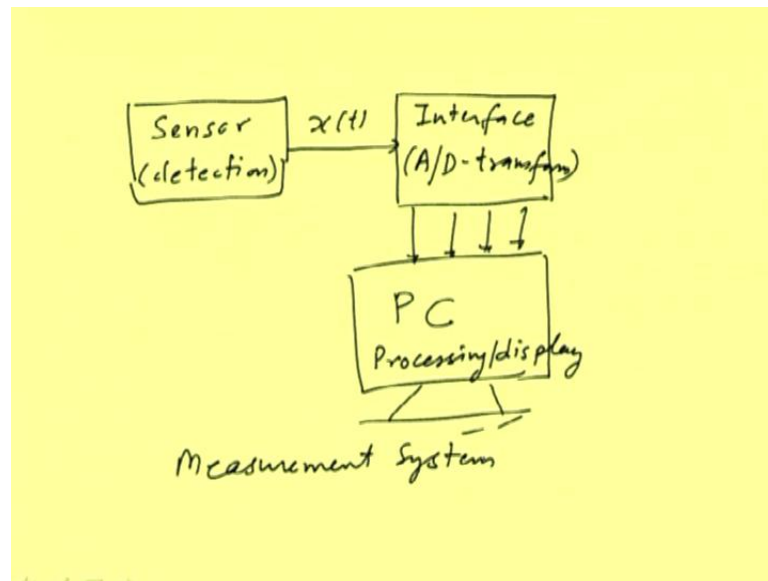
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Which is called water fall diagram in this is the frequency of the vibration our spin speed is here in this axis and magnitude of the vibration is in this direction. So, we can have different speed lines like this and this is nothing but, the something like we measured the time domain signal and then we did the fft a for a particular speed. So, one particular data will give us a dot like this in which these are the: peaks corresponding to the either first natural frequency or the rotational speed of the rotor.

And as we will increase the speed you may find that these amplitudes may increase or decrease depending upon at with which rpm where rotating the rotor these also can have, various peaks. So, it is giving a some kind of cascade of history of the amplitude as we are changing in the speed of the rotor. Now, we will see the measurement and the signal processing what are the errors involved in that? What precautions we should make during the measurements. And the possible causes of the error and how to avoid them during the measurement. So, first let us see what is the measurement system?

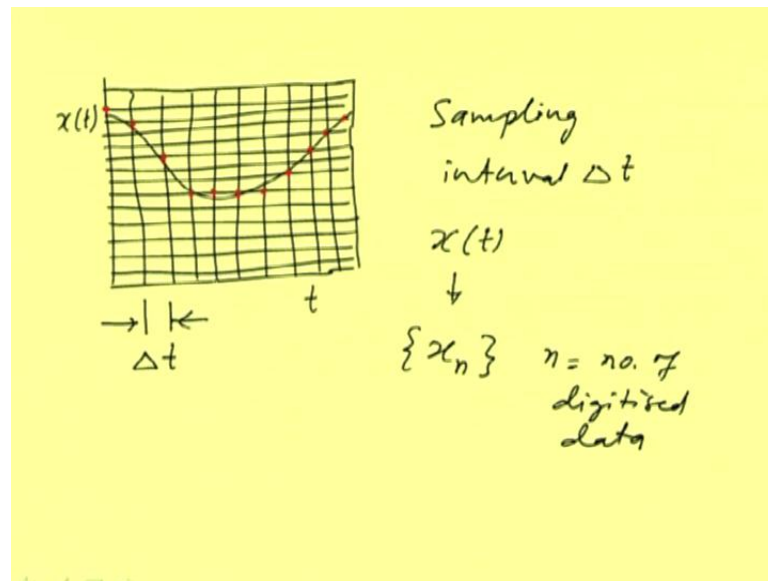
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So, in measurement system we have a sensor which detect the vibration is the detection of the vibration and it gives in the form of displacement with respect to time then we have, the interface which converts the analog signal which is measured by the sensor to the digital form A to D transform. Then we feed this to this signal the digitized signal to a personal computer and there we do the processing of the measured signal processing and display of the signal.

So, this is the measuring measurement system. So, when we measure the analog signal through sensor and transform that to a digital data in the into the computer then those signals are in the digitized form. So, there will be in the form of numbers. So, let us see how the errors comes in during the digitization of the data.

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So, let us take a signal which is measured in time domain is having some this form this is a analog signal. Now, if we want to digitize it what we have to do let us see, we are making a grid the grid size depends upon the accuracy of the measurement instrument these are the grids. So, the instrument is having capability to measure the data or digitize the data on these grids. So, these are the it depends this is the nothing but, the accuracy of the instrument. So, I am just overlapping the grid on to the signal.

Now, when we are digit rising it so whatever, the data's are there they will be at the corner of these grids. So, during digitization it will take the nearest grid points. So, we can able to see that when I am a digitizing the data in the grids we are getting the same signal as the actual signal, signal is or deviating these red points indicating the data which will be getting after the digitization. So, during digitization of the data generally there is sampling interval will take as constant.

So, the sampling interval which we represent as delta t in this figure is given this is the sampling interval. And once we have digitized the data from analog signal then we have these data in a form of numbers n is the number of data where n is the number of digitize number of digitized data during the during this digitization we do the digitization of the amplitude and you can able to see in the figure that we take the nearest node of the analog signal.

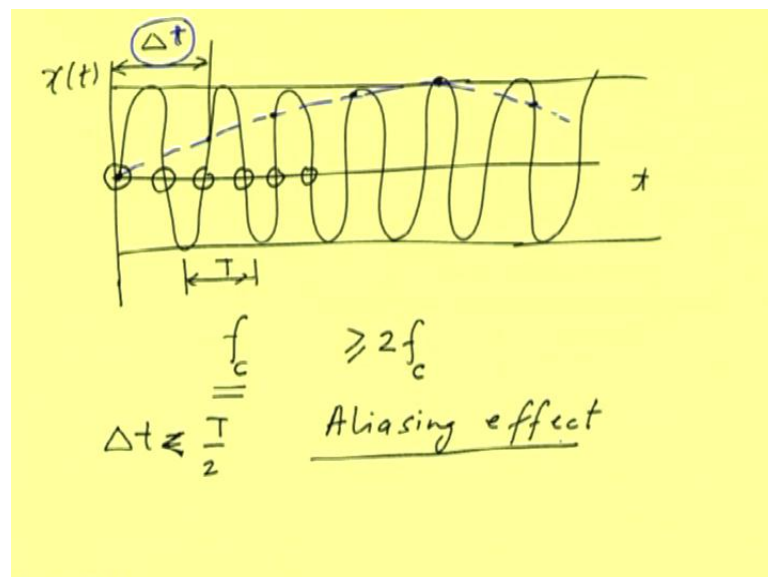
So, there are some error involved in the sampling of this data and generally when we transfer this data to computer we do 2 basic operations, one is the there is signal

extraction and another is the our data transformation. So, data transformation is something like when we digitize the data it is there in the binary form. So, we transform them in a more convenient form So, that we can able to display them in a graphical form. Now, we will see some other kind of problems which we face during the sampling of the signal and specially when the signal is fast.

The frequency of the signal is fast and if you are sampling that signal with slow then we may have effect called Aliasing effect in which we will not able to measure the actual signal. So, through one example I would like to explain this phenomena you must have see in the old movies in which the horse cart wheel you must have seen that rotating in the opposite direction actually it rotates in the same direction but whatever, the sampling frequency of those video frames are there they are slow at that time.

So, you could not able to see the wheel of the cart moving with the correct direction it looks like it is rotating in the opposite direction So, similar effects may occur in the vibration signal if we capture with the slow sampling rate through one example I would like to show this

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This is a time domain signal, $x(t)$ and let us there is the amplitude of that let us say, this is a perfectly sinusoidal curve of same frequency. So, frequency is not changing amplitude is also not changing if we sample this particular signal, let us say with this period this is the sampling interval. So, second point will be coming somewhere here third will be

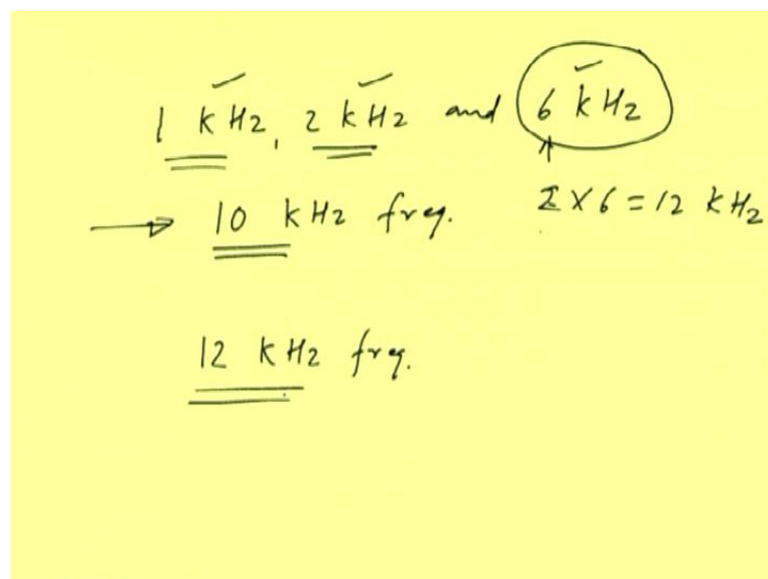
coming here fourth will be coming somewhere here fifth will be coming somewhere here.

So, we can see that signal will appear to us like this, because of the choice of the sampling interval as different actually it is more than the that time period of the signal So, this blue color signal will appear is a measurement there is a something like virtual signal, but actual signal is entirely different and the theorem behind this is if let us say we have a signal in which frequency components are less than f_c .

All the signals which are there in a particular measurement frequency component is less than f_c . If we measure the sample rate is more than $2 f_c$ then we will able to measure the signal correctly. It should be more than $2 f_c$ like I will give an another example in the same signal. Let us see we are having the sampling period as equal to time period by 2 of the signal time period of the signal is from here to here this is the time period of the signal. So, if we take the sampling period half of that. So, we will be getting let us say we are starting from here.

So, if these points will be getting as the measurement and if you join them we will get a signal with 0 mean, so we will not able to see anything in the signal. So, always we should have the sampling interval less than half, so that we can get the correct picture of the signal this is called Aliasing effect.

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Another example in this if let us say we have a signal which is having one kilohertz 2 kilohertz and 6 kilohertz signals and if we sampling with 10 kilohertz frequency then we

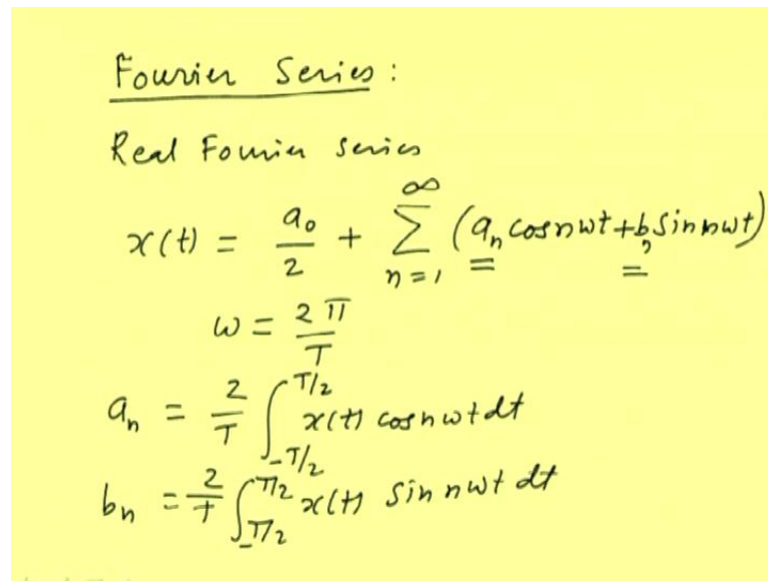
may not get a correct picture of the signal, because 10 hertz is less than twice of the maximum frequency which the signal is containing that is 2 into 6 is 12 kilohertz. So, unless we have sampling frequency more than 12 kilohertz will not get all these 3 frequencies correctly.

So, if we are using 10 kilohertz that is the frequency we may not able to get the correct picture of this instead of this we may get 4 kilohertz signal these we will able to measure using the 10 kilohertz, but if we are using 12 kilohertz frequency then we can able to measure all 3 components of the frequency that is 1 kilohertz 2 kilohertz and 6 kilohertz. In practical situation, when we are we do not choose the sampling interval arbitrary generally what we do we have some low pass filters.

So, we have a some frequency below which where are the signal to pass, because we know what is the our requirement of this signals frequencies based on the our knowledge on the analytical a formulations we will be knowing up to what frequency we are interested in our signal So, we will be putting a a low pass filter for that and the sampling interval we will be decided by then twice of that low pass filter frequency Now, we will be studying some of the basics regarding their signal processing like: Fourier series, Fourier transform, Digital Fourier Transform and Fast Fourier transform what is the mathematical background behind that?

And, because unless we know the mathematical background behind that whatever, during the usage of that we may misinterpret there the signal which we are capturing. So, let us start with the first with the Fourier Series as you know that Fourier Series is nothing but, when there is a periodic signal. And using Fourier Series that periodic signal can be split into their fundamental harmonic and higher harmonics. That means we can have that particular signal in various harmonic component form. That means the cos and sin terms of the fundamental frequency which that signal is carrying also there higher harmonics.

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Fourier Series:
Real Fourier series
$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$
$$\omega = \frac{2\pi}{T}$$
$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \cos n\omega t dt$$
$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \sin n\omega t dt$$

So, let us see the basic formulae regarding the Fourier Series which you must have studied in the mathematical in the mathematics courses, but our orientation will be toward the signal processing how these can be utilize? So, first let us see the real Fourier Series and which a particular signal which is continuous signal is split into a constant term and summation of various cos and sin terms like this where this omega is nothing but, the frequency where T is the time period of that particular signal.

And these constants can be obtained if we multiply or this formula by cos an omega t and sin omega t and integrate over the domain, so it is like this. So, we are multiplying this by cos omega t dt So, we are getting an similarly bn can be obtained minus to plus t by 2 xt sin n omega t dt.

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$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\Rightarrow x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\omega n t}$$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-j\omega n t} dt$$

$$n = 0, \pm 1, \pm 2, \dots, \infty$$

Complex Fourier Series

Now, there is another convenient form of the Fourier Series is there that is complex representation of the Fourier Series we have the Euler relation which can be use to convert the real Fourier Series to the complex Fourier Series, because complex Fourier Series have is having some inherent advantage in handling the mathematical terms is more compact. So, this signal which is periodic can be expressed as summation of minus infinity to infinity plus infinity $c_n e^{j\omega n t}$.

So, it is the complex representation of the Fourier Series where the complex coefficient now is given like this where this n is 0 plus minus 1 plus minus 2 etcetera up to infinity, and this particular equation is called Complex Fourier Series Complex Fourier Series (Refer Slide Time: 27:10)

$$\rightarrow c_n = \frac{a_n - j b_n}{2}$$

$$c_0 = \frac{a_0}{2}$$

$$\rightarrow c_{-n} = \frac{a_n + j b_n}{2}$$

Re(c_n) Symmetric

Im(c_n) Skew-symmetric

And this is the complex coefficient of that series which is related with the real coefficients like this a_n and b_n 's are the real Fourier Series coefficients and C_n is given by this and a_{-n} is given as $a_n + jb_n$ by 2. So, from here we can see that when we want to have the real part of C_n that is both in the n when it is positive direction or negative direction the value of this will be same or in either of this direction because that is you can see these two terms real part of that is nothing but, a_n by 2.

So, it will be symmetric about the vertical axis all the components will be symmetric about the vertical axis and conversely if we see the imaginary part of C_n if you want to plot for n is equal to plus a negative you will find that from first expression this is the negative term minus b_n by 2. But, here in the second term is b_n by 2 positive. So, if this term is negative this will be positive or if we have this term as positive it will be negative. So, it will be skewed symmetric about the vertical axis. So, here skew symmetric, but for real case that is symmetric. So, this is some of the characteristic which we will like to or keep it in mind.

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$$C_n = |C_n| e^{-j\theta_n}$$

$$|C_n| = \sqrt{a_n^2 + b_n^2}$$

$$\theta_n = \angle C_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

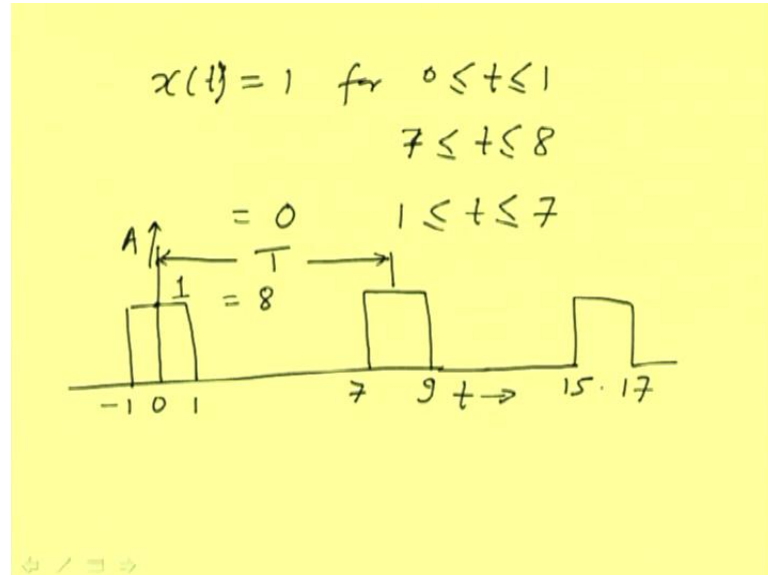
amplitude spectrum

$|C_n|^2 \rightarrow$ power spectrum

even the Complex Fourier coefficient can be express as the magnitude of that and the phase where the magnitude itself in terms of the real Fourier Series coefficients or given like this and the phase of the C_n is given as $\tan^{-1}(b_n/a_n)$ and this is if we plot with respect to n if we plot this one this is called amplitude spectrum and this θ_n this is called phase spectrum and if we have a plot of C_n modulus square that is then called power

spectrum which is very common in the Fourier Transform, we will be taking one example of square wave and we will obtain the Fourier Series of that.

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So, let us see the how the signal is $x(t)$ is equal to 1 for a time period let us see 0 to 1 and for 7 to 8 units and it is 0 for from 1 to 7 and then it is a periodic. So, repeats after this. So, if we want to show this on a picture it will look like this, this is the time this is the amplitude this end. So, signal is something like this. So, minus 1 0 1 then let 7 which is having a amplitude, amplitude this amplitude to let us say 1 this is 9. So, time period of this signal is from here to middle of this which is 8 units. And this signal repeats after this. So, is a continuous signal in both direction positive and negative. Now, if we want to obtain the Fourier coefficients of this.

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$$\begin{aligned}
 C_0 &= \frac{1}{T} \int_{-T/2}^{T/2} x(t) dt \\
 &= \frac{1}{T} \left[\int_{-4}^{-1} 0 dt + \int_{-1}^1 1 dt \right. \\
 &\quad \left. + \int_1^4 0 dt \right] \\
 &= \frac{1}{8} [2] = \frac{1}{4} = 0.25
 \end{aligned}$$

So, let us say complex coefficient C_n then we are to use the formulae which we have already given previously where $x(t)$ is the signal. So, this will take the form now we are substituting the value of the $x(t)$ for various reasons. So, let us see minus 4, because T is 8. So, minus 4 to minus 1 there is magnitude of this signal is 0 there then minus 1 to 0 or even up to 1 is having magnitude 1 we can see the. So, minus 1 to 1 is having magnitude one and prior to that is having 0 magnitude plus from 1 to 4 again it is having 0 magnitude yes. So, this will give us because T is 8. So, this will be 2 or 1 by 4 which is nothing but, 0.25.

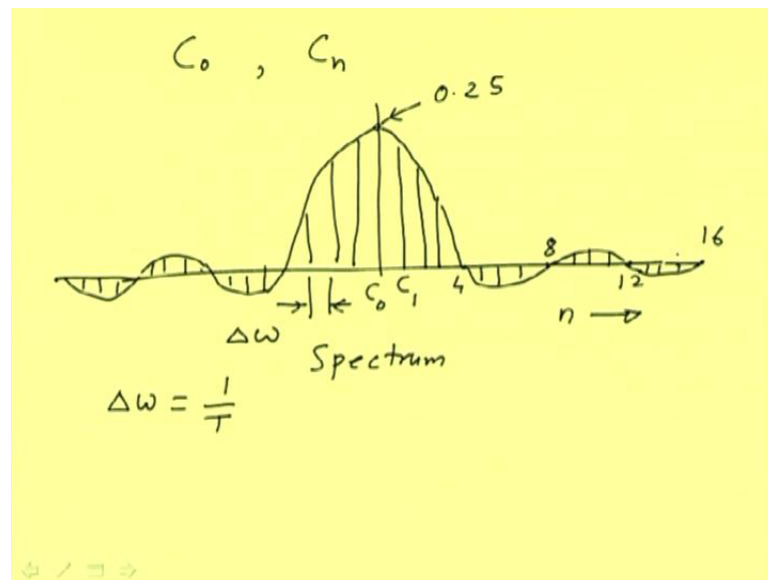
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$$\begin{aligned}
 C_n &= \frac{1}{T} \int_{-T/2}^{T/2} x(t) dt e^{-jn\omega t} \\
 &= \frac{1}{T} \left[\int_{-4}^{-1} 0 dt + \int_{-1}^0 e^{-jn\omega t} dt \right. \\
 &\quad \left. + \int_0^1 e^{-jn\omega t} dt + \int_1^4 0 dt \right]
 \end{aligned}$$

$$\boxed{C_n = \frac{2 \sin n\omega}{T n \omega}}$$

Then other coefficients can be obtained like this. So, here again from minus $T/2$ to $T/2$ by $2 \times t \, dt$ and if we start substituting for various period will be having minus 1 to minus 1. 0 value of this signal then minus 1. 0 is having 1 value but, I think here will be having minus $j \omega n t$ or here also we have another terms $j \omega n t \, dt$ then from 0 to 1 similar term is there and there for the remaining period again its value is 0. So, if we integrate these two terms and simplify I am giving the direct solution this take this form. So, C_n is having this value.

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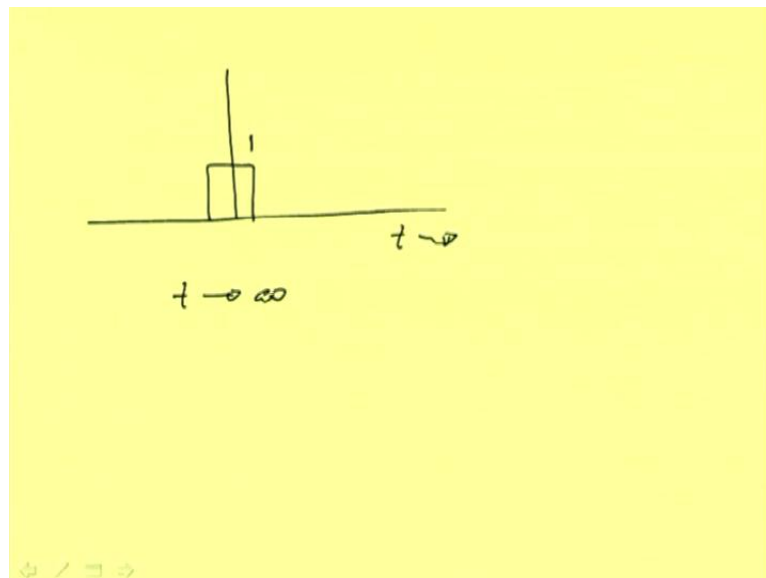
And if we want to plot these C_0 and C_n will be getting the spectrum of the signal the signal will be something like this and which we have let us say this is C_0 having some value which is we already obtained earlier. Let us say this 2. 0.25 then this is C_1 similarly other values will be having C_2, C_3, C_4 there is fourth then this is 8 and this is 12 there is 16, so if we obtain the envelope of this we will get something like this one is the bottom similarly this side also will be having this kind of plot.

Now, these are the spectrum these are discrete amplitude and we have obtain the envelope of that, so it is looking like a smooth curve and in this particular this is nothing but, spectrum in which here we have kept the n and, because we know at one particular any one we can choose this is nothing but, ω that change in the ω that itself can be obtained yes $1/T$. So, be basically the horizontal axis represent in some hence the frequency and the vertical axis represent the amplitude of the spectrum.

Now, we have already seen the Fourier Series. Now let us see the Fourier Transform how it works specially in the previous example we have seen that there is square wave which is periodic in nature we have done the Fourier Series or of that we obtain that now if instead of a square periodic signal. Let us say we have only a square pulse and. So, it is not periodic in nature, so how to obtain the spectrum of that, that will be seeing through the Fourier Transform.

And in this case the basic premises that, once we have is let us say square pulse we assume that pulse that repeats after infinite time. So, let us see the signal of that pulse first.

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So, this kind of pulse we have which is, so it is not repeating after time after some time interval, so we assume that of at t tends to infinity it repeats that is the assumption based on that we will try to obtain the spectrum of this. So, before that let us go to the how there expressions of the Fourier Transforms how we can able to do?

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$$x(t) = \sum_{n=-\infty}^{\infty} \left[\frac{1}{2\pi} \int_{-T/2}^{T/2} x(t) e^{-jn\omega_0 t} dt \right] e^{jn\omega_0 t} \cdot \frac{2\pi}{T}$$

$$\omega = \frac{2\pi}{T} = \omega_0$$

n^{th} order $n\omega_0 = \omega_n$

$$T \rightarrow \infty \quad \omega_{n+1} - \omega_n = \Delta\omega = \omega_0 = \frac{2\pi}{T}$$

We have already expression of the Fourier Series in that itself we are combining that Fourier Series and its coefficient in one expression like this. So, it is the coefficient which we are substituted in the Fourier Series expression. So, within the square bracket is the coefficient C_n and here we have the term of the Fourier Series. Now, because we have we have 2π term. So, there will be extra term coming here like this where is ω is the frequency it is the time period the fundamental wave frequency which we expresses like this ω_0 .

And n^{th} order frequency can be written as ω_n and if we want the difference in the frequency between 2 order that is nothing but thus that itself will be ω_0 or 2π by T . So, because this particular pulse is repeating after infinite time. So, if we put the limit this expression we are putting the limit.

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$$x(t) = \int_{-\infty}^{\infty} \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \right] e^{j\omega t} d\omega$$

$\omega_n \rightarrow \omega$
 $\Delta\omega \rightarrow d\omega$ $T \rightarrow \infty$
 $\Sigma \rightarrow \int$

$$x(t) = \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

So, let us see how takes the form $x(t)$ is equal to minus infinity to infinity 1 by 2π , so this summation terms become integral minus j omega t omega is nothing but, n into omega naught term. And 2π by t is become delta or d omega. So, basically we have replace this with omega and delta omega as d omega and summation with the integral, because once we put the limit T tends to infinity the period of the signal T is equal to infinity this expression can be written in more compact form like this d omega.

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$$X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Fourier transform $x(t)$

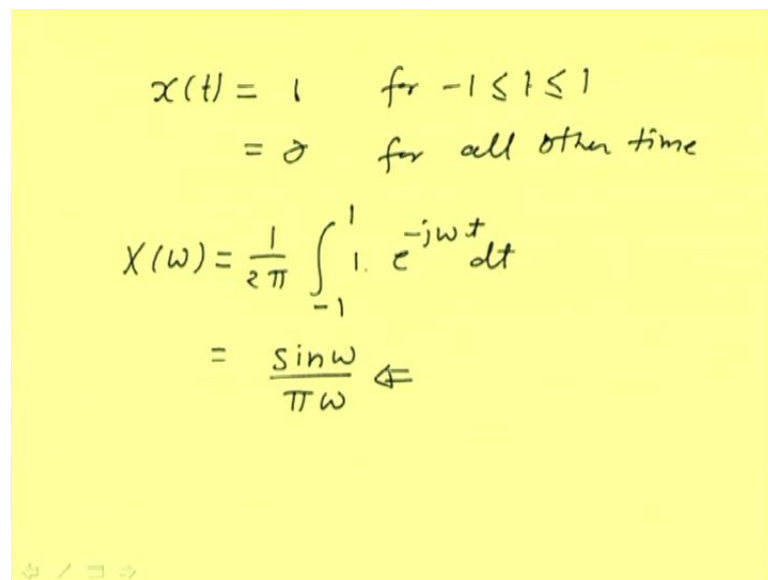
Inverse f 1

Where this term is expressed as 1 by 2π minus infinity to infinity $x(t) e$ raise to j omega t dt . So, this term is nothing but, Fourier Transform of the signal $x(t)$ there is the Fourier

transform of the signal $x(t)$ and the previous expression this expression is called Inverse Fourier transform, Fourier transform which converts the frequency domain signal back to the time domain signal. So, using Fourier Transform you are converting the time domain signal to the frequency domain signal and using Inverse Fourier Transform in the vice versa.

So, on let us see a with a simple example of a square pulse how the spectrum can be obtain using the Fourier Transform and we will be comparing that spectrum with the Fourier Series which we earlier obtained of a square periodic signal, and will try to correlate them and that will give us some insight how we can able to do the Fourier Spectrum of some other kind of complicated signals.

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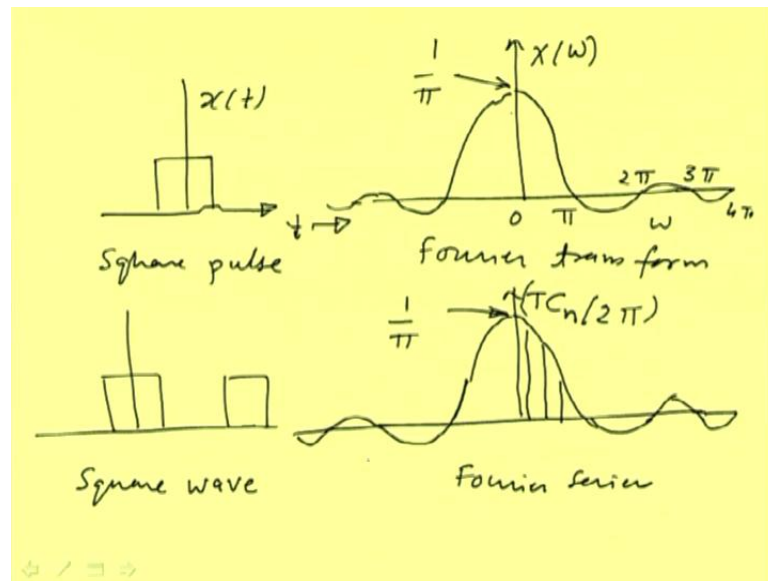


The image shows a handwritten derivation on a yellow background. It defines a rectangular pulse $x(t)$ as 1 for $-1 \leq t \leq 1$ and 0 for all other time. The Fourier Transform $X(\omega)$ is then calculated as $\frac{1}{2\pi} \int_{-1}^1 e^{-j\omega t} dt$, which simplifies to $\frac{\sin \omega}{\pi \omega}$. A double arrow points to the final result.

$$\begin{aligned} x(t) &= 1 \quad \text{for } -1 \leq t \leq 1 \\ &= 0 \quad \text{for all other time} \\ X(\omega) &= \frac{1}{2\pi} \int_{-1}^1 e^{-j\omega t} dt \\ &= \frac{\sin \omega}{\pi \omega} \Leftarrow \end{aligned}$$

So, the pulse is having this characteristic is having magnitude 1 for minus 1 to 1 unit of time and is having 0 value for all other time. So, the Fourier Transform of that as we have defined will be given as because most of the time it is having value 0 and during minus 1 to 1 it is having unit value. So, we can written this term rest of that terms will be 0. So, this gives us after solving this is the Fourier Transform of the pulse and if we plot this.

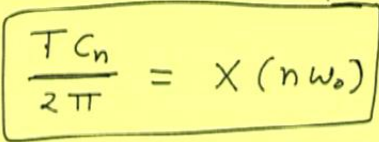
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Let us say this is the pulse this is the pulse and the whatever, the Fourier Transform which we obtained If we plot we will get a curve something like this we have this is 0 pi 2 pi 3 pi and 4 pi and other side also similar symmetric variation will be there now, if we want to compare this signal with the this spectrum with the previous one in which we obtain the Fourier Series of a square wave. So, in that also we got the similar variation of the spectrum only thing was we had that in terms of the discrete values.

And we obtain the envelope of those spectrum where joining the peaks to get the similar curve as we obtain using the Fourier Transform. So, this is we have got from the Fourier Series of the square wave there is periodic in nature and this is the Fourier Transform of a square pulse this qualitative matching is there between them but, if we want to see the magnitude of this, this is basically C_n and if we plot this we multiply this C_n by time period divided by 2π . Then the magnitude of let us say, this peak is having 1 by pi value and this is also having 1 by pi by value that means we have a definite relationship between these two.

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$$\frac{T C_n}{2\pi} = X(n\omega_0)$$

That is $T C_n$ which is Fourier Series coefficient divided by 2π is related with the Fourier Transform like this. So, if we multiply the Fourier Series coefficient with time period and divided by 2π we will get the Fourier Transform. So, now this relation is holding at each and every spectrum is not necessary is holding at the peak we can able to check that it valid at each and every our locations. And this particular relation we will be using or to have spectrum of some kind of our signal which is neither a pulse nor they are periodic in nature but, they are finite in length but, they are not periodic.

So, there is nothing but, discrete Fourier Transform we call it or because we obtain the frequency domain data from time domain data of a signal which is neither it is periodic neither it is infinitely long and that is in between so whatever the relation we develop, now we will be using for obtaining the spectrum of a particular signal which generally we measure through the sensors.

So, and now we have already studied the Fourier's and Fourier Transform and we already seen the relationship between them of Fourier Series is applicable to a periodic signal and we can transform that period signal from time domain to frequency domain, and in Fourier Transform we have already seen that a pulse which reach with the assumption that it repeats after infinite time has been converted in the frequency domain and whatever the characteristic of these two spectrums were there we related in the previous discussion.

Now, the same characteristic will be using for obtaining the spectrum of a arbitrary signal generally in machinery when we measure the signal they will not be periodic in nature, because so many inherent noise effect, and so many frequency components are there in the system which we do not know also they will be there in the system. So, is very difficult to find out what is the period of the particular signal. So, we take the measurement of that vibration signal for sufficiently long time and we assume that particular signal contains most of the frequency component of the machine.

Now, in Fourier Transform we need a particular signal which is infinite in time but, during measurement this is not practical to measure a signal of infinite time interval. So, in that sense it is finite in the whatever, the time we measure or that is finite also it is not periodic. So, directly we cannot able to I have that do the Fourier Series of that or Fourier transform of that, but there is a in between there is called Discrete Fourier Transform which utilizes both the Fourier Series and Fourier Transform relation which will be using it for this purpose.

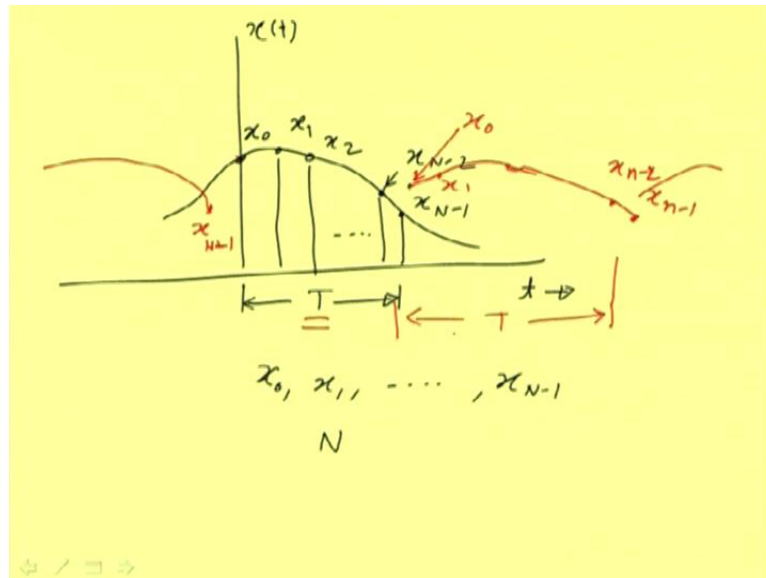
And the basic premise is that we measure particular signal and whatever, the length of the signal we assume that is sufficiently long may not be infinite and we assume that, that signal repeats after next interval there is the assumption based on that we obtained the Fourier Transform of that particular signal, and because that signal is discrete in form. So, whatever the spectrum we get it is called Discrete Fourier Transform of in this when we will be doing the Discrete Fourier Transform there are certain error scrip in into the signal as one we already seen that there Aliasing effect.

That means if whatever, the frequency components are there in the system if we are sampling that of signal with slow rate we cannot able to capture them correctly. So, this is one of the aspect another is, because the length of the signal is finite and, because of that what will happened the period of this signal will not coincide with the that signal. So, when we do the Fourier Transform we may find that there will be some errors which is called leakage error will occur.

So, whatever the frequency components are there in the system apart from that some more frequency component there are nothing but, leakage error will appear in the spectrum this one another is, so it let yeah another is, because we are taking the measurement length of the signal is finite they are not infinite. So, we will be having problem in that also some error will be creping in into the spectrum. So, let us see how

the sample data is the how we do the fft or the Fourier Transform of that. So, for that may be let us see first the signal itself which we have captured signal has been captured in the form of data from x_0 up to x_{N-1} .

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So, there are total N number of data, are there and let us say the signal is something like this, this is the signal and this is the time. Now, in on this we have obtain these are the discrete data which we have gathered these are the discrete data which we have sampled and this is x_{N-1} this is x_{N-2} in between some other data are there. Now, the solid line represent the actual signal, but because we have captured up to this as a time period of the signal we have captured up to x_{N-1} only.

So, during when we will be converting this into the frequency domain for that what we do this particular signal from x_0 to x_{N-1} we repeat after the I can use different color from here like this. So, this is representing the x_0 and x_1 like this and this is x_{N-1} x_{N-2} and again we repeat the signal like this. This side also this side also we have the repetition of the signal. So, this will represent x_{N-1}

So, a similar to that the signal which is there from x_0 to x_{N-1} is being cut and past at subsequent before and after the signal. So, now we are assuming that this particular signal is having period T , because now as I have shown now, this particular signal repeats after time T , because this is another time period and with this. Now will be doing the Fourier Series coefficients will be obtaining of this particular signal.

And then using the analogy between the Fourier Series and Fourier Transform will try to get the Discrete Fourier Transform coefficient of this particular signal. In the next class also, we will continue to explore the dft that is a Discrete Fourier Transform and Fourier Fast Fourier Transform, because there are some limitation of their discrete Fourier Transform. So, how the Fast Fourier Transform to take over that limitations and then, we will see some more detail what are the problems associated with the signal processing when we do the signal processing what are the errors involved and how we can...