Mechanical Vibrations
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Module - 11
Finite Element Analysis
Lecture - 2
Finite Element Formulation for Beams: Galerkin's Method, Beams Element Mass and Stiffness Matrices, Element Force Vector

Today, we will study regarding the transverse or bending vibration of shaft. You have already studied about what is the transverse or bending vibration. Here mainly we will be focusing on the development of the equation of motion specifically for the finite element analysis or of the transverse vibration. In this particular case specially we will be using the Galerkin method for development of the elemental equation of the shaft element for mass and stiffness, and forces internal and the external forces. And through examples we will see the applicability of the method for one case. Especially in that the assembly procedure the matrices and application of the boundary conditions will be explained. And so we will be starting with let us say a beam which is getting into the bending vibration due to the transverse loads. Loads can be either distributed load or concentrated load.

And in this particular case we are assuming the Euler Bernoulli hypothesis or the theory of beam. In which we assume that a particular plane of a beam remains plane after bending also that particular plane which is perpendicular to the neutral axis of the beam, after binding also it remains perpendicular to the neutral axis.

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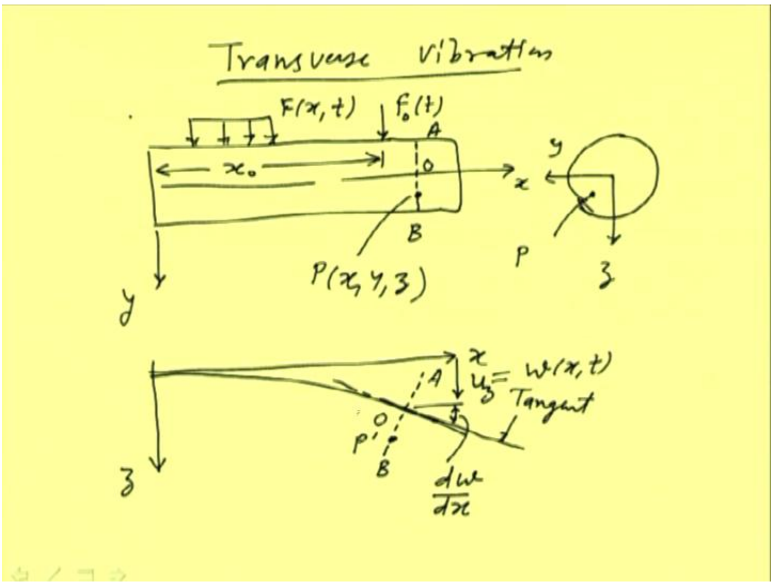

So, let us see this through the diagram. So, we are dealing with the transverse vibration, and in this say there is a beam, there is axis of the beam this is the vertical axis and on this beam there are various kind of loads acting. Let us say this is the distributed load, which is function of x and t there is concentrated load also, which is function of time and the location of the concentrated load is let us say x 0.

And if we consider 1 plane let us say AB and, a point on this P this particular point P the coordinate of that is x y and z. And in the side view, this is the cross section of the beam. And that is circular in nature, and this is the axis point P is somewhere here. Now, let us draw the another diagram in which we want to show the elastic line of the shaft in the bend configuration.

So, this is the elastic line of the shaft which is bend. Now, this particular plane AB is here because, it remains plane. And even it is perpendicular to the neutral axis after bending also. And if we draw a tangent to the neutral axis this is the tangent, to the neutral axis at point O, this is the point O. So, you can see that the there are 2 displacements which is taking place 1 is the linear displacement of the point O. That is let us say we are representing.

Because, that displacement is in the z direction uz is equal to w which is xt it is function of x and t. And this particular slope which is d w by d x and, because of this slope you can see that particular point P which was originally here has occupied this position P here. Let us say P prime.

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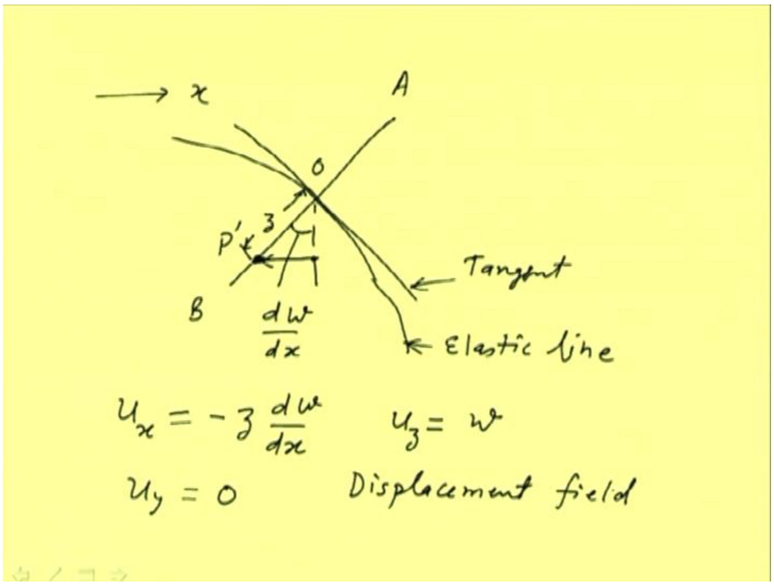

Now, we will draw this particular diagram separately for more clarity. So, let us say this is the elastic line of the shaft this is the line AB. And this is the tangent at the elastic line at O, this is the tangent this is the elastic line of the shaft. Point P is somewhere here P prime let us say, and the location of that was set. Because, initially the coordinate of P toward the downward direction was z.

And this tilting of the plane AB is dw by dx. So, we can see that the displacement of P has taken place from here to this point. So, the displacement of point P in the x direction or x direction is this direction is minus z, this because minus, because it is opposite to the displacement positive x direction. And this much displacement is this angle into the length of this radial distance.

So, it is the displacement of the point P in x direction. According, to hyper Euler Bernoulli hypothesis the displacement in the y direction. That is a plane which is perpendicular to the plane of the board or the screen is 0.

So, when we are applying a transverse vertical load there is no displacement in the horizontal direction. And uz is w. So, this is the displacement field. This is the displacement field and once we have the displacement field we can able to obtain the strain and stress. And then, we can obtain the strain energy, kinetic energy and work done on to the beam. And that will be useful for developing the equation of motion of the beam.

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$\epsilon_{xx} = -z \frac{\partial^2 w}{\partial x^2}$ and rest are zero

$\sigma_{xx} = -E z \frac{\partial^2 w}{\partial x^2}$ and rest are zero.

$$U = \int_V \left[\frac{1}{2} \sigma_{xx} \epsilon_{xx}\right] dV$$

$$= \frac{1}{2} \int_0^l E\, I_{yy} \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx$$

$$I_{yy} = \int_A z^2 \, dA \quad \text{——}$$

So, with this is the strain field is given as only 1 strain will be nonzero. Then others will be 0 and rest are 0. Similarly, the stress now we can able obtain. Stress also only 1 components component will be non zero, because this will be 0. Using this we can get the potential energy of the system, which is summation of the stress and strain, because other component are 0. So, we will not be having those components. So, if you substitute this 2 expressions here, and simplify you will get for these terms. Where Iyy is given as z square dA. So, this is the potential energy of the system. Now let us see the kinetic energy of the system.

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$$T = \int_0^l \int_0^A \frac{1}{2} \rho \left(\dot{u}_x^2 + \dot{u}_y^2 + \dot{u}_z^2 \right) dA\, dx$$

$\dot{u}_x \approx 0 \qquad \dot{u}_y = 0 \qquad \dot{u}_z = \dot{w}(x,t)$

$$= \frac{1}{2} \int_0^l \rho A \left(\frac{\partial w}{\partial t}\right)^2 dx$$

$$W = \int_0^l \left[F(x,t)\, w(x,t) + F_0(t)\, \delta^*(x - x_0)\, w(x,t) \right] dx$$

Direc delta fn.

$\delta^*(x - x_0) = 1$ for $x = x_0$

$= 0$ otherwise

So, kinetic energy will be given as various velocity components. And only this velocity component is there sorry, the in this velocity or this is approximately 0.uy is already 0, and uz will be wxt. If you substitute this we will get, the kinetic energy of the system like this. Now, work done by the distributed force and the concentrated force can be written as, this is the distributed force this is the displacement, plus this is the concentrated force, and we will be using a direct delta function for this. So, this is the work done. Regarding direct delta function it has certain properties, let us see these properties. It is equal to 0 x minus x naught is 1 for when x equal to x naught and it is 0.

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$$\int_{-\infty}^{\infty} g(x)\,\delta^{*}(x-a)\,dx = g(a)$$

$$L = T - U - W$$

$$\int_{t_1}^{t_2} \left[\delta (T-U) - \delta W\right] dt = 0$$

Otherwise, also it has integration property which we will be using it. If there is a function gx, let us say, and we are integrating a direct delta function along with it. So, this will give us this. So, this property we will be using it for this purpose. So, once we have got the expression of the kinetic energy, potential energy and work done. We can able to use the Lagrangian and you with the help of Hamilton principle we can able to obtain the equation of motion.

Let us see, what is the expression for Lagrangian t minus u t minus t w. And the Hamilton principle stated like this, t minus u delta w dt is equal to 0. So, using Hamilton principle in the earlier lecture we already explained, How to get the equation of motion. So, I will be is skipping these steps, and finally I will be giving the final equation of motion which will be getting from this energies.

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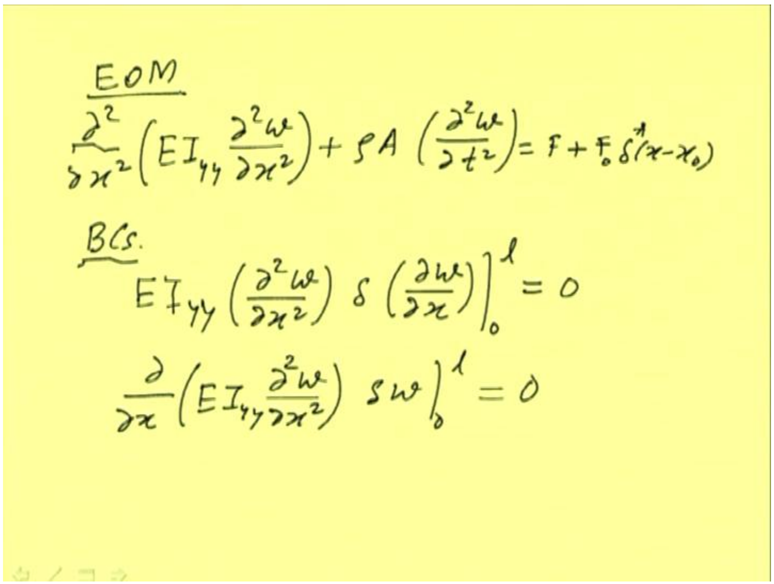

So, the equation of motion which we will be getting. So, this is the term corresponding to the elasticity of the system then, this is the term which is corresponding to the inertia, and this is the external force continuous and the concentrated force with the direct delta function. And boundary conditions, and the boundary conditions are our first term is the bending moment. And this is the variational operator, this is the slope these are the limits.

And second boundary condition, this is the shear force and this is the displacement. Now we will be analyzing these equations through finite element method. As you know this finite element method is approximate method in which we assume the solution in the form of polynomial, and we substitute these solution in the equation of motion. And obviously, because these solutions are approximate. So, some residual will be left out all the equations will not be satisfied. And using Galerkin method we will minimize this residual to get the elemental equation for the finite element of the transverse vibration of the shaft.

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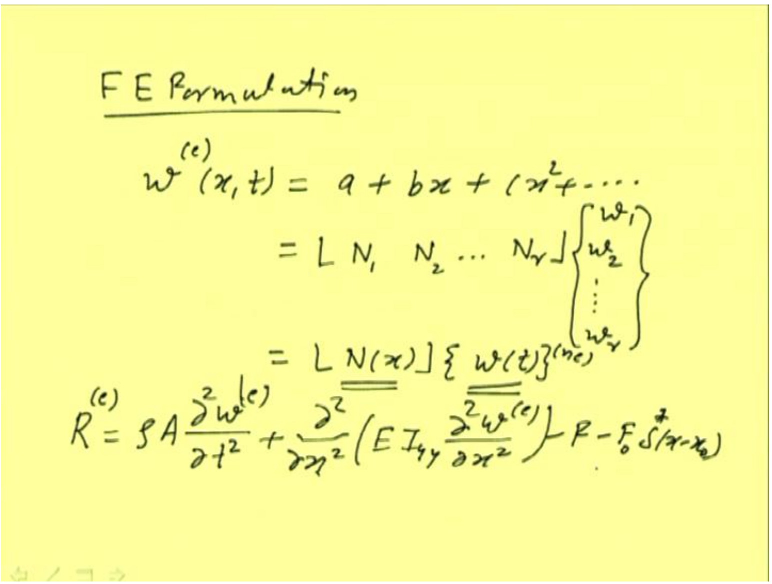

So, for this first we assume the solution the displacement for a element in a polynomial form. Let us say a plus bx plus cx square can be of several terms. Number of terms finally, it depends upon the type of problem we are handling at present we are keeping this in the general form. And this can be expressed as various functions. Let us say, they are r in number and corresponding displacements at various nodes we have. And in more general form it can be written as

So, here N is function of the x only and w is function for time. So, once we substitute this approximate solution into the equation of motion the residual will be, this is the residual, rho A this is the inertia term. Now, in all displacement terms we are writing for the element, this is the this is the elastic term. Then, forces…

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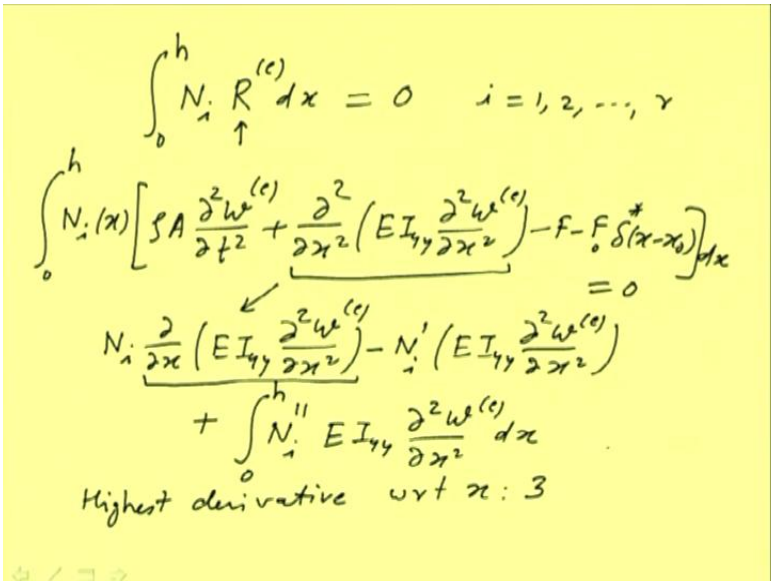

Now, we will be minimizing this particular residual over the element by multiplying by shape function. And we will get this to 0 and where i is from 1,2 r. Now, we are substituting the residual here. So, this is the residual terms, this I am giving all the steps intermediate steps. So, that all the steps are clear is equal to 0. Now, in this particular term this particular term will be doing integration by part. So, that we can have less tangent compatibility in the completeness condition for the assumed solution which, we are assuming in a polynomial form.

So, let us do the integration by part of the this particular term. So, this will give us rest of the terms will remain same. First term, take integration of the second term. Then, The differentiation of the first term, and integration of the second term, plus differentiation again of the first term and second term as it is. And in this limit is from 0 to h. h is the element length. So, this particular second term we have expanded like this. And, so from here you can see that now the highest derivative in the expressions are now, are here that is three. Highest derivative with respect to x is 3 with respect to x is 3.

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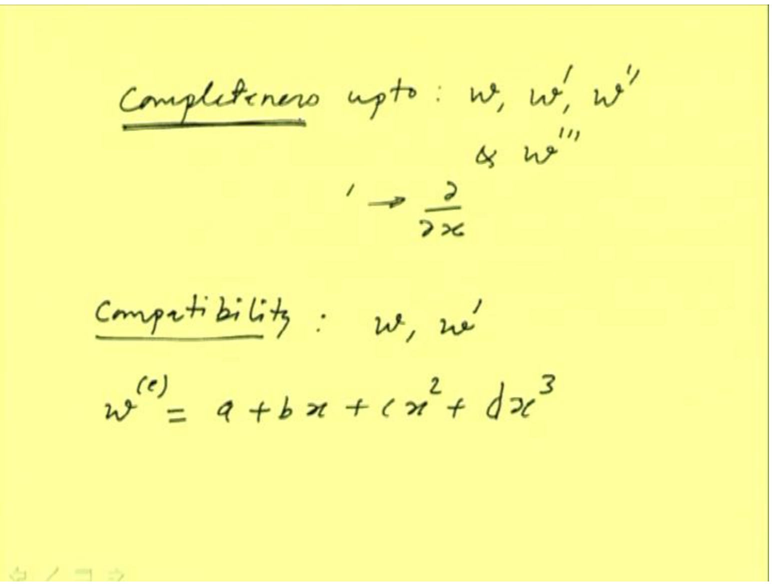

So, we require the compatibility the completeness up to the I can go to next slide. So, completeness up to w 1, w prime, w double prime and w 3 primes. This primes represent derivative with respect to x. So, you can see that a cubic polynomial will able to satisfy this completeness condition. Also, in the previous term if we see the inside the integral. The maximum derivative is second, inside the integral. After, taking the integration that part. So; that means the compatibility condition we require of w and w prime; that means, 1 order less. Because, is having derivative up to second.

So, 1 order less compatibility. That means, w 1,w prime compatibility is required for the shape function to satisfy. So, as we have seen that for completeness the cubic polynomial is sufficient. Because, it will not vanish if we take the third derivative of w a constant term d will be remaining. So, it satisfies the completeness condition. Now, let us because these abc are the constants that need to be obtained. And that we will be obtaining through the boundary condition of the element.

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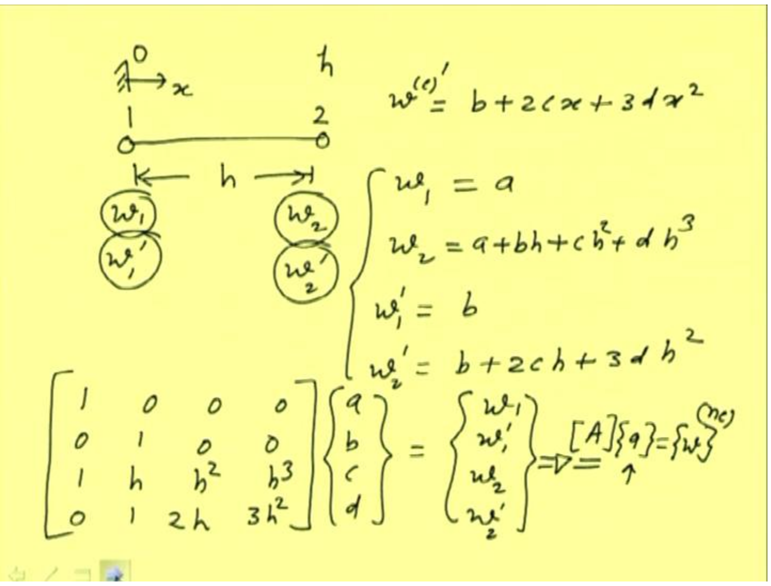

So, let us say this is the element the length of the element is h. It is the beam element and nodes are: 1 and 2 and, each node is having these are the conditions. Now, here the local axis it can able to assign x. So, this is x equal to 0 and this is x equal to h. So, we have these 4 conditions which we need to satisfy into the polynomial. Polynomial we had of the w but, the prime of that; that means, the derivative with respect to x of that can also be obtained.

Because, it is there in the boundary condition. Now, we can satisfy this 4 boundary conditions this 1 will give us w1 as a. Then, second boundary condition will give us w2 as a plus bh. Because, we have to substitute for x as equal to h. Then, the third boundary condition of this will give us w1 prime equal to b, and the fourth will give us b plus ch plus 3 d h square.

So, we are not 4 equations and 4 unknowns are there. These expression can be put in a matrix form like this. w1, w1 prime, w2, w2 prime the ordering has to be taken care. Ordering is not important thing because, we can arrange these equations in any form. So, here we will be having terms like this: 0 0 0 0 1 0 0 1 h h square h cube 0, 1, 2h 3h square. Now, to get a, b, c, d we have to need to invert this matrix. Let us say, this particular matrix we are writing in more compact form as A a is equal to w. So, we need to obtain this. So, we need to the invert the A matrix.

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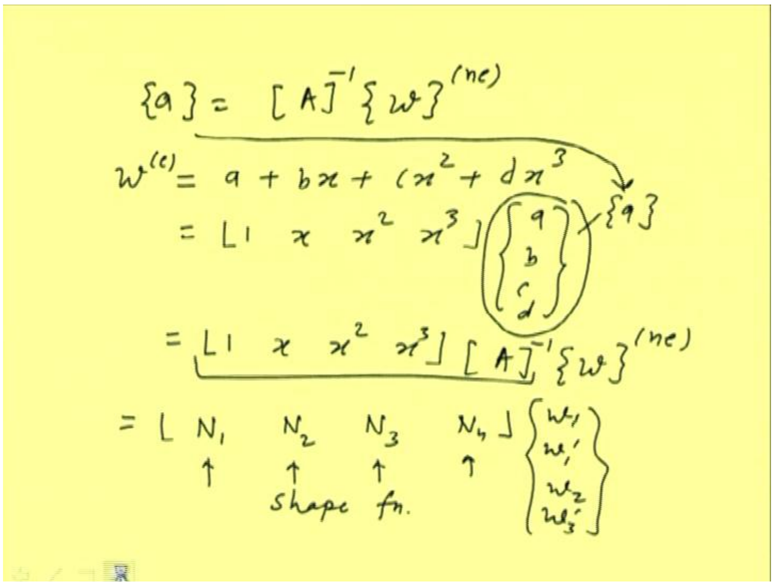

So, a vector we can able to write if we invert the A matrix and multiply by the w vector. And, this polynomial which we had earlier that is a, plus bx, plus cx square, plus dx cube. This can be written as 1 x, x square, x cube and a b c d or which can be written as, 1 x, x square, x cube. Because, this quantity is nothing but, small a vector. So, this we can able to substitute here.

So, once we substitute that there we will get A inverse A inverse w ne. Now, we will combine these 2. So, we will be getting 4 terms I am writing N 1,N 2,N 3,N 4 and these are w 1,w 1 prime, w 2, w 3 prime Now, I will be a giving expression for these shape function they are called shape function, they are function of x.

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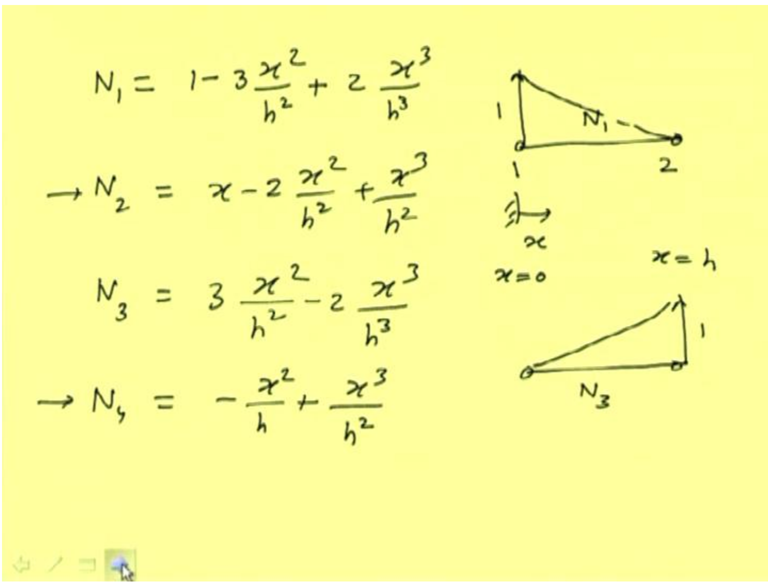

So, these functions can be expressed as If we multiply those vectors. They are in terms of polynomial, these shape functions are having unique property which we will see subsequently. So, this is the third shape function and the fourth shape function is x square by h, plus x cube by h square. Now, this shape functions have unique property. Let us say, a particular element is node 1 and 2.

So, if we want to plot the N 1 over this you can see that when x is equal to 0. This is the x axis here x is equal to 0 here x is equal to h. So, when x is equal to 0 N 1 is having value 1. And when x is equal to h if we substitute here it becomes 0. And in between it varies cubic variation. Similarly, N, N 2 this is N 3 Let us say, N three. N 3 is having opposite property when x is equal to 0 it becomes 0 and when x is equal to h it becomes 1 in between it is having this property.

Similarly, N 2 and N 3 there derivatives are having similar property that we will be using it in subsequent analysis. So, once we obtain the displacement now, we should substitute those into the equation of motion and that will give was the elemental equation.

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$$\int_0^h N_i \underset{\uparrow}{R^{(e)}}\, dx = 0 \qquad i = 1, 2, \dots, r$$

$$\int_0^h \underline{\underline{N_i(x)}}\left[\rho A \frac{\partial^2 w^{(e)}}{\partial t^2} + \underbrace{\frac{\partial^2}{\partial x^2}\left(EI_{yy}\frac{\partial^2 w^{(e)}}{\partial x^2}\right)} - f - F_0^*\,\delta(x-x_0)\right]dx = 0$$

$$\underbrace{N_i \frac{\partial}{\partial x}\left(EI_{yy}\frac{\partial^2 w^{(e)}}{\partial x^2}\right)} - N_i'\left(EI_{yy}\frac{\partial^2 w^{(e)}}{\partial x^2}\right) + \int_0^h N_i'' EI_{yy}\frac{\partial^2 w^{(e)}}{\partial x^2}\, dx \qquad \underline{i = 1, 2, 3, 4}$$

Highest derivative wrt x : 3

So, we will be using these equations which we have already earlier obtained. Here you can see that the I subscript is there which is varying now from 1, 2, 3 to 4. Because, now we have 4 shape functions. So, it will be varying up to 4. So, these are the 4 such equations. So, let us combine them first and then we will substitute the displacement here.

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$$
\int_0^h \rho A \{N\} \frac{\partial^2 w^{(e)}}{\partial t^2} dx + \int_0^h E I_{yy} \{N''\} \frac{\partial^2 w^{(e)}}{\partial x^2} dx
$$

$$
= - \begin{Bmatrix} N_1 \left(E I_{yy} w^{(e)''}\right)' \Big|_0^h \\ N_2 \left(E I_{yy} w^{(e)''}\right)' \Big|_0^h \\ N_3 \left(E I_{yy} w^{(e)''}\right)' \Big|_0^h \\ N_4 \left(E I_{yy} w^{(e)''}\right)' \Big|_0^h \end{Bmatrix} + \begin{Bmatrix} N_1' \left(E I w^{(e)''}\right) \Big|_0^h \\ N_2' \left(E I w^{(e)''}\right) \Big|_0^h \\ N_3' \left(E I w^{(e)''}\right) \Big|_0^h \\ N_4' \left(E I w^{(e)''}\right) \Big|_0^h \end{Bmatrix}
$$

$$
+ \int_0^h \{N\} \left[F + F_0 \, \delta^*(x - x_0) \right] dx
$$

So, now after combining those 3, 4 equations it will take this form. So, now in place of Ni it is having a vector form. Here we have double derivative of N. And the internal forces I am writing in the expanded form double prime inside and outside another prime 0 to h N 2, E, Iyy, wt double prime, single prime outside the bracket 0 to h. So, this can be, so these are basically shear forces N fourth term.

Similar, terms are there for the bending moment, 0 to h this for bending moment. These terms will get simplified once we start substituting the boundary condition 0 and h condition. Because, we know the shape functions are having some inherent property either, they become 1 or 0 at these ends. And apart from this the external force components this is the distributed force and this is the concentrated force and this is the direct delta function. Now, as we know the property of N it is 1 for x is equal to 0

But, it is 0 for x is equal to h. So, those property we will be using here in all other places. So, these equations will get simplified. So, these equations will get simplified and apart from this will be substituting the displacement which we obtained earlier here in all other places. So, let us substitute that.

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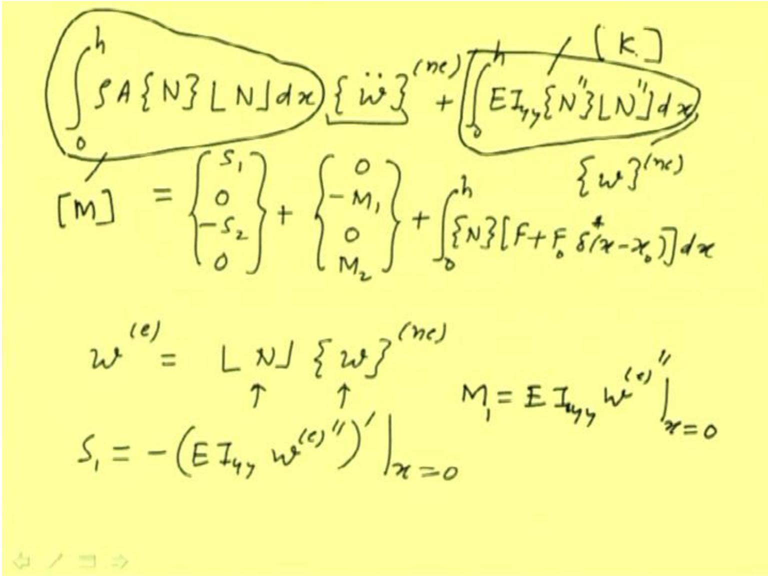

So, we will get 0 to h rho A N which is vector then rho vector of N dx or w will take the time derivative. Actually what we are doing we are substituting for w which is like this. So, this is function of x and this function of time. So, time derivative is taken by the w ne and this is inside. Because now this not function of x. So, it can be taken out. Similar, terms are there for the inertia or for the similar terms are there for the elastic force elastic term. Here and is having double derivative this is equal to and here we have w ne.

So, earlier the expressions after substituting the value of N1 and N2 another terms for x is equal to h and x is equal to 0.They will take this simple form these are nothing but, shear force and binding moment at node 1 and 2.And this is the external force term which remains as it is as previous 1. Regarding, the shear force S1 is given as E, Iyy, w e double prime whole single prime at x is equal to 0.

Because, x is equal to h N1 was 0. So, that term will not contribute in the first term. This is coming from the previous expression here, so N1 is 0 for x is equal to h and it is 1 for x is equal to 0. So, that gives us the S1 and this is the S2 term. Similarly, this is M1 term, M2 term other terms are getting vanished. So, M1 is E, Iyy, w e double prime at x is equal to 0.So, now you can see that this particular expression. We will be calling as a M matrix which will call as mass matrix this term we will call it as a K matrix stiffness matrix and let us, it those expression separately.

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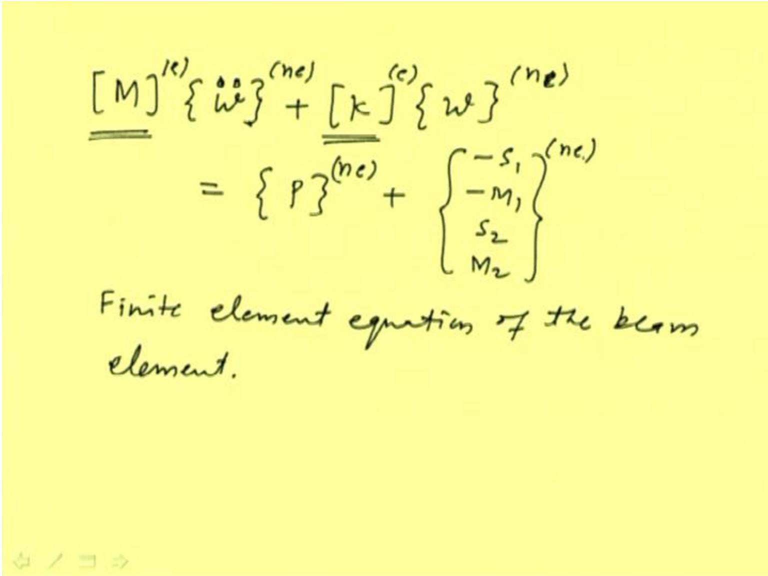

So, this is the element matrix this is the an acceleration term. Then, stiffness matrix this is the displacement term at various nodes. Is equal to the external force plus the internal force and moments. Because, this is the elemental equation for element 1 having node 1 and 2. So, these 2 nodes are appearing here. So, this is the elemental finite element equation of the beam element. Now, we will see these term in the expended from they are all elemental equation this at the node.

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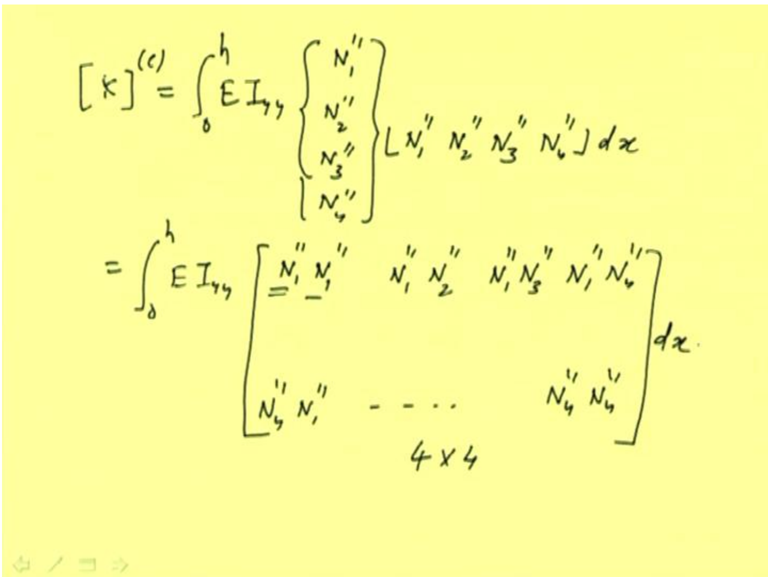

So, let us see the stiffness matrix first. Which is given as now, I am expanding this N this is a rho vector of N dx. So, if we multiply this we will get N 1 double prime, N 2 double prime, N1 double prime. This will be N1 itself this will N2 double prime, N1 double prime, N3 double prime, N1 double prime, N4 double prime. So, it will go like this here it will be N4 double prime. So, other terms similarly we can able to obtain. This is a basically 4 by 4 matrix. So, because we know N1 and other terms there derivatives can be obtained and it can been integrated.

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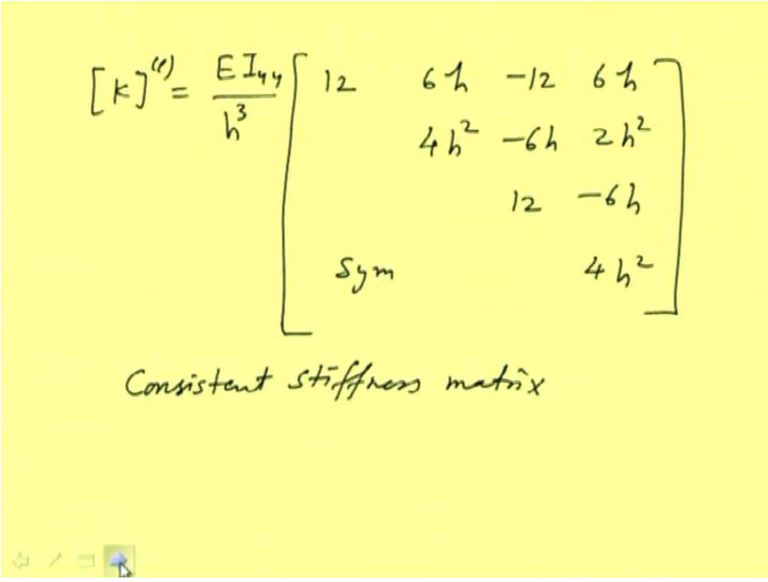

So, if we integrate this the K matrix finally, will of this form.12, 6h, minus 12, 6h, 4h square minus 6h, 2h square, 12, minus 6h 4h square and this is symmetric matrix. So, this is the consistent stiffness matrix.

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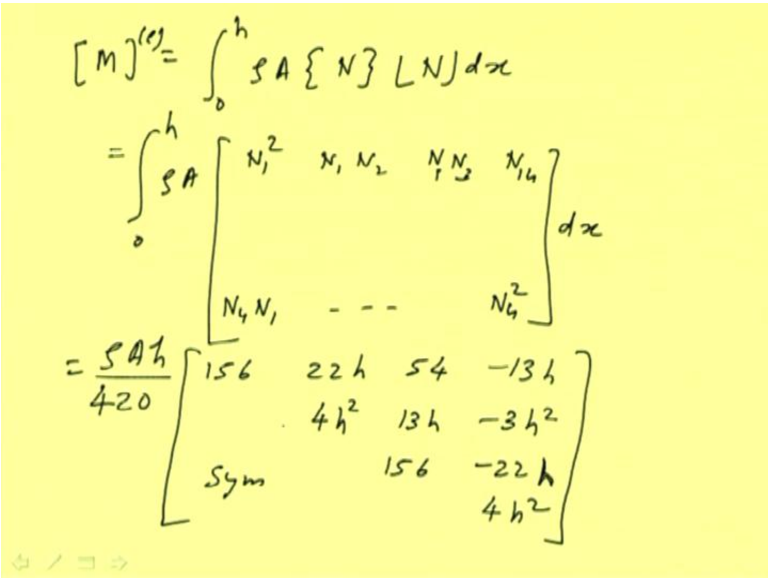

And on the same line we can able to obtain the mass matrix which is given by this. So, if we expand this as we diagnosis before this will be N1 square, N1, N2 primes will not be there in this case. And if we integrate this we will get rho Ah by 420,156, 22h, 54, minus 13h. In this also it is symmetric. So, I writing 1 of the matrix this is h, 4h square the symmetric matrix.

So, this is the elemental mass matrix once we have seen the expanded from of the mass matrix and stiffness matrix also the internal force vector that contain shear force and bending moment. Let us see, how the force of matrix the external force matrix takes the form. Specially, when we are having different kind of load distribution either, it is linearly varying or it is that is uniformly distributed load or concentrated load how the elemental or equations for that can be developed.

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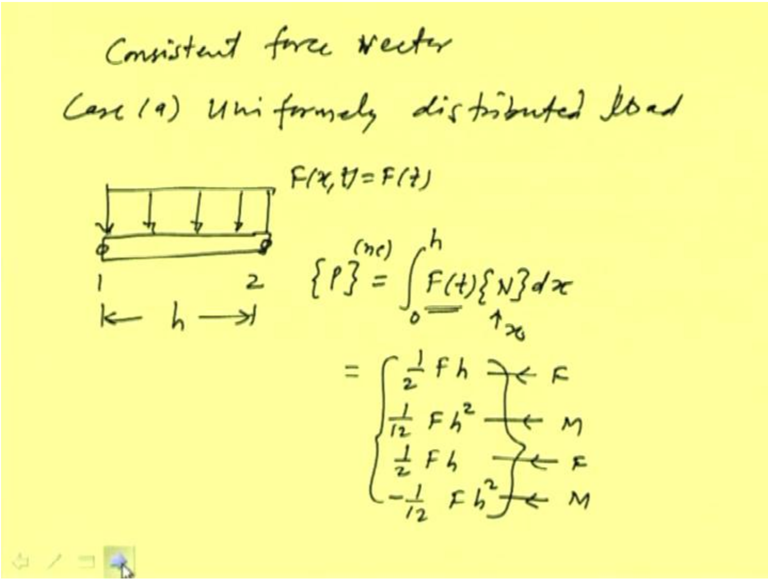

Let us say, first case we are considering which is uniformly distributed load uniformly distributed load. So, in this case this is the element 1 and 2 nodes are there this is the element length load is acting uniformly from here to here. And this is let us say, F some constant value. Now, the external load vector can be written as 0 to h integration over the element Ft. Because, now it is constant N dx this is coming from the previous equation of motion of the element.

And if we substitute because, this is the constant term it will go out. If we because, this is function of x. So, if we integrate it we will get half Fh, 1 by 12Fh square again half Fh and minus half Fh square. So, this is the consistent force vector when distributed load is there. You can see that these are corresponding to the forces and they are corresponding to the moments. These are forces and these are moments as we add in the internal load also these where the shear forces and these where the moments.

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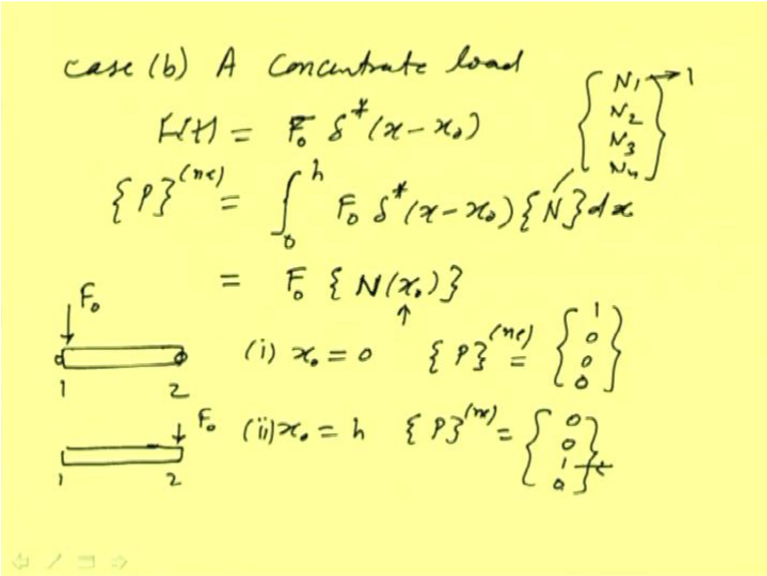

Now, let us see case b in which a concentrated load is acting. So, for concentrated load Ft is given as F star x minus x naught. And P vector is given as integration 0 to h F naught this is the F naught direct delta function N. And, so basically as we see the property of the direct delta function by which we can able to say that. This we will get we have to just substitute in all N's x is equal to x naught.

So, we will get the integration of this quantity. So, let us see this particular thing for different cases of concentrated load. So, let us say a force is acting here at node 1. So, for this case x naught is 0.Because, this is the position of the concentrated load. So, if we substitute x naught equal to 0 the P vector will be 1, 0 0, 0. Because N is nothing but, it contain N1, N2, N3, N4.

And you can be able to check that at x equal to 0 only N1 is having 1 value all others are having 0 value. Similarly, if F naught is acting at node 2 then x naught is equal to h and P will take the form of 0 0 1 0.So, because only N 3 will be 1 others will be 0.

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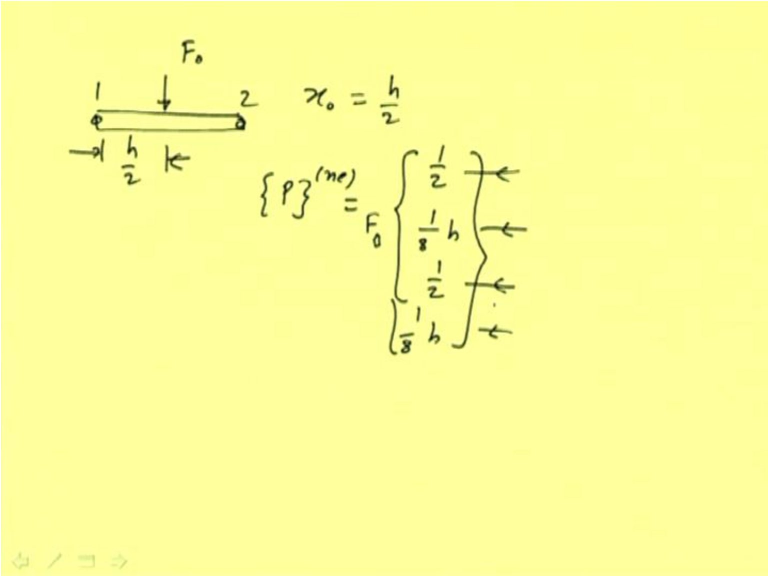

So, in third case when the load is acting in between the node. Let us say, at middle of the element h by 2.So, in this case x naught is h by 2 and if we substitute this in the previous expression we will get this as half 1 by h again half 1 by 8h.So, you can see that when load is acting the middle not only the forces also the moments will also be appearing at the nodes. We have seen the how the load vector takes the form when a uniformly distributed load is there or when a concentrated load is there.

Here, we are talking about consistent force vector and consistent mass and consistent stiffness matrices. These are there are another version of these matrices that is called lump mass or lump stiffness matrix or generally lump mass and the lump force matrices are there. They can they simplify the analysis because of this specially when we are doing the lump mass analysis various forces various inertia forces get decoupled and we have more simplified version of the equation of motion So, especially now let us see another form of the force which is distributed force. And linearly varying how the we can able to up get the load it is load matrix load vector how we can get the load vector.

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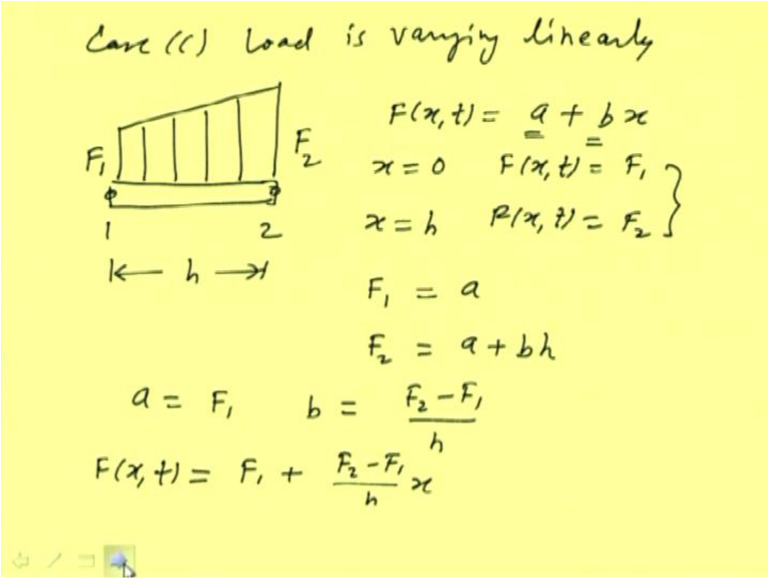

So, this is the case c in which load is varying linearly. So, for this case this is the element and load is varying from 1 into another end let us say amplitude it is series F1 here it is F2 and in between it is varying linearly. So, because force is varying linearly with the element length. So, we can able to write this as a linear function Here a and b is some other constant they are not to related with previous a and b.

So, we have boundary condition that x is equal to 0 we have F, x, t as F1 and we have at x is equal to h F,x t is F2. So, these 2 conditions can be used to obtain the a and b. So, let us substitute 1 by 1.So, first condition will give F1 is equal to a second condition will give F2 is equal to a plus bh and these 2 can be solved for a and b. So, we will get a as F1 and b as F2 minus F1 by h. So, the force which is varying linearly can be written as F1 plus F2 minus F1 by h x.

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$$F^{(e)}(x,t) = F_1 + \frac{F_2 - F_1}{h} x$$

$$= \left(1 - \frac{x}{h}\right) F_1 + \left(\frac{x}{h}\right) F_2$$

$$= \left\lfloor \left(1 - \frac{x}{h}\right) \quad \frac{x}{h} \right\rfloor \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$= \lfloor N_f \rfloor \{F\}^{(ne)}$$

$$\{P\}^{(ne)} = \int_0^h \underbrace{\{N\} \lfloor N_f \rfloor} dx \, \{F\}^{(ne)}$$

Or it can be simplify or it can be simplified as F, x, t for the element as F1 plus F2 minus F1 by h x. So, the F1 and F2 can be separated. So, we will be getting expression 1minus x by h F1 plus x by h F2 which can be written as 1 minus F x by h and x by h F1 F2.Now, this can be written as the shape function for the force and this is the nodal forces. So, if we substitute this in the force vector N was already there now another shape function from the force which is varying linearly will come. And they will be a force ne term.

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$$\{P\}^{(ne)} = \begin{bmatrix} \frac{7}{20} h & \frac{3}{20} h \\ \frac{1}{20} h^2 & \frac{1}{30} h^2 \\ \frac{3}{20} h & \frac{7}{20} h \\ -\frac{1}{30} h^2 & -\frac{1}{20} h^2 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$= \begin{Bmatrix} \frac{7}{20} h F_1 + \frac{3}{20} h F_2 \\ \left(\frac{1}{20} F_1 + \frac{1}{30} F_2\right) h^2 \\ \left(\frac{3}{20} F_1 + \frac{7}{20} F_2\right) h \\ \left(-\frac{1}{30} F_1 - \frac{1}{20} F_2\right) h^2 \end{Bmatrix}$$

So,now, if we substitute these 2 and integrate we will get a term of P like this even this is coming from the integration. 3 by 20h,7 by 20h and 1 30h square,1 by 20h square. Here force vector is also there at nodes F1 and F2.If you multiply them we will get 7 by 20h F1 plus 3 by 20h F2.Other terms can be obtained on the same lines even we can able to simplify them in more compact form.

So, thus we can see that now they contain terms in all places even at the force and moment place they contain the terms. And contribution of the F1 and F2 amplitudes at either end of the linearly varying force are appearing at every place. So, on the same line if the force is varying with some other function like: parabola then we can able to take the polynomial as quadratic a nature there three constants will be there and shape function for same can be obtained.

So, only thing another node will be requiring in the in the middle of the element. So, that the three constants can be obtained. In the subsequent lecture we will see with some examples application of these element matrix which we have developed. So, today we have already seen that for the transverse vibration or the bending vibration. Especially, for the Euler Bernoulli hypothesis how the elemental matrices of the mass, stiffness and forces takes the form. And in the subsequent lecture we will be explaining through example the assembly procedure of the application of this method even the application of the boundary condition and how we can use this for obtaining the natural frequency or the force vibration of a beam.