Mechanical Vibrations
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Module - 10
Approximate Solutions for Continuous and Discrete Systems
Lecture - 2
Matrix Iteration Method

So, in the last class, we have studied about the approximate methods. And we know they are different approximate methods available to find the natural frequencies of the distributed and discrete mass systems. So, in case of distributed systems and discrete mass systems we have seen the methods are Rayleigh method, Rayleigh Ritz method Galerkin method. Also one may apply for discrete system this matrix iteration method and for discrete system also one may apply this Dunkerley method to find the natural frequency if several masses are acting on a system. So, Holzer method and transfer matrix method also can be used.

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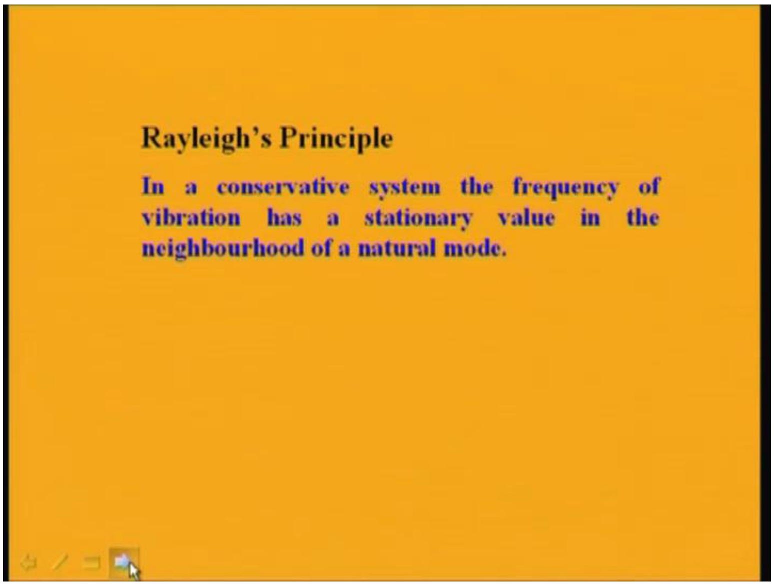

So, we have started with this Rayleigh method and the Rayleigh principle is in conservative system, the frequency of vibration as a stationary value in the neighborhood of the natural modes. So, we have seen, so in the neighborhood of the natural modes the Eigen frequencies are concentrated. So, in a conservative system the frequency of the vibration, as a stationary value in the neighborhood of the natural mode.

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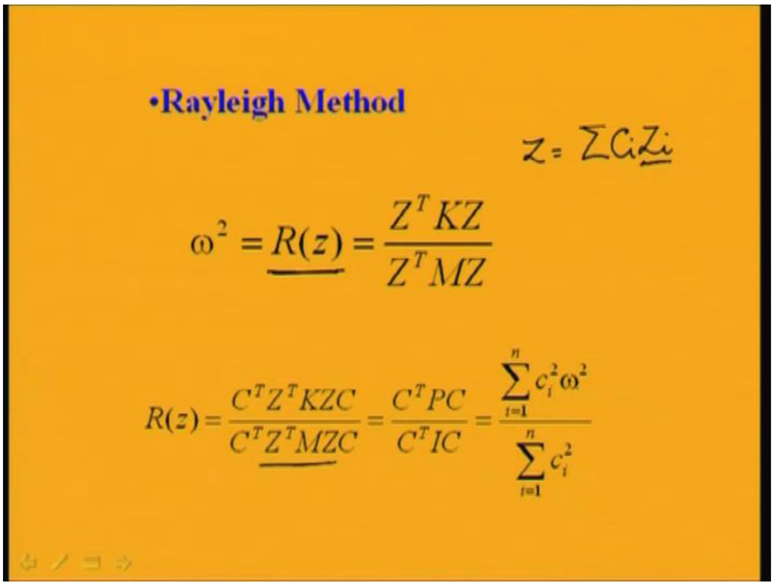

So, without taking the approximate without taking the exact Eigen value which is not known in most of the time one may take an approximate vector and find the Eigen values of the system. So, this is written by this Rayleigh coincident Rz. So, omega square this is the approximate value to find the approximate value we can take a approximate function z. So, by taking that function I can write this Rayleigh coincident equal to z transpose KZ by Z transpose MZ where K is the stiffness matrix M is the mass matrix of the system. So, this is for a discrete system you can write and this Z Z can be taken in such way that this Z transpose MZ can be unity matrix and this Z transpose KZ can be a diagonalized matrix.

So, by taking that way, so one can write this Rz that this is Rayleigh coincident equal to. So, I can write this z as the summation of a vector. So, this is equal to I can write this is summation of C i Zi. So, this Zi is a matrix this is a diagonal matrix and I can write this Rz equal to C transpose Z transpose KZC by C transpose Z transpose MZC. So, the ZR ZR calculated in such way that this Z transpose MZ can be unity matrix and this Z transpose KZ if a diagonal matrix written by P. So, that way if you write, so this becomes C transpose PC by C transpose I into C. So, this P is a diagonal matrix which contain the Eigen vector or the approximate Eigen vector in the diagonal. So, it will be C i omega I square I equal to 1 2 n C i square omega square by summation C i square. So, by using this formula, so we can find.

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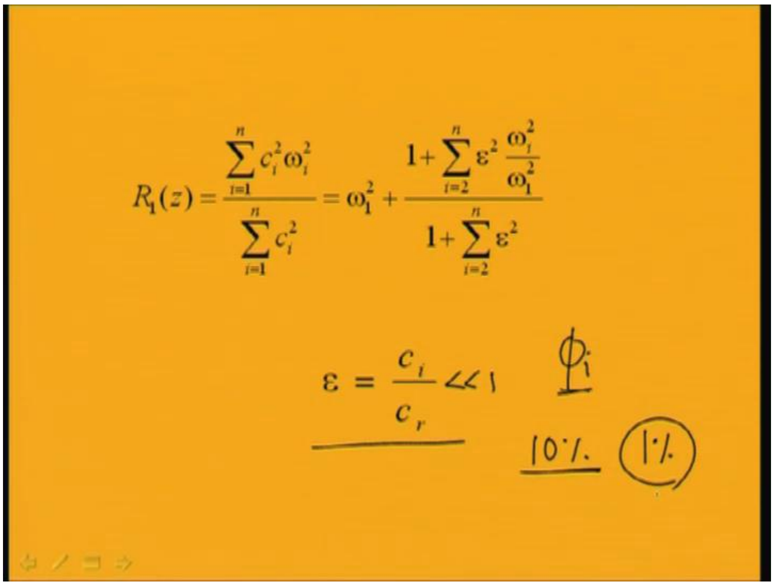

So, let us write for the first mode. So, R 1 z can be written as C i square omega I square by C i square. So, it can be taken c 1 square plus omega 1 square plus c 2 square plus omega 2 square by writing this as omega. So, this can be written in this form this is equal to omega 1 square plus 1 plus summation I equal to 2 to infinity epsilon square omega I square by omega 1 square by 1 plus epsilon square where epsilon is the C i by Cr. So, here we are assuming that the r th mode has a variation from the phi i. So, the exact Eigen value is phi i, but we are taking this as Z R. So, that there is a variation in this phi. So, let the variation due to this variation the C r will be large.

So, the C i by C r for all the modes except r equal to I will be very small and that thing can be taken as a small parameter epsilon which is very, very less than 1. So, epsilon equal to very, very less than 1. So, this R 1 z 1 can find this is equal to omega 1 square plus 1 plus this term. So, this is the additional term or one can tell that this is this will give omega 1 square plus some torque in this decimal term. So, if you taken an error in ten percent in the, so it can be shown that if you take error in ten percent in the Eigen vector. Then there will be a 1 percent error, because it is epsilon square error will be in the order of epsilon square. So, it will be 1 percent error in the Eigen value. So, we have.

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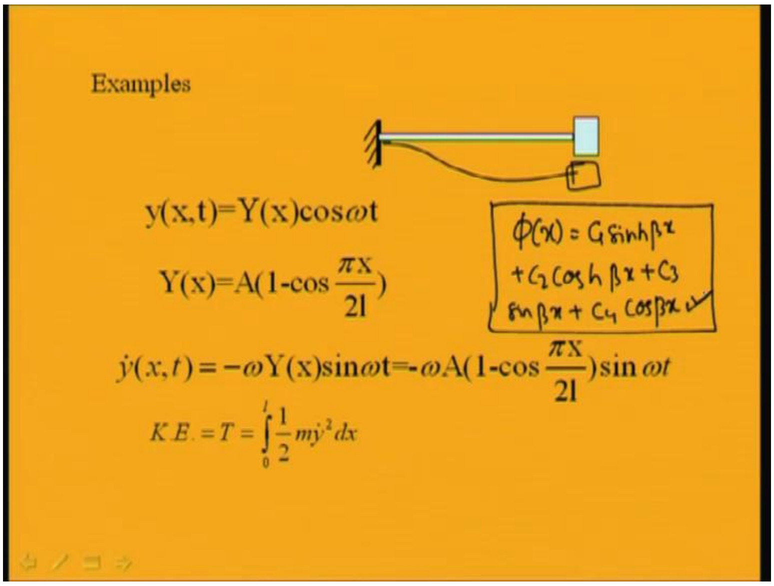

So, using this thing we haves we can solve many problems or we can find the in Eigen values or we can find the natural frequency of the different systems. So, let us consider this system. So, this is a cantilever beam and at the end there is a attached. So, let us find the frequency of the system. So, in this case we can assume the solution to be in this form that is equal to Yx into cos omega t and you can you know that its deflection will be. So, as this end is fixed, so the slope will be 0 and the deflection may takes place like this. So, this is the deflection of the system. So, you can assume this deflection in this form Yx equal to A into 1 minus cos pi x by 21. Let us assume, so by assuming this thing, so if you put this x equal to 0 you can find this Yx equal to 0.

Also if you differentiate this thing then it will becomes we can find that the y dash x also equal to 0 at x equal to 0. So, this satisfy this left hand boundary condition that is include have a slope 0 and displacement 0. So, by taking this function yxt equal to Yx cos omega t where Yx equal to A into 1 minus cos pi x by 21. So, we can find the natural frequency of the system already from the exact solution. You can find the equation you can find the frequency equation and the mode safe from this equation. So, by solving this Euler Bernoulli equation you can find. So, there you can take this phi x equal to phi x equal to C 1 C 1 sin hyperbolic beta x plus C 2 cos hyperbolic beta x plus C 3 sin beta x plus C 4 cos hyperbolic C 4 cos beta x. So, this is the generalized mode safe. So, in this generalize mode safe if you put the boundary conditions.

So, the boundary conditions are at the left end pi x will be equal to 0 and pi dash x will be equal to 0. That is the displacement equal to 0 and slope equal to 0 and in the right end this bending moment will be equal to 0. And the shear force will be equal to the shear force will be equal to the inertia force of this system. So, as you have added a mass, so let this is m, so for this mass. So, this will be inertia force will be m y double dot. So, this m y double dot will be equal to the shear force. So, shear force equal to EI del qy by del xq. So, by taking this generalized pi equation modes of equation and substituting this boundary condition you can find the exact frequency function and the mode safe.

So, in that case you may have to go for the computational method to find the frequencies of the system. As the frequency equation will not be a simple equation you have to go for the computational numerical methods. But we can assume in this approximate method we can assume the solution Yx equal to in this form Yx equal to A cos pi x by 221. So, y dot xt can be written in this form minus omega Yx sin omega t equal to minus omega A 1 minus cos pi x by twenty- 1 sin omega t. So, the kinetic energy of the system it will contain 2 parts 1 due to this beam and other due to this mass.

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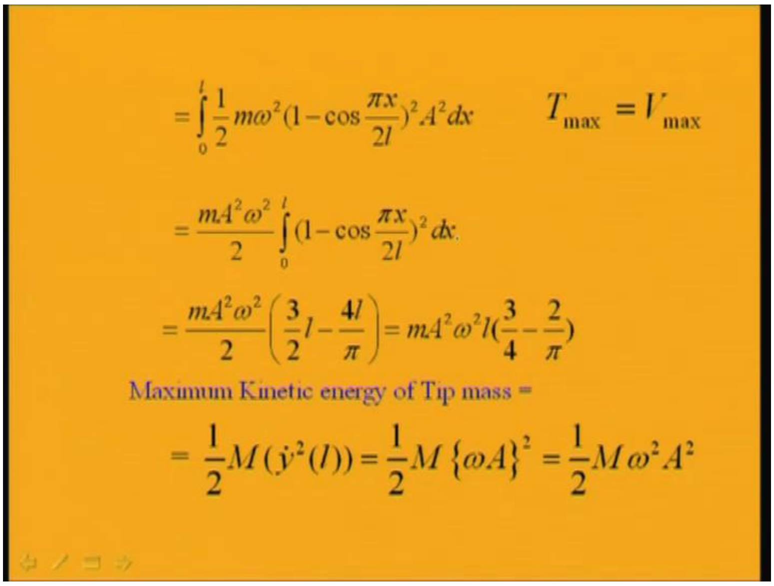

So, kinetic energy due to this beam part will be equal to half m y dot square dx. And due to this mass you can also find. So, this due to this beam let us find first. So, y dot expression by substituting this y dot expression in this integral. So, you can write it equal to 0 to l half M omega square 1 minus cos pi x by 21 square A square dx. So, then you can find this equal to mA square omega square by 2 0 to l 1 minus cos pi x by 2 l into 2 l square dx. So, here it is taken Yx equal to A 1 minus cos 2 pi 1 minus cos pi x by 2 l. So, this is 2 l and so this can be written mA square omega square by 2 into 0 to l 1 minus cos pi x by 2 l square dx. So, this is equal to mA square omega square 2 by integrating this thing you can find it equal to mA square omega square by 2 3 by 2 l minus 4 L by pi. So, this is equal to mA square omega square l into 3 by 4 minus 2 by pi. So, this is the maximum kinetic energy obtained due to the beam vibration. Similarly, we can find the maximum kinetic energy at the tip of the mass.

So, at the tip of the mass we have to find the velocity and half into mass into velocity square will give you the kinetic energy at this tip and we can find the maximum kinetic energy. So, which is given by this equal to half M omega A square as the velocity maximum velocity will be maximum velocity at this end. So, this is the expression for the expression for the velocity. So, maximum velocity at the end will be equal to A omega. So, this part will be equal to 1. So, maximum velocity will be equal to A omega. So, you can write the maximum kinetic energy of this mass will be equal to half M omega square A square where M is the mass tip mass. So, M is the tip mass. So, maximum kinetic of the tip mass already you have obtained and this is the maximum kinetic energy of the beam.

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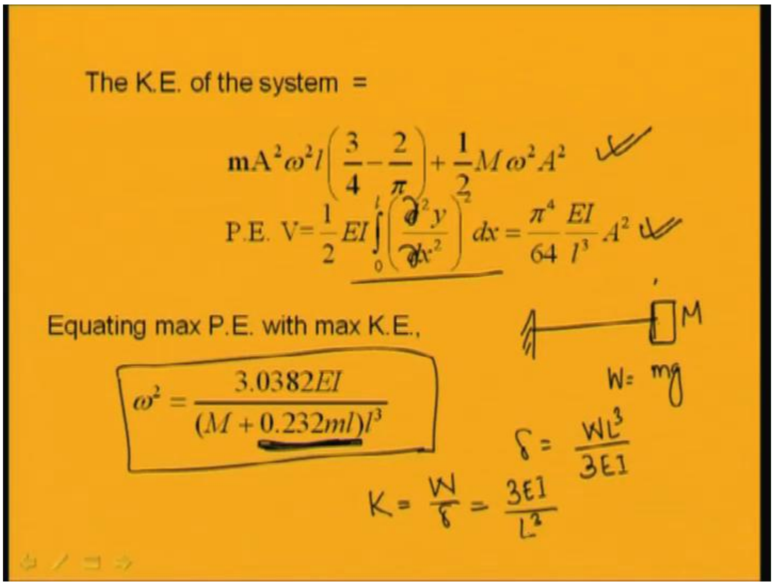

So, by adding these 2. We can find the total energy kinetic energy of the system. So, total kinetic energy of the system equal to mA square omega square l 3 by 4 minus 2 by pi plus half M omega square A square. Similarly, we can find the potential energy and the maximum potential energy potential energy expression can we given by half EI 0 to l d square y by dx square dx. So, this integration you can do by it taking this y equal to. So, already the function y is given to us or know we have taken we have assume this function A into 1 minus cos pi x by 2 l. So, by assuming that function I can that find this potential energy equal to half EI 0 to l del square y by del x square into dx.

So, the maximum value will be equal to pi fourth by sixty-four EI by l cube A square. So, now by equating this maximum kinetic energy with this maximum potential energy you can find omega square equal to. So, omega square equal to you can find this is equal to 3.0382 EI by M plus M plus 0.232 M L into l cube. So, this is the expression for the cantilever beam with an end mass. So, if the end mass is not there or due to the end mass this is the additional term you can you may observe that this is the addition term due to the end mass or this is the additional term due to the mass of the beam. So, we have consider the mass of the beam. So, this is the additional term if you can find or you can do it in this way.

So, if you have only, a mass at the end and this beam is mass less than by considering this weight equal to mg. So, the deflection here I can find equal to this is W equal to mg this is the weight acting at this end. So, the deflection equal to at the end equal to WL cube by 3 EI. So, the deflection at the end equal to WL cube by 3 EI. So, the stiffness of this beam can be written as W by delta. So, W by delta will give me 3 EI by 3 EI by L cube. So, this is the, or this is the stiffness of the beam or the system. So, from this deflection and from the mass tip mass, so this is the tip mass M. So, the natural frequency omega square will be equal to root over K by M. So, root over K by M will give 3 EI 3 EI by l cube by M. So, this omega square you can write.

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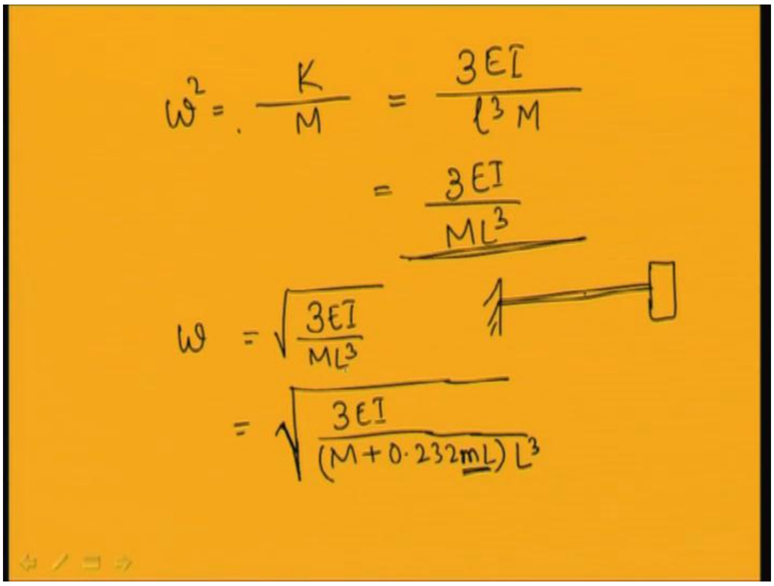

So, omega square will be equal to K by M K by M. So, this K equal to already we obtained this is equal to 3 EI by l cube. So, this is 3 EI by l cube and M equal to M. So, this becomes 3 EI by M L cube. So, this is 3 EI by M L cube, so this is the or the frequency equal to frequency equal to root over K by M. So, this becomes 3 EI by M L cube and when you are taking into account the mass of the beam. So, here we have consider the mass of the beam. So, when you are taking the mass of the beam then we are getting this frequency equal to root over. So, this I can take it equal to 3.

So, this is equal to 3 EI by M plus, so you have derived it. So, this is 3 EI by M plus 0.232 ml, 0.232 into L cube where this M L is the total mass of the beam m is mass for unit length of the beam. So, by taking into mass of the beam into account is expression for the cantilever beam with a tip mass is reduced to this form. So, this is now the frequency of this cantilever beam with end mass becomes 3 EI by M plus 0.232 M L into l cube when you are taking the mass of the beam into account. And this is equal to root over 3 EI by M L cube when the mass of the beam is not considered.

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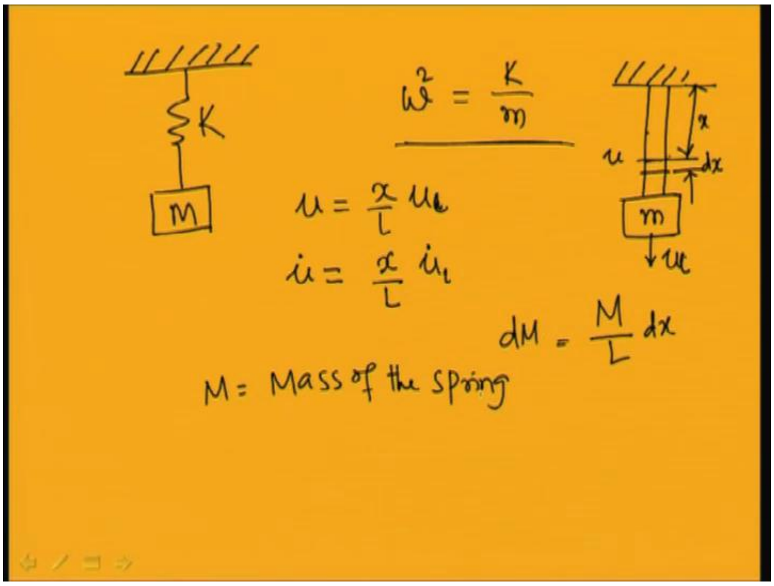

Similarly, you can find for the simpler case of a spring mass system. So, this is spring with stiffness K and this is mass M. So, let us consider the mass of the spring. So, if you consider the mass of the spring. So, without considering the mass of the spring we have obtained the kinetic we have obtained the frequency equal to omega square equal to K by m. So, now considering the mass of the spring we can find also the natural frequency of the system. So, or, so this system is also similar to the system this is the longitudinal vibration of the rod and with an end mass. So, by taking this this system, so these 2 systems are also equivalent. So, let us consider the mass of the spring or in this case the rod. So, in this case let us take at a distance x from this fixed end. So, let us take a small element here, but in this case in the spring also we can take a small element. So, the small element dx let us take a small element dx at a distance x and let mass per unit length equal to m.

So, mass per unit length equal to m, so mass of this element equal to mdx, so mass of this element equal to mdx let u be the deflection at this point. So, if u is the deflection at a distance x. So, I can write this u equal to. So, this u can be written as the. So, if the displacement here equal to ui or ul at this tip. So, I can write this u at any distance will be equal to x by L. So, this is equal to x by L into ui ul let me write equal to ul. So, ul is the displacement at this or distance x a distance L or at the tip. So, this u at any point will be equal to ul by L into x, so this is u. So, I can write this u dot to find the kinetic energy I can write this u dot u dot equal to. So, I can write this is equal to u l dot. Similarly, I can write this mass of this element equal to mass of this element will be equal to let m is the total mass of this rod or of the spring. So, in that case it will be M by L into dx. So, this is the mass of the elemental mass. So, the elemental mass equal to M by L into dx where M is the total mass of this mass of the spring. Or in this case the mass of the rod mass of the spring or the case of in this case it is the mass of the rod.

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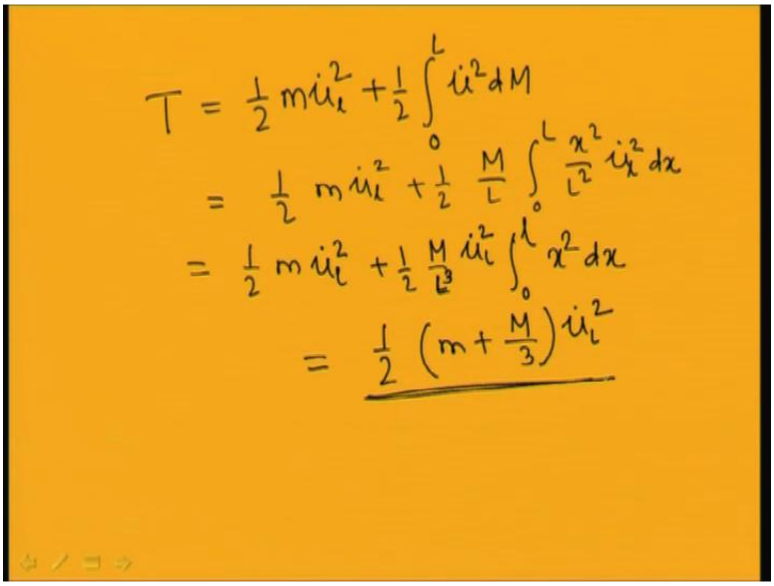

So, the kinetic energy I can write equal to. So, T is the kinetic energy T equal to half. So, this kinetic energy will be due to 2 components 1 due to the spring mass of the spring. And other due to this tip mass for in this case it will be due to this rod and in this case it is due to this mass. So, you have to add these 2 or here you have to add for these 2 effect. So, taking these 2 kinetic energy, so it can be written as half. So, tip mass I am writing a small m, so ul dot square. So, this is the kinetic energy for the tip mass plus for this elemental mass I can write and I can find for the whole beam. So, it will be 0 to l u dot square u dot square dM.

So, this thing can be written as half mul dot square plus M by L 0 to Lx square by L square ul dot square dx. So, we can write it equal to half m. So, this u dot already we have taken equal to x by L ul dot. So, u dot I can write x by L ul dot. So, this becomes mul dot square plus half M by L and this will be ul dot square ul dot square I can take it out. So, this will becomes integration 0 to l and this is l into l square. So, this become l cube, so this becomes 0 to lx square dx. So, this integration will become x cube by 3, so by substituting 0 to l. So, it will be xl cube by 3. So, I can add these 2 terms, so by adding these 2 you can write this is equal to this is m plus M by 3 into uL dot square. So, this is the maximum kinetic energy; this is the maximum kinetic energy of the system.

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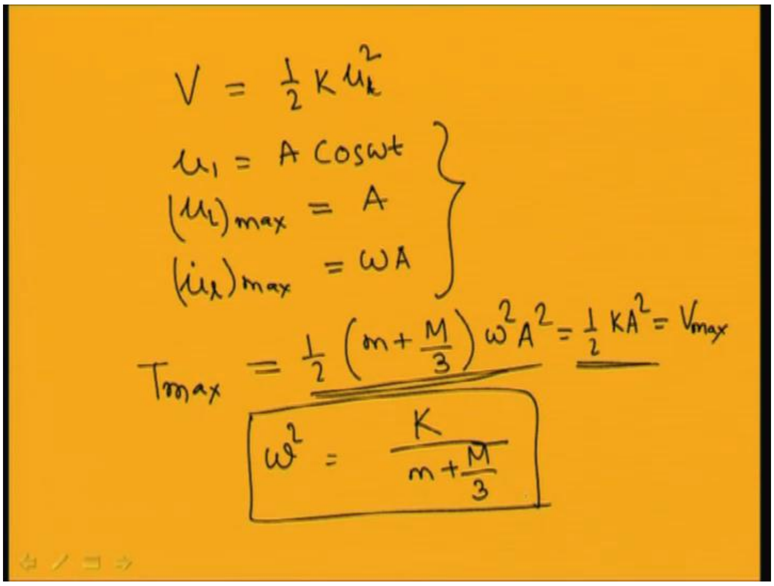

Similarly, the potential energy can be written in this form. So, potential energy V equal to half K ui or ul square ul is the maximum displacement. So, the maximum potential energy equal to half k into ul square. So, let us take this u 1 equal to A or u with time I can take it equal to A cos omega t A cos omega t. So, this ut maximum ul maximum will be equal to. So, the maximum value will be equal to A. Similarly, the maximum velocity ul dot maximum will be equal to. So, this will be A sin omega A omega sin omega t. So, the maximum value will be equal to omega A. So, here we are assuming the time function in this way. So, ul equal to A cos omega t, so ul will be maximum value will be equal to A and ul dot maximum value will be equal to omega A.

So, by substituting these terms in this potential energy and kinetic energy, so T max maximum kinetic energy will be equal to. So, maximum kinetic energy equal to half m plus. So, there we got capital M by 3 omega square A square this is equal to half KA square. This is the maximum potential energy and this is the maximum kinetic energy. So, this is equal to V max, so by equating these 2 I can write this omega square equal to. So, this will be equal to K by m plus M by 3. So, omega square equal to K by m plus M by 3. So, by taking into account the mass of the spring in the natural frequency 1 should take this factor into account that is M by 3.

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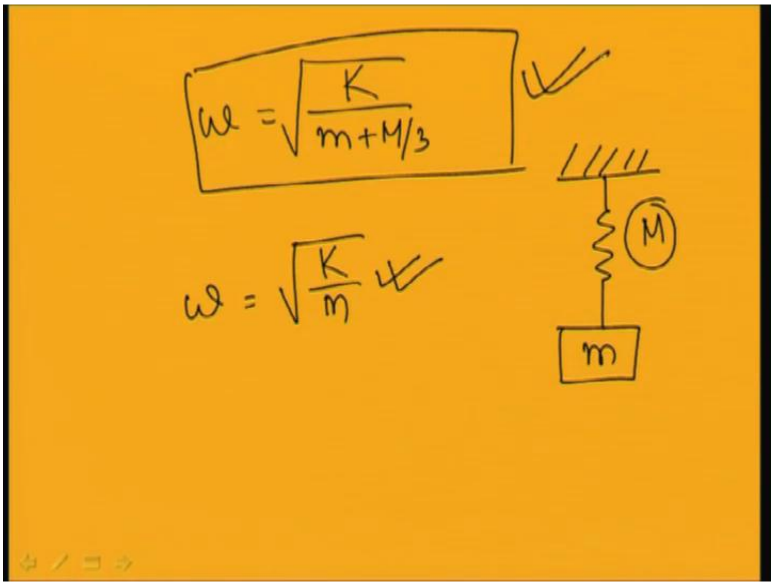

So, by taking into account the mass of the spring one may write this omega that is the natural frequency of the system equal to root over K by this is m plus M by 3. So, where M is the mass of the tip mass, so m, so this is the spring mass system. So, in the spring mass system m is the mass of the mass at the tip and M capital M is the mass of the spring. So, if one consider the mass of the spring then he has to modify the equation for the system without when we are not considering the mass of the spring. So, this is without considering the mass of the spring and this is considering the mass of the spring

So, when mass of the spring is considered. So, one has to take or one has to add one third mass to the tip mass to find the frequency of the system. Similarly, in case of cantilever beam, so we have already derived that in case of the cantilever beam if you consider the mass of the beam. Then factor of 0.232 to be added to the mass of the tip mass at the tip. So, the effective mass this can be considered as the effective mass of the system in that case. So, the effective mass of the system in when you consider the mass of the beam will be equal to M plus 0.232 mass of this beam. Similarly, in case of the spring when we are considering the mass of the spring then it will become one third of the mass of the spring.

So, by considering one third of the mass of the spring you can find the natural frequency of the spring mass system without considering the mass of the spring. So, when we are not considering the mass of the spring then this is equal to root over K by m. And when you are considering the mass of the spring just add one third of the mass of the spring to this mass tip mass and you can find the frequency of the system. So, in this way you can apply this Rayleigh method to find the natural frequency of different systems. Let us consider another example.

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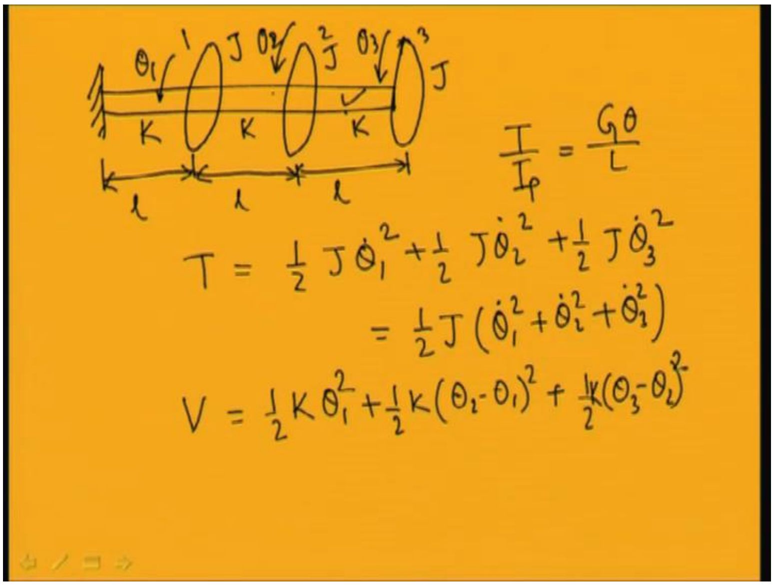

So, in this example we will take a torsional system, so in this torsional system. So, let this is a shaft. So, in the shaft led there are 3. So, let us consider 3 disc mounted on the shaft. So, there are 3 disc mounted on the shaft this is disc 1; this is disc 2 and this is disc 3. So, let us consider the inertia of this at same that is J and this rotation I can write equal to theta one. So, let us write this as theta, so we can write this as theta 3. So, this is theta 2 and. So, this is or let me write let me start from this end. So, this will be in this case it will be equal to theta 3 this is theta 2 and this is theta 1.

So, this is 1; this term 1; this is 2 and this is the third mass we are considering. So, and the shaft let us assume the shaft has equal stiffness in this 3 range this is K. The shaft stiffness and the rotors are mounted at equal distance. So, this is l this is also l and this is also l. So, in this case we are we want to find the natural frequency of the system. So, already you know this K the stiffness of the system can be written from this formula T by

J equal to C theta by L or G theta by L or T by Ip. So, polar moment of inertia of the shaft, so where Ip is the polar moment of inertia of the shaft and T is the applied torque and G is the rigidity modulus and theta is the rotation and L is the length of the shaft.

So, in case of continuous system you have solved similar problems by considering the wave equation. So, for a fixed shaft you have count. So, in that case fix shaft and shaft with one end fixed also you have found and you in that case you have found by using this wave equation. So, solving this wave equation you are found the frequency of that case. So, in this case by taking this approximate method or now we will assume that we will assume the mode safes of the system and we will find the natural frequency of the system. So, if we are interested for finding only the fundamental frequency we can proceed in this way.

So, can assume that when, so the rotation of this or this rotation theta 1 theta 2 theta 3 will be proportionate to the applied torque or it is it will be proportionate to it will be proportionate to the moment of inertia of this rotors. So, now, let us find the kinetic energy of the system. So, this kinetic energy T will be equal to half J theta 1 dot square plus half J theta 2 square plus half J theta 3 dot square. So, this thing can be written equal to as we are considering same J for all the 3. So, it can be written half J into theta dot square plus theta 2 dot square plus theta 3 dot square. So, this is the kinetic energy of the system and the potential energy of the system to find it. So, we should know what is the rotation of each segment?

So, in the segment 1, so it is rotating with theta 1 about this between in segment 2 the shaft will be rotated this side left side by theta 1 and the right side be theta 2. Similarly, in case this segment the left side is rotated by theta 2 and the right side is by theta 3. So, there will be relative motion theta 2 minus theta 3 in this case and theta 3 minus theta 2 in it is this position. So, the potential energy can be written as this potential energy V will be equal to half K theta 1 dot theta 1 square plus half K. So, for this portion I can write it equal to theta 2 minus theta 1 square and for this portion I will write this equal to half k theta 3 minus theta 2 square. So, now I can assume. So, as I do not do not know the exact model function.

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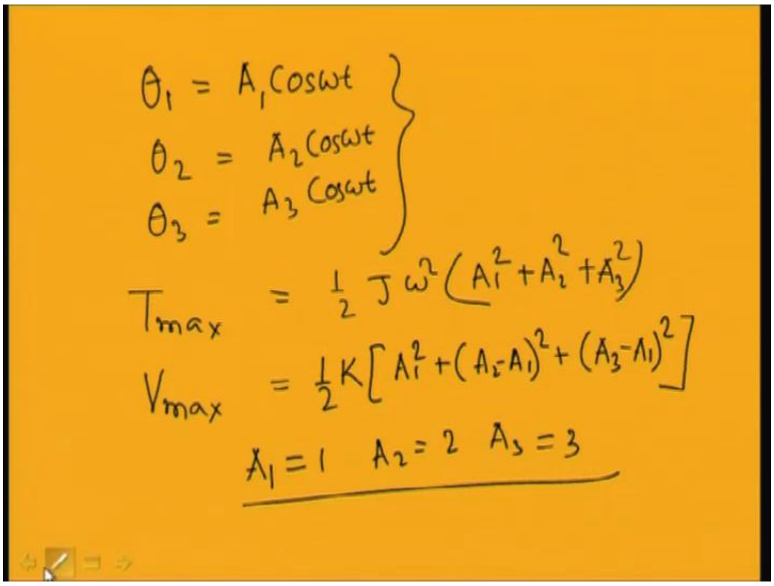

So, I can assume this thing. So, I can assume that this theta 1 equal to A cos omega t or A 1 cos omega t and theta 2 equal to A 2 cos omega t and theta 3 equal to A 3 cos omega t. So, by assuming this thing I can write the maximum kinetic energy is. So, T max will be equal half J omega square. So, this is the expression for kinetic energy. So, this becomes half J theta 1 dot square theta 2 dot square plus theta 3 dot square. So, maximum will be A 1 omega square A 2 omega 2 square and A 3 omega square. So, I can write this equal to, so half J omega square. So, A 1 square plus A 2 square plus A 3 square.

Similarly, V max that is, this potential energy of the system. So, this is the expression for V potential energy here by substituting this maximum theta will be equal to theta 1 will be equal to A 1 theta 2 will be equal to A 2 and theta 3 will be equal to A 3. So, by substituting this thing, so I can write this equal to half. So, this will be equal to half K. So, I can take this K common, so this will become. So, A 1 square plus A 2 minus A 1 square plus A 3 minus A 1 square. So, in this case we can assume this A 1 A 2 A 3 in this ratio. So, I can assume this A 1 equal to 1 and A 2 equal to 2 and A 3 equal to 3. So, I can assume in this case in this rotor.

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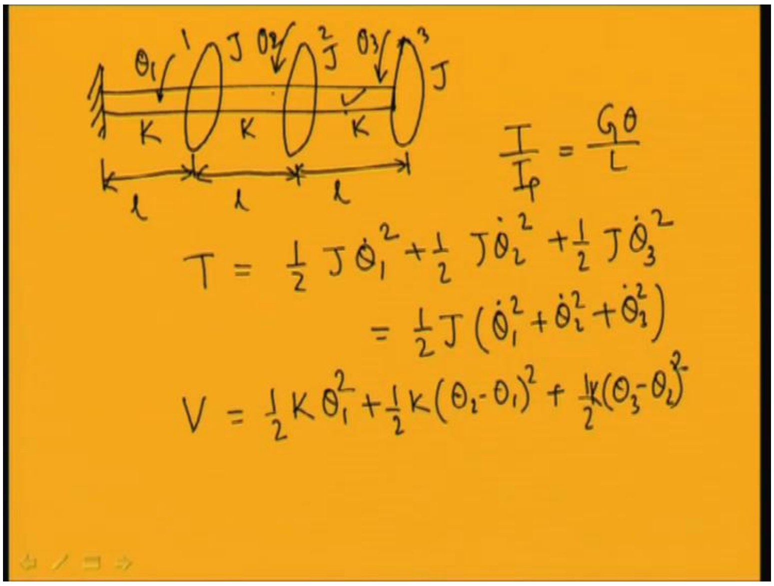

So, when it is rotating this, this is rotating one. So, you know this free end will have more rotation than this and this will this will have the least rotation. So, I can assume this rotation to be equal to 1. So, here it is equal to 2 and the rotation of this 1 equal to 3 unit.

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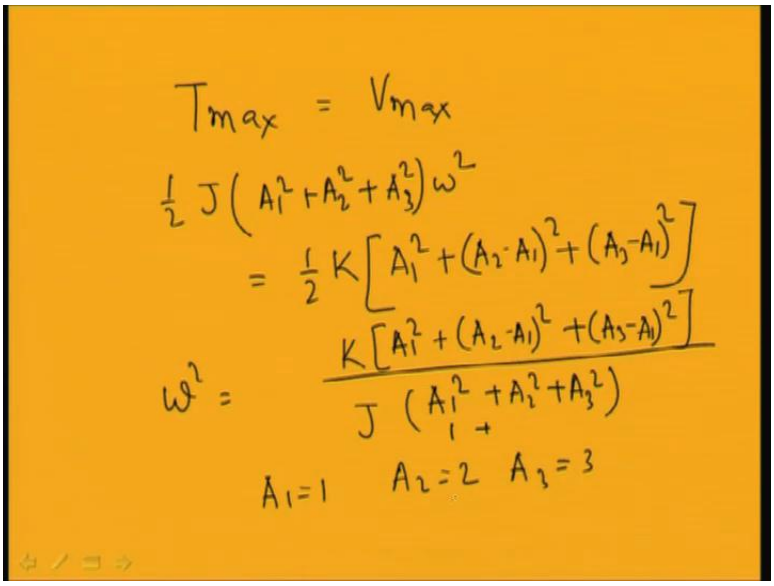

So, by taking this thing I can write. So, I can write this expression omega square or I can write this T max equal to V max. So, maximum kinetic energy equal to maximum potential energy. So, by equating that thing, so I can write this half J omega square. So, half J omega square into A 1 square plus A 2 square plus A 3 square into omega square equal to half K into A 1 square plus A 2 minus A 1 square plus A 3 minus A 1 square. So, this omega square will equal to, so I can divide, so this half, half will get cancel. So, it will be equal to K into A 1 square plus A 2 minus A 1 square plus A 3 minus A 1 square by A 1 square plus A 2 square plus A 3 square into J. So, by taking this A 1 equal to 1 A 2 equal to 2 and A 3 equal to 3, so it will be equal to 1 plus. So, this becomes 1, so 1 plus 1 2 and here we have A 3 minus A 1 equal to 2. So, square will be equal to 4; so 4 plus 1 plus 1; so this become 6. So, and in the here we will have, so this will becomes that upper side become 6 K and here you have 1 plus 4 plus 9. So, 9 plus 4 13 plus 1, so you have 14; so you will have this thing equal to 6 by 14. So, 6 by 14 becomes, so you can find 6 by 14.

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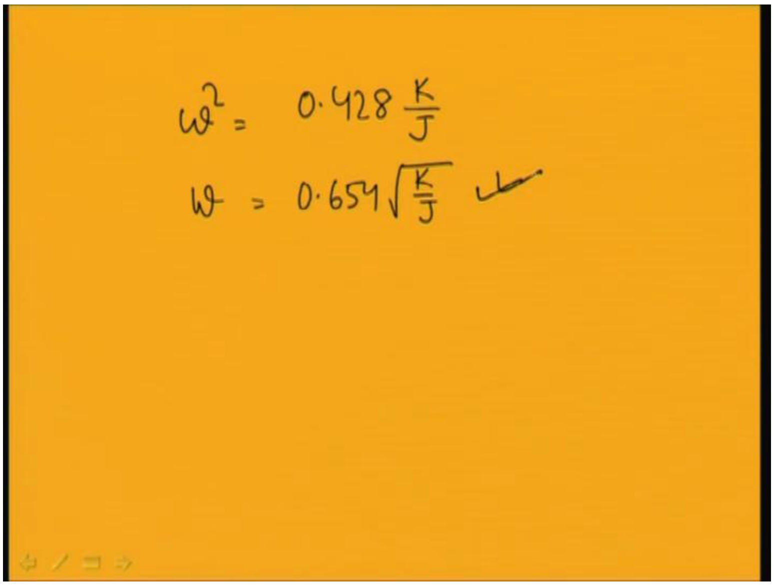

So, this will becomes 0428. So, you can write this equal to, so omega square equal to 0.428 K by J. So, we can find this omega, so it will be equal to root over 0.428. So, this becomes 0.654 root over K by J. So, let us calculate it again, so this becomes 1 and then this is 1 is 1 plus 1 2 and here this is A 3 minus A 1 this is 2. So, 2 square 4, so 4 plus 1 this 5 plus 1 6 and here it is 1 1 plus this 4 plus 9. So, 13 plus 1 14 and here equal to 6, so 6 by 6 by 14. So, 6 by 14 equal to 0.428 0.428 and root over that equal to 0.654 root over K by J. So, in this way you can find the natural frequency or for this is the fundamental frequency and this is the fundamental frequency of the system. So, you can find the fundamental frequency by using this Rayleigh method. So, for a long parameter system also you can find the fundamental frequency.

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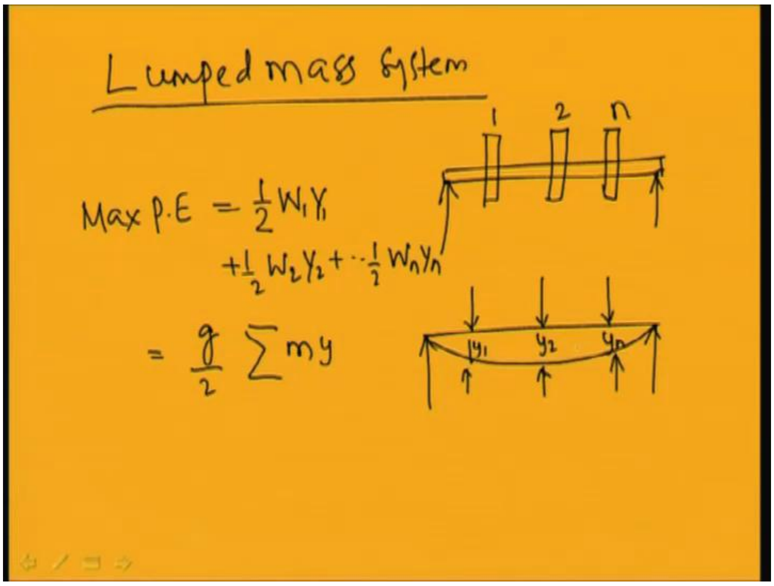

Let us have a shaft it is supported like this and you have several masses attach to this shaft. So, in this case you can find you can find the natural frequency of the system. So, by equating the maximum kinetic energy of the system to the maximum potential energy of the system you can find the natural frequency. So, let us assume that under this mass 1 2 and 3 or let you have n mass, so this n mass. So, if you have n mass then the elastic deflection. So, let this is the elastic line of the system. So, in this case I can take let under this mass m 1 let the displacement is y one. So, this displacement is y 1. Similarly, under this mass 2, so let the displacement is y 2 and under this yn. So, let this is the nth mass placed here. So, in this case nth mass let it will be yn.

So, let these are the displacement under each mass. So, we can find this maximum potential energy of the system will be equal to. So, maximum potential energy will be equal to mgh. So, I can write this maximum potential energy equal to. So, as this displacement will takes place from 0 to this value gradually. So, I can take this equal to half. So, it can be taken equal to. So, the maximum I can find the maximum potential energy of the system. So, by finding this maximum potential energy I can write this equal to as the as it takes place from 0 to y 1. I can write this equal to W 1 Y 1 plus half W 2 Y 2 and similarly half Wn Yn, so where W equal to mass into gravity. So, this can be written equal to g by 2 into summation m into y.

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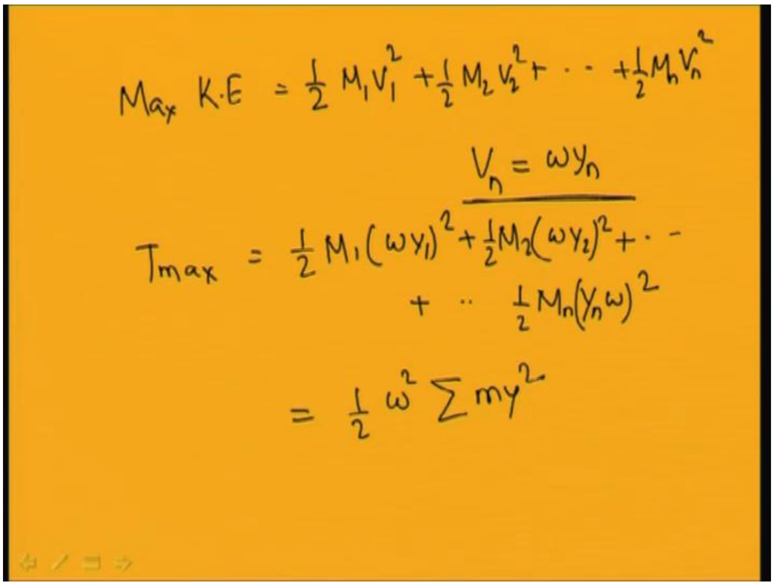

So, this is the maximum potential energy or y will be equal to y max. We can take similarly I can find the maximum kinetic energy. So, the maximum kinetic energy turn maximum kinetic energy can be given by this. So, maximum kinetic energy will be equal to half M M 1 V 1 square plus half M 2 V 2 square. So, plus half Mn Vn square, so this V equal to. So, as I am assuming this y in the form of A cos omega t or A sin omega. So, the maximum maximum will be equal to A omega. So, it will equal to or Vn will be equal to omega yn. So, substituting this in this equation you can write this T max equal to. So, T max will be equal to half m 1 into omega y 1 square plus half m 2 into omega y 2 square and in this way you can write it is equal to half Mn I am writing this as capital M.

So, this is capital M and this will be equal to half Mn Yn into omega square. So, this is the maximum kinetic energy and already you got the expression for the maximum potential energy. So, by equating this maximum kinetic energy with maximum potential energy, so you can write the expression or you can find the expression for this omega n. So, omega n square will be equal to. So, by equating this we can write this expression this kinetic energy expression as half. So, I can write this maximum kinetic energy as half omega square into summation m y square.

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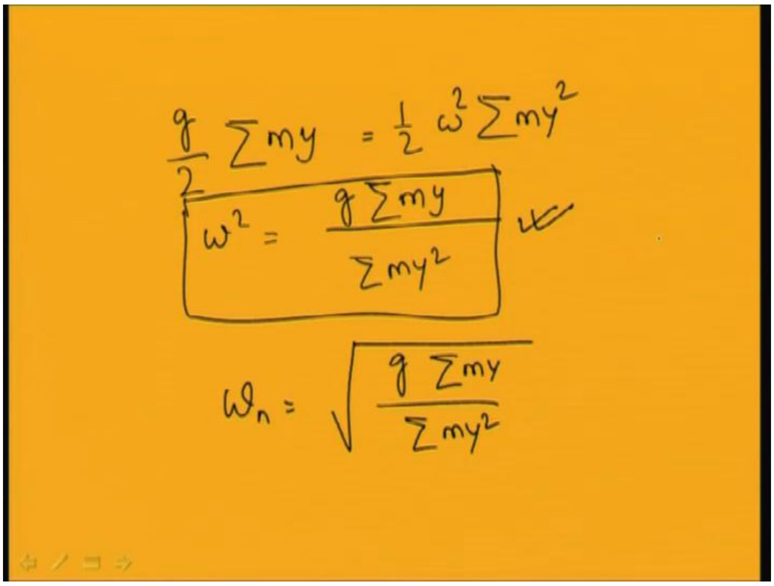

So, now dividing this I can write expression or I can write this half g by 2 summation m y will be equal to half omega square summation m y square. So, I can write this omega square equal to summation m y by summation m y square. So, omega square that is the frequency square of the natural frequency can we obtained in this way. As g into this is g is the acceleration due to gravity into summation of the mass at each position you know the mass at each position and if you know the displacement corresponding displacement. So, the frequency square can be obtained or the frequency can be obtained by equating the maximum kinetic energy with potential energy. And the expression can be given by this for this omega n it will be equal to root over g summation m y by summation m y square. So, in this way when a lumped mass system is there you may find the natural frequency of the system.

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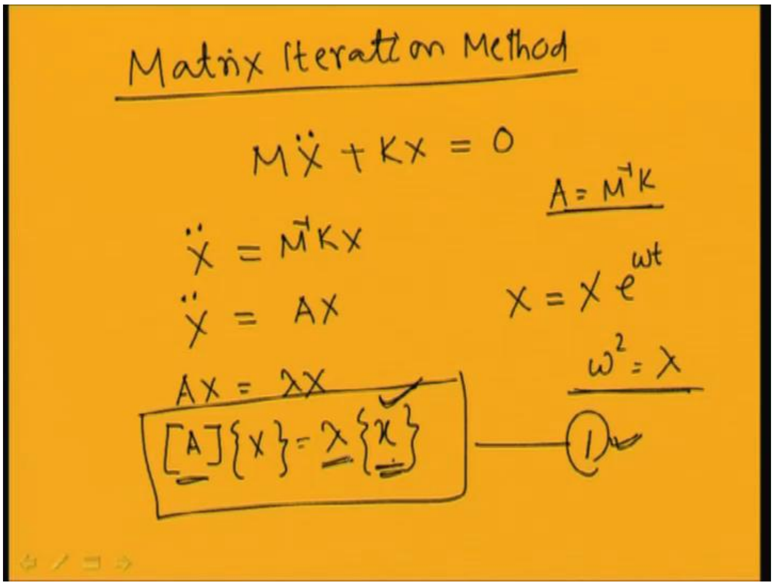

So, let us consider some let us consider or let us go to the other types of approximate method. And now let us study about this matrix iteration method matrix iteration method is used for this multi degrees of freedom system. In case of multi degrees of freedom let us consider a multi degree of freedom with n number of degrees of freedom. So, in that case you can write the equation motion of the system in this form MX double dot plus KX equal to 0. So, this is the pre vibration equation for this case of a multi degree of freedom system. So, if x you can write as the mode safe of the system. So, you can write this equation also in this form by using this m inverse K as A by writing A equal to M inverse K. I can write this equation in this form X double dot X double dot will be equal to M inverse KX or X double dot will be equal to AX.

Now, by considering the normal mode vibration X equal to I can take X equal to X e to the power I omega t. So, X equal to I can take Xe to the power I omega t. So, by taking e to the power I omega t or sin omega t I can get this X double dot equal to minus omega square X. So, X double dot will be equal to minus omega square X. So, minus omega this omega square I can take equal to lambda or the Eigen value of the system. So, this expression I can write in this way. So, it will be equal to AX AX equal to lambda X or AX equal to lambda X or in matrix and vector form if I am write. So, this is A matrix into X vector will be equal to A matrix into X vector will be equal to lambda into this x vector.

So, you just note that using this dynamic matrix A and this normal mode or the mode safe of the system you can observe that this mode shape repeats itself. So, when I am taking the mode safe of the system and if multiplying with the dynamic matrix then this mode safe repeat itself. So, the normal mode when multiplied with the dynamic matrix will reproduce itself. From this equation we are observing that when this normal mode is multiplied with this dynamics matrix it is repeating itself. So, this is the basis of this matrix iteration method. So, if we start with an assumed value of this x and go on iterating by knowing this dynamics matrix A, so we can find some value of X which will satisfy this equation. So, that value will be the normal mode of the system. So, taking that normal mode of the system we can find this value the Eigen value lambda of the system.

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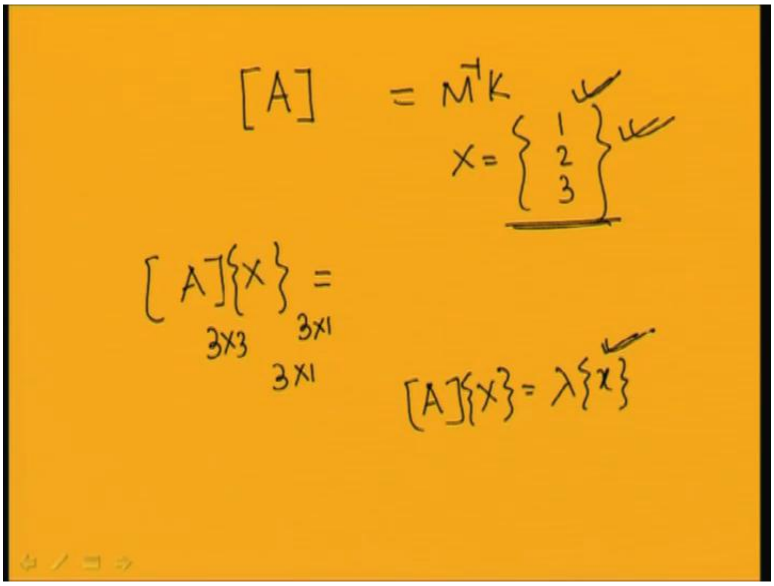

So, the steps you can follow to in this method. So, first you find this dynamic matrix A. So, dynamic matrix A can be found by M inverse K. So, by finding this dynamic matrix and assuming 1 mode safe, so this mode safe X you can assume. So, let for a 3 degree of freedom system you can assume the mode in the is way let me write this is 1 2 3. So, this is the assumed value I have taken, so by taking this thing. So, I will multiply this assumed value with this dynamic matrix. So, by multiplying this assumed value by this dynamic matrix I can get.

So, let this A will be a 3 is to 3 matrix and this is 3 is to one. So, this gives me a 3 is to 1 vector. So, in this 3 is to 1 vector. So, I can normalize that vector and I can find this normalized mode safe of the system. So, I can find this normalized mode safe of the system and after finding this normalized mode safe of the system. If it is equal to the previous or the assumed mode safe then it will satisfy this equation AX equal to lambda x. So, if it is satisfying this equation then I can take this x value or the assumed value as the mode safe and I you can calculate this value of lambda from this expression.

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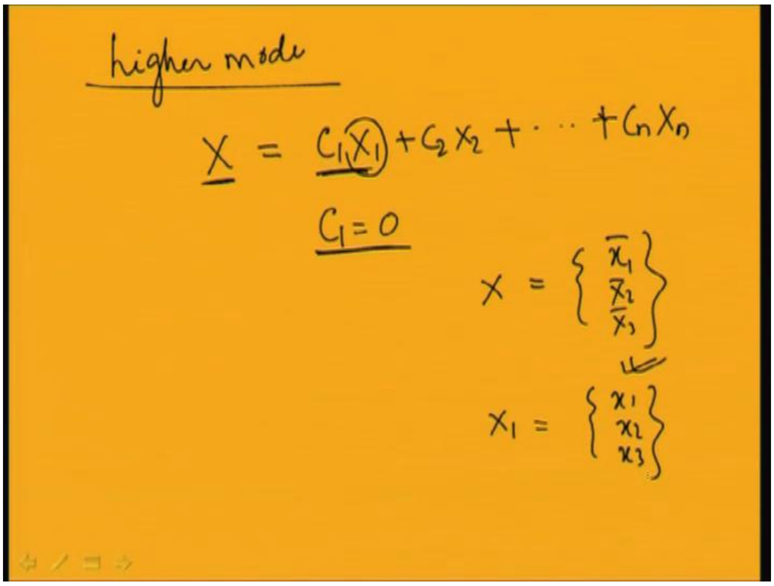

So, to find the higher modes you can. So, to obtain the higher mode you know the any vibration can be written as this C 1 X 1 plus C 2 X 2 plus Cn Xn. So, where this X 1 X 2 Xn are the normal mode vibration of the system and X is the free vibration resulting free vibration of the system. So, this X can be written as C 1 X 1 plus C 2 X 2 plus Cn Xn. So, this is the assumed mode we are taking for our calculation. So, if this assumed mode contain this first mode then the resulting vibration or the resulting natural frequency or Eigen value what we are, what we want to obtain will leads to the fundamental frequency only.

So, to find the higher mode frequency we have to make the C 1 equal to 0 from this resulting vibration. So, to make the C 1 equal to 0 we may use this orthogonality principle. I can write this X equal to X equal to X 1 bar X 2 bar and X 3 bar. So, these are the. So, these are the assumed mode and this X 1 X 2 X 3 or x 1 x 2 x 3 are the Eigen values of this or the values of the I th mode of the system. So, X 1 will be equal to this X 1 will be equal to x 1 x 2 x 3 and this X that is the assumed value I am taking in this form that is equal to X 1 bar X 2 bar and X 3 bar of the system. So, applying this orthogonality principle I can write I can write this X 1.

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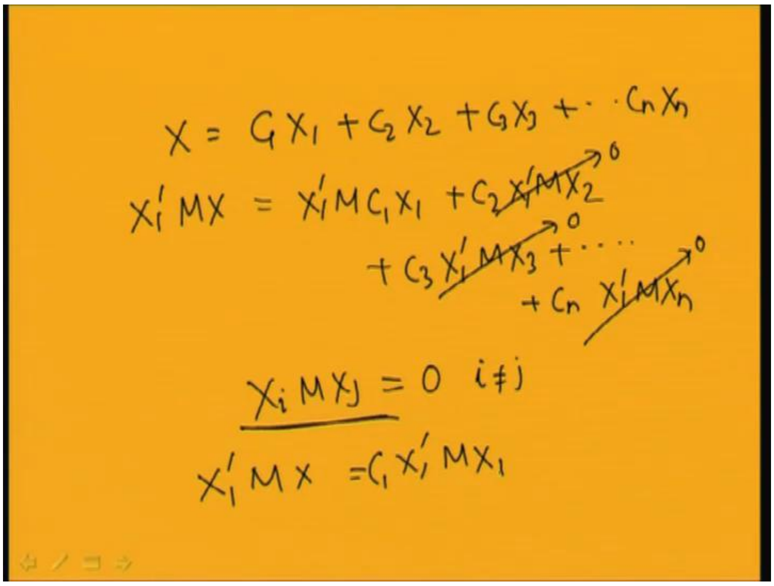

So, as this X equal to C 1 X 1 plus C 2 X 2 plus C 3 X 3 plus Cn Xn. So, applying this orthogonality principle I can write I can write X 1 dash X 1 dash MX will be equal to X 1 dash MC 1 X 1 plus C 2 into X 1 dash M X 2 plus C 3 into X 1 dash MX 3. And similarly I can write this is Cn Cn X 1 dash MXn. So, from the orthogonality principle you know Xi MXj equal to 0 if X if I not equal to j. So, when I not equal to j, so this becomes 0. So, this is the orthogonality principle. So, applying this principle you can check that all these terms becomes equal to 0. So, this X 1 dash MX 2 equal to 0 X 1 dash MX 3 equal to 0. Similarly, all these terms equal to 0 and you are left with X 1 dash MX equal to X 1 dash C 1 into X 1 dash MX 1. So, as C 1 to be equal 0 to find for the higher modes, so putting C 1 equal to 0.

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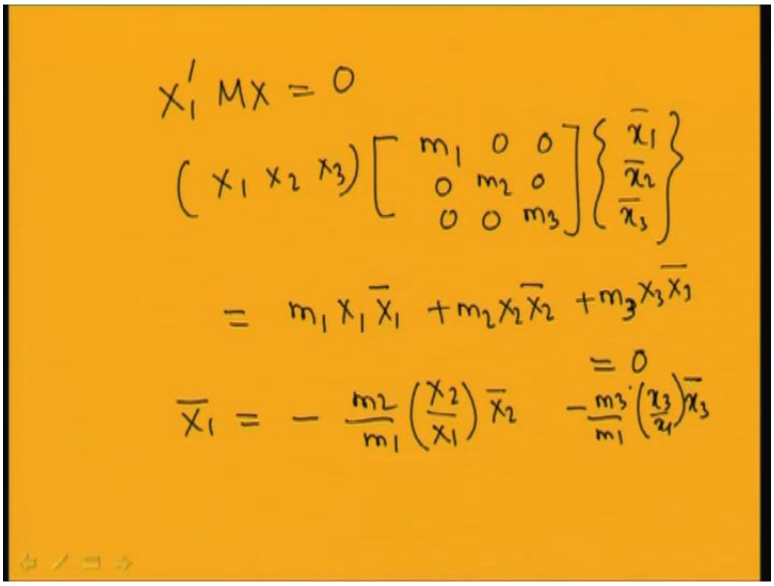

So, you can find or you can write this left hand side should be equal to 0; that means, X 1 dash MX 1. So, X 1 dash MX will be equal to 0. So, this X 1 dash you can write in this form that is equal to X 1 X 2 X 3 the transpose of X 1 X 2 X 3 into let me take a 3 is to 3 3 degrees of freedom systems then it will be m 1 0 0 0 m 2 0 0 0 m 3 into this is x 1 bar x 2 bar and this is x 3 bar So, if you multiply this thing. So, you are getting this is equal to m 1 X 1 X 1 bar plus m 2 X 2 X 2 bar plus m 3 X 3 X 3 bar. So, this would be equal to 0. So, I can write this X 1 bar equal to X 1 bar equal to minus m 2 by…

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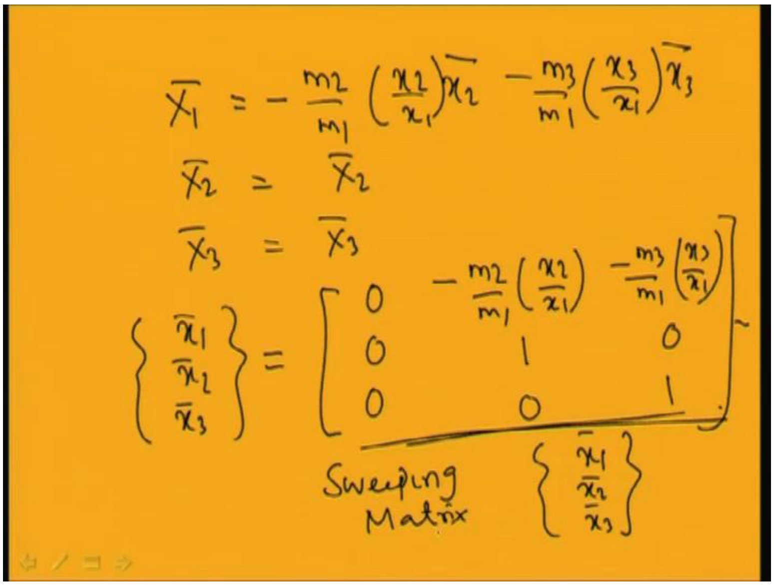

So, X 1 bar will be equal to minus m 2 by m 1 X 2 by X 1 into X 2 bar and this is equal to minus m 3 by m 1 into x 3 by x 1 into x 3 bar or I can write this equal to X 1 bar equal to minus m 2 by m 1 into x 2 by x 1 into x 2 bar minus m 3 by m 1 into x 3 by x 1 into x 3 bar. Now, I will write this X 2 bar equal to X 2 bar and this X 3 bar equal to X 3 bar. So, I can write this left side as x 1 bar x bar x 3 bar. So, this is the assumed mode I am taking now. So, this will be written as, so as this does not contain x 1 bar. So, this will be 0 0 0 and this is equal to minus m 2 by m 1 into x 2 minus x 1 minus m 3 by m 1 x 3 by x 1. Similarly, this will become 1 this is 0; this is 0; this is 1; this into x 1 bar x 2 bar x 3 bar. So, you can see, so in this case the fast mode is not present in the system.

So, the fast mode we have eliminated by using this matrix. So, this matrix is known as the sweeping matrix. So, today class we have studied or we have taken some example for the Rayleigh method and found the natural frequency of the systems. Also we know the principles of matrix iteration and also we have found or we have studied the procedure for finding the frequencies for the higher mode in case of this matrix iteration method. So, next class, we will complete this matrix iteration method. And also we will study the Rayleigh Ritz method and Galerkin method and Dunkerley method which are the approximate methods for finding the natural frequency of a system.