Mechanical Vibrations
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Module - 10
Approximate Solutions for Continuous and Discrete Systems
Lecture - 1
Rayleigh's Energy Method

So, welcome to this module of vibration engineering**.** So, in this module we will study about the approximate numerical methods used in this vibration. So, you have studied about the discrete system and distributed mass systems. So, in discrete systems, that a definite number of degrees of freedom and definite number of frequencies associated with the system. And in case of distributed mass system or continuous systems there are infinite number of frequencies associated in the system. But all these frequencies are not of importance to this to the engineers when the designing the system. So, most of the systems are vibrating with in the lower frequencies. So, if the engineer knows about the few frequencies few lower mode frequencies then he will able to design the system. So, also we studied the methods how to determine these mode shapes and the frequencies of the systems.

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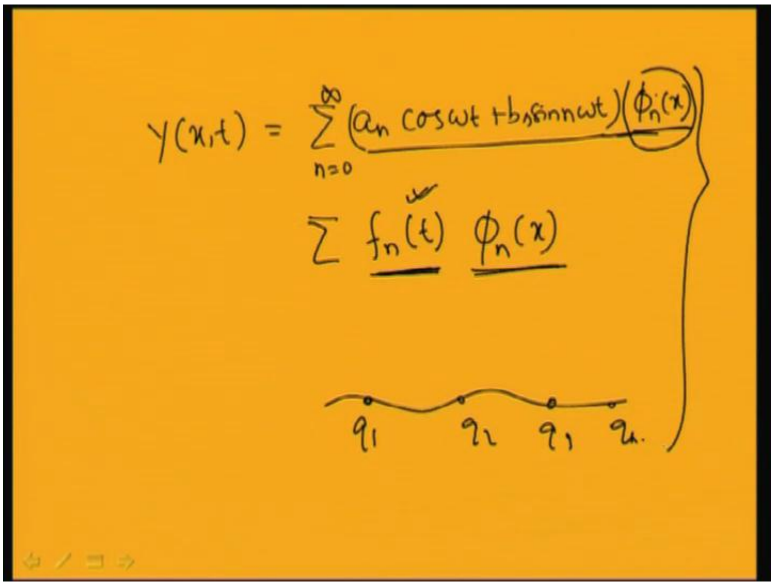

So, in case of continuous system you can write the solution in this form. That is Y x t equal to n equal to 0 to infinity an cos omega t plus bn sin n omega t into phi t phi x phi n x, so for n th mode. So, the free vibration response of the system you can write in terms of this model matrix in case of a discrete system or this codal function in case of a continuous system and by this time modulation. So, this is the time modulation this modulation is an cos omega t plus bn sin omega t. So, in this case in case of this continuous system you may take infinity number of modes to find the solution of the system. But for practical application one need not to have take these infinity number of modes and he may truncate this series to a finite number of finite number. So, by truncating these thing the error introduced in the system.

So, one can see that there will not be much error in the system. So, for practical application some approximate methods can be used without finding this exact solution of the system. So, if by taking these approximate method our results is within 5 percent or within a limited value then it can be used for the engineering purpose. So, for that purpose this approximate numerical methods has to be studied, so in the exact methods. So, you are you are finding the Eigen values of the system and Eigen vectors of the system. And in case of the continuous systems you are finding this Eigen functions and the Eigen functions are the mode shapes of the system. So, in this approximate method we are going to find only the few frequencies for example, we will find only the fundamental or first and second mode frequencies in these systems.

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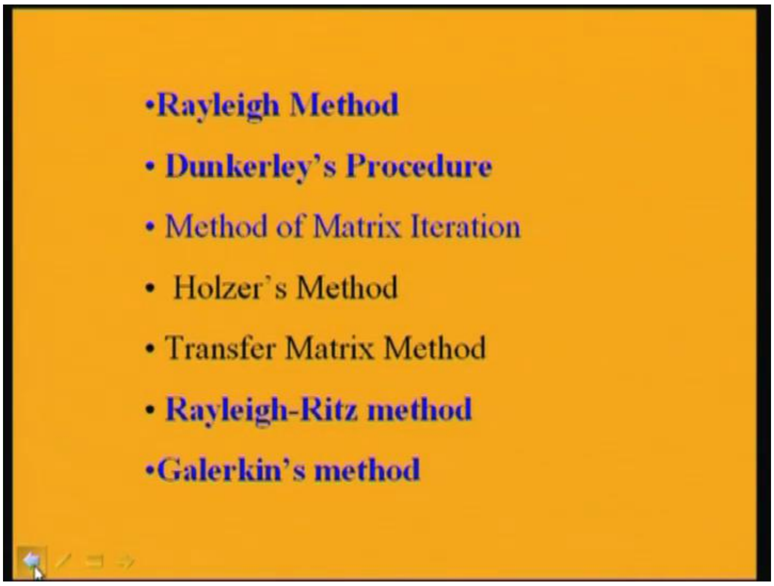

So, we will use different methods. So, those methods include the Rayleigh method, Dunkerley's procedure, method of matrix iteration Holzer method, transfer matrix method, a Rayleigh-Ritz method and Galerkin's method. So, these methods can be applied both to the continuous and discrete systems. So, in the series solution we have seen by we can approximate the function by truncating this infinity series to a finite number of series. And also the formulation and computation effort can be reduced by truncating the series and we can do it in 3 different ways. So, the first way is to retaining n natural modes only and considering them as the generalized coordinates and computing n weighting function to best feed the initial condition or the forcing function.

In the second case substituting for the n modes an equal number of known function phi n x that satisfies the geometric conditions of the system then compute the function phi n. So, these equation I can write it in this from. So, it can be written as phi nt fnt and phi nx. So, I can take a function this phi nx will satisfy only the geometric boundary conditions. And I will find these fnt which will best feed the initial conditions and other boundary conditions or the forcing functions. Already you know in exact method we are finding these Eigen function of the system. But instead of finding these Eigen functions will satisfy both these which satisfy both the differential equation and all the boundary conditions.

I can take some function who will satisfy only the geometric boundary conditions. And then I will find this fnt which will best feed the other initial conditions the differential equation and the forcing function. So, in that way I can approximate the system. The third way I can approximate it by. So, let this is the system, so in the system I can take several points on the system and taking this physical coordinates or the generalized coordinates of the system or the displacement of this points. I can take let it is q 1 q 2 q 3 and qn. Like that I can take these function by taking this physical coordinates as the generalized coordinates and considering them as function of time.

Then I can compute and feed the differential equation and find a find an equation or expression which will feed the initial and boundary conditions. So, in these three different ways I can approximate the system. So, in the first way I can take the generalized coordinate and generalize I can take these as the generalized coordinates and approximate or find the weighing function fnt. In the second I can take the function which is will satisfy only the geometric boundary conditions and find this fnt. And in the third case I can take these physical parameters as function of time and feed the function to find. Or find the function which will satisfy the boundary conditions and the differential equations at initial and boundary conditions. So, in this 3 way I can approximate the series.

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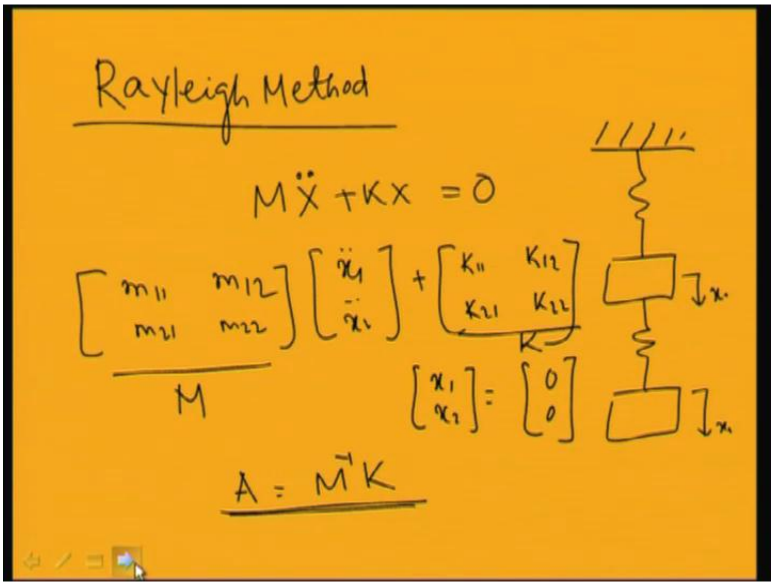

So, let us see the Rayleigh method. So, in case of Rayleigh method let us consider this discrete system let us start with a discrete system. So, let this Mx double dot plus Kx equal to 0 is the equation for this discrete system. So, for the discrete system for example, you just take the spring and mass system this is another spring and mass system. So, this is a 2 degrees of freedom system and in this case you can write this equation in this from m 1 1 m 1 2. So, the equation becomes m 1 1 m 1 2 m 2 1 m 2 2. So, this is x 1 this is x 2.

So, I can write this as x 1 double dot x 2 double dot plus similarly k 1 1 k k 1 1 k 1 k 2 1 k 2 2 into x 1 x 2 equal to 0 0. So, this for the free vibration of the system and we are interested to find the frequency of the system. So, already you know in the exact method I was I can find it by finding this dynamic matrix that is M inverse k. First finding this M this is M matrix mass matrix and this is the stiffness matrix K matrix. So, by finding this M inverse K I can find the dynamic matrix A. So, after finding this dynamic matrix I can find the Eigen value of this dynamic matrix to find the Eigen values of the system.

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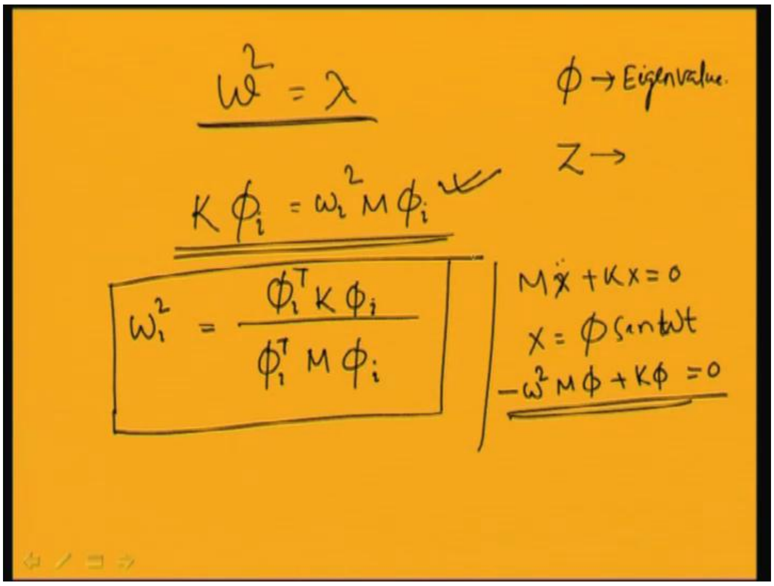

So, these Eigen values correspond to the square of the nature frequency. And also after finding this Eigen value I can find the mode shape or normalized modes of the system which are the Eigen functions of the system. So, I can find the Eigen values and Eigen function of the system. So, if I am taking this Z as the Eigen value of the system I will take Z as the approximate value let me take initially this phi as the Eigen value of the system. So, phi if phi is the Eigen value of the system. So, from this expression MX double dot plus KX, so it will satisfy this equation. So, you know, so it will satisfy the equation K. So, then it will be K phi i. So, for the I th mode. So, it will be K phi i equal to omega I square.

So, it will be equal to omega I square phi i omega I square M phi i. So, for the i th mode I can write this equation will satisfy thus this MX double dot plus KX equal to 0 will satisfy that equation because this x I am assuming it to be harmonic. So, x can be written as phi sin x x can written as phi sin t and... So, by substituting this x equal to phi sin t you can write this as minus M omega square phi plus K phi equal to 0. So, minus, so in this equation; so this is your equation M MX double dot plus KX equal to 0 substituting x equal to. So, let me substitute x is equal to phi sin t phi sin omega t then it becomes. So, this equation becomes minus omega square M phi plus K phi equal to 0 or I can write this as for I th mode K phi i equal to omega I square M phi.

So, from this expression you can write this or from this I can write the I th mode I can write it as like this. So, omega I square will be equal to. So, by pre multiplying I can pre multiply this by phi I transpose and by applying this orthogonal deference pull I can write this as. So, this will be equal to phi I transpose. So, phi I transpose K phi i by phi I transpose, so T for transpose M phi i. So, this omega I square omega I square equal to phi I transpose K phi i by phi I transpose M phi i. So, here phi I is the exact Eigen value of the system. So, if you do not know the exact value we can assume some other value which will approximate this phi. So, in that case by substituting that approximate function. So, I can write this expression in this form.

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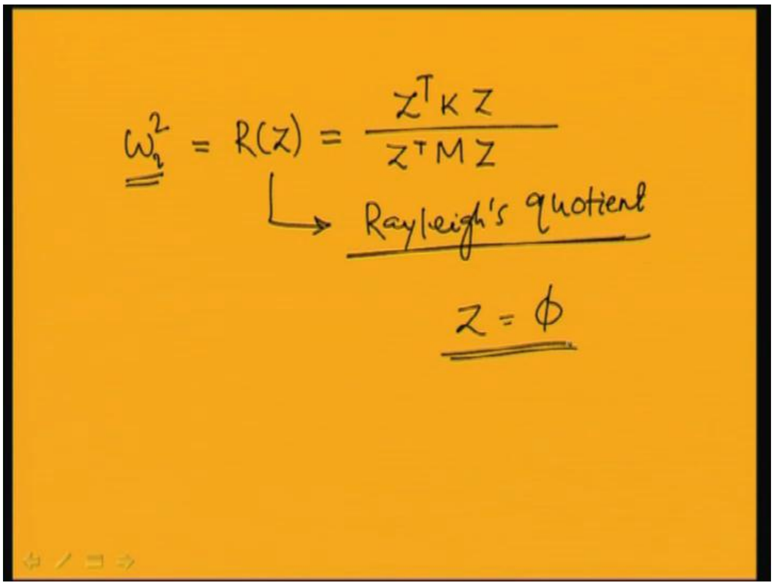

So, I can write this omega I square equal to RZ let us substitute the Z as the function arbitrary vector let us considering this as Z at the arbitrary vector. So, I can write this as RZ. So, then will be Z transpose KZ by Z transpose MZ. This RZ the, this quotient this RZ is known as the Rayleigh quotient of the system Rayleigh. So, this is known as Rayleigh's quotient. So, when you are taking this Z as the approximate function or approximate vector then this is known as the Rayleigh quotient. So, this Rayleigh quotient will be equal to the actual Eigen value of the system when this Z equal to phi. So, when Z equal to phi. So, then this function will reduce to or this Rayleigh quotient will be reduced to the Eigen value of the system.

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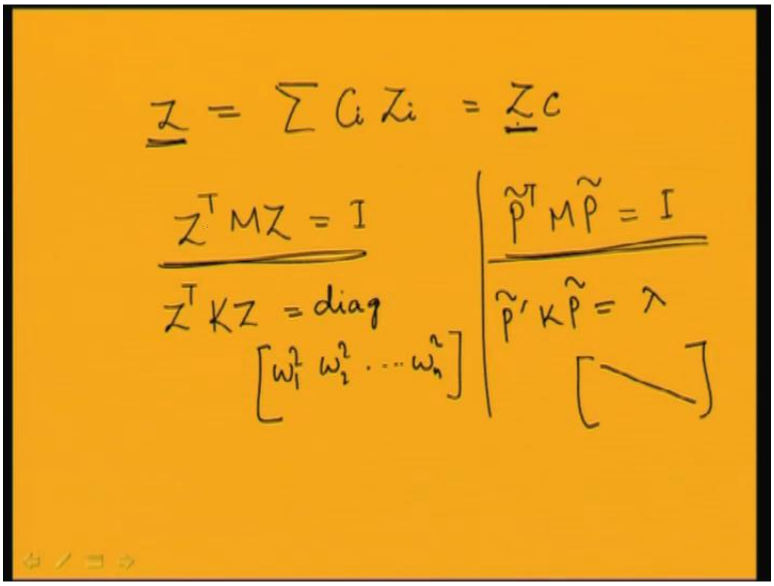

So, we know from the linear algebra that any function Z or any vector Z can be expressed. As a linear combination of n linearly independent vectors. So, this is the expansion theorem. So, you can write this Z as Zi. So, I can write this as Ci into Zi or I can write this as Zc where Z is the square model matrix. So, Z is the square model matrix and by normalizing the thing 1 can write this as Z transpose MZ 1 can normalize the square transfer this matrix. So, Z transpose MZ equal to I already you have seen similar cases in case of weighted model matrix. So, in case of the weighted model matrix P weighted model matrix transpose.

So, P weighted model matrix transpose MP weighted model matrix was I. So, here this P was the model matrix and P weighted model matrix was the, was obtained from this weighted model matrix. So, 1 can find a Z such that this Z transpose MZ will be equal to I. So, when it satisfies these equations that time 1 can write this Z weighted model Z transpose KZ. So, that time Z transpose KZ will be equal to a diagonal element. So, if you recall this P weighted model matrix KP weighted model matrix in case of the discrete system. So, it you have seen that this is this matrix is that lambda matrix or this matrix is the diagonal matrix containing Eigen value in the diagonals. So, in this case. So, if you are taking this Z transpose KZ.

So, we will obtain a diagonal matrix. So, we will obtain a diagonal matrix with with omega square term for omega 1 square omega 2 square and omega 1 n square in the diagonal. So, let us assume a vector Z. So, this Z can be written in this form where Z if the matrix square matrix capital Z is the square matrix. So, this capital Z transpose MZ we can take this Z in such way that this Z transpose MZ equal to I that is unit matrix. And Z transpose KZ equal to diagonal element omega 1 square omega 2 square and omega n square.

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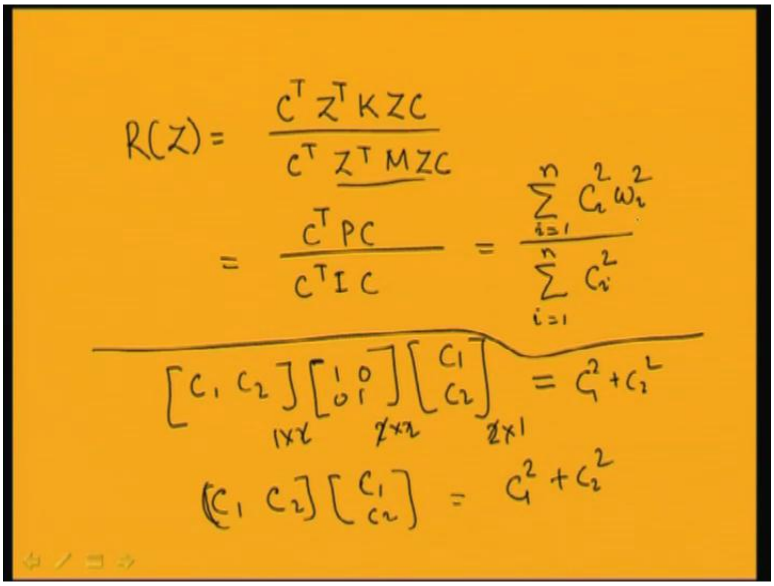

So, if you can find in this way. So, I can write this RZ that is the Rayleigh quotient in this case can be written in the form. So, Rayleigh quotient can be written. So, C transpose Z transpose KZC by C transpose Z transpose MZC. So, already you know this Z transpose KZ can be written in this as a diagonal matrix and this Z transpose MZ is the I matrix. So, this is this is equal to I and this is a diagonal matrix with omega square in the diagonal terms. So, this thing can be written as. So, this will be equal to C transpose. So, if I am writing this matrix as P matrix this Z transpose KZ which is a diagonal matrix let me write this as the P matrix. Then I can write this as C transpose PC by C transpose IC, but this is equal to C transpose PC will nothing but this it will equal to summation I equal to 1 to n. So, this becomes CI square omega I square by summation of CI square.

So, this is I equal 1 to n. So, this is summation CI square. So, you can verify this thing by taking 2 is 2 matrix. Let me the take this C equal to C 1 C 2 then it becomes C 1 C 2 multiplied by. So, you have a unit matrix 1 0 0 1 let us take and this will be equal to C 1 C 2. So, if you multiply this thing then it becomes C 1. So, let us multiply the first 2. So, this is 2 is to 1. So, this is 1 row 2 column. So, 1 row 2 column this 2 row 2 column and this is 2 row 1 column. So, this becomes. So, final expression will becomes 1 cross 1. So, if you multiply this become C 1. So, this is you will get 1 cross 2. So, if you multiply this become C 1 and this become C 2. So, this final expression, so this multiplication will be C 1 square plus C 2 square. So, this multiplication will be C 1 square plus C 2 square. So, if you multiply this then this becomes 1 row and 2 column C 1 C 2 into C 1 C 2 into C 1 C 2. So, this will become C 1 square plus C 2 square.

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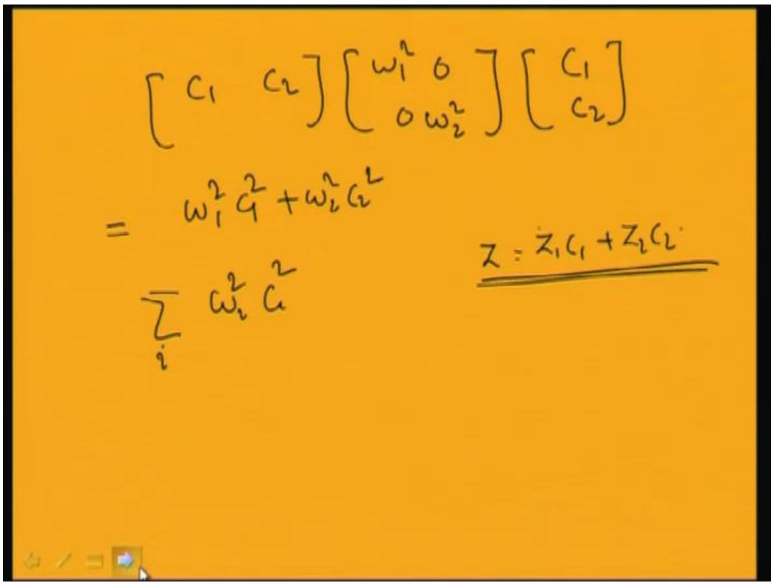

So, this is the lower part. Similarly, in the upper part you have a diagonal matrix this is equal to. C 1 C 2 multiplied by omega 1 square 0 0 omega 2 square and let me multiplied by C 1 C 2. So, these are the coefficient of that Z we have taken. The Z vector we have written in terms of Z 1 C 1 plus Z 2 C 2 like that we have written. So, from that this C 1, C 2 coefficients you are getting. And if you multiply this thing then if will becomes omega 1 square C 1 square plus omega 2 square C 2 square. So, this thing you can write it has summation omega I Ci square omega I square Ci square. So, this RZ now you can write in this form. So, this becomes. So, the expression for RZ you can write Ci square omega I square I equal to 1 to n Ci square. So, in this way you can find. So, in this way you can find the expression for this RZ, so in this expression, so you can now, if we select. So, let us assume that we have made some error in assuming this Z.

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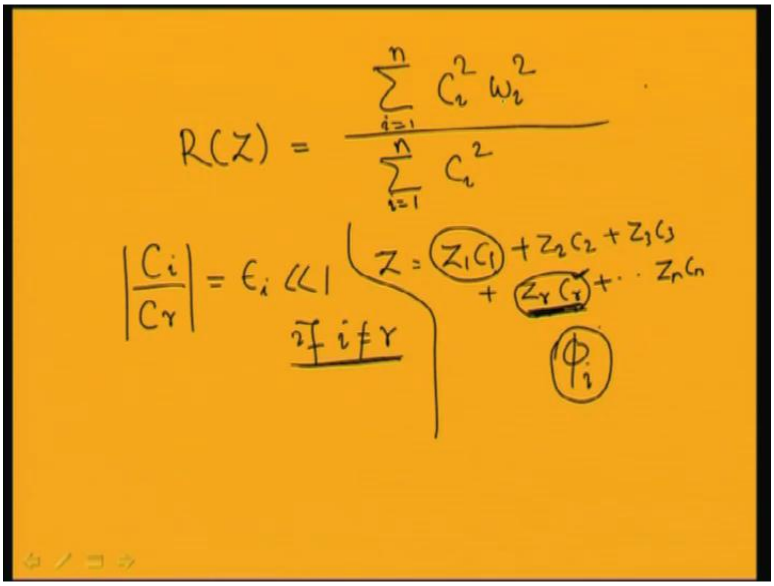

So, already you know that this Raleigh quotient you are writing in this form. So, this is equal to I equal to 1 2 n Ci square omega I square and this is equal to Ci square. So, let us assume that this n th mode or some mode differ from this actual Eigenvalue we are assuming this Z. And 1 of the mode or this Z 0 is or Z differs very little away from the certain Eigen vector phi r then the corresponding. So, let we are writing this Z in this form. So, Z equal to Z 1 C 1 plus Z 2 C 2 Z 2 C 2 plus Z 3 C 3 let me write this is equal to Zr Cr plus Zn Cn. So, here we can write this Z vector as a linear combination of this Z 1 Z 2 Z 3. So, in this way, so if this let this r th Eigen value, so r th Eigen value. So, correspond to this Zr. So, if it differs from this then I can write that the Cr differs from this actual value so Ci by Cr.

So, let us assume written in this way. So, let it is equal to epsilon I which is very very less than 1. So, when it differs from the particular Eigen value phi i. So, let this r mode differ from this phi i. So, when it differs from this phi i. So, I can, so when it differ little from this phi I the corresponding Cr term. So, you note that these terms. So, these correspond to different modes and. So, this term this when it differs from this phi I then the Cr corresponding Cr term will be a large value. So, as the Cr term will be large then Ci by Cr we can write it equal to epsilon I as the Cr is a larger term. So, we can write the Ci by Cr equal to epsilon I where epsilon less than equal to r.

So, this expression we can write if I not equal to r. So, when I not equal to r. So, we can write. So, we can write this Ci by Cr equal to 1. So, here we are assuming that this r th mode. So, this Zr Cr is not equal to phi I and in that case as it is differing from this particular Eigen value for Eigen mode. So, we can assume that this Cr is very large. So, when you are assuming the Cr very large then the Ci by Cr will be a small term. So, this will be equal to epsilon and this epsilon is less than equal to 1. So, we can, so from this RZ expression we can substitute. So, let us substitute this expression here and in this RZ expression.

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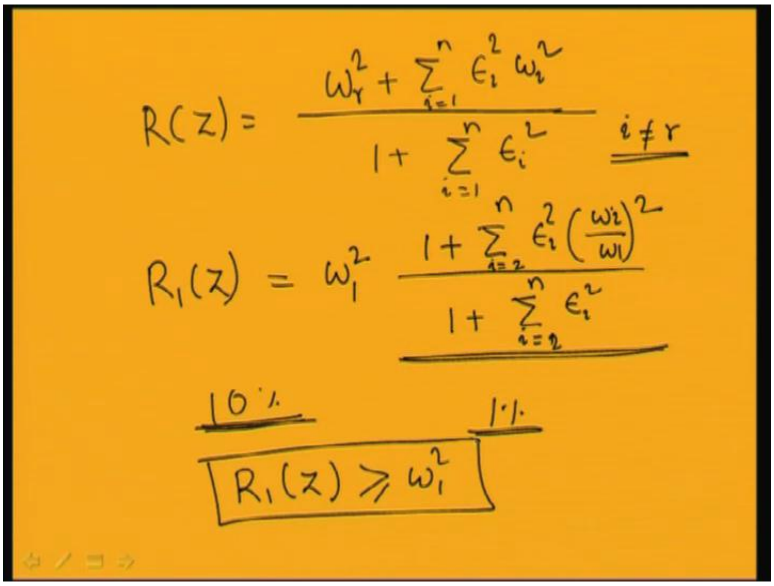

So, we can write this expression RZ equal to now. So, it will be equal to. So, if we expand this thing. Then it will be C 1 square omega 1 square C 2 square omega 2 square C 3 square omega 3 square and Cr square omega r square and this part it is equal to C 1 square plus C 2 square plus up to Cn square. So, now dividing by Cr square, so we can write this will be equal to. So, by dividing that thing by Cr square I can write this equal to omega r square plus. So, this is equal to omega r square plus I equal to 1 2 n. So, this will be equal to epsilon I square into omega I by omega r square and this is 1 plus I equal to. So, I equal to n epsilon I square, so this is equal to epsilon. So, this expression I can write it correctly.

So, this becomes C 1 square omega 1 square plus C 2 square and similarly Cr square omega r square I am dividing it by Cr square. So, this becomes omega r square and other terms in other terms I will write it Ci by Cr whole square and omega I square. So, it becomes epsilon I square omega I square this become 1 plus epsilon I square, so this equal to when I not equal to r. So, when I not equal to r this Rayleigh quotient I can write in this form. So, you can observe from this let us for let us take this I equal to 1. So, you can note that this epsilon I square is a very small quantity and we can write this R 1 Z for the first mode I can write Rayleigh quotient R 1 Z I can write R equal to 1.

So, this becomes omega 1 square. So, I can take this omega r square common. So, this is omega n square into 1 plus. So, when I am taking this common. So, I equal to as I am finding for r equal to 1. So, this will not contain this first terms. So, it will be I equal to 2 to n then epsilon I square omega I by omega 1 square by here I can write. So, this is equal to 1 plus summation I equal to 2 to n epsilon I square. So, from this expression you can note that when we are making an error of let we have we make an error of 10 percent in taking the correct value of Z. So, in that case the error will be. So, this part will be the, this part will be the error.

So, for actual Eigen value this R 1 Z should be equal to omega 1 square, but if you are taking a 10 percent error in finding this value 10 percent error in this value. Then you can see that the error will be of the order of epsilon square. So, the error will be only 1 percent in this case. So, a 10 percent error in the form of Eigen vector or natural mode will result in only 1 percent error in this Eigen value or the natural frequency. So, you just observe that this in this case I can write that this R 1 Z R 1 Z always greater than equal to omega 1 square. So, you can observe this that the Rayleigh quotient is never will never have a value less than this first mode Eigen frequency.

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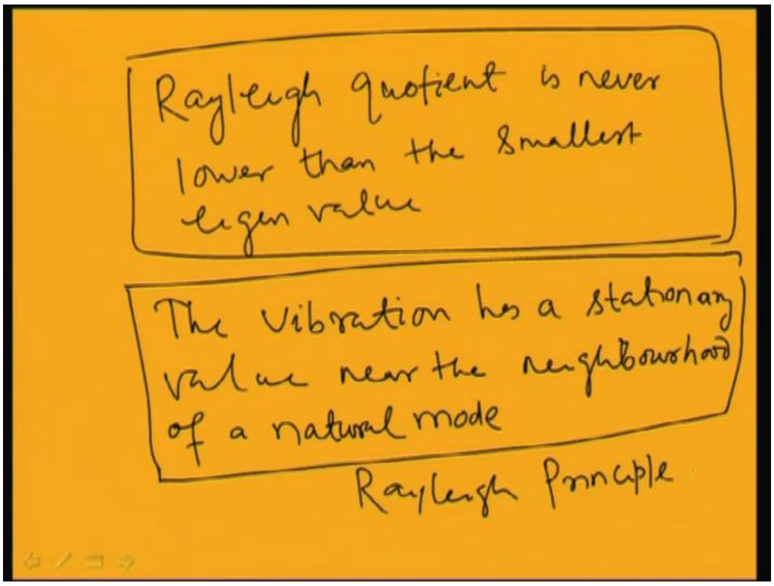

You can note that this Rayleigh quotient is never lower than the smallest Eigen value. Also you can see that in a conservative system the frequency of vibration has a stationary value in the neighborhood of the natural mode. So, from this expression you can see that if you take any other value of this Z. So, always you will get a value nearer to this omega 1 square. So, you can tell that the vibration has a stationary value in the neighborhood of the natural mode. The vibration has, so you can note this, so the vibration has a stationary value near the neighborhood of a natural mode. So, this is known as the Rayleigh principle.

So, this is known as, so the Rayleigh principle states that the vibration has a stationary value near the neighborhood of the natural mode. So, in this case you have found this RZ for R 1 is nearly equal to or always have a value slightly greater than omega 1 square. Similarly, by taking a mode which is equal to the second mode you can find this R 2 Z equal to omega 2 square. Or if the function or the vector what we have chosen is nearer to the second mode. So, we will get the second natural frequency or we will get a frequency which is slightly closer to the second natural frequency. So, by using this Rayleigh quotient method we can get the approximate value or approximate Eigen value of a system. So, also we can proceed or we can find this value in some other way.

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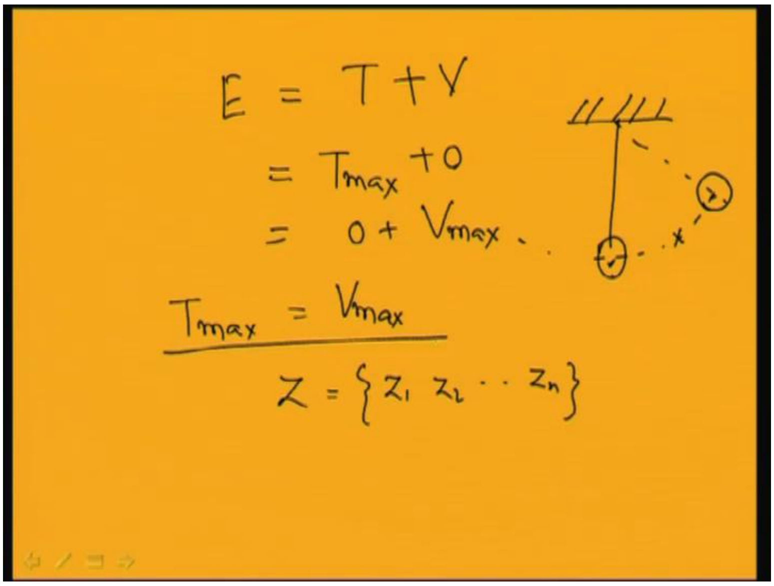

So, let us write this expression in this form. So, the total energy of the system can be written in terms of T plus V that is kinetic energy of the system plus potential energy of the system that you take in the case of a simple pendulum. So, in this case of a simple pendulum, so you know that when it is passing through the mean position the system the velocity of the system is maximum or velocity of the pendulum is maximum. And when it is at this end position the velocity is 0 and all the energy are converted to this potential energy. So, at this position the system has only potential energy and at the system or at this instant the system has kinetic energy. So, this is maximum kinetic energy and this is maximum potential energy and at any other the system has a energy which is a mixture of potential energy and kinetic energy.

So, you can write this E equal to T plus V that is the total energy of the system also you can write this as T max maximum kinetic energy plus 0 or you can write this as 0 plus V max maximum potential energy. So, you can write this T max equal to maximum kinetic energy will be equal to V max maximum potential energy of the system. So, by using this expression also you can find this natural frequency of the system. So, T max equal to V max. So, from this you can write. So, let us assume a function certain mode. So, if you assume a certain mode that is Z. So, let Z equal to I am assuming in this form Z equal to Z 1 Z 2 Zn. So, let me assume this mode. So, if I will substitute this in this kinetic energy and potential energy I can write this kinetic energy term or you know the kinetic energy can be written as half V transpose MV.

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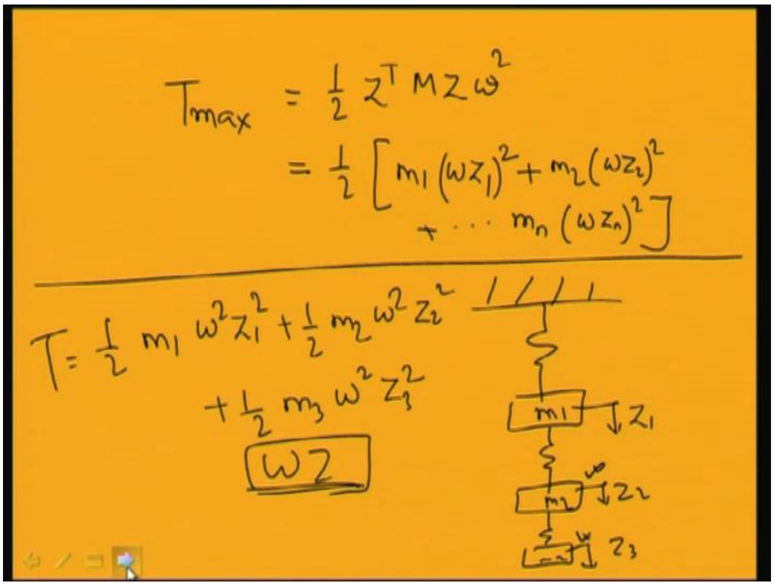

So, I can write this T max maximum kinetic energy equal to half Z transpose MZ omega square. So, as it is equal to VM V transpose MV and velocity can be written in terms of the displacement that is omega Z. So, by substituting that thing you can write this T max that is maximum kinetic energy of the system equal to half Z transpose MZ omega square. Similarly, if by taking or choosing this Z matrix in such a way that it will yield this as a unit matrix and you can write in this way. So, in that case you can write this will be equal to half m 1 if I will x 1 this thing then it can be written in this form. M 1 Z Z 1 square I can write for the Z mode I can m 1 into Z omega into z 1 square plus m 2 into omega into Z 2 square. Similarly, you can go on writing this will be equal to mn. So,. So, this will be equal to mn into omega into Zn square.

So, this is the T max that is the maximum kinetic energy of the system. So, let you have a system with 3 mass. So, this is 1 mass this is the second mass. So, let me add a third mass also to the system. So, this is m 1 this is m 2 this is m 3. So, the kinetic energy of the system will be equal to. So, this is Z 1 this is Z 2 and you are assuming this is to be Z 3. So, the kinetic energy will be equal to half m 1 Z 1 dot square plus half m 2 Z 2 dot square plus half m 3 Z 3 dot square. So, assuming Z to be in the form of Z sin omega T or harmonic. So, you can write this equal to half m omega square Z 1 square half m half m 2 omega 2 square Z 2 square plus half m 3 half m 3 omega 3 half. So, you can write this way.

So, half m 1 omega 1 square Z 1 square plus half m 2 omega 2 square Z 2 square plus half m 3 omega 3 Z 3 square as omega you are taking for a particular value of omega. So, then this is equal to half m 1 omega. So, omega Z is the velocity and. So, for the first mass the velocity is omega Z 1 second mass it omega Z 2 and third mass it is omega Z 3. So, it becomes this kinetic energy becomes half m 1 omega square Z 1 square plus half m 2 omega 2 square Z 2 square and for this half m 3 omega square Z 3 square. Similarly, for n system you can write this kinetic energy in this form. So, now, we have to guess this value Z 1 Z 2 Z 3. So, in this case I can take the static vibration or this this will be proportional to m 1 g this will be proportional to m 2 g the displacement of this will also proportional to m 3 g. So, in this static deflection, so we can find this potential energy of the system.

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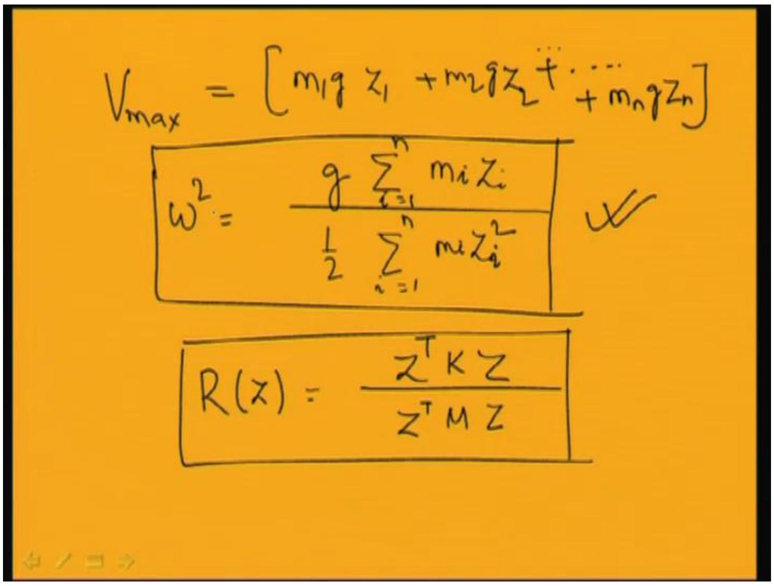

So, the potential energy I can write in this form. So, V max will be equal to, so m 1 g 1 Z 1 plus m 2 g 2 Z 2 m 2. So, m 1 g Z 1 plus m 2 gZ plus mngZn, so this way we can write this kinetic energy and we can write the potential energy of the system. Now, we can equate this and we can write this omega square in this form. So, it will be equal to g into I equal to 1 to n. So, this will be miZi and this will be equal to half I equal to 1 to n miZi square. Already we got this kinetic energy in this form that is equal to half omega square into m 1 Z 1 square plus m 2 Z 2 square plus mnZn 1 square. And this potential energy if you writing in the form m 1 gZ 1 plus m 2 gZ 2 and for n th mode you are writing this equal to mn. So, for other modes also you can add it.

So, you can add and for you can write mngZn then this equating this kinetic energy and potential energy you can write this equal to gmiZi. So, summation I equal to 1 to n miZi half miZi square. So, in this way you can find for a discrete system. You can find the Rayleigh quotient or the Eigen value of the system or the approximate Eigen value of the system by equating the maximum kinetic energy equal to this maximum potential energy or by directly writing this formula that is RZ equal to. So, you can write this as Z transpose KZ by Z transpose MZ. So, by writing from this expression or from equating the kinetic energy maximum kinetic energy equal to maximum potential energy you can find the frequency of the system.

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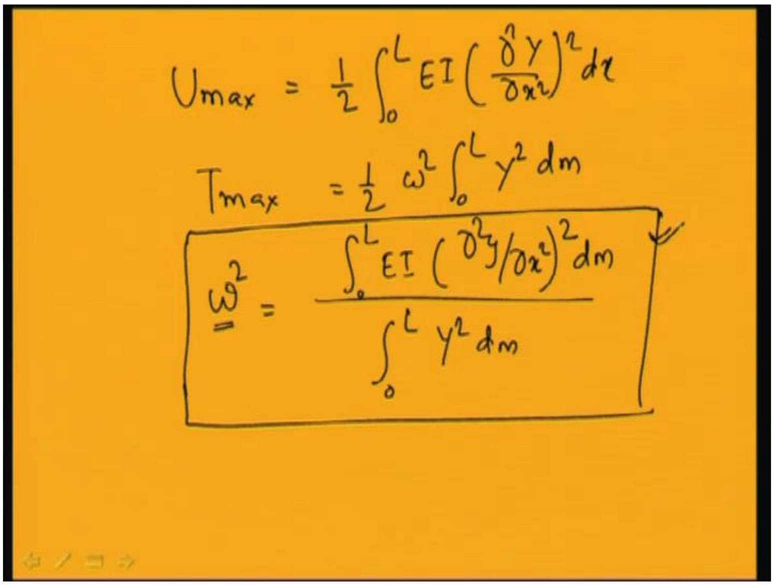

So, for a continuous system. You can write this U max that is the maximum potential energy like this. So, it is equal to half 0 to L EI del square I y by del x square whole square dx and this T max equal to half omega square 0 to L y square dm. So, here you may take this y as the static deflection of the continuous system. So, by taking the static deflection term by equating this U max equal to T max or maximum potential energy equal to maximum kinetic energy. So, you can write this omega square equal to. So, this is equal to 0 to L EI del square y by del x square whole square by 0 to L y square dm.

So, when this y matches with the Eigen value of the system then you will get the exact Eigen from Eigen value. And when it is not matching with this Eigen value you will get the approximate value and already we have seen that this approximate value. So, if you make a error 10 percent error in taking this Eigen vector then there will be 1 percent error in finding this Eigen value or this Eigen value you can great it of the order of epsilon square. So, 1 can predict accurately by using this Rayleigh method the fundamental frequency of the system.

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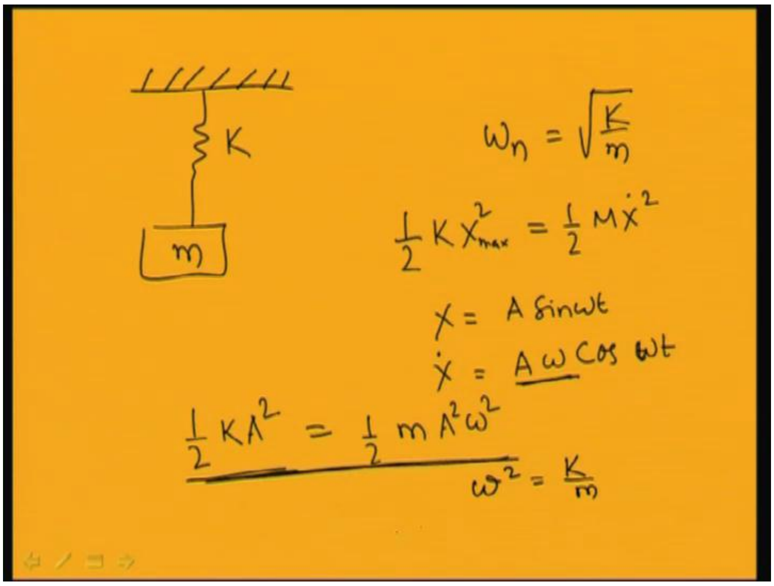

So, let us take some example. So, as a first example let us take a discrete system. So, let in this discrete system we want to find the Eigen value of the system. For example let us first take the simple system with K and m. So, in this case already you know the Eigen value or the natural frequency of the system equal to root over K by m. So, the same thing you can find it using this Rayleigh method. So, in the Rayleigh method you can have this maximum kinetic energy equal to half KX max square. So, this will be equal to half MX dot square, so X dot, so assuming this X to be in the form of a sin omega t. So, you know your X dot equal A omega sin omega t.

So, this is X dot will be equal to A omega cos omega t. So, as X equal to A sin omega t you have X dot equal to A omega cos omega t. So, this maximum potential energy will be equal to half KX square. So, X X will have a maximum value that is equal to A. So, this becomes half KA square. So, this is the maximum potential energy of the system. So, this will be equal to maximum kinetic energy. So, the maximum kinetic energy. So, maximum velocity will be A omega. So, maximum kinetic energy of the system will be equal to half m A square omega square. Now, equating these 2 terms, so you can find. So, half half cancels, so this A square A square cancels. So, this omega square you can write equal to omega square equal to K by m.

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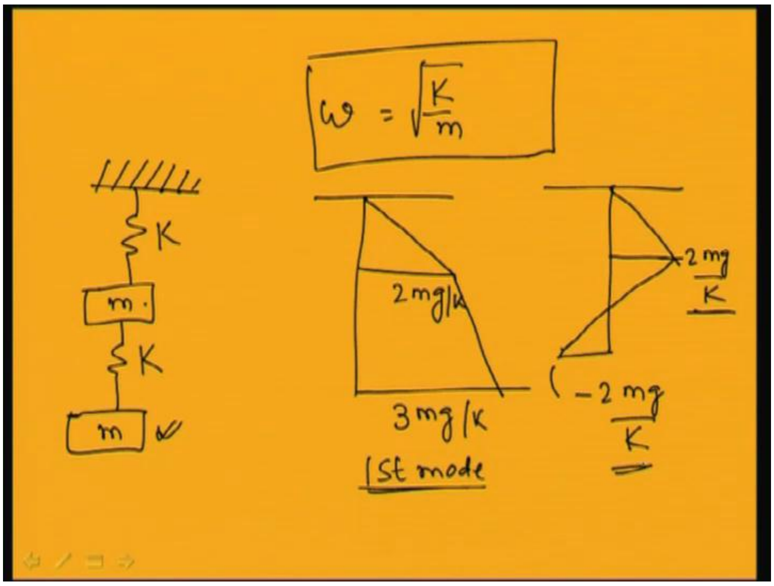

So, omega equal to, so or omega equal to in this case you have found. Omega equal to root over K by m. So, this is the natural frequency that we have written n. So, in this way you can find and let us take a system with 2 mass. So, this it is first spring and mass let it is K and m let us take same mass and stiffness, so this is m and K. So, in this case I can take the static mode like this. So, due to, so this will have a weight of mg and this is this has a weight of mg. So, 2 mg weight is acting on this spring K. So, it will have displacement of 2 mg by K. So, let me take this is the line. So, the displacement will be 2 mg by K in this case. N. And this mass will have a displacement of. So, this spring a subjected a weight of mg. So, this will have displacement of mg by K and already you have displacement of 2 mg by K.

So, total displacement of the spring second spring will second mass will be equal to 3 mg by K, so by taking the static deflections. So, I can take this is equal to 3 mg by K. So, the deflection plot I can. So, it in this way, so this is 2 mg by K and this is 3 mg by K. So, I can take this Z 1 equal to 2 mg by K and Z 2 equal to 3 mg by K in this case. So, the first mode when it is vibrating in the first mode I can take the, I i can take this approximate vector as 2 mg by K and 3 mg by K. So, when it is vibrating in the second mode. So, this is for the first mode you these are the approximate function I can take in the second mode I know that one. So, in the first mode both the masses are moving in the same direction and in the second mode the first mass and the second mass will move in opposite direction.

So, when they are moving in opposite direction. So, I can I can let me keep the first mass same that is 2 mg by K. But the second mass will move in the opposite direction. So, I can take this is 2 mg by K and for the second mass I can it as minus 2 mg by K. So, in this case, so I have taken that static deflection for the mass 1 as 2 mg by K and for the second mass is equal to 3 mg by K for the first mode. And for the second mode as I know both the masses are moving in opposite direction I can choose it in this way. So, by choosing it in this way, so I can write the kinetic energy of the system and the potential energy of the system.

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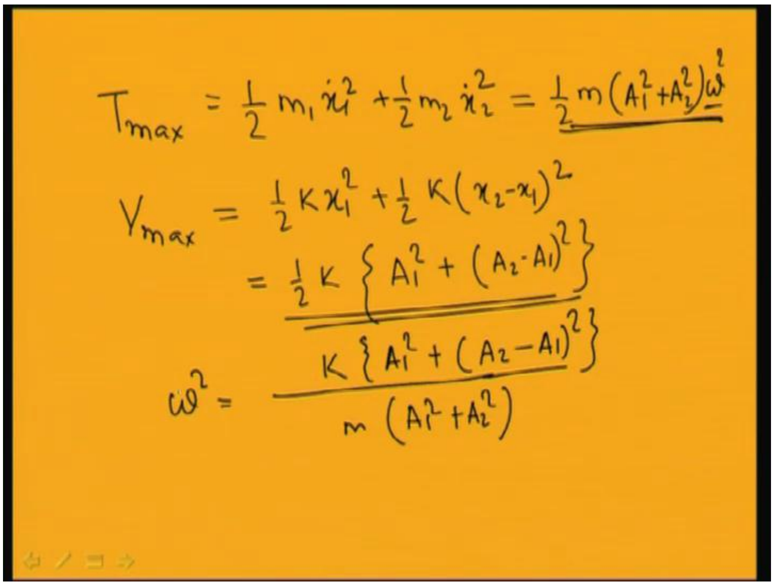

So, I can write T max maximum kinetic energy will be equal to half m 1 x 1 dot square plus half m 2 x 2 dot square. So, this will be equal to, so the maximum velocity equal to x 2 dot square maximum velocity will be equal to A omega. So, I can write for the first mass maximum velocity equal to A 1 omega and for second mass it is equal to A 2 omega. So, the maximum will be equal to as I am taking this mass as same it will be equal to half m A 1 square plus A 2 square. Similarly, I can find this V max maximum velocity maximum potential energy. So, that will be equal to half K x 1 square plus half K x 2 square. So, let us see the, so this spring has displacement of. So, this is x 1 this is x 2. So, this spring will have a displacement of x 1, but this spring will have a relative displacement of x 1 and x 2. So, this will be x 2 minus x 1.

So, it will be, so in case of second spring the displacement is K x 2 minus x 1 square. So, the total potential energy of the system becomes half K x 1 square plus half K x 2 minus x 1 square. So, by substituting the maximum value of x 1 equal to A 1 and x 2 equal to A 2 I can write this as half K this will become A 1 square plus. So, this becomes A 2 minus A 1 whole square. So, in this way I can write or I can equate this. So, this is this kinetic energy is multiplied by half m A 1 square plus A 2 square omega square as velocity equal to velocity maximum velocity equal to A omega. So, it can be written half m A 1 square plus A 2 square omega square. So, I can equate this maximum kinetic energy with this maximum potential energy. Here I am assuming A 1 equal to.

So, A 1 I have assumed equal to 2 mg by K and 3 mg by K equal to A 2 for the first mode. And for the second mode A 1 I have assumed equal to 2 mg by K and for the second mass it is equal to minus 2 mg by K. So, I can write this omega square expression for omega square this will be equal to half K A 1 square plus A 2 minus A 1 square by half m A 1 square plus A 2 square. So, this half half cancels, so this becomes A 1 square plus A 2 square I can delete this half. So, this becomes omega square equal to this K by m A 1 square plus A 2 minus A 1 square by A 1 square plus A 2 square.

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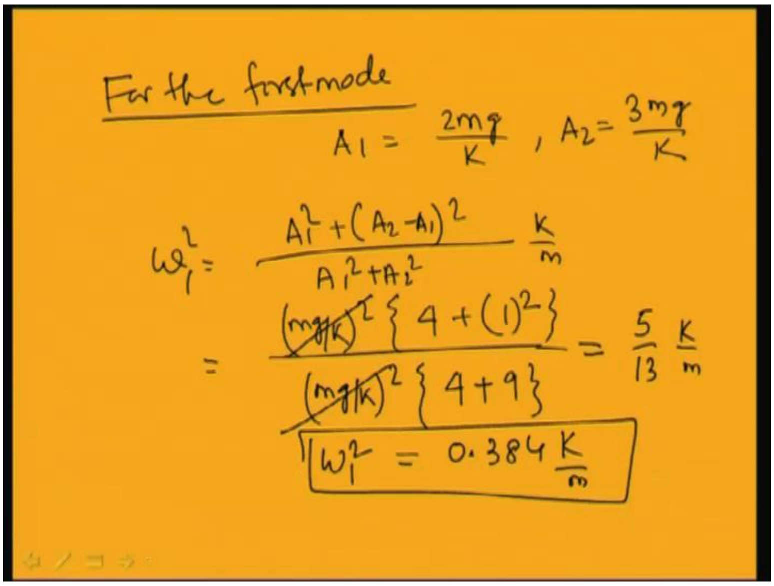

So, now substituting for the first mode. So, let us find for the first mode, so I can substitute this A 1 equal to 3 mg by. So, A 1 equal to 2 mg by K and A 2 equal to 3 mg by K. So, by substituting it in this expression then this becomes A 1 square plus. So, I have this expression omega square equal to omega square equal to A 1 square plus A 2 minus A 1 whole square by A 1 square plus A 2 square into K by m. So, I can have it like this. So, for A 1 I will put 2 mg by K. So, this becomes 4. So, if I will take this mg by K mg by K common mg by K square common then it will become A square. So, it will become 4.

So, then plus 4 plus A 2 minus A 1 A 2 minus A 1 will give. So, this is 9 3 minus 2 this is 1. So, 3 minus 2 square it is equal to 1 1 square. And below it is equal to A 1 square that is mg by. So, this becomes mg by K whole square into for A square I can write this is equal to 4 plus this is equal to 9. So, in this way I can cancel these 2 terms. So, this becomes, so in the, so this becomes 5 by 13. So, this 5 by 13 will give me, so 5 by 13. So, you are getting a value of 0.384. So, this is equal to 0.384 K by m. So, this omega you can. So, this is omega square or omega 1 square.

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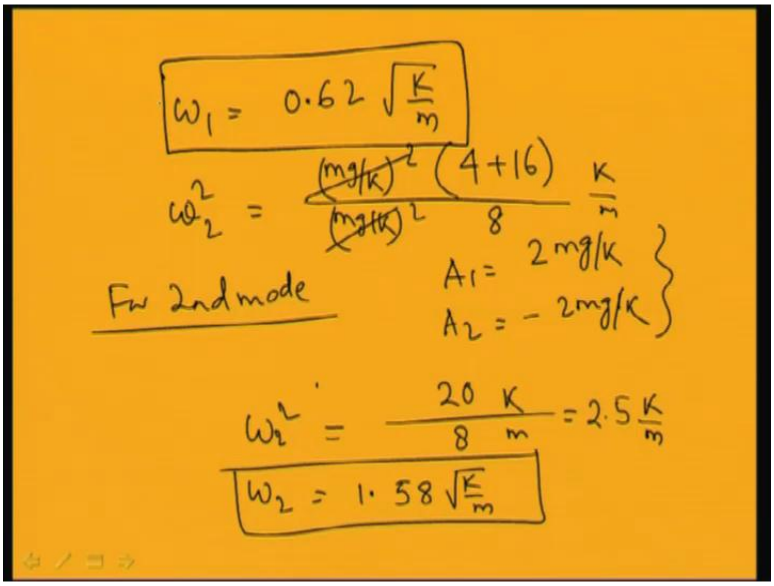

So, you are finding it for the first mode or omega 1 will be equal to omega 1 will be equal to root over this. So, this becomes 0.62 root over K by m. Similarly, you can find omega 2. So, in this case omega 2 square will be equal to mg by K whole square mg by K whole square into. So, for second mode you are taking this A 1 equal to 2 mg by K and A 2 equal to minus 2 mg by K. So, it will becomes then it will become this A 1 square A 1 square will be 4. So, this will be 4 plus A 2 minus A 2 minus A 1 that is minus 2 minus 2 minus 4. So, minus 4 whole square this becomes 16. So, 4 plus 16 here you are getting and below you are getting this is 4 plus 4. So, 4 plus 4 this is 8.

So, for the second mode you are getting omega 2 square. So, this is also mg by K whole square. So, this cancels. So, omega 2 square you are getting equal to 20 by 8 K by m. So, 20 by 8 will give you 20 by 8 will give you 2.5. So, you are getting this is equal to 2.5K by m. So, in this case this omega 2 this is equal to 2.5 root over. So, it becomes 1.58 root over K by m. So, you are getting the second mode frequency equal to 1.58 root over K by m. And the first mode frequency equal to. So, the first mode frequency we got it equal to 0.62 root over K by m.

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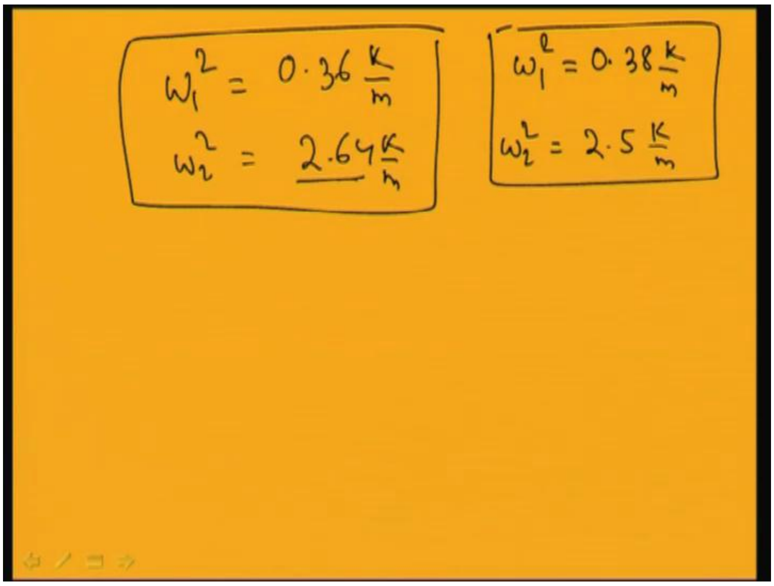

So, you can see this omega once exact value are like this omega 1 square equal to 0.36 K by m and omega 2 square equal to 2.64 K by m. So, these are the exact value. And in this case we obtain this omega 1 square equal to 0.38 K by m and omega 2 square we obtained equal to 2.5 K by m. So, you just note that by taking the static deflection we obtain very close to this first mode and second mode frequency. Here the first mode is 0.36 and here we are getting it is equal to 0.38. And in case of the second mode the Eigen value is 2.64 exact Eigen values is 2.64 and here we are getting 2.5. So, if you calculate the errors you can find 2.5. So, minus 2.64 by 2.64 you can divide that thing and you can obtain the error only. So, 0.05 you are getting.

So, if you multiply this thing. So, you are getting in the second mode 5 percent error. And in the first mode, so the, it is 0.02. So, 0.36 minus 0.38, so 0.025 0.38 you can make. So, this is similar, so you can see the error only in the second decimal only. So, in this way you can find the fundamental frequency of the system by using this Raleigh method.