Mechanical Vibrations

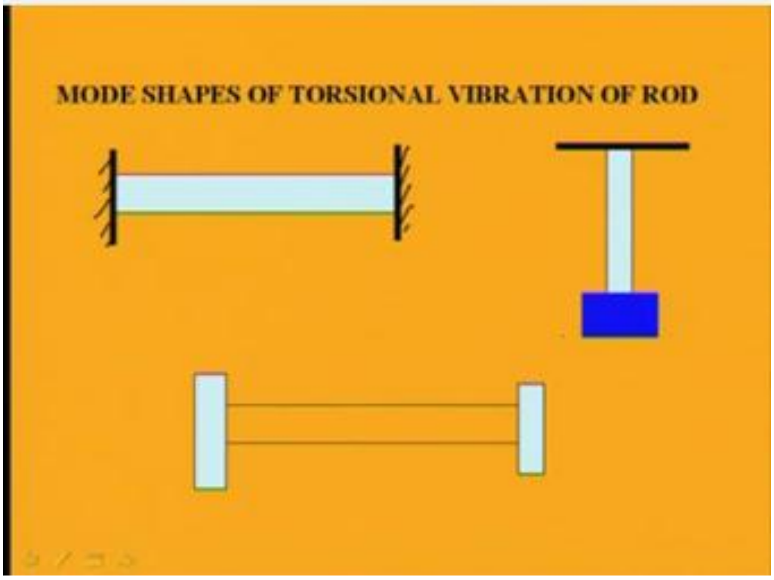
Prof. S. K. Dwivedy
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 9
Continuous systems: Closed form Solutions
Lecture - 6
Transverse Vibration of Beams: Natural Frequencies and Mode Shapes

In the last few classes, we are studying about the continuous system. So, we have seen 2 different types of equations in this continuous system of 1 dimensional system. One is the wave equation for which we have taken the systems of lateral vibration of the torque string. Then we have taken longitudinal vibration of rod and the torsional vibration of the shaft. And we have seen another type of equation that is the Euler-Bernoulli equation where you have a fourth order equation and in case of wave equation you have a second order equation.

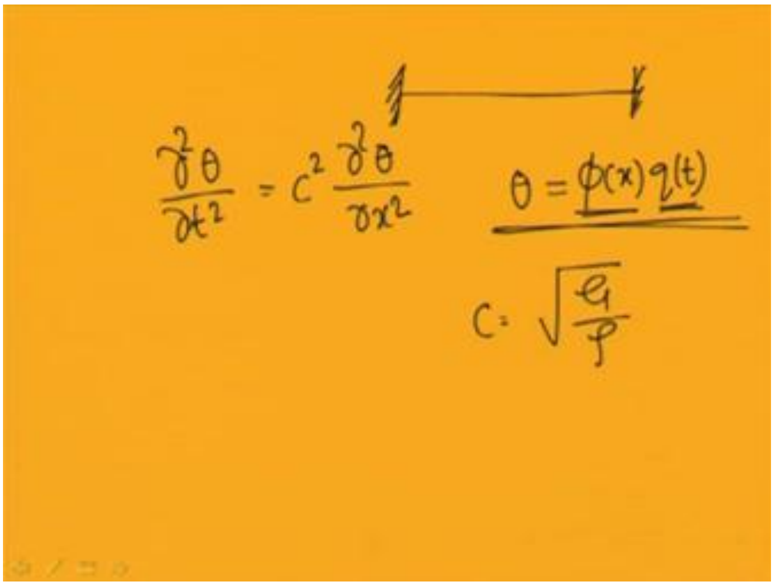
So, in the previous classes, we have studied about the Euler-Bernoulli beam with different boundary conditions such as simply supported boundary condition clamped boundary condition cantilever beam and free-free beam. Also we have studied about the longitudinal vibration of rod and the lateral vibration of torque string. Already we have solved this wave equation and found the general solution for this wave equation and applying to different boundary conditions. We have studied the modes shapes of longitudinal vibration of rod and the lateral vibration of torque string. So, today class I am going to tell about the mode shapes or the torsional vibration of rod for example.

(Refer Slide Time: 02:23)



I will show you the vibration of or mode shapes of a fixed-fixed rod and a rod with an attached mass and a rod within which in the 2 ends you have the inertia. So, in this cases you can see that for this only for this fixed case will have a very simple frequency equation. But with increase in this complexity of the system that is when you are adding only 1 mass or when you are adding only 2 mass the equation frequency equations are getting complicated. And you cannot solve it by manually. So, you may have to go for the computational or you have to use the computer to find the solution for the systems. And in addition to this I will tell you about this initial value problems and also briefly I will tell you about these force vibration. And by how by using this model analysis method you can study the force vibration of a continuous system. So, to start with, so let us find the mode shape of this fixed rod. So, in case of the fixed rod.

(Refer Slide Time: 03:43)

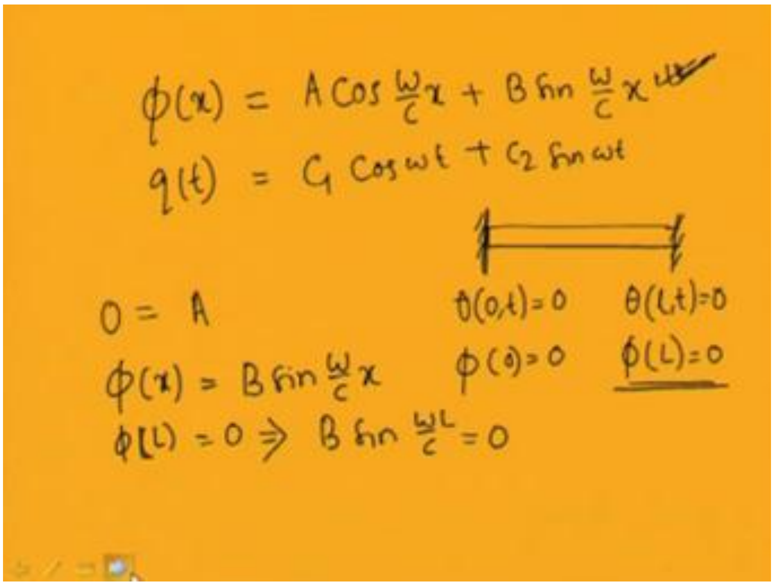


So, already you have found the equation motion for this rod. So, in case of a rod torsional vibration of a rod the equation motion can be written in this form del square phi by del t square equal to C square del square phi by del x square del square. So, here let me take theta let del square theta by del t square equal to C square. Del square theta by del x square where theta is the torsional displacement or the rotation of the shaft and this rotation. I will express in terms of as a function of it will be I will express this as a function of phi x and q q t. So, here phi x is the mode shape of the system and q t is the time modulation of the system.

So, already you have derived this equation by using Hamilton principle. And also you can derive this equation also by using this Newton's method. So, in case of Hamilton principle you take the kinetic energy of the system and take the potential energy of the system. Potential energy is the strain energy stored due to the shearing effect of the shaft by taking these things. You can apply you can find the Lagrangian of system l equal to t minus u by taking that Lagrangian then you can apply the Hamilton principle to find the equation motion.

So, the equation motion is in this form del square phi by del t square equal to C square del square theta by del x square where C square equal to G by row. So, this is C square. Equal to G by rho or C you can write equal to G by rho. So, where G is the rigidity modulus and rho is the inertia for unit length of this system. So, by taking these del square theta by del t square equal to C square d square theta by d x square which the wave equation and this is the general solution for this case which we have already obtained. So, let us phi let us find this frequency equation for this case.

(Refer Slide Time: 06:20)



So, the general from the general expression you know this. Phi x can be written in this form. So, phi x equal to A cos omega by C x A cos omega by C x plus B sin omega by C x here q t is the time modulation. And it can be written in this form and C 1 cos omega t plus C 2 sin omega. So, by taking this time modulation C 1 cos omega t plus C 2 sin omega t, and this general mode shape phi x equal to A cos omega by C x plus B sin omega by C x. So, now, you can apply the boundary conditions in this case. So, in case of the fixed beam or in this case of the fixed rod, so as the as both ends are fixed. So, there will be no rotation in this fixed ends.

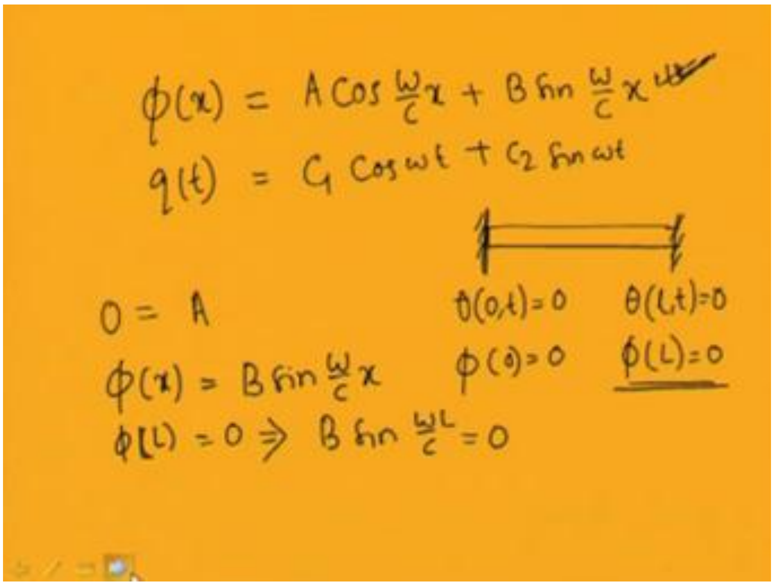
So, was there no rotation in this fixed end; that means, theta zero t equal to 0 and in this side theta l t equal to 0. So, theta 0 t equal to 0. So, this implies for all time this theta equal to 0 so; that means that this phi 0 will be equal to 0. As you are substituting theta equal to phi x into q t, so as for all times this boundary condition or at this boundary rotation will be equal to 0. So, you can take phi 0 equal to 0 similarly here you can take phi l equal to 0. So, now, substituting these 2 boundary conditions in this equation of mode shape, so you can write, so phi x. So, if I will write this phi x equal to, so by substituting x equal to 0. So, cos 0 0, so you will have 0 equal to A plus.

So, sin zero equal 0, so B into zero, so this means this A equal to 0. Similarly, now by putting this boundary condition phi L equal to 0 as already A equal to 0 you can write phi x equal to B sin omega by C x already A equal to 0. So, by substituting A equal to 0 in this equation, So, you can write phi x equal to B sin omega by C x now, substituting this phi L equal to. So, phi L equal to 0. You can write B sin omega by C L equal to 0. So, this is equal to, so this implies already as I told in the previous cases this B sin omega C omega L by C equal to 0. So, B cannot be equal to 0, because when B equal to 0 this will lead to the trivial solution of the system.

(Refer Slide Time: 09:30)

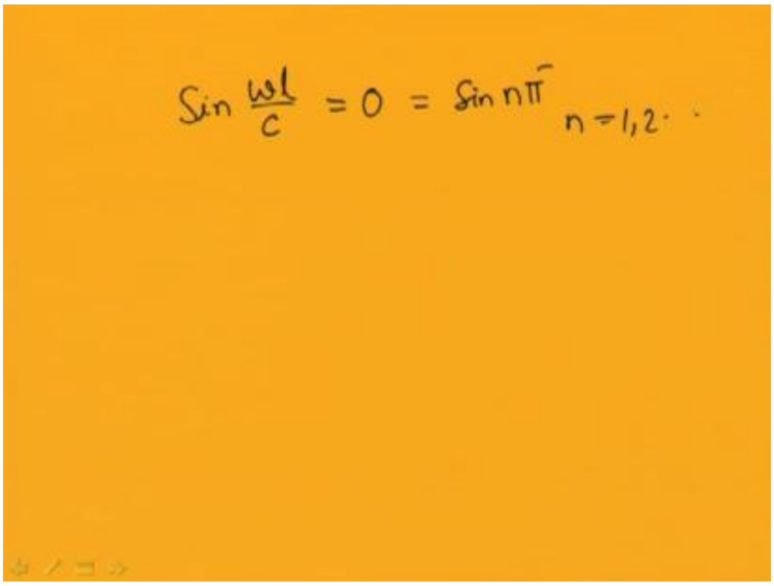
So, in this case of the rod, so this is the rod. So, in this case, so in this rod you are fixing at the both the ends of the rod. And. So, when both ends are fixed. So, at this ends there is no vibration at this ends. So, you have taken this phi or theta 0 equal to 0. And theta L equal to 0. So, the mode shape phi 0 equal to 0 and phi L equal to 0 you have taken, so when you are taking B equal to 0. So, the total expression phi x will be equal to 0. So, that will lead to the trivial states solution; that means, there will be no vibration of this rod or the rod will not rotate. But as you are considering the rotation of this rod, so we have to take B not equal to 0.

(Refer Slide Time: 10:17)



So, when you are taking B not equal to 0. So, in that case expression will reduce to sin omega L by C equal to 0.

(Refer Slide Time: 10:25)

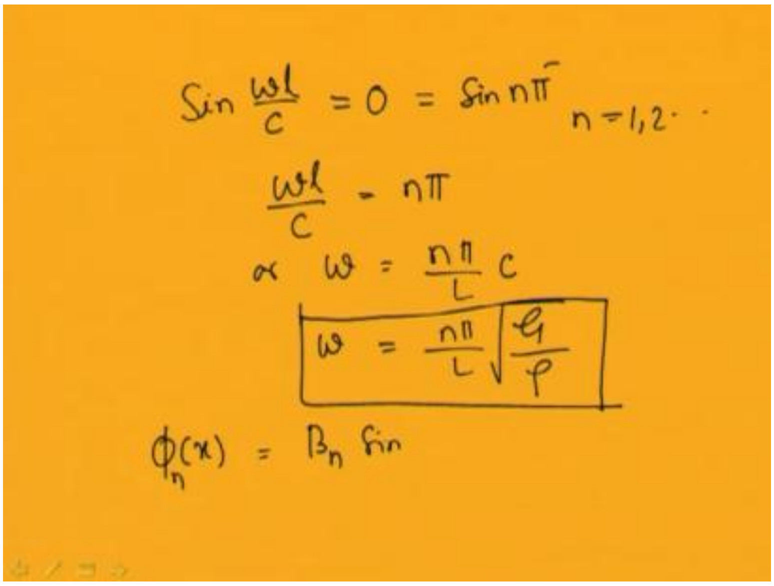


So, we have sin omega l by sin omega l by C or omega by C l equal to 0 this implies that sin omega l by C equal to sin n pi. So, this is equal to sin n pi. So, n equal to 1 2 3, so you can put different value of n. So, n equal to you can put 1 2 and other values. So, n equal to 1 correspond to the fundamental mode n equal to 2 correspond to the second mode and n equal to 3 higher modes N equal to 3 third modes and similarly n. So, you can go upto n number of mode or infinity number of modes.

(Refer Slide Time: 11:11)

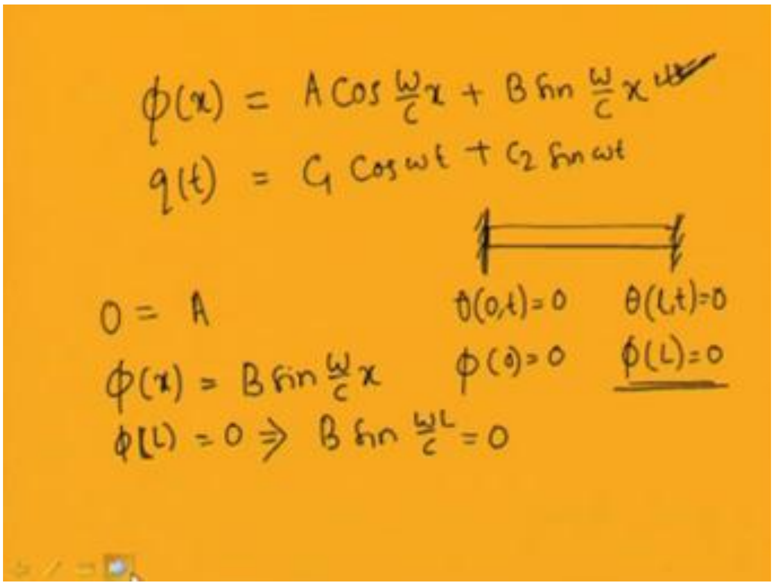
So, the shaft or the shaft this shaft will have infinity number of infinity number of frequencies.

(Refer Slide Time: 11:20)



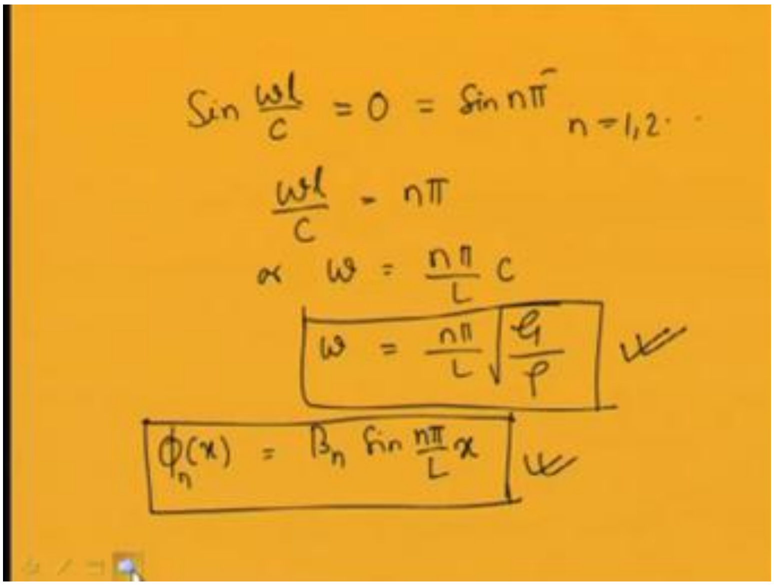
And. So, in this case you can write this omega l by C equal to n pi or this omega you can write equal to n pi n pi by L into C, so where C equal to n pi by L into root over G by rho. So, this is the expression for omega. So, you can observe that. So, in this case you can observe that the nth frequency is n times of the fundamental frequency of the system. So, in case of torsional vibration of this fixed rod the nth vibration will be n times the first mode vibration

(Refer Slide Time: 12:18)



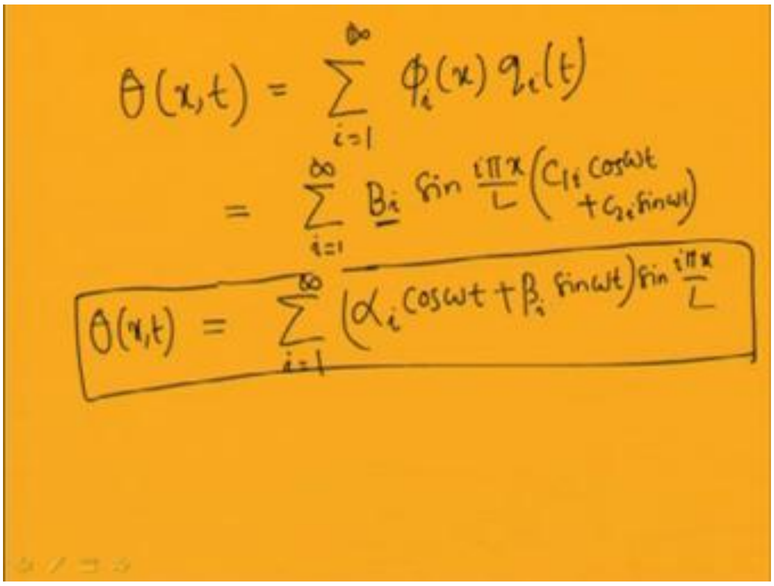
So, the corresponding mode shapes you can write the phi x will be equal to phi x equal to or phi n x equal to Bn sin omega by C. So, you can write from for this omega by C equal to n pi.

(Refer Slide Time: 12:22)



So, omega by C equal to n pi by L, so this is n pi by L into x, so this is the mode shape of the system. And this is the frequency equation or the expression for the frequency of the torsional vibration of the fixed rod.

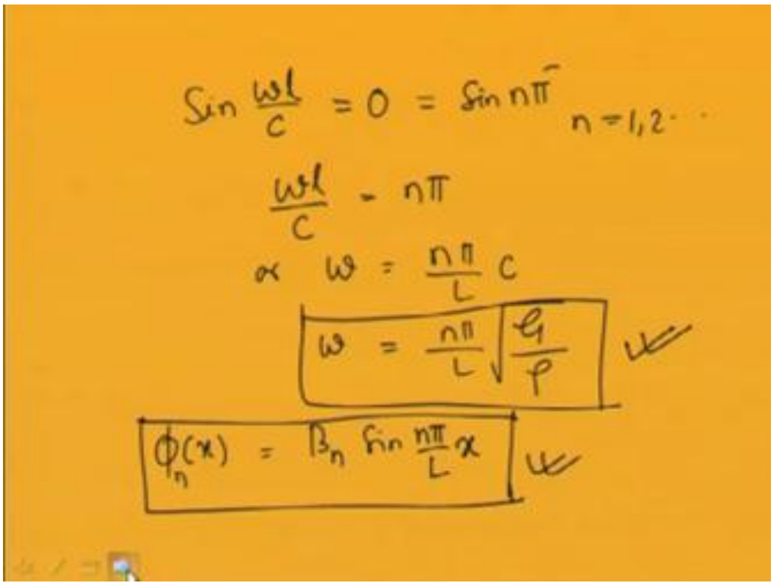
(Refer Slide Time: 12:49)



So, the general expression or the general vibration will be general vibration or general theta x t I can write in this form. So, theta x t as contains n modes, so I can write this is equal to summation of I equal to 1 to infinity. So, product of, so this will be phi n or phi ix I can write phi ix into qit. So, here I can write this equal to. So, this will be equal to I equal to 1 to infinity. So, this is already I got this equal to Bi sin I pi x by L. So, for ith mode I can write this is equal to Bi sin I pi x by L and for qit I can write this as. So, any 2 this will be a function of sine and cosine term. So, I can write this as C 1 I cos omega t plus C 2 I sin omega t. So, now, you observe that these constant can multiplied with these 2 constant and this equation can be reduced or can be written in this form.

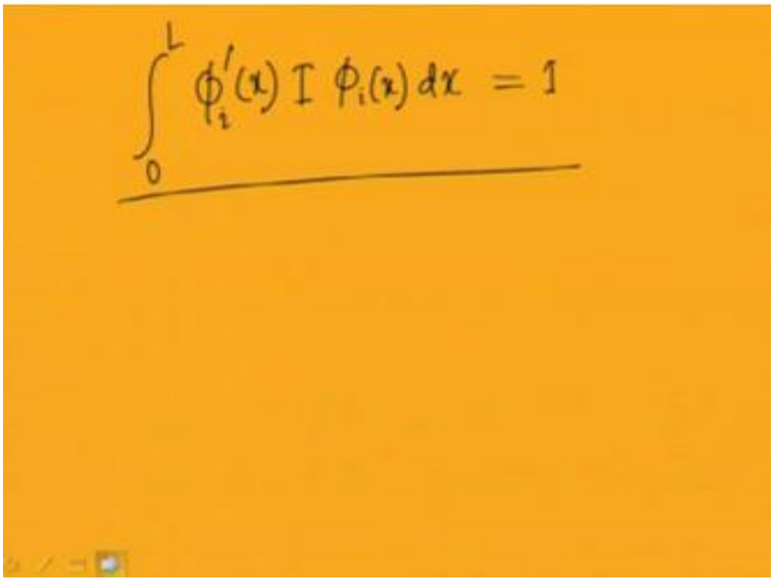
So, you can write this is equal to I equal to 1 to infinity. So, I can write this as using some general constants let I am writing this is in terms of C or in terms of alpha and beta. So, I can write this is equal to alpha I cos omega t plus we can write this as beta I sin omega t into sin I pi x by L. So, this is the general expression for this free vibration of this rod. So, this alpha I and beta I alpha I will be equal to Bi into C 1 I and beta I equal to Bi into C 2 i. So, these 2 constants can be obtained from the initial conditions. So, let in today's class we will see the initial value programs. So, how to find these initial conditions? We will find.

(Refer Slide Time: 15:20)



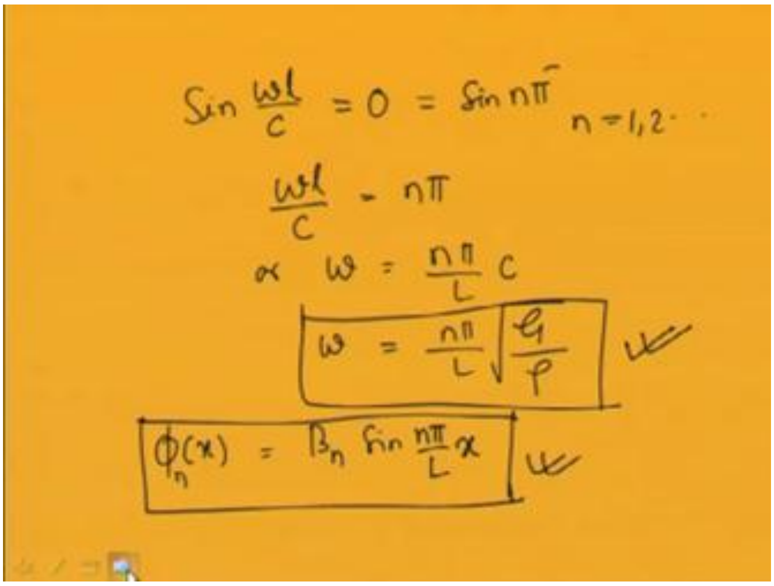
So, you just observe that this Bi for different mode this Bi will be different. So, you will have a for n mode you will have n number of constants B 1 B 2 B n. So, like in case of multi degree of freedom systems also here you can normalize this and you can find the normalized value for this. So, this normalized value to find this normalized value you can apply this orthogonality the principle. So, to apply orthogonality principle already you know.

(Refer Slide Time: 15:51)



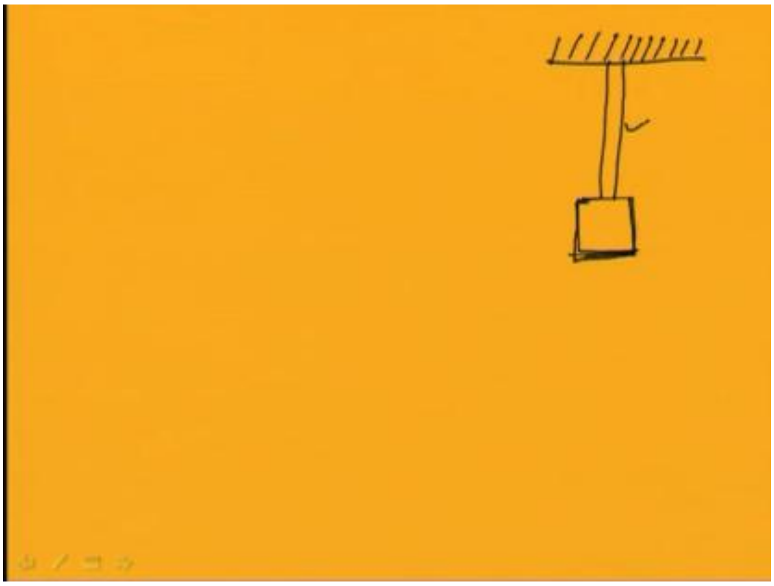
So, the orthogonal different full can applied in this way. So, phi, so you can write this as phi ix into. So, you can multiply with this mass matrix of this. So, phi I into I this moment of inertia of the system into phi I, so this is phi I transposing to phi ix dx. So, you make it equal to 1. So, if you make it equal to 1 this integral. So, in that case you will find the value for this Bn. So, for ith mode you can make equal to 1 and find the constraint B.

(Refer Slide Time: 16:46)



So, in this way by using this orthogonality principle you can find this constant B. And you may use that thing for finding the response of the system. So, let us consider the other system in this case.

(Refer Slide Time: 16:56)

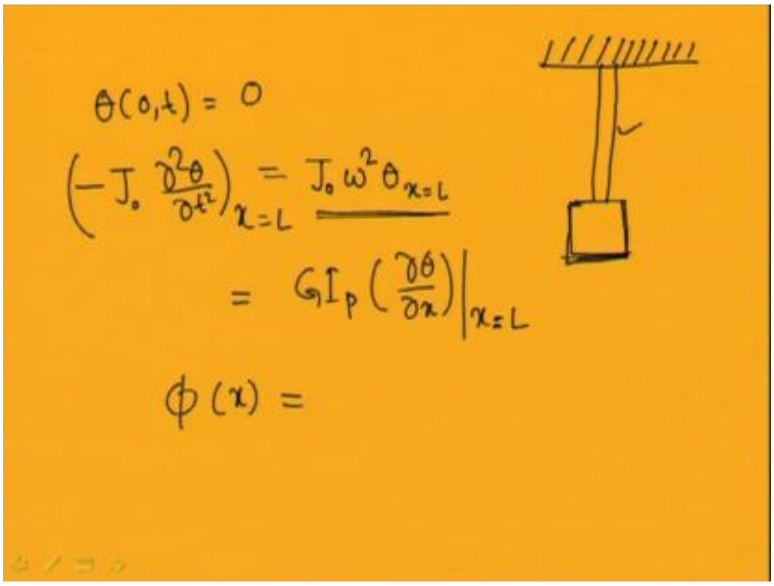


We all consider. So, let us make the system slightly more complex that is let us consider a rod consider a rod with an inertia. So, in this case if I am adding this inertia to this end of the system then I have to find the frequency equation for the system. So, this case can be used as a practical example you can consider a well. So, from this oil well you have to use this drill bit or a tool to. So, this is the drill and this is pipe attached to the drill. So, in this case it will. So, we have to find the natural vibration of the system. So, to find this thing, so in this case the boundary conditions will be. So, as it is fixed at one end. So, let this is the rod or the pipe. So, it is fixed at the upper end and in the lower end you are attaching one mass or cutter.

(Refer Slide Time: 18:05)

So, in that case, so you can consider this top end there is no vibration to the top end or the rotation is 0 at this top and the, at this end. This load will be equal to the inertia load of the system as you are adding a or attaching a inertia at this end. So, the load will be equal to the inertia load at this end.

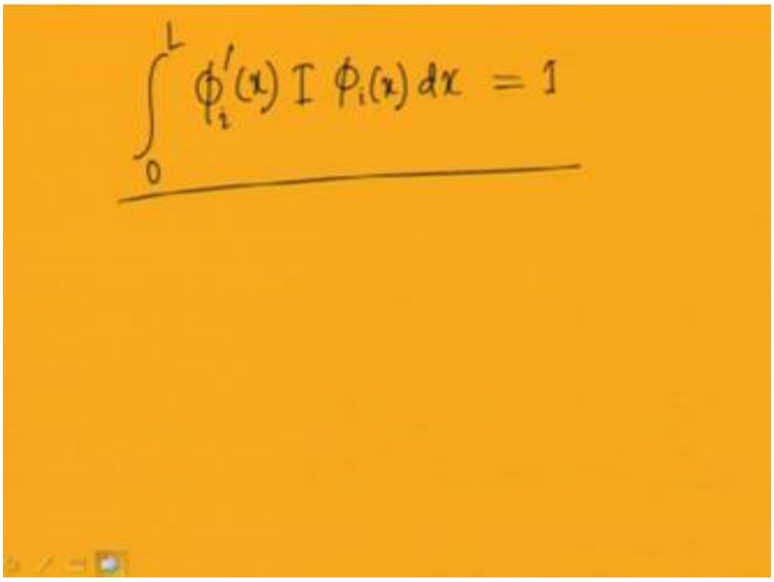
(Refer Slide Time: 18:24)



So, in this case you can find the equation motion in this way. So, you can write the boundary conditions theta. So, I can take this is equal to 0, so theta 0 equal to. So, 1 boundary condition will be theta 0 t equal to 0. And the other boundary conditions you can write equal to. So, at as it is added to the right end already I told you about these boundary conditions. So, in this boundary condition it will be equal to. So, these minus J zero del square theta by del t square del t square at x equal to L. So, this will be equal to at x equal L will be equal to this inertia force. So, this will be equal to J 0 mega square theta at x equal to L.

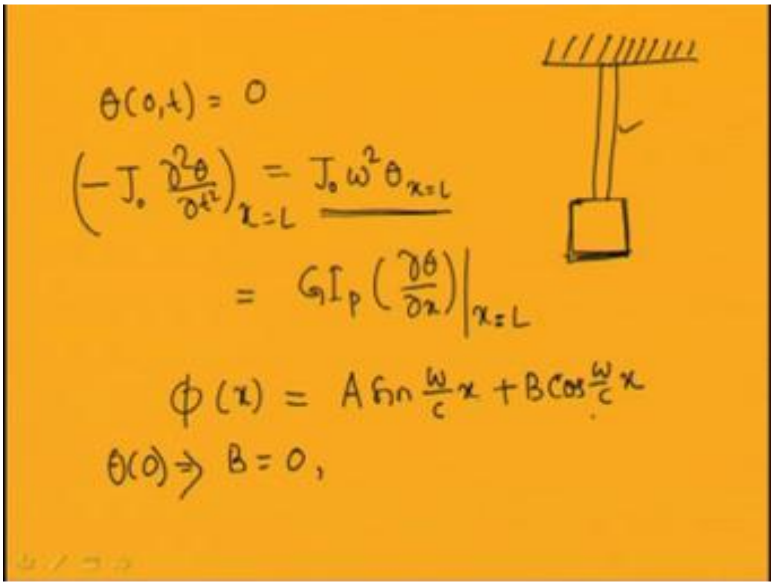
So, this is the inertia force. So, inertia equal to J theta double dot as we are assuming this theta to be harmonic. So, I can write this equal to J 0 omega square theta. So, this inertia force J 0 omega square theta at x equal to L will be equal to this minus J 0 del square theta by del t square. So, that is the. So, I can write. So, this inertia force equal to G. So, this will be equal to G Ip del theta by del x at x equal to L. So, this inertia load will be equal to the load at this end. So, this will be equal to. So, I can equate this at this boundary I can equate these 2 terms to find the solution of the system or to find the mode shapes of the system.

(Refer Slide Time: 20:39)



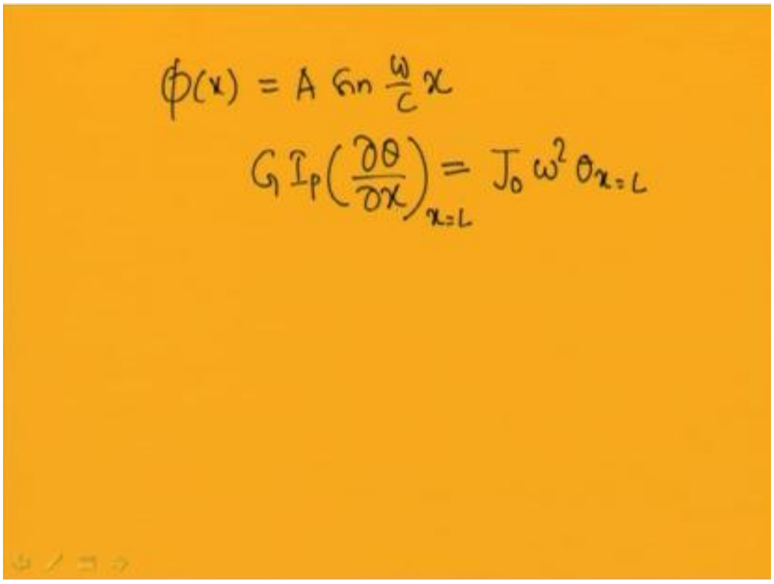
So, in this case now, I can put phi x equal to already I have written this phi x.

(Refer Slide Time: 20:41)



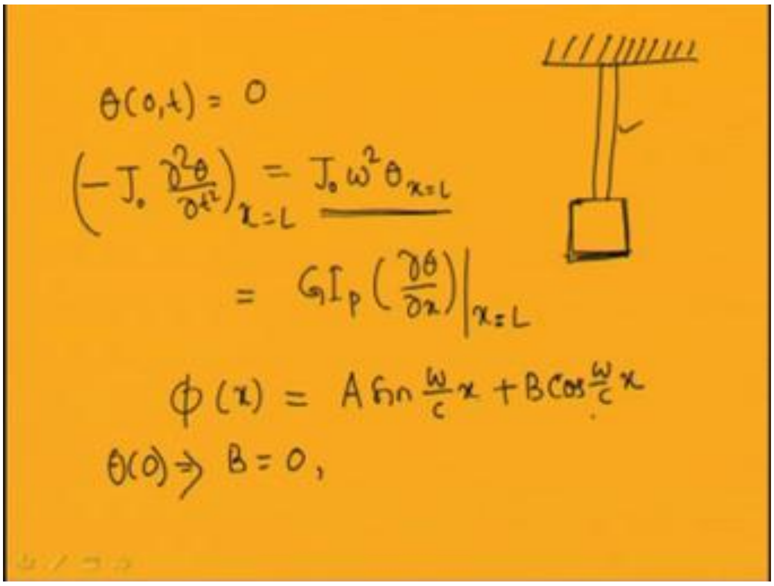
So, phi x equal to I can put it as A sin omega by C x plus B sin omega by C x, so by substituting this theta equal to 0. So, this A becomes substituting theta equal to 0. So, this A sin, so this term goes and. So, this will be let write it correctly. So, this is A sin omegas by C x plus B cos omega by C x this is the general expression. So, in this case when theta equal to 0, so this B into 1 will be equal to 0. So, theta equal to 0 implies phi x or phi 0 equal to 0. So, it implies B equal to 0. So, now, as B equal to 0 we will where left with this term phi x equal A sin omega by C x.

(Refer Slide Time: 21:44)



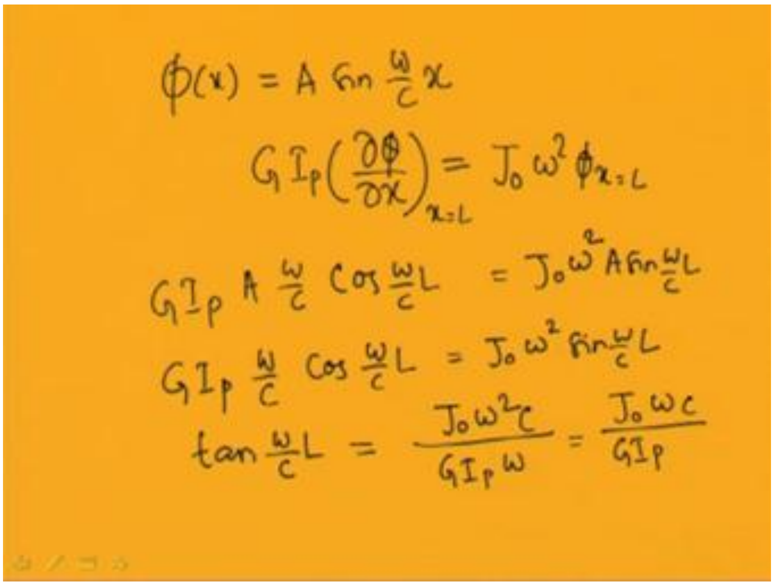
So, phi x equal to A sin omega by C x, so in this expression let us substitute G Ip into del theta by del x del theta by del x. So, this is del theta by del x is the strain and multiplied by these this is the stress and by multiplying these we are finding this load of the system. So, this be equal to this inertia torque. So, this is the torque applied. So, this is equal to inertia. So, this is equal to J 0 omega square theta at x equal to L. So, this is x equal to L.

(Refer Slide Time: 22:34)



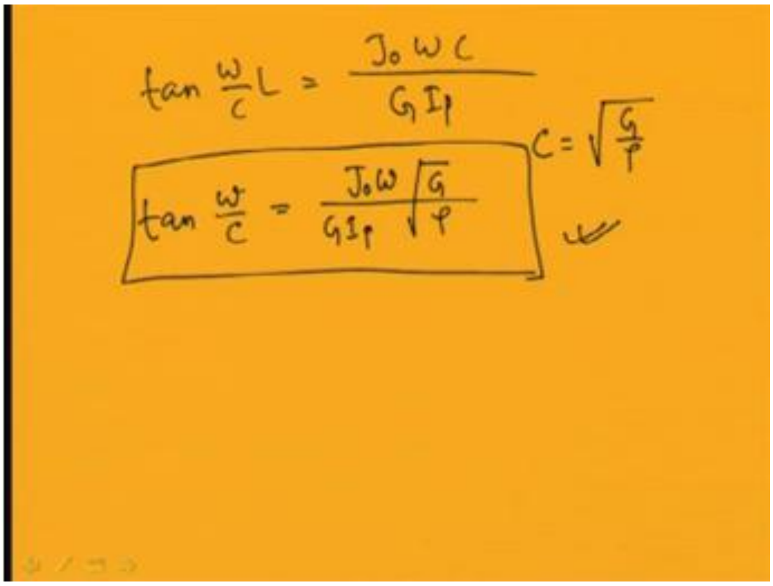
So, by substituting this thing in this equation, so let us find this is del theta by del x. So, del theta by del x as theta del phi.

(Refer Slide Time: 22:36)



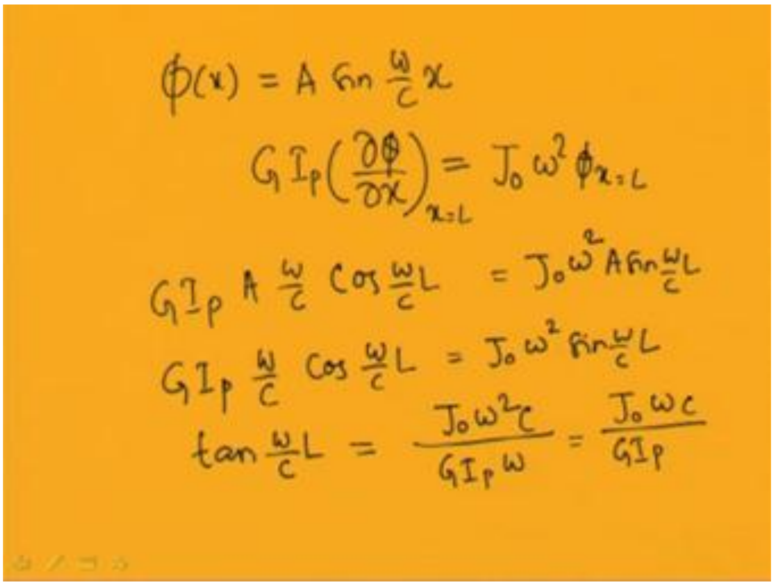
So, this is this will be del phi and this also phi as q is same. So, you can have this is equal to phi and from this by differentiating this equation. So, del phi by del x will give you A cos A omega by C into cos omega by C x. So, I can write this G Ip A omega by C. So, this is A omega by C A omega by C into cos omega by C x. So, this will be equal to for x equal to L I can substitute this equal to L. So, this will be equal to J 0 omega square. So, for phi x I can write this as a sin omega by C x X equal to L. So, in this case A A will get canceled. So, I can have this expression. So, I can write this thing this expression in this way this substituting this G Ip omega by C cos omega C L equal to J 0 omega square sin omega by C L. So, or I can write this as tan the sin omega by C L by this cos omega by C L. So, I can write this as cos or tan omega by C L equal to. So, this will be equal to J 0 omega square J 0 omega square by G Ip omega by C.

(Refer Slide Time: 24:44)



So, this will give rise to J 0 omega G Ip. So, this is C and also you know. So, this is tan. So, I can write this as tan omega.

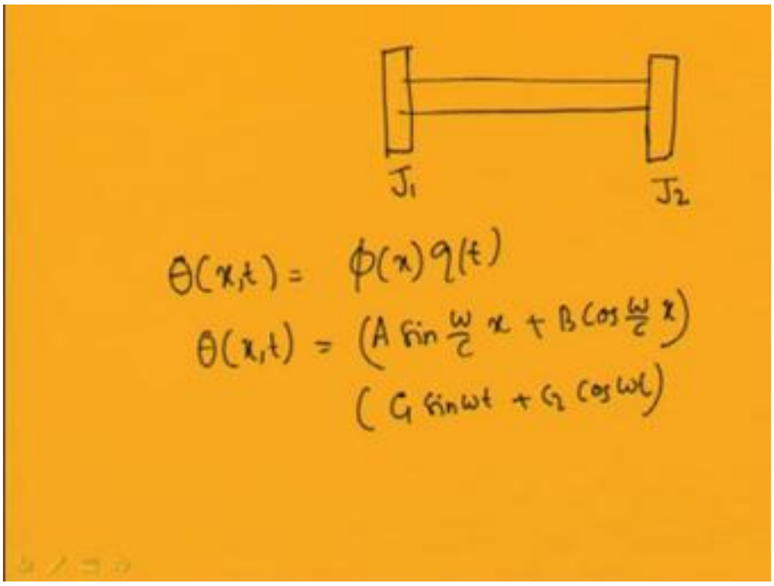
(Refer Slide Time: 24:47)



So, this tan omega by C L equal to, so from this I can write J 0 omega this expression J 0. Omega C by G Ip, so I can substitute this value of C; where C equal to square root over G by rho. So, I can substitute that thing and you can find this expression. So, this expression will give me. So, that is J 0. So, this is J 0 into for the C I can substitute it equal to root over G by rho. So, this G is by rho and this is G Ip. So, in this way you can find this expression and these expressions you can solve for a practical problem. You can solve this expression to find the frequency equation and corresponding mode shape.

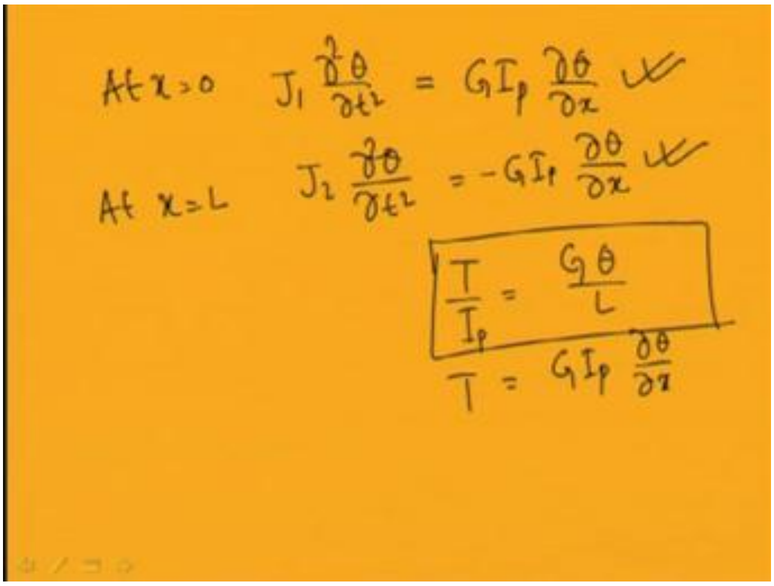
So, here J 0 is the inertia of the moment of inertia of the rod; G is the rigidity modulus; Ip is the inertia of the attached mass and this omega is the frequency. So, from this expression you can find this omega by C equal to tan inverse this right side term and you can find this frequency of the system. So, there is a omega term here also J 0 omega root over this. So, in this way you can find the frequency equation. So, from the frequency equation you can find the mode shape of the system. So, let us take the third example.

(Refer Slide Time: 26:48)



So, in this third example, let me take or let me put 2 masses or 2 inertia at this end of the rod. So, for example, this is the shaft is connected to 1 side to the generator and other side to a motor or a pump. So, if the shaft is connect to this generator and a pump, so due to their inertia. So, you have to find what is the torsional vibration in the shaft? So, in this case let me take it equal to. So, I have 2 inertia, so 1 let me write it J 1 and this is J 2 and due to this I can write the general expression theta x t equal to. So, this theta x t general expression you can write as phi x into qt phi x into qt and here your equation you can write in this form. So, theta x t let me write this theta x t in this form. For phi x I will write it equal to sin omega by C into x plus B cos omega by C into x these multiplied by the time modulation. So, this time modulation equal to C 1 sin omega t plus C 2 cos omega t. So, in this case the boundary conditions will be, so at both the ends you half inertia attached. So, you can write the boundary condition like this.

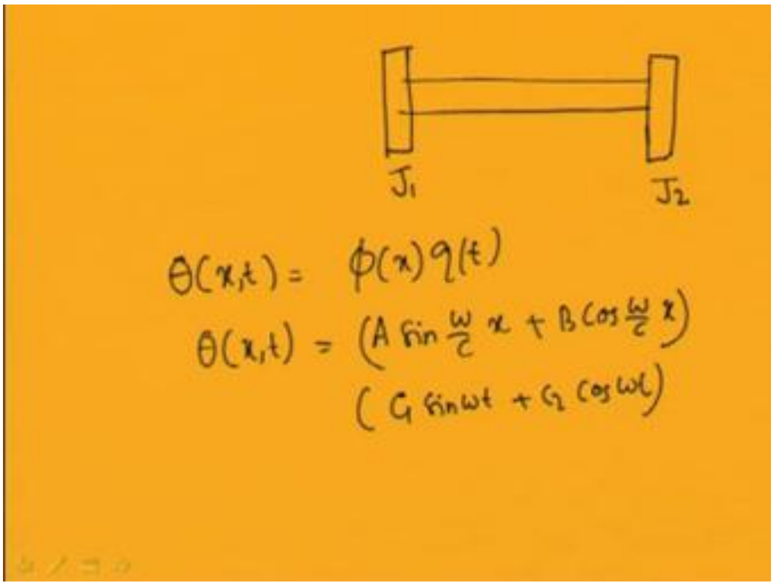
(Refer Slide Time: 28:33)



So, at x equal to 0, so you have J 1 del square theta by del t square this is the inertia torque. So, this will be equal to. So, for the left end it will be equal to G I 1. So, it will be equal to G. So, I can write this is equal to Ip. So, this is the polar moment of inertia of the shaft into del theta by del x. And for the right end at x equal to L I can write this as J 2 del square theta by del t square equal to minus G Ip into del theta by del x. So, already you, so this expression came from this expression already you know this T by J equal to T by J J I can take this as Ip, so T by J as G theta by L. So, from this you know this torque. So, this torque will be equal to G Ip and for small rotation.

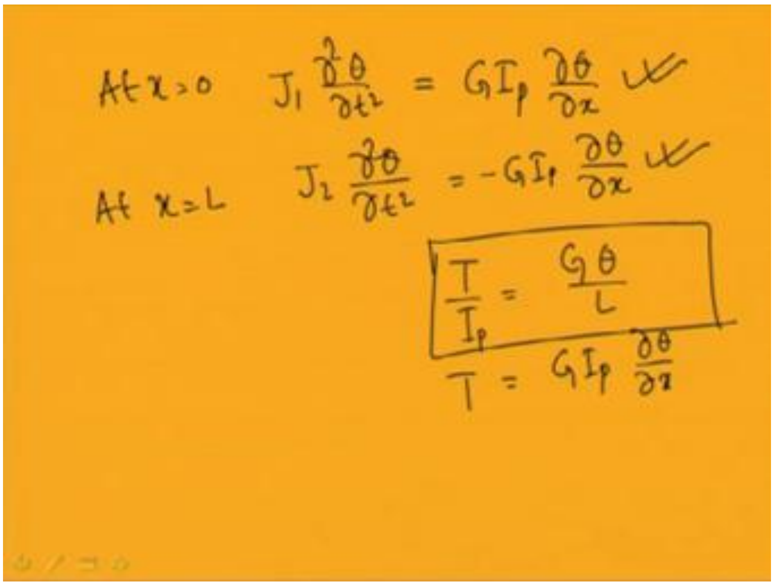
So, this will be equal to. So, this torque will be equal to G Ip into del theta by del x. So, this is general expression you know this is G by G theta by L. So, this is the general expression for torsional vibration of a shaft. So, the torque at this end can be represented by this. So, this will be G Ip del theta by del x. So, for small strain, so del theta by del x. So, this torque will be equal to the inertia torque at these 2 ends. So, at this left end the inertia torque will be equal to G Ip del theta by del x and at the right end it will be equal to minus G Ip into del theta by del x.

(Refer Slide Time: 30:37)



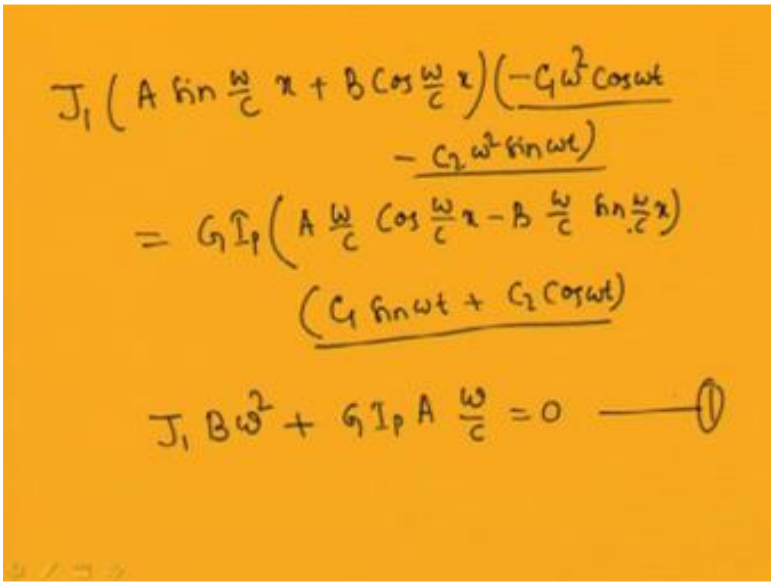
So, in this way, so after getting this boundary conditions, so you can substitute these thing. In this generalized mode shape that is theta x t or theta x t equal to A sin omega by C x plus B cos omega by C x into this time modulation that is C 1 into sin omega T plus C 2 into cos omega T so. By substituting the things, so you can write. So, if you differentiate it.

(Refer Slide Time: 31:00)



Twice and right, so the expression will be comes like this. So, you can write at substituting for the left end you can write.

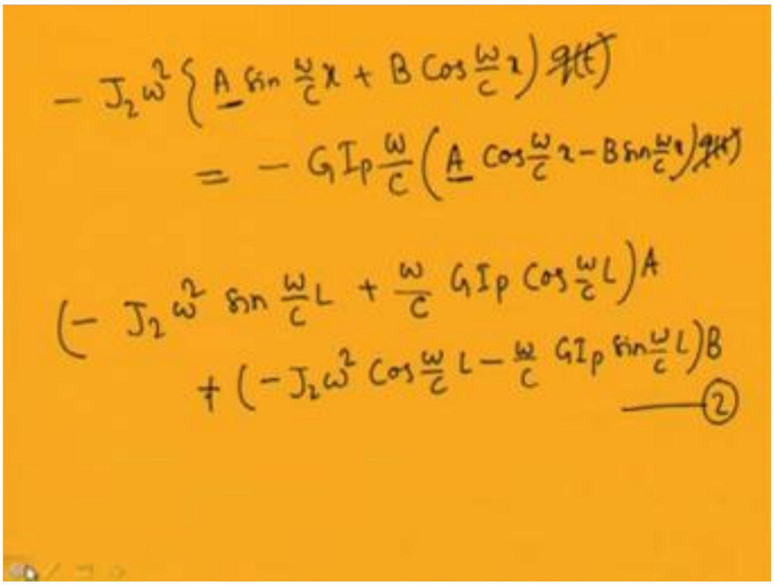
(Refer Slide Time: 31:11)



J into A sin omega by C x plus B cos omega by C by x into, so as you are differentiating a twice with respect to time. So, it will be minus C 1 omega square cos omega t and then it is minus C 2 omega square sin omega t. So, this is the left side that is J 1 into. So, this is J 1 into del square theta by del T square. So, this will be equal to G Ip. So, this is equal to G Ip into del theta by del x. So, if you differentiate it once. So, becomes A omega by C cos omega by C x minus B omega by C into sin omega by C x. So, this into, so del theta by del x you are finding. So, you have differentiated that and this into it become C 1 sin omega t plus C 2 cos omega t. So, just you have differentiated with respect to x.

So, this is the term you are getting del theta by del x and as you are not differentiated with respect to T. So, this is this qt term remains same. So, you can note that this term becomes. So, this is equal to this term is equal to minus omega square into this term. So, you can cancel it out. So, you can write this expression in this way. So, this expression by after canceling you can write, so and by substituting theta x equal to 0. So, when you substitute x equal to 0 this sin term will go and here also the sin term will not be there. So, the expression, so from this expression you can write this expression equal to J 1 B omega square plus G Ip A omega by C equal to 0. So, this is 1 expression you are getting from the left end boundary conditions. Similarly, from the right end boundary condition, so by substituting that thing you can write in this way.

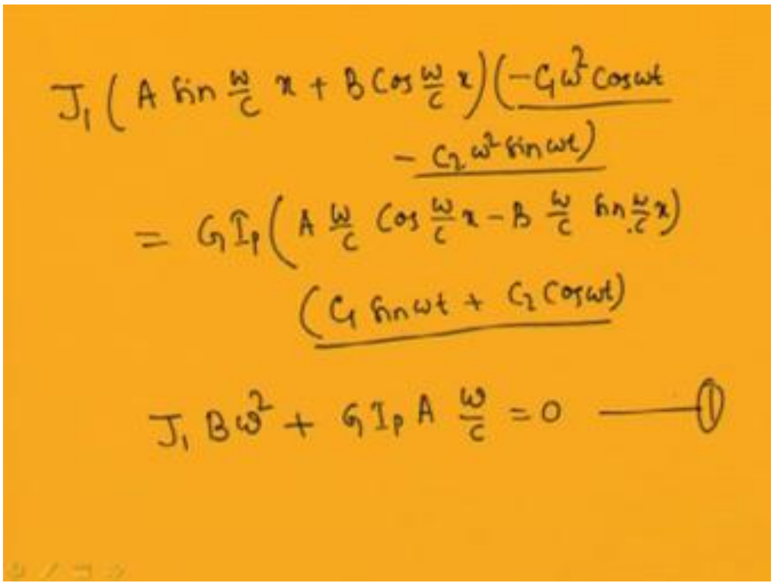
(Refer Slide Time: 33:34)



So, J 2 omega square into A sin omega by C x when you are finding J 2 del square theta by del t square. So, you differentiate that theta twice with respect to time; that means, you have to differentiate time modulation term that is qt twice and keeping this phi x constant, so A sin omega by C x plus B cos omega by C x into. So, by differentiating it twice you have taken this minus omega square common. So, the remaining term. So, it will become qt I you can write. So, the expression for qt you can write here. So, this will be equal to minus. So, you just note that here you have a minus sin. So, this is equal to minus G Ip del theta by del x. So, del theta by del x for that I can write this is equal to. So, when we are differentiating with respect to x. So, this omega by C term you can take it out. So, then it will becomes A cos omega by C x minus B sin omega by C x into similarly that qt term will be there.

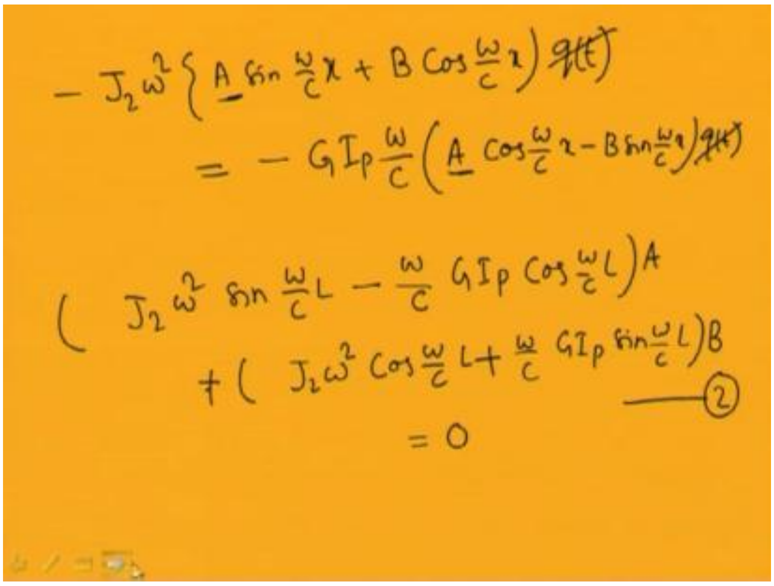
So, this qt qt term you can cancel. So, now, you can substitute x equal L. So, if you substitute x equal to L this equation or this equation you can write as minus you can write this equation as minus J 2 omega square sin omega by C L by omega by C G Ip minus J omega square sin omega by C L plus omega by C G Ip cos omega by C L into A. So, I am collecting the terms with coefficient A and B. So, here you have this term and here this term. So, I am collecting the coefficients of these. So, I will write this minus this equal to 0 after substituting x equal to L. So, from that I can write this expression. So, this equal to or this plus minus J 2 omega square cos omega by C L minus omega by C G Ip sin omega by C L to B. So, you have 2 equations.

(Refer Slide Time: 36:25)



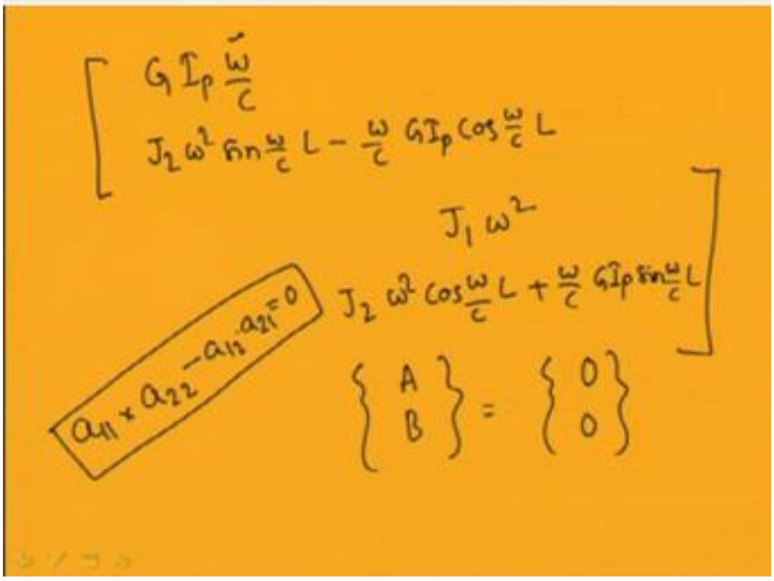
So, in the first equation you can write you have written this thing G Ip omega by C into A plus G 1 omega square B this is equal to 0 and in the second expression.

(Refer Slide Time: 36:36)



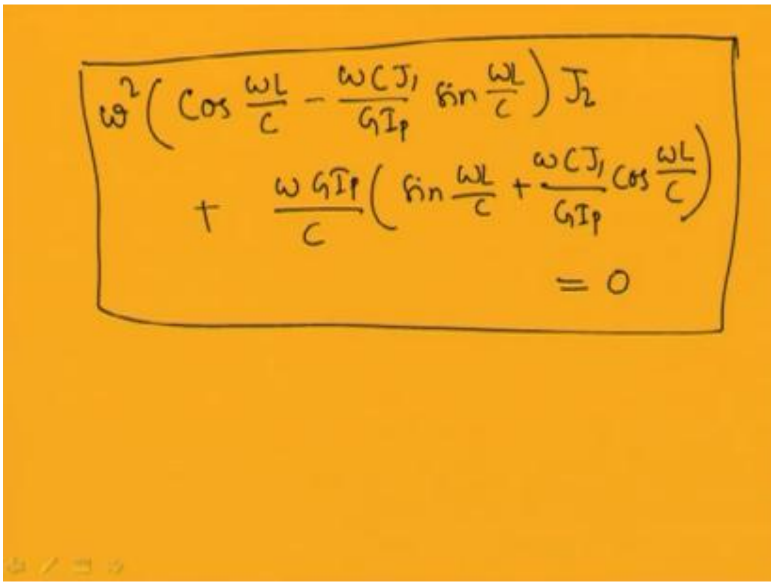
You have this minus J omega square sin omega by C. So, this is equal to 0. So, I can take a negative sign common. So, I can write, so this positive. So, I can write this positive and this minus and this and these terms are positive by taking a negative sign common I can write this expression in this way. So, I have 2 expressions, so with unknowns A, so I can write this matrix in matrix form. So, I can write this expression.

(Refer Slide Time: 37:11)



So, I can write this in this way. So, it will be G Ip omega by C then J 2 omega square sin omega by C L minus omega by C G Ip cos omega by C L. And in the second column it will be J 1 omega square and this is J 2 omega square cos omega by C L plus omega by C G Ip sin omega by C L So, this into A B. So, this is equal to 0 0. So, from this expression you can find or you can see that these are nontrivial solution. So, for non trivial solution that is when both A and B are not equal to 0 the determinant of this matrix will be equal to 0. So, you can find the determinant of this matrix. So, the determinant will be these multiplied that is A 1 1. So, you can multiply this first column with this A 11 into A 2 2 minus A 1 2 into A 2 1. So, you can make it equal to 0. So, this way you can find the determinant of the system. So, by finding the determinant of the system you can write the final expression for this case in this way.

(Refer Slide Time: 38:55)

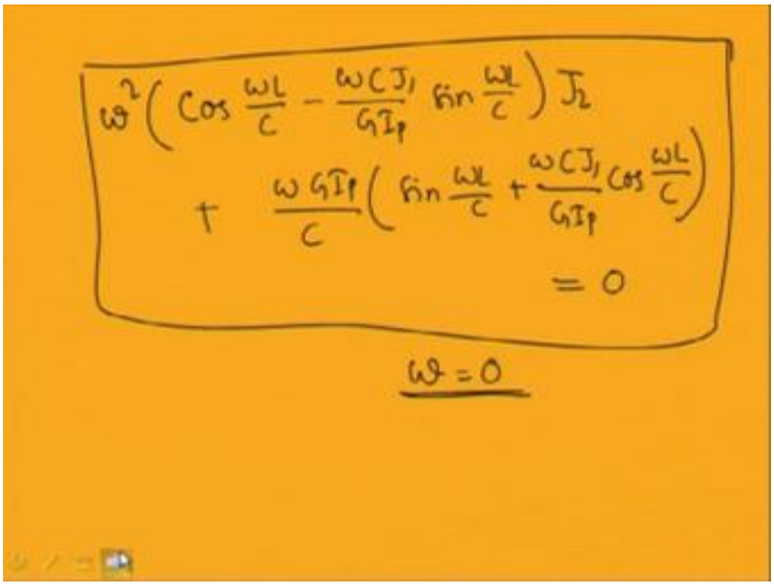


So, the final expression will be omega square into omega square into cos omega L by C minus omega C J 1 G Ip sin omega L by C into J 2 plus omega G Ip by C into sin omega L by C plus omega C J 1 by G Ip cos omega L by C. So, this is equal to 0. So, this is the frequency equation you are getting from getting for the system. So, you can note from this equation that for 1 omega value equal to 0. So, omega equal to 0 correspond to the rigid body motion of the system. So, if you have a rod. So, in this rod if in the both the ends you have 2 masses or 2 inertia are attached.

(Refer Slide Time: 39:59)

Let one side you have this fly wheel other side you have some other inertia mass you have attached. So, in that case in that case it may have a free motion like this. So, the body can move open down. So, this is the rigid body motion of the system. So, in this case of the rigid body motion, so omega equal to 0 as there is no vibration of the system. But there is only pure pure or pure displacement that is the displacement equal to 0 omega equal to 0. The whole system will move simultaneously. So, there will be no relative motion between any particles of this body. So, this is the rigid body motion in this case

(Refer Slide Time: 40:48)

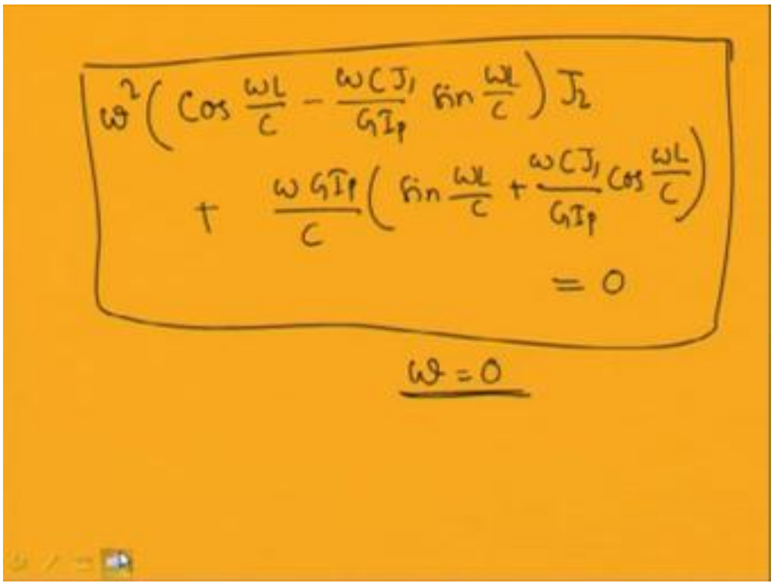


So, when omega equal to 0. So, the system will have a rigid body motion and or the system. So, in this case the whole system will rotate without.

(Refer Slide Time: 40:57)

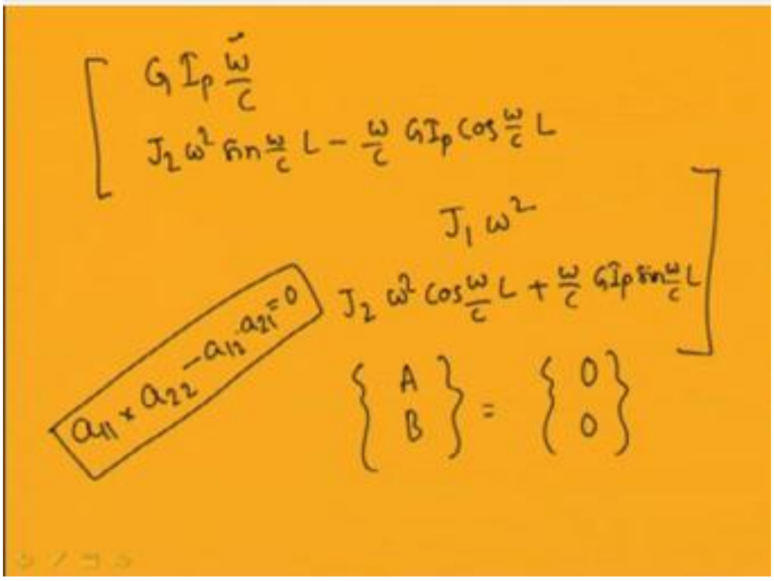
So, the whole system will freely rotate. So, in this case of torsional vibration the whole system will freely rotate and you will have this omega equal to 0. So, when, so the other frequency you can find from this.

(Refer Slide Time: 41:17)



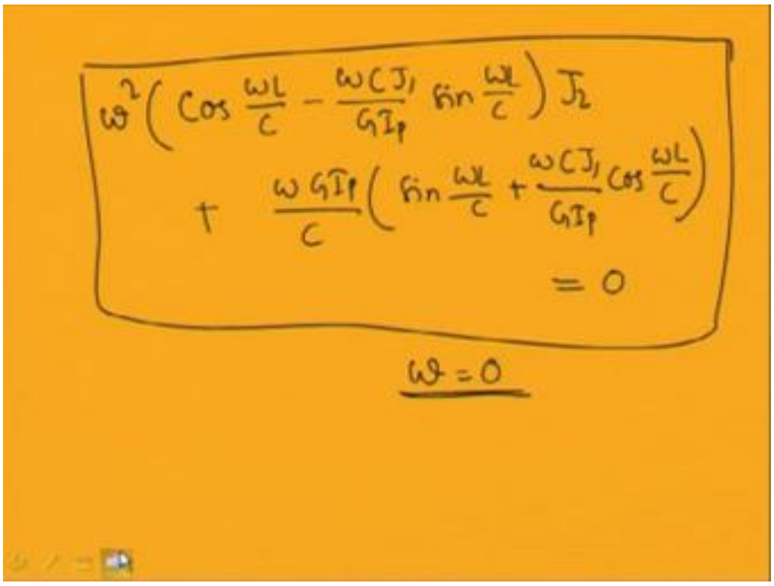
By solving this equation after substituting this numerical value of different parameters, so in this way you can derive the frequency equation of the system. So, after deriving this frequency equation of the system, now you can write from this matrix.

(Refer Slide Time: 41:36)



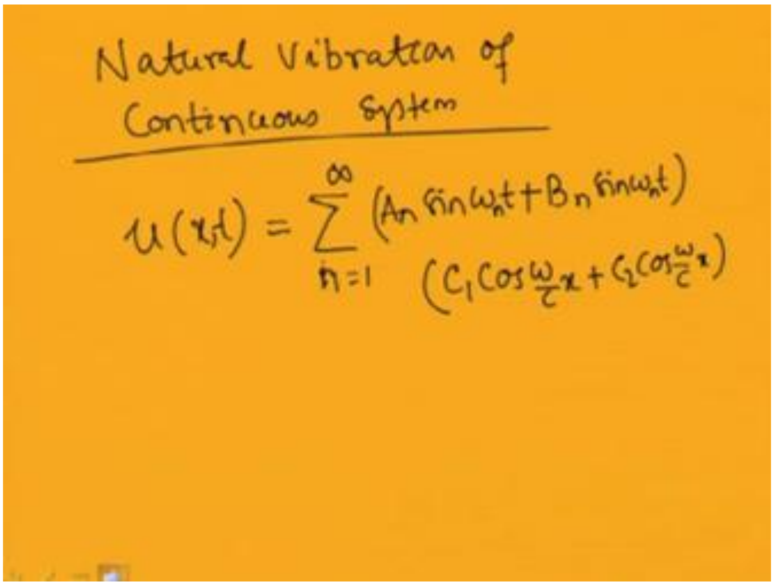
A in terms of B. So, after getting this A interns of B you can substitute that expression in expression for phi x. So, the final expression will contain A into some function x.

(Refer Slide Time: 41:52)



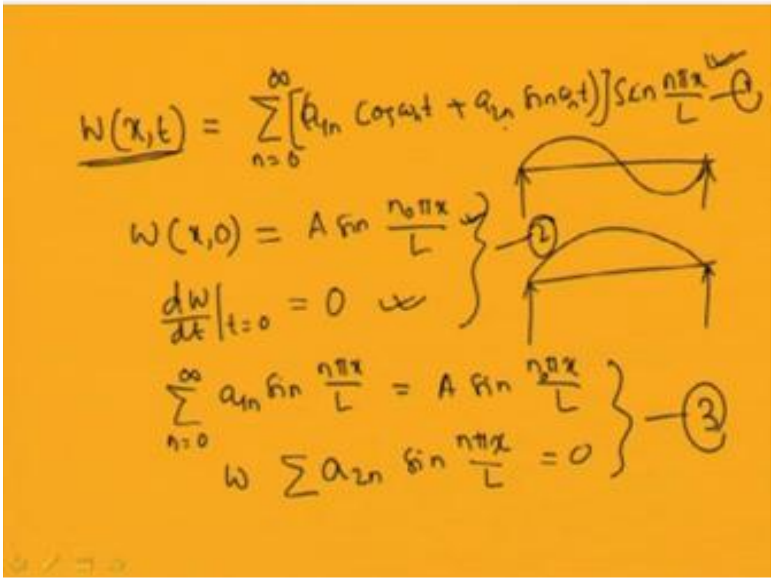
So, by normalizing, so you can find this A value and you can write the expression for different modes in a normalized way. So, in this way you can find the torsional or mode shapes of the torsional vibration of this torsional system. So, in this case I have studied I told 3 differential systems 1 is both end are fixed in other case 1 end is fixed and other end contains a mass or inertia and in the third case both end contain inertia terms. So, in these 3 cases you have seen only in case of the only in the first case that is fixed. So, you have a expression or you have a closed form solution for this system. Or you can manually you can determine this solution of the system. But in other 2 cases you have to do this computational or we have to take the numerical help of numerical analysis to find the solution of these systems. Or find the solution for these frequency equations of the system. So, after finding the frequency equation you can find the mode shapes and normalized mode shapes of the system.

(Refer Slide Time: 43:16)



So, let me briefly tell you about this natural vibration of the system. Natural vibration of the continuous system in case of this continuous system. So, already you know that the vibration can be written as or the free vibration can be written as the summation of all the model vibrations. Or you can write these u x t that is u is the displacement at any time at any position x. So, this thing you know that you can write it in this form. So, you can write it it in this form I equal to or n equal to 1 to infinity. So, An sin omega n t plus Bn sin omega n t into. So, these into you can write. So, this is the time modulation and you can use this mode shape. So, the mode shape you can use in this form. Let the mode shape let me write. So, this is equal to I can write it has C 1 cos omega by C x plus C 2 cos omega by C into x. So, this way you can write the generalized solution of the system. So, let for example, you take the case of a simply supported beam.

(Refer Slide Time: 44:57)



So, in case of the simply supported beam. You can write the displacement transverse displacement of w x t in this way. So, you have written this thing as a 1 n cos omega n t plus a 2 n sin omega n t, so this into mode shape. So, in this case of simply supported beam the mode shape has reduced to the form of sign n pi x by L. So, this thing you can write sin n pi x by L. So, this is the transverse vibration for a simply supported beam. So, in this case, so the total free vibration is expressed as the summation of all modes vibrations. So, if a particular mode is dominant you can tell that the resulting vibration will be that of the, that particular mode. Otherwise it will be the summation of the all modal frequencies.

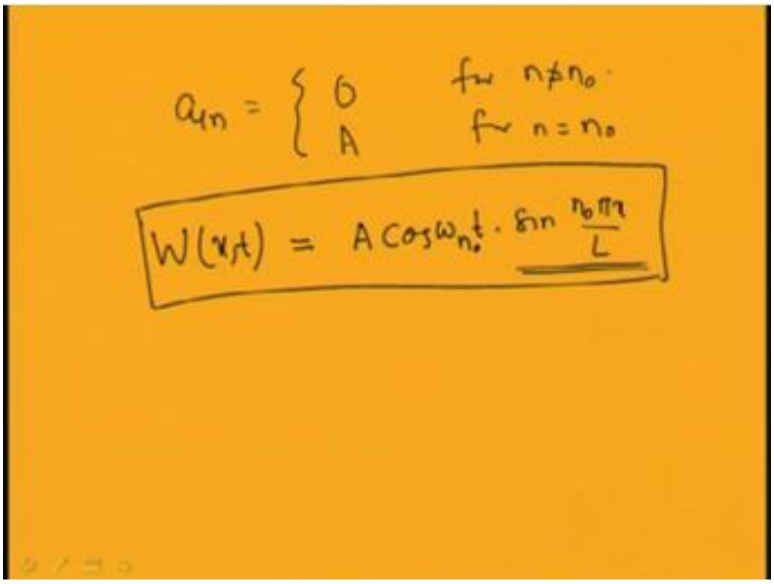
So, let us consider let us consider the initial condition in this way. So, I can consider the initial condition w x 0 let at a particular time this w x 0 equal to A sin n n 0 pi x by L. So; that means, at a particular time let the beam is vibrating with a particular node or the initially the beam is vibrating with a particular mode. That is the, at t equal to 0 let us consider that the beam is vibrating with the second mode. So, this is the mode shape of the second mode already you know the mode shapes of the first mode can be given in this way. So, the, so this is the first mode this is the expression for second mode similarly third and fourth mode you know. So, let for a particular mode that is n 0. So, the beam is vibrating at t equal to 0 with this mode. And there is the slope is 0 this; that means, the dw by dt at t equal to 0.

So, that is the velocity let us consider it is it has a velocity 0. And at beginning it vibrates with a particular mode or it is vibrating with n zero modes. So, now substituting these 2 initial condition in this expression we can find this A 1 n and A 2 n and from that we can find or we can obtain this general solution of the system. So, this is the initial value problems. Say this is the natural vibration of the system can be considered from this initial value problem. So, in this particular case, so now, substituting this w x 0 equal to A sin n 0 pi x by L. So, I can write this as summation. So, this summation, so this is n equal to I you can put it n equal to 0 to infinity. So, n equal to 0 to infinity I can take. So, this will be equal to a 1 n sin n pi x by L, so at x equal to 0.

So, let me first write this expression. So, a 1 n sin n pi x by L will be equal to A sin n pi x by L. So, I can write this other expression as omega into alpha 2 n sin n pi x by L equal to 0. So, from the second expression from the second by differentiating these and substituting this x equal to. So, for any value of x we are finding. So, we are substituting at t equal to 0, so the initial condition t equal to 0 if you substitute in this case. So, w x zero equal to A sin n pi x by L, so substituting t equal to 0. So, this expression sin A 2 n becomes 0. So, this becomes A 1 and only. So, this is the summation A 1 n. So, here you have sin n pi x by L. So, this is the initial condition that is A sin n 0 pi x by L. Similarly, differentiating this expression I can write dw by dt at t equal to 0. So, then differentiating it once, so you have this cos will become sin.

And, so this A 1 n term will go A 1 n into 2 0. So, this becomes A 2 n. So, this as it is cos. So, it will become 1. So, A 2 n. So, this is A to n. So, let me write A to n sin n pi x by L. So, this is sin n pi x by L equal to 0. So, this is your expression let this is equation 1 this is 2 and now, this is 3 equation. So, from this expression you can see that this equation will be valid let us consider this fast expression. So, this equations will be valid only if and only if. So, this n will be equal to n 0, because sin n pi L in the left hand side you have and in a right hand side you have this is sin n 0 pi x by L. So, this equation will be valid only if this n equal to n 0 and this alpha or A 1 n equal to…

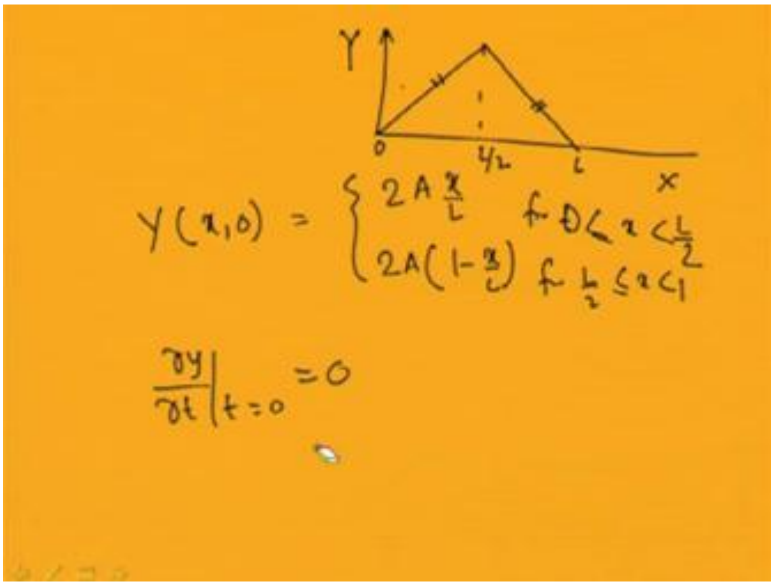
(Refer Slide Time: 50:49)



So, this n equal to n 0 and this a 1 n will be equal to 0 for n not equal to n 0 that is those terms will not be there and equal to A for n equal to n 0. So, for n equal to n 0 this A 1 n should be equal to A and for other time this term should vanish otherwise this left side will not be equal to right side. So, in this case I can write this expression also this A to n from this to A 2 n will be equal to 0 A 2 n equal to 0 and A 1 n equal to.

So, you obtain A 1 n equal to 0 for n not equal to n 0 and equal to A for n equal to n 0. So, by substituting this thing, so you can write this w. So, this is w is the displacement. So, this w x t w is the transverse displacement w xt equal to A cos omega n 0 t into sin n 0 pi x by L. So, you just see that the resulting vibration in this case. So, if you have started with a particular mode that is you have started with n 0 mode of the system then the resulting vibration will be in this form. That is A cos omega n 0 t into sin n 0 pi x by L. So, when you are starting with a particular mode or the initial condition is such that it is vibrating with a near a particular mode then that mode will be dominant and other modes will be not, so dominant. So, the resulting vibration will contain that mode and similarly you can find for other cases for example.

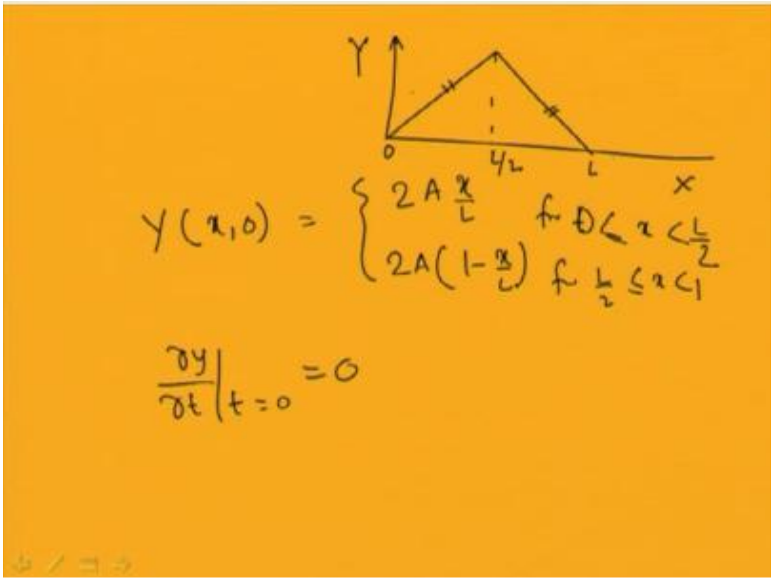
(Refer Slide Time: 52:56)



So, you can take this as an exercise problem you can do, so for the torque string. So, let us take a string and in this case let initially you have. So, this w is the string in tension initially you have take it up and then left. So, when you take it up. So, this side I can take this equation in as. So, this is your X direction this is the Y. So, I can write this Y x 0. So, the initial condition for the string I can write in this form. So, Y x 0 equal to I can write this as let it is 2 A. So, this position let this position is. So, this is 0 this is L. So, this is A 2 A x by L let me put. So, this is for t for 0 less than equal to x less than equal to L by 2. So, this is L by 2. So, for L by 2 this equation is 2 A A 2 A x by L. And for the shaft the equation is.

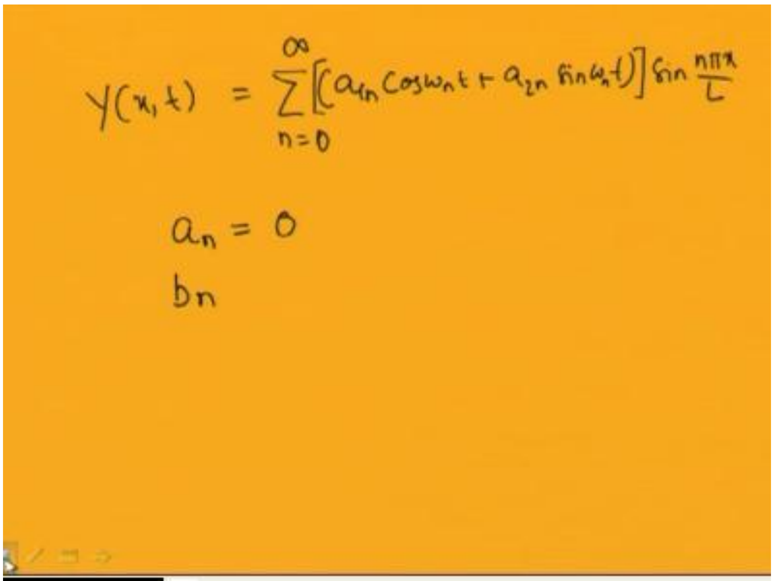
So, you can write this expression for this equation that is equal to 2 A 1 minus x by L. So, this is for L by 2 less than equal to x less than equal to 1. So, in this case you can see. So, with this initial condition you have to find what is the resulting vibration of the string? So, by taking these 2, so this is the expression for this displacement. So, you required another initial condition also let the velocity that is del y by del t at t equal to 0 equal to 0. So, let you have given 1 initial velocity and no initial displacement to the string and left. So, at t equal to 0 velocities equal to 0. So, with this you can find the solution of this system.

(Refer Slide Time: 55:18)



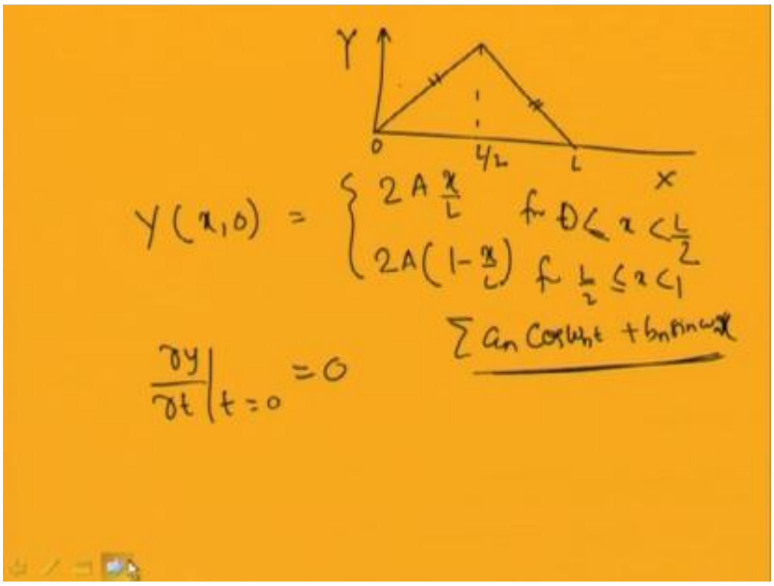
So, to start with you just take the expression for general expression for the string vibration. That is Y x t equal to pi x into. So, summation of n equal to 1 0 to infinity, so you just take in that same way. So, that is equal to An. So, you can write this expression in this form this equal to A 1 n. So, let me write in that way. A 1 n cos omega n t plus a 2 n sin omega n t. So, into, so in this case also if the, if it is fixed at both the ends then you can take this is equal to sin n pi x by L. Sin n pi x by L already you have seen for the fixed string or the string fixed at both ends. So, you can write this expression in this way.

(Refer Slide Time: 56:21)



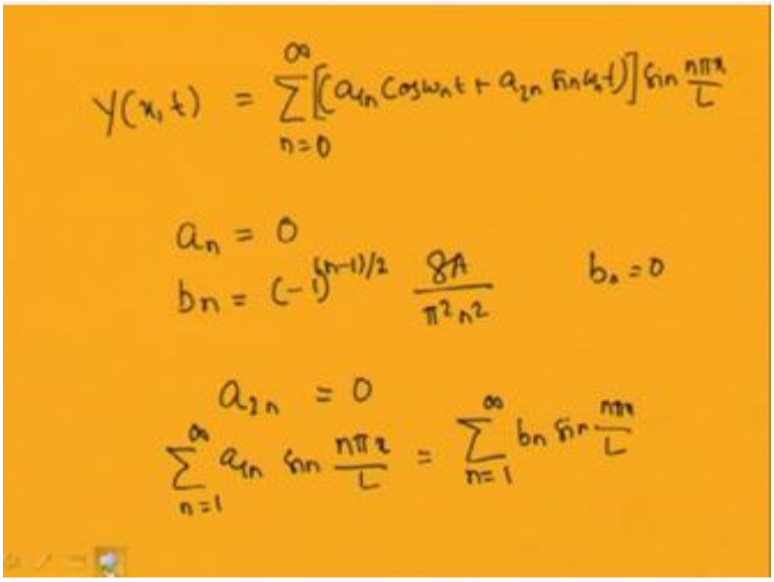
So, now, applying these boundary conditions, so before applying these boundary conditions you can write this in the terms. Applying this Fourier series you can write you can write this expression and to apply this Fourier series you can from the Fourier series. You can find that this will be equal to an will be equal to 0 and the bn terms of that expression.

(Refer Slide Time: 56:41)



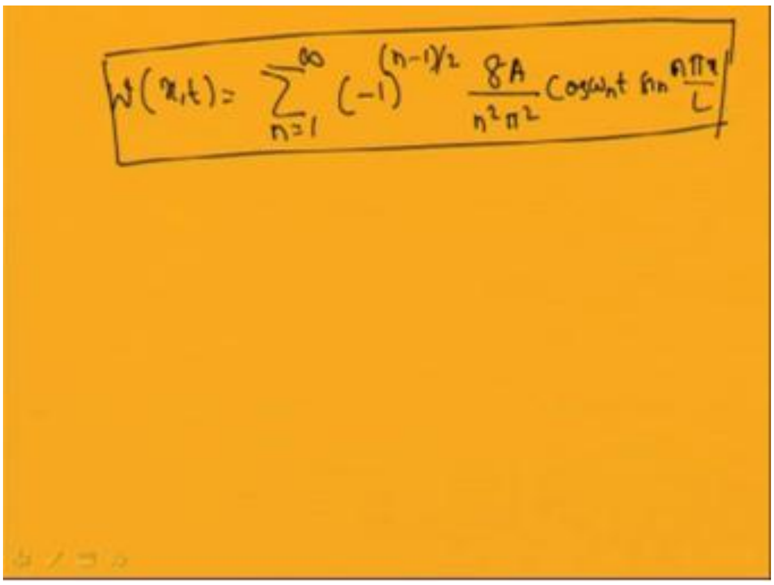
So, this expression, you can write it as summation an cos omega n t plus bn sin omega n t. So, you can express this y x t in term of harmonic function. And so in this case this is omega n x, this is omega n x an sin omega n x plus bn sin omega n x.

(Refer Slide Time: 57:14)



So, after writing these in terms of the harmonic functions, so you can find that for this particular case an equal to 0 and bn equal to minus 1 to the power n minus 1 by 2 into eight A by pi square n square. And this b 0 equal to 0, so now, by taking these initial conditions. So, you can see that this a 2 n a 2 n equal to 0 and from the other condition you will get that is n equal to 1 to infinity a 1 n sin n pi x by L equal to summation n equal to 1 to infinity bn sin n pi x by L. So, these similar to the previous case this equation will be true if and only if this alpha this 1 n equal to bn when this 1 n equal to bn. So, this expression will be true.

(Refer Slide Time: 58:34)



So, in that case you can write this expression, W x t. So, w x t this is the displacement. So, this displacement relation you can write in this form. So, w x t equal to n equal to 1 to infinity minus 1 n minus 1 by 2 into eight A pi square n square pi square cos omega n t sin n pi x by L. So, in this way you can find the free vibration response of this continuous system by applying these initial value or initial conditions. So, in this way you can study the free vibration response of the continuous system.