Mechanical Vibrations
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Module - 9
Continuous Systems Closed Form Solutions
Lecture - 5
Transverse Vibration of Beams: Equations of Motion and Boundary Conditions

In last class, we have studied about the vibration of continuous system in which, we have taken the Euler-Bernoulli beam. And we have found the mode shapes for fixed-fixed beam, simply supported beam and cantilever beams. So, in those cases, we have seen the vibration of the beam can be written in terms of this Euler-Bernoulli equation.

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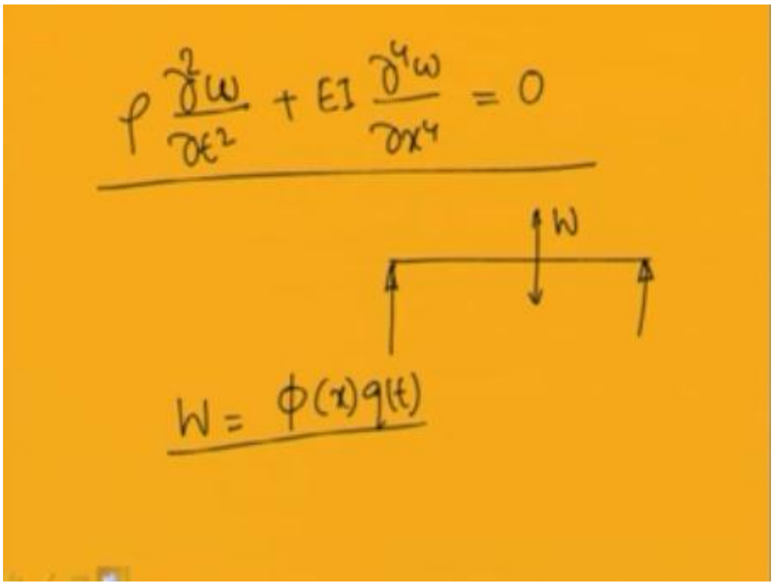

That is del square u del square w by del t square plus, so rho into EI del fourth w by del x fourth equal to 0. So, in this case this is the governing equation, that is the Euler-Bernoulli equation, here rho is the mass for unit length EI is the flexural rigidity of the system w is the deflection, so if this is a cantilever beam. So, in this cantilever beam, sothe vibration will takes place in the transverse direction. So, this transverse direction deflection I have taken it equal to w. So, this w is a function of x and t and, soyou have found this W, we have written it in terms of phi x and qt. So, phi x is the mode shape and qt is the time modulation of the system. So, for simply supported case we have seen the, mode shape in this form.

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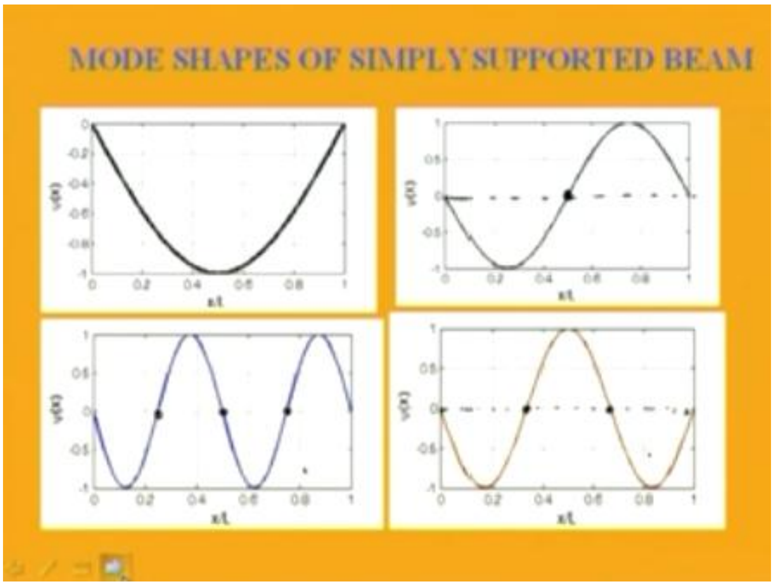

So, I have shown the mode shape, this is the mode shape for the first mode, this is for the second mode, this is for the third mode, and this one is for the fourth mode. So, you can observe that there is a node at l by 2 or at the middle of this in case of a simply supported beam, when it is vibrating with first mode. So, in case of first mode, there is no node and in case of second mode, there is a single node from here. So, this is the beam, soin the first mode, this is the vibration of the system and the second mode this is the vibration of the system. So, this is along the length of the beam and here you can find at this point, there is no vibration. So, this point node, this point is the nodal point, so initially for the second third modal. So, this is the line and you have this and this point will have the 0 vibration. Similarly, in case of fourth mode, these are the points, where there will be 0 vibrations. So, you can see the simulation of this by using, you can write a simple program in matlab to find these things. So, in the program you can write the expression for this phi x.

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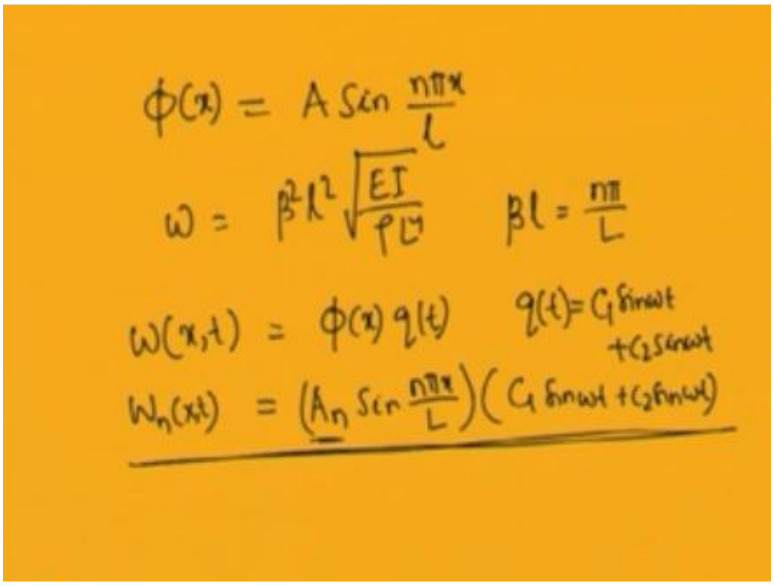

So, already you have derived, this phi x equal to A sin n pi x by l and this omega equal to beta square l square into root over EI by rho L fourth. And in case of simply, supported beam, you have found this beta l value equal to n pi by L. So, using this expression, you can write a simple code in Math lab to simulate the vibrations of the simply, supported beam. So, in this case you can write this vibration W x t W x t will be equal to phi x into qt. So, qt is nothing but this is the C 1 sin omega t plus C 2 sin omega t, so where C 1 and C 2 depends on the initial conditions. So, this W xt you can write equal to A, so for nth mode, you can write this Wn xt equal to An sin n pi x by l into C 1 sin omega t plus C 2 sin omega t, so this A in term is arbitrary. So, you can normalize, this term to get the actual value of A 1. So, let me show the simulation for this, so you can write this expression and you can simulate this value.

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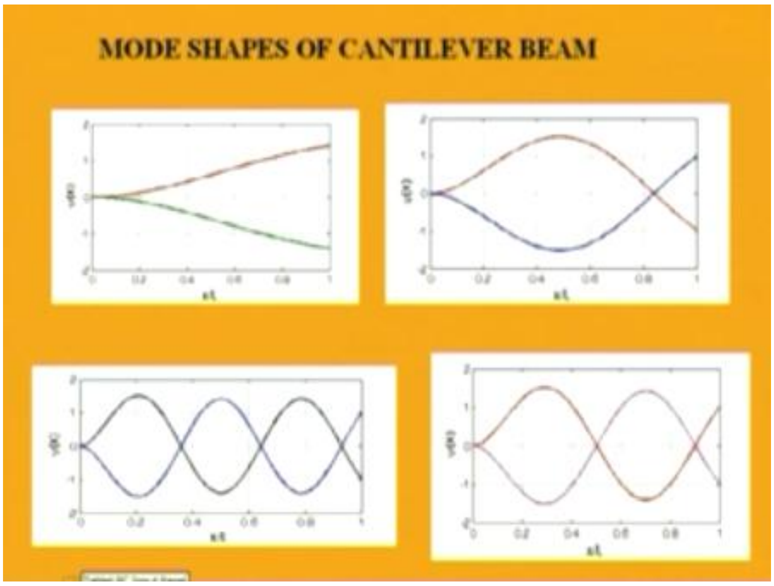

So, let us, do the simulation for this case.

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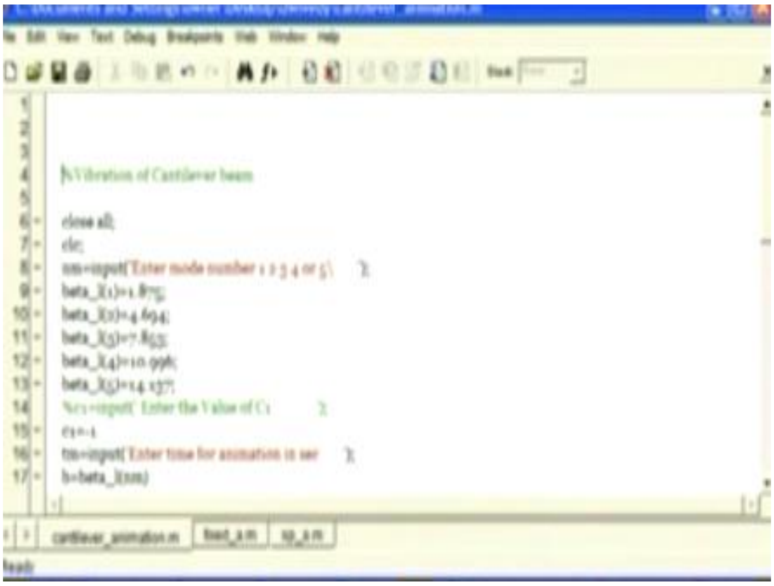

So, in this case of a simply, supported beam, so this is for…

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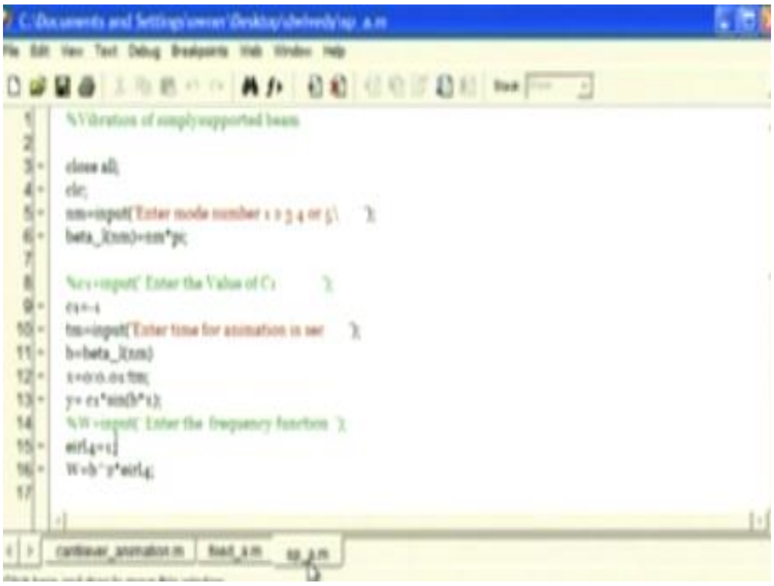

A simply, supported beam the program is written for a simply supported beam.

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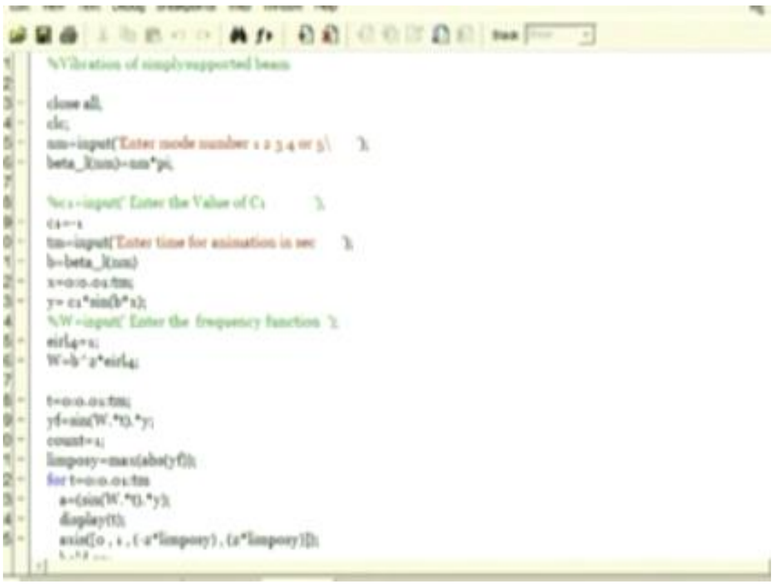

So, in this case, you can see here in this program, I asked for the number of mode. So, whether it is first second or third forth and, so the beta l value for nth mode is known to us. So, for beta l equal to n pi, so this beta l for nth mode will be n pi and we can write taking this EI by rho l forth equal to 1, you can find this W function.

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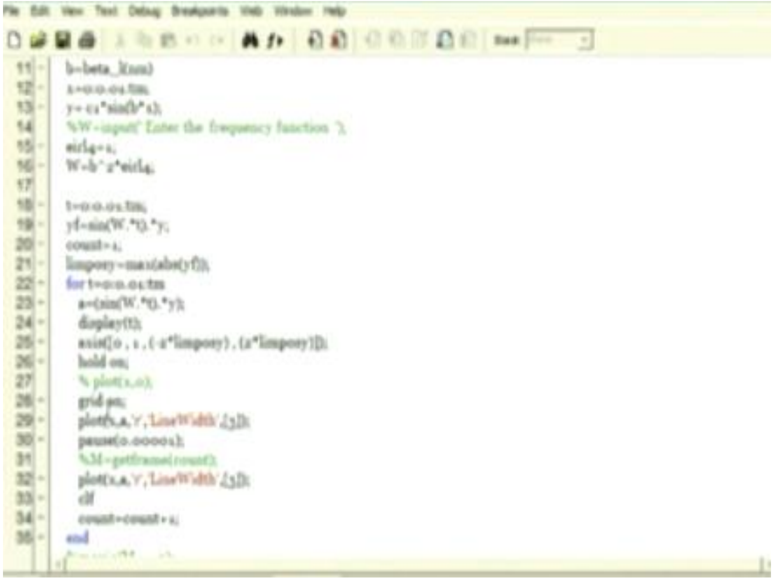

This W is written in terms of yf this is sin omega t and this is the mode shape C 1 sin beta x. So, to simulate this thing, so you just run this and check.

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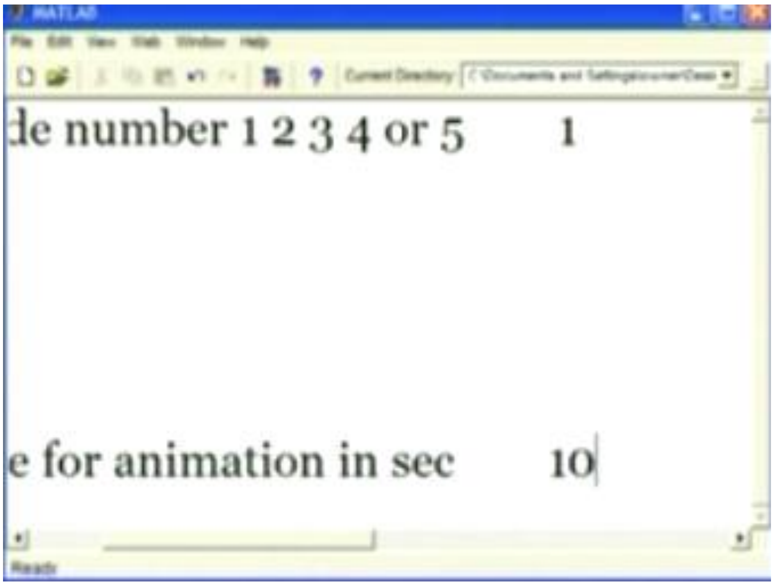

So, for different modes, you can find for different mode, let us find it for the first mode. So, for the first mode, so if you give the time let us simulate it for 10 second.

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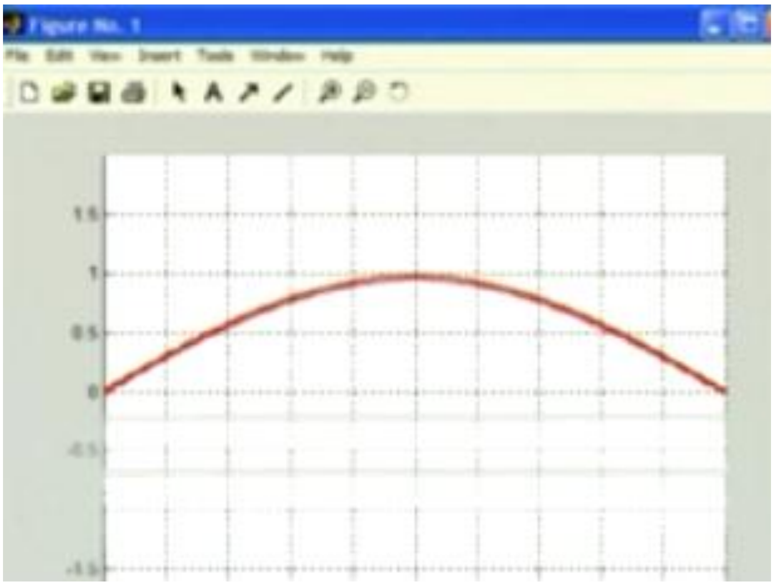

And you can see for the simply, supported case, so the vibration takes place like this. So, at these 2 points the vibrations are 0. So, at these 2 points it is supported, so similarly, we can find for the second mode. So, for the second mode…

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So, for the second mode you can find it.

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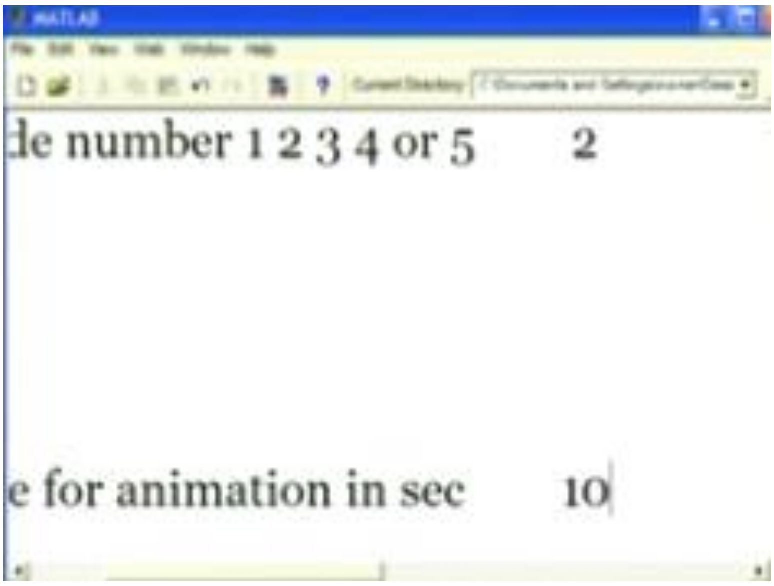

So, let us, take for the second mode and you can observe that. So, for the second mode we can find, so let us run it for this.

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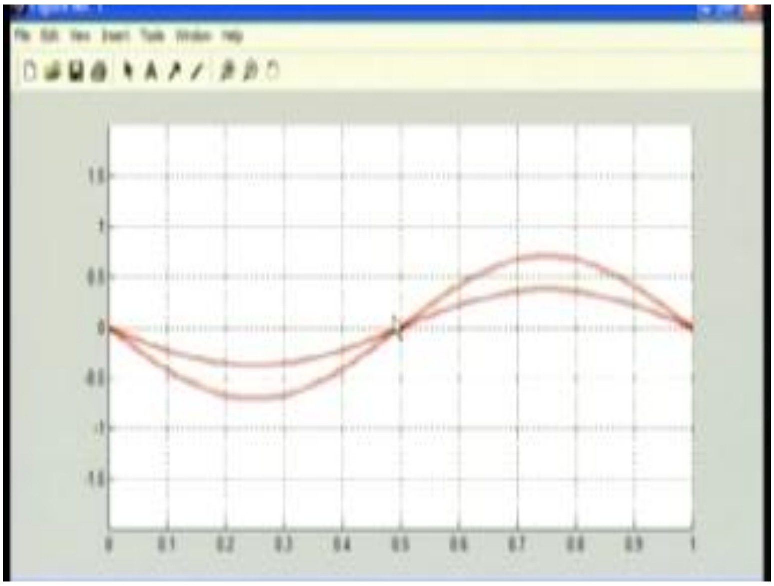

So, in case of second mode, you can observe that at this point at 0.5 there is no vibration of the beam. So, there is a no vibration of the beam at l equal to 0.5 in case of the second mode and similarly, you can find for the third mode. So, for the third mode case.

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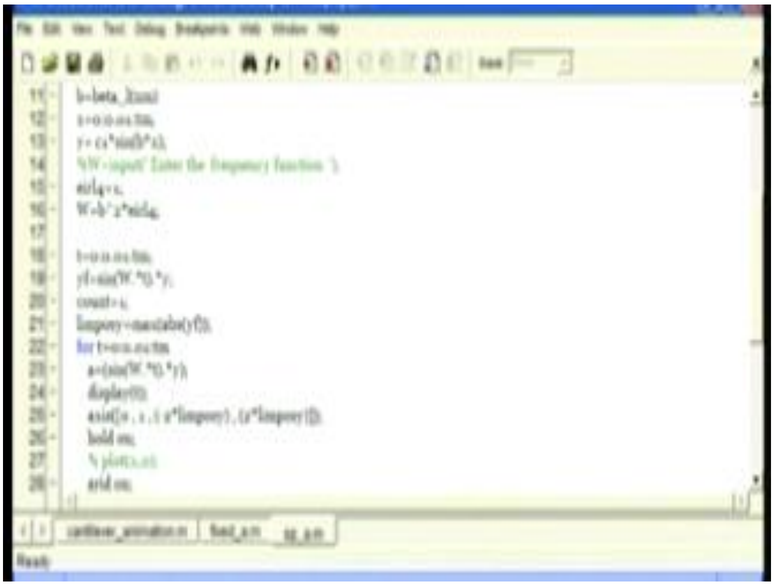

For this case, let us see for the third mode. So, in this case…

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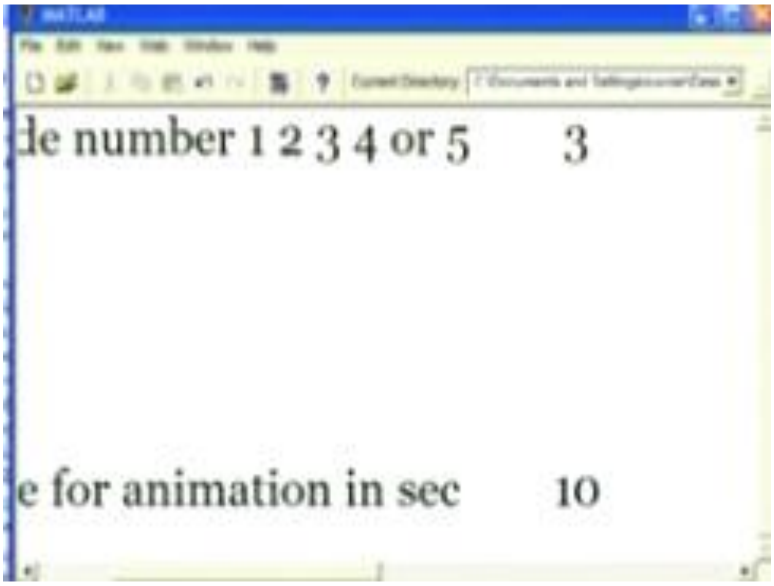

I am writing it for third, so if you simulate it for…

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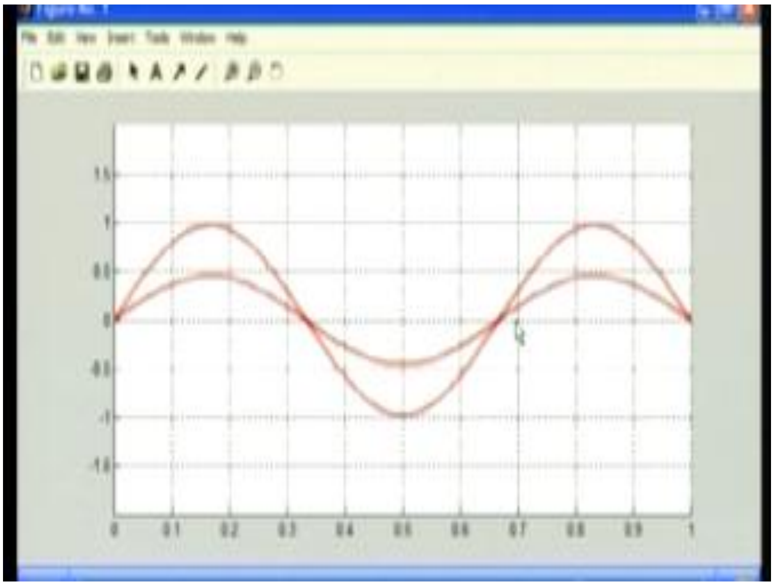

You can see clearly, you can visualize that there are 2 nodes present in this case. So, at this 2 nodes that is no vibration and the beam vibrates in sin n pi x by l. Similarly, you can simulate it, for forth mode and other modes also. So, let us see the simulation for the case of a cantilever beam.

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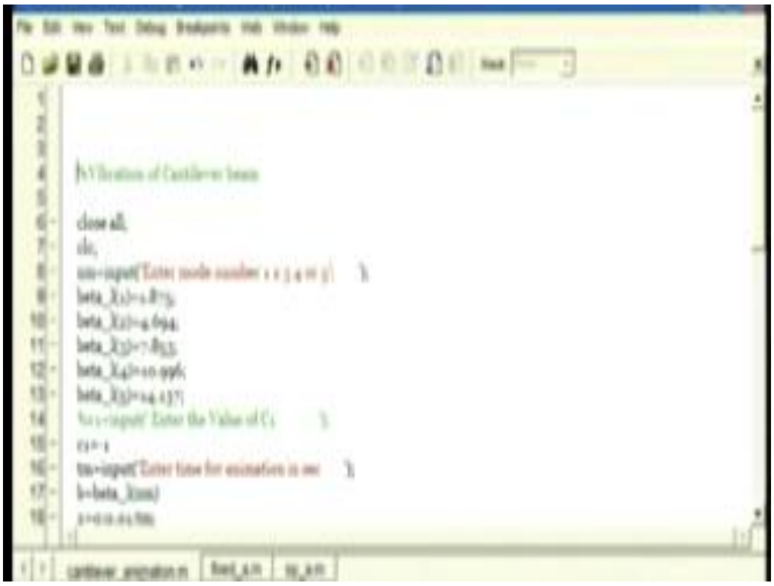

So, in case of a cantilever beam, you can find the vibration.

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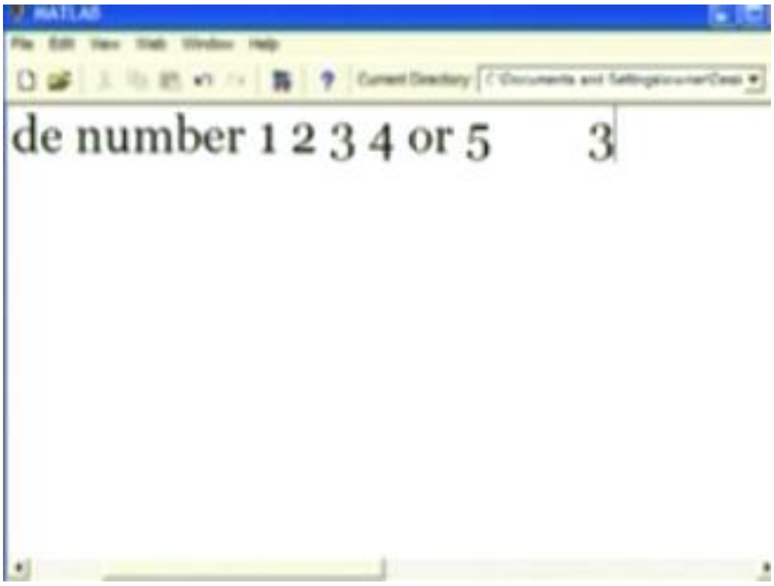

So, for the first mode, let us see say for the first mode. So, in case of the first mode…

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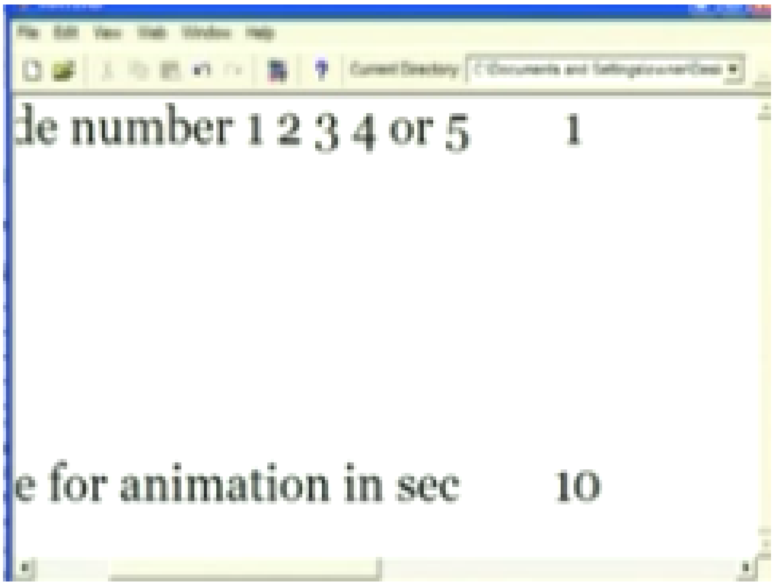

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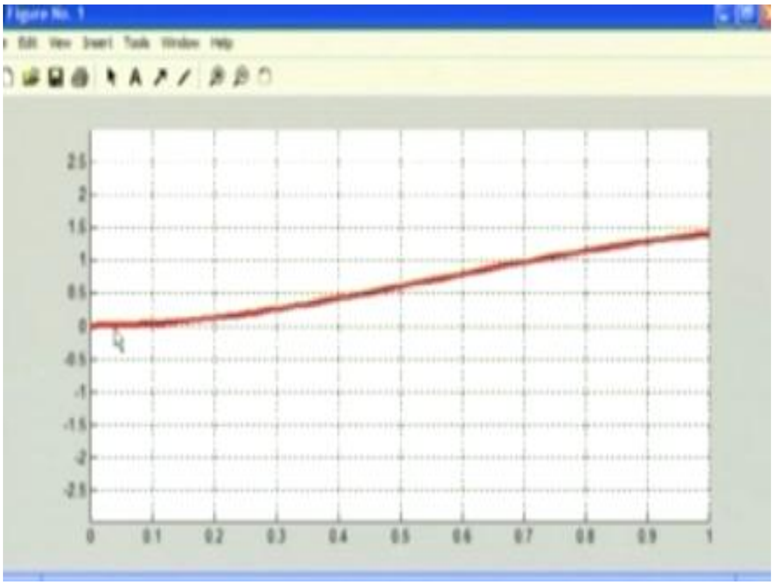

So, you can see that the free end is moving up and down. And there is maximum deflection in the free end and in the fixed end you can observe up to certain extent, there is no vibration of the free end. So, up to certain extent the vibration in case of free fixed end, it is slope will be equal to 0, Similarly, for the second mode.

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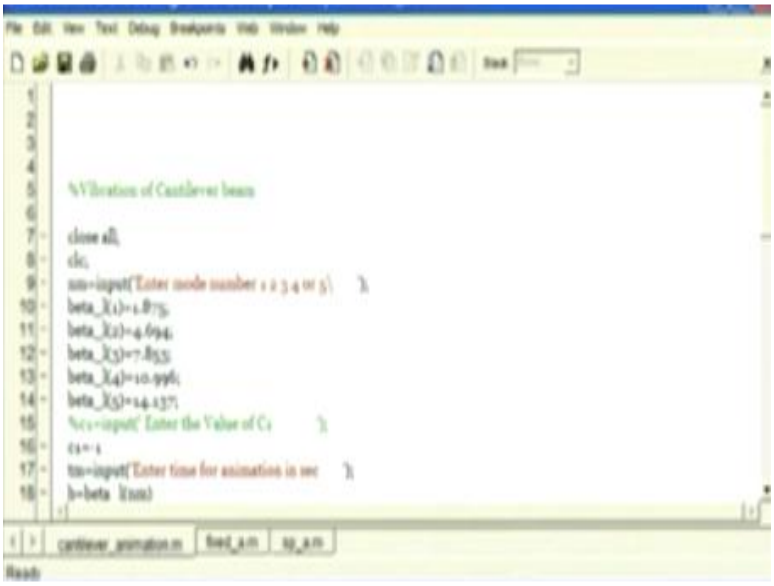

So, if you want to find for the second mode in case of a simply, supported beam. So, let us find.

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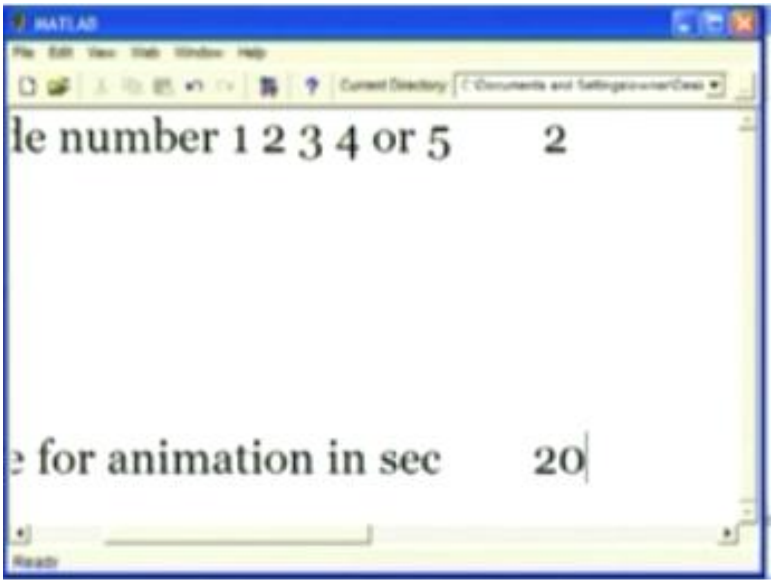

For the second mode, so in this case you can. So, in this case, you can observe that the beam is vibrating. So, that is a node formation here and you can similarly, simulate for the third and fourth mode in this case. So, in case of third mode, you can observe the error 2 nodes present in case of third node third mode of vibration, so in that case.

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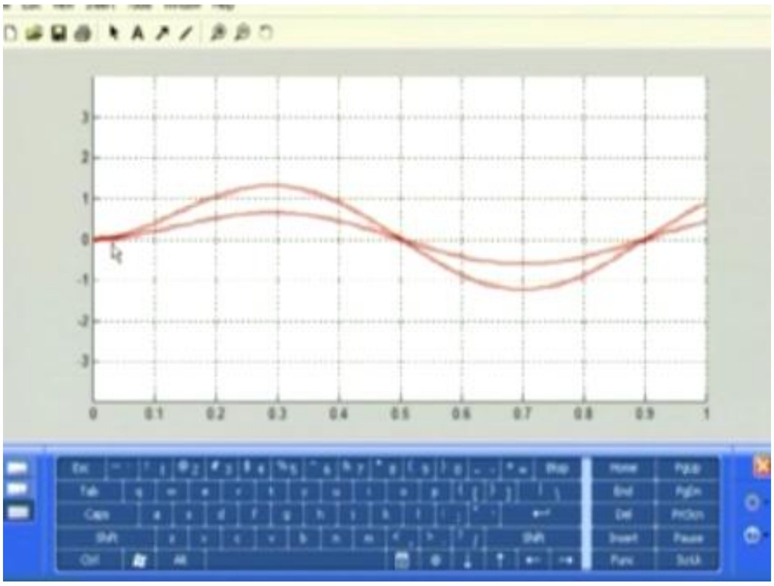

So, you can clearly, observe there are 2 nodes 1 node is here, and other node is here in case of third mode of vibration in case of a cantilever beam and in the fixed end, you can clearly observe the slope is 0 and displacements are also 0. So, to see the forth mode, so in that case also you can find.

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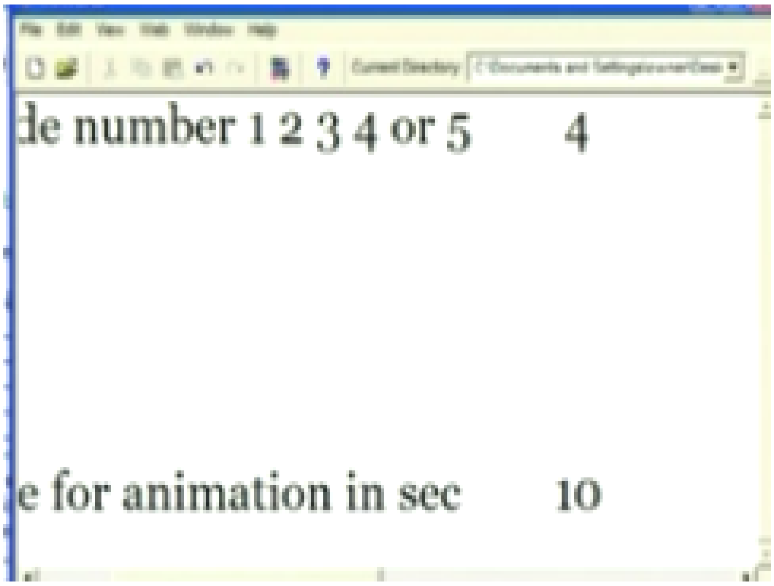

So, in case of forth mode, so clearly you can observe 3 nodes are found in this beam in the cantilever beam. Similarly, for higher nodes n minus 1 number of nodes, you can find and the free end will vibrate with a maximum and the fixed end that is 0 deflection and the slope is 0 also in this case.

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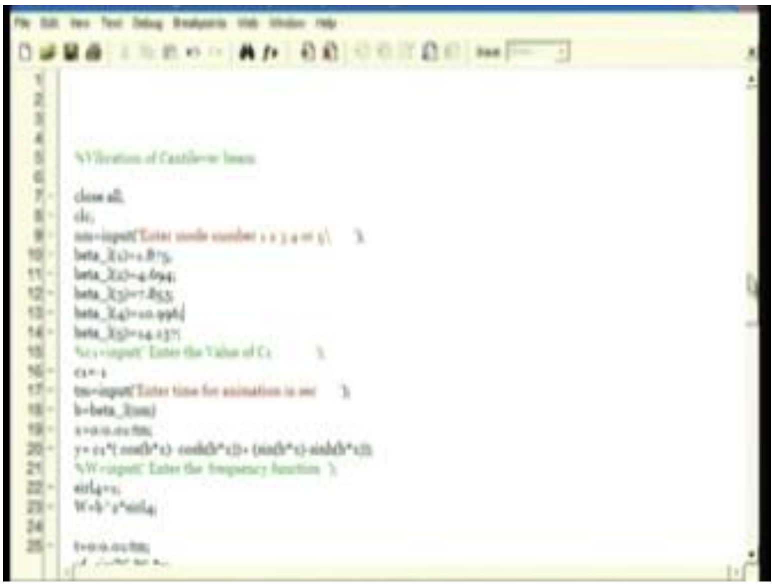

So, in case of a cantilever beam, you can observe that first frequency correspond to beta l that is equal to 1.7 5 the second modal characteristic function. That is beta l 2 that equal to 4.6 9 4. And for the third mode, it is equal to 7.8 5 3 and fourth mode, 10.9 9 6 and fifth mode, it is equal to 14.1 3 7. So, taking those value, so you can simulate for different modes and you can find.

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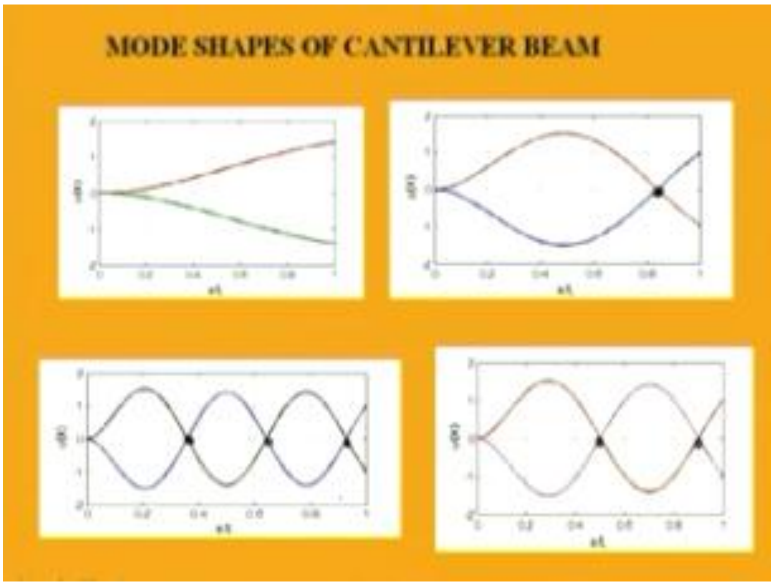

The mode shapes and the response of the system. So, in case of cantilever beam already, you have seen that the vibration. These are the mode shapes this is for the first mode and this is for the second mode and in the second mode a mode is found at this position. In case of third mode, there are 2 nodes and in case of forth mode, there are 3 nodes present in this beam. So, in these cases, you have found that for nth mode there will be n minus nodes, that is at n minus node n minus 1 points on the beam there will be no vibration.

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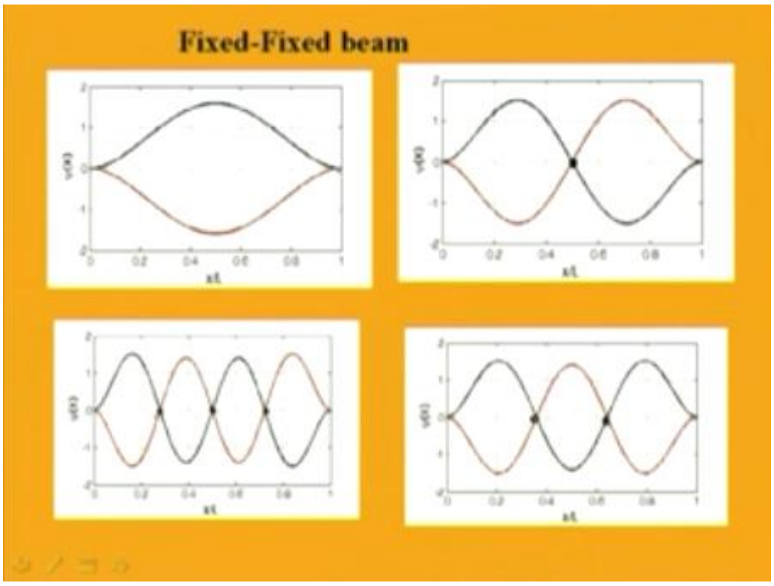

In case of fixed-fixed beam also if you find, so this is at the mode shapes. So, this is this is a Euler-Bernoulli beam. So, in this case, both the ends are fixed and slopes are 0 here and in case of second mode, you can see there is 1 mode in case of third mode that is 2 nodes. And in this case you have 3 nodes present, so the systems represented by wave equations are.

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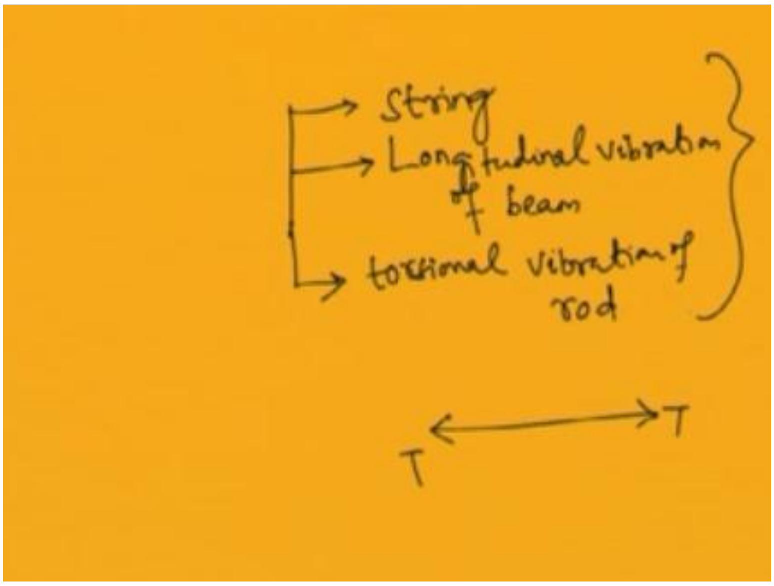

So, we have studied about 3 systems in the first system. So, you have studied, about the string. So, the torque string, so in this case the strings in tension, so the vibration of the string in tension and the longitudinal vibration of beam, longitudinal vibration of beam and the torsional vibration of rod.

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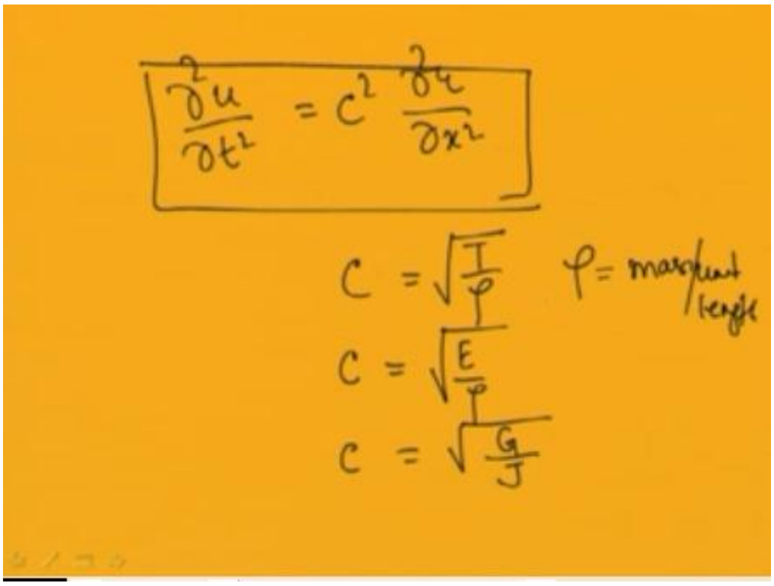

So, these 3 systems, you can write the equation motion for 3 systems by using this wave equation that is Del square u by Del t square equal to C square del square u by del x square, so in these cases, so the C. So, in case of a torque string C equal to or equal to T by rho, where rho is the mass per unit length. So, mass per unit length and in case of this longitudinal vibration of rod C equal to root over E by rho and in case of torsional vibration of rod it is equal to G by J. So, here in case of, so in all these cases, you can check that the C represent the represent a velocity term and already, you have solved this wave equation and you have found.

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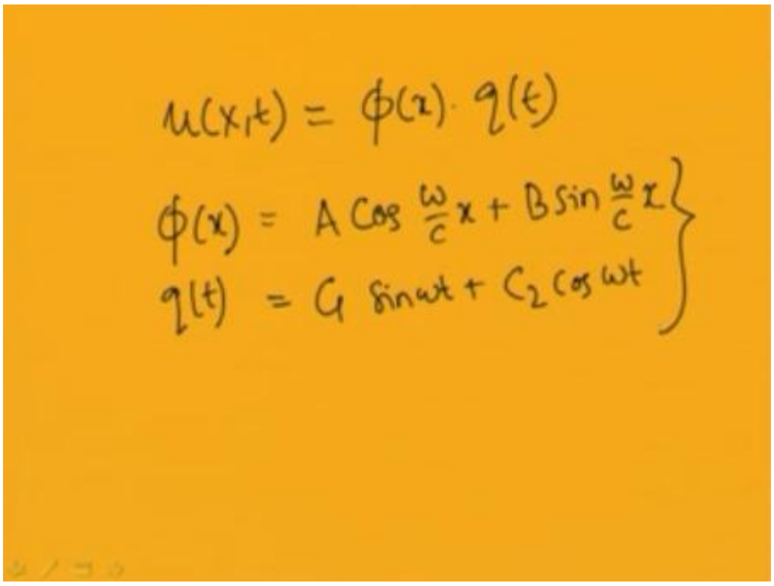

The response u xt can be written in this form u xt equal to phi x into qt, where this phi x you have written as A cos omega by c x plus B sin omega by c x. And qt, you have written in this form qt equal to C 1 sin omega t plus C 2 cos omega t. So, for different boundary conditions, today will find the mode shapes for the systems. So, let us first take the system of a string for this case for the fixed-fixed spring.

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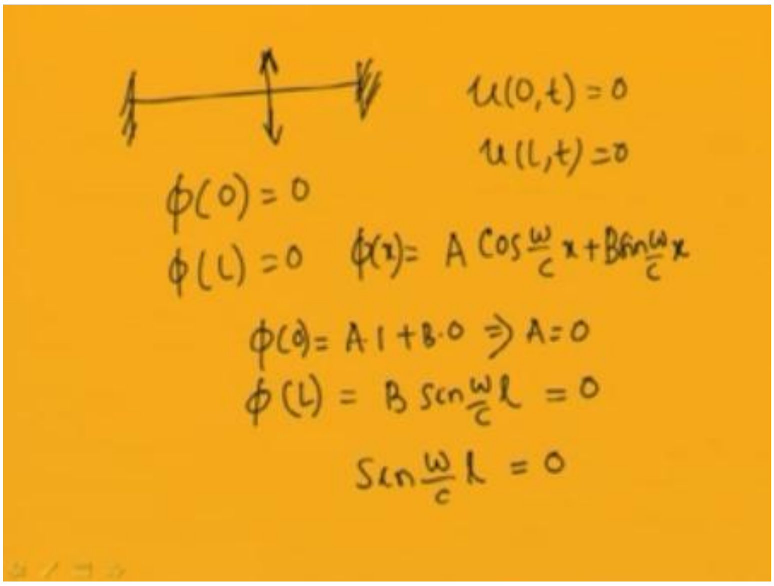

We have already, found the mode shapes and in this case, when it is fixed as both ends are fixed in this string. So, the displacements are 0, so phi so your u 0 t equal to 0 and u lt also equal to 0. So, as u 0 t equal to 0 and the u Lt equal to 0 for all time, so you can write this phi 0 equal to 0 and phi l equal to 0. So, when phi 0 equal to 0 in that case A cos omega by c x plus B sin omega by c x equal to. So, at x equal to 0, this will be equal to 0, so this is expression for phi x. So, I can write this phi 0 equal to, so this is equal to A into 1 plus B into 0 implies this A equal to 0.

Similarly, phi L will be as A equal to 0 phi x becomes, B sin omega by C x now, substituting x equal to l. So, you can write this is equal to B sin omega by C l equal to 0. So, as omega sin omega by C l equal to 0, so this equal to sin, so B will not be equal to 0 as B to when B will be equal to 0 phi x will becomes, 0. So, the case will be a trivial solution of the system. So, as we are interested for the non trivial solution of the system is, when the beam when the string is vibrating as we are interested to see the vibration of the string. So, in that case, we should B's will not be equal to 0. So, as B not equal to 0. So, our frequency equation will be sin omega by C l equal to 0.

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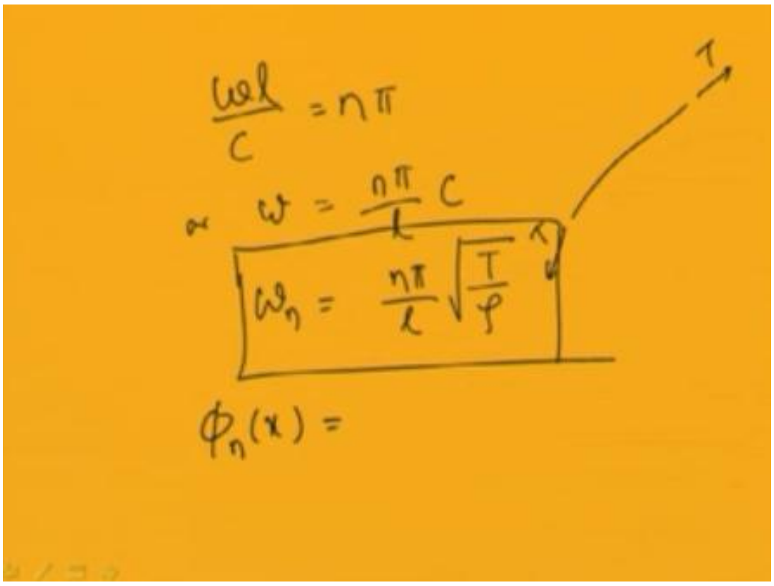

So, as sin omega by C l equal to 0, so this omega l by C will be equal to n pi or omega will be equal to n pi by l into C. So, for this case of string you found this as root over T by m or T by rho. So, rho is the mass per unit length and T is the tension in the string. So, if this is the string and this is the tension tension in the string, so this omega, so as you have seen this is n. So, it depends on the number of modes, so as you will go on increasing this n, so you will have different values of omega. So, this can be written as the nth modal frequency of the system. So, the nth modal frequency of the system can be written as n pi by l into root over T by rho. So, the modal or the mode shape of the nth frequency. So, phi n x can be written as.

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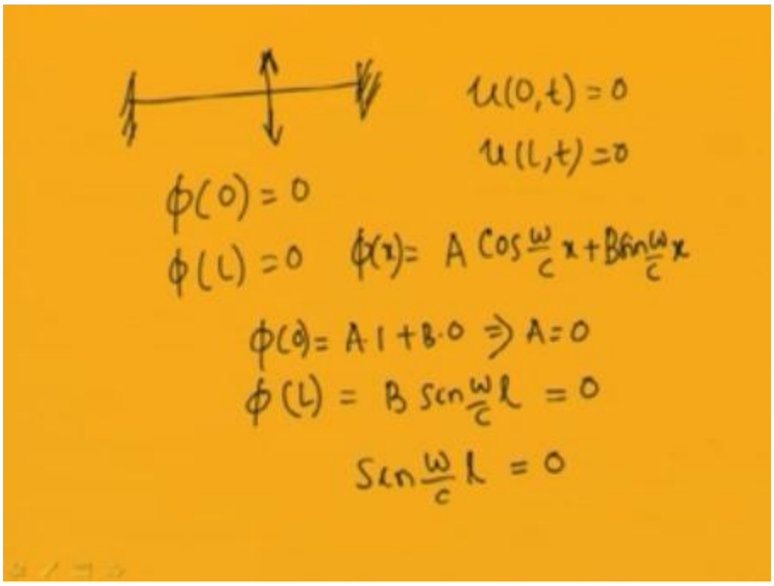

So, phi x equal to A cos omega by C, so already you have seen this A equal to 0, so phi x equal to B sin omega by C x.

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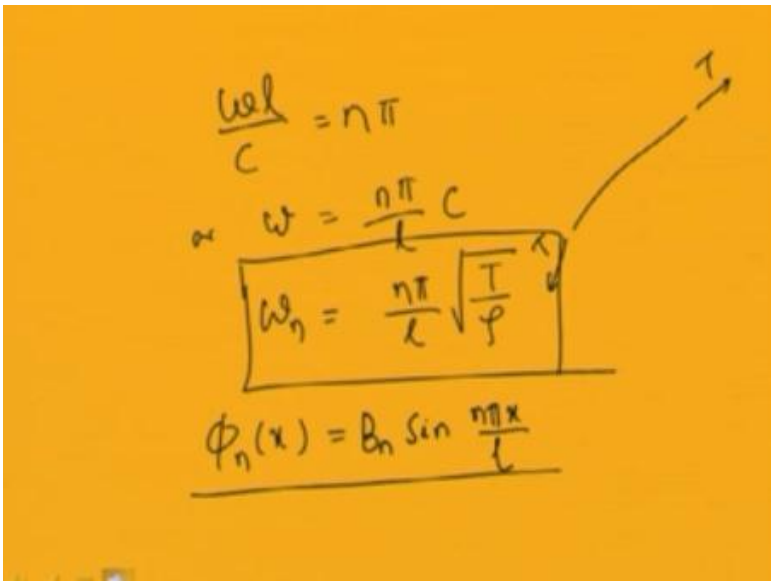

So, for nth mode, I can write this equal to Bn sin n pi x by l, so phi n equal to Bn sin n pi x by l, so for different modes, as this phi n equal to Bn sin n pi x by L.

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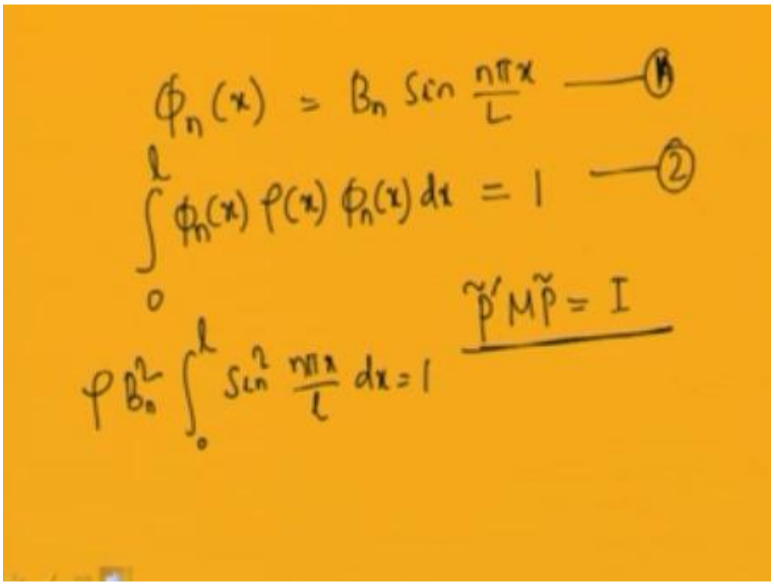

So, for different mode this Bn is different, so for first mode this is B 1, second mode B 2 and nth mode it is Bn. So, this is similar to the mode shapes, you are you obtain in case of multi degrees of freedom systems. So, in that case you have normalized that mode shapes and you have found the normalized mode shape of that system. You can write this nth mode amplitude equal to 0 or you can write this phi n x rho x into phi n x dx, so0 to l, so this should be equal to 1. So, phi, so you can normalize this mode shape by using this formula, similar to the things. You have done in case of the multi degree of freedom system here; you have taken the weighted modal matrix and found this. So, let P is the weighted modal matrix. So, you have found this thing P weighted modal matrix transpose MP.

So, you have seen this equal to or you have found this equal to I, so in this case, you have got the diagonal term is 1. So, similarly, by making this normalization, you are making the mode shapes. So, you can normalize the mode shapes, of the nth mode, so to normalize this thing. So, you can write this phi n x into rho x into phi n x dx equal to 1. So, for the string this mass per unit length, I can take it constant. So, this is equal to 1, so this is equal to rho. So, this mass per unit length equal to rho I can take it out, similarly, substituting this equation this equation in the second equation, this first equation in the second equation. I can write rho into Bn square integral 0 to l sin square n pi x by l dx equal to 0 equal to 1. So, when I am normalizing this thing, so I can write rho Bn square integration 0 to l sin square n pi x by l dx equal to 1. Already, you know the sin square theta.

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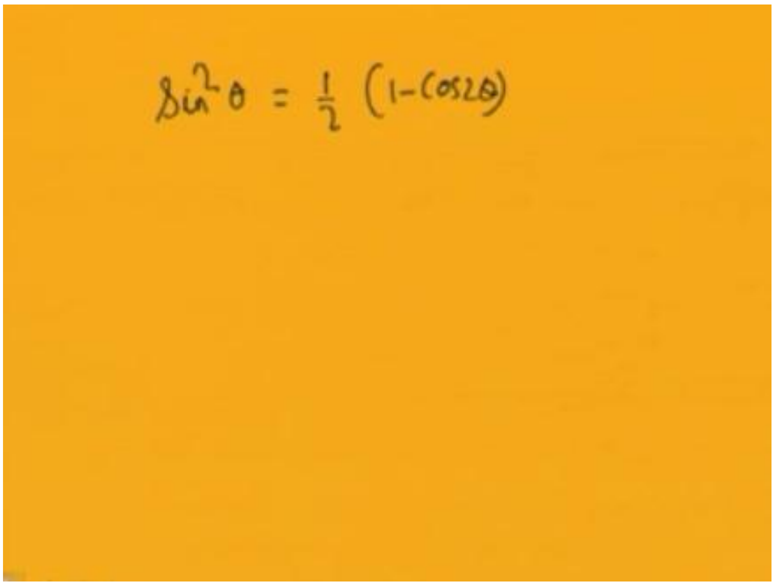

You can write in this form half 1 minus cos 2 theta, so this integration, previous integration.

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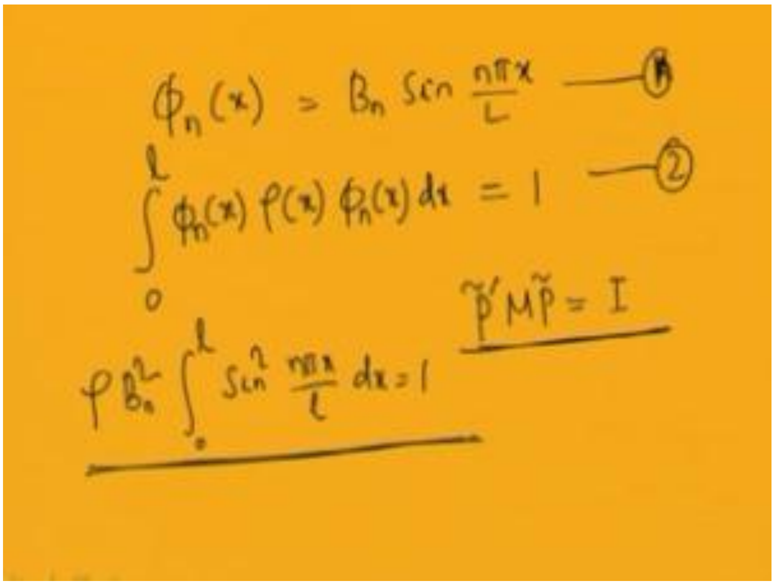

So, this integration you can find, by writing this way.

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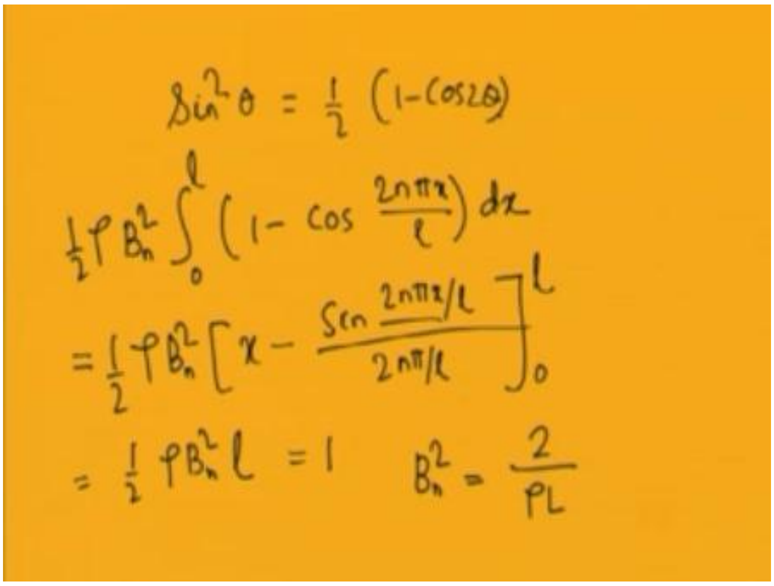

So, rho beta n rho B n square, so integration the sin square n pi x, you can write this is equal to 0 to l 1 minus cos 2 n pi x by l dx. So, this integration will become rho Bn square, so this is x minus this cos integration will be sin. So, this is sin 2 n pi x by l divided by, so you can divide this by 2 n pi by l. So, this is from 0 to l. So, this integration, you can find that this is equal to. So, you can substitute x equal to l, so there is a half term here, as we have sin square theta equal to half 1 minus cos 2 theta. So, I can have this half term, so this is half rho Bn square. So, the sin 2 pi nx for value of sin 2 pi n x by l for x equal to l, this is equal to sin 2 n pi. So, this sin 2 n pi equal to 0 similarly, when x equal to 0 this is sin 0. So, sin 0 equal to 0, so you will have this term only, equal to half rho Bn square l. So, this is equal to, so you can substitute this,, and you can write this equal to 1. So, you can write this Bn square, so Bn square will becomes, equal to 2. So, this is equal to 2 by rho L or Bn you can write Bn.

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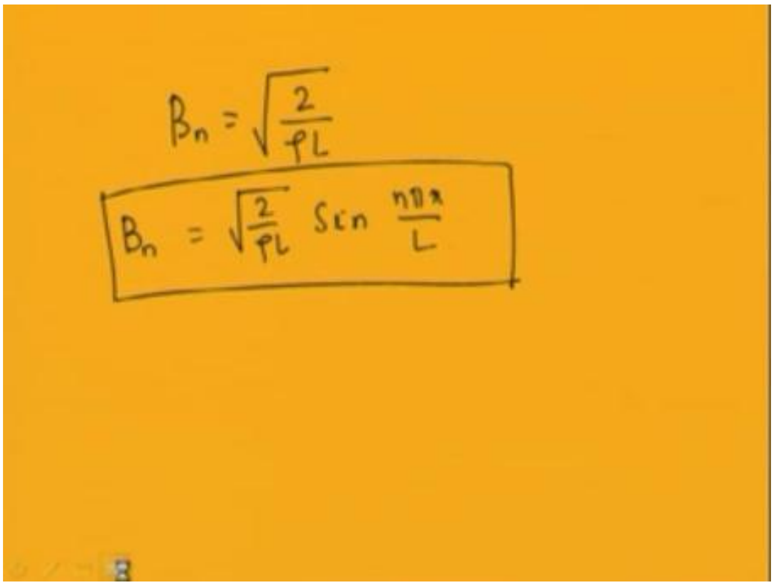

You can write equal to root over 2 by rho L. So, the normalized mode shape, you can write as Bn equal to root over 2 by rho L sin n pi x by L . So, this is a normalized mode shape or Eigen function of the string. So, now, you can show that this Eigen functions or this mode shapes are orthogonal. To show mode shapes are orthogonal.

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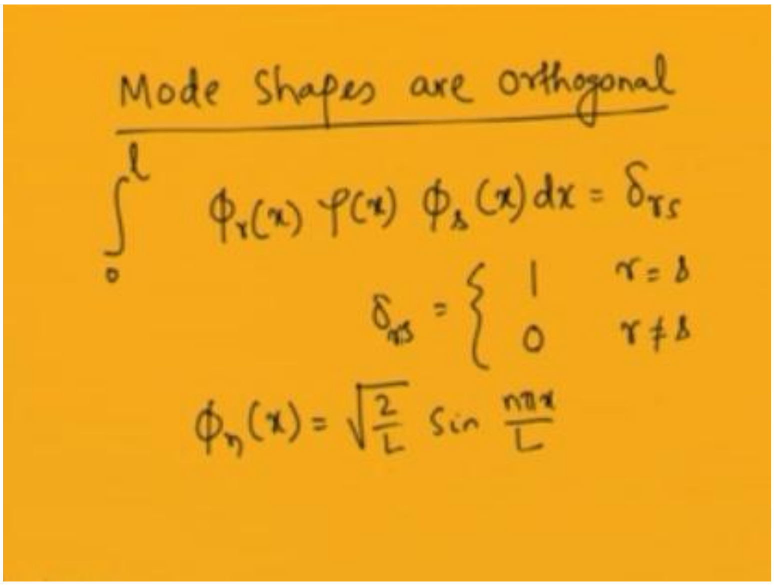

So, we can, so this mode shapes are orthogonal with respect to this mass matrix in case of multi degree of freedom system, you have seen. So, in this case also, you can show that the mode shapes are orthogonal by writing these things. So, 0 to l rho phi r, so let us take the rth mode and this phi r into rho x into phi s mode, let us take r and s mode. So, we should show that, if the mode shapes are orthogonal. So, this integration should be equal to delta rs, where delta rs is the Kronecker delta term. So, delta rs equal to, so this will be equal to 1 when r equal to s and it will be equal to 0 when r not equal to s. So, if this satisfy this equation, then we can show that or we can tell that systems are this mode shapes are orthogonal, so to prove these things. So, already, we have found the mode shape for the nth mode. So, phi n x equal to root over 2 by l sin n pi x by l, so by substituting this expression in this expression, so we can show that.

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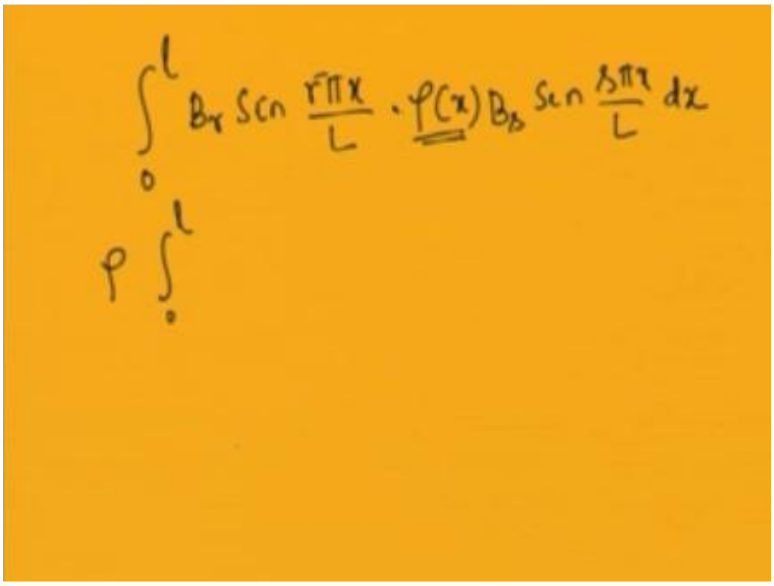

So, when r not equal to s, so this becomes, 0 to l for phi r I can write this is equal to Br sin n pi or r pi x sin r pi x r pi x by l into rho x into for s mode. I can write in this form sin s pi x by l dx, so I can take this rho x constant by taking rho x constant equal to rho. So, this becomes, 0 to l and for this Br I can write this equal to…

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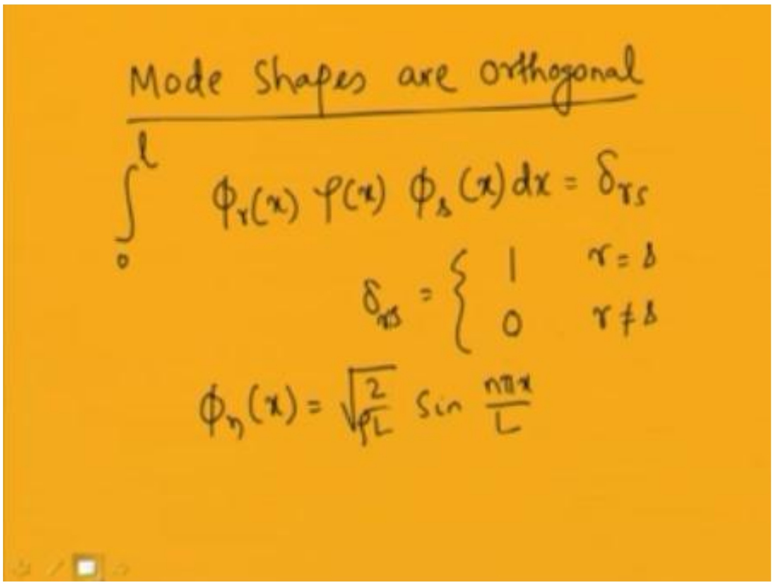

Already, we have found this Bn, it is equal to root 2 by rho L. So, 2 by rho l, I can put it here. So, this becomes.

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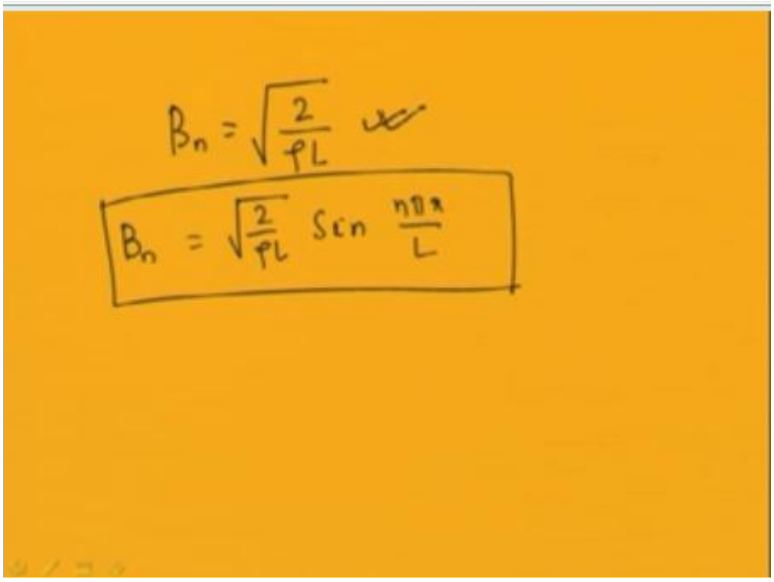

So, you obtain this Bn equal to 2 by rho L. So, you can substitute that root over 2 by rho L and you can find.

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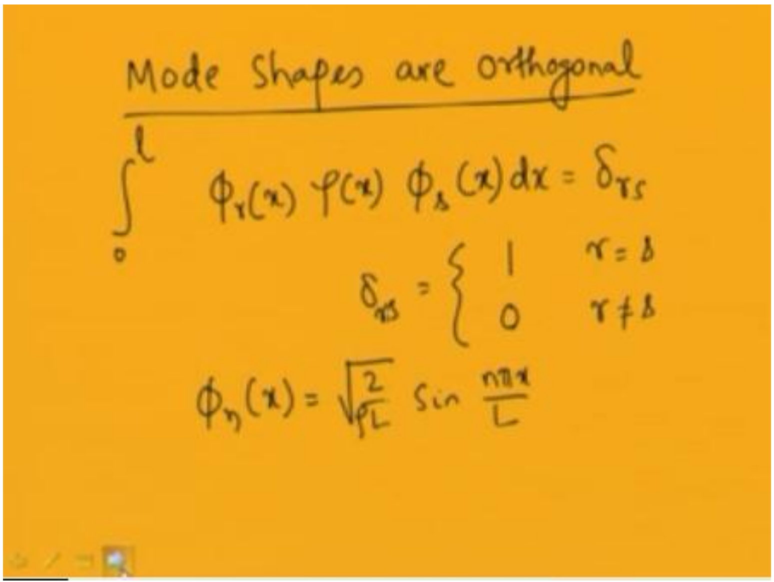

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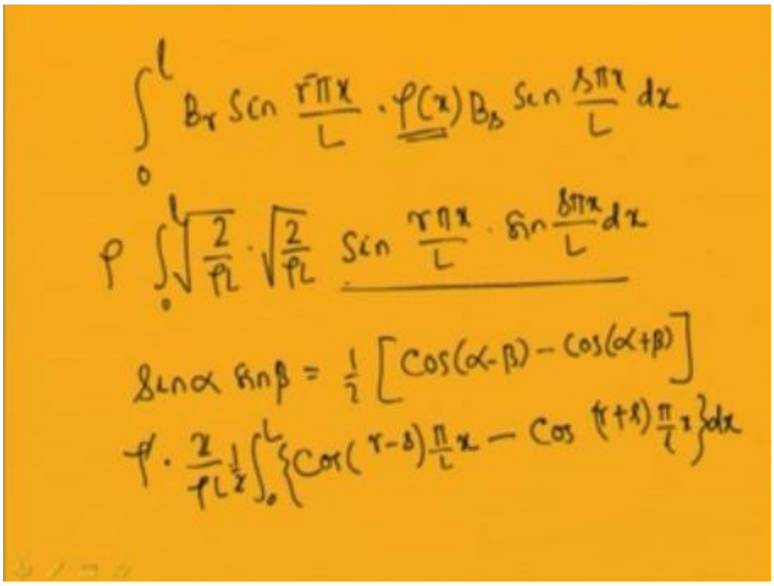

So, this will becomes 2 by rho l root over into again another 2 by rho l for this Bs. So, you will have sin r pi x by l into sin s pi x by l dx. So, you know the integration or you know this term sin r pi x by l into sin s pi x by l can be written. Or you know the term sin alpha into sin beta sin alpha into sin beta can be written as half cos alpha minus beta minus cos alpha plus beta. So, if you substitute this thing sin for the sin alpha into sin beta. So, you can find for the from this equation rho into 2 by rho l integration 0 2 l and another half will come here, so this is half. So, this is cos alpha minus beta, so for this alpha and beta, you can write this as r minus s into pi by l to x minus cos r. So, this is r plus s into pi by l into x into dx. So, you have reduce this product of sin terms in terms of cos terms. So, this becomes, so you can divide this 2 2 and rho rho, but canceled. So, this becomes.

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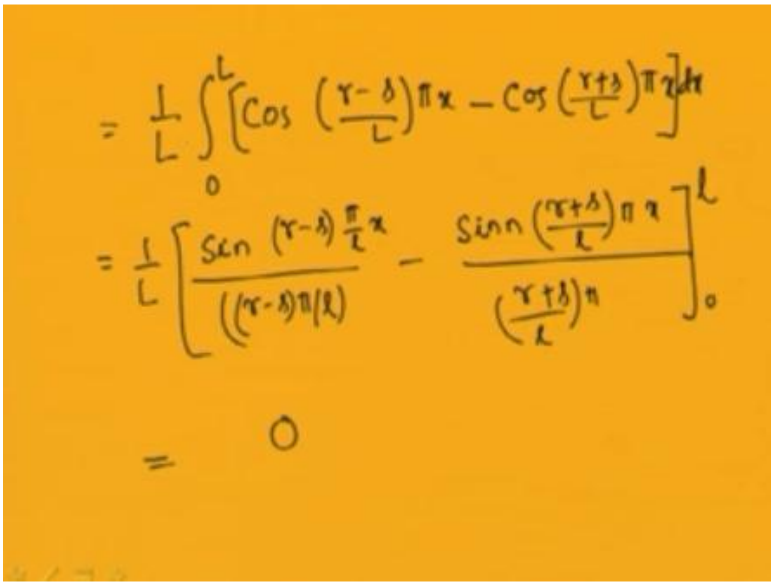

This is equal to 1 by L integration 0 to L s cos r minus s by l into pi x minus cos r plus s by l into pi x dx. So, this integration becomes, 1 by L, so for this cos, you can write this is integration is sin. So, this is sin r minus s into pi by l x by, this is r minus s pi by l minus similarly, this becomes, sin r plus s by l pi x by r plus s by l into pi. So, this is from 0 to1. So, you can see that these terms at x equal to l and x equal to 0 it vanishes and this becomes 0. So, when r not equal to s, so you have checked that this integral will be equal to 0.

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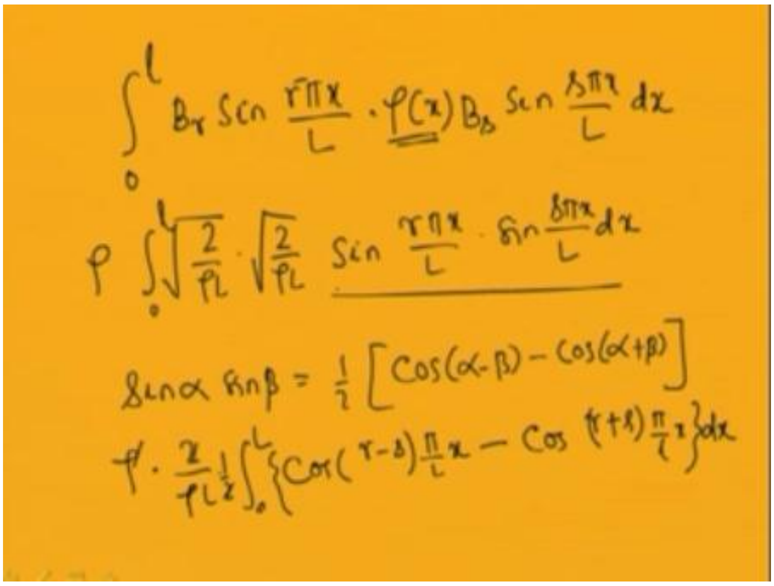

And when r equal to s, so this term, you can write this as Br square sin square. So, when r equal to x.

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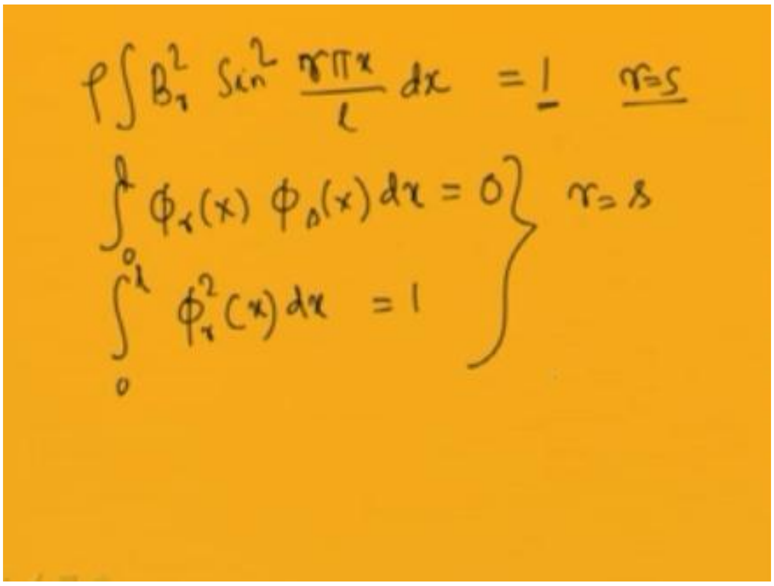

Your integration term becomes, Br square sin square, so rho into sin square r pi x by l dx. So, already you have integrated this thing, in case of r equal to n and that integration becomes, equal to. So, this integration equal to this integration equal to l by 2, you have seen and the total integration will be equal to 1. So, you have observed that, this is equal to 1 and, so you can prove that, when r equal to s, so when r equal to s. So, this integration become 1 and when r not equal to s this becomes 0. So, this proves the orthogonality principle of this mode shapes. So, the mode shapes are orthogonal to each other. So, the mode shapes are orthogonal to each other; that means, so when r not equal to s. So, the mode shapes, this phi r s into phi s x phi r x into phi s x equal to 0 when r not equal to s and when r equal to s 0 to l. So, that case it becomes, phi square x dx. So, this will becomes 1. So, this proves the orthogonality property of the mode shapes similarly, if you recall.

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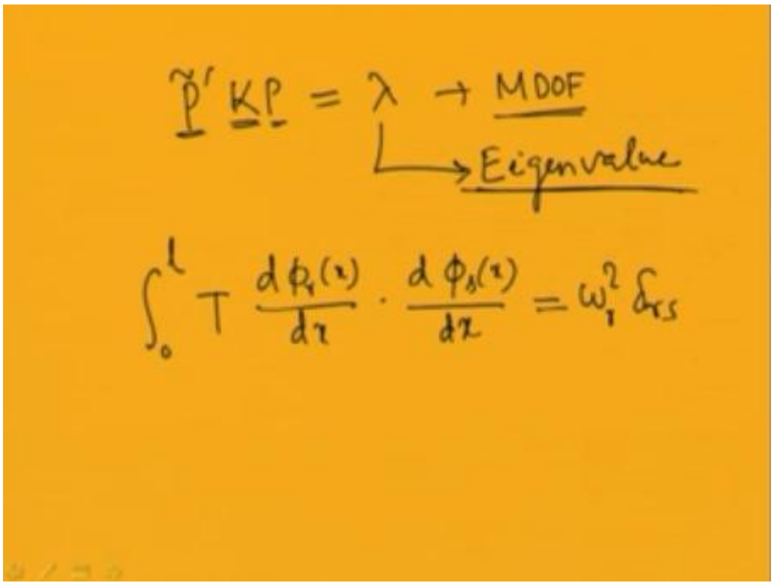

This P weighted modal matrix KP, you have checked that, this will becomes the Eigen function Eigen values of the system. So, using this modal weighted modal matrix, this is the stiffness matrix and this is the weighted modal matrix. So, P weighted modal matrix transpose, this is stiffness matrix and this is P matrix for a multi degree of freedom system. So, this becomes, equal to lambda that is the Eigen value of the system. Similarly, in this case also you can prove that or you can show that, this integration 0 to l rho, so integration 0 to l. So, this T d phi r x by dx into d phi s x by dx, so this will be equal to omega r square delta rs, where omega r is the rth mode frequency. And omega r square is equivalent to the Eigen, value of the system. So, in this case will prove this thing, so already we got the expression for phi x. So, phi r x is the d phi r by dx is the derivative of phi x. So, you can write this thing this equation in this form, so this will becomes T into.

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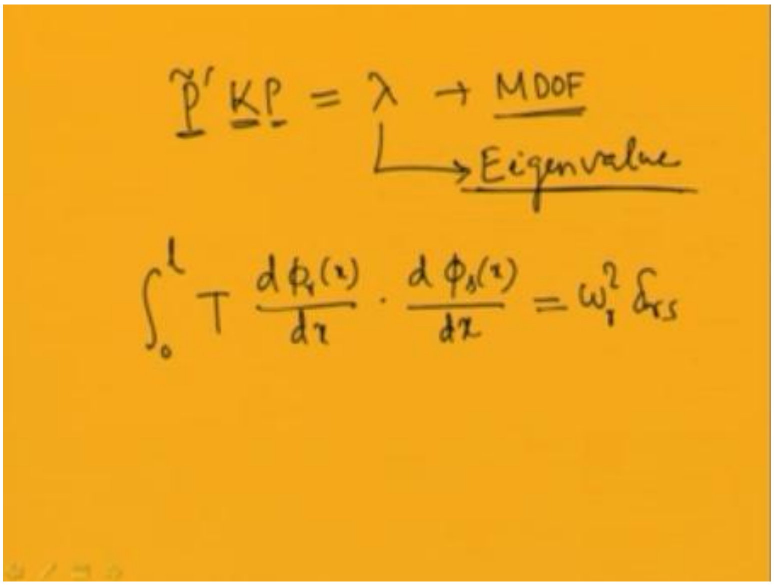

So, by using the Normalized mode I can write this as.

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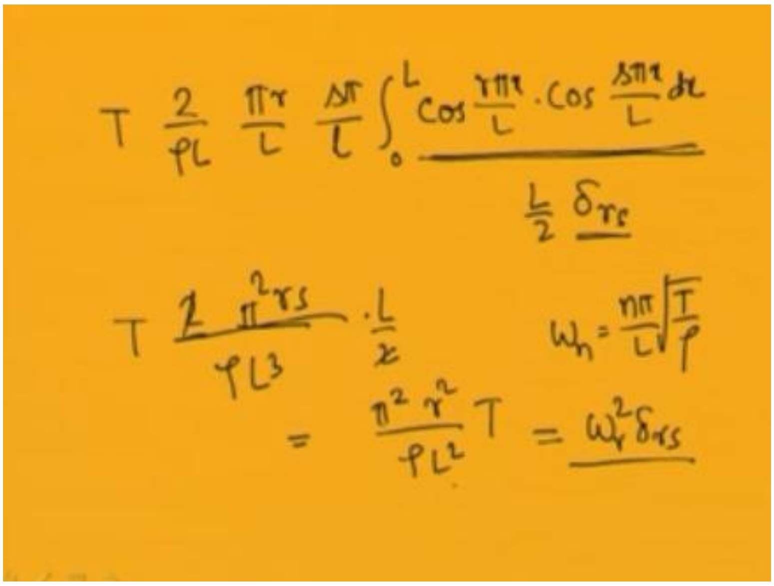

T into 2 by rho L pi r by L into s pi by l integration 0 to L cos r pi x by L into cos s pi x by L dx, so this thing can be written equal to. So, this is, so this integration you can find this equal to l by 2 delta rs. o, this is equal to L by 2 delta rs; that means, when r equal to s this value equal to 0 and when r when r equal to 0 this value becomes, delta rs equal to L and when r not equal to s delta rs equal to 0. So, from this when r equal to s, you can find this value equal to 2 by rho L into pi square rs by l square. So, this is pi square r l by, so 2 pi square rs by rho l cube. So, this thing can be written as rho l cube into l by 2, so this becomes, pi square, so this as r equal to s. So, it can be written as pi square r square, so this r into s equal to r square pi square r square by rho L square into T.

So, as omega are already, you know this is equal to omega r equal to n pi by l into root over T by m. So, this omega n, so this is omega n, so omega r will becomes, r pi by l into root over T by rho. So, I am using rho, so this omega r square term, you can write equal to n square pi n square r square by l square into T by rho. So, already we have T by rho and this is pi square r square by l square. So, this thing can be written as omega r square omega r square. So, I can write this integration equal to omega r square delta rs, so when r equal to s this integral will reduce to omega r square delta omega r square and when r not equal to s this value will become 0. So, we have seen mode shapes are orthogonal and from this also.

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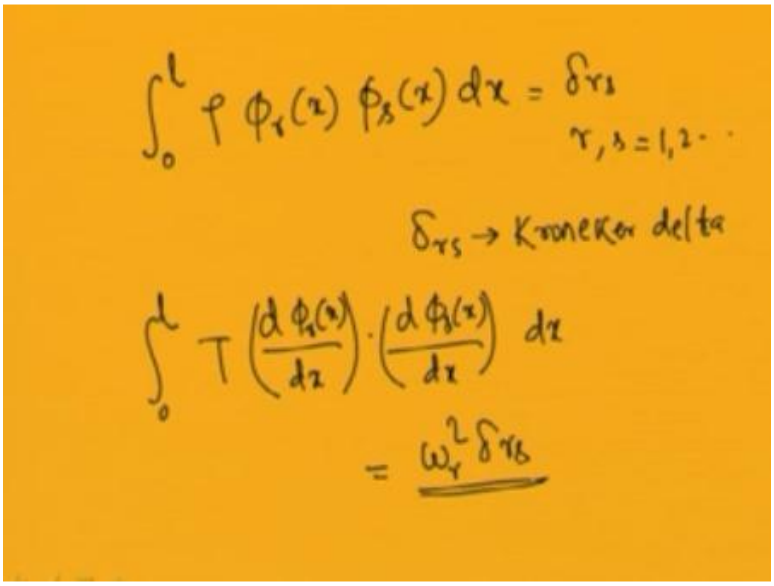

We have checked that this integration 0 to l rho phi r x phi s x, where phi r and phi s are rth mode shape and s mode shape. So, this is equal to delta rs, when r s equal to 1 2, so you can take any number of modes, so this delta rs is the Kronecker delta. And also we have proved that 0 to l, this omega or T d phi r x by dx into d phi s x by dx equal to omega r square delta r s. So, in this way, you can find or you can show that different modes are orthogonal to each other. And, so you can using this property of orthogonality, you can find the free vibration response of the system, so a general free vibration response of the system.

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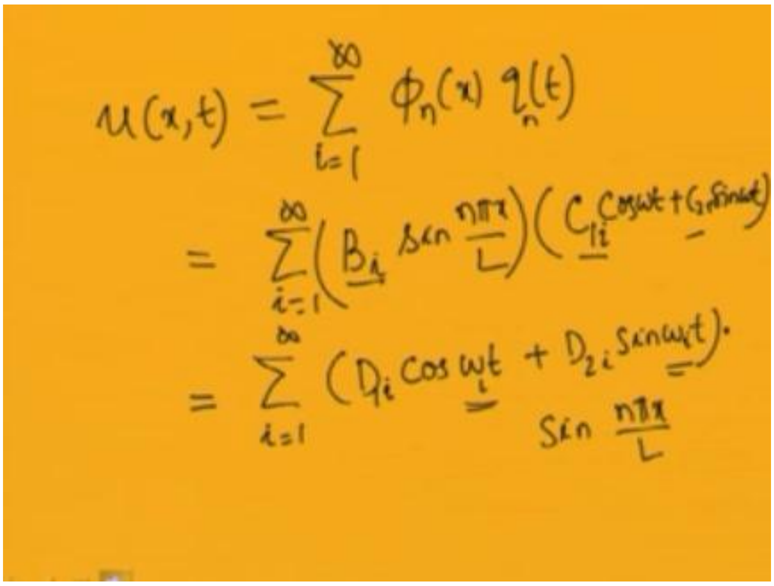

That is u x t will be the summation of all the modal frequencies all or you can find this u, x, t from the contribution of different modes. And you can write this u, x, t at summation I equal to 1 to infinity phi n x into qn t. So, this equation can also be written like this, so already you have seen for a string this is equal to. So, this is equal to I equal to 1 to infinity, so this is equal to Bi. So, for phi x I can write Bi sin n pi x by l and for qn, I can write this is equal to C 1 I C 1 I cos omega t plus C 2 I sin omega t. So, I can multiply this B with the C and these 3 constants can be reduce to 2 constants.

And So, I can write this expression for the free vibration response of the system like this. So, this will be equal to I equal to 1 to infinity. I can write this as Ai or let me write in terms of some C. So, some other C term or D term I can use, so this becomes Di cos D 1 I cos omega t omega I t plus D 2 I sin omega I t. So, where this omega I are the natural frequencies of the system. So, this into, so this into this mode shape term also will be there. So, this into sin n pi x by l, so already you know the relation between this omega and omega for different mode shapes different modes. So, for different modes you have seen this.

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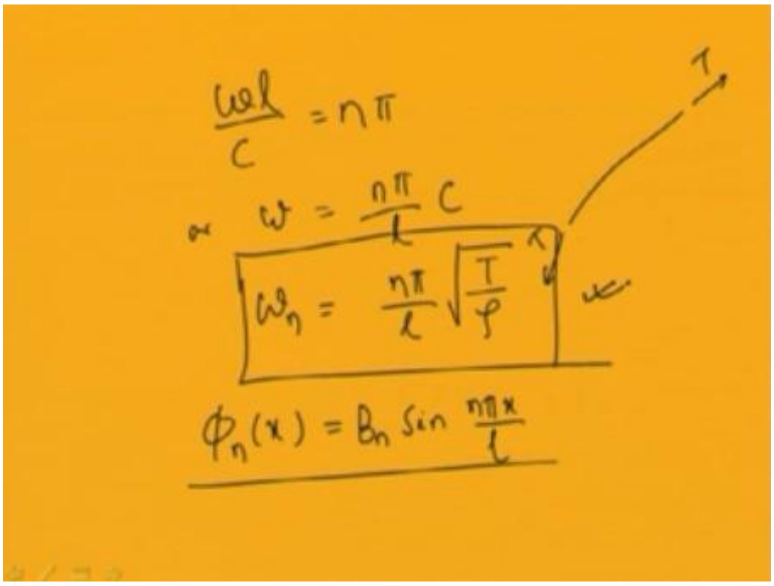

Omega equal to already, you have found the expression for omega for this transverse vibration of the string and this can be written in this form. So, omega n equal to n pi by l root over T by rho. So, this is for the nth mode and the final expression for this for the case of the string you can write in this form.

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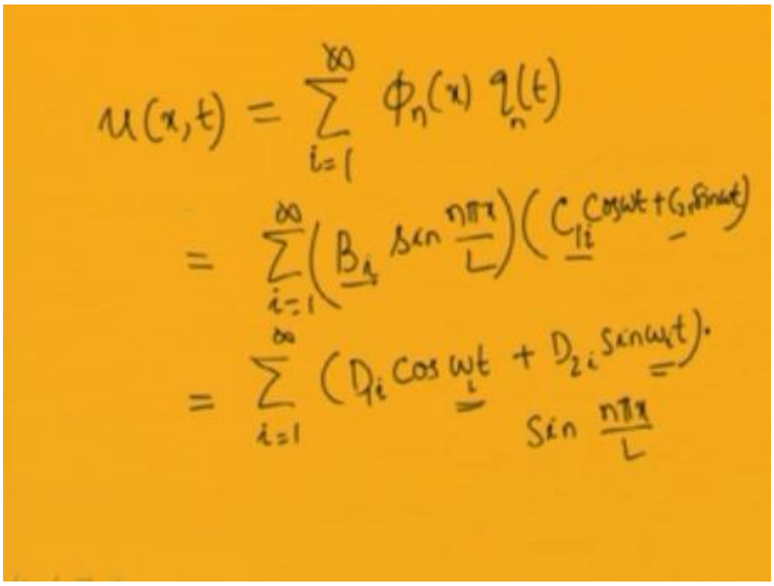

So, u x t equal to D 1 i cos omega i t plus D 2 i sin omega i t into sin n pi x by l now, let us find the mode shapes for the case of the longitudinal vibration of rod.

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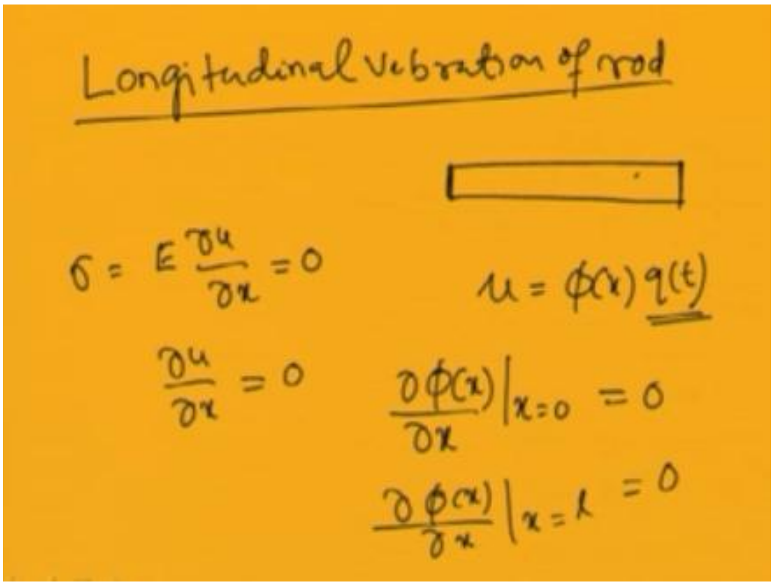

So, in this case will take different boundary conditions and will find the frequency equation and the mode shapes of the system. So, let us take the case of a free-free beam. So, in case of the free-free rod, so this is the free-free rod. So, in this case both the ends are free. So, when both ends are free this ends are not stress, so there is no stress in this in this ends as there is no stress in the ends. So, I can write that the stressed or stress equal to E del u by del x equal to 0 as del u by del x equal to 0. So, I can write E del u by del x that is the stress sigma equal to 0.

So, I can write this del u by del x that is the strain equal to 0 at both the ends. So, this u you can write in this form u equal to phi x into qt . So, as u equal to phi x into qt and this expression is valid for all the time this boundary conditions. That is del u by del x equal to 0 at the free end and at the at both the free end are 0 or the slopes this del u by del x equal to 0 at both the ends for all time. So, you can write this phi x at x equal to 0 or del phi x by del x at x equal to 0 equal to 0 similarly del phi x by del x at x equal to l also equal to 0.

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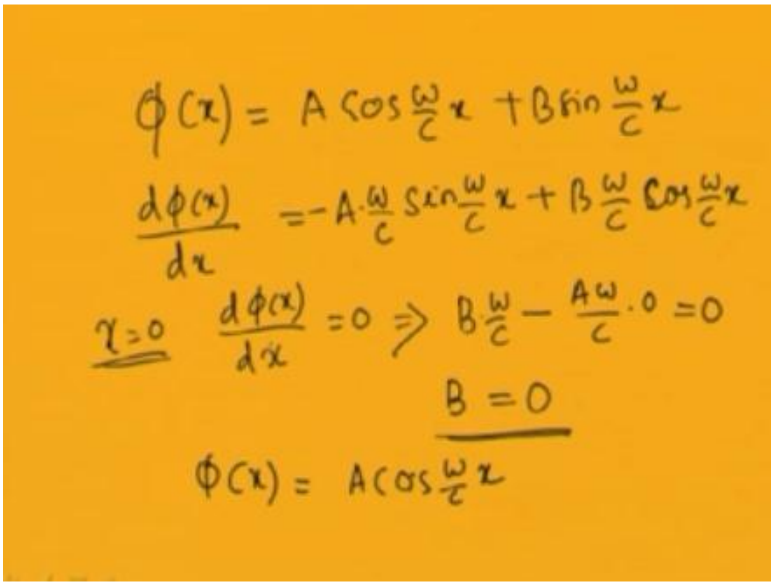

So, as phi x you can already got the expression for phi x that is equal to A cos omega by c x plus B sin omega by c x. So, you can find this d phi x by dx, so that is equal to A into omega by c. So, this becomes if you differentiate this thing this becomes sin omega by c x minus plus B into omega by c cos omega by c x. So, when x equal to 0, so for x equal to 0 d phi x by dx equal to 0 d phi x by dx equal to 0. So, this implies that this B into this cos term B into omega by c A omega by c into 0 sin 0 equal to 0. So, this becomes equal to 0. So, this implies this B term equal to 0. So, from this boundary conditions you have seen that this B equal to 0. So, as B equal to 0. So, here phi x now becomes phi x equal to A cos omega by c x, so now d phi by dx at the right hand.

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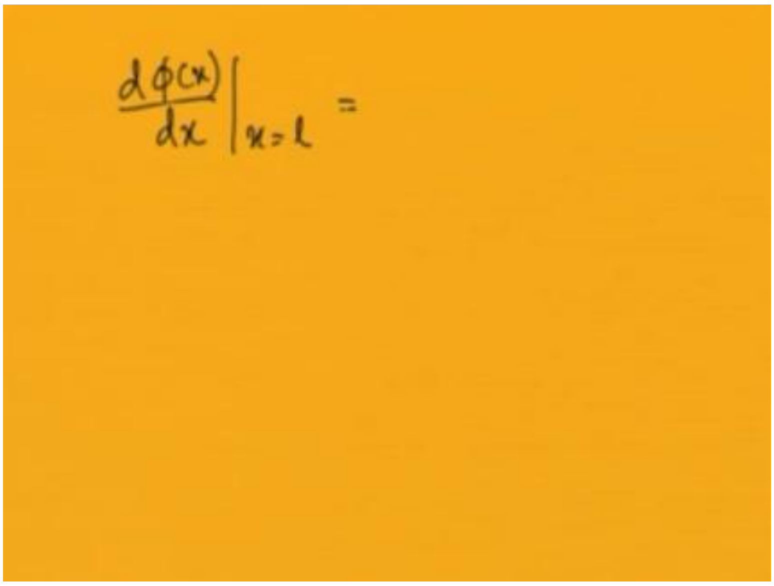

D phi x by dx at the right hand that is x equal to l can be written as...

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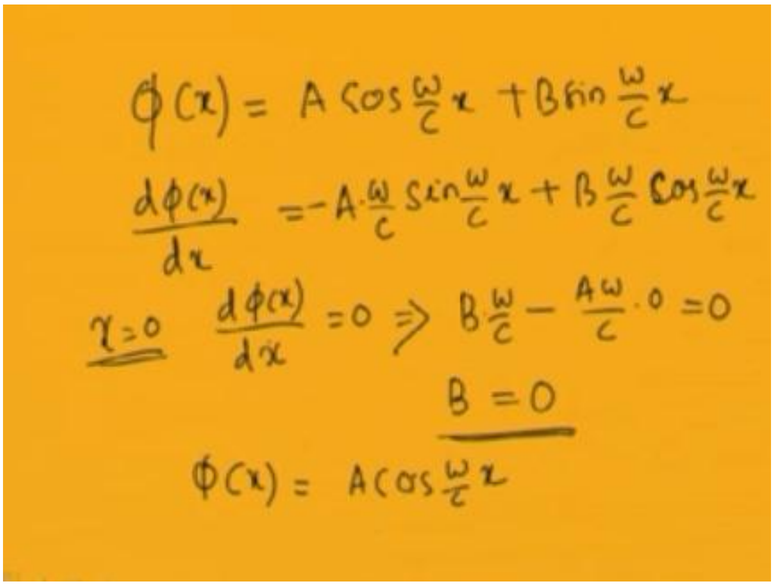

So, this becomes A omega by c into.

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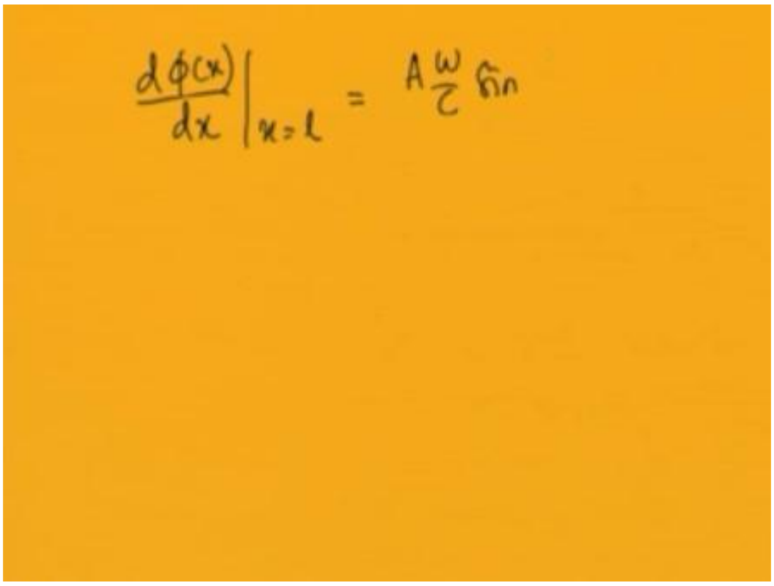

A omega by c sin.

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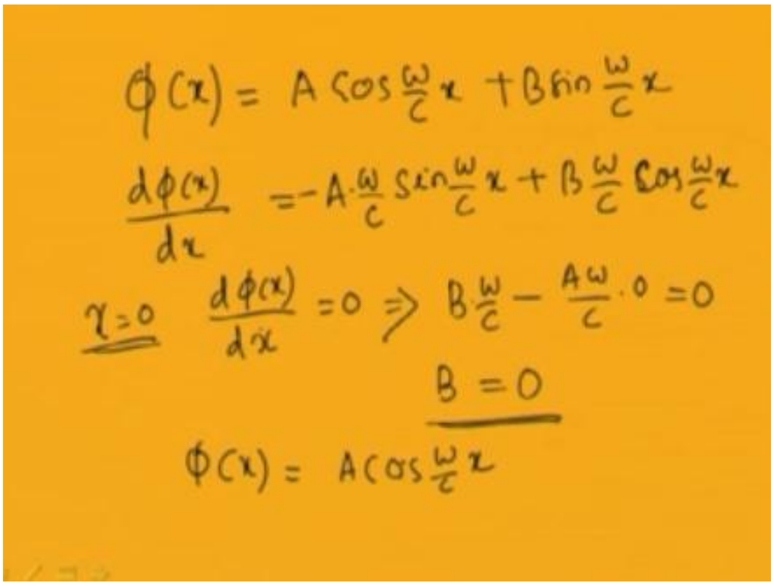

Sin omega by c x.

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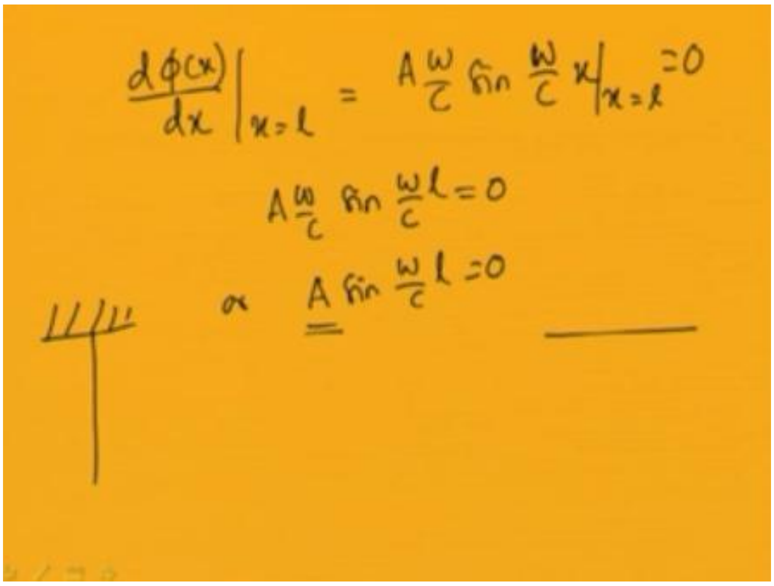

So, this is equal to at x equal to l equal to 0. So, I can write this as A into omega by c sin omega by c l equal to 0 or I can write this as A sin omega by c l equal to 0. So, from this as A cannot be 0, because when A equal to 0 you will have a trivial state solution; that means, there we will be no vibration of this rod this may be rod. So, there will be no vibration of this rod. So, you can take this as a free-free beam. So, in this case you can take this as a, this as a free-free rod.

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So, in this both the ends are free, so it is not stressed. So, as this is not stressed in this case you can find that the stress are 0. So, E into strain will be equal to 0 or strain will be equal to 0 that is del u by del x will be equal to 0 and as you know this u can be written in terms of pi x and qt. So, for all times to come strain at this 2 ends will be equal to 0. So, for that reason, so you can find this expression of the mode shape like this A sin omega c by l equal to 0 if I am making this A equal to 0 then there will be no vibration of this rod. So, for this free-free rod there will be no vibration when A will be equal to 0, but as the rod is vibrating we are or we are studying the vibration of this rod. So, A cannot be equal to 0. So, as A will not be equal to 0.

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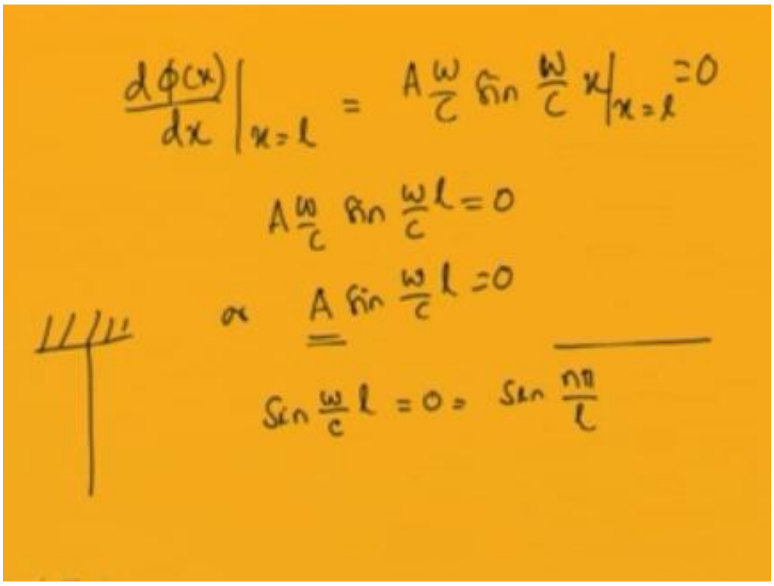

So, in that case I can write this sin omega by. So, this is omega sin omega by c l will be equal to 0 equal to sin n pi by l. So, as this is equal to sin n pi by l.

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So, I can write this omega L by. So, omega l by c.

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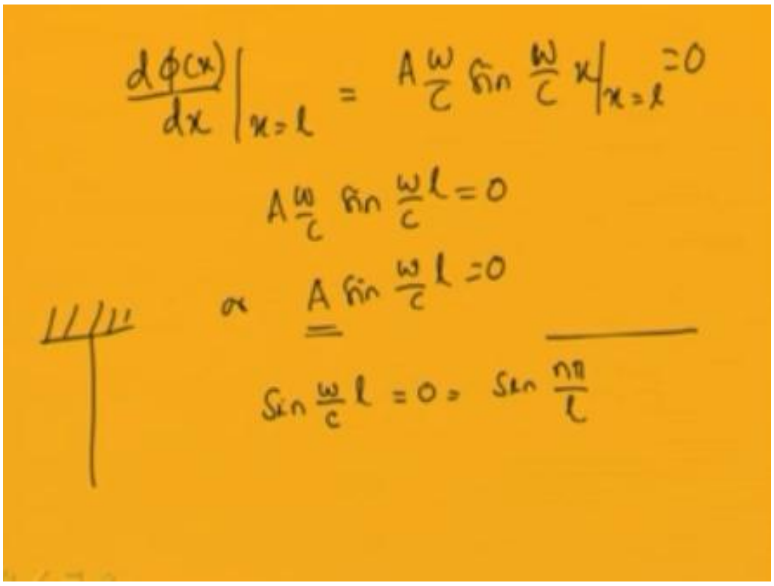

Omega l by c equal to n pi.

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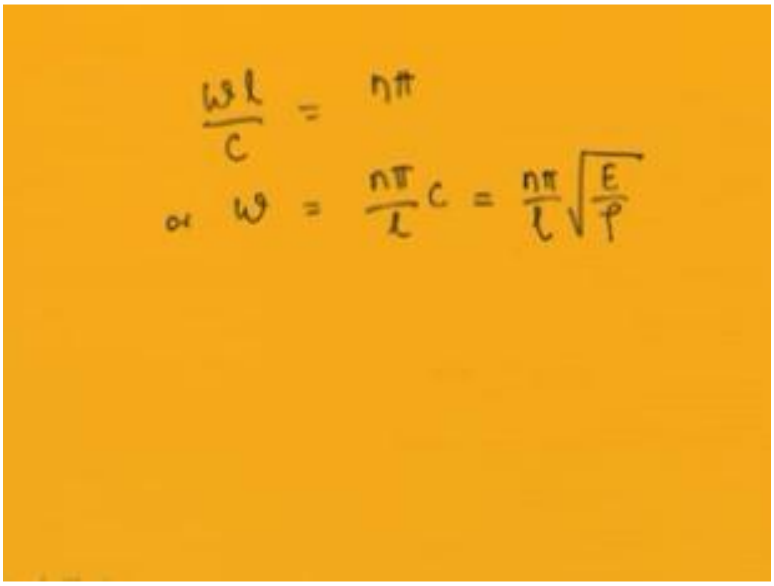

Or you can write this omega equal to n pi by l into n pi by l into c. So, this is equal to. So, in case of this rod this is can be written this c equal to E by rho. So, where E is the Young's modulus and rho is the mass per unit length of this rod.

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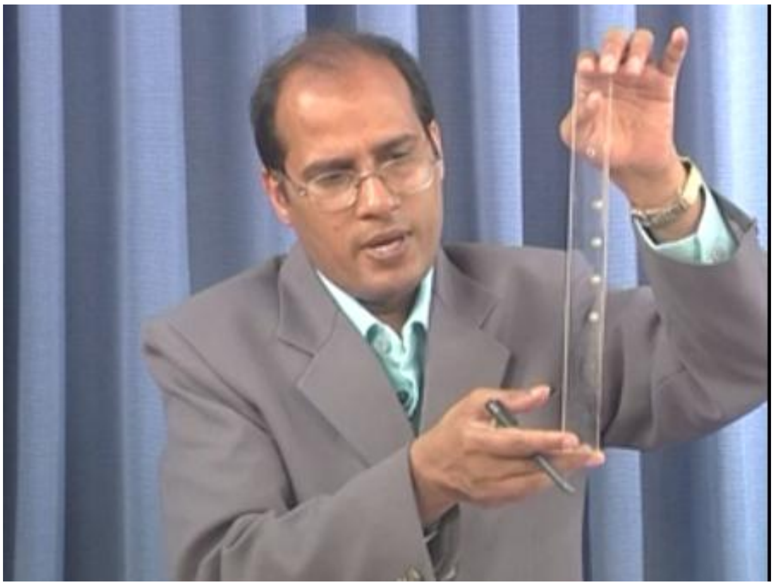

So, in this rod the mass per unit length I am taking equal to rho E is the young's modulus. So, in this case when it is vibrating that time the frequency the nth mode you can write in this form.

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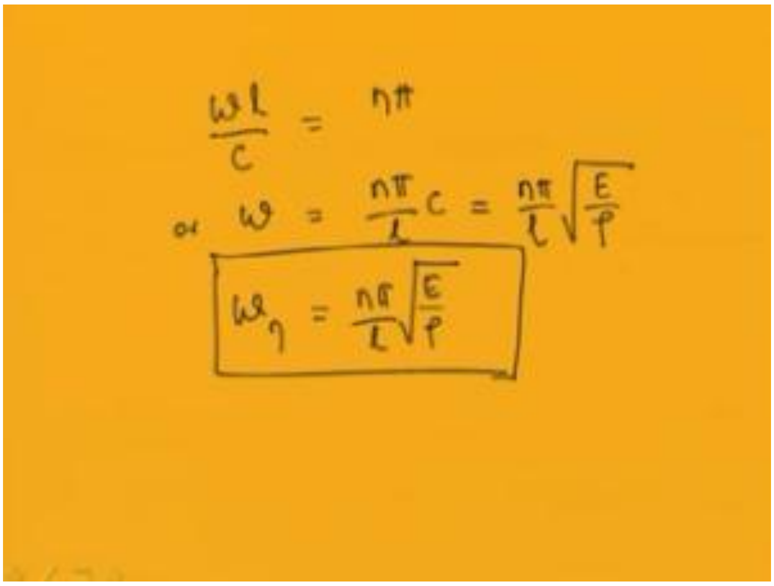

So, this is equal to n pi by l into E by rho and the mode shape you can write or Eigen function you can write in this form.

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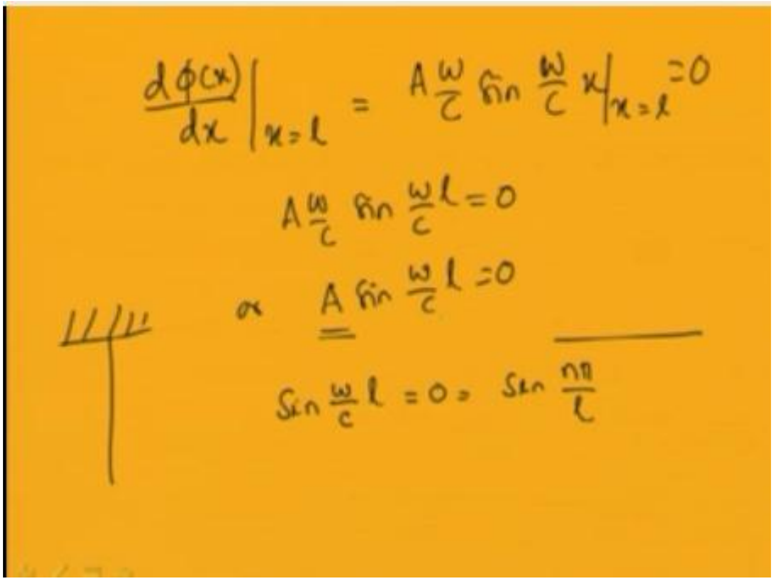

So, the Eigen function will become A sin.

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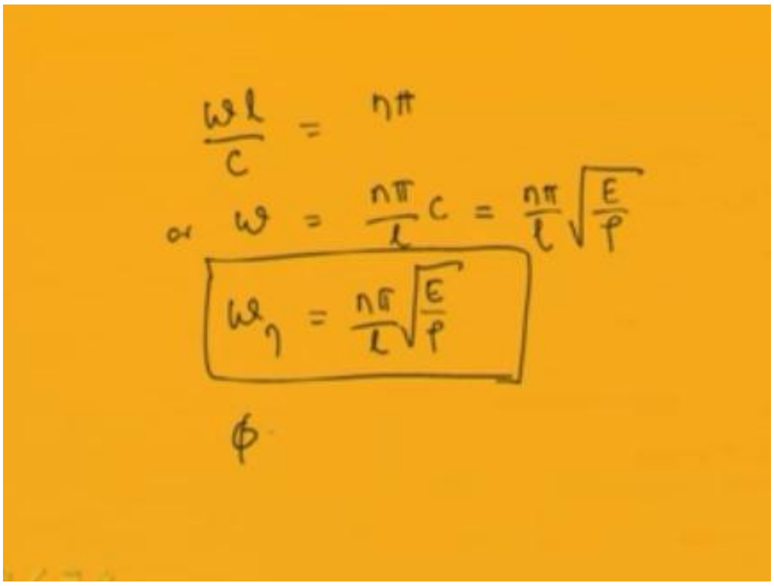

So, phi becomes.

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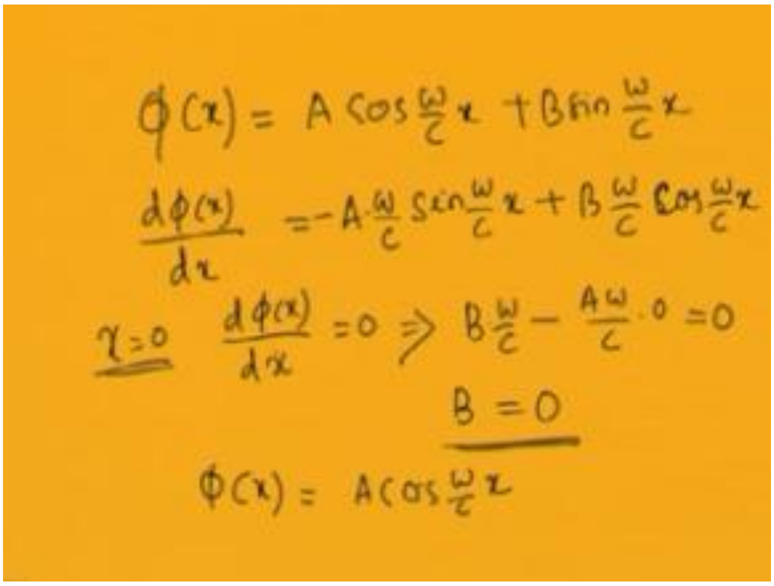

So, as A not equal to 0. So, this phi x B equal to 0 you can write phi x equal to A cos omega by cx.

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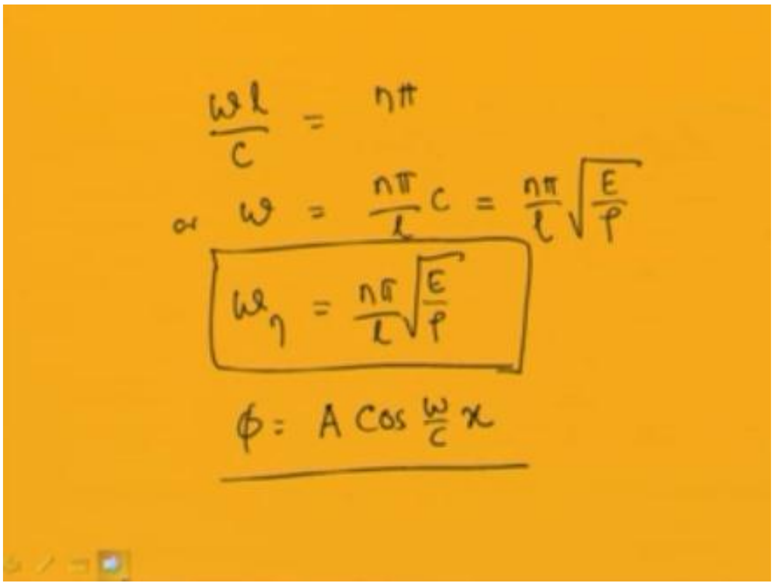

So, phi equal to A cos omega by c x and for omega I can substitute this value equal to n pi by l into root over E by rho. So, this omega by c can be written or this omega by c equal to n pi by l.

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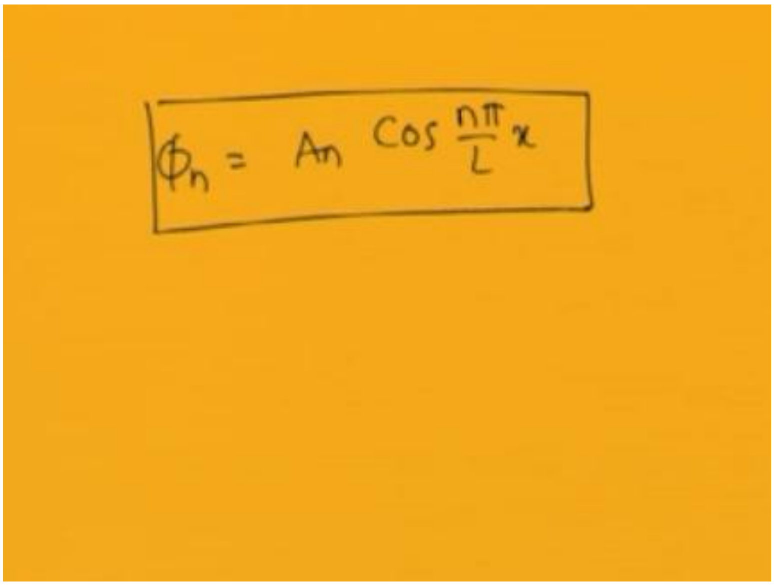

So, this expression for phi equal to An for phi n can be written An cos n pi by l x. Cos n pi by l x. So, you can see that this free-free rod vibrates in a cosine as a cosine form.

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So, in this rod the vibration will takes place. So, in this case the vibration will takes place in this longitudinal direction. So, in this longitudinal direction the wave propagation will takes place in a cosine will, so this pi n equal to A n cos n pi n pi l by x.

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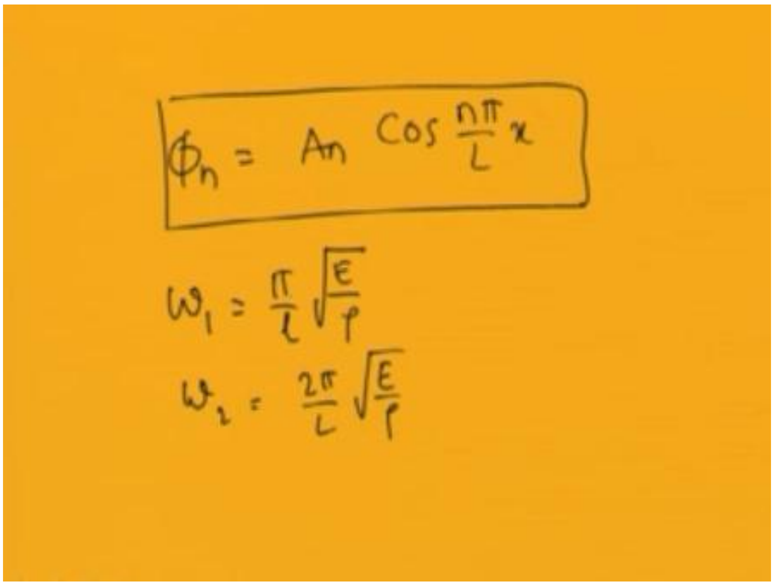

So, you can write this first modal frequency equal to pi by l into E by rho and second modal frequency will be equal to 2 pi by l into root over E by rho And similarly the nth modal frequency will be n pi by l into root over E by rho; that means, the nth modal frequency is n times the first modal frequency of the system. So, the general vibration or the free vibration response of the system you can write.

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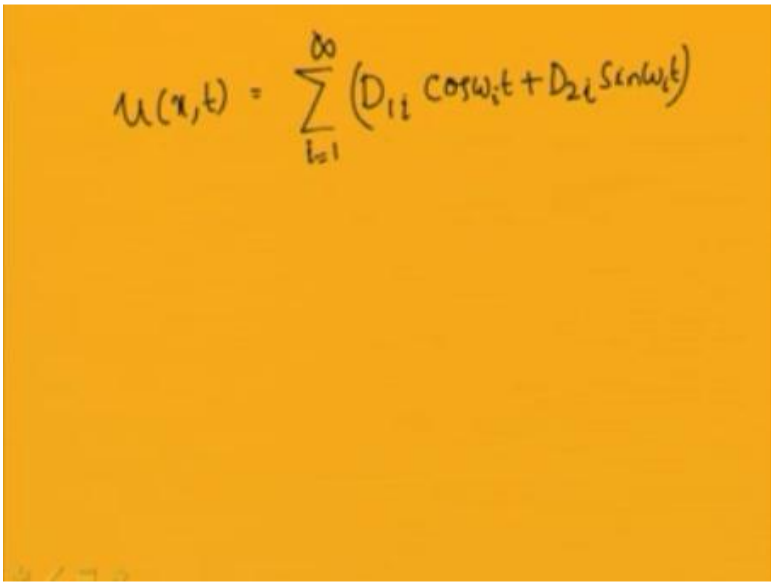

This u x t will be equal to summation. So, this will be equal to summation like in the previous case of string I can write this as D 1 I equal to 1 to infinity. So, D 1 I cos omega I t plus D 2 I sin omega I t into.

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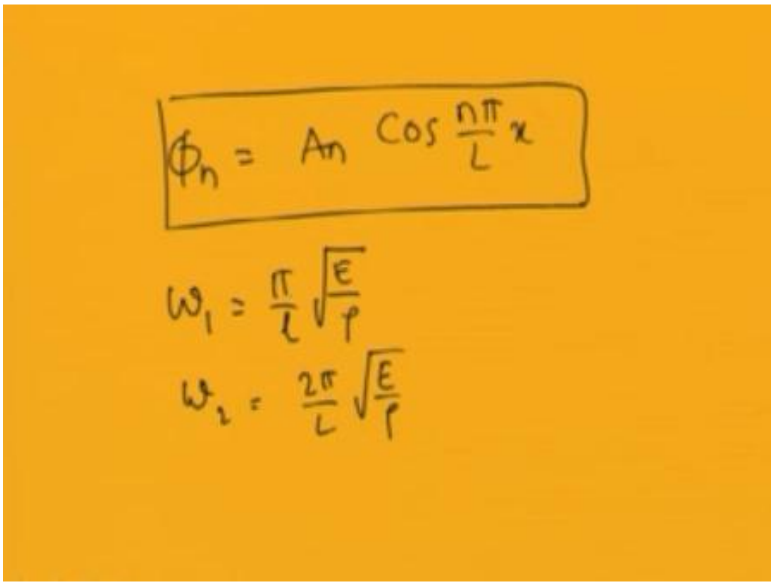

So, in this case, it becomes cos…

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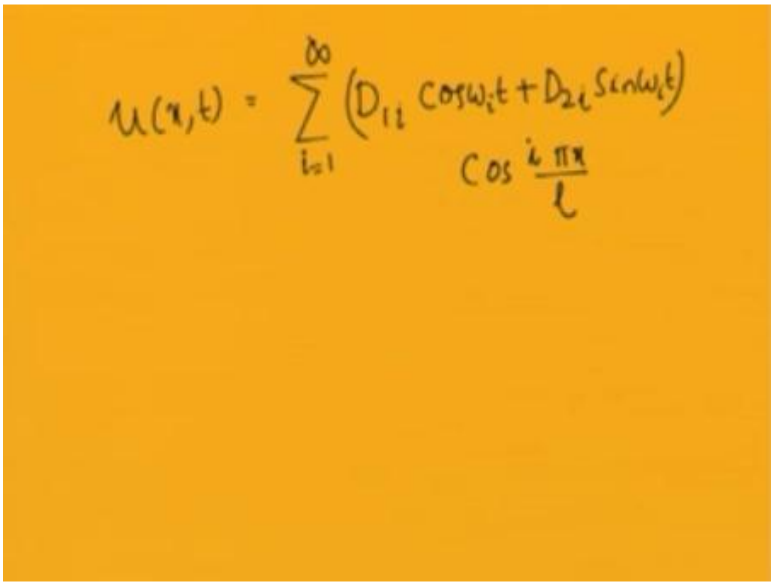

So, it will be cos n pi x by cos n pi x by l or. So, this is equal to omega n by x by l is your. So, for n I can substitute this equal to I . So, I pi x by l. So, u xt that is the free vibration response of the system becomes I equal to 1 to infinity D 1 I cos omega I t plus D 2 I sin omega I t into cos I pi x by l. So, in this D 1 I and D 2 I are the modal participation of the ith mode. So, these 2 this D 1 I and D 2 I you can obtain it from the initial conditions of the system. So, let us find the mode shapes for the case of a clamped free beam. So, let one in this clamped and other end is free.

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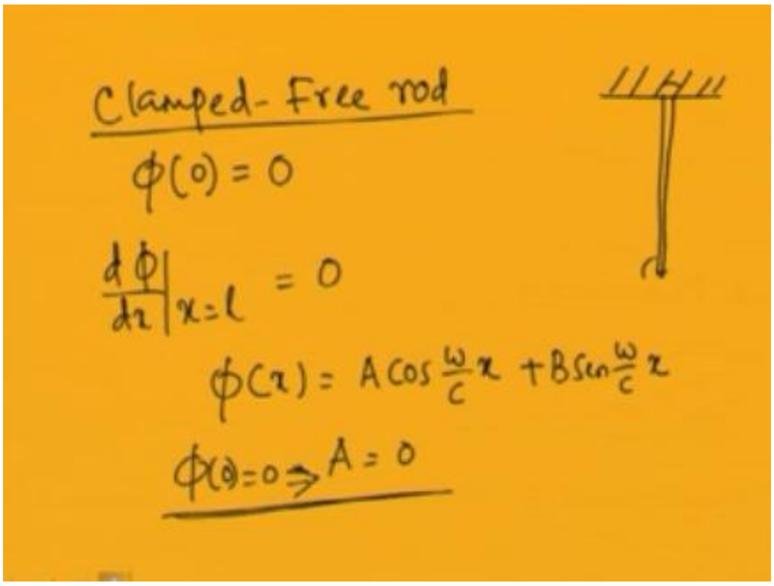

So, this end is clamped and other end is free. So, in this case you have to find the mode shapes of the longitudinal vibration of the rod. So, in case of a clamped free beam clamped free. So, this is a clamped free rod ao, we are interested to find the longitudinal vibration of this rod. So, in this case the boundary conditions are. So, at the fixed end displacement will be equal to 0 and at the free end the stress will be equal to 0. So, are the stress equal to 0 already we have seen that it can be written at the free-free free end d phi by dx equal to 0 and at the fixed end phi x equal to 0. So, our boundary condition will be phi 0 equal to 0 and d phi by dx at x equal l will be equal to 0. So, we can write this phi x in this form. So, phi x will be equal to A cos omega by c x plus B sin omega by c x. So, in this case I can substitute this phi x equal phi 0 equal to 0. So, that will give B equal to B into 0 plus A equal to 0 or A equal to 0. So, that will give A equal to 0 phi 0 equal to 0 implies A equal to 0. So, now differentiating it once, so I can write this.

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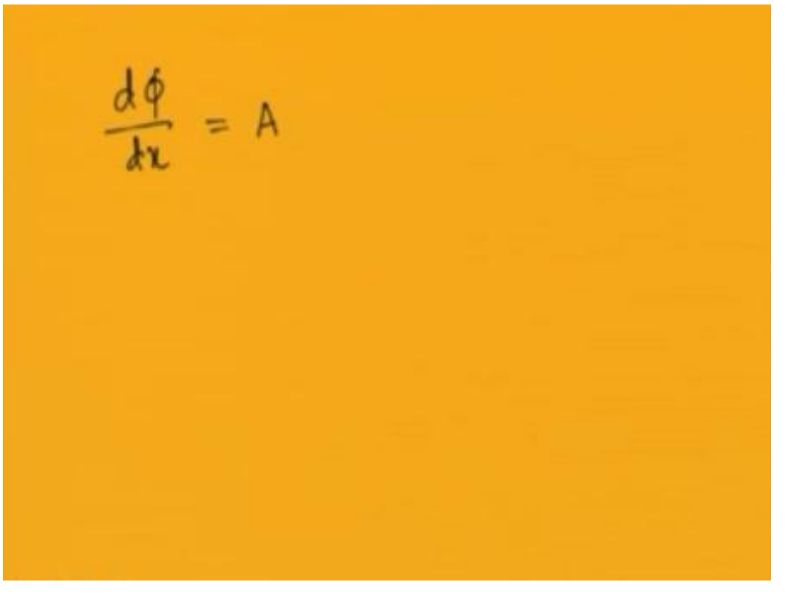

D phi by dx d phi by dx becomes A omega. So, now the…

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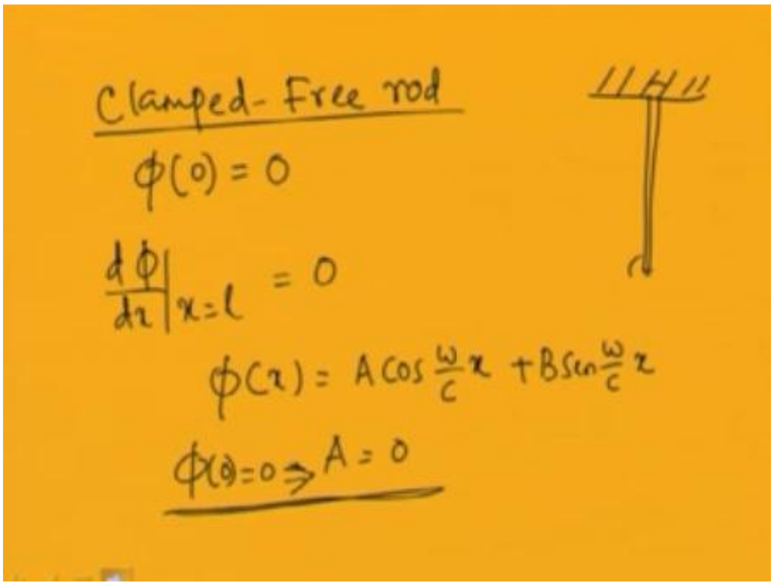

Remaining terms as A equal to 0. So, will have phi x equal to B sin omega by c x.

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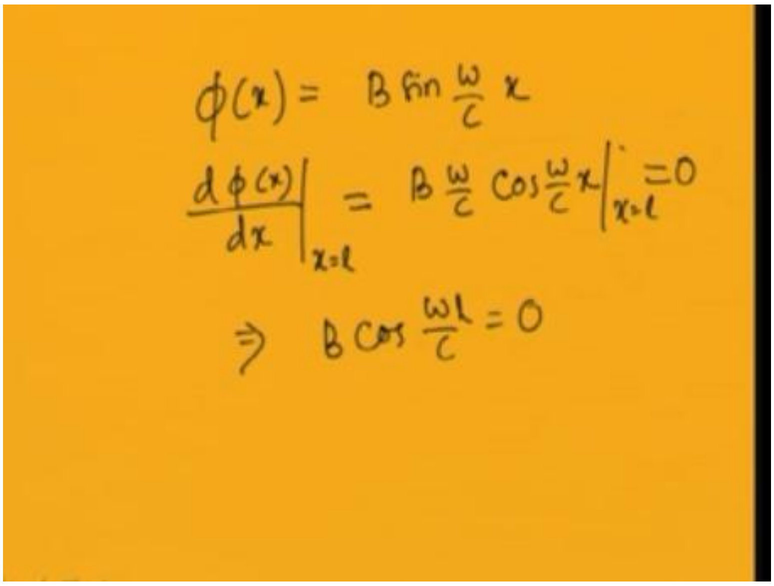

So, we have phi x equal to. So, I can write this phi x equal to, so phi x equal to B sin omega by c x. So, d phi x by dx, so d phi x by dx will be equal to B omega by c cos omega by c x. So, now, at x equal to l equal to l this equal to 0. So, this implies that this B cos omega by c l equal to 0. So, today class we have studied the free vibration response of string and also for the longitudinal vibration of the rod. Or the case of string we have shown that the Eigen functions or orthogonal and for the case of fixed-fixed string. We have found the frequency equation and the mode shapes and also for this longitudinal vibration of rod. We have found the frequency for the case of the free-free rod. Next class, we are going to study about the mode shapes of torsional vibration of rods and will summarize the free vibration of continuous systems.