Mechanical Vibrations

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Module - 9
Continuous Systems: Closed Form Solutions
Lecture - 4
Longitudinal and Torsional Vibration of Rods

So, last class we have derived, the solution of Euler-Bernoulli equation, and also we have found the solution of the wave equations. So, in case of Euler-Bernoulli equation, we have seen the mode shape. We have found the mode shape and we have found the mode shape or the case of a simply supported beam.

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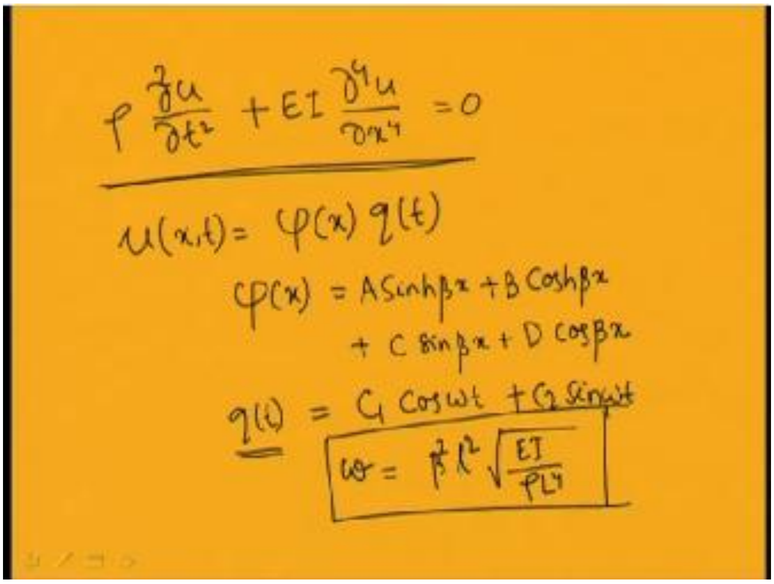

In case of Euler-Bernoulli beam the equation can be written in this form, that is del square you by del t square. Rho into del square by u by del t square plus EI del fourth u by del x fourth equal to 0 and we have found the solution for this equation and we have written that u can be written in this form. So, u which is a function of x and t can be written as psi x into q t. So, in this case, psi x is written in this form, psi x equal to A sin hyperbolic beta x plus B cos hyperbolic beta x plus C sin beta x and D cos beta x. Similarly, this q t, we have found it can be written in this forms, it can be written C 1 cos omega t and plus C 2 sin omega t. So, by multiplying these 2, that is mode shape pi x are mode shape psi x and this time modulation qt, we can find the generalized solution for this Euler-Bernoulli beam. So, we have found, the response for the, or we have found, the mode shape for a simply supported beam and the mode shapes, can be given in this way.

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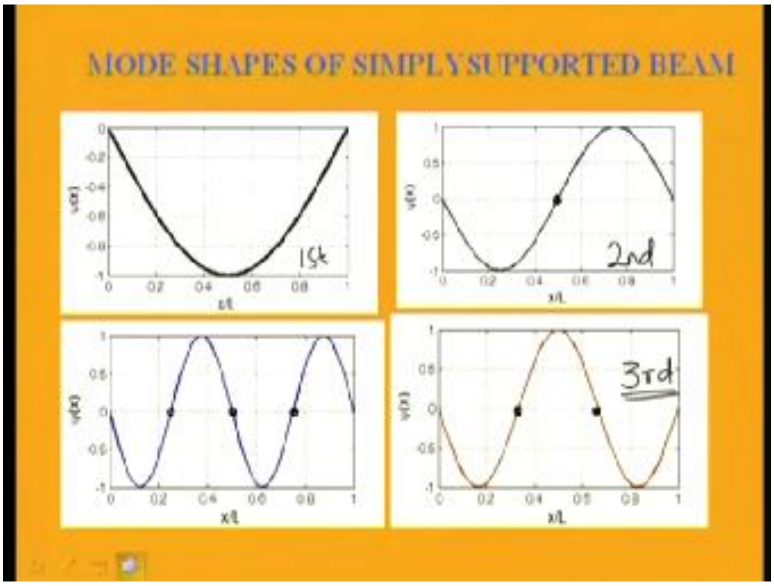

So, this is the first mode. So, this is the first mode shape, so in the first mode the beam is vibrating like this or the mode shape is given by this or the second mode, you can find the mode shape is this way and there is 1 node here. So, this is the node point; that means, in case of second node second mode this point will not vibrate. Similarly, for the case of third mode, so you can see there are two nodes in case of third mode, there are 2 nodes present. So, this is for the third, so this is this correspond to third mode. So, in case of third mode, so there are 2 nodes and in case of fourth mode you can find there are 3 nodes.

So, in this case, there is no node present and in case of second mode, this is the fundamental case first or fundamental frequency. So, the fundamental frequency the expression for the fundamental frequency also we have found. So, the expression can be written in this form omega equal to beta square l square into root over EI by rho one fourth. So, we have found the expression of the frequency in this way. So, omega the frequency can be, written as beta square l square into root over EI by rho one fourth. And we have found for this case of the simply, supported beam this beta l equal to n pi. So, we have found this expression for beta l.

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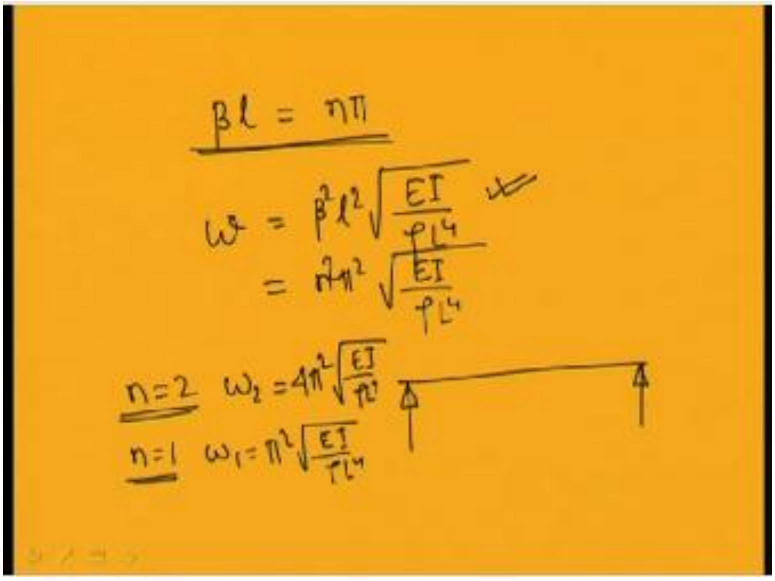

So, this is equal to n pi in case of the simply, supported beam. So, this omega as omega equal to beta square l square into root over EI by rho l fourth. So, for a given material or for a given system, this can be written as n square pi square into EI by rho one fourth as for a given system, EI and rho and l are constant. So, let this is the simply, supported beam. So, in case of simply supported beam we have taken the boundary conditions, as displacement equal to 0 and bending moment equal to 0. So, by taking displacement equal to 0 and bending moment equal to 0. We have found the mode shapes of these and also we have found the frequency equation. So, in this case the frequency from the frequency equation, we are writing this omega equal to n square pi square root over EI by rho l by fourth. So, you can see for n equal to 2, let me take the second mode.

So, in this case omega 2 will be equal to 4 pi square, so this becomes 4 pi square root over EI by rho one fourth when I am taking n equal to 1 that is fundamental frequency, this omega 1 equal to pi square root over EI by rho one fourth. So, you may note that the second mode frequency is 4 times the first mode frequency. Similarly, the third mode frequency will be 9 times the first mode frequency. So, the nth mode frequency is n square time the first mode frequency. So, in case of a simply, supported beam the nth mode frequency is n square time the first mode frequency and the expression for. So, we have already, seen the expression for this mode shape. So, you can find the mode shape, so the mode shape can be written in this form and we have obtained these concern ab and d are 0. And only the remaining things psi x the expression for the psi x in case of the simply, supported beam, last class we have found that it is equal to C sin beta x. So, psi x equal to C sin beta x I can write psi x equal to.

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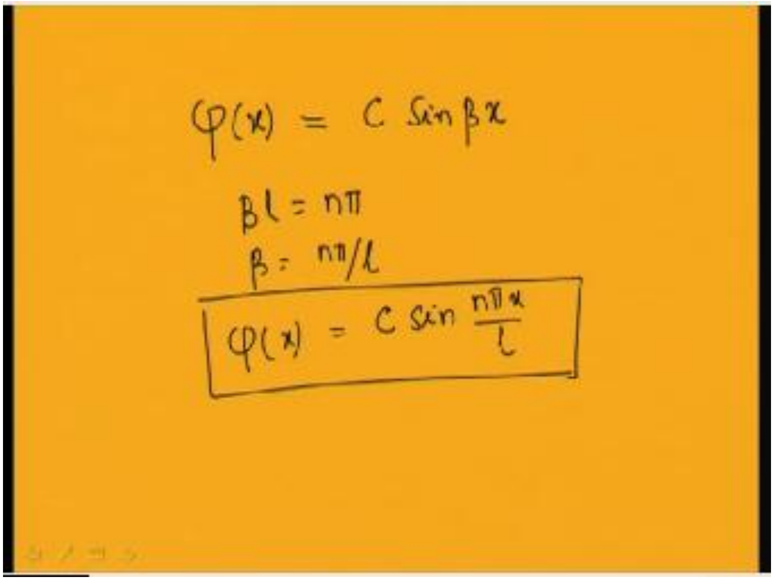

So, in case of simply supported beam psi x equal to C sin beta x and already, I got beta l equal to n pi. So, this beta equal to n pi by l, so this psi x I can write this equal to C the sin C sin n pi x by l. So, this is the expression for the mode shape from which we have obtained all the modes and last class also, I have shown you the animation of this simply supported beam. So, let us find the same thing for a cantilever beam.

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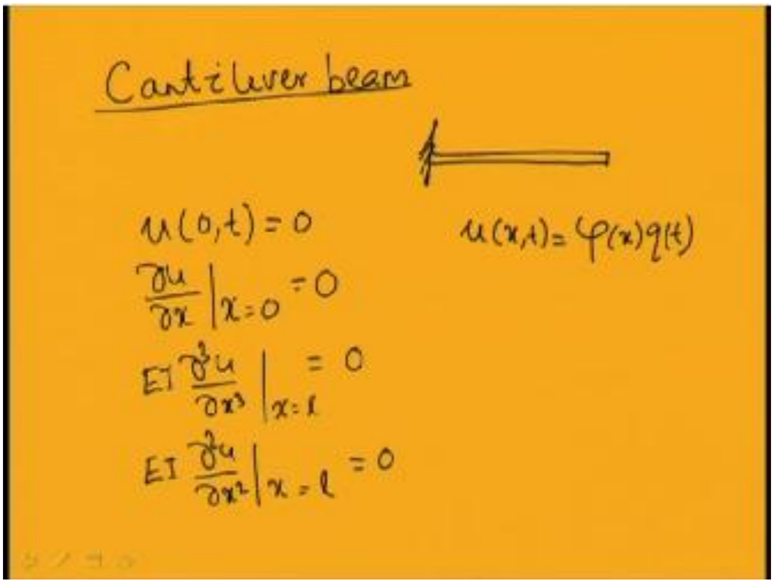

So, let us take the boundary condition of a cantilever beam and let us, find the mode shape for this case. So, in case of a cantilever beam, the left end or one end is fixed and other end is free. So, the free end or the fixed end in the in case of the fixed end you have the natural you have the geometric boundary conditions, the geometric boundary condition are the displacement equal to 0, and the slope equal to 0. So, the boundary conditions are u 0 t equal to 0 and du by or del u by del x as del u by del x at x equal to 0 equal to 0.

So, the both displacement equal to 0 and slope equal to 0 similarly, in the right end or the free end free end the shear force and bending moment equal to 0 shear force, can be written in this form shear force equal to EI del cube u by del x cube. So, these at x equal to l equal to 0 similarly, the bending moment also 0 at this end. So, bending moment can be written in this form EI del square u by del x square at x equal to l equal to 0. So, as we have taken this u equal to u xt equal to psi x into qt. So, u 0 t equal to 0 implies, that this psi x equal or psi 0 equal to 0 equal to 0.

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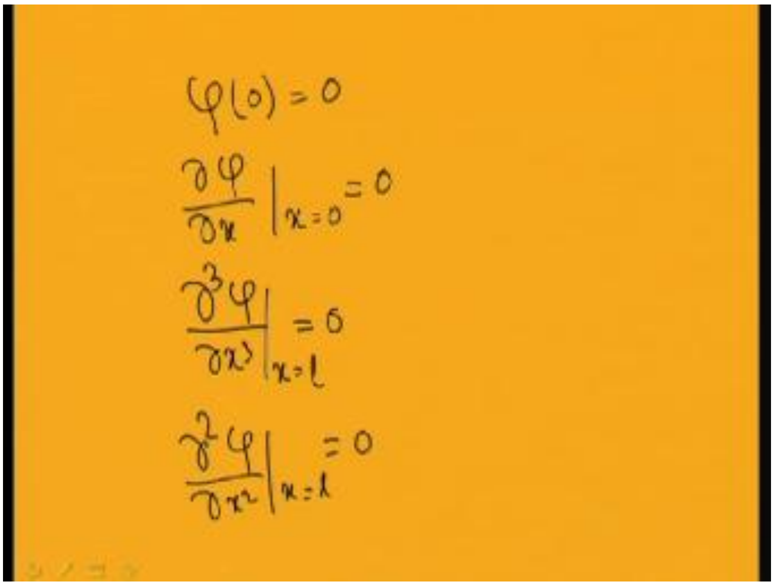

So, from this I can write the condition. Psi 0 equal to 0, from the second condition, that is del u by del x at x equal to 0 equal to 0 will give by substituting this expression there, I can find this del psi by del x equal to 0. So, del psi by del x at x equal to 0 equal to 0, Similarly, for the third condition. So, I can find this, del q psi by del x q equal to 0 del q psi by del x q equal to 0 at x equal to l and del square psi by del x square at x equal to l equal to 0. So, these are the 4 boundary conditions, which should be solved to find the constants in the expression for psi x.

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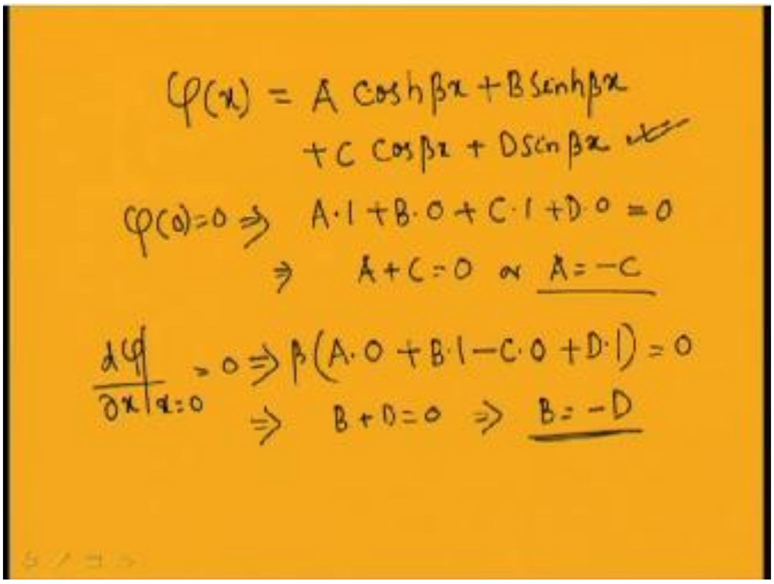

So, in case of psi x. The expression for psi x can be written in this form. So, this is equal to A sin A cos hyperbolic let me write in this form. So, this is A cos hyperbolic beta x plus B sin hyperbolic beta x plus C cos beta x plus D sin beta x. So, by substituting x equal to 0 that is displacement equal to 0, so psi 0 equal to 0 implies. So, A into, so this cos hyperbolic 0 equal to 0 cos hyperbolic 0 equal to 1 and the sin hyperbolic x when I am substituting x equal to 0 this is 0. So, A into 1 plus B into 0 plus similarly, C into 1 plus D into this is 0, so this is equal to 0. So, this gives rise to A plus C equal to 0 or from this, I can write A equal to minus C. Similarly, by taking this slope equal to 0 that is del psi by del x equal to 0. So, we can differentiate this expression once and we can substitute x equal to 0 by doing that. So, d psi 0 d psi by dx at x equal to 0 you can write at x equal to 0 equal to 0 implies.

So, if I will do the first derivative this cos will become, sin and the sin will becomes, cos sin hyperbolic will become cos hyperbolic into beta. So, I can write this, beta into A into for cos hyperbolic this becomes, sin hyperbolic. So, by substituting 0 x equal to 0 this becomes 0, Similarly, if plus B. So, this becomes, cos hyperbolic this cos hyperbolic, when I am substituting x equal to 0. So, this becomes, 1 then plus this derivative of cos beta this becomes minus sin. So, this becomes, minus C minus C. So, so this becomes, C into 0 and plus in case of D. So, if I will differentiate this, sin beta x this becomes cos beta x. So, I have already, taken beta common, so D into 1. So, this becomes, cos, so this becomes this, so in this way I can find. So, this is B plus D equal to 0. So, this implies, B equal to minus D, so I got 2 conditions in the first conditions A equal to minus C and in the second condition B equal to minus D By substituting these 2 in this equation in the equation for psi x. So, I can rewrite this psi x expression in this form.

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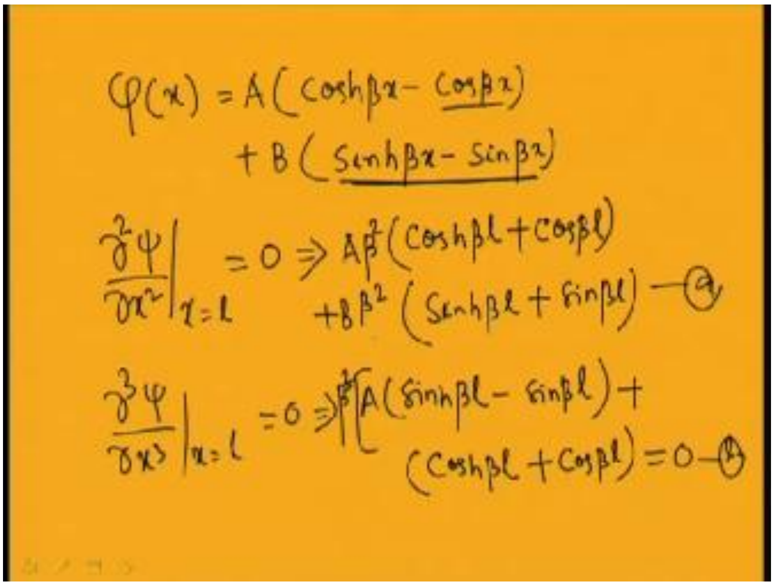

So, psi x equal to, so I can take a common, so by taken a common these becomes, cos hyperbolic beta x and for these I can write cos beta x. So, cos hyperbolic beta x minus cos beta x plus B into, so in this case it becomes, sin hyperbolic beta x and this becomes, minus sin beta x. So, sin hyperbolic beta x minus sin beta x, so psi x equal to a cos hyperbolic beta x minus cos beta x plus b into sin hyperbolic beta x minus sin beta x. So, now, substituting the other 2 remaining boundary conditions, that is at x equal to l. We have 2 boundary condition shear force equal to 0 and bending moment equal to 0. From which we obtained this, equation del square psi by del x square is corresponding to the bending moment at x equal to l equal to 0.

So, this implies, so I have to differentiate this twice. So, where while differentiating it twice, then I will have this beta square term. So, I can take beta square term common, so this cos hyperbolic beta, twice if I will differentiate this, then it will remain same, this cos hyperbolic beta x by substituting x equal to 0. So, this becomes, 1 now for this cos beta x by differentiating twice. So, it remain this, so cos while differentiating it, once this becomes, minus sin, so this, minus minus become plus plus sin. Then again, it differentiating it becomes cos beta x, so it becomes, plus cos beta x by substituting this cos beta x. So, now, we have to substitute this x equal to l. So, if I differentiate it twice, then this becomes, cos hyperbolic beta x. So, by substituting x equal to l I can write this equal to cos hyperbolic beta l similarly, by differentiating it twice it becomes plus cos beta x.

So, by substituting these, this becomes cos beta l similarly, here B into beta square b into beta square into this become, sin hyperbolic beta l and this sin beta x if I differentiate twice, then it becomes, plus sin beta l. So, this is 1 equation I am getting now, by taking this del cube psi by del x cube at x equal to 0 equal to 0. So, this will imply, so I have to differentiate this thrice. So, this imply A, so I can take this beta square common. So, this becomes A into, so if I will differentiate it thrice. So, then this becomes, sin beta l A into sin hyperbolic beta l A into sin hyperbolic beta l then this term, when I am differentiating it twice then cos becomes minus sin. So, this becomes, plus sin plus sin then again differentiating this becomes plus cos and differentiating it becomes minus cos. So, this becomes minus cos beta l.

So, this cos beta x cos beta x cos beta x; differentiating it, once it becomes, minus sin, so minus minus cos, so plus sin beta x again it differentiating. So, this becomes, minus cos beta x and for minus cos beta x. So, we have to differentiate it thrice, So, by differentiating it thrice we get. So, this as sin beta l, so this become sin beta l. So, beta, so as we are differentiating twice, thrice. So, this becomes, beta cube, so beta cube a into sin hyperbolic beta l minus sin beta l similarly, by differentiating these. So, thrice we will get beta cube or beta cube I have taken common. So, I can write this plus by differentiating this thrice I can write this as. So, this will becomes this thing I have to differentiate it thrice. So, by differentiating this becomes, cos hyperbolic beta l cos hyperbolic beta l plus cos beta l. So, now, this is equal to 0, so from this expression let this expression is a and this is b. So, from a and b I can write A by B equal to.

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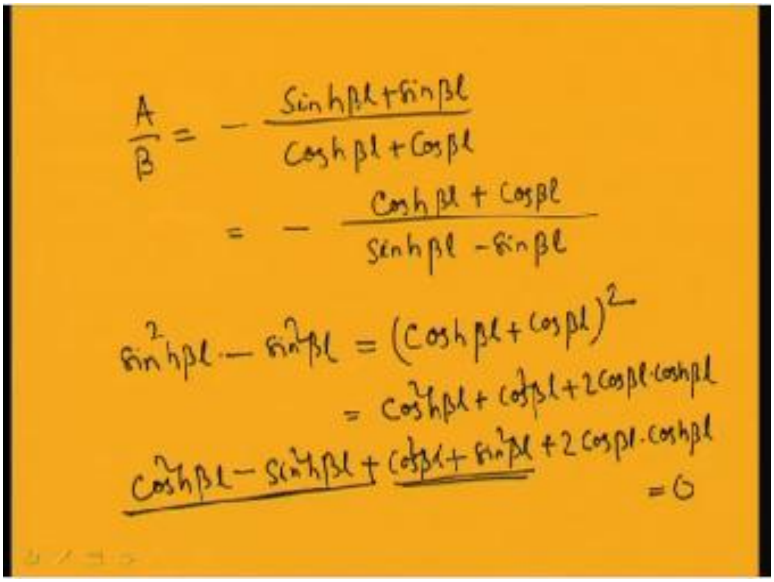

So, I can write from the first equation a A by B equal to minus, so minus sin hyperbolic beta l plus sin beta l by cos hyperbolic beta l plus cos beta l. And from the other equation I can write this, equal to minus cos hyperbolic beta l plus cos beta l by sin hyperbolic beta l minus sin beta l. So, I can write these 2 expression in this form, so A by B I can write. So, A by B in this case sin hyperbolic beta l plus sin beta l by cos hyperbolic beta l plus cos beta l and in this case it becomes cos hyperbolic beta l plus cos beta l by sin hyperbolic beta l minus sin beta l. So, by writing this, so as they are equal, so I can find the frequency equation from this. So, by cross multiplying this equation, I can write this sin hyperbolic beta l. So, these multiply this, this is sin hyperbolic beta l plus sin beta l and this is sin sin hyperbolic beta l minus sin beta l.

So, product of these 2 gives, sin square hyperbolic sin hyperbolic sin square hyperbolic beta l minus sin square beta l equal to this cos beta l plus cos hyperbolic beta l. So, this is also cos hyperbolic beta l plus cos beta l, so this becomes, cos hyperbolic beta l plus cos beta l whole square. So, now, expanding this thing you can write this becomes, cos square hyperbolic beta l plus cos square beta l plus 2 cos beta l into cos hyperbolic beta l. So, rearranging this thing I can write this cos square hyperbolic beta l. So, this thing I will take right sides, so minus sin square hyperbolic beta l plus cos square beta l plus sin square beta l minus 2 cos beta l this is plus. So, plus 2 cos beta l into cos hyperbolic beta l, so this becomes equal to 0. So, you know this cos square hyperbolic beta l minus sin square hyperbolic beta l equal to 1 similarly, cos square beta l plus sin square beta l also equal to 1. So, this 1 plus 1 this becomes 2. So, 2 plus. So, we have 2 plus.

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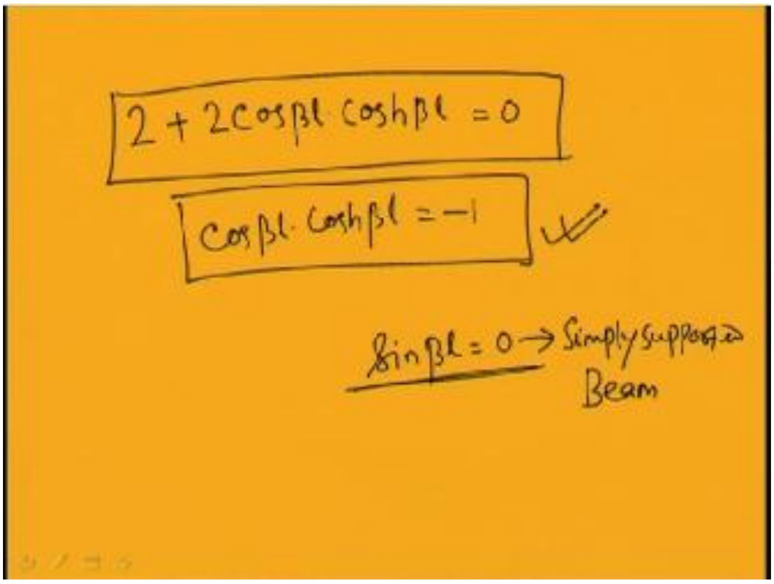

So, I can write this expression as 2 plus 2 cos beta l into cos hyperbolic beta l equal to 0. Or, so this is my frequency equation or by simplifying this thing also, I can write the frequency equation as cos beta l into cos hyperbolic beta l equal to minus 1. So, this is the frequency equation, one can obtain in case of the cantilever beam. So, if you recall you have got obtained the frequency equation sin beta l equal to 0, so you got these expression for a, so this expression for simply supported beam.

So, in case of simply supported beam, so you got the expression sin beta l equal to 0. So, this leads to this equation is very simply and you have obtained this beta l equal to or sin beta l equal to 0 equal to sin n pi. So, beta l equal to n pi by l, but in this case, you can note that this equation is not. So, simple, so you have to solve this equation numerically, to find the value of beta l. So, you have to solve this equation numerically to obtain the value of beta l. So, by solving these, you can see that the values are, so the first 5 values of beta l, you can find by solving are first 1.875.

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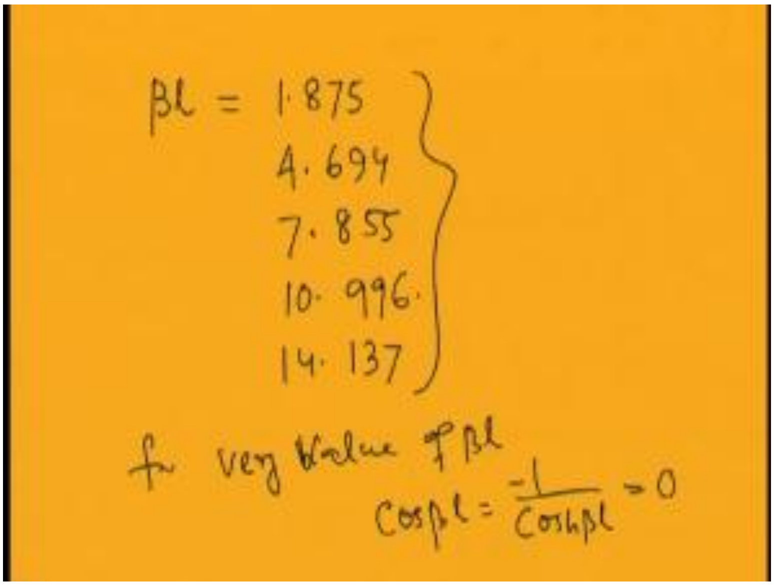

Then 4.694, 7.855 and 10.9, so these values are 10.996, 14.137, and so there are the first 5 values of beta l. So, this 1.875 correspond to the first mode or fundamental frequency 4.694 corresponding to the second mode. And 7.8 5 5 correspond to the third mode 10.996, it is the fourth mode and 14.137 is the fifth mode. From these, you can note that for hire mode this beta l value will be very high and this cos hyperbolic beta l will have a very large value. So, if this cos hyperbolic beta l value is very large, then you can write this cos beta l cos beta l equal to 0. So, for high value of for very high value of beta l value of beta l; that means, you just take beta l greater than the fifth mode. So, in that case, you can write this cos hyperbolic beta l is very high. So, you can write cos beta l equal to 1 by cos or minus 1 by cos are minus 1 by cos hyperbolic beta l. So, as this is very high, so this tends to 0.

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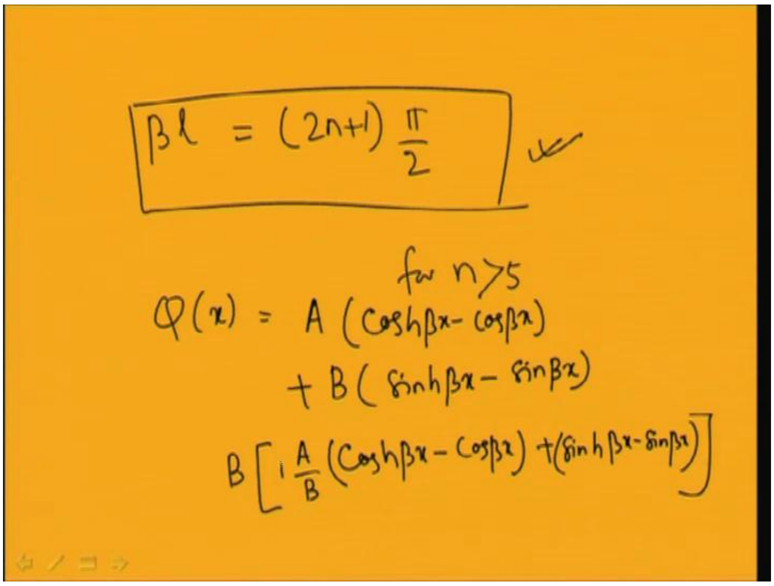

So, cos beta l tends to 0 or beta l. You can write beta l equal to 2 n plus 1 into pi by 2. So, for n greater than 5 for n greater than 5 you may take beta l equal to 2 n plus 1 into pi by 2. So, in this case, as we have obtained before the mode shape, we have to write the um. So, this is the frequency equation beta l into cos hyperbolic beta l equal to minus 1 the frequency equation and we should find the mode shape. So, already, we have found the, or we have written the mode shape in this form.

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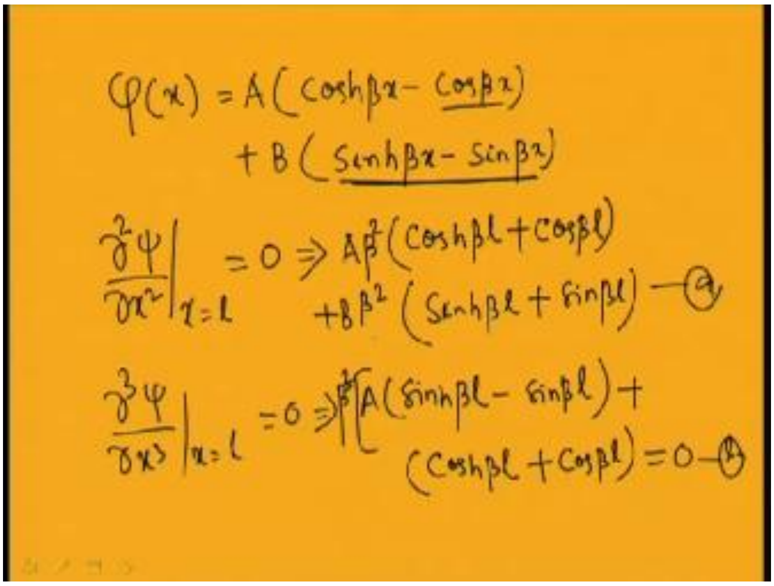

Psi x equal to a into cos hyperbolic beta x minus cos beta x plus b in to sin hyperbolic beta x minus sin beta x. And we have found the expression for A by B from these 2 boundary conditions.

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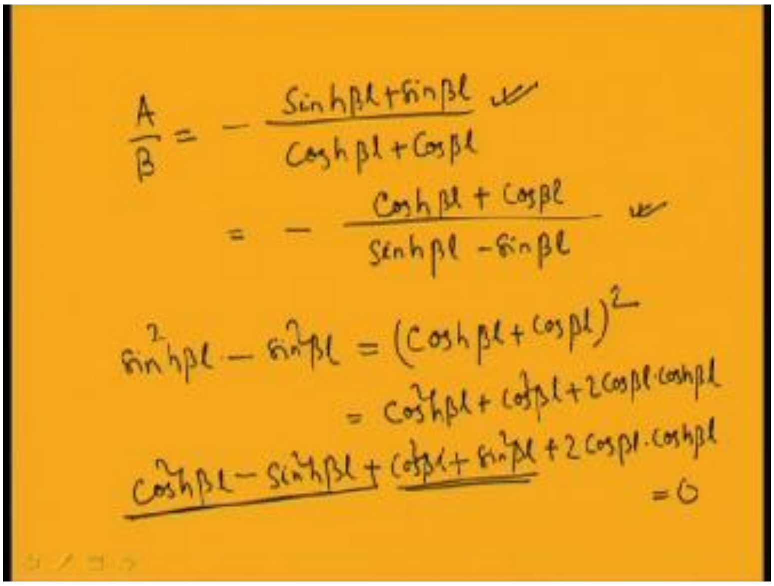

So, either we can write A by B equal to this or A by B equal to this term. So, by writing that, we can write the expression for. So, we can write the expression for this pi x, so pi x will be, so I can take the common. So, pi x you can write in this form. So, phi x equal to a into cos beta x minus cos cos hyperbolic beta x minus cos beta x A into cos. Hyperbolic beta x minus cos beta x plus B into sin hyperbolic beta x minus sin beta x and you can take this B common.

So, if you take B common, so you can write this is equal to, so this phi x equal to B into, so this becomes A by B, A by B into cos hyperbolic beta x minus cos beta x plus sin hyperbolic beta x minus sin beta x. So, in this expression, you can substitute the value of A by B from these expression A by B. Already, we have found this is equal to sin hyperbolic beta l plus sin beta l by cos hyperbolic beta l into cos hyperbolic beta l plus cos beta l. Either you can substitute A by B equal to this or this expression and for different modes. So, that is for first mode. You can have the value of beta l equal to 1.8 7 5 and you can obtain the response. So, the mode shapes are plotted.

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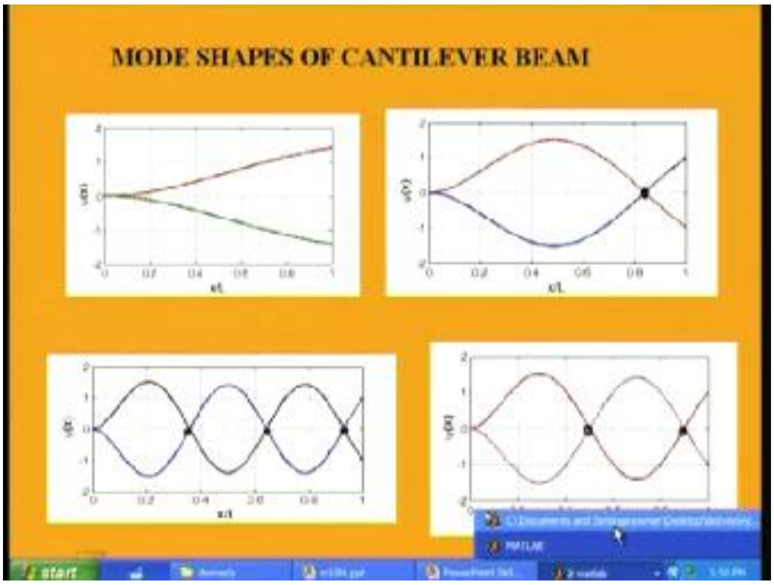

So, you can see the mode shapes by plotting the mode shape, you can see these are the mode shapes for 5 4 different modes. So, in the first, you can see the beam is vibrating. So, the cantilever beam is vibrating either upward are downward in the second mode, it will form a node at this position. And in the third mode, you can see the node are formed at 2 positions and in the fourth mode, you can find there are 3 nodes present in the system. So, in the first mode, there is no mode and the cantilever beam is moving upward or downward. In the second case, it will have 1 node in the third case it will have 2 node and in the fourth case or in the fourth mode it as 3 nodes.

So, if this is a cantilever beam. So, in this case, this beam is fixed at this side. So, in the first mode it will move either upward or downward and in the second mode a node will form at this position at near nearly, about 0.8 1. And in case of third mode or in case of third mode you will have 2 position at which nodes will be formed and in case of fourth mode, they are 3 position in which nodes will form. So, these nodes are nothing, but the points at which there is no displacements of the beam. So, let us see the simulation of this are the phi 5 different modes for this cantilever beam. So, a code as written in matlab using this and you can see the simulation of different modes in this case.

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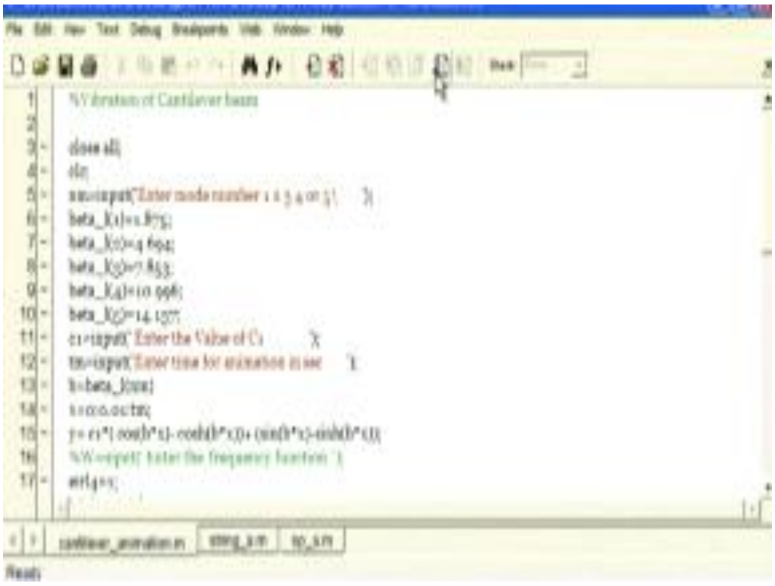

So, by using this matlab we can, so this is the simple code, written for this cantilever beam and now, by running for different modes we can see the mode shapes.

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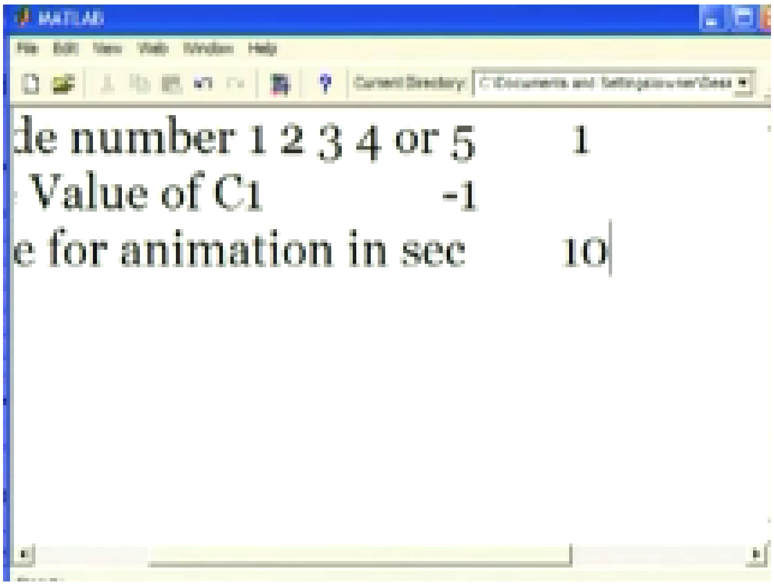

So, let us see the first mode, so for the first mode. So, I can a enter the value of the C 1 and let us see the animation for 10 second, so for 10 second if we want to see.

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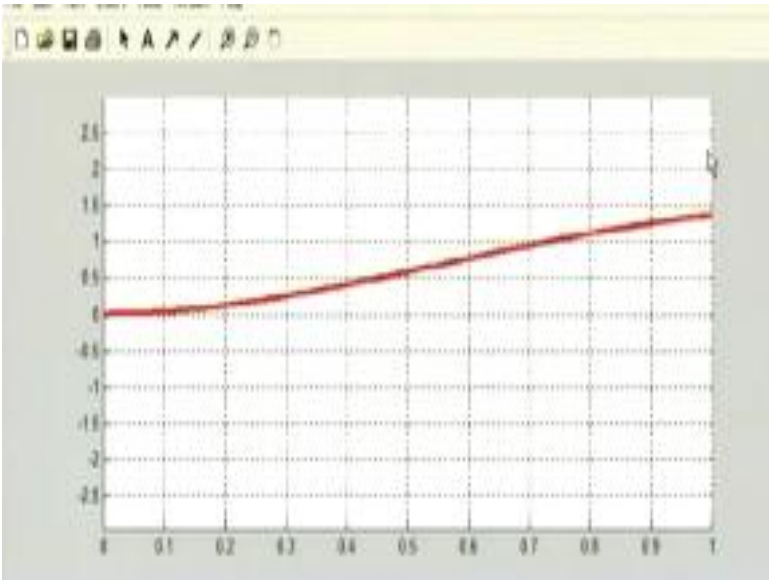

So, you can see the beam is vibrating or the cantilever beam, if moving up and down in the first mode. So, in the first mode, you have seen the frequency equal to 1 point beta l equal to 1.875.

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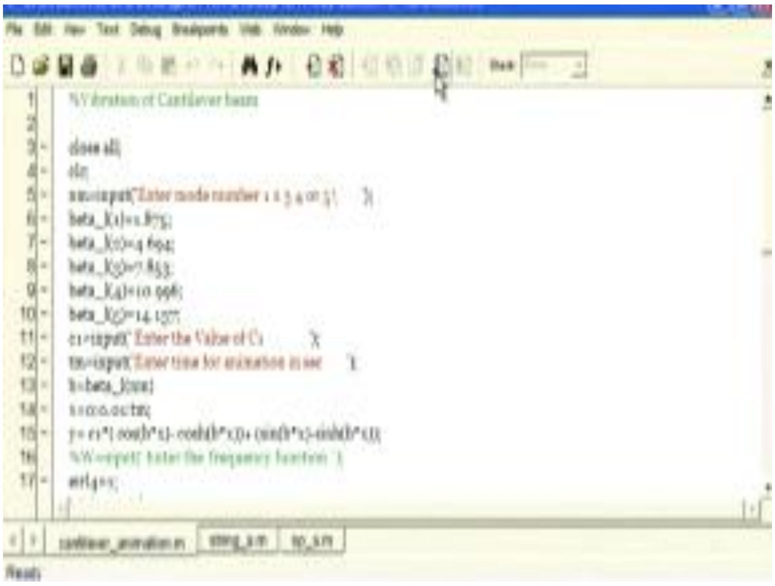

So, let us see for the second mode, so in the second mode. So, we can learn it for the second mode, so in this case we can find.

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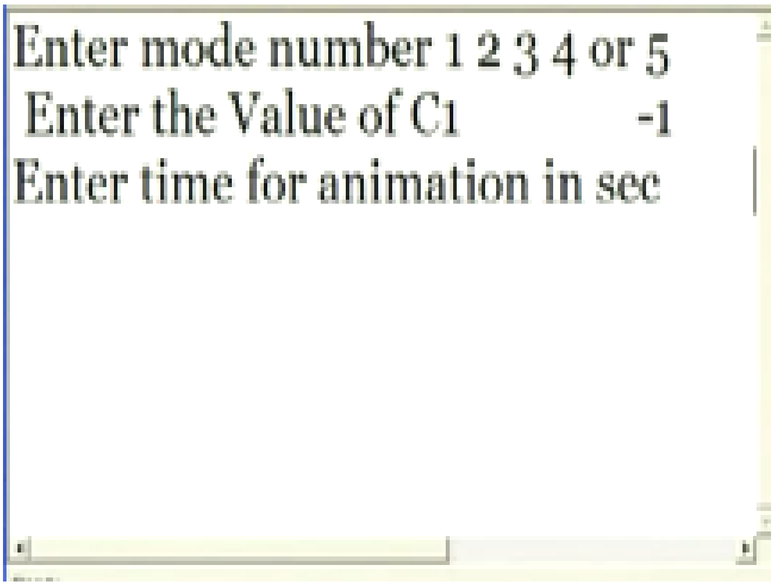

Let us, write the mode number equal to 2. So, if the mode number equal to 2 then, let us simulate it for 10 second, let us see for 10 second.

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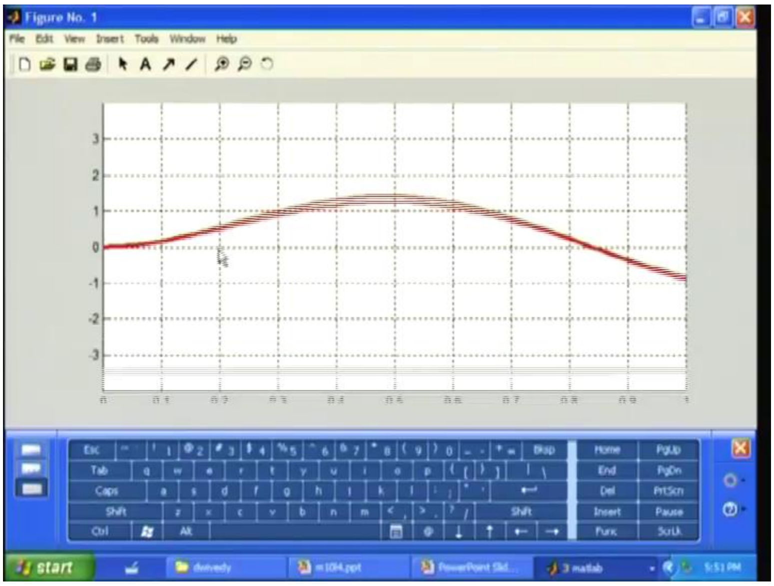

The vibration of the beam, so in this case clearly, you can see that there are 2 points at in case of second mode clearly; you just see this is the point at which there is no vibration of the beam. And also you just note that at the beginning that is at the fixed end the slope is 0.

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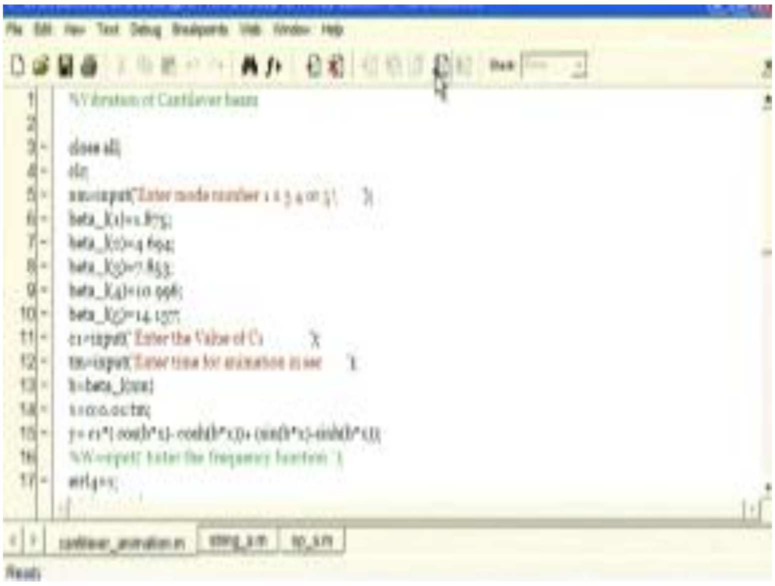

So, let us see for the third mode. So, in case of the third mode.

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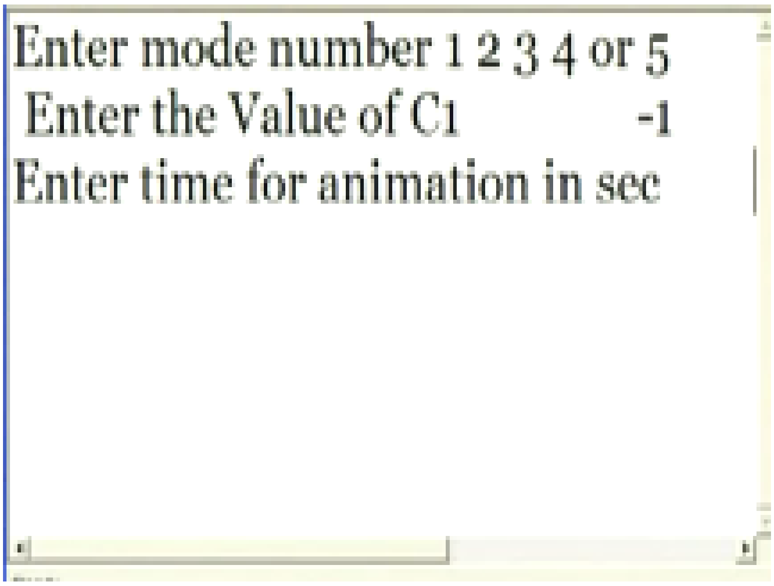

So, you can we will be able to see that there are two different nodes, present in this case. So, in case of third, so let us simulate it for 10 second.

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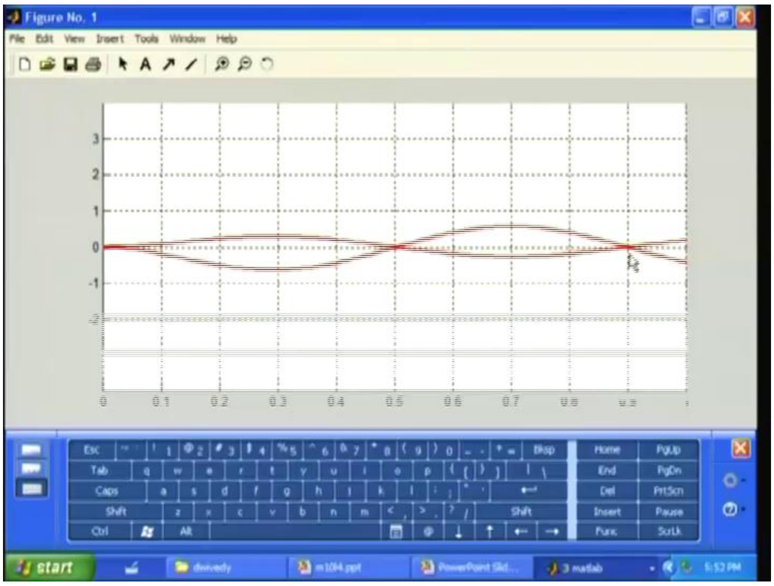

And see, so we just note that there are 2 points at which there is no vibration that is at 0. 5 and at 0.9 there is no vibration. And at 0 at 0 slope the slope is equal to 0, so we will see for the fourth mode let us see for the fourth mode.

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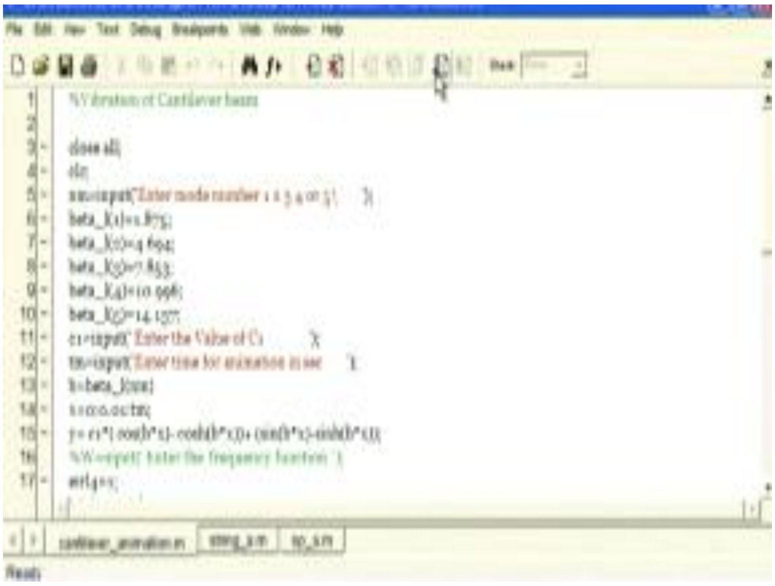

So, in this case in case of fourth mode this is 4 mode.

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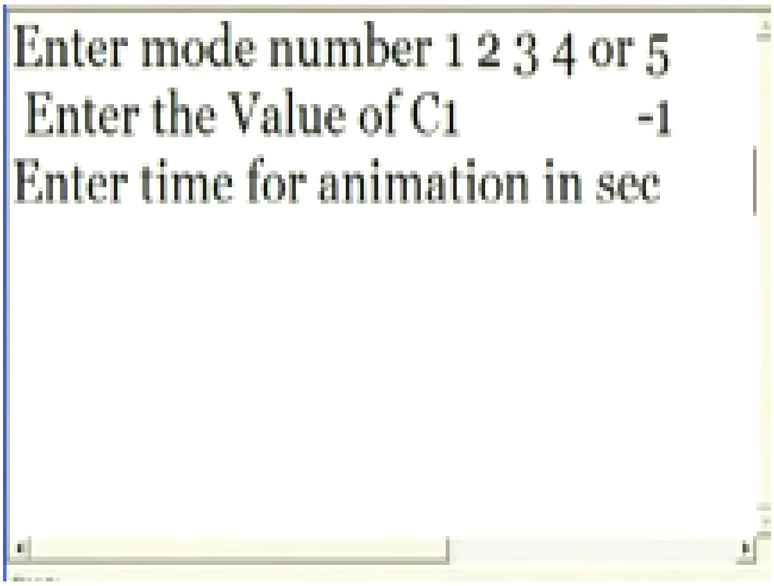

Number I written it is equal to 4 and let us, see this animation for twenty second, and so we can see this animation. So, let us, this let us see this animation for the fourth mode, so n number of mode equal to 4 minus 1 let me put see 1 equal to minus1. This constant term, I am putting it minus 1 and let the time for animation I am putting ten second and this is.

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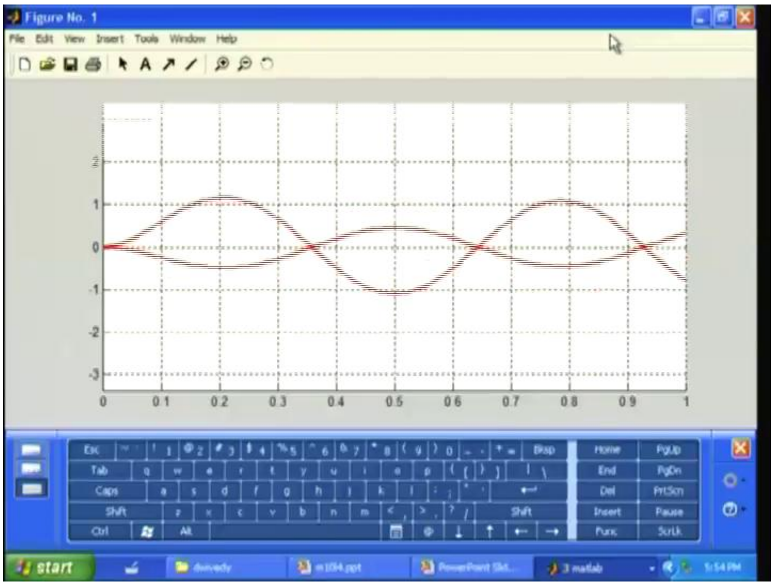

So, you can see that there are 3 distinct point at which there is no vibration of the system and at the beginning the slope is equal to 0. So, the beam will vibrate in fourth mode like this.

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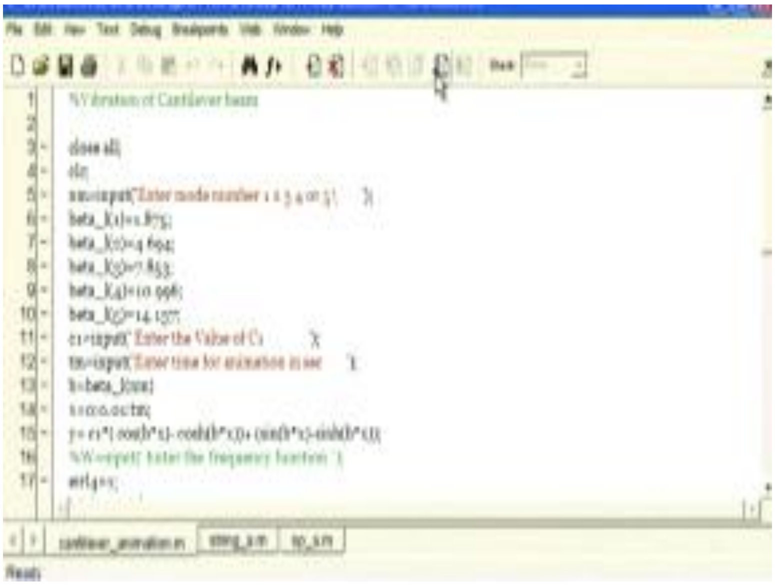

So, if we want to see for the fifth mode. So, let us see for the fifth mode.

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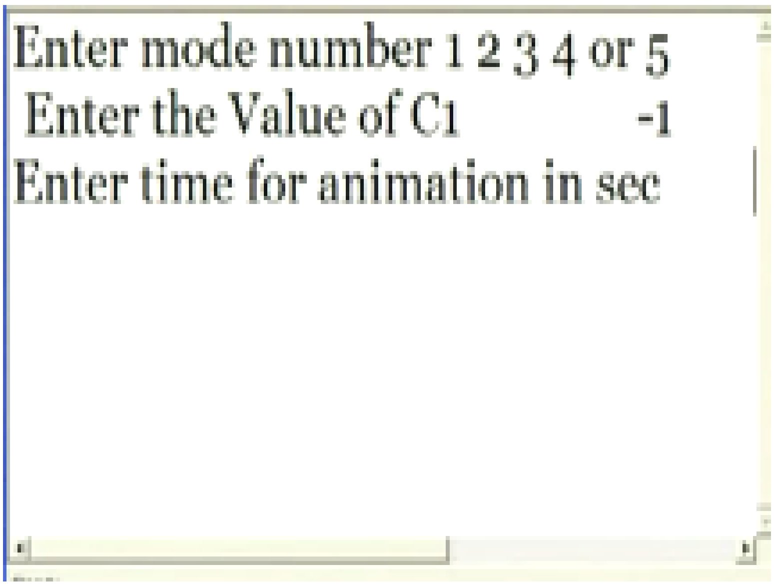

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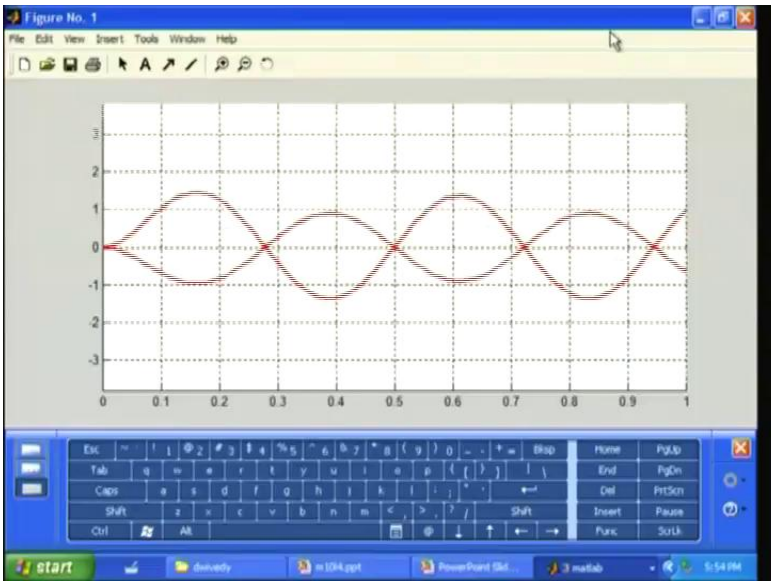

So, in case of… So, you can clearly, observe that there are 4 points at which there is no vibration of the system. So, the resulting vibration, so these are the normal modes present in a system, but. So, this, so if we are if want to find the normal mode of the, so these are the, these mode shape correspond to the normal mode of the system. So, in this normal modes.

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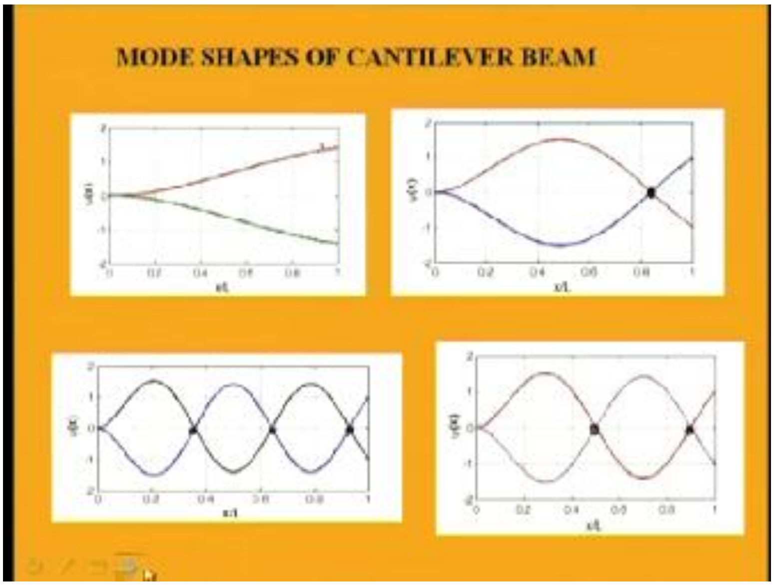

So, you have seen for the first mode, the beam is moving either upward or downward in the second case, that is in the second mode. You have only one node present in the system and in case of third mode, there are 2 nodes and in case of fourth mode there are 3 nodes present in this case. So, the resulting vibration, resulting free vibration, if you want to find, so this as this mode shape correspond to the normal mode frequency are normal vibration. So, the free vibration will be a mixture of all these normal modes. So, similar to the discrete system here, also this normal mode concept can be applied.

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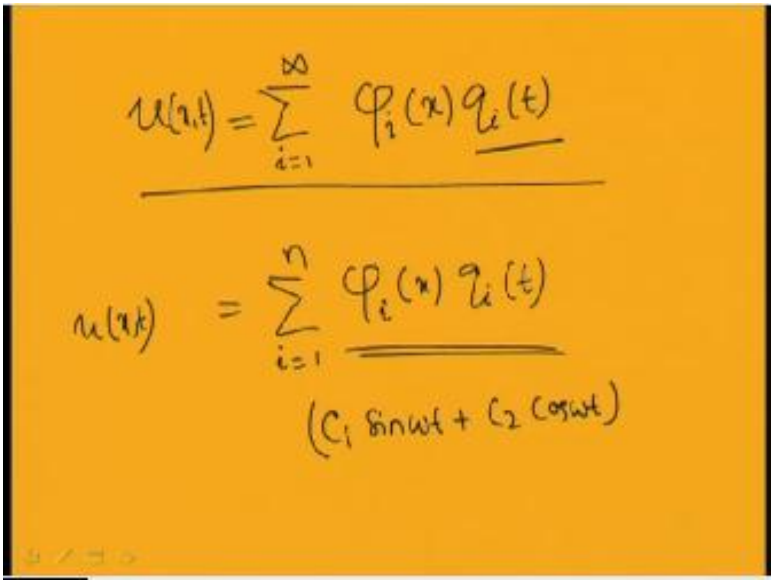

So, here also we can write the free vibration response of the system that is u. Will be e will be summation of different modes. So, that is phi I x into qi t, so where I equal to 1 to infinity the resulting free vibration of the system at any time that is xt can be written similar to these discrete system. We can write in this case, it is summation of the different modes. So, where phi I represent the ith mode of the ith mode shape and qi t represent the ith time modulation. So, the resulting free vibration is the summation of all the model vibrations are model mode shapes all the different mode shapes and time modulations. But when you know that the system is exited at a particular frequency, then we may truncate this equation and we can write instead of infinity we can truncate it 2 a finite number of modes.

So, that case, we may take say 2 or 3 or depending on the problems, we may take n. So, this equation, we can write in this form I equal to 1 to n psi I x and qi t. So, depending on the frequency of excitation, so we can truncate this system. So, this we can truncate this and we can write this expression u xt that is the resulting free vibration of the system as psi ix into qi t. So, in case of this cantilever, so already, we have found this phi x and qi to also, we know that can be written in this form, C 1 sin omega t plus C 2 cos omega t and we have obtained the expression for phi ix. So, u xt we can write or the product of this phi ix and qi t. So, let us find the mode shape.

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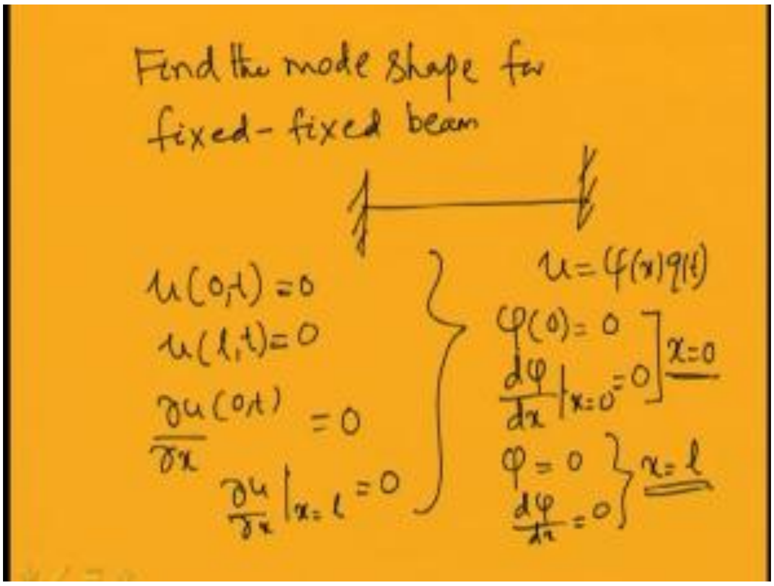

So, let us take the second example, and find the mode shape for find the mode shape for the fixed-fixed beam or fixed fixed or sometimes it is known as clamped clamped beam also, so in case of fixed-fixed beam. So, the beam is fixed at both the ends. So, if I will take this as the beam. So, this beam should be fixed at both these ends, so in that case if the beam is fixed at both the ends. So, that time there will be no displacement at both the ends and also there will be no slope at both the ends. Already, you have seen this simulation for this cantilever beam. So, in case of cantilever beam you have seen that at the fixed end there is no vibration up to certain distance; that means, the slope is 0 up to certain distance.

So, in case of fixed fixed beam at both ends the displacement and slopes will be equal to 0, so by taking the boundary condition as you 0 t equal to 0 and u lt equal to 0. Similarly, so these are the displacement boundary conditions at both the ends. Similarly, the slope boundary condition is del u 0 t del u by del x at 0 t equal to 0 and del u by del x at x equal to l equal to 0. So, these are the 4 boundary conditions, by taking these 4 boundary conditions and substituting this u as already, we now, we are writing u equal to psi x into qt. So, we can write that, so as it is present for all times at this boundaries these terms becomes, 0. So, we can write this psi 0 equal to 0 and d psi by dx d psi by dx equal to 0 d psi by dx at x equal to 0 equal to 0. So, the, this is at the left end that is at x equal to 0. Similarly, at the right end psi equal to 0 and d psi by dx equal to 0 at x equal to l. So, the generalized or the general mode shape expression already we know.

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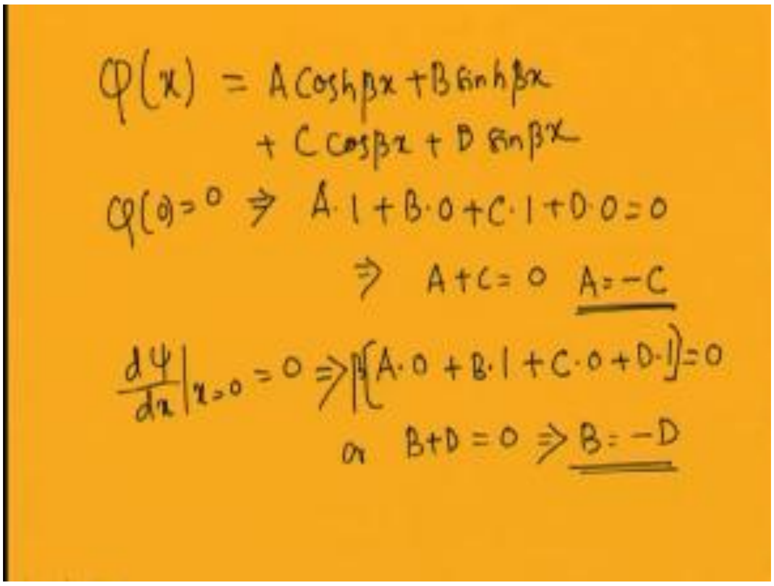

So, that is can be written as psi x equal to A cos hyperbolic beta x plus B sin hyperbolic beta x plus C cos beta x plus D sin beta x. So, now, by applying these boundary conditions, that is at psi 0 equal to 0. So, psi 0 equal to 0 implies that, so in this case I can write this A into. So, cos hyperbolic 0 equal to 1 similarly, plus B into 0 plus C into 1 plus D into 0 equal to 0, so this implies, A plus C equal to 0, so as a plus c equal to 0 I can write A equal to minus C. Similarly, d psi by d psi dx at x equal to 0 equal to 0. So, I can differentiate, it once and I can write this is equal to 0.

So, if I will differentiate. So, this implies A into, so beta will be there So, A into, so this becomes sin. So, this multiplied by 0 plus B B into sin hyperbolic this becomes, cos hyperbolic. So, this becomes, 1 and then plus C into C into this becomes, cos beta x becomes, sin beta x. So, sin beta x becomes, 0 plus d into sin differentiation becomes, cos, so in that case this becomes 1. So, this equal to 0 or from this, I can write this B plus D equal to 0. So, as B plus D equal to 0, so B equal to. So, implies B equal to minus D.

So, I can write B equal to minus D and A equal to minus C in this case. So, now, substituting these 2 expression here, I can write this phi x equal to.

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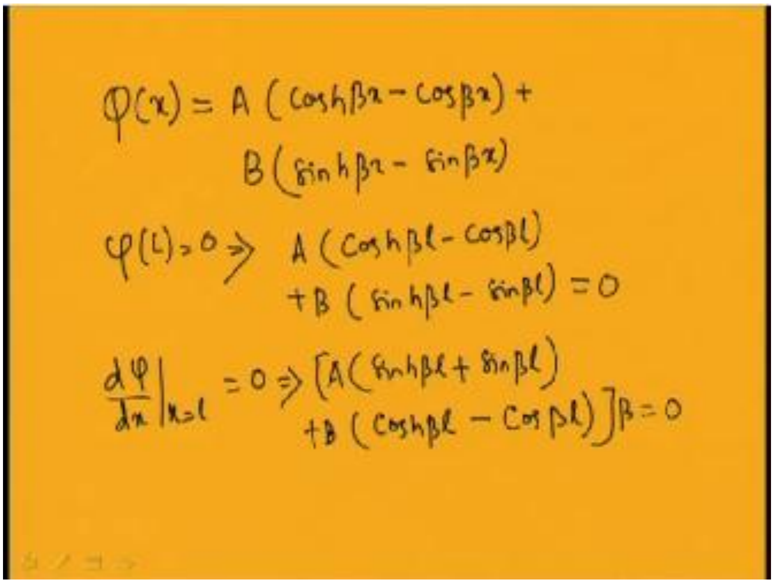

So, phi x can be return as A cos hyperbolic beta x minus cos beta x plus B into. So, plus B into sin hyperbolic beta x minus sin beta x, so this is the expression for phi x after substituting the boundary condition, in the left side. And now, substituting the boundary condition at the right side that is psi l equal to 0, so this implies, so I can substitute this x equal to l. So, this becomes, A into cos hyperbolic beta l minus cos beta l plus B into sin hyperbolic beta l minus sin beta l. So, this equal to 0 similarly, taking d psi by dx at x equal to l equal to 0. So, this is the slope boundary condition, so in case, of slope boundary conditions, I can differentiate it once and I can write this equal to 0. So, this becomes, so this implies A into, so sin hyperbolic beta l.

So, minus cos beta x, so differentiating this, this becomes, plus sin beta l plus B into for sin hyperbolic this becomes, cos hyperbolic beta l and sin this becomes, minus cos beta l. So, this whole multiplied by beta equal to 0 as we have differentiated it once. So, it is multiplied by beta, so this becomes, equal to 0, so like the previous case, here also we can write A by B from this expression and also A by B from this expression and we can equate these 2. So, by writing A by B I can write this, A by B will be equal to minus sin hyperbolic beta l minus sin beta l by cos hyperbolic beta l minus cos beta l. Similarly, for this expression, A by B becomes, minus cos hyperbolic beta l minus cos beta l by sin hyperbolic beta l plus sin beta l.

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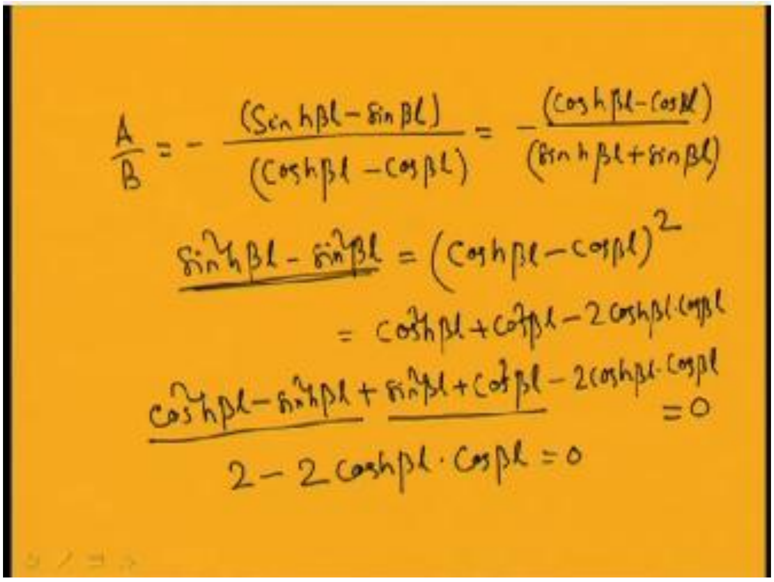

So, I can write this, as A by B equal to minus sin hyperbolic beta l minus sin beta l by cos hyperbolic beta l minus cos beta l. So, this will becomes, minus cos hyperbolic beta l minus cos beta l by sin hyperbolic beta l plus sin beta l. So, proceeding in thus previous way, so as this equal to this I can make the cross multiplication; that means, this term into this term. So, this will be equal to this term into this term, so now, by multiplying this I can write. So, this sin hyperbolic beta l minus sin beta l minus sin beta l this is sin hyperbolic beta l plus sin beta l. So, this becomes, sin square hyperbolic beta l minus sin square beta l.

So, this will becomes, cos hyperbolic beta l minus cos beta l whole square or this is equal to cos square hyperbolic beta l plus cos square beta l minus 2 cos hyperbolic beta l into cos beta l. So, by taking this to the right side, so I can write this becomes, cos square hyperbolic beta l minus sin square hyperbolic beta l plus sin square beta l plus cos square beta l minus 2 cos hyperbolic beta l into cos beta l equal to 0. So, you know already, this cos square hyperbolic cos square hyperbolic beta l minus the sin square hyperbolic beta l equal to 1. Similarly, this sin square beta l plus cos square beta l equal to 1. So, I can write this becomes, 2 minus 2 cos hyperbolic beta l into cos beta l equal to 0.

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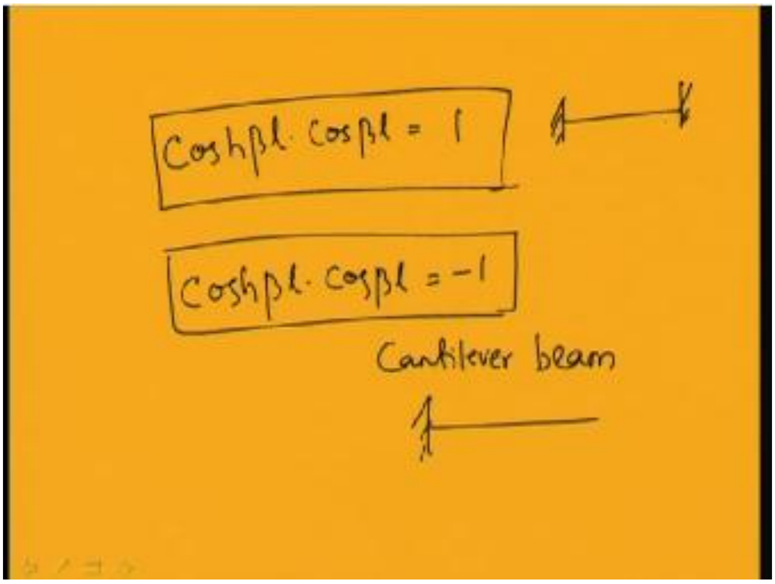

So, I can write this cos hyperbolic beta l into cos beta l this becomes, 1. So, this is the frequency equation, in case of this fixed fixed beam. So, if you recall for the case of cantilever beam, this cos hyperbolic beta l into cos beta l was minus 1. So, in case of cos beta l into cos, so this becomes, minus 1 in case of cantilever beam. So, in case of cantilever beam, that is when 1 side is fixed. So, this expression, was minus 1 and in case of fixed, fixed beam the expression is cos hyperbolic beta l into cos beta l equal to 1. So, in this case, you can note that you cannot get the expression for beta l easily. So, in that case also you have to solve it numerically, to obtain the solution of this beta l. So, if you solve this expression, so you can find the first mode first frequency 5 mode frequencies as. So, first mode frequency, we can find this lambda l equal to 0.

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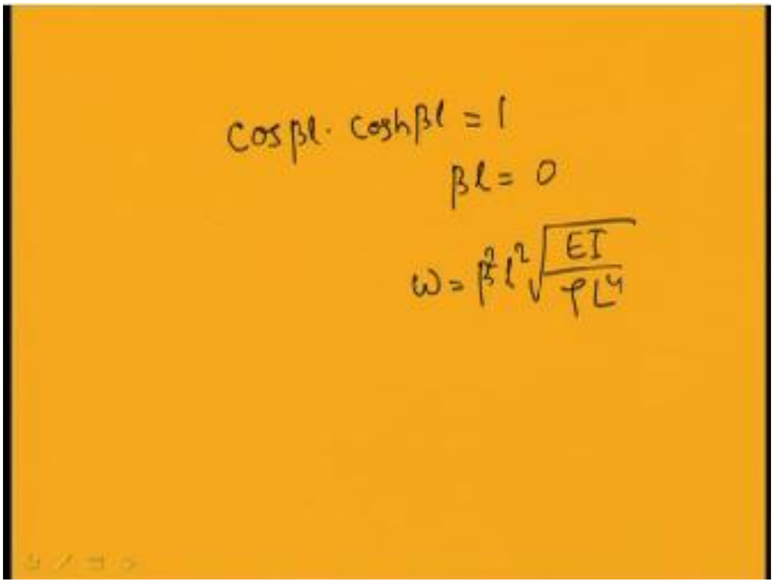

So, just see this expression. Cos lambda l into cos beta l into cos hyperbolic beta l cos beta l into cos hyperbolic beta l equal to 0. So, when beta l equal to 0 you can see that. So, this is equal to one. So, when beta l equal to 0 you can note that this expression is valid. So, beta l equal to 0 beta l equal to 0 correspond to the rigid body motion as we already know this omega formula for omega. So, omega equal to beta square l square root over EI by rho one fourth. So, when beta l equal to 0. So, omega equal to 0 so; that means, there is no frequency of fixed vibration. So, as the frequency of vibration is 0, so in this case the body will have a rigid body motion.

So, the rigid body motion means, so the, so the body or the beam will fixed at both the end it will move up and down. So, in this case, this is the rigid body motion of the system. So, there is no vibration of the beam actions for the beam, will move up and down or will have a rigid body motion all the points in this body will have the same displacement. So, so in this rigid body, so when beta l equal to 0 you have the rigid body motion and so let us see the other motions. So, by solving this expression you can see that you can get this beta l value.

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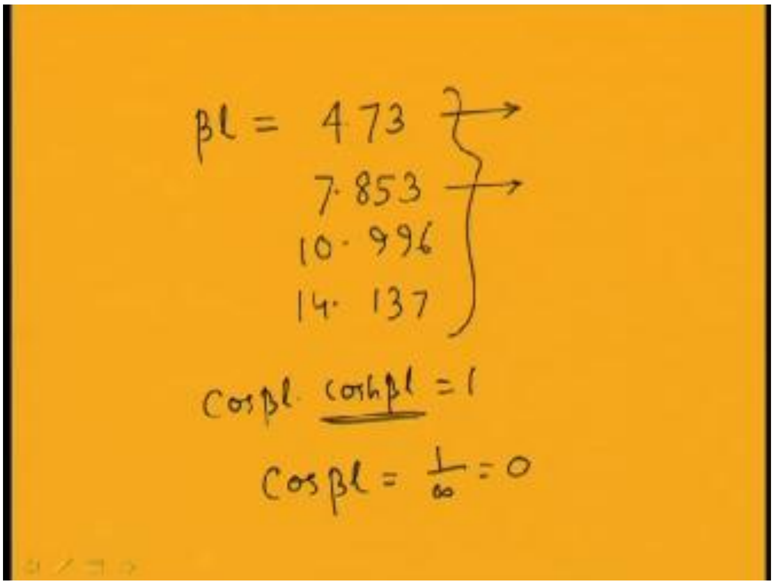

So, beta l equal to 4.73, 4.73 then 7.853, 10.996, 14.137. So, these are the first 4 mode frequencies. So, by solving that equation numerically, you can find the expression for this beta l. So, here this 4.73 correspond to the first or fundamental frequencies 7.853 is the second mode frequency, 10.996 is the third mode frequency. And this 14.137 is the fourth mode frequency. So, in this case if you, so for higher modes, so similar to the previous case also, in this case cos beta l into cos beta l into cos hyperbolic beta l equal to 1. So, as it is equal to 1 for beta l for higher value of beta l this cos hyperbolic beta l tends to infinity.

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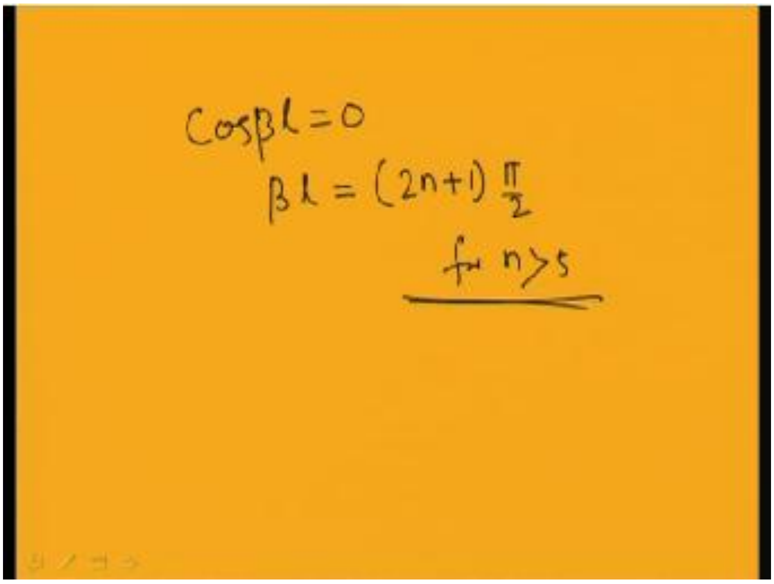

So, this cos beta l cos beta l equal to 1 by infinity, so this becomes, 0, so as cos beta l equal to 0. Cos beta l equal to 0, so you can find beta l. So, beta l it is similar to that in case of cantilever beam. So, this is equal to 2 n plus 1 pi by 2 for n greater than 5. So, for n greater than 5, you can take this beta l value equal to 2 n plus 1 in to pi by 2. So, in this case, you can see that for higher modes the cantilever beam and this fixed-fixed beam at the same frequency. So, if you want to plot the mode shape of this case, so you can see that the mode shapes will be like this, so for the first mode.

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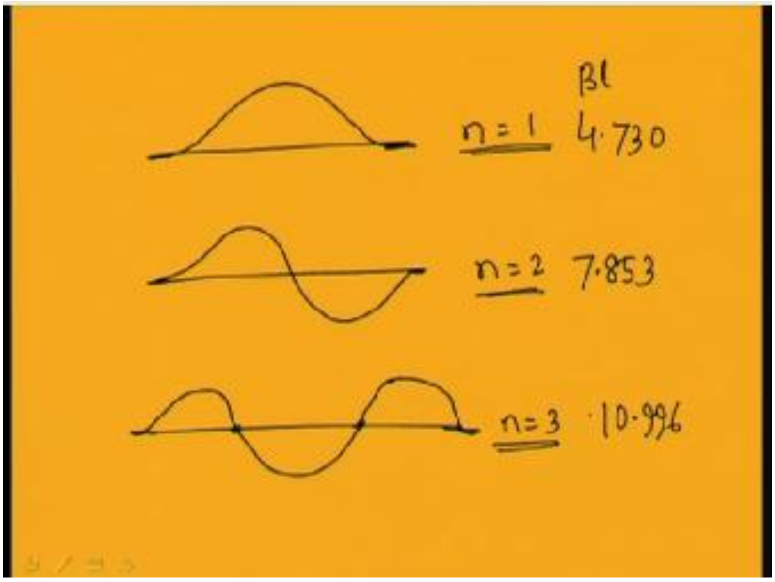

The beam will vibrate like this, so at both the ends it will we have 0 displacement. So, this is for the first mode and in case of the second mode you will have. So, in the second mode, you will have 1 node and in case of the third. So, you will have 2 nodes present in the system. So, you will have 2 nodes in the beginning, there will be no slope and at the end also, you will have 0 slope in the beginning and end you will have 0 slope. So, in this case, you have 2 nodes.

So, n correspond to this 1 this n equal to 2 and this n equal to 3. So, n equal to 1 correspond to that is 4 point beta l equal to fourth point beta l value equal to 4.730 and in this case this correspond to 7.853 and in the third case it becomes, 10.0 beta l equal to 10.996. So, you can also similarly, plot the fourth mode and fifth mode and you can simulate also to see the vibration of the fixed-fixed beam. So, as both ends are fixed, so can, note that at this to ends the displacements are 0. So, the slope also are 0 in this 2 cases in this case the displacement and slopes are 0. So, let us now, see another boundary condition.

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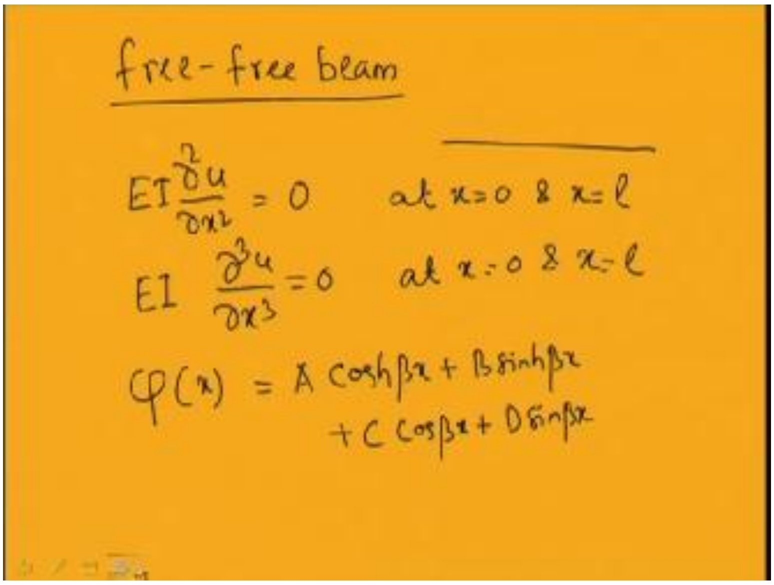

So, that is free-free beam. So, in case, of free-free beam, so already, I told you the examples, of the free-free beam are the space craft or aeroplanes. So, as they are not supported at the ends, they can be considered as the free-free beams.

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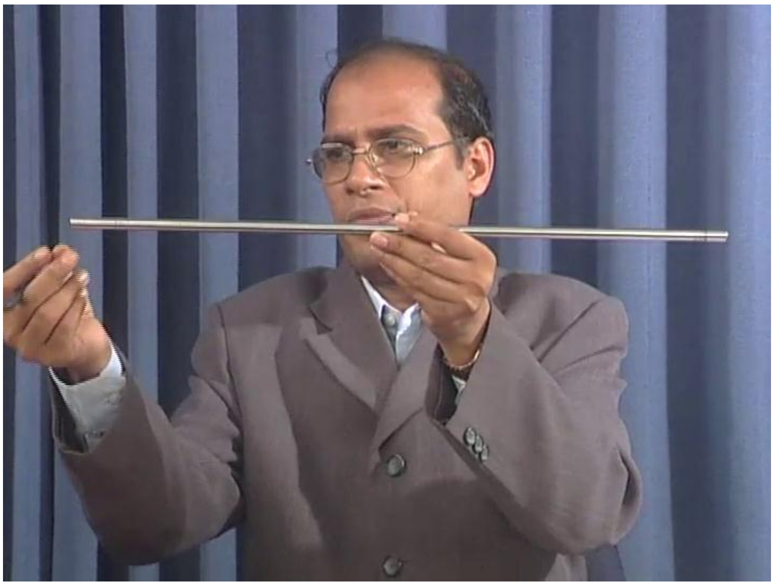

So, at both the ends of this beam, so both the ends are free. So, as both ends are free, so in this case the, so the both ends will be subjected to some deflection or there will be vibration at both the ends. And they are may be slopes also at the end, but the bending moment and shear force at both the ends will be 0. So, by taking this beam or this is a rod I have shown. So, you can take the beam also, so in case of this beam. So, if it is free at both the ends, so this end it is free. So, it may have motion like these in this end, this end also as there is it is free, it may have a motion this end. So, there will be some displacement at this end and at this end. So, the displacement and slopes are not 0 at both the ends, but the bending moment and shear force are 0 at both the ends.

So, if the beam is free-free, so the boundary conditions will be shear force and bending moment at 0. So, for bending moment, I can write this is equal to EI del square u by del x square equal to 0. Similarly, for, so this is bending moment and equal to 0, so shear force equal to 0, so EI del cube u by del x cube equal to 0. So, at x equal to 0 and x equal to l similarly, this is also at x equal to 0 and x equal to l. So, you can substitute this expression, in the original expression for psi x that is equal to A cos hyperbolic beta x plus B sin hyperbolic beta x plus C cos beta x plus D sin beta x. So, by substituting this expression in this by substituting this boundary condition, so you may write by substituting these boundary conditions, you can find the expression for this. So, in this case, you can write as this bending moment and shear force are 0.

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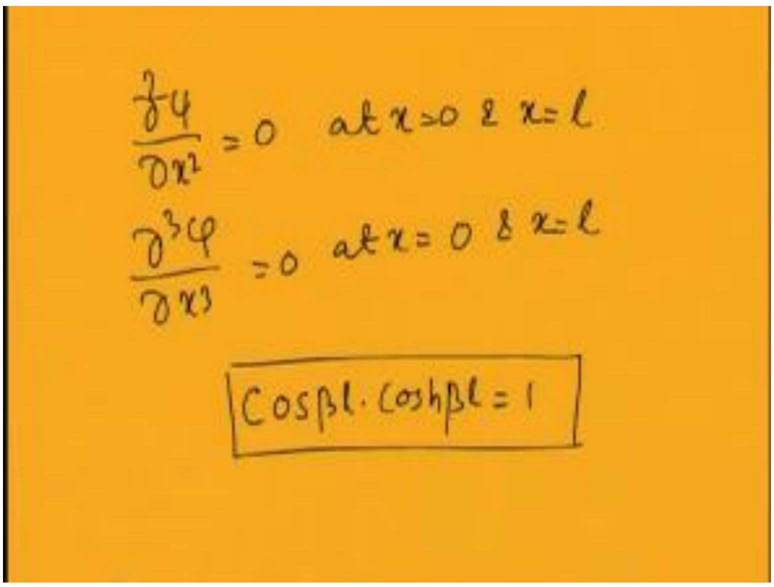

So, I can write del square psi by del x square equal to 0 at x equal to 0 and x equal to l, similarly del cube psi by del x cube equal to 0 at x equal to 0 and x equal to l. So, preceding in the similar way you can see that in this case, you will find the frequency expression. So, the frequency, expression will be same as that of the fixed-fixed beam. So, in this case you can see that the frequency equation will be cos beta l into cos hyperbolic beta l equal to 1. So, the fixed-fixed case and in this case, you can find the frequency equation are same, but the mode shapes, you can see that the mode shapes will be different. So, in this case the mode shapes for the first mode.

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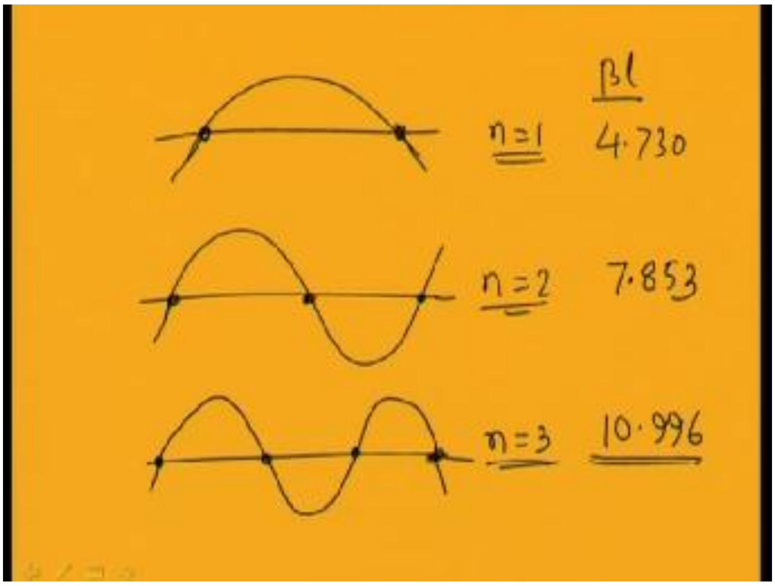

So, the, for the first mode. You will have the displacement like this. So, at both ends, the displacements or slopes are not equal to 0 similarly, this, so this is n equal to 1 that is the first mode. So, in the first mode the frequency that is beta l equal to 4.730 and in the second case or in the second mode you can plot it like this. So, you will have, so you can plot it like this. So, you can see that there are there is 1 node. So, for n equal to, so you have a node here you have a node here and also a node here. So, in the first case, you have 2 nodes. So, at these points on the beam there is no vibration for second mode, you can see that there are 3 points at which there is no vibration.

So, in third mode, also this curve is the second mode correspond to the frequency of 7.853. So, in the next case, you will have 4 points at 4 points there will be no vibration. So, this is the first point second point, so at these 4 points there is no vibration. So, these correspond to. So, this is the third mode, so this correspond to 10 point, so 10.996. So, in this way, you can find different mode of the different types of beam. So, next class, we will study some other different examples, regarding this wave equation and Euler-Bernoulli equation.