Mechanical Vibrations
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Module - 9
Continuous Systems: Closed Form Solutions
Lecture - 3
Vibration of Strings

In the last class, we have studied about the derivation of equation of motion of continuous system by using Hamilton principle, also we have derived the equation by using Newton's second law and D or D Alembert principle. So, we have already derived the equation motion for longitudinal vibration of a rod and lateral vibration of string. We have found the torsional vibration of rod and the transverse vibration of beam.

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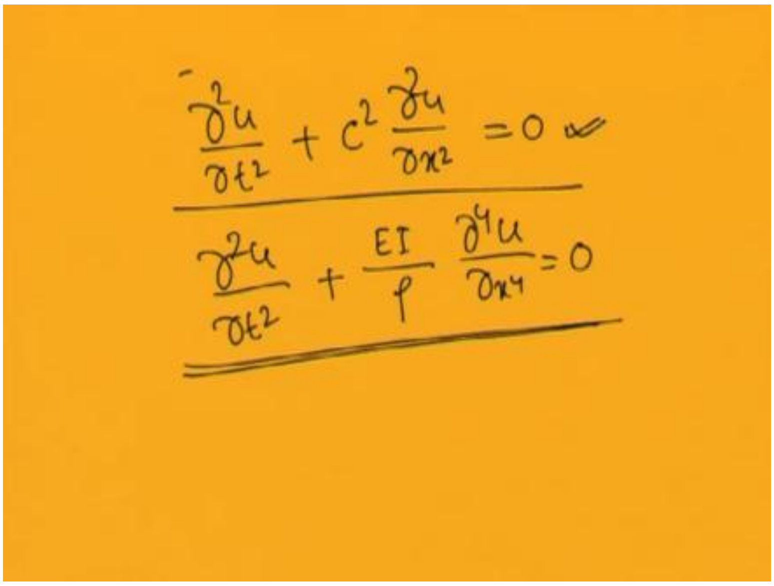

In case of longitudinal vibration of rod or the torsional vibration of beam and for the string, we have found the equation motion, which can be written del square u by del t square plus C square del square u by del x square equal to 0. So, this equation is known as the wave equation and today class, we are going to find the solution or we are going to find the response of the system for the systems, which whose equations are written in terms of the wave equation. Also last class, we have derived the equation motion for the Euler beam. So, in that case the equation is written in this form del square u by del t square plus. So, del square u by del t square plus EI by EI by m or EI by rho EI by rho into del fourth u by del x forth equal to 0. So, in this case this is a forth order equation and in the previous case the equation is in the form of a second order equation. So, today class, we are going to find the response of the system with wave equation.

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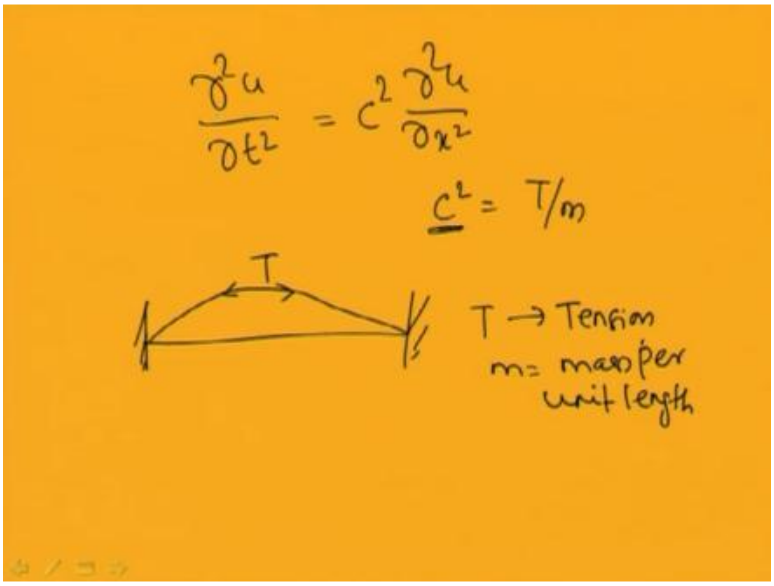

So, in this case the equation also can be written in this form del square u by del t square equal to C square del square u by del x square in the case of string, we have seen that the C square equal to T by m in case of, so where T is the tension. So, this is a string, so in case of the string. So, in this case of string, we have found that the vibration will takes place. So, where the T is the tension and m is the mass of mass per unit length, mass per unit length. So, the C square, you can find or the C the unit of C you can find it is in meter per second or that of velocity. So, later we will see that the C represents the velocity of wave in the string. So, the for the string, we have found already, the equation motion in this form del square u by del t square equal to C square del square u by del x square, also for the longitudinal vibration of rod.

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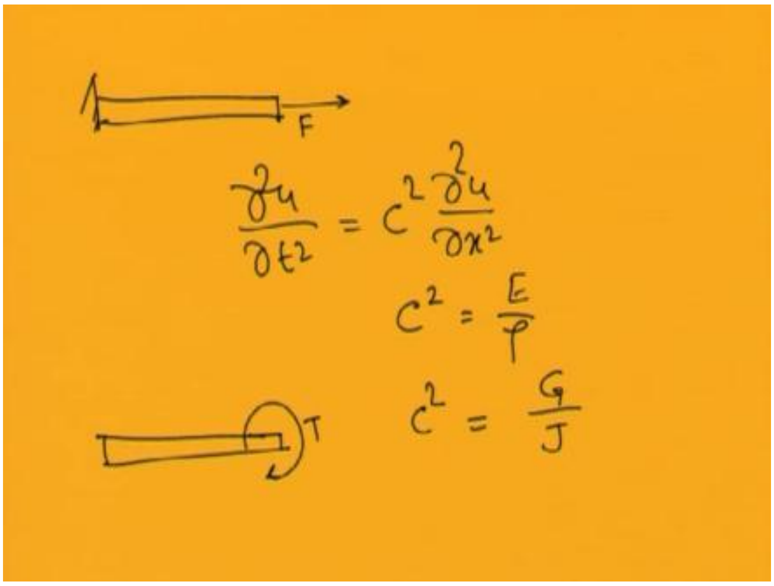

So, in this case, so this is a rod. So, in this case also, we have found the equation in this form, that is del square u by del t square equal to C square del square u by del x square, so here, C equal to or C square equal to. So, this was E by rho or C equal to root over E by rho. Similarly, for torsional vibration of the rod, so for the torsional vibration of rod, we have found, so it is torque. So, the torsional vibration of rod we have found, C square equal to. So, that is equal to G by J. So, in all these cases the equation motion is written in this form.

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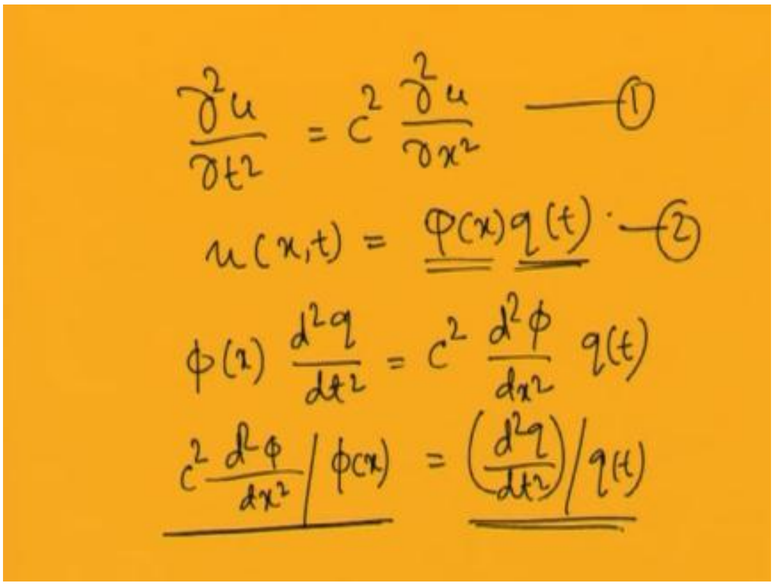

That is del square u by del t square equal to C square del square u by del x square. So, here u is the displacement in case of the longitudinal vibration of the rod or string. And it is the rotation in case of the torsional vibration of the rod, so here u is a function, so u is a function of x and t. So, I can write this u equal to psi x into I can write this is equal to phi x into qt. So, where phi is the function of x and q is the function of t only. So, you can substitute this equation in this wave equation. So, I can find the equation in this form. So, it will becomes del square or phi x into del square q by del t square or del as this is q is a function of t only. So, I can write this is ordinary differential, that is d square q by dt square.

So, this will be equal to C square into C square into del square u, for del square u by del x square, I can write this is will be equal to d square phi by dx square into qt. So, this equation, I can write also in this form. So, phi x or I can write this, d square phi by dx square d square phi by dx square into C square by this phi x will be equal to I can take this q left side. So, I will write, so this will be equal to d square q by dt square by qt. So, you just see the left side is a function of x only, and the right side is a function of t only. So, as they are equal, so they must be equal to a constant. So, this constant let me for the time being let take.

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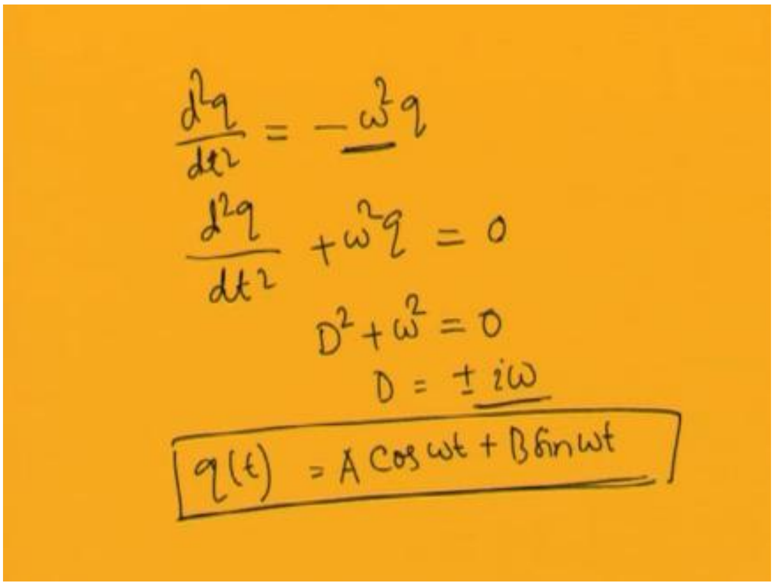

d square q by dt square equal to will be equal to minus omega square q. So, I have taken this negative minus, omega square as the constant. So, this is due to the fact that this d square q can be represented as the acceleration term and q as the displacement term, as the displacement acceleration is proportional to displacement. So, this is the case of a simple harmonic motion and if you are considering stable solution or stable response of the system, then it should be minus omega square. So, if I am taking this as minus omega square, this equation can be written in this form d square q by dt square plus omega square q will be equal to 0. And the auxiliary equation in this case becomes, D square plus omega square equal to 0 or the roots will be D will be equal to plus minus I omega. So, the solution qt will be equal to A cos omega t plus B sin omega t, because as the roots are plus minus e to the power I omega. So, as the roots are plus minus I omega, so the solution can be written in this form.

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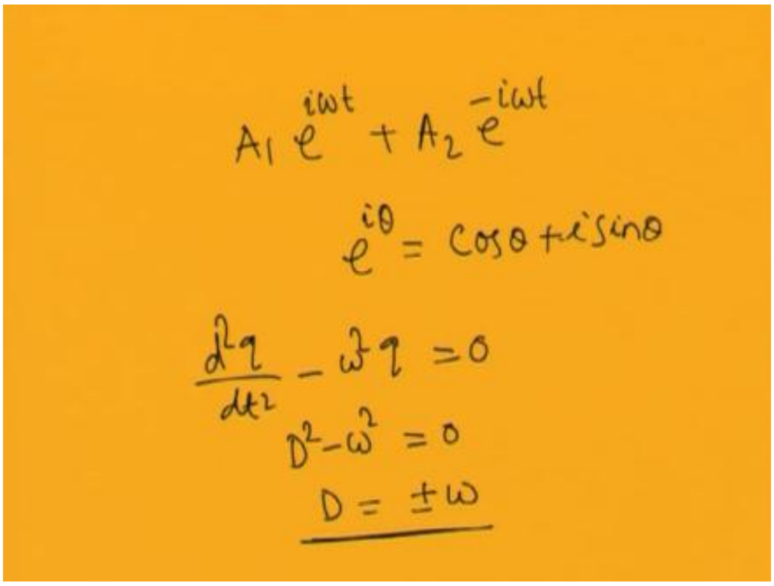

Solution should be written A 1 e to the power I omega t plus A 2 e to the power minus I omega t already, you know e to the power I theta can be written in the form of cos theta plus I sin theta. So, if you substitute this in this equation, the equation can be written in this form. So, qt will be equal to A cos omega t plus B sin omega t, so instead of taking this negative omega square, if someone takes this is positive omega square then the equation will becomes, d square q by dt square minus omega square q equal to 0. So, in this case the auxiliary equation becomes D square minus omega square equal to 0 or D equal to plus minus omega.

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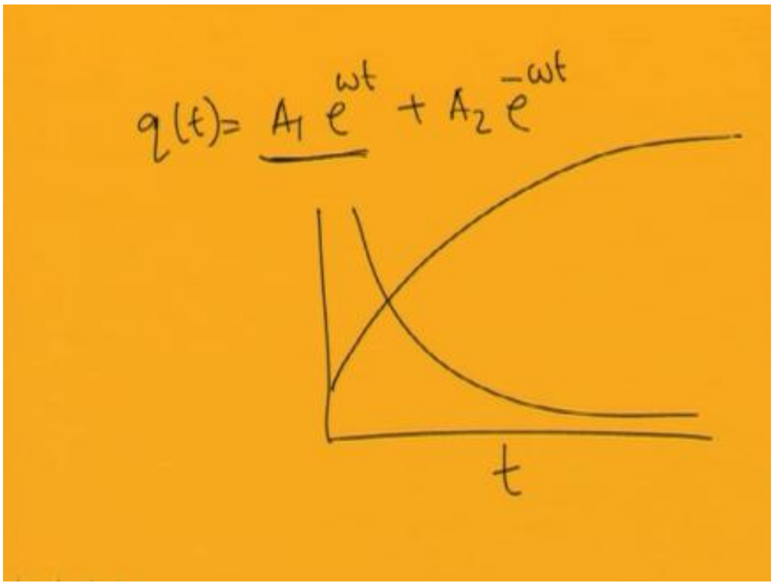

So, if D equal to plus minus omega, then the solution can be written as Qt equal to A 1 e to the power omega t plus A 2 e to the power minus omega t. So, in case of e to the power minus omega t, the response will die down the response will die down and come to 0. So, for t equal to 0 this is A 1. So, it will start with A 1 at A 1 and it will go on increasing as t tends to infinity. So, in this case the response, so the response will goes on increasing as t tends to infinity. So, the system will be unstable, as we are considering a stable system for this reason, we should not take this constant to be positive omega square. So, for that reason, we should take the constant to be minus omega square and the solution of this equation is qt equal to A cos omega t plus B sin omega t.

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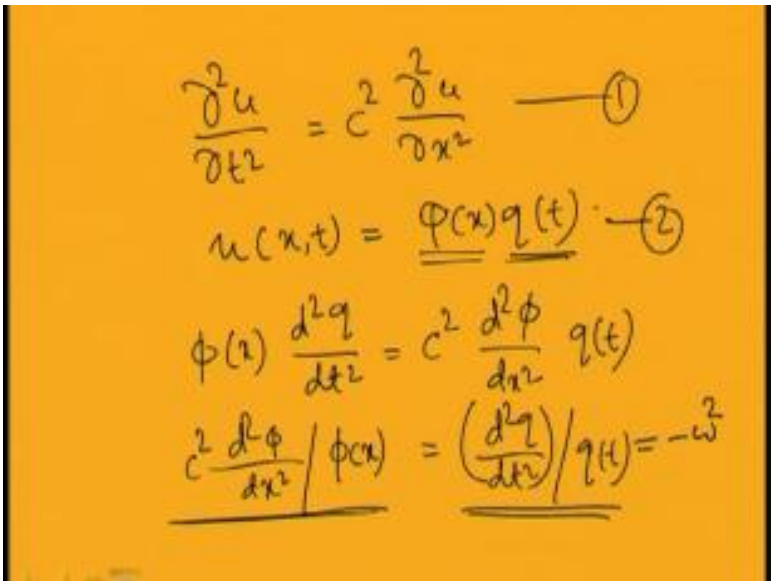

Now, taking the other term, that is C square d square phi by dx square by phi x equal to minus omega square, I can write this equation d square phi by dx square.

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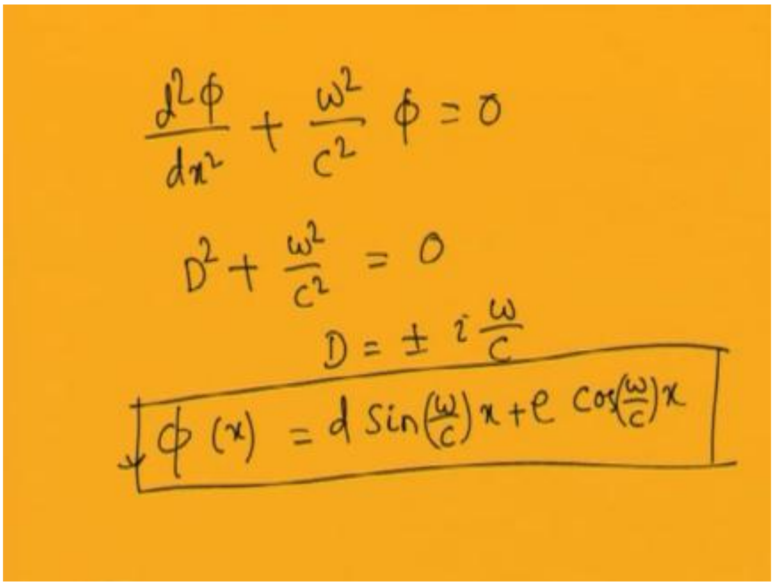

d square phi by dx square plus or, so plus omega square by C square phi equal to 0. So, in this case similarly, the auxiliary equation becomes, so I can write D square plus omega square by C square equal to 0 or D will be equal to the roots of this equation, will be equal to plus minus i C I omega by C. So, the solution can be written the in the form. So, phi x can be written equal to A 1 e to the power I omega by C into x plus A 2 I omega by

C into x. As you know, this e to the power I theta equal to cos theta plus I sin theta 1 can write this directly in terms of sin or cos. So, one can write this, phi x equal to C sin omega by C x plus d sin d cos omega by C x. So, here phi x equal to C sin omega by C, so instead of writing C let me write, so phi x will be equal to. So, phi x will be equal to d sin omega C by x plus e cos omega by C x. So, here C is the, so for case of string the C equal to root over t by m and longitudinal vibration of beam. It is equal to root over E by rho and in torsional vibration of the shaft it is equal to root over G by J. So, we got the expression for phi x and also we got the expression for u qt.

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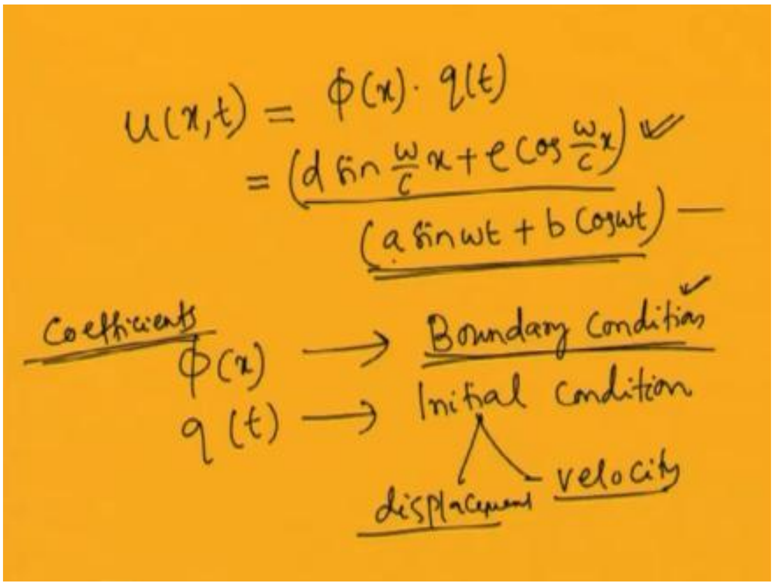

So, I can write this, U x t. So, the displacement of the string, can be written as phi x into qt. So, in this case phi x I can, write in terms of d sin omega C by C x plus E cos omega by C x into for qt, I can write this as a sin omega t plus b cos omega t. So, by using this expression, I can represent the vibration of the string or the rod or the longitudinal vibration of the beam. So, you may note that, this d e depends on the boundary conditions and a and b depends on the initial condition of the system. So, this time concern, that is a sin omega t plus b cos omega t the coefficient of this that a and b depends on the initial conditions at t equal to 0 1 has to know, what is the displacement of the system? And what is the velocity of the system? To determine this a and b and also. One should know the boundary conditions, to determine this d and e. So, by knowing the boundary conditions, so one can find this phi x.

So, phi x can be or coefficient of phi x, can be determined from the boundary conditions and coefficients of this time concern can be determined from the initial conditions. So, it is from boundary condition and coefficients of. So, this coefficients of phi that is d and e from the boundary condition and similarly, coefficients of this q that is a and b can be obtained from the initial condition. So, we can have 2 initial condition, that is displacement and velocity and second 1 is velocity. So, from this 2 displace 2 initial condition, you can find 2 equations which will give a and b and in the boundary conditions in this, one dimensional system at the left end. You have one boundary condition and at the right end, you have the other boundary condition. So, you have 2 boundary conditions, so taking those two boundary conditions we can determine the constants d and e.

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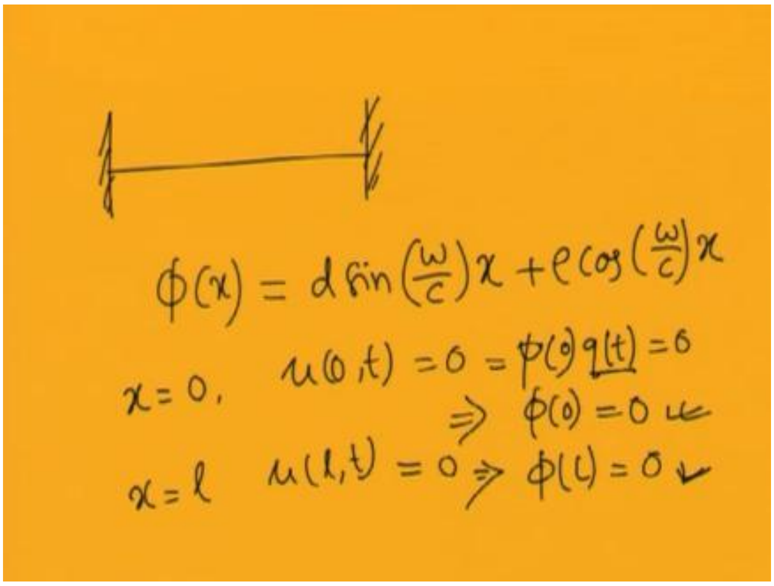

So, let us take the case of the fixed beam fixed-fixed string. So, let this is the string, it is fixed at both the ends. So, in this case we have this phi x equal to. So, in case of string, we can write this phi x equal to let me write. So, this is I have written d sin d sin omega by C x plus e cos omega by C x plus e cos omega by C x. So, in this case at left end, that is x equal to 0, so I can take this is equal to x equal to 0. So, in this case u x t equal to 0 and u x for x I can substitute this x equal to 0, you u 0 t equal to as 0 t equal to 0. So, this is u 0 t, I can write this as phi 0 and qt equal to 0. So, this implies that phi 0 will be equal to 0 as qt, t can take any value then qt cannot be 0. So, phi 0 will be equal to 0 similarly, at x equal to l I can write u l t. So, as it is fixed here, so the displacement will be equal to

0, so in that case u l t equal to 0. Similarly, I can, write this implies that phi L will be equal to 0. So, by taking these 2 boundary conditions, that is phi 0 equal to 0 and phi L equal to 0. Now, you can determine the value of d and e and also, we will obtain the frequency relation from this. So, let us substitute this phi 0 equal to 0.

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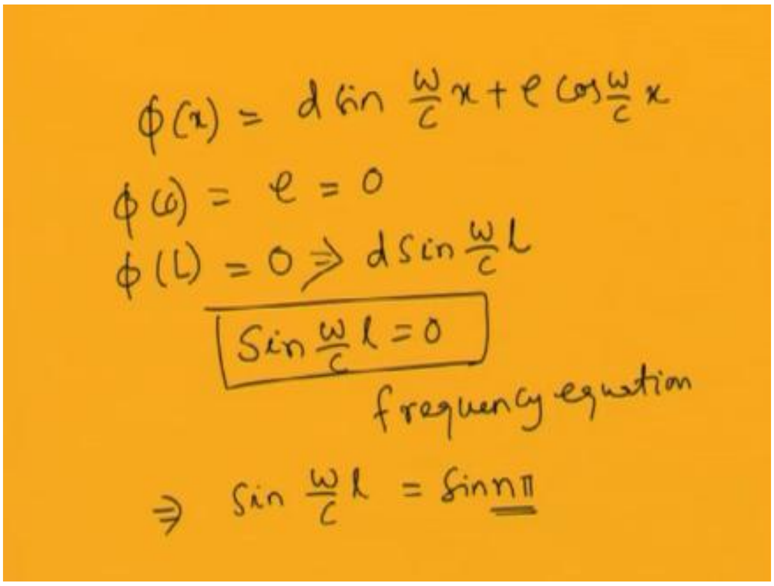

So, as phi x equal to I am writing phi x equal to d sin omega by C x plus e cos omega by C x. Now, phi 0 equal to, so by substituting x equal to 0. So, sin 0 becomes 0, so this d into 0 that is 0. So, then e cos omega cos omega by C x, so this becomes, cos 0, so cos 0 equal to 1. So, this e equal to 0 at phi 0 equal to 0. So, e equal to 0, so now, at phi L substituting phi L equal to 0 1 can write phi L equal to. So, d sin, so I can write this, as d sin omega by C l already, this e equal to 0. So, this term is not there, So, 0 equal to d sin omega by C into l. So, either from this either, you can write d equal to 0 or sin omega by C equal to 0. So, if you write, d equal to 0 in that case the whole solution that phi x will become, 0 phi x equal to 0 means that the system has a trivial response that is the system response is always 0.

So, the system is in equilibrium position always, but as we are considering the vibration of the system, the system may be the string or may be the beam. So, in that case d cannot be 0 as we are considering the non trivial response of the system. So, as d not equal to 0, so in that case, we should have sin omega by C into l equal to 0. So, this equation is known as the frequency equation of the system. So, this is the frequency equation, so this is the frequency equation of the system. So, from this frequency equation, we can find the expression for this omega. So, as sin omega by C l equal to 0, so this implies that sin omega by C l should be equal to sin n pi. So, for sin n pi equal to 0 n equal to 1 2 3 4, you can start n equal to 0 also. So, n equal to 0 refers to the rigid body motion of the system otherwise, if you take n equal to 1. So, this expression becomes.

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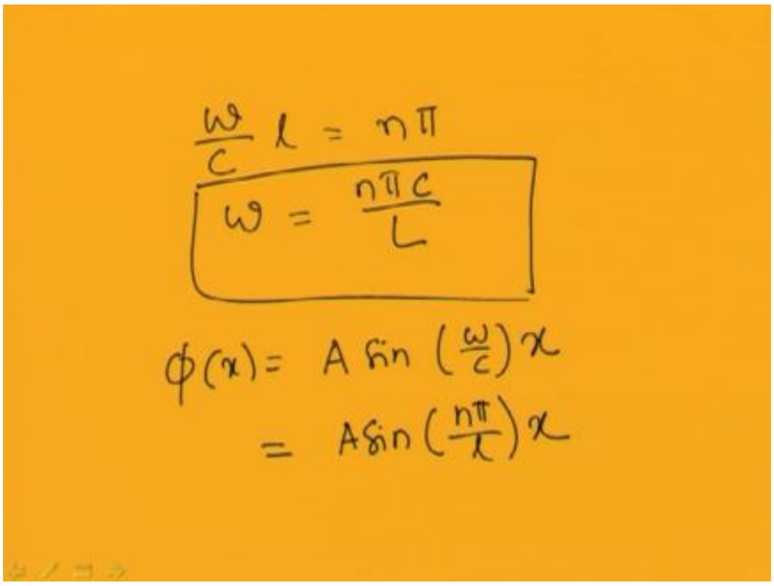

Omega by C into omega by C into l equal to n pi, so I can write this, omega equal to n pi l by C. So, omega equal to n pi l by C in this case, so by writing this omega equal to n pi by C we can find the mode shape of the different systems. So, here we got sin omega by C l equal to sin n pi or we can find the response of the system. So, now, phi x equal to d sin omega by C x into e cos omega by C into x; already, we got e equal to 0. So, the response of the system becomes, phi x equal to d sin omega by C into x. So, I can write this phi x equal to now, I can write phi x equal to A sin A sin omega by C into x, already, I got the value of omega. So, this omega equal to n pi l by C. So, you can substitute it in this equation, so this becomes A sin. So, this is A omega by C l equal to n pi.

So, omega will be, omega will be n pi into C by L. So, this is into C by L, so here omega by C I can substitute it equal to n pi by L. So, this is n pi by l into x, so n is the number of modes. So, for the first mode n equal to 1, so omega will be equal to pi C by L. Similarly, for 2, so this will be equal to n will be equal to 2. So, this way one can substitute the value of n to obtain different mode of vibration. This pi x is known as the mode of vibration. So, now, one can find different value of pi. So, pi equal to 3.14. So, the first mode it is equal to 3.14. So, one can plot this pi x.

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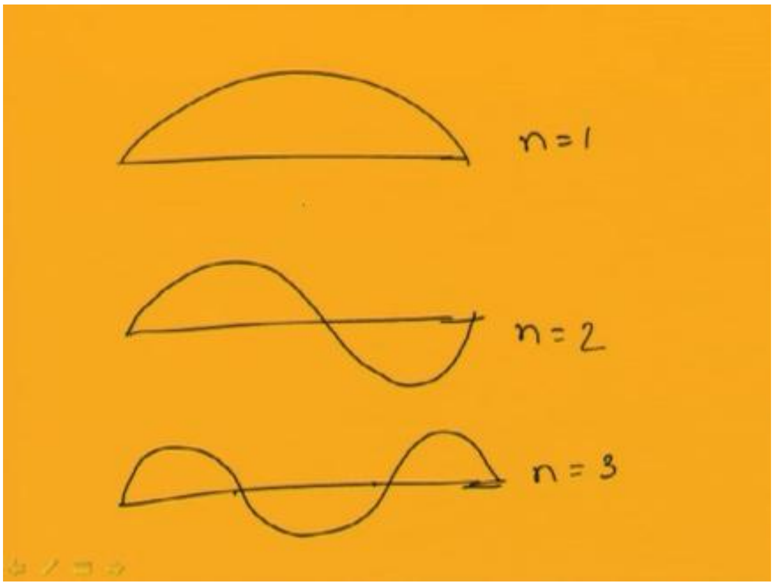

So, the response will be equal to, so for the first mode the response will be this and because this is equal to A sin n pi by L. So, the response, will be sin this sin n pi by pi by L. So, this is length of this string, so this is the first mode for second n equal to 2. So, 2 pi by l sin 2 pi by l if when plot. So, then the response will be, so it will show 1 node in this and in case of, so this is n equal to 1. So, this is n equal to 2 similarly, for n equal to 3 1 may get 2 nodes. So, at n equal to 3 1 will get 2 node, similarly, by increasing the number of nodes the deflection pattern, will be different. So, let us see the simulation of this.

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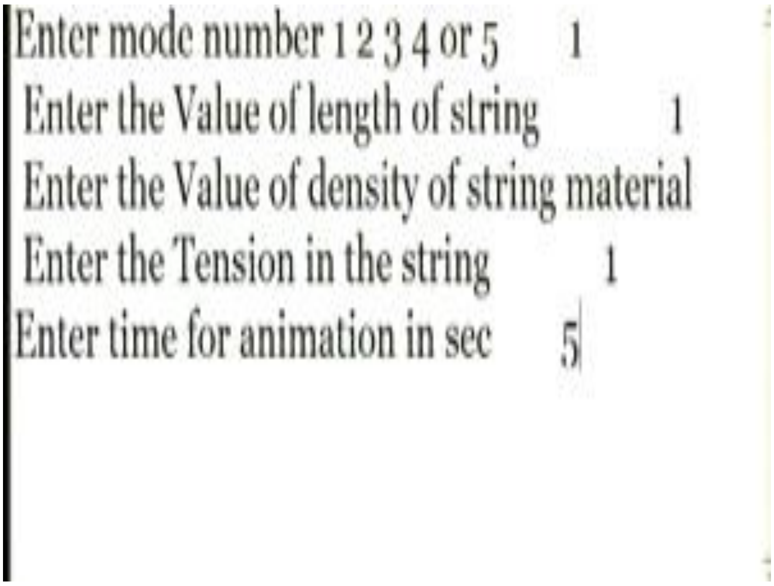

So, let me see the first mode, let me write the different modes of this uh. So, let I am interested for the first mode. So, if I am putting mode number 1, then in this case let me take length of the string to be 1 and then density also one and the tension and… So, unit tension for unit tension and let me see for 5 second the animation.

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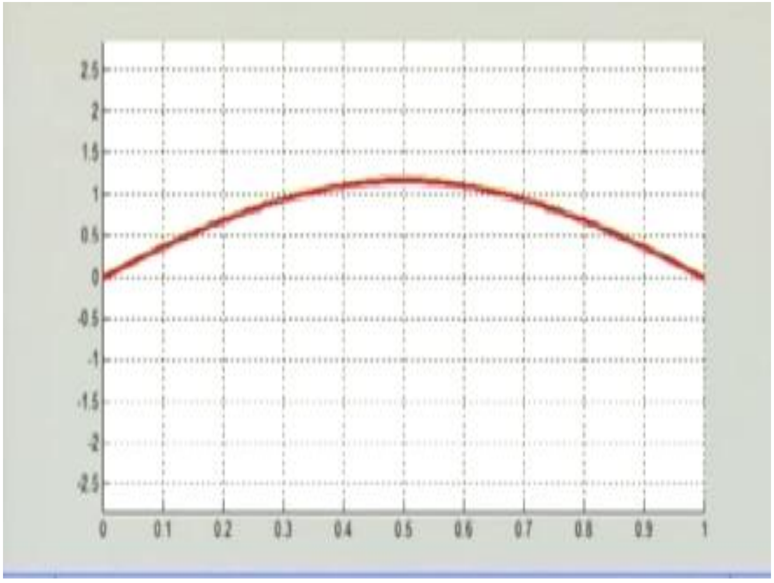

You can see the beam is vibrating or the string hydration will takes place in this form. So, the string vibration you can see the string will vibrate in that way.

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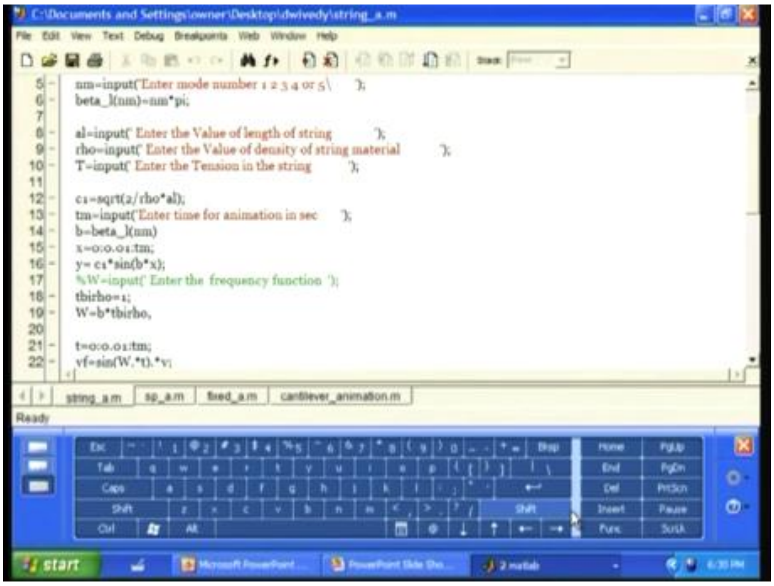

So, I can find the response, for this second mode. So, in that case, so let us enter number of mode equal to 2. So, we can take unit length and tension also, unit and let us find the animation for 3 second.

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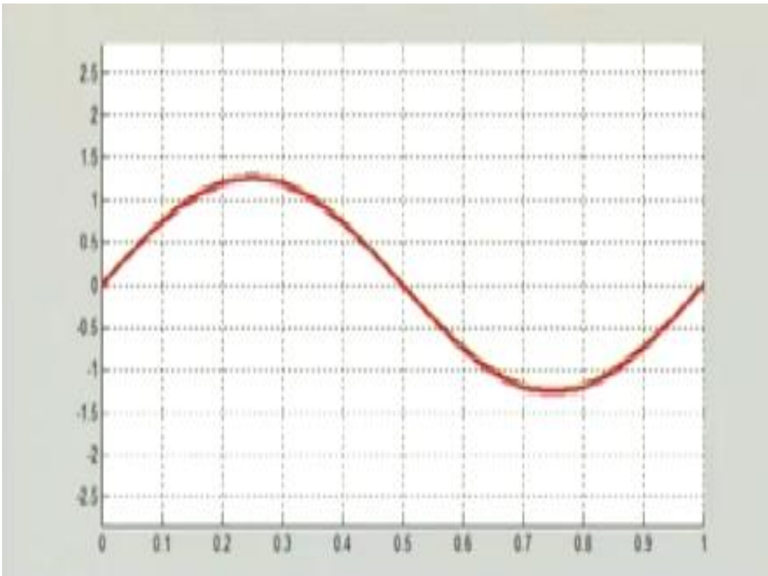

So, you can see clearly, you can visualize the node point at middle.

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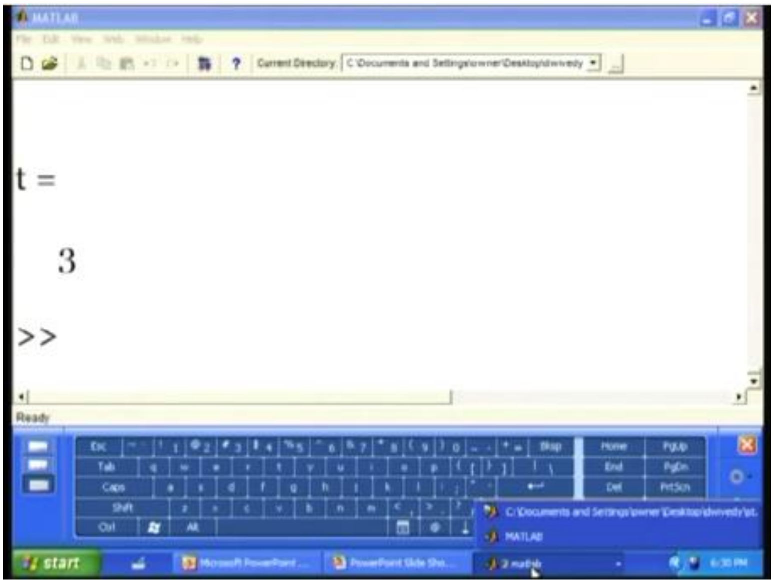

So, in this case you can see also for the third mode. Let us see for the third mode animated for the third mode. So, in case of third mode, let us substitute number of mode equal to 3 and then you can choose the length of the string density of the material and also the tension. So, by substituting all these things and if you want to see the animation.

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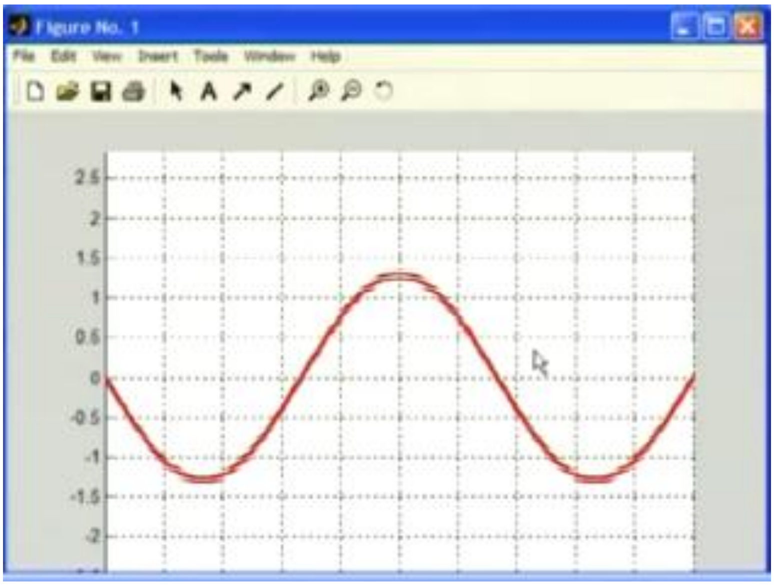

For the third mode, you can observe clearly 3 nodes are there. So, these 3 nodes are operating and you can find similarly for the forth mode. So, the fourth mode, so the fourth mode you can take in this way.

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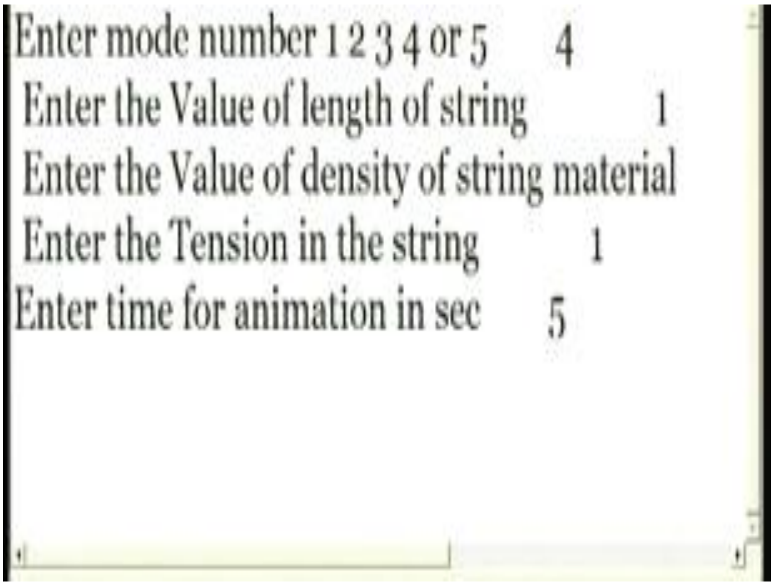

So, let us find for the fourth mode. So, for the forth mode putting this material density and the tension, so you can put different tensions, different length and different density of the material.

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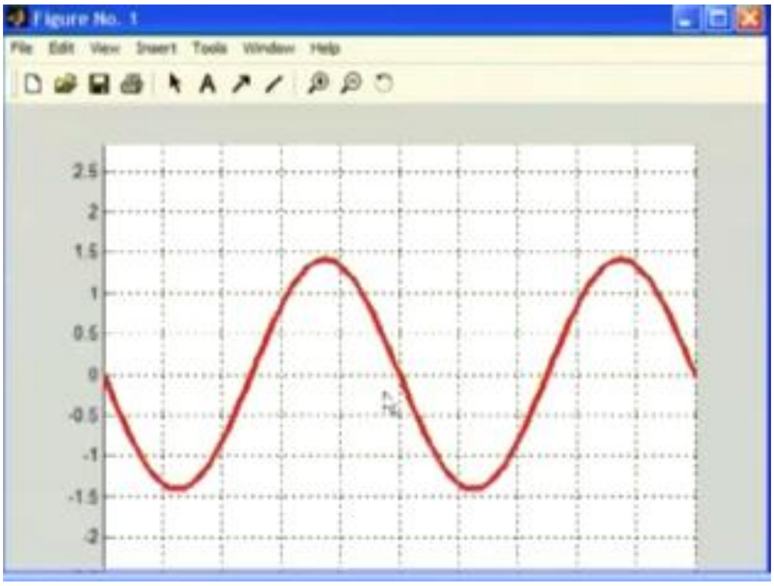

So, you can animate and you can see clearly, there are 4 different points, at which there is no displacement of the system. So, in these points these are known as the node points of the system. Similarly, you may simulate for the fifth mode of the system. So, for the fifth mode let us see.

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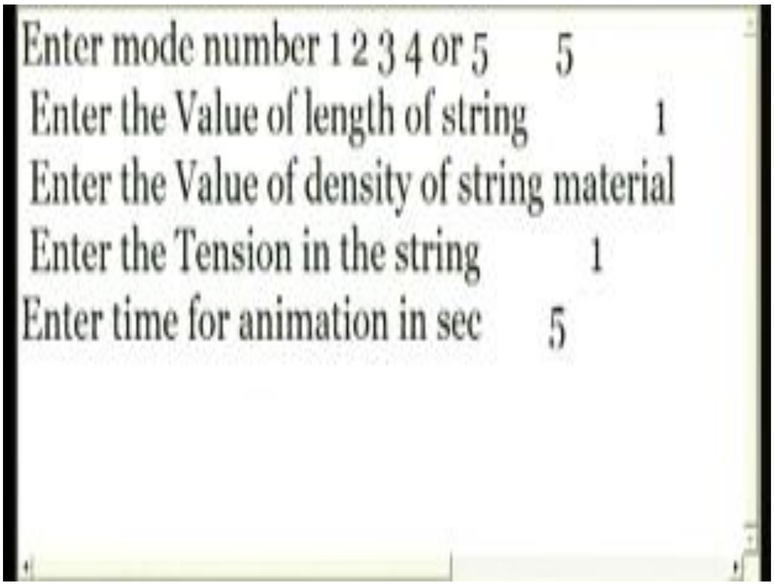

So, I can substitute the number of mode 5 and taking the length, to be unity density to be unity and the tension to be unity, I can find or I can show you the response of the string.

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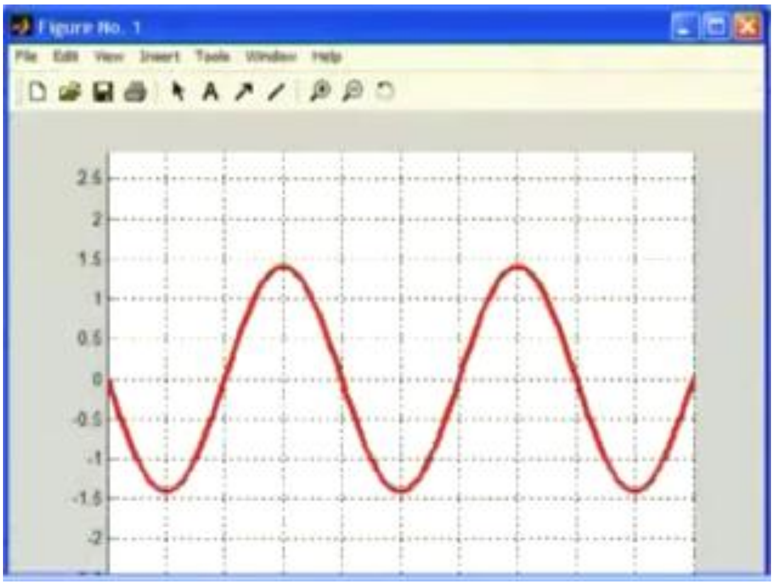

So, the string vibration will be like this, so at 5 different points. You can find that there is no vibration at 5 different points of the system.

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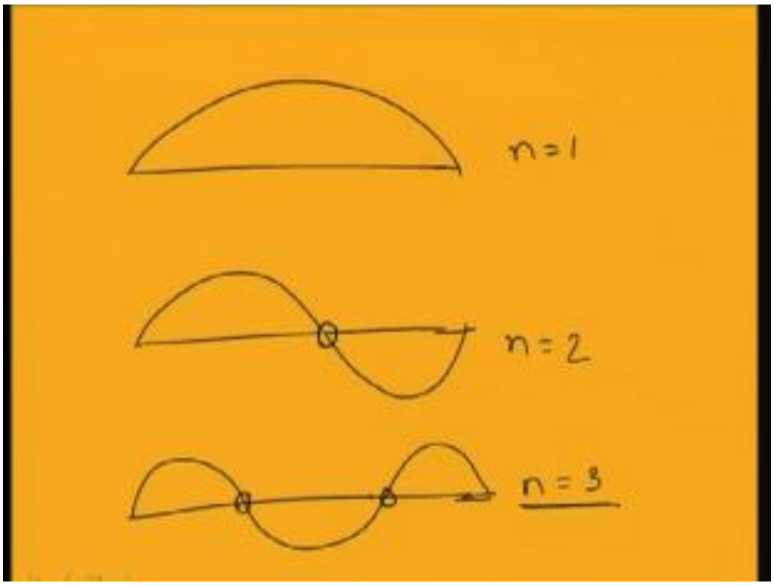

So, you have seen for the first mode, there is no node present in the system and for the second mode; there is a node at the middle and for n equal to 3. So, you can see that there are 2 nodes in the string. So, this node point represent the points at, which there is no vibration of the system. So, now similarly we can find the response of the system for a Euler-Bernoulli beam. So, in case of Euler-Bernoulli beam we have found the equation motion in this form.

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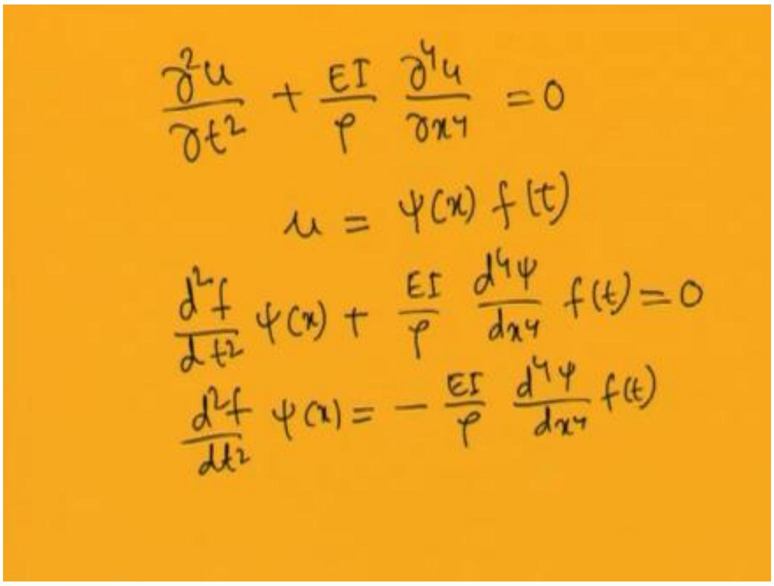

That is del square u by del t square plus EI by rho del fourth u by del x fourth equal to 0. So, this is the Euler-Bernoulli beam. So, in this case also proceeding in the case of wave equation, so we can find the response of the system. So, in this case let me write, u equal to phi or let me write this is equal to psi x into ft. So, now substituting this equation in this equation, I can write del square f by del t square into psi x plus, this EI by rho we can write EI by rho into del 4 for. So, this will be d fourth psi by dx fourth into ft. So, in this case as f is a function of time only. So, instead of writing partial derivative, I can write this is d square f by d t square psi x plus EI by rho d fourth psi by dx fourth ft equal to 0. Or I can write this, equation in this form d square f by dt square into psi x equal to minus EI by rho d fourth psi by dx fourth into ft or I can write this d square f by dt square by ft.

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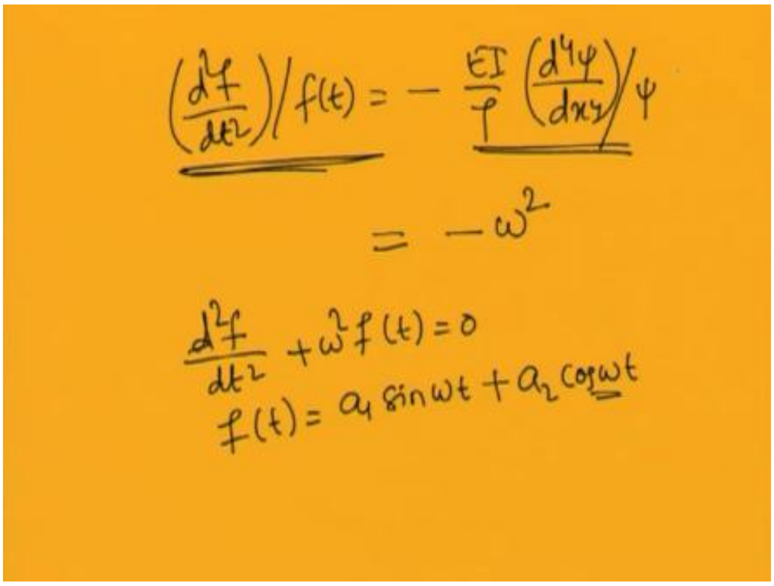

D square f by dt square by ft equal to minus EI by rho d fourth psi by d s x fourth by psi. So, as similar to the previous case here, also we can observe that, the left side is a function of time and right side is a function of space coordinate that is x. So, following the previous wave or previous condition, I can write this will be equal to minus omega square, so as this is equal to minus omega square. So, I can write this, ft equal to or, so this is acceleration by displacement equal to this minus omega square. So, one can write this, d square f by dt square plus omega square ft equal to 0 or the solution of this equation, one can write this ft equal to a 1 a 1 sin omega t plus a 2 cos omega t. So, here omega is the frequency of oscillation of the system. Now, in the other equation that is, minus EI by rho d fourth psi by dx fourth by psi equal to minus omega square.

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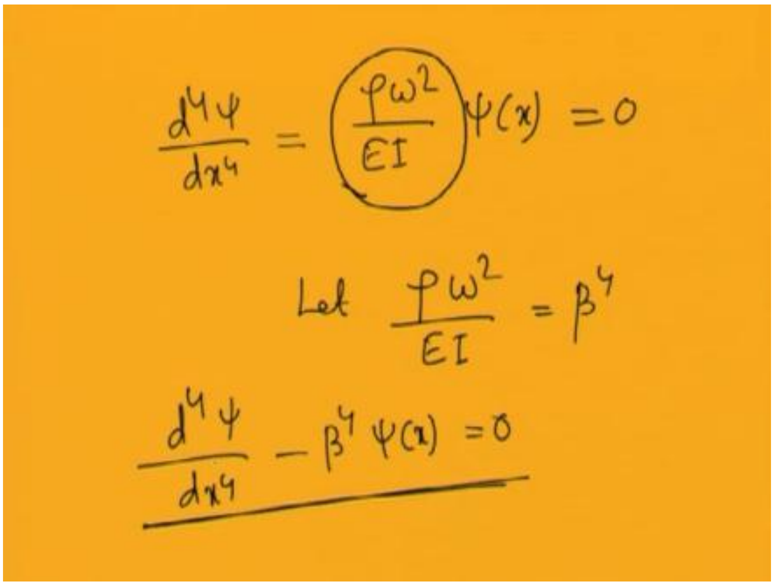

So, this equation if one can write, so one can write d fourth psi by dx fourth will be equal to, so this is equal to minus omega square by minus EI by rho. So, this becomes, rho omega square by EI. So, this becomes, rho omega square by EI into, so rho omega square by EI into psi x, so this is psi x. So, into psi x equal to 0 or let us take this term, this rho omega square by EI equal to beta fourth, so let rho omega square by EI equal to beta fourth. So, I can write this, equation in this form d fourth psi by dx fourth minus beta fourth psi x equal to 0. So, this fourth order equation now, we have to solve the fourth order equation. So, this auxiliary equation for this becomes D fourth minus beta fourth equal to 0.

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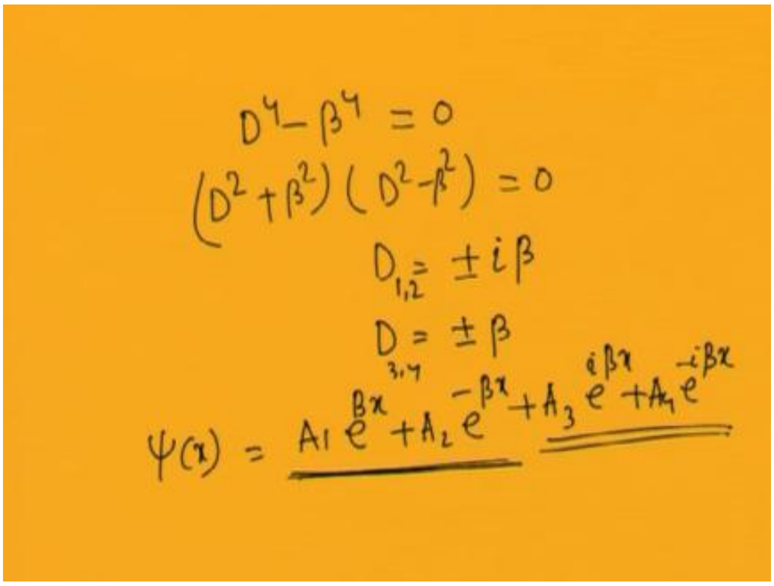

So, the auxiliary equation becomes, D fourth minus beta fourth equal to 0 or I can write this as D square plus beta square into D square minus beta square equal to 0. So, I can write this, D equal to plus minus I beta and similarly, D square equal to minus beta square D will be equal to. So, D 1 2 equal to plus minus I beta D 3 4 equal to, so D square equal to beta square. So, D will be equal to plus minus beta, so when the roots are plus minus beta. So, that time the solution will be A 1 the solution can be written in the form of A 1 e to the power beta t plus a 2 e to the power minus beta t. So, in this case it is x. So, the solution will be in the form of, so one can write the solution, this is solution for phi x. So, phi in this case we are writing psi.

So, the psi x can be written as A 1 e to the power beta x plus A 2 e to the power minus beta x plus A 3 e to the power I beta x plus A 4 e to the power minus I beta x. So, you may note that e to the power beta x plus e to the power minus beta x by 2 that is equal to cos hyperbolic beta. And similarly, we know one can write this e to the power beta x minus e to the power minus beta x by 2 as sin hyperbolic beta x. Similarly, e to the power I beta x can be written as cos beta x plus I sin beta x and e to the power beta x can be written as cos beta x minus I sin beta x. So, these 2 terms will reduce to hyperbolic function cos hyperbolic and sin hyperbolic function. And these 2 terms will reduce to harmonic terms, that is sin beta x and cos beta x. So, one can write this expressions psi x in this form.

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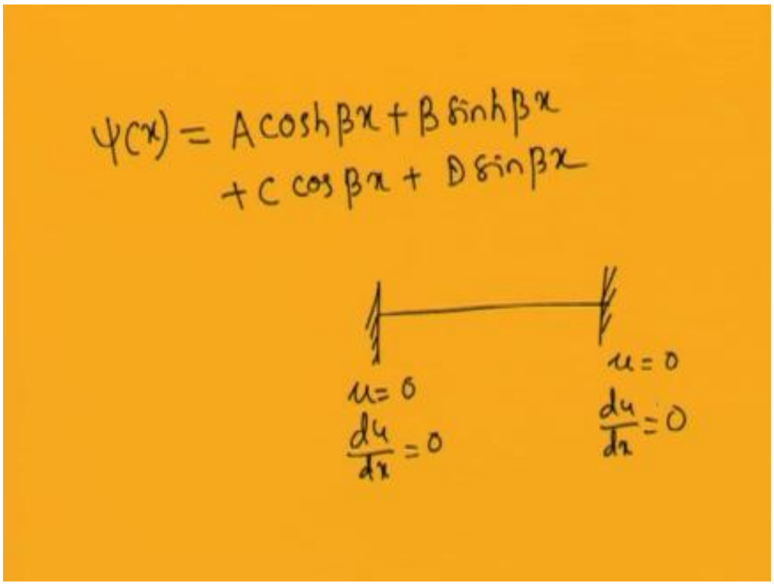

So, psi x can be written as A. So, I can write this as A sin A cos hyperbolic beta x plus B sin hyperbolic beta x plus C cos beta x plus D sin beta x. So, in this expression you can find there are 4 constants. So, these 4 constants can be obtained from the 4 boundary conditions in the transverse vibration of beam. The Euler-Bernoulli beam is for the transverse vibration of the beam. So, if you have a beam, so this is the beam let us, take this fixed, fixed beam. So, in this case the boundary conditions are in the left side, the boundary condition that is u equal to 0 and also the displacement that is du by dx equal to 0. Similarly, in the right side displacement equal to 0 and the slope also equal to 0.

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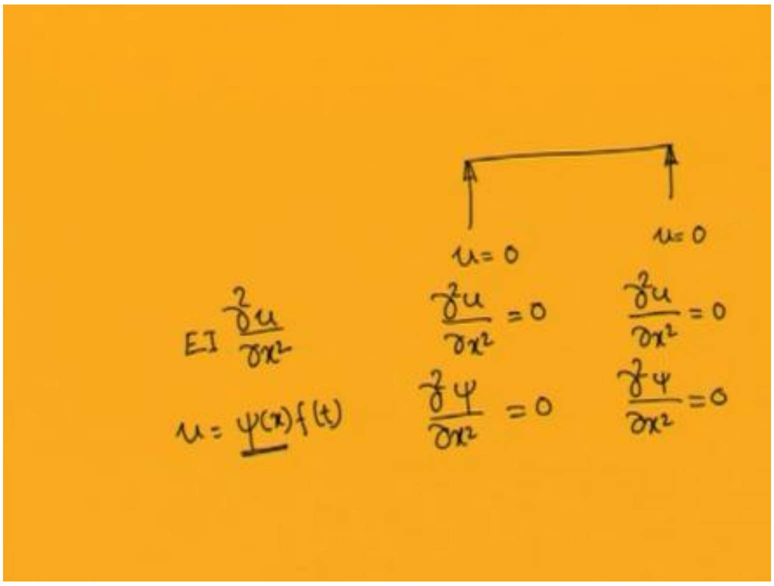

So, in case of simply supported beam, the boundary conditions will be displacement, will be equal to 0 at both the ends. So, this is the left end, so in left end displacement will be 0 that is u equal to 0 at x equal to 0 and also at x equal to l u equal to 0 also the bending moment will be equal to 0 at both the ends. Bending moment will be given by EI del square y. So, bending moment will be EI del square u del x square. So, this will be equal to 0 at both the ends, so del square u by del x square will be equal to 0. So, del square u by del x square equal to 0 at the left end and also del square u by del x square will be equal to 0 here. So, as u equal to u is a function, we are taking u is a function of psi x and ft or qt. So, in this case I can take this, del square u by del x square equal to 0 as del square psi by del x square equal to 0 and del square psi by del x square equal to 0 at both the ends. Similarly, for a cantilever beam, so this is a cantilever beam.

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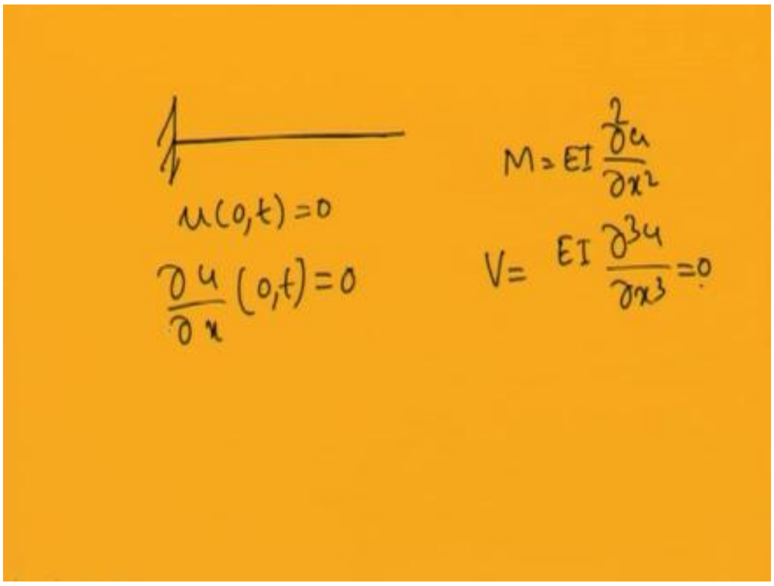

So, in case of a cantilever beam, the boundary conditions are at this fixed end, u 0 t equal to 0. And displacement equal to 0 and slope also equal to 0 that is du by dx at 0 t equal to 0, but in the free end displacement not equal to 0 and slope not equal to 0. So, in this case the displacement and slope you can write it equal to that is these 2 are not 0, but shear force and bending moment are 0. Shear force can be written as, EI del cube u by del x cube, that is the rate of the rate of change of the bending moment already, you know that bending moment is written in terms of EI del cube u by del.

Bending moment is written M equal EI del square u by del x square and shear force. You know that is written as EI del cube u by del x cube shear force V equal to EI del cube u by del x cube. So, shear force equal to 0 gives del u by del x cube equal to 0 similarly, bending moment equal to 0 gives del square u by del x square equal to 0. So, in this case in the left side, we have displacement u 0 t equal to 0 and the slope where du by dx. So, this is not du, so this will be del as u is a function of x and t. So, it should write in terms of partial derivative. So, del u by del x at 0 t equal to 0. Similarly, in case of a free free beam.

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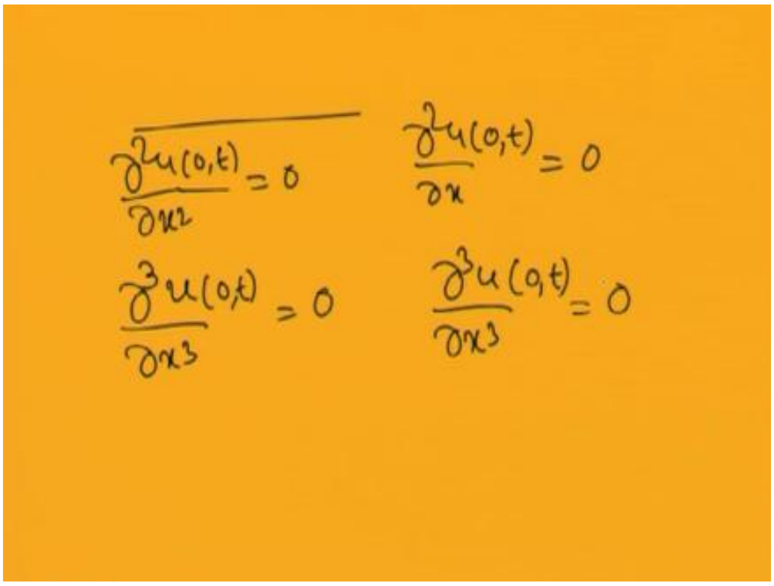

Already, I told you the free, free beam can be modeled or if the model of the space craft or aero plane flying in the sky. So, that time there is no support conditions, so it can be considered as a free beam. So, in case of free beam already, we have seen in the free end of a cantilever beam, the bending moment and shear force are 0. Similarly, in this case, you should write del square u by del x square equal to at 0 t equal to 0. So, that is bending moment equal to 0 for shear force equal to 0 del cube u by del x cube 0 t equal to 0. Similarly, in the right side also del square u by del x square 0 t this is equal to 0 and shear del cube u by del x cube 0 t equal to 0. .Similarly, if you have some other conditions like.

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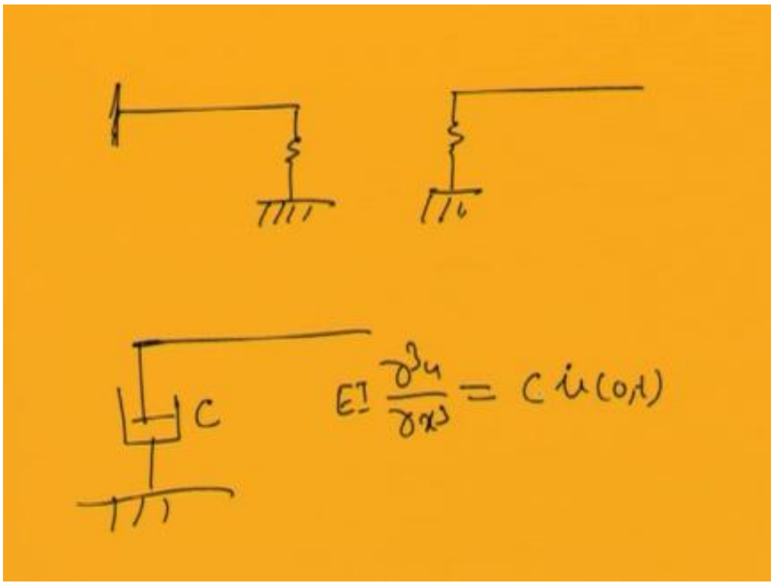

Let you have some spring support. So, if you have some spring support at the right, so in this case or if you have some spring support at the left. So, in that condition the shear force will be equal to the spring force. Shear force already, you know this is equal to EI del cube u by del x cube. So, the shear force EI del cube u by del x cube will be equal to spring force ku, in this case it will be equal to minus ku and the case where you have damper. So, in place of spring if if you have damper in this end. So, in this case the shear force will be equal to this damping force, shear force equal to EI del cube u by del x cube. So, this is the shear force. So, this will be equal to, so let C is the damping. So, it will be equal to C u 0 t dot, so this is the velocity u dot 0 t is the velocity and multiplying by the damping factor C, this will be the shear force here. Similarly, in the right, end it will be negative in the right end, if you have a damper then it will be EI del cube u by del x cube equal to minus C u dot 0 t and if you have a mass, supported at the end.

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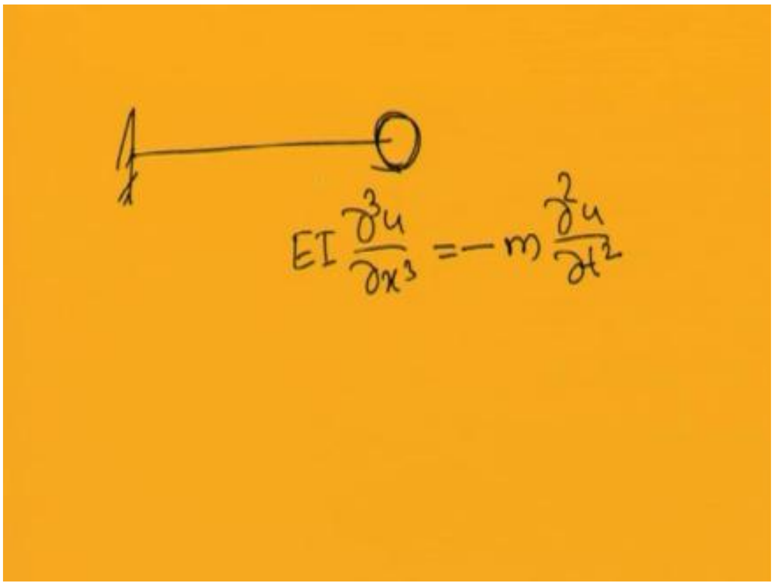

So, let us, take a mass at the end, so in this case the mass the weight or the mass will give the inertia force. So, the inertia force will be equal to shear force at this end. So, EI del cube u by del x cube will be equal to mu double dot in this case. So, you can write EI del cube u by del x cube this is the shear force, should be equal to inertia force. So, this is equal to minus m del square u by del t square. So, by taking different boundary conditions, so you can see that, you have 4 boundary conditions. So, in this particular case, shear force equal to inertia force bending moment equal to 0 and in the left end your deflection and slope are 0. So, in all the cases, you are finding 4 boundary conditions. So, using those 4 boundary conditions, you can find the equation for the mode shape of the system. So, already you got the general expression.

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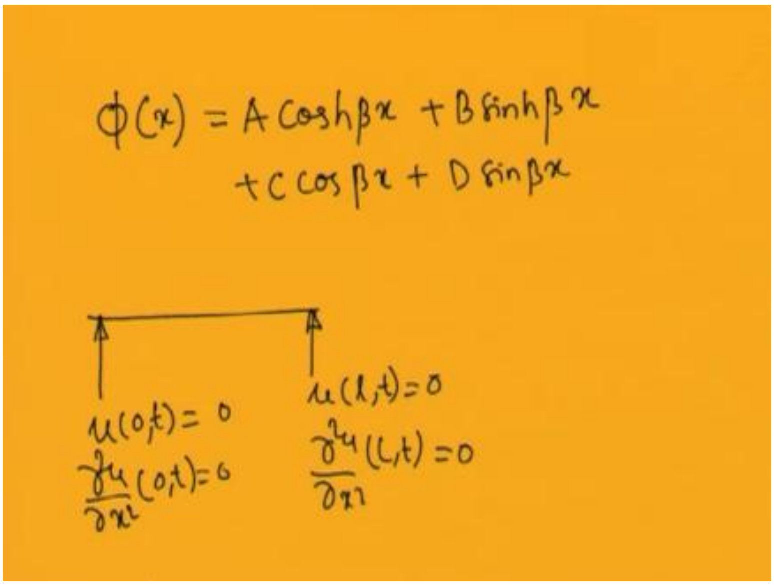

For the mode shape, that is phi x phi x equal to A cos hyperbolic beta x plus B sin hyperbolic beta x plus C cos beta x plus D sin beta x. So, let us take the case of a simply supported beam. So, in case of the simply supported beam, both end are simply supported. So, in this case, displacement at 0 t equal to 0 and here, also displacement at l t equal to 0. And we have seen that the bending moment equal to 0 that is del square u by del x square, at 0 t equal to 0 similarly, del square u by del x square l t equal to 0. So, using these 4 boundary conditions, let us find this constant ABCD. So, in this case, I can find, So, as u 0 t equal to 0. So, I can write u 0 t.

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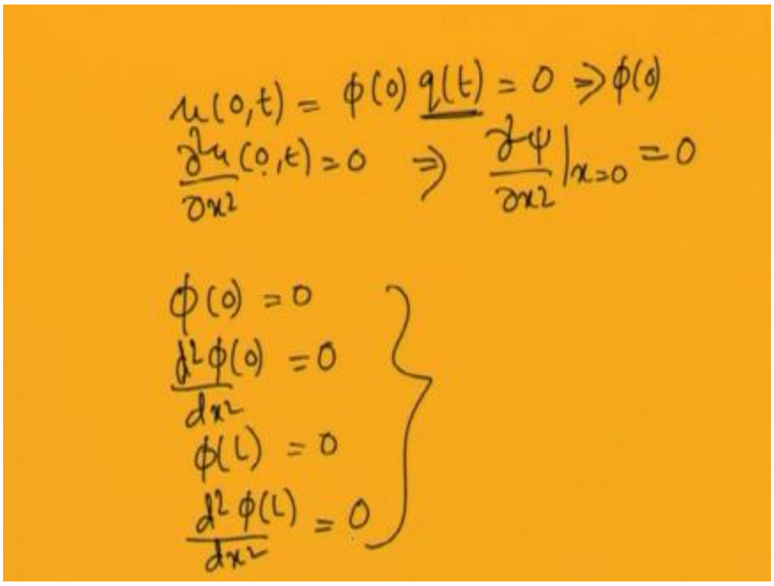

so, you know that U 0 t equal to. So, phi 0 into qt equal to 0, so as qt not equal to 0 at t can take any time. So, you can take any time, so phi 0 equal to 0. Similarly, we can tell that d square or del square u by del x square equal to 0 t equal to 0. So, it will implies, that del square psi by del x square at 0 del square psi at x equal to 0 equal to 0. So, we have 4 boundary condition, the first boundary condition phi 0 equal to 0, second is d square phi by dx square at 0 equal to 0. The third boundary condition become, phi L equal to 0 and the fourth one I can write d square phi by dx square at x equal to L equal to 0. So, now, from the general expression phi x.

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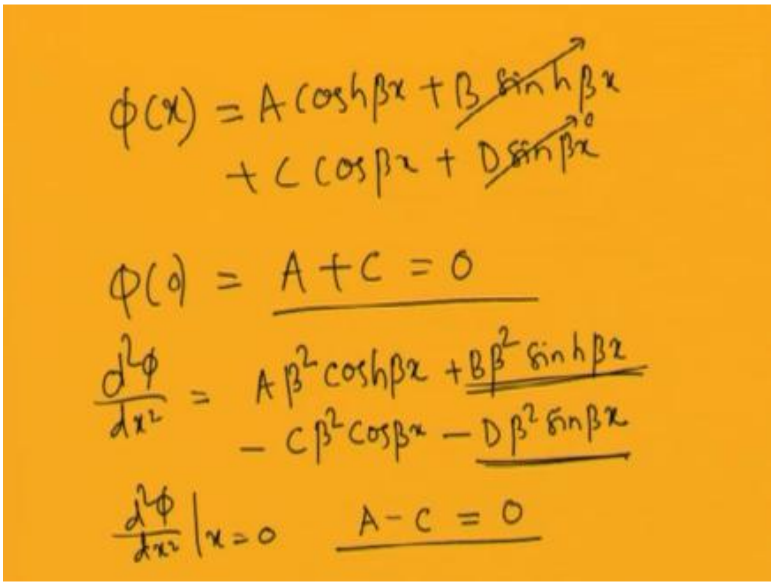

We can find phi 0, so phi, so already, you know phi x equal to A cos hyperbolic beta x plus B sin hyperbolic beta x plus C cos beta x plus D sin beta x. So, phi 0 will becomes, so phi 0 you can substitute that x equal to 0 cos 0 1 and sin 0 0. So, this B term, so this expression this becomes 0 and D term also becomes 0. So, term containing B and D are 0 for phi equal to 0, so I can write A plus. So, from this A plus C equal to 0, now, if I will substitute d square phi by dx square. So, if I will differentiate it twice, so then it becomes, so phi d square phi by dx square. So, this is equal to, so as A equal to C I can write A equal to A plus C equal to 0. So, A equal to minus C. Now, by differentiating twice, so I can write this becomes, A beta square, so A beta square cos hyperbolic will remain cos hyperbolic cos hyperbolic beta x.

Similarly, this becomes B B into beta square plus B into beta square sin hyperbolic beta x differentiating this cos beta x twice this becomes minus minus C beta square cos beta x and differentiating sin this becomes minus D beta square sin beta x. So, now, d square phi by dx square at x equal to 0 will yield, so d square phi by dx square at x equal to 0. So, if I will substitute x equal to 0 then this term becomes, 0 and this hyperbolic B hyperbolic term containing B also 0. So, in this case I can obtain A minus. So, cos hyperbolic beta x at x equal to 0 is 1 similarly, cos beta x at x equal to 0 equal to 1. So, this becomes, A minus C equal to 0, so from this, we got A plus C equal to 0 and A plus C equal to 0 and A minus C equal to 0 this gives that 2 A equal to 0. So, A equal to 0 and you can subtract. So, as A equal to 0 A plus C equal to 0, so C is also equal to 0, so A and C both are equal to 0. So, as A and C both are equal to 0. So, the remaining term, are phi x equal to B sin hyperbolic beta x.

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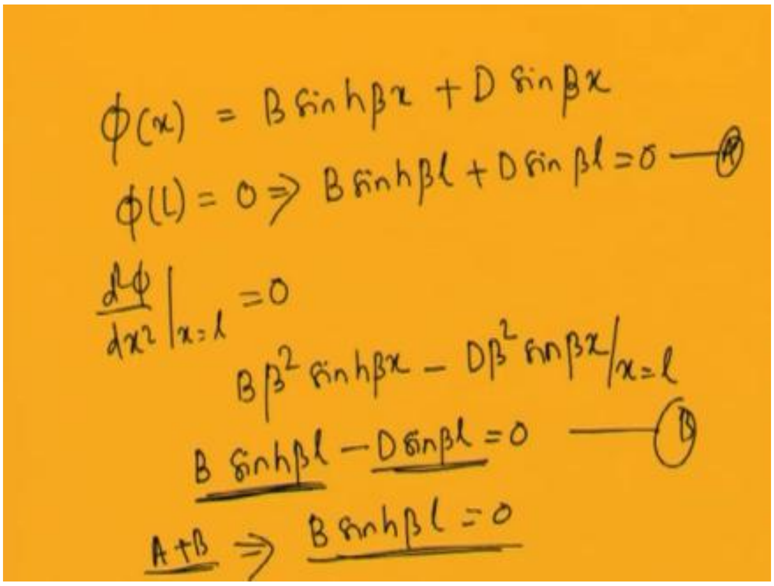

So, I can write this phi x equal to B sin hyperbolic beta x plus D sin beta x. So, these are the 2 remaining terms, now, I will apply the other 2 boundary conditions that is phi L equal to 0. So, phi L equal to 0 gives, so if I will put phi L equal to 0 then it becomes, B sin hyperbolic. So, B sin hyperbolic beta l plus D sin beta l equal to 0. Similarly, if I will put d square phi by dx square at x equal to l equal to 0, so this is for from the bending moment. So, this will imply that, so if you differentiate this equation twice. So, this becomes, B beta square sin hyperbolic beta x, but this becomes minus D beta squares sin beta x. So, when I will put this equal to 0. So, this, so this expression becomes.

So, let me write, so this is B into, B into beta square sin hyperbolic beta x. So, this becomes, plus D into beta square minus this becomes, minus, minus D beta square sin beta x. So, when I put x equal to l, so this is x equal to l this term becomes, D into sin hyperbolic beta l minus d into sin beta l equal to 0. So, as this trigonometry function this is hyperbolic function, so they becomes 0. So, it implies that this B into sin beta l should be equal to 0 or D into sin beta l should, will be equal to 0. So, when B into sin hyperbolic beta l or from I can take from these 2 equations.

So, this equation and this equation, so let it is equation A and it is equation B. So, phi l equal to B sin hyperbolic beta l plus D sin beta l equal to 0 and the other equation, what I got this? Is B sin hyperbolic beta l minus D sin beta l equal to 0? So, if I will add these 2, so i am getting this 2 into B sin hyperbolic beta l equal to 0. So, by adding this A and B by adding A plus B, you can get you can get B sin hyperbolic beta l equal to 0, so as B sin hyperbolic beta l equal to 0, so as hyperbolic function cannot be 0. So, this D equal to 0, so now, we got B equal to 0, so our expression, so from these equation, either from A or B as B equal to 0. So, you can write D sin beta l equal to 0.

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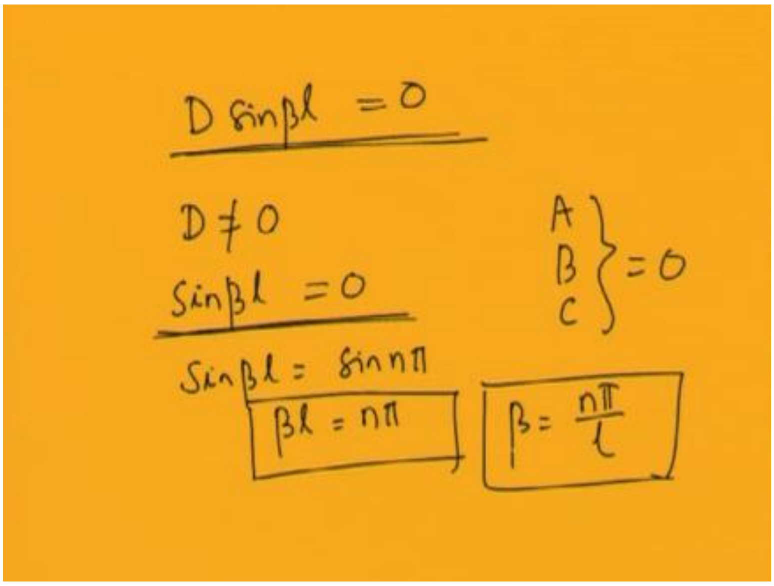

So, D sin beta l equal to 0 as D sin beta l equal to 0, so similar to the previous case, we should not write this D equal to 0 as already, we got that A B and C are 0. So, we should not D equal to 0, if I writing D equal to 0, then it will leads to the trivial solution. But as you are interested for the non trivial solution then these should not D equal to 0. So, D not equal to 0, so the expression becomes, sin beta l sin beta l equal to 0 as sin beta l equal to 0. So, I can write, sin beta l equal to, so sin beta l equal to sin n pi. So, for sin n pi this becomes 0, so I can write this beta l equal to n pi or I can write beta equal to n pi by l or beta equal to n pi by l. So, this is the expression, for the, or this is the frequency equation beta equal to n pi by l for a simply supported beam. Already, we know the expression for beta, we have written.

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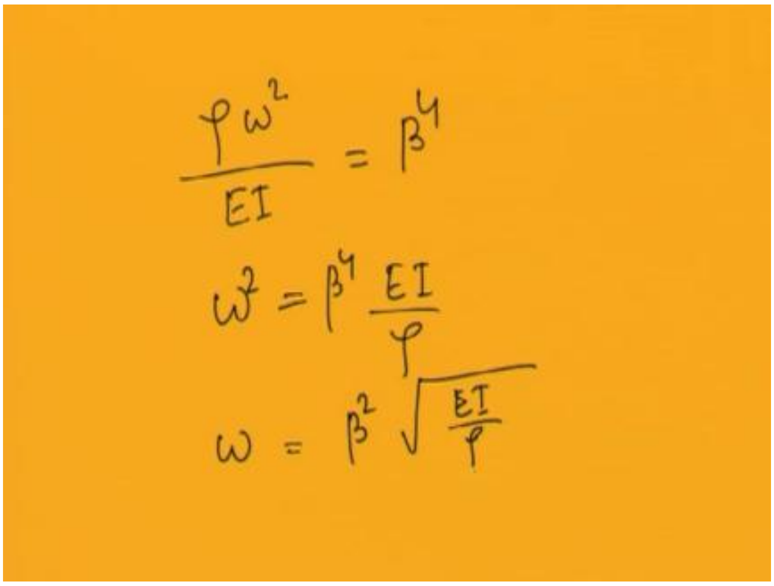

The rho omega square rho omega square by EI equal to beta fourth, we have written rho omega square. So, let us see the expression for beta fourth, what we have written? So, in a.

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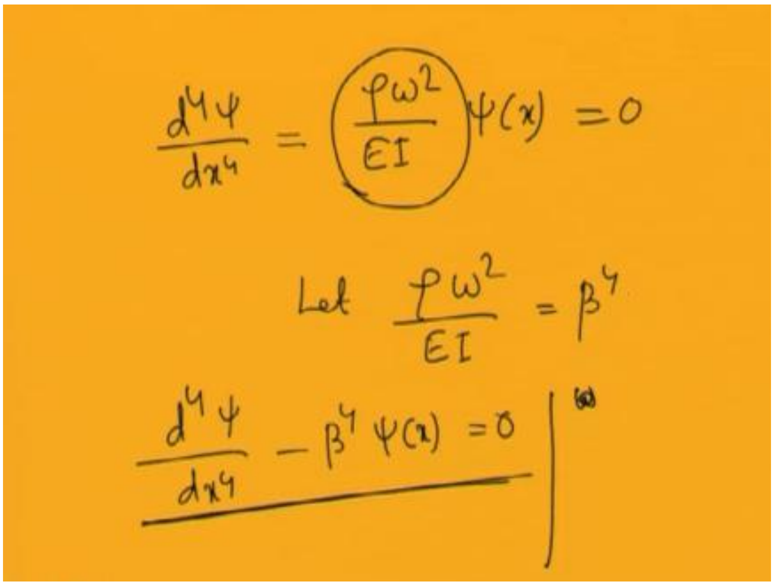

So, we have written rho omega square by EI as the beta fourth. So, now, from this, you can write this omega square or omega square equal to EI beta fourth into EI by rho or omega square equal to beta fourth into EI by rho. So, let me write beta fourth equal to rho omega square by EI or omega square equal to EI beta fourth into EI by rho. So, from this, you can obtain the frequency of the system that is omega equal to beta square into EI by rho. So, if I will multiply l fourth here.

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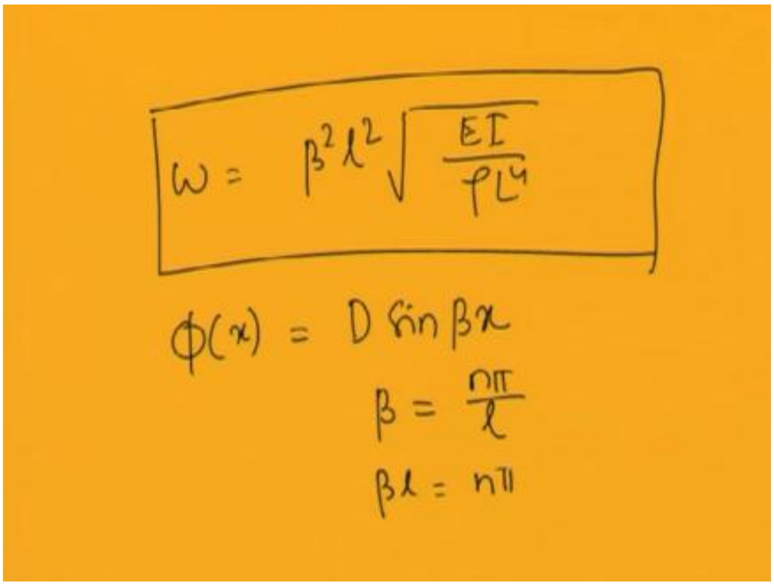

So, this omega expression will becomes. So, this omega expression will becomes omega equal to beta square l square root over EI by rho l fourth. You may note that this EI by rho l fourth is a non dimensional number and this omega equal to beta square l square by EI fourth. So, for the simply supported beam, we obtain the mode shape phi x equal to, so phi x equal to D sin beta x D sin beta x, where beta can be written, so already, we got the expression for beta. So, beta equal to n pi beta we got n pi by l, so we got we have written this beta equal to n pi by l or beta l I can write beta l equal to n pi. So, for this, case, I can write this omega equal to n square pi square.

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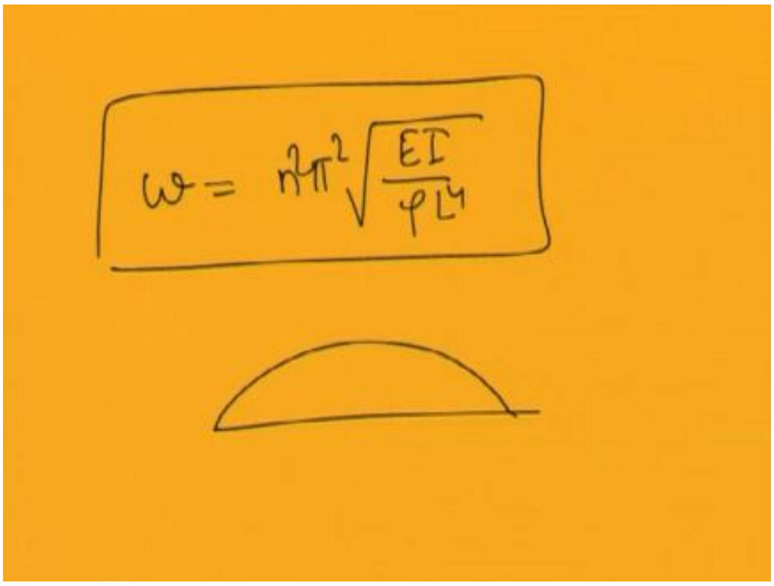

So, for the simply supported case I can write omega equal to n square pi square root over EI by rho one fourth. So, you may observe that the system has n number of natural frequency in this case; we can vary the number n from 1 to infinity. So, it can the system can have infinity number of frequencies or it can vibrate in a infinity modes. So, this is this one can consider this as a simply supported beam, so in this case in this case in the simply supported beam. So, one can find infinity number of modes. So, for the first mode, if you substitute n equal to 1 the response or the mode shape will be like this and for n equal to 2.

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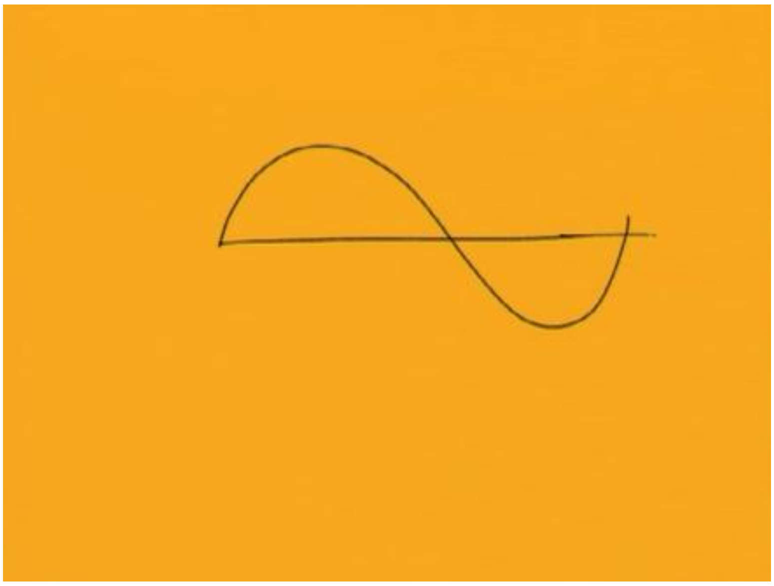

You may see you have this is for n equal to 2, and similarly you may have different modes. Next class, we are going to study about the other boundary conditions, also I am going to show you about the animation of the free vibration of different boundary conditions. So, these boundary conditions will include free-free boundary condition fixed-fixed, simply supported and clamped free boundary conditions.