Mechanical Vibrations
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Module - 9
Continuous Systems: Closed Form Solutions
Lecture - 2
Derivation of Equations of Motion Part 2
Newton's and Hamilton's Principle

Last class we have studied about the continuous system, where we have derived the equation motion of string, and longitudinal vibration of rod, and torsional vibration of rod. And we have found that those equations can be written in the form of wave equations unlike, the distributed mass system here in continuous system. So, the equation motions can be written in terms of partial differential equation. In case of discrete systems the equations are written in terms of ordinary differential equation. Also in case of discrete or lumped mass system. It does not depend on the boundary conditions, but we have seen in case of continuous or discrete mass systems, it depends on the boundary conditions. So, I told you there are 2 different types of boundary conditions.

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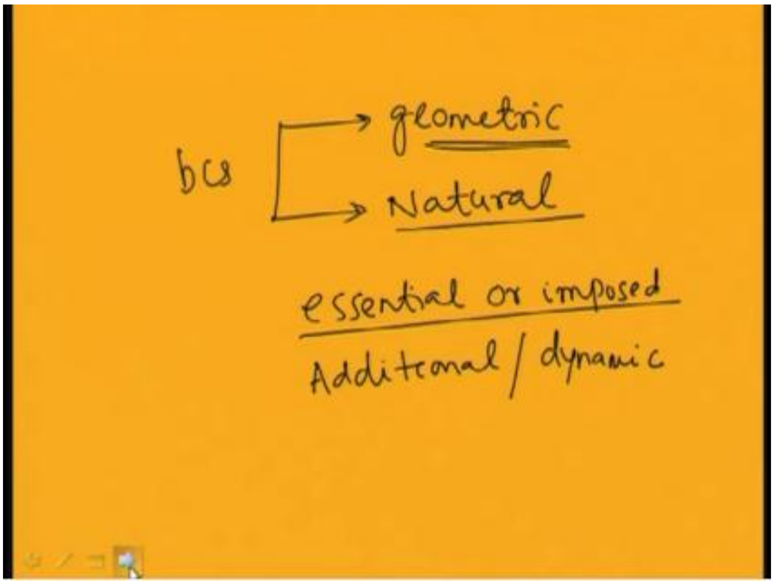

So, geometric type boundary conditions and second one is the natural boundary condition. So, this geometric boundary conditions are also known as the essential or imposed boundary conditions, geometric or essential or imposed boundary condition.

And this natural boundary conditions are also known as dynamic or additional boundary conditions. This natural boundary conditions are known as additional or dynamic boundary conditions. So, in case of geometric boundary condition or essential, or imposed boundary conditions, the boundary conditions are of the type displacement or slope. And in case of natural boundary condition or additional and dynamic boundary conditions the dynamic the boundary conditions we are consider are force or moment in nature. So, in case of a string or in case of a rod fixed.

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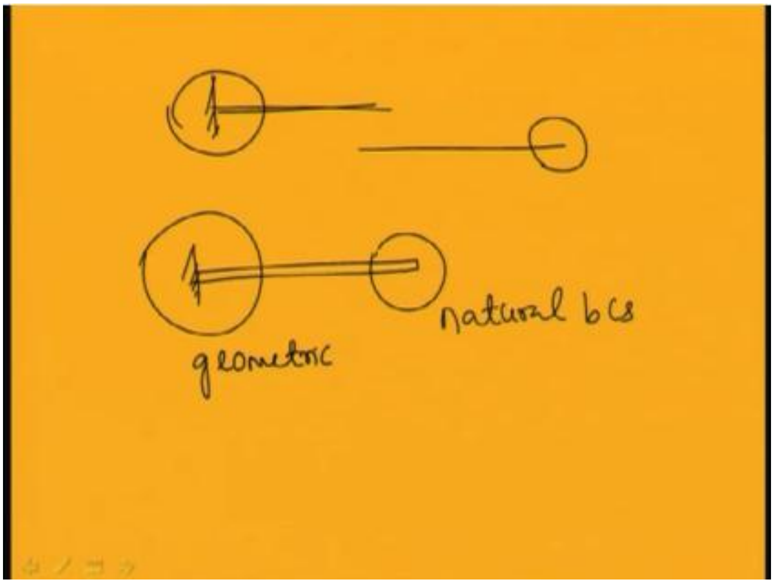

At one end, and fixed at one end, can be the boundary condition can be. So, if the fixing end the boundary condition will be the geometric type where it is the displacement equal to 0 and slope equal to 0. So, if the boundary condition is free type. So, let this side is free, so when it is free in case of the longitudinal vibration of a rod or if you consider the transverse vibration of a rod, transverse vibration of a beam which we are going to study now. So, in that case the boundary conditions will be shear force and bending moment will be 0 at this free end. So, at this free end the boundary condition is of natural boundary condition or additional boundary condition or dynamic boundary condition. So, in this case of a cantilever beam in transverse vibration the left side the left side boundary condition. It will be slope equal to 0 and displacement equal to 0. So, this is geometric boundary condition and in the right side this is natural boundary condition. So, this is natural boundary conditions and here, it is geometric boundary condition. In some cases you can find both the boundary conditions are also present for example, in case of simply supported beam.

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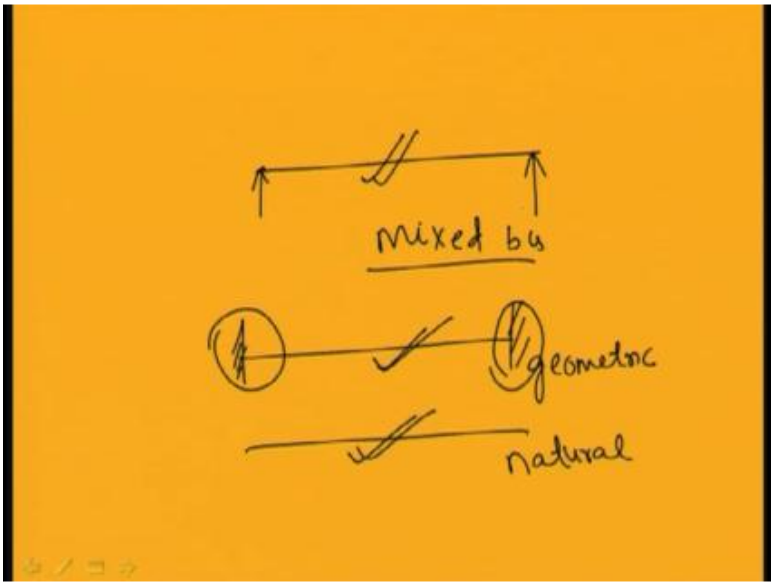

In case of simply supported beam, the boundary conditions are the displacement will be 0 and the bending moment also will be 0 as displacement equal to 0. So, it gives rise to this geometric boundary condition and when the bending moment equal to 0 that is the natural boundary condition. So, in this case you have a mixed boundary condition. So, in this case you have mixed boundary conditions in case of. So, this is the case of a simply supported beam. So, you have mixed boundary conditions, and in case of this fixed-fixed beam you have seen the boundary conditions are force. So, boundary conditions are geometric in nature. So, in this case both the side the boundary conditions are. So, this left side the displacement and slope is 0 slopes are 0 and here displacement and slopes are also 0. In case of free-free.

So, let you take a free-free beam or rod. So, free-free beam of rod. So, the physical application is the space craft or a ship sailing on the sea that time. So, it can be considered as a free-free beam or free free rod. So, in that case the boundary conditions are in case of a fixed free-free beam the boundary conditions are shear force and bending moment will be 0 at both the ends. So, this is natural boundary conditions. So, in both the, so in this case you have a natural boundary condition, in this case you have a purely geometric boundary condition, and in this case you have both geometric and natural boundary conditions. So, this is a mixed boundary conditions. So, already we have derived the equation of motion for transverse vibration of a string, longitudinal vibration of rod and torsional vibration of rod. So, in these cases we have found the equation motion to be of the form. That is known as wave equation. So, in that case we have written the equation in this form. So, we have written.

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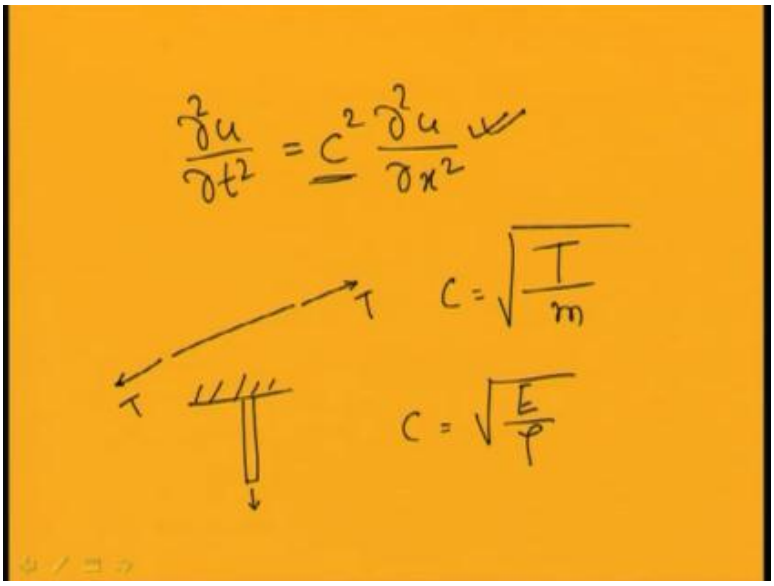

Del square u by del t square equal to C square del square u by del x square. So, where u is the displacement in displacement or in case of torsional vibration it is the rotation and this t is the time and C depends on the systems what you are considering in case of a string? So, it is equal to T by rho or T by m, where m is mass per unit length T is the tension. So, this is a string. So, in case of a string subjected to constant tension in both the sides we have derived that the equation of motion is in this form and we have seen this T equal C square equal to T by m or C equal to root over T by m, and in case of longitudinal vibration of a beam or rod. So, this is the beam subjected to longitudinal vibration. So, in this case we have seen the C equal to root over E by. So, this is equal to E by rho and in case of.

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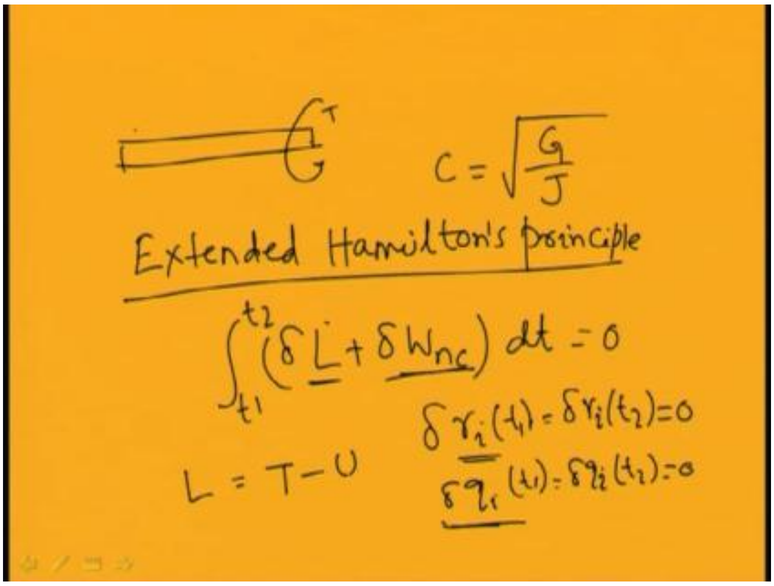

So, this is a rod subjected to torsion. So, it is subjected to torsion T. So, we have seen that in this case C square or C will be equal to this rigidity modulus by J, J is the polar moment of inertia and G is the rigidity modulus. Today, we will derive some of these equations and also the equation for Euler-Bernoulli beam by using Hamilton principle. So, using the Hamilton principle or extended Hamilton principle, we can derive the equation motion. So, extended Hamilton principle, so this principle tells that the expression for this principle can be written in this form.

So, t 1 to t 2 del of del L plus del W n c d t equal to 0. And delta r i at t 1 equal to delta r i at t 2 equal to 0. So, in this expression L is the Lagrangian of the system. Where Lagrangian can be can be written as T minus u T is the kinetic energy of the system. U is the potential energy of the system delta W n c this W n c delta W n c is the virtual work down by the non conservative forces and t is the time and delta r i are the physical coordinate of the system. Or I can write in terms of the generalized coordinate delta q i delta q i at t 1 equal to delta q i at t 2 equal to 0. So, this delta q is the variation in this parameter variation in the generalized coordinate at time t 1 and t 2.

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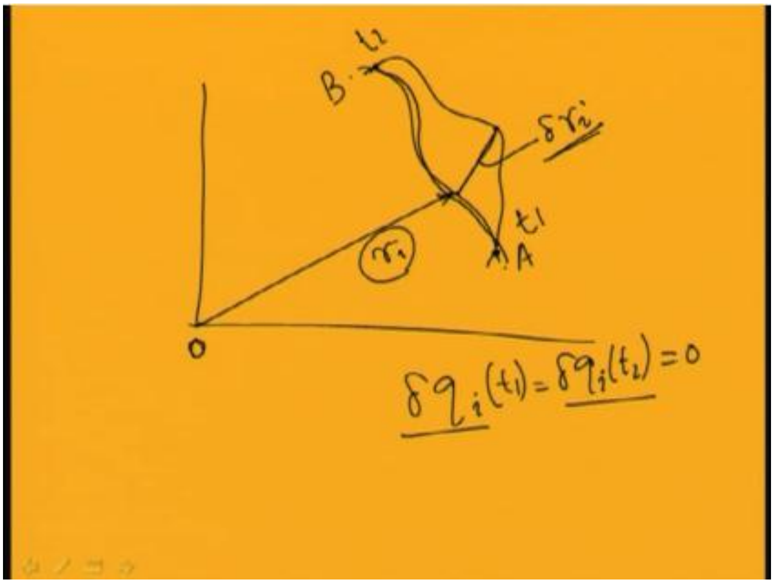

So, if we consider a system let it is moving from 1 position to other position. So, if i consider this as the inertia frame. So, let it is moving from this position to this position. So, I can write this as r i. So, if this is the true path I can take some other path, which are the varied path. So, there may be n number of path between this point to this point let point A to point B. So, there may be infinity number of paths from point A to B, so if A to B. So, this path the paths 1, if this path is the actual path other paths can be considered as the virtual or varied path. So, this delta r i this thing can be written as a r i is the virtual displacement as we are considering this is the varied path. So, this delta r i is the virtual displacement at any time t.

So, we can assume those paths for which at time t 1 and t 2 this varied path equal to this actual path. We can consider those paths in this Hamilton principle or extended Hamilton principle. That means at t 1 delta r i equal to 0 and at t 2 also delta r i equal to 0 or here ri is the physical coordinate. Similarly, if one take the generalized coordinate q i we can write this delta q i at t 1 equal to delta q i at t 2 equal to 0. That means there is no variation at time t 1 and t 2 and that variation equal to 0. So, by using this Hamilton principle or extended Hamilton principle we can derive the equation motion generally this Hamilton principle is used for the conservative system.

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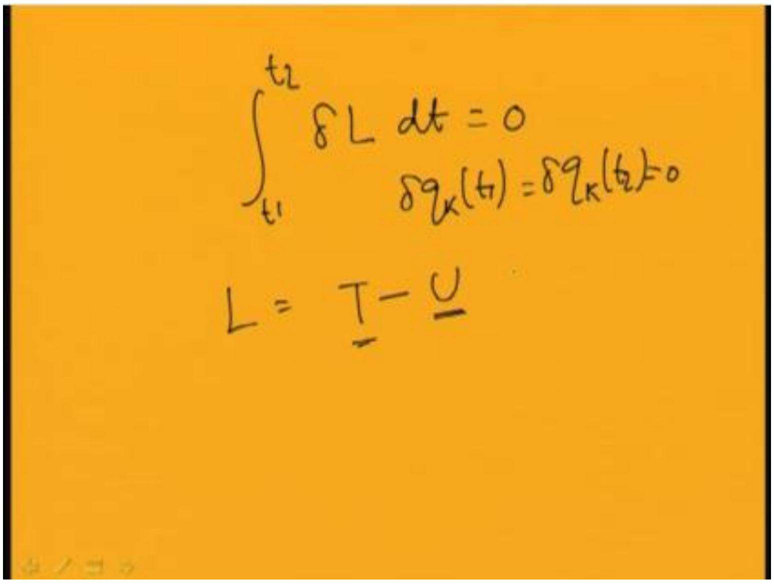

So, in that case this delta W n c equal to 0 and the Hamilton principle will be integral t 1 to t 2 integral T 1 to t 2 delta L d t equal to 0, where delta q k t 1 equal to delta q k 2 equal to 0. In case of non conservative force applied to the system we have to use this extended Hamilton principle. So, here L is the Lagrangian that is equal to T minus U. So, if you know the kinetic energy and potential energy of the system which are scalar parameters we can derive the equation motion. The drawback of Newton's method is that it is a vectorial approach method. So, when the number of systems goes on increasing that time application of this Newton's law will be very difficult. So, we know when we are using Newton's second law or D'Alembert principle we have to draw the free body diagram for each particle. So, when the number of particle goes on increasing. So, developing this free body diagram and finding the equation motion is very difficult.

That is why one can use this Lagrange principle or Hamilton principle for multi degree of freedom system and continuous systems. So, as you can see that this for continuous system. The equations involve this kinetic energy and potential energy equation involve integral and differential terms. So, application of Lagrange equation will be slightly difficult. So, once we would go for this Hamilton principle to derive the equation motion. So, only for simple cases one may find the equation motion by using this Newton's approach. Newton second law, but for complicated cases 1 can go for this Hamilton principle or extended Hamilton principle to find the equation motion. That is why in today class the application of Hamilton principle initially for the simpler cases will be discussed and later it will be extended to some complicated systems.

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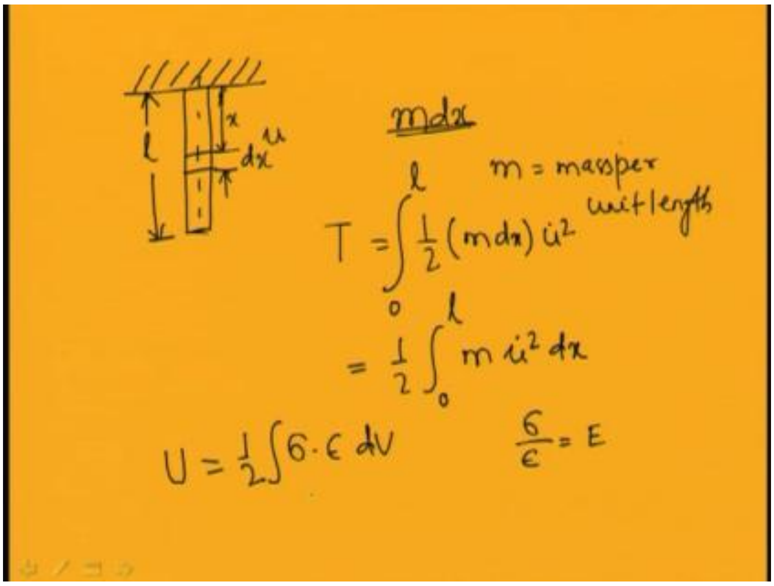

So, let us find or use this Hamilton principle to derive the equation motion for Longitudinal vibration of the rod, already we have found this longitudinal vibration by using Newton's law now let us find it by using this Hamilton principle. So, in this case., so the vibration takes place along the longitudinal direction that is along the length of the beam. So, I can write first the let me take a small element at a distance x. So, from the fixed end let me take at a distance x a small element of length d x. So, if m is the mass per unit length of this rod then I can write the mass of this element equal to m dx. So, m is the mass per unit length mass per unit length. So, mass of this small element equal to mdx, so the kinetic energy of that small element can written as half into mass of the that small element into velocity square of that small element. Let u is the displacement of that element at the section x. So, the velocity will be u dot. So, the kinetic energy will be half mass into. So, it will be half mass that is equal to m d x into u dot square. Now, we have to derive.

So, this is the kinetic energy for that small element. So, the kinetic energy for the total beam will be integration of this from 0 to l for the total length of the beam. Let l is the length of that beam, so total kinetic energy equal to half integration 0 to l m u dot square d x, so where u dot is the velocity at this section which is at a distance x from the fixed end. So, now to derive the potential energy, so potential energy is the strain energy stored in this rod. So, the strain energy is can be found from this expression strain energy equal to half. So, it is equal to half stress into strain stress let sigma is the stress and epsilon is the strain. So, half stress into strain into dv, so half stress into strain into dv dv is the volume or elemental volume of the small element what we have taken. So, in this case I can write the sigma or stress by strain equal to Young's modulus E. So, this epsilon can be replaced.

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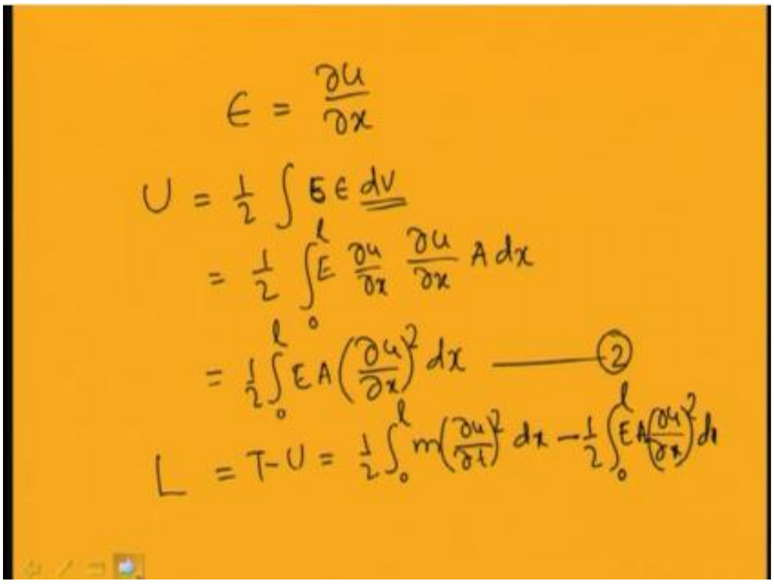

Already we know the epsilon epsilon equal to if U is the displacement then epsilon the strain will be equal to del u by del x. So, epsilon equal to del u by del x. So, this potential energy U can be written as half integration sigma into epsilon into dv equal to half. So, for epsilon I can write this is equal to del u by del x and sigma sigma is the stress. So, at stress by strain equal to E, so one can write the stress equal to E into epsilon, so as one can write it is equal to E into epsilon. So, this becomes E into for epsilon I can put this is equal to del u by del x again and this dv. So, dv can be written as A into dx. So, A is the area of cross section and dx the small element what we have taken. So, the potential energy can be written as half E into A then del u by del x whole square into dx.

So, we have found the kinetic energy of this small element initially that is equal to m into dx into u dot square. And the total kinetic energy of the system becomes half 0 to l m u dot square dx. And now, we have got the potential energy equal to half EA del u by del x square dx. So, the Lagrangian of the system will be equal to L equal to T minus U. So, this will be written as half. So, this is integral 0 to l. So, half 0 to l m del u. So, u dot equal to del u by del t del u by del t square into dx. So, this is equal to m u dot square that is del u by del t square into dx, you may note that U is a function of both space that is x and t that is why one can use this partial derivative in this case. So, this L equal to T minus U that is equal to half 0 to l m del u by del t square into dx as we are assuming only the free vibration in this case or we are considering the free vibration. So, we are not considering the force. So, the work done by the non conservative force equal to 0.

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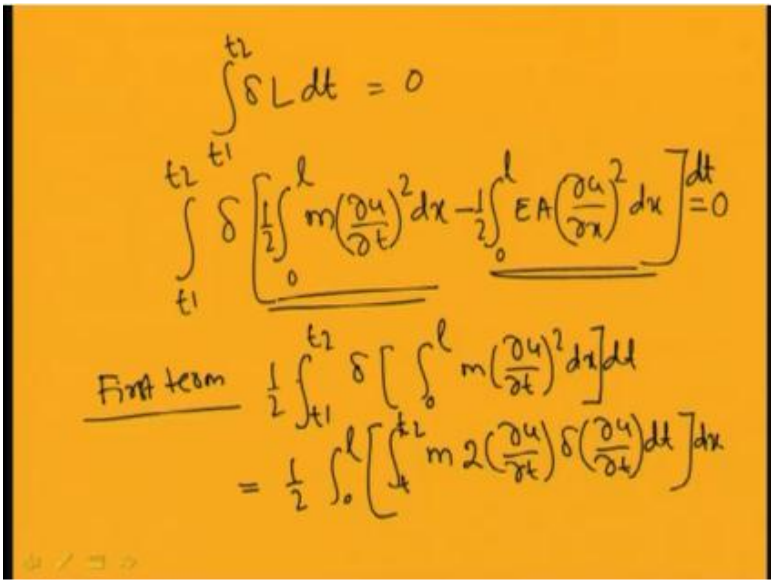

So, the Hamilton principle becomes del of t 1 to t 2 t 1 to t 2 del L dt equal to 0. So, in this case I can write this is t 1 to t 2 del of, so 0 to l. So, I can write this 0 to l half I will take common. So, this becomes m u dot or del u by del t square dx minus 0 to l. So, for this potential energy i can write. So, this l becomes T minus U. So, this is the term for T and for U 1 can write this is equal to 0 to l EA del u by del x whole square dx. So, this term I will write there. So, minus half, so this becomes minus half EA del u by del x whole square dx. So, this is the total term and this would be equal to 0. So, let us take, so it contains this expression contain 2 terms.

So, let us take this as the first term and this as the second term and do this integration by parts. So, by taking this first integral, so the first integral, so the first term I can write this equal to integration t 1 to t 2 del of. So, half I can take it out. So, del of 0 to l m del u by del t whole square dx. So, outside dt is there. So, dt will be. So, this is the first term. So, I can take this del operator inside. So, before taking this del operator inside. I can interchange between this this integral and this integral term. So, I can write this equal to half 0 to l del of or I will take this inside this del. So, t 1 to t 2 t 1 to t 2 m, so this by applying this del operator I can write this equal to 2 into del u by del t into del of del u by del t dt and dx.

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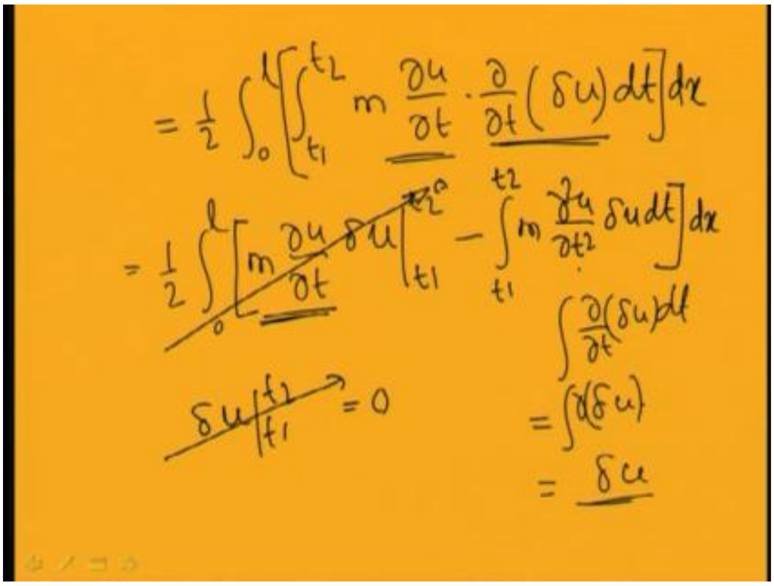

So, this becomes this is equal to half Integral 0 to l t 1 to t 2. So, t 1 to t 2, so this 2 2 cancel. So, m into del u by del t m into del u by del t into. So, this del, so I can interchange between this del and del by del t operator. So, that time I can write this equal to. So, I will write it del by del t into del u into del u dt dx dt into dx. So, now I can integrate this by part. So, I can take this as the first function and this as the second function. So, by taking this as the first function, so I can that write this expression as integration 0 to l. So, this integration I can write as m. So, the first function remain as it is then integration of the second. So, the second term is del by del t of del u.

So, integration of del by del t of del u dt is nothing, but this del u because it is integration of del of del u. So, that is equal to del u. So, I can write m del u by del t into this. So, this is from t 1 to t 2 then minus integration put the bracket here, so minus t 1 to t 2 m. So, derivative of this or differentiation of this into this term, so it becomes m del square u by del t square into del u into dt and whole into dx. So, this term becomes m del u by del t del u t one to t 2 minus t 1 to t 2 m del square u by del t square del u dt dx and already I told you that del q or generalized generalized coordinate variation of the generalized coordinate at t 1 and t 2 equal to 0. So, in this case del u at t 1 and t 2. So, this becomes 0. So, del u at t 2 it will become 0 and del u at t one becomes 0. So, this first term, so these becomes 0. So, now, this first term equal to…

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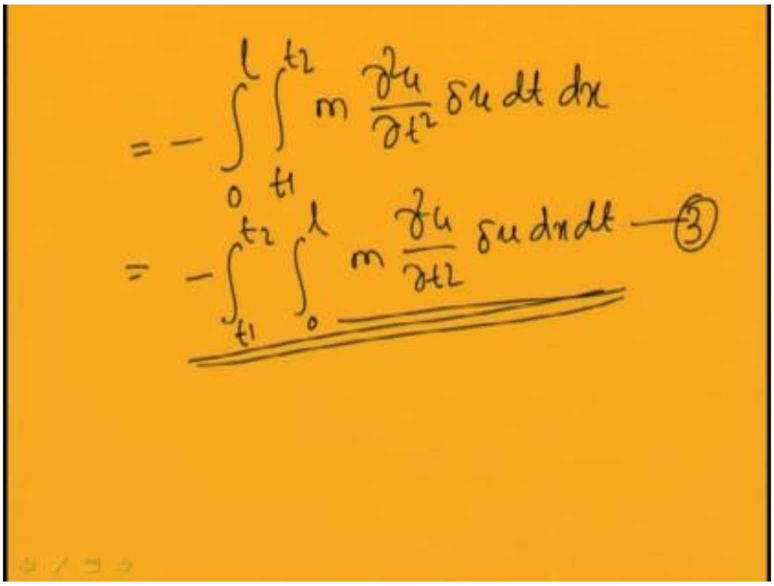

So, the first term I can write equal to minus. So, this is minus integration 0 to l. So, this half term was not there. So, already this half and half canceled. So, the half term was not there here. So, this becomes minus integral 0 to l then t 1 to t 2 m del square u by del t square into del u dt and dx. So, I can interchange now and I can write it this way t 1 to t 2 minus 0 to l m del square u by del t square into del u dx dt. So, this is the obtained after the integration by parts of the first term. So, now taking this second term the second term is this is the first term I have taken and this is the second term and the second term you just note that there is a negative sign. So, I can write this second term as minus. So, l will take the second term.

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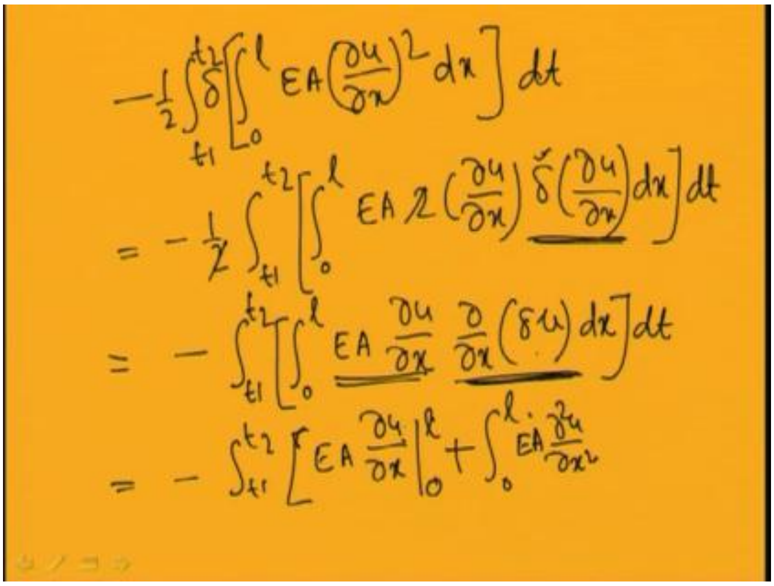

And write this as minus integral t 1 to t 2 half. So, this is 0 to l EA del square. So, this is del u by del x del u by del x whole square. So, I have to operate this del operator here on this. So, into dx dt. So, this is the second term. So, now, applying this del operator I can write this as this is half t 1 to t 2 0 to l EA. So, this term can be written as 2 del u by del x into this del operator on this del u by del x into. So, this is dx and finally, it is dt. So, this thing can be written as minus half. So, this term, so this 2 2 cancel. So, this half term is not there. So, minus integration t 1 to t 2 0 to l EA del u by del x into like the previous case here also we can interchange between this operator and this operator. So, I can write this as del by del x of del u dx dt.

Like the previous case here also I can apply the integration by parts. So, taking this as the first function and this as the second function I can write this equal to minus t 1 to 2 0 t 1 to t 2 into EA del u by del x first function remain as it is. So, it is from t 1 to t 2 it is from 0 to l this integration is taking place from 0 to l. So, it is from 0 to l minus, so minus minus plus, so plus integration 0 to l 0 to l EA. So, differentiation of this, so the differentiation of this becomes del by del x of del u by del x. So, if I am assuming this EA to be constant then I can write this as EA del square u by del x square. And integration of this equal to already proceeding in the previous way we have seen that del u by del by del x of del u integration of this term equal to equal to del u. So, I can write it again.

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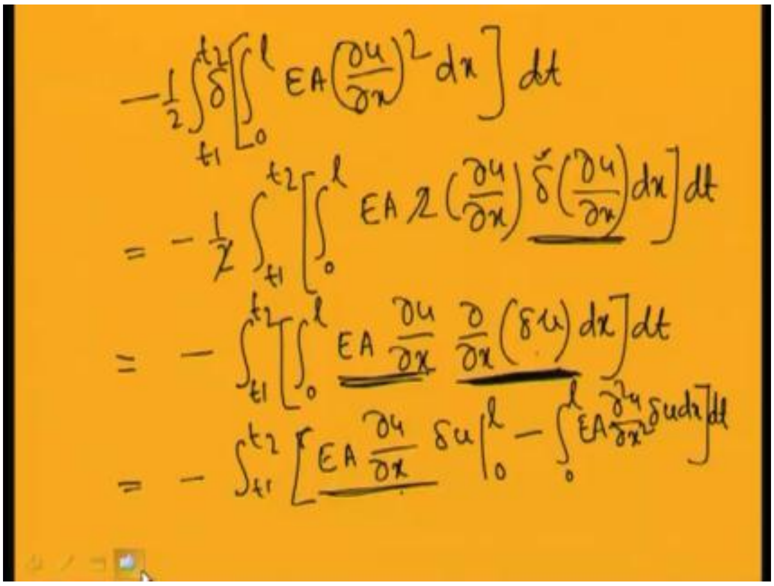

So, I have to integrate it by parts. So, this is the first function and this is the second function. So, while I am doing integration by parts. So, this is the first function remain as it is then integration of the second. So, the integration of the second as previously you have seen. So, this integration is nothing but del u. So, this is from 0 to l then minus integration 0 to l. Differentiation of the first function that is EA del square u by del x square if you are assuming EA to be constant then it will be this otherwise it will be del by del x of EA del del u by del x into del u into dx into dt.

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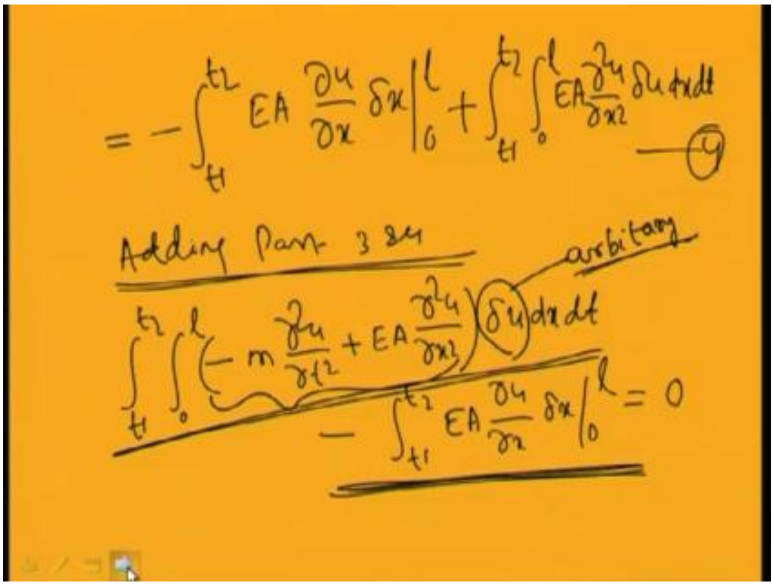

So, this term can be written as minus integration t 1 to t 2. So, t 1 to t 2 EA EA del u by del x into into this. So, with boundary condition 0 to l then minus minus plus I can put. So, this is equal to t 1 to t 2 0 to l EA del square u by del x square del u dx dt. So, now, I have got this equation 4 and this is the for the third part second part and equation 3 for the first part. So, applying this Hamilton principle, so I can add this part 3 and 4 adding. So, adding part 3 and 4 I can write, so t 1 to t 2 0 to l t 1 to t 2 0 to l minus. So, in this case you can see this term equal to minus m del square u by del t square. So, I can write this as minus m del square u by del t square plus EA del square u by del x square into del u dx dt minus t 1 to t 2 EA del u by del x into 0 to l equal to 0.

So, applying Hamilton principle we have found the expression or the equation to be reducing to this form. So, in this equation you can note that the first part. So, as this plus this part first part plus the second part becomes equal to 0. So, individually they will be equal to 0. So, this part you can note it this is the boundary condition it gives the boundary condition and if you note this part the first part. So, as del u is arbitrary. So, this is arbitrary it can take any value as this is the virtual displacement it can take any value. So, as this part is arbitrary. So, the integral will be equal to 0 if and only if this part equal to 0.

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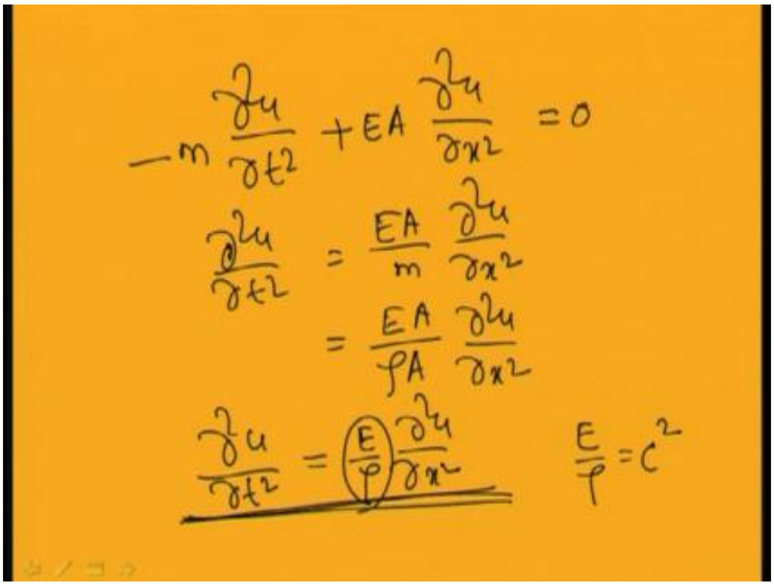

So, this part equal to 0 equating this part equal to 0 that means minus m minus m del square u by del t square plus EA del square u by del x square equal to 0. We can write this a del square u by del t square equal to EA by m del square u by del x square. So, this m is mass per unit length. So, I can write this equal to rho into A, so mass per unit length. So, this can be written as E into A and this mass per unit length will be density into area. So, if rho is the density of that rod. So, it will be density into area into del square u by del x square or I can write this as del square u by del t square equal to E by rho del square u by del x square. So, previously we have seen or we have derived this expression and we have written this E by rho equal to C square. So, E by rho equal to C square previously we have written. So, I can write this equation.

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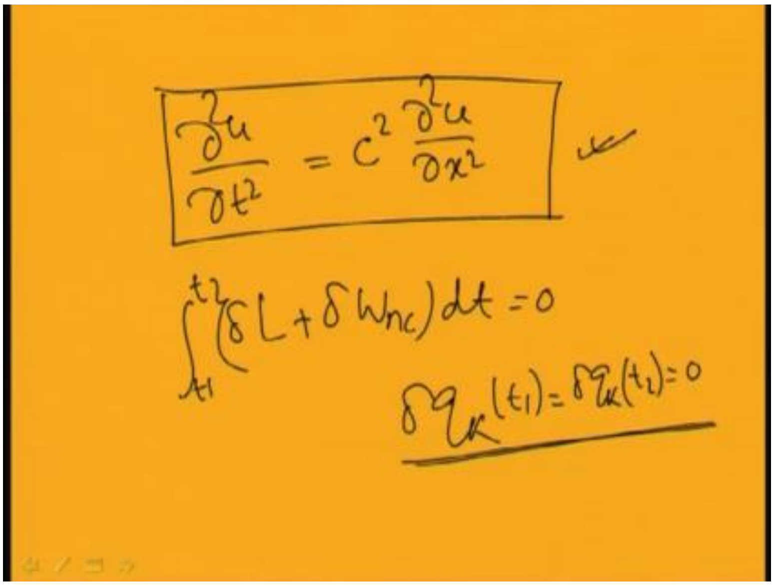

In this form del square u by del t square equal to C square del square u by del x square. So, in this way by applying the Hamilton principle you can derive the equation motion of the longitudinal vibration of a rod. So, when we are applying the Hamilton principle first derive the kinetic energy expression for the kinetic energy expression for the potential energy and work work function. So, after finding all these 3 parameters you can apply the Hamilton principle by writing t 1 to t 2 or finding this t 1 to t 2 this del of del l plus del W nc dt equal to 0. So, where del qk at t 1 equal to del qk that is virtual virtual displacement at t 1 and virtual displacement at t 2 equal to 0. So, by applying this rule you can derive the equation motion of the system also in this in this case you can find along with the equation motion you are finding the boundary conditions of the system.

So, this term the second term in this. So, when you are adding this part 3 and 4 the second term that is t minus t 1 to t 2 EA del u by del x into del x 0 to l equal to 0 will give the boundary condition of the system. So, in this case you can note that either this EA del u by del x equal to 0 or this. So,. So, you have a del u term here. So, or this del u equal to 0 for both are 0 at this l or 0 to satisfy this equation. So, to satisfy this condition this EA del u by del x will be equal to 0. That is or del u by del x will be equal to 0; that means, the slope equal to 0 or the displacement equal to 0. So, in case of longitudinal vibration of the rod, so either the slope will be 0 or the displacement will be 0 or both slope and displacement will be 0 at the ends. So, if you do not know the boundary conditions by applying this Hamilton principle, you can find the equation motion along with the boundary conditions of the system

So, the actual boundary conditions will depends on the the system you are taking. So, as we have taken a very general system that is why we are getting all the boundary conditions which is written in this form. So, all the boundary conditions are either at L del u by del x will be equal to 0; that means, the slope will be equal to 0 or del u that is the virtual displacement that will be equal to 0. Similarly, at x equal to l that is at the left end the slope will be equal to 0 or the displacement equal to 0. Last class we have seen all the boundary conditions or all possible boundary conditions for the longitudinal vibration of rod, torsional vibration of the shaft and transverse vibration of the string. So, now let us derive the equation for beam subjected to pure bending.

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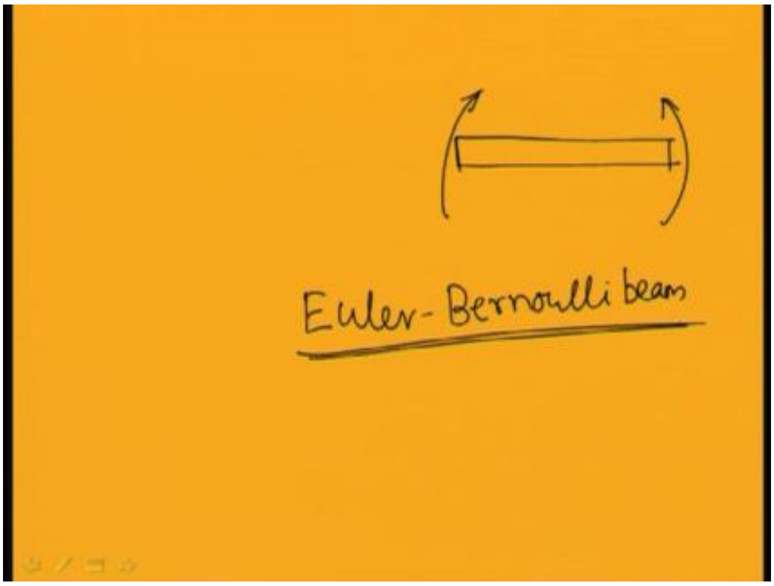

So, when a beam is subjected to pure bending. So, beam is known as Euler-Bernoulli beam, the Euler-Bernoulli beam the beam subjected to pure bending is known as the Euler-Bernoulli beam. So, we have to derive the equation for a vibration free vibration equation for the Euler-Bernoulli beam. So, in this case by taking a small element of the beam we can derive the equation motion by applying the Newton's second law, but now we are going to derived this equation from the Hamilton principle. So, to apply the Hamilton principle.

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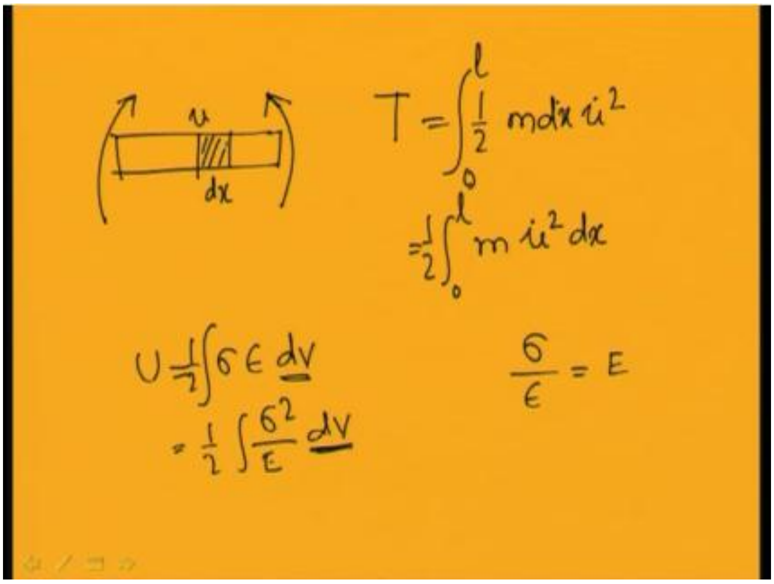

In case of this beam subjected to bending. So, when this beam is subjected to bending we can write the kinetic energy of the system we can write the potential energy of the system and then we have to find the equation motion. So, by taking a small element similar to the previous case let the small element of length dx. So, for the small element the kinetic energy will be equal to half m into let u is the displacement at this end. So, it will be equal to u dot half m dx m dx is the mass of that small element into u dot square that is the velocity square. So, the kinetic energy of that small element equal to half m dx u dot square. So, the kinetic energy for the whole beam it will be equal to integral of this half m dx u dot square from 0 to l.

So, I can write this T equal to T equal to integral 0 to l. So, half I can take it out. So, this becomes m into u dot square dx now, to derive the potential energy. So, potential energy is same as the strain energy. So, the strain energy I can derive it similar to the previous case. So, this is equal to half. So, this is equal to half stress into strain into dv. So, where v is the dv is the elemental volume I have considered sigma is the stress and epsilon is the strain of that element. Already we know this the relation between stress and strain. So, stress and strain can be written as stress by strain equal to E. So, I can write this expression equal to. So, this this will be equal to for the strain I can substituted by sigma by E. So, this becomes sigma square by E dv. So, this volume elemental volume, so during bending.

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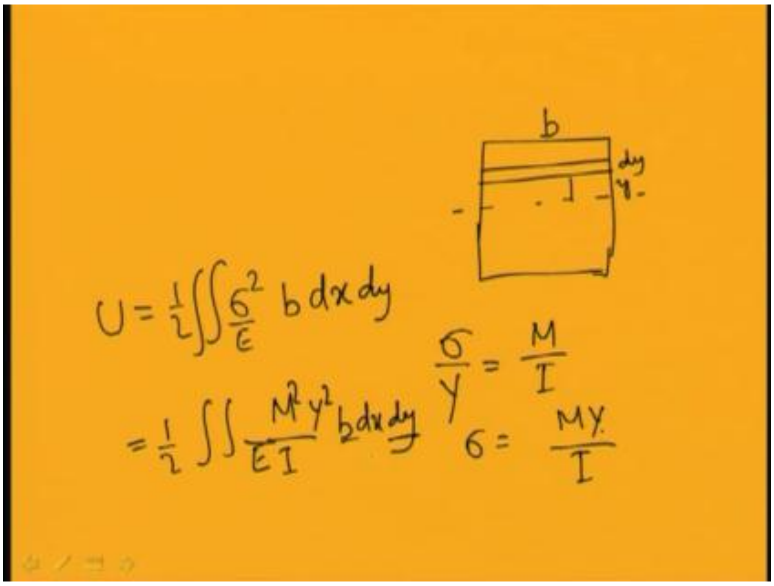

So, I can draw the section cross section of the beam during bending. So, this is the cross section of the beam. So, this is the neutral axis if I am considering a small element here that is at a distance y from the neutral axis let me consider this is dy. So, this width b you can assume this width b is constant. So, this dv will becomes d into dy into dx. So, I can write this U equal to. So, integral, so I can write this U equal to integral half sigma square. So, this is sigma square. So, I can write equal to half sigma square by E into. So, for this dv I can replace it by. So, dv as I am writing dv equal to b dx dy. So, I can put another integral. So, this will be equal to I can I can write the integral here. So, this will be equal to b into dx dy.

Now, the sigma for pure bending we know the relation between this stress and moment. So, sigma this sigma by I, so this I zz, so sigma by I will be equal to M by I. So, sigma by Y equal to. So, we know this expression sigma by Y equal to M by I or I can write the sigma will be equal to sigma will be equal to MY by I. So, I can substitute this in this expression. So, this becomes half sigma square for sigma square I can write this equal to M square Y square by I into b dx dy. I can write this b dy equal to da and this expression can be written. So, there is E term is also there. So, M square by Y M square Y square by EI this. So, I can write this expression equal to Half.

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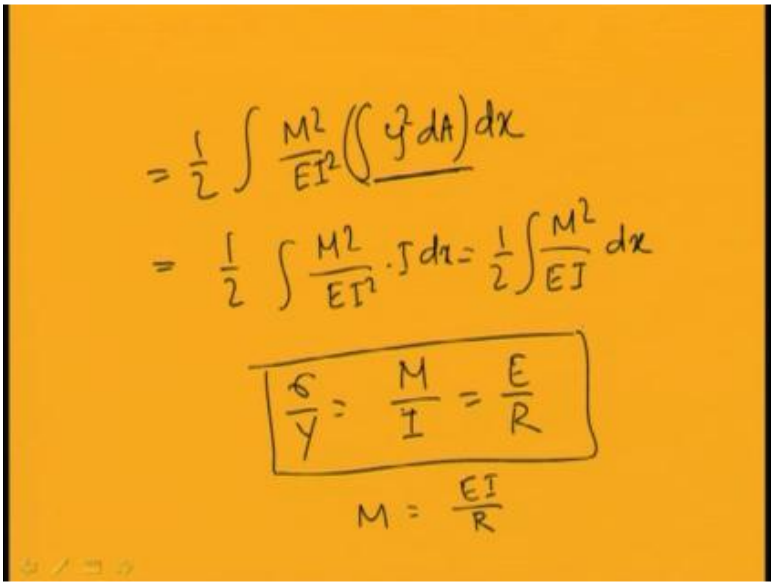

So, this is equal to M square by EI I can take it out M square EI integral. So, this becomes y square dA into dx, but this. So, this is M square. So,. So, for sigma I can substitute this is equal to MY by I, so MY by I. So, this becomes square of this will become M square Y square by EI square, so in this case M square by EI square. So, this integral of y square dA is nothing but the moment of inertial that itself. So, I can write this equal to. So, M square by EI square into i dx or this is equal to half M square by EI dx, but already we know this M by i equal to or M by Y sigma by Y equal to M by i equal E by R. So, this is the bending equation well known bending equation you have studied in case of strength of material. So, this for this M I can substitute this is equal to EI by R. So, this M equal to EI by R, but this 1 by R, that the curvature can be written equal to.

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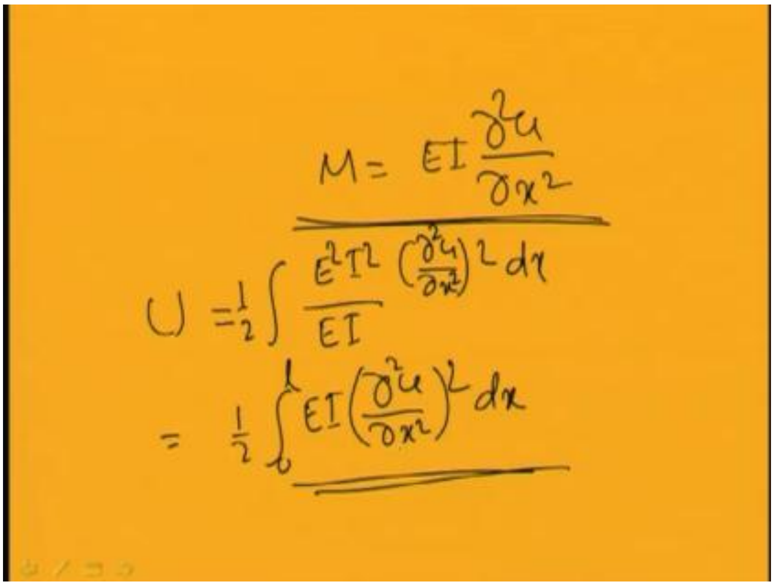

So, this M can be written as EI del square u by del x square. So, for M I can substitute this is equal to EI del square u by del x square. So, this U potential energy or strain energy of that section can be written as half for M square i can substitute it equal to M square by EI. So, I can write this is equal to half E square i square del u by del x whole square by EI. So, this becomes half integral 0 to l I can substitute EI del u by del square u by del x square whole square del square u by del x square whole square dx. So, in this way you can find the expression the potential energy of the beam in bending. So, this becomes U equal to half 0 to l EI del square u by del x square whole square dx.

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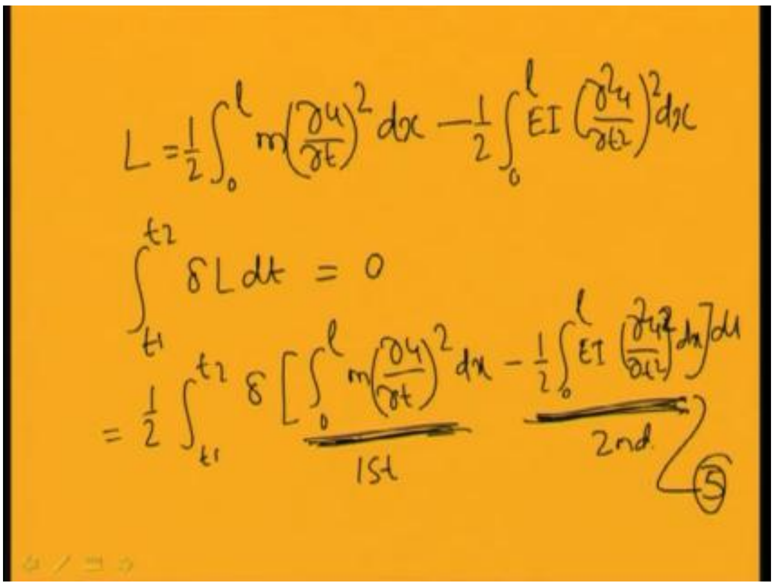

So, now the Lagrangian of the system becomes L equal to integral 0 to l. So, that is half. So, mass this m is mass per unit length m into del u by del t u dot equal to del u by del t square into. So, I can write this into dx. So, this is the t that is the kinetic energy of the system minus the potential energy of the system can be written as 0 to l. So, this is equal to EI del square u by. So, this is EI del square u by del t square whole square into dx. So, this is the Lagrangian of the system. So, now by taking this Lagrangian of the system when you are studying the free vibration we can take this work done or virtual work done equal to 0. So, the Hamilton principle can be written as this t 1 to t 2 del L dt equal to 0.

So, I will apply this del operator to this L and I can write this expression as t 1 to t 2 del of. So, I can take this half out. So, del of in bracket I can write. So, this is 0 to l m del u by del t whole square dx minus half 0 to l EI del square u by del t square whole square dx into dt. So, you may note that this already you have done this part in case of the longitudinal vibration of the rod. This part is similar to that in in case of longitudinal vibration of rod, but only this part is different. So, let me derive only the second part the first part can be written as. So, if you write the first part. So, this is the first part and this is the second part. So, the first part you can note it from this which is written in expression number 3. So, the first part can be written like this.

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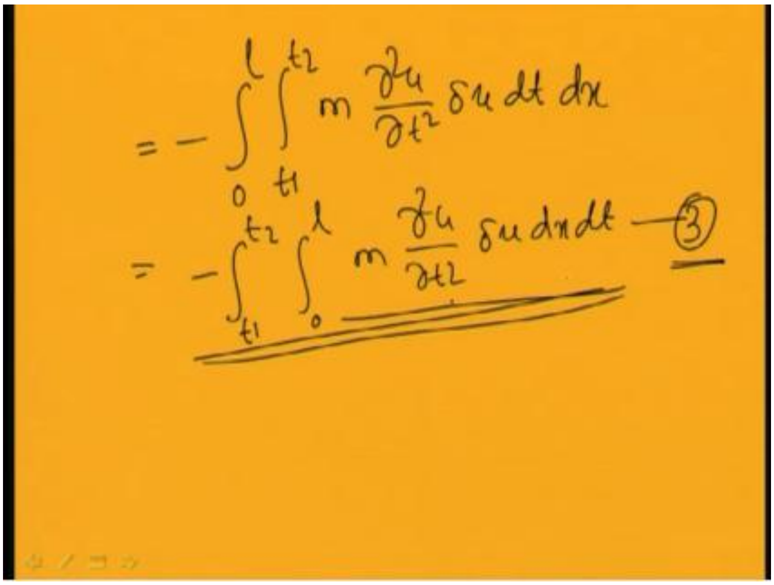

First part equal to minus t 1 to t 2 0 to l m del square u by del t square del u dx dt. So, I can write the first part in the similar way. So, the first part is written as minus half t.

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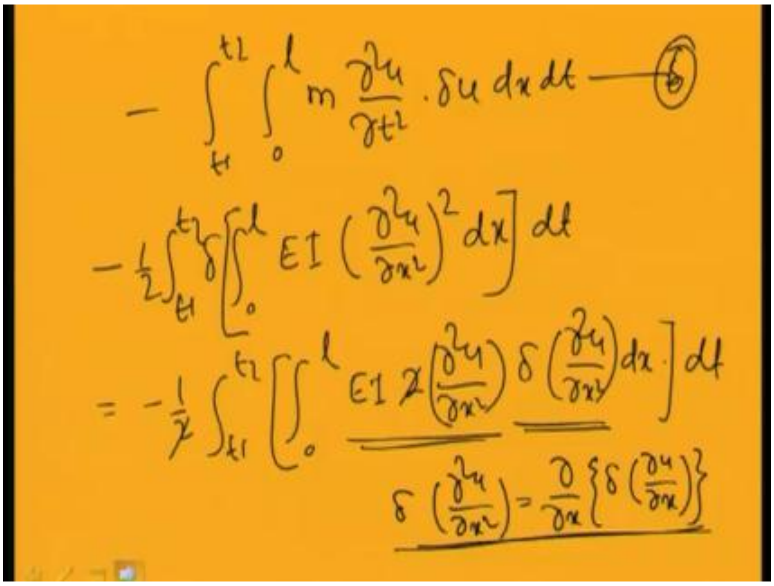

So, that is no half. So, t 1 to t 2 0 to l m del square u by del t square into del u dx dt. So, this is the first part now I can write. So, this is let me put this equation number 5. So, this is equation number 6. So, the second part of the equation number 5. I have to derive. So, this becomes half of. So, I have taken this half out. So, there is no half here. So, this becomes minus integral t 1 to t 2 0 to l EI del square u by del x square whole square dx dt and I have to apply this del operator here. So, there is a half term here. So, this becomes minus half. So, i will use this del operator inside. So, this becomes minus half t 1 to t 2 0 to l then EI. So, applying this del operator inside. So, this becomes 2 into del square u by del x square into. So, this part I can write del of del square u by del x square into dx into. So, outside I can write dt. So, this 2 2 cancel. So, this becomes minus half.

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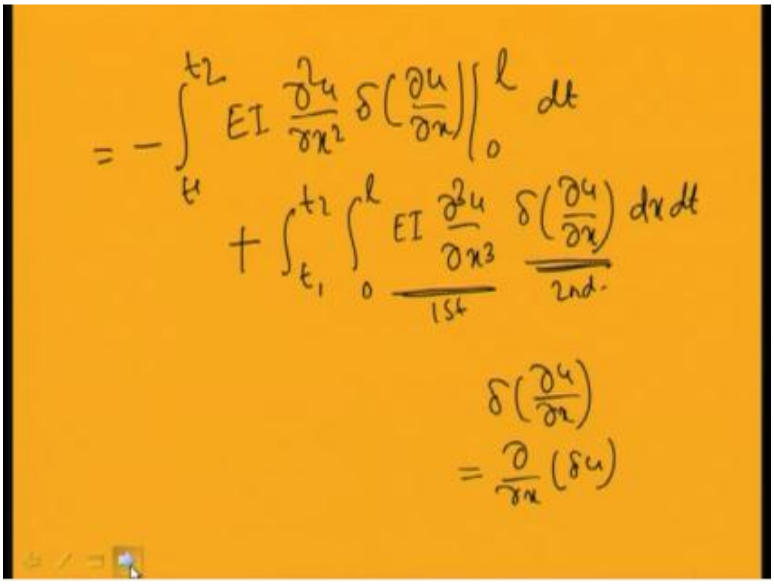

So, half is at there, so integral t 1 to t 2 now I will use this as the first function and this as the second function. So, in this case I can interchange between this and I can write this del del square u by del x square as del by del x of del of del u by del x. So, by writing this way and applying this integral integration by parts I can write this expression equal to half t 1 to t 2. So, this is half t one to this becomes half t 1 to t 2 and this term becomes EI. So, first term remain as it is. So, EI del square u by del x square into. So, you can note that integration of the second term del by del x of del of del u by del x.

So, this becomes. So, del of del u by del x del of del u by del x at the boundary 0 to l minus, so this minus minus plus t 1 to. So, this is dt minus t 1 to t 2 0 to l. So, I can write this differentiation of this. So, that is EI del cube u by del x cube into del of del u by del x into dx dt. Now, again you can take this term as the first function and this as the second function and you can interchange between this del and the separator. So, this del of del u by del x you can write this as del by del x of del u. So, in that case this is the first function and this is the second function. So, this integration becomes I am writing only this part.

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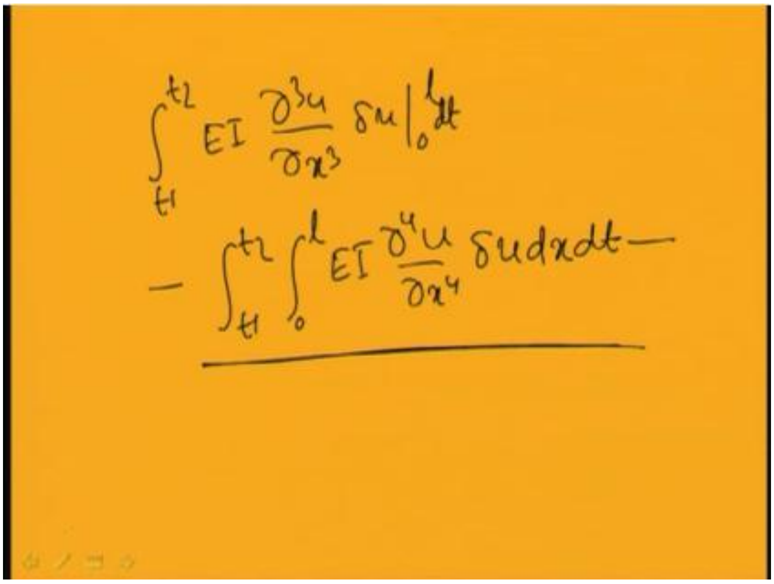

So, this integration or the, this integration I can write equal to T 1 to t 2 t 1 to t 2 EI del cube u EI del cube u by del x cube into this integration will becomes del u into del u 0 to l minus t 1 to t 2 0 to l. So, this is dt is there. So, 0 to l I can write this expression below. So, this is minus t 1 to t 2 0 to l. So, differentiation of the first term that will give me EI del four u by del x four into del u dx dt. So, this becomes EI del four u by del x four into del u dx dt. So, this is. So, I have to add for this expression for this expression I have found this expression. So, the total expression will become. So, this, so the total expression will contain this term plus these 2 terms and the expression given in 6. So, by adding these terms I can write the integral.

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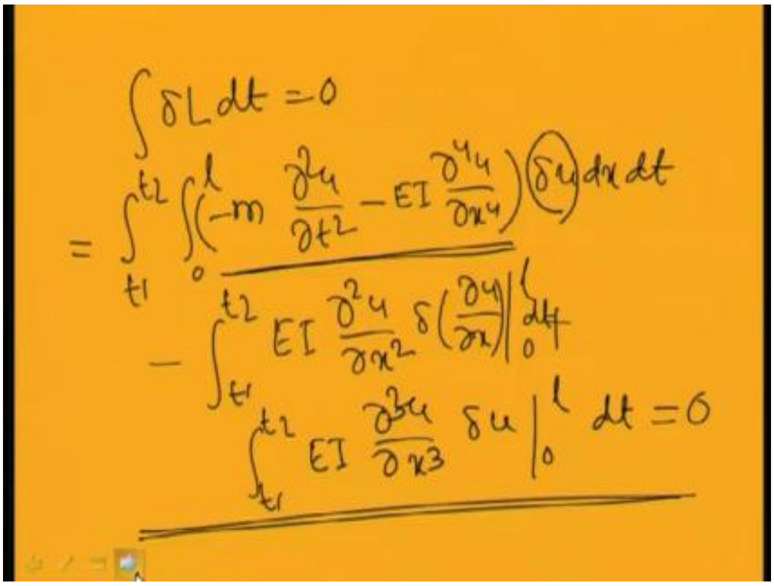

So, the integral del L dt equal to 0 can be written as. So, this will be equal to t 1 to t 2 0 to l m del square u by del t square. So, there is a negative sign here plus. So, I can write this plus or I have a negative sign here again. So, this is minus minus EI del four u by del x four into del u dx dt. So, this is the term and I have to add these 2 more terms that is minus t 1 to t 2 minus t 1 to t 2 minus integral t 1 to t 2 EI del cube u by del x cube into del of del u by del x 0 to l. So, this term and another term is there. So, this is the other term. So, this is plus. So, I can write this is plus plus integral. So, this t 1 to t 2 I can write here.

So, this is t 1 to t 2 integral t 1 to t 2 EI del cube u by del x cube.So, here, so in the first expression this becomes del square u by del x square and here it is. So, this is del square u by del x square and in the next term it becomes EI del cube u by del cube this is cube del x cube into del u at 0 to l dt equal to 0. So, this becomes the final expression or after applying this Hamilton principle we obtain this expression. So, this is equal to 0. So, now, in the like the previous case here also this del u that is the virtual displacement is arbitrary. So, this whole term will be equal to 0 when this integral term or this terms between this bracket will vanish. That means, this minus del square u by del t square minus EI del four u by del four equal to 0.

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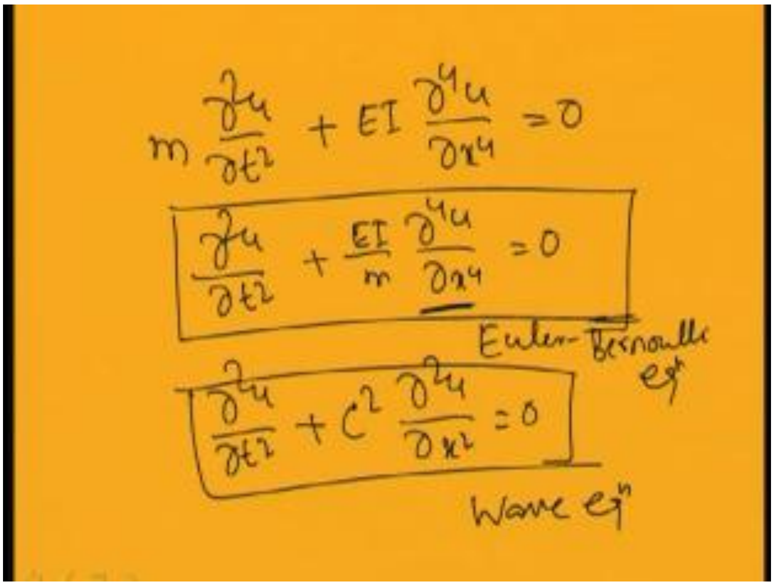

So, writing that term. So, I can write m del square u by del t square plus. So, that is a minus minus. So, it will becomes plus. So, EI del four u by del x four equal to 0. So, this expression can be written in this form also del square u by del t square plus EI by m del four u by del x four equal to 0 this equation is known as the Euler-Bernoulli equation. So, unlike the wave equation you can see here this is a forth order equation. So, this is del four u by del x four and in case of wave equation the expression was del square u by del t square plus C square del square u by del x square equal to 0. So, this is the wave equation and this is Euler-Bernoulli equation.

So, here we have derived the Euler-Bernoulli equation by applying this Hamilton principle along with this as we have taken the beam to be of general type we can find all possible boundary conditions also. So, from these 2 expressions you can find the boundary conditions. So, for this general case we have found the equation motion by applying this Hamilton principle. So, in this case we have seen by applying Hamilton principle you can get the equation motion and the boundary conditions. So, today class, we have studied or we have found the equation motion for longitudinal vibration of rod and transverse vibration of beam by using Hamilton principle. And next class, we will study how to solve this wave equation and Euler-Bernoulli equation for different boundary conditions.