Mechanical Vibrations
Prof. S.K. Dwivedy
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module – 9
Lecture - 1
Derivation of equations of motion part 1
Newton's and Hamilton's Principle

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Today, we are going to study and the vibration of continuous systems. Previously we have studied about the single degree of freedom systems, 2 degree of freedom system and multi degree of freedom system. So, you have seen previously. We have modal all the systems like this spring and mass systems. So, this is the mass attached to this spring. So, these are the springs, so we are modeling the stiffness of the system by the springs and this mass. So, already we have studied about the single degree of freedom system. So, this is a single degree of freedom system. So, any machines you can model as a single degree of freedom system with the help of spring and mass and damper. So, when this system is subjected to some force. So, when it will vibrate you can model these as a single degree of freedom system with force vibration and to observe that vibration already you have seen. So, you can add another spring and mass system to it.

So, the secondary spring and mass systems you can add such a way that the frequency of excitation of the external or the disturbing force should be equal to the natural frequency of the secondary system. So, in the secondary system let the mass is m and the stiffness is k or mass is m 2 and stiffness is k 2. Then the frequency of excitation should be equal to root over k 2 by m 2 to observe all the vibration of this primary system. So, this is a 2 degrees of freedom systems and we have studied that this 2 degrees of freedom system can be modeled by a 2 is to 2 mass matrix and 2 is to 2 stiffness matrix. And 2 is to. So, 2 rows and 1 column force vector.

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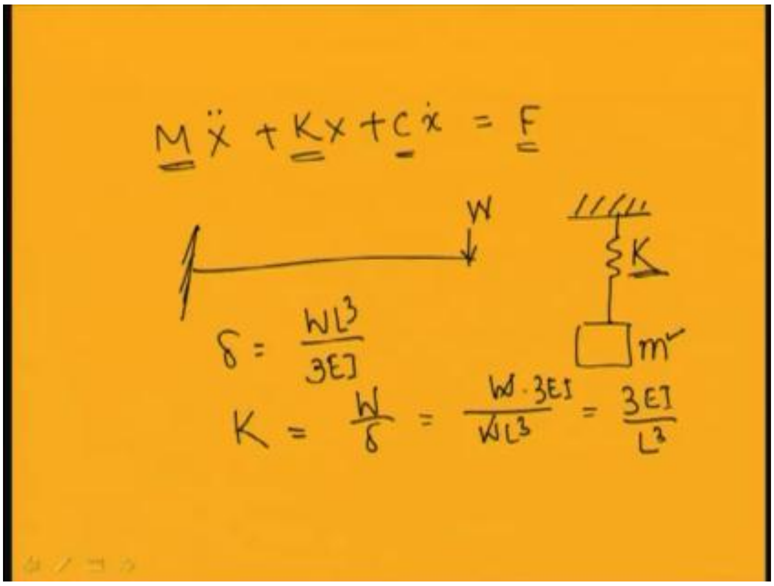

So, you can write the equation motion of 2 degrees of freedom system by using this equation MX double dot plus KX or if you have damping present then you can add this damping matrix also equal to F. And in case of a multi degree of freedom system, so M will be replaced by the n cross n. So, we are n is the degrees of freedom of the system. And this K matrix also will be any n cross n matrix and C is also a n cross n matrix and for this F which is a n cross 1 vector. So, in this way already you have seen to model a discrete mass system.

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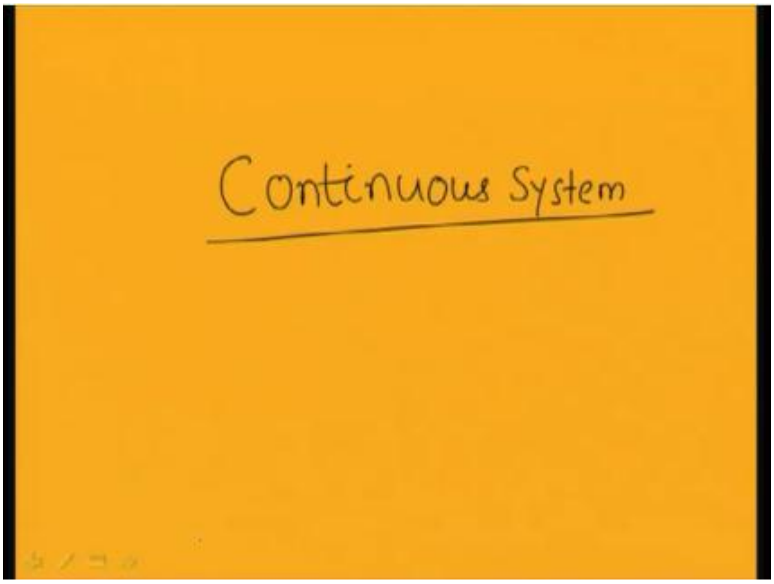

So, you can have a mass matrix stiffness matrix, and damping matrix and a force vector, but always you cannot model all the systems as this discrete mass and damper system. Let us take this system

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This beam, so this beam initially you have modeled this as a, so this is beam. So, if I will take a beam cantilever beam let me take a cantilever beam. So, in case of a cantilever beam this side is fixed and 1 side is fixed and other side is free. So, this is the figure of a cantilever beam and I have attached a mass here. So, this is a cantilever beam with a mass. So, its vibration so you are considering this equivalent to a spring and damper system previously. So, you are considering this as a spring and damper system with stiffness k and mass m. So, mass of the stiff mass you are taking m and this stiffness you are calculating for this system. So, in this system you know if you are applying a load W then it will have a deflection at this end delta equal to WL cube by 3 EI. So, you found this stiffness k equal to W by delta. So, this will be equal to W by WL cube by 3 EI. So, this becomes W W cancels. So, it becomes 3 EI by L cube. So, you have taken the stiffness and this mass and calculated the natural frequency of this system.

But you may note that this natural frequency is only the approximate natural frequency, because you have not considered you have you have not considered all the properties of the system. So, this whole system can be modeled as infinity number of mass and spring. So, if there are infinity number of mass and spring in the system, this continuous system then you has infinity number of natural frequency associated with this continuous system.

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So, you should not model it has a single spring and mass system and found the natural frequency or found or find the response of the system, so to model it completely you should model this as a continuous system and to model the. And in today class will see how to model this as a continuous system and also you may have some rod.

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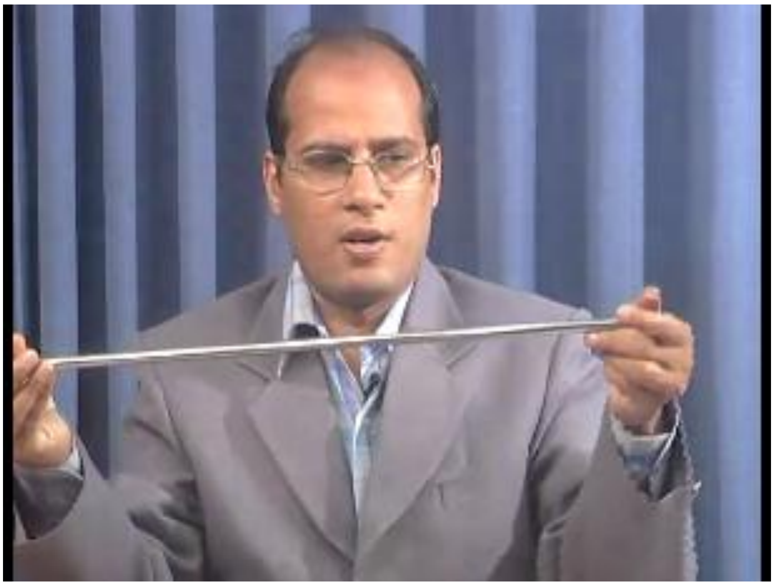

So, in this rod, it is subjected to some torsional vibration. So, when it is subjected to this rotational or torsional vibration then we can also find the vibration or the response of the system. So, in this case initially, if when we have studied about the wheeling of shaft we have model these as a single degree of freedom system, but now, we should model this as a continuous system and study its vibration.

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So, this and also we may take the longitudinal vibration of a beam or rod, so in this rod I can find the longitudinal vibration of this rod also. So, by fixing 1 end and by applying a load at the other end by applying a load at the other end by fixing 1 end I can find the longitudinal vibration of this rod. In case of longitudinal vibration, the vibration will takes place along the length of this rod. And in case of transverse vibration the vibration will takes place transverse in a transverse direction that is perpendicular to the direction of this rod. So, either it will vibrate in this direction or it may vibrate in this direction. So, depending on the radius of gyration the vibration will take place.

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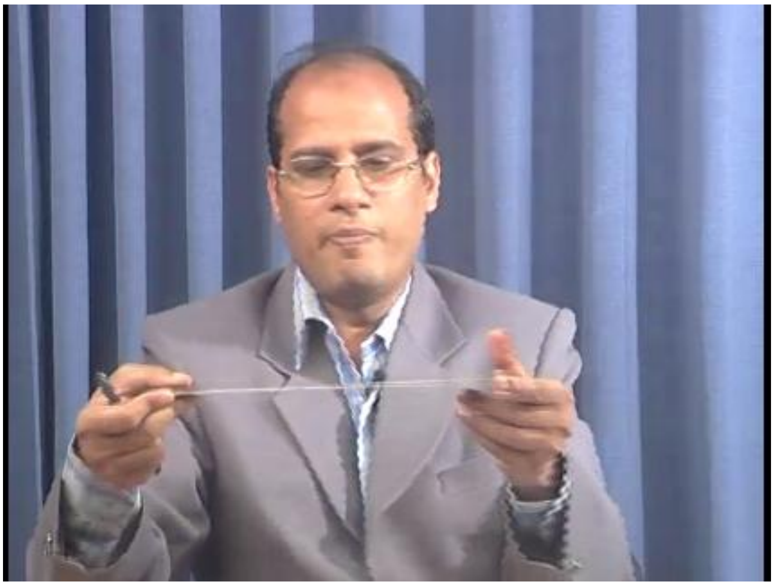

So, let us take this simple aluminum beam, so in this case the vibration. So, if I will push it the vibration will take place in this transverse direction and if I will take this way and push it will not vibrate. So, you can note that in this direction its inertia is more compared to in this direction that is why in this direction it has a vibration, but in this direction when I am rotating this thing or I am keeping it in this there will be no vibration in this direction. But it will have a vibration in this lateral direction. So, this lateral direction it has a vibration, but in this direction there is no vibration and in the longitudinal direction also there is no vibration.

So, the vibration takes place in a direction in which. So, it has least radius of gyration or least moment of inertia or least resistance is given by the system. So, we can, today class we will go to study about this type of continuous system. So, we may take the case of the strings also. So, already you have seen in musical instrument the strings or the strings you can find as cable in the electrical wire. So, those string systems also we can consider them has a continuous system and we will study that thing. So, there are many different types of continuous systems also. So, in the in some systems you may have.

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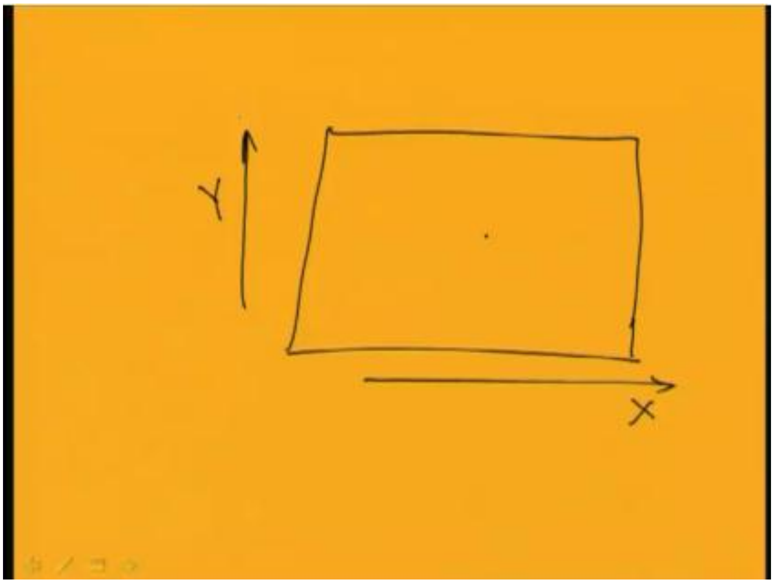

So, you can take a plate type of system; so in case of a plate, so it is a 2 dimensional system. In this case the vibration may takes place in X direction or the vibration may takes place in Y direction also. So, if you have a thick plate the vibration may take place in X Y and Z direction, but in case of a thin plate the vibration you can assume that the vibration will take place in this Z direction. And this transverse vibration taking place in Z direction will very along the X axis and also it will vary along the Y axis. But we are limiting our study to only 1 dimensional system like string.

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So, we will study about the vibration of the string and we will study also the longitudinal vibration of rod longitudinal vibration of rod and lateral vibration of beam lateral or transverse vibration of beam. And vibration of beams also we are going to study the torsional vibration of rod torsional vibration of rod. So, in these cases we can see. So, all these cases are single degree of or 1 dimensional system and we are going to find the equation motion in all these cases and we are going to study the response in these cases.

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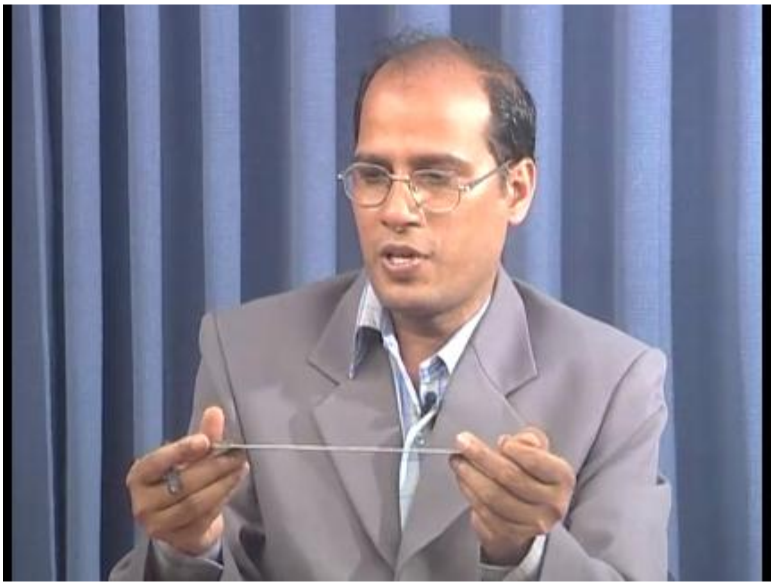

So, as you can see in these cases in this continuous system the vibration will depends on 2 parameters on like in case of the spring mass system.

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So, in the spring mass system the response of a point the response of a point only depends on the time, and the initial conditions initial velocity and displacement of the system. But in this case in case of the continuous system let us take this beam.

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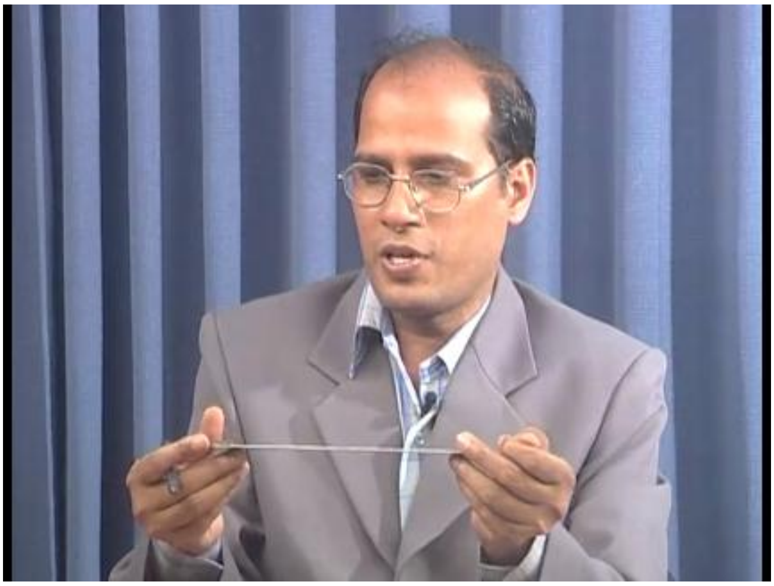

So, in this case the vibration will depends on the position. That means, we have to find at which position we are interested to find the vibration whether it is taking place at this end whether a taking place at the middle or at the at this point. So, it depends on the position of the system position or location of the points on this beam and also it depends on the time. So, it depends on 2 parameters; one is the location or the space parameter or the space coordinate and second one is the time. In case of discrete system it depends only on time. But in this case of continues or distributed mass system is depend on the space coordinate and the time. And we can find, so using these 2 coordinates that the space coordinates and time.

So, we have to develop the equation motion. So, you have seen in case of discrete systems. So, the equation motions are written in terms of ordinary differential equations and later it was converted to the Eigen value problems by using matrix algebra. So, we have converted that thing the equation motion to a set of algebraic equations and solving those algebraic equations. Or finding the Eigen values of the system we have determined the natural frequency of the system. But in this case in case of continuous system as it depends on 2 parameters that is the space coordinate and the time. So, in this case the equation motion will be written in terms of partial differential equations.

So, in place of ordinary differential equation in case of discrete system in case of distributed or continuous systems we will have partial differential equations. And these equations or we have to find the Eigen functions in this case in case of the discrete system we have found the Eigen values for the natural frequencies or the square of the natural frequency where the Eigen values of the system. But in this case we have to find the Eigen functions of the system also you can note that in case of continuous system, it depends on the boundary conditions. So, how the boundary are maintained it the equation motion depends on that. So, in case of a cantilever beam.

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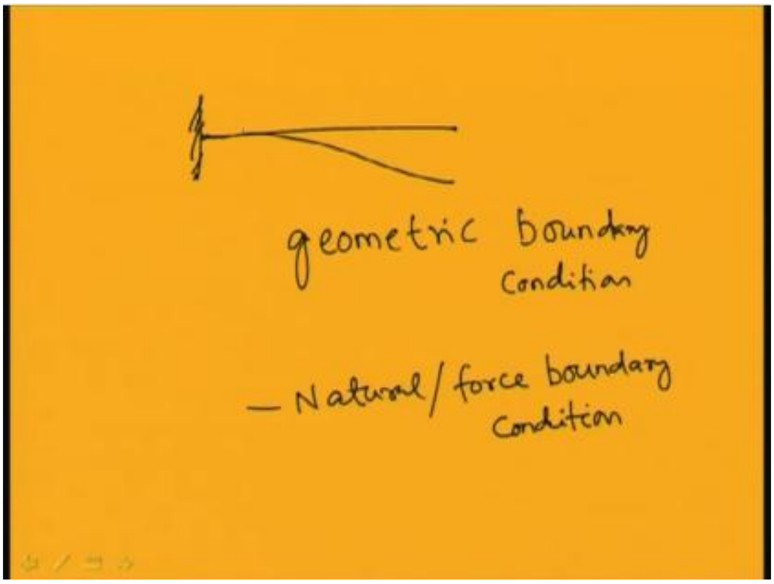

So, in case of a cantilever beam when one end is fixed and other end is free. So, the boundary conditions will be, so in the free end the boundary conditions will be the bending moment and the shear forces are 0. And in the fixed end the displacements and the slopes are 0. So, when it vibrates. So, the, so the displacement will be like this. So, the displacements and slopes are 0 here and the bending moment and shear forces are 0 are the other end. So, you have seen there are 2 different types of boundary conditions available one is the geometric boundary conditions one is the geometric boundary conditions.

And the second one is the forced or the natural boundary conditions natural or forced boundary condition. So, in case of geometric boundary conditions, the boundary conditions are of geometric type that is the displacement or slope. In case of natural or forced boundary conditions the boundary conditions will be force in nature force and moment may be there. So, force bending. So, in this case of cantilever beam, so it is shear force and bending moments are 0, but in the fixed end the slopes and the displacement are 0.

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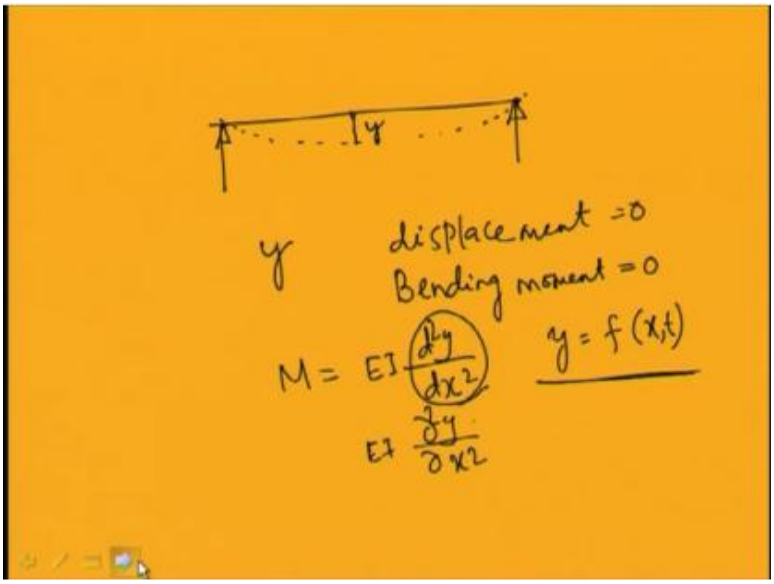

Similarly, in case of a. Simply supported beam, so this is a simply supported beam. So, the beam is simply supported at both the ends. So, here the deflection, so from your strength of material you have read about the deflection of these simply supported beam. So, in this case you can see that the displacement. So, at these ends the displacements are 0 at the support points the displacement are 0, but the slopes are not 0. And the bending moment is 0 the bending moment and the displacements are 0, but the slopes are not 0. So, in this case in case of simply supported beam, so you have a mixed type of boundary conditions, so both it contains both geometric boundary condition that is the displacement is 0 and the forced boundary condition or the natural boundary condition that is the bending moment equal to 0.

So, displacement equal to 0 in this case displacement equal to 0 and second is bending moment is 0. And already you know that the displacement if I am writing the displacement equal to y then the bending moment will be proportional to d square y by dx square or you know this bending moment can be written as EI d square y by dx square. And if I am writing this y y is the deflection of this beam already I told you that this y is a function of x. So, it is a function of x and t. So, this d square y by dx square I will replace it by partial differential instead writing the ordinary differential. So, actually it should be EI del square y by del x square. So, instead of writing d square y by dx square 1 should write it del square y by del x square as y the deflection is a function of x and t. So, in contrast to the descript systems you have seen.

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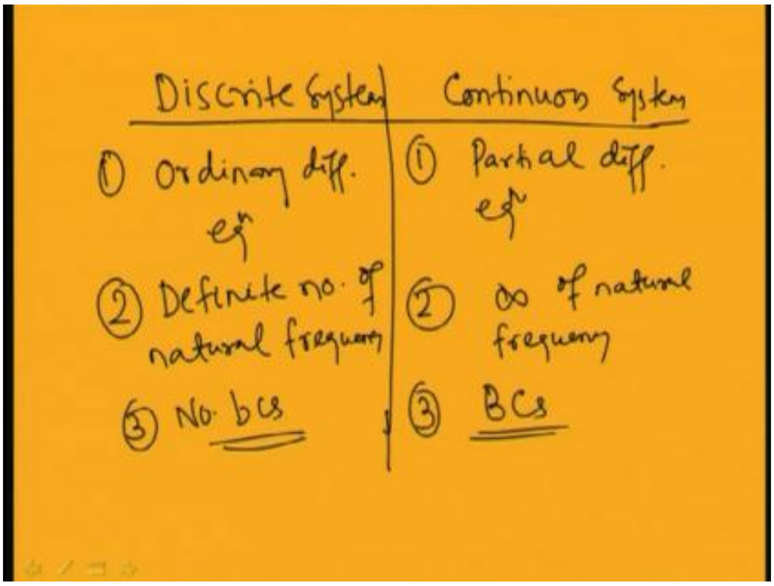

So, if I will distinguish between the discrete system and the distributed mass system. So, this is discrete system vibration of discrete system and continuous systems will see. So, the difference you can note. So, first difference is the equations can be written in case of discrete system as a ordinary differential equation ordinary differential equation, but in this case you can write this as partial differential equation. So, you have in case of discrete system, so you know the number of degrees of freedom of the system and you can write your natural frequency the natural frequency or the Eigen values are discrete.

So, discrete number or discrete number of natural frequency or a definite definite number of natural frequency, so definite number of natural frequency you can get in this system, but in case of continuous system. So, you will get infinity number of natural frequency. So, in this case in case of continuous system it depends on the boundary conditions. The system depends on the boundary conditions, but in this case no boundary condition is required. So, in this case no boundary conditions are required, but in case of continuous system you require boundary conditions to find the response of the system. So, now will consider different cases.

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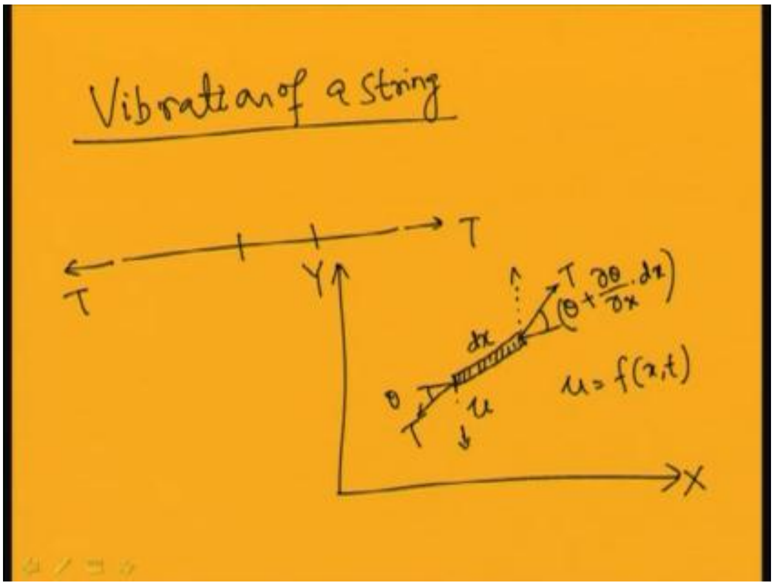

so, first let us see the vibration of a string vibration of a string. So, you have seen this strings in case of the in musical instruments and. So, in that case let this is a string. So, which is in tension, so let T is the tension in the spring I can take a small element of that spring small element of that spring let dx is the small element of that string. So, this is the small element I have taken. So, as I am taking it a small element I can tell that the tension in both the sides I can tell this tension in both the side same. So, let the tension is T here and tension is T here. So, the slope is here theta. So, I can write the slope here theta plus del theta by del x into dx. So, I have taken a small distance or small elemental length of the system as dx. So, in the string which is subjected to tension in both the sides I can take a small elemental string and I can write the equation motion or I have to find the equation motion of the system.

So, in this case let u is the deflection at this end. So, if u is the deflection at this end I can write this deflection. So, this is the deflection. So, I can write this deflection is a function of x and t. So, it is a function of x and t. So, to apply the Newton's second law of motion to find the equation motion in this case I can equate the forces let me take the coordinate system like this. So, this is the X coordinate this is the Y coordinate. So, I can write the equation motion by equating the forces in the Y direction, because in the X direction I can assume the forces are same. So, in the Y direction the forces will be t into. So, it it'll have a component along this direction and this side it'll have a component along this direction.

So, this forces this component in the right side the component equal to T sin of this angle. So, these angle is theta plus del theta by del x into dx. And in the left side in the left side it it is equal to T into sign theta. So, T into sign theta, so as theta is small. So, I can write it equal to T theta and this side it becomes T into theta plus del theta by del x into dx. So, the net force acting on this element will be the difference between T into theta plus del theta by del x into dx minus T into theta. So, this force will be equal to the inertia force of the system. The inertia force of the system will be equal to mass into acceleration of the system as we have taken u is the displacement of that element. So, the acceleration will be mass of that element into the acceleration of that element. So, mass of this element let m is the mass per unit length of the string.

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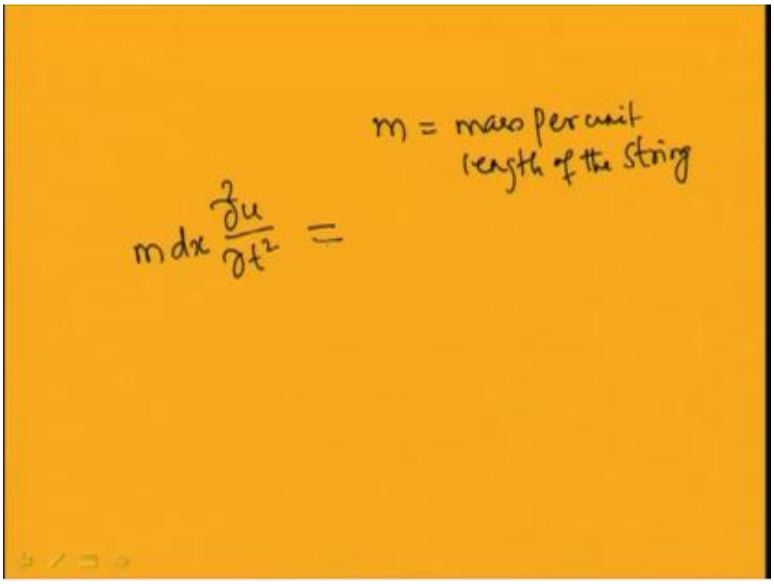

So, if m is the mass per unit length of the spring string mass per unit length unit length of the string. So, in that case the mass elemental mass will be equal to m dx. So, this is the elemental mass of the string and I can write the acceleration as del square u by del square u by del t square as already you know that this u is a function of x and t. So, the derivative with respect to t nothing, we can write as a partial derivative. So, this becomes m dx into del square u by del t square this is the mass into acceleration. So, this will be equal to net force in this vertical direction.

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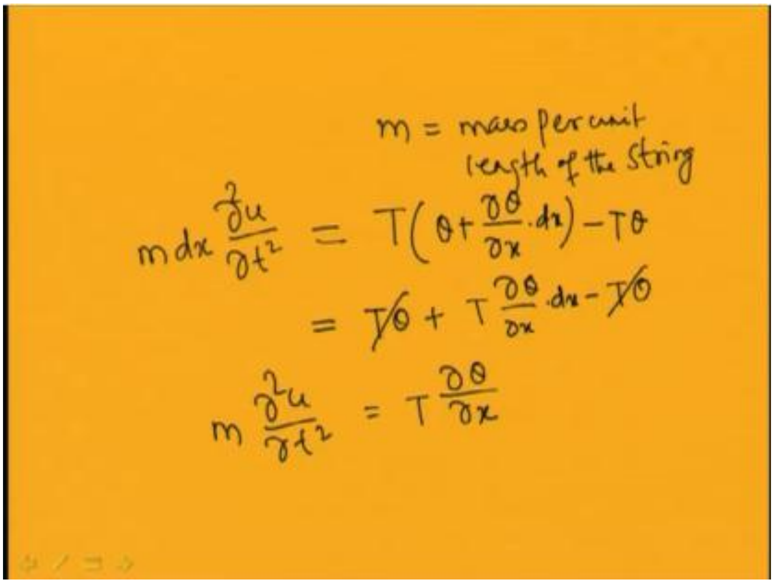

So, net force in the vertical direction equal to t into sign theta plus del theta by del x into dx as I am taking this theta plus del theta by del x dx this angle to be small. So, I can write this equal to T into. So, this becomes T into theta plus del theta by del x into dx minus T theta. So, I can simplify this thing. So, this becomes T theta plus T into del theta by del x into dx minus T theta. So, this T theta T theta cancels. So, you can write this equal to and this dx dx also cancel. So, I can write m del square u by del t square equal to T into del theta by del x. But what is this x? x the slope.

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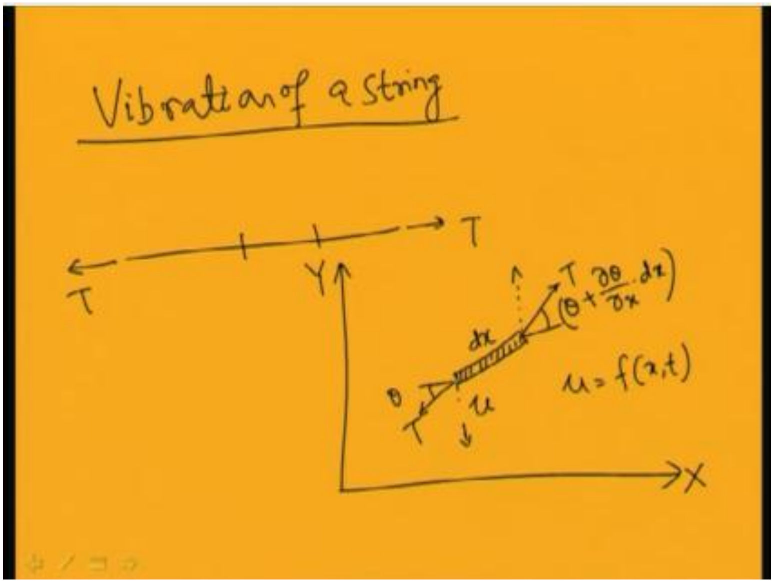

At this point is nothing but this is the differential of this deflection at that point. So, deflection is u, so the slope will be equal to…

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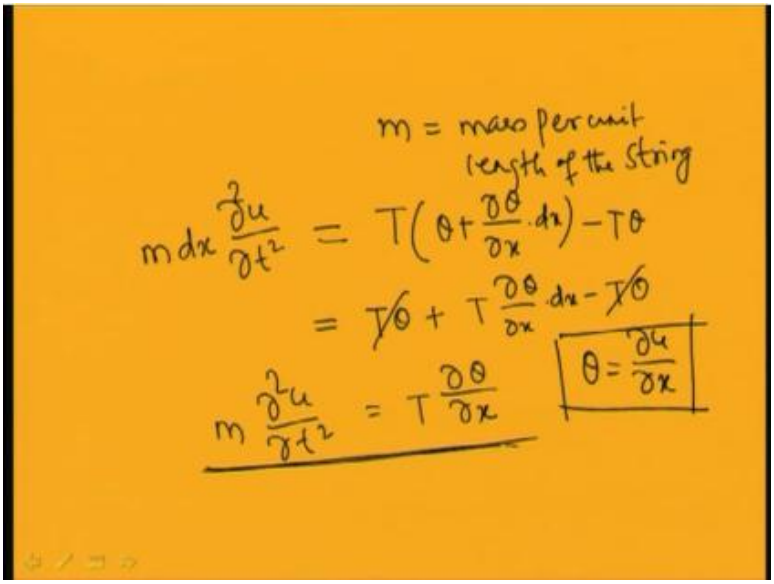

So, del slope theta will be equal to. So, slope theta I can write this is equal to del u by del x. So, as slope theta equal to del u by del x. So, I can write this expression m del square u by del T square equal to T into del theta by del x.

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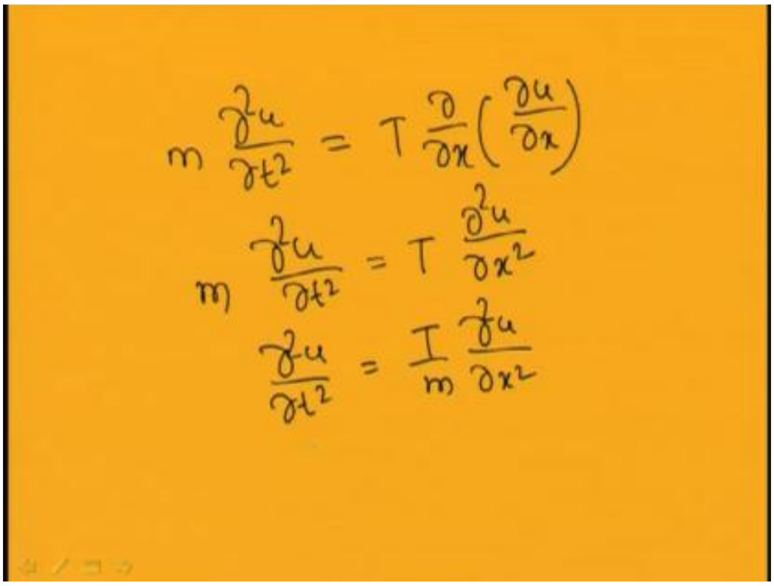

As so m del square u by del T square will be equal to. So, this will be equal to T into del by del x del by del x per theta I will write it is equal to del u by del x. So, now I can write this equation in this form. So, m del square u by del t square equal to T into del square u by del x square. Or I can write this equation as del square u by del t square equal to T by m del square u by del x square. Or this equation also can be written in this form.

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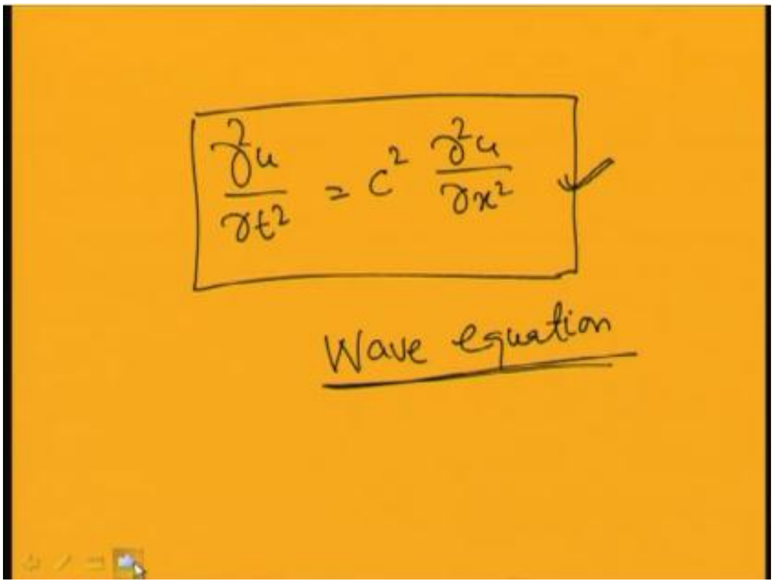

So, it can written as del square u by del t square equal to C square into del square u by del x square. This is a famous equation and it is known as the wave equation. So, you can see or you have now learnt the equation motion of the transverse vibration of a string can be written in terms of del square u by del t square equal to C square del square u by del x square. And in this case the boundary conditions we can we will discuss it later and before that let us find the equation motion for similar types of system.

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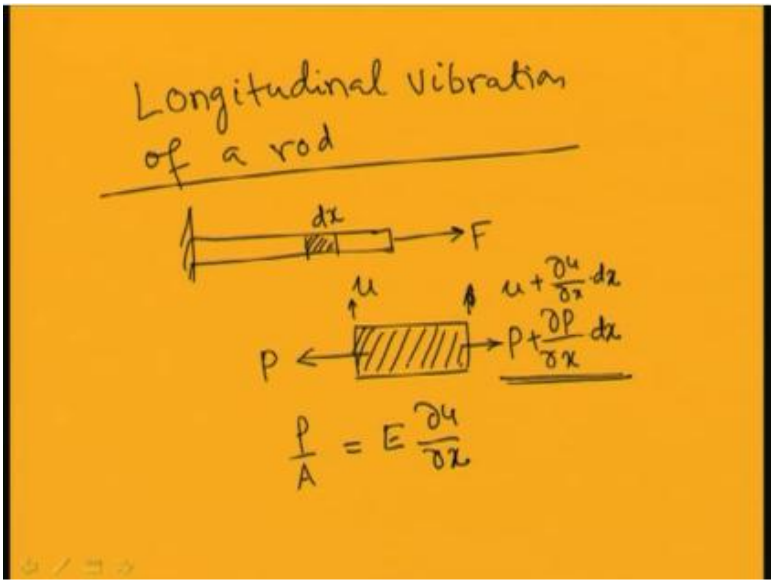

Let us see the longitudinal vibration of a rod. So, in case of longitudinal vibration of a rod vibration of a rod, so the rod is subjected. So, this is a rod let me draw a rod. So, this rod is subjected to a let it is subjected to a force F. So, I can take a small elemental length dx here. So, by taking a small elemental length dx I can study the longitudinal vibration of the rod. So, in case of longitudinal vibration of the rod, the rod will vibrate along the vibration of the rod will takes place along the length of the rod. So, in this case I can find the equation motion. So, let me take the small element dx. So, in the small element I can which is at a distance x from this fixed end. So, let it is at an distance x from the fixed end.

So, in this case I can write the forces acting on this like this. So, this is the small elemental length and the forces acting are. So, this side let me apply force P and the side the force will be P plus del P by del x into dx. So, this is the force acting here and also if I am assuming the displacement to be x displacement to be u in the side. So, the displacement in the side longitudinal displacement in this side I can write equal to u plus del u by del x into dx. So, the strain of this section will becomes due to this vibration the strain in the section will be equal to del u by del x.

So, the strain is del u by del x. So, I can write this P or the stress P by A is the stress. So, the stress by strain equal to E. So, I can write the stress equal to stress equal to E into strain that is equal to del u by del x. Now, to apply this second law of Newton second law I can find or I can write the inertia force equal to the net external force acting on this section. The forces acting on the section are in the right side P plus del P by del x into dx and in the left side it is P. So, I can equate the force and I can write this force equal to this inertia force. And if I will take similar to the previous case.

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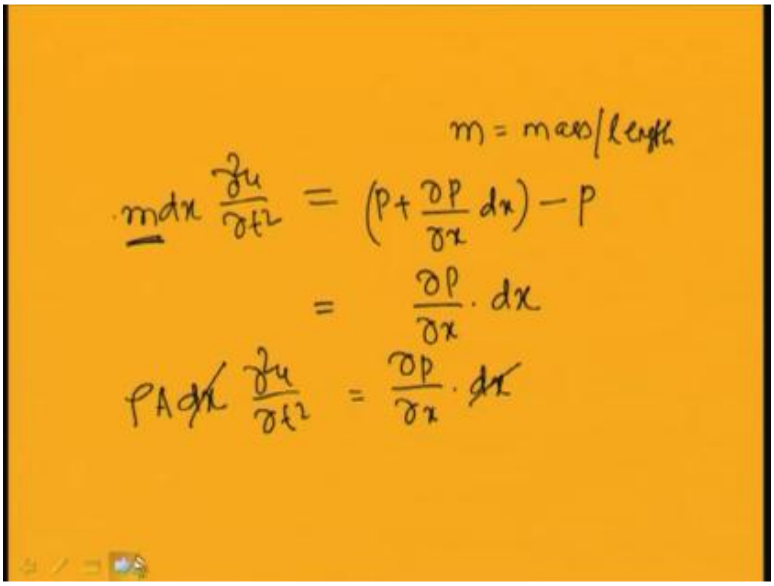

If I will take let M is the mass per unit length of this, so mass per unit length. So, in this case, so I can write mass. So, I can m into dx. So, this is the mass of that element into del square u by del t square. So, this is the mass into acceleration that is the force.

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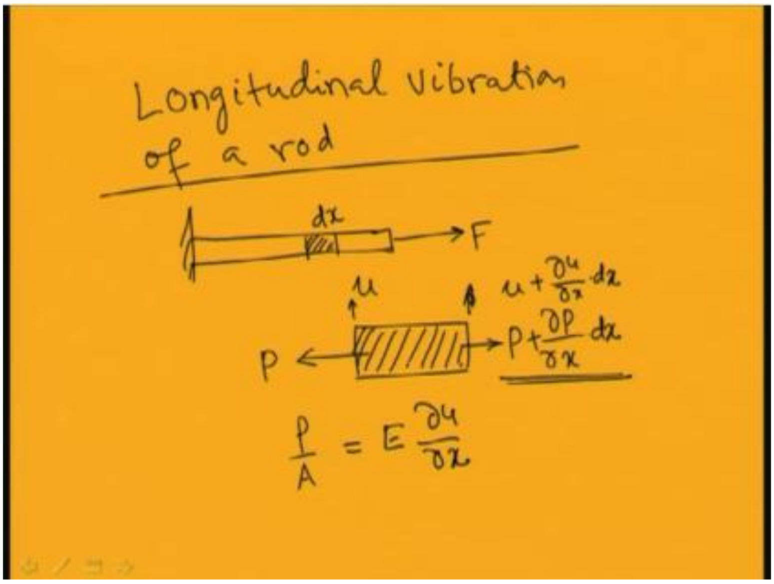

So, this will be equal to the net force acting on the system. So, this will be equal to P plus. So, P plus del P by del x P plus del P by del x into dx minus P. So, this will be equal to del P by del x into dx. So, I can write this equation in this form. So, this mass I can write rho. So, this thing mass I will write mass of this element will be equal to rho into A. So, this is rho A into dx where rho is the density. So, rho A dx into del square u by del t square equal to del P by del x into dx. So, I can cancel this dx from the both side.

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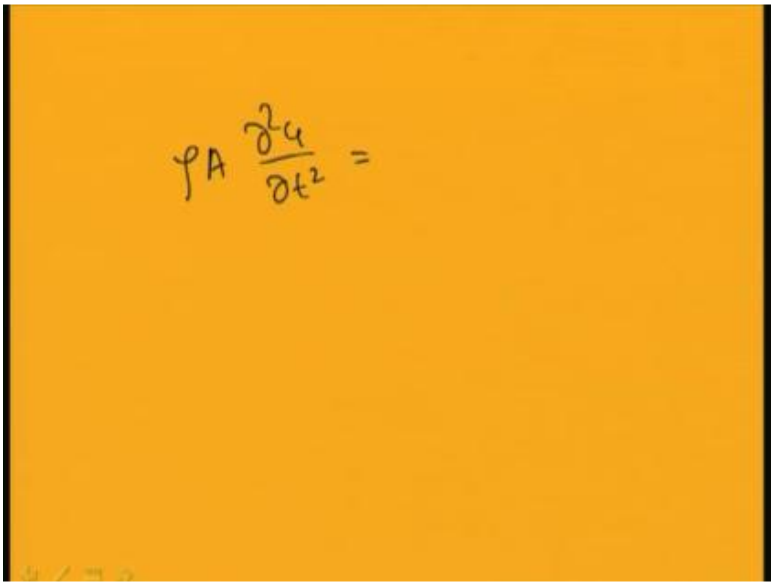

So, the equations is reduce to rho A rho A del square u by del t square equal to…

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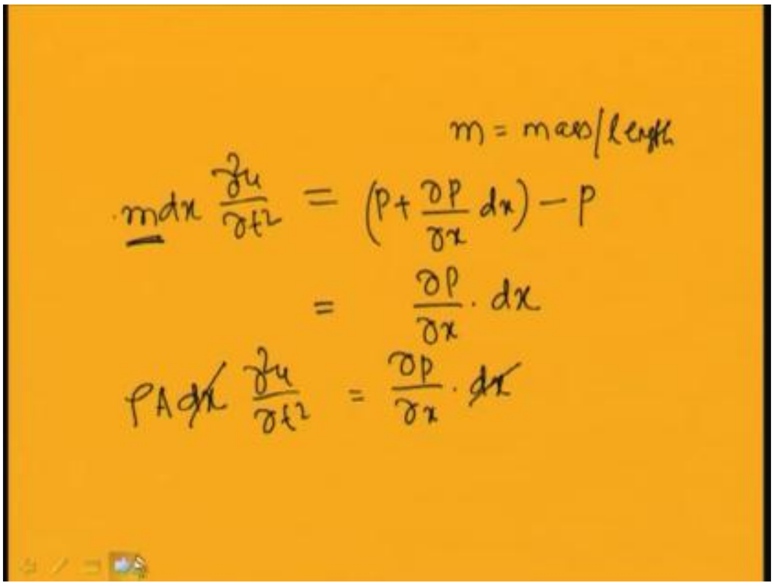

This is equal to del P by del x.

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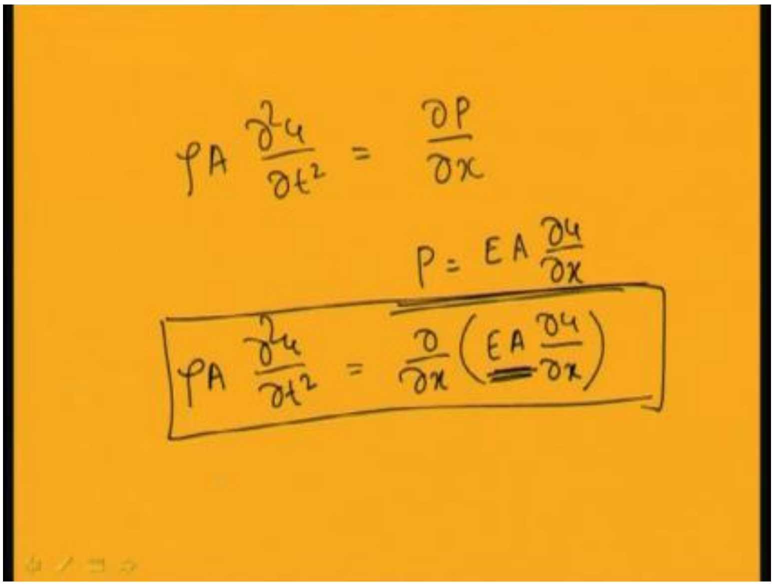

Del P by del x, but, so this P we have found equal to EA del u by del x where E is the young’s modulus A is the area cross section and this del u by del x is the strain where u is the displacement at a distance x. So, this is the longitudinal vibration of this the rod we are finding. So, I can write substituting this P equal to EA del u by del x. So, I can write rho A del square u by del t square equal to del by del x del by del x of EA del u by del x. So, if the rod has uniform cross section I can write or, so if the rod has uniform cross section and it is a homogenous rod then I can write this is E constant and A is constant. So, if it is not homogenous then E is not constant and if the area is or the cross section is not uniform then I cannot take this A as constant and I will write this is the equation of the system. But when E and A are constant; that means, area of cross section is constant that is you are taking a rod of uniform cross section and it is a homogenous beam where E is constant. Then I will take I will take this equation like this.

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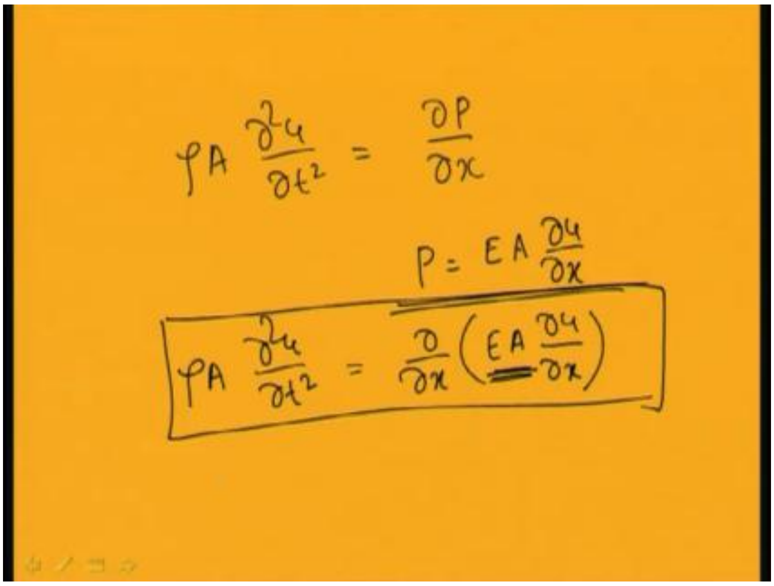

Rho A del square u by del x square equal to So, this will be equal EA.

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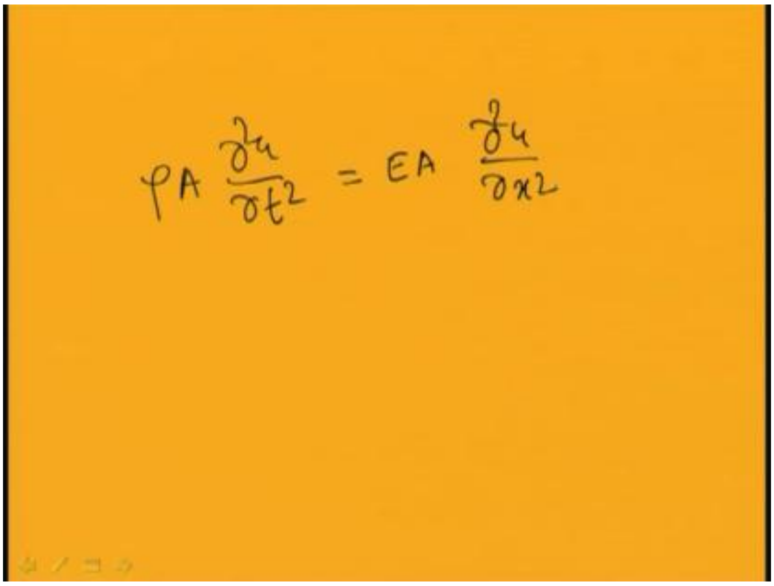

EA in del square u by. So, this is del square u by del t square t is equal to del square u by del x square.

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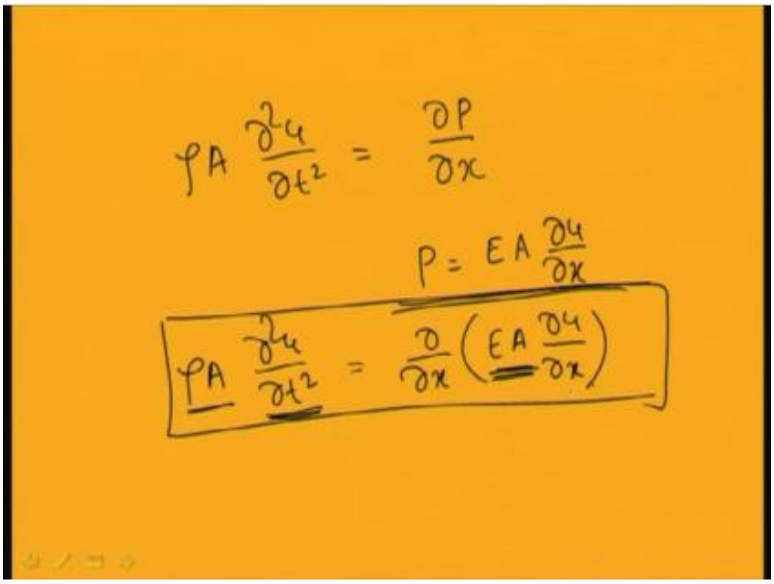

Del square u by del t square is the acceleration rho A is the mass. So, mass into acceleration this is the inertia force or the force acting on the system.

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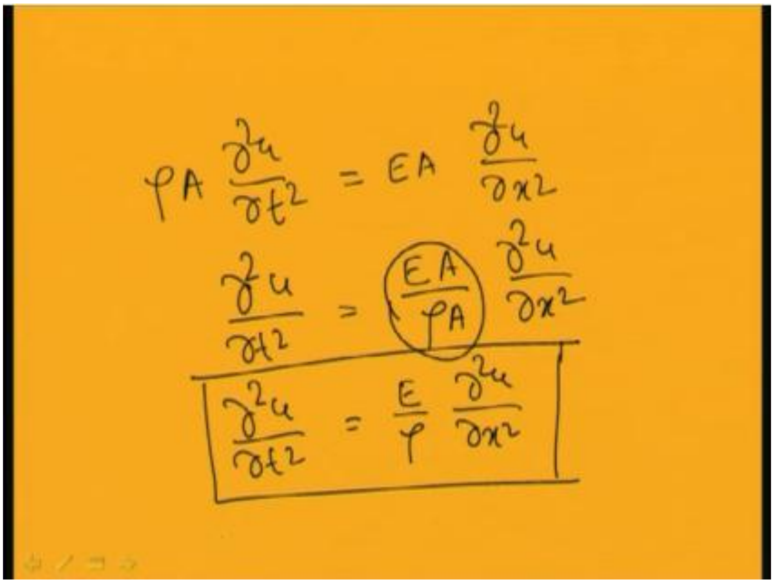

So, this will be equal to EA del square u by del x square or the same equation can write in this form. So, that is del square u by del t square equal to EA by rho A into del square u by del x square. So, in this case also by substituting this EA by rho A. So, that is equal to E by rho I can write del square u by del t square equal to E by rho del square u by del x square. So, this is the equation of the longitudinal vibration of a rod. So, this equation also I can write in this form by substituting this E by rho.

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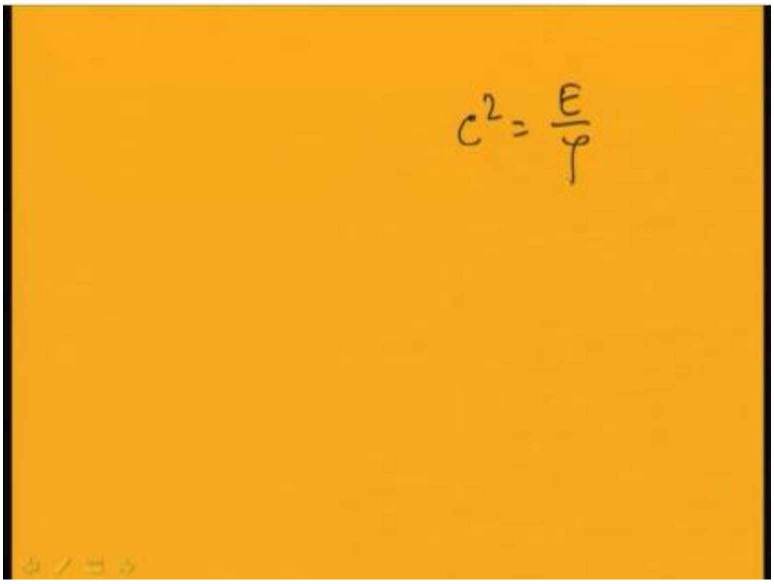

Are C square let C square equal to E by rho.

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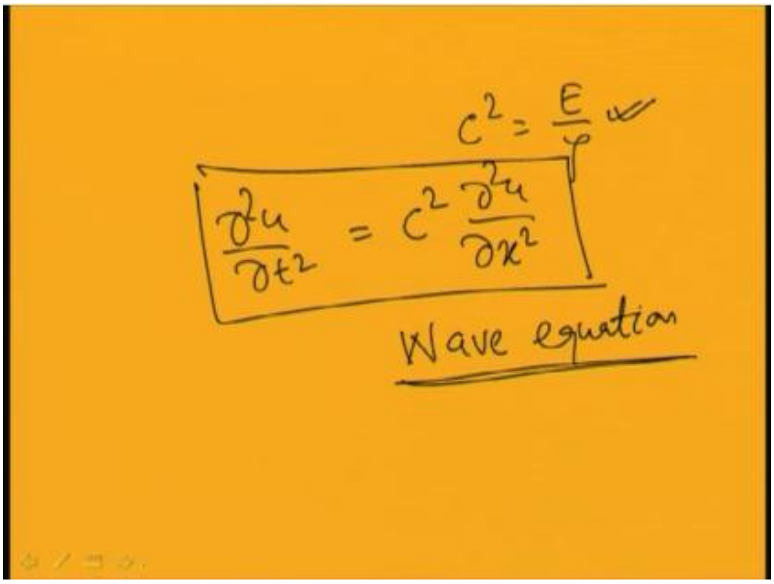

So, this equation will becomes del square u by del t square del square u by del t square equal to C square into del square u by del x square. So, this equation already you have derived in case of the transverse vibration of a string. So, in case of transverse vibration of the string also you have found the same equation where the C square was equal to T by m. But in this case in case of this longitudinal vibration of this rod C square becomes T by E by rho. So, this is also the wave equation. So, in both the cases you have seen the equation motion did you say to that of the wave equation later will see how to find the response of the system from these wave equation? So, let us now, see the other condition or other systems. So, in case of a torsional vibration of a shaft.

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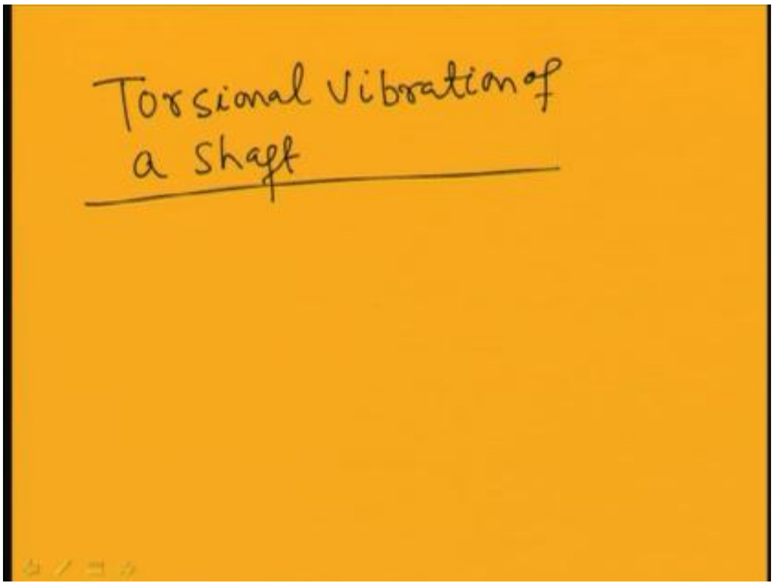

So, let us derive the equation for the torsional vibration of a shaft. So, in case of a shaft, so this is a shaft already you have seen this rod or this is a shaft which can rotate about its longitudinal axis. So, in this case when it is rotating or when it is coupled with coupled at the both the ends it may be subjected to some torque.

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And in case it is subjected to torsion already you know the equation for the torsional vibration of torsion in case of pure torsion. So, in case of pure torsion you know the formula the well known formula at is equal to T by J equal to G theta by.

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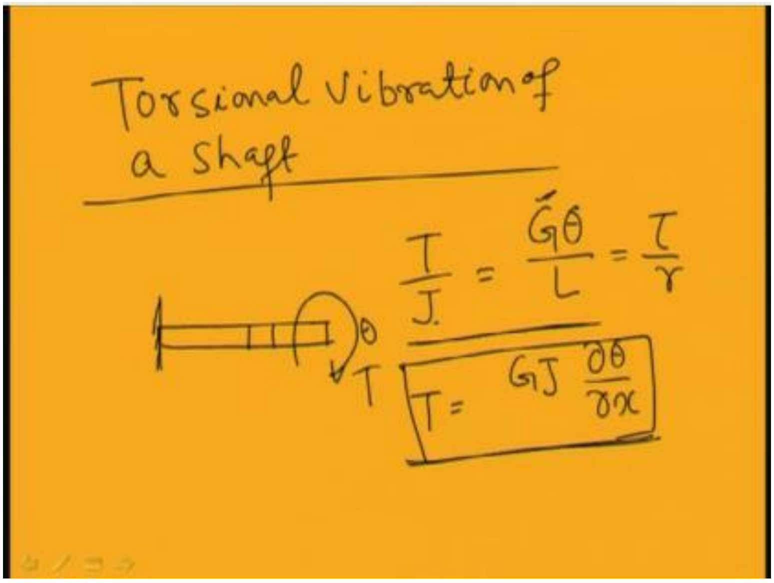

So, you know this formula T by this is the polar moment the torque applied. So, if you have a shaft subjected to the torque. So, let it is subjected to a torque. So, when it is subjected to a torque. So, let it is subjected to a torque like this and this end let me fix it. So, it is subjected to a torque T when you can write this T by J equal to C or G theta by L equal to tau by r. So, where T is the applied torque; J is the polar moment of inertia of the cross section and G is the modulus rigidity modulus and theta is the rotation. So, let it has a rotation theta and L is the length of this beam or rod or the shaft and tau is the shear stress and r is the radius of this shaft. So, if I will take small sections from the shaft to derive the equation motion I can take a small section of the shaft and I have to find the torsional vibration of the shaft. So, let me take a small section of the shaft.

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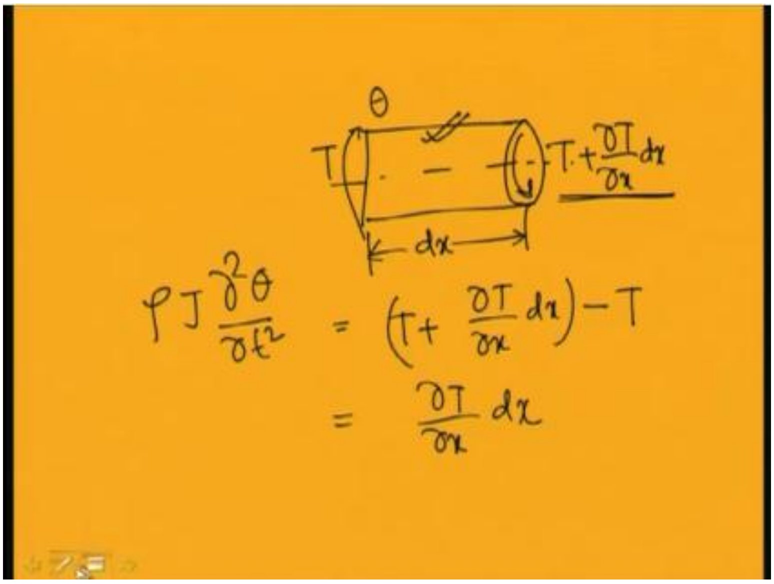

So, in the small section, so this is a small section I have taken. So, in the small section it is subjected to a torque. So, this side if the torque is T. So, this side the torque I can write let this side the torque is T. So, this side the torque will be T plus del T by del x into dx, because I have taken an elemental length of dx. So, taking an elemental length of dx let us derived the equation motion for case of the torsional vibration of this shaft. So, in the left side the torque I am writing equal to T and in the right side the torque equal to T plus del t by del x into dx. Similarly, I can write this theta. So, this side the displacement is theta or I can write this equation motion by using this Newton's second law. So, by applying Newton's second law I can write for this elemental length the inertia force or inertia torque equal to inertia moment of inertia into theta double dot that is let the let theta is the displacement or rotation of the shaft.

So, the inertia force will be equal to rho. So, I can write this is equal to rho into J into del square theta by del t square. So, this is the inertia of torque. So, this inertia torque will be equal to. So, this inertia torque will be equal to T plus del T by del x into dx. So, this is the torque in the right side minus the torque in the left side, so this equal to T. So, this becomes del T by del x into dx, but already we know this T can be written as So, from this previous. So, if I will take a elemental length dx then this T I can write equal to. So, it will be written as GJ del theta by del x del theta by del x. So, del theta is the deformation or change in theta. So, this T can be written like. So, I have written this T for this elemental length T will be equal to GJ del theta by del x., so substituting these equations here. So, I can write rho J del square theta by del t square

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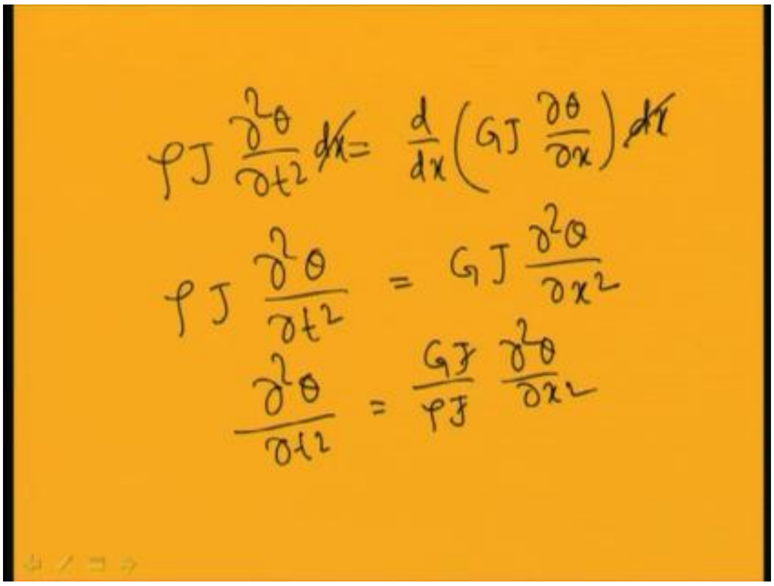

Rho J del square theta by del t square. So, this will be equal to, so here I have taken an elemental length of dx and. So, here rho J into dx rho J is the moment of inertia for unit length and when I am multiplying these things. So, this is the mass moment of inertia for this small elemental length and it is multiplied by del square theta by del t square this is giving the total inertia torque. So, now, taking these total inertia torque equal to the torque at the right side minus the torque at the left side. So, then I can write into dx. So, this will become D by dx. D by dx of for T I will substitute. So, the expression for T GJ del theta by del x I'll substitute this. So, this is GJ del theta by del x into dx. So, this dx dx cancel. So, I can write this rho J del square theta by del t square equal to GJ del square theta by del x square. Or I can write this del square theta by del t square this is equal to GJ by rho J GJ by rho J del square theta by del x square. So, I can write this deal. So, J J get cancel.

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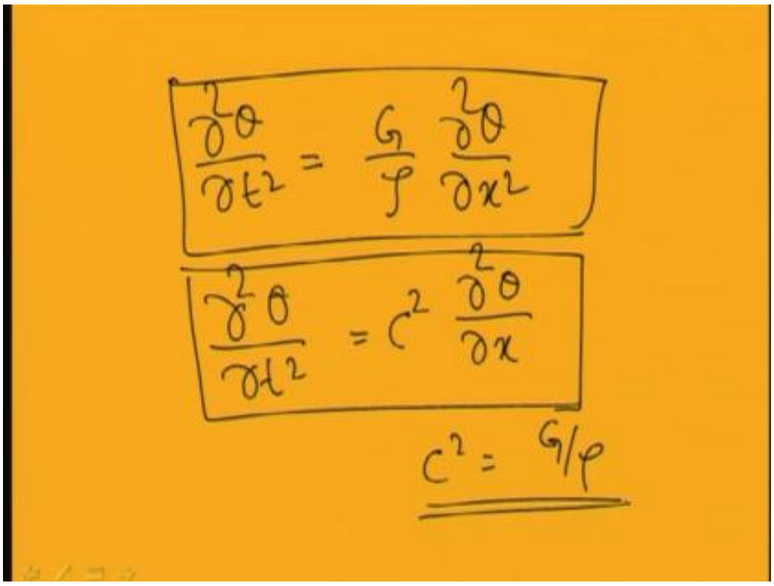

So, del square theta del square theta by del t square I can write this equal to G by rho where rho is the mass density. So, G by rho into del square theta by del x square. So, in this case also I can write this equation in this form del square theta by del t square equal to C square del square theta by del x square. So, in this case you can note the C Square equal to C square equal to G by rho. So, in this way you can or you have derived the equation motion of the one dimensional vibration of string.

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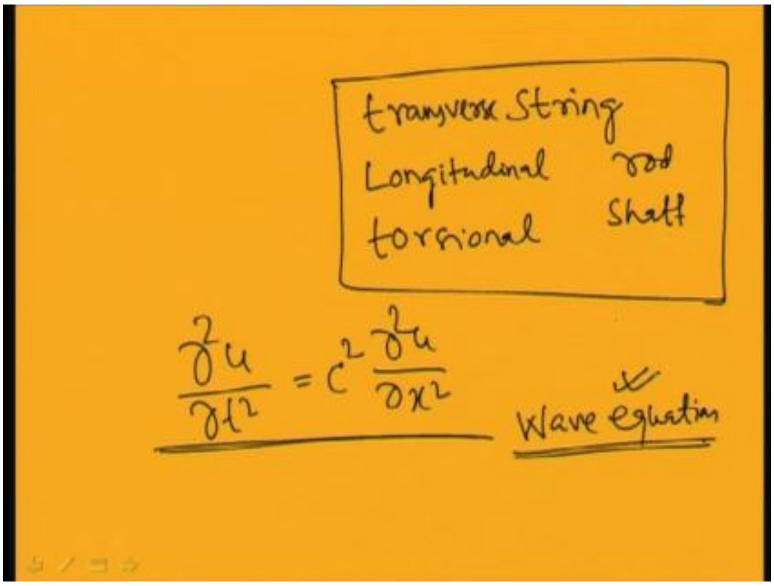

So, for the string you have found the equation motion and for so this transverse vibration; transverse vibration of string and then longitudinal vibration of rod and this torsional vibration of shaft. So, in these 3 cases you have found the equation motion to be of the same type where you can write the equation motion in this form Del square u by del t square equal to C square del square u by del x square to which is known as the wave equation. So, this equation is known as the wave equation. And now, we are going to find the response of the system from this wave equation. So, before proceeding further, so you should know that the response of the system will depend on the boundary conditions of the system. In case of a string it will depend how the ends are fixed in case of the rods and in case of the shaft also it will depends how the end conditions are there. So, in case of the longitudinal vibration of rod.

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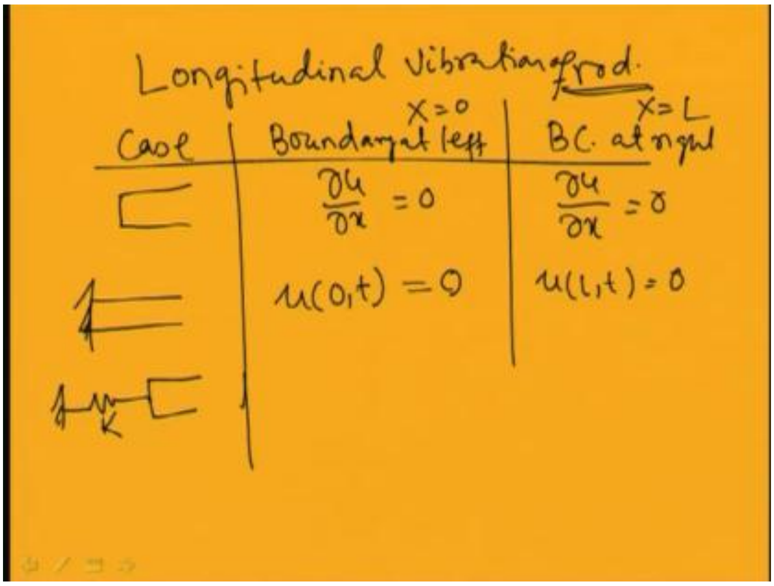

Let us see what are the boundary conditions? So, in case of longitudinal vibration of rod, so let us see the left end. So, will consider different cases. So, boundary at the left end let us see at left and boundary at the right. So, in, so let us see the different types of boundary conditions. So, let the rod is free. So, if the rod is free; that means, it is not supported at the ends. Then the boundary conditions will be. So, at the left end that is X equal to 0; at the right end that is X equal to L. So, you can write if the if it is free at this end then this theta or the slope at the ends will be equal to 0; that means, del u by del x will be equal to 0. Similarly, at the right also you will have del u by del x equal to 0. So, let us take the case when it is fixed at the left end or when it is fixed. So, if the end condition is fixed then the displacement at this end will be equal to 0.

So, the longitudinal vibration or the displacement at the end will be equal to zero; that means, u 0 t u at position x equal to 0 for any times t will be equal to 0; similarly at the right end also u l t will be equal to 0. And if the end let the end is spring loaded. So, let us put a spring at the left end in case of the longitudinal vibration of the rod; that means, when the rod is vibrating along its length let us put a spring k at the left end or right end. So, if we are putting a spring at the end then the force exerted by the spring will be equal to the force given by this rod. So, you know the displacement at this position equal to at the left end the strain will be equal to del u by del x. So, the stress will be equal to E multiplied by del u by del x and the force at the left end P will be equal to So, EA del u by del x. So, this EA del u by del x will be equal the spring force. The let me explain you will again.

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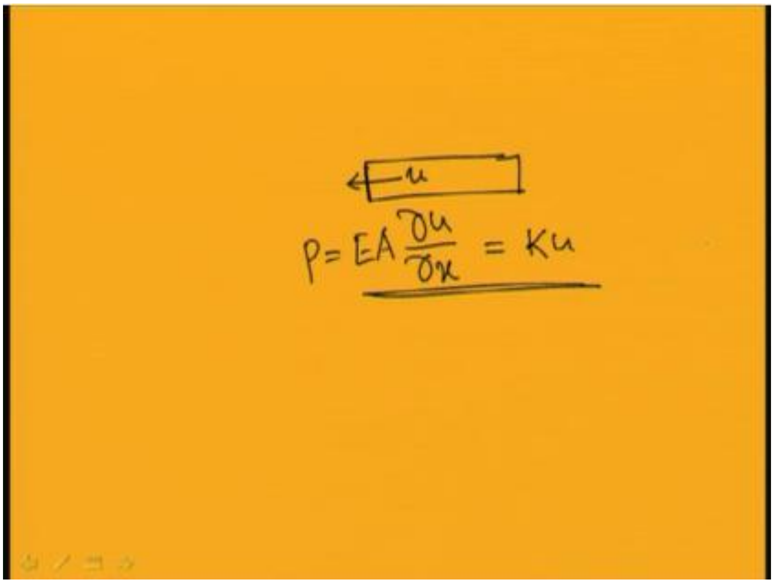

So, in the longitudinal vibration of the rod the rod is having a displacement u along this. So, at the left end if as it has a displacement of u. So, strain will be del u by del x is the strain at this end. So, if you multiply these by E this is the stress. So, stress multiplied by area of cross section at this end this will give the force and this force will be equal to the spring force at this end. So, spring force at the displacement is u the spring force will be equal to Ku. So, the spring force will be equal to EA by del u by del x. So, at the left end, so Ku will be equal to EA del u by del x and in the right end. So, this EA del u by del x will be equal to minus ku.

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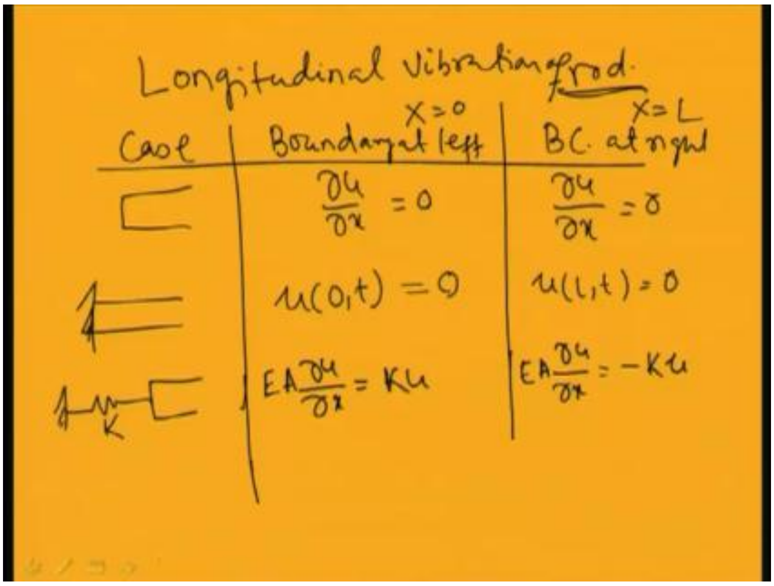

So, at left end I can write this EA del u by del x EA del u by del x will be equal to ku and in the right end it will be equal to EA del u by del x equal to minus ku.

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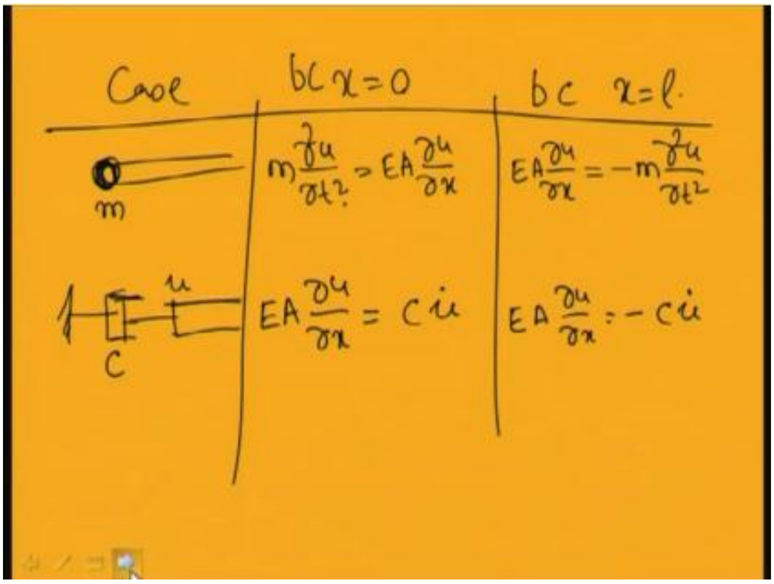

Similarly, if the end condition in the end let us put a mass at the end, so in case of this let us put a mass, so we are considering 3 cases. So, this is case and this is x equal to 0 boundary condition at x equal to 0 and boundary condition at x equal to l. So, if we are putting an end mass at this end. So, let this mass is m if we are putting a mass m at the left end. So, the force EA del u by del x, so this force will be equal to the inertia force in this case, so the inertia force equal to m del square u by del t square. So, this will be equal to EA del u by del x. So, the force at this point that is equal to EA del u by del x will be equal to m del square u by del t square So, at the right end, so this EA del u by del x that is the force at the right end should be equal to the inertia force, but the negative of that, so m. So, it will be minus m del square u by del t square. So, instead of putting a mass if you put a end damper.

So, let us put a end damper at this end. So, if we are putting a damper at this end. So, this damper is connected to this rod or the rod in longitudinal vibration a damper is connected at this end. So, in that case the force EA del u by del x will be equal to the damping force the damping force if this point has a displacement of u. Let C is the damp damping coefficient, so the damping force will be equal to Cu dot. So, the Cu dot will be equal to EA del u by del x. So, I can write at x equal to l this EA del u by del x will be equal to Cu dot similarly in the right end it will be equal to EA del u by del x will be equal to minus Cu dot. So, these are the boundary conditions in case of a longitudinal vibration of a rod.

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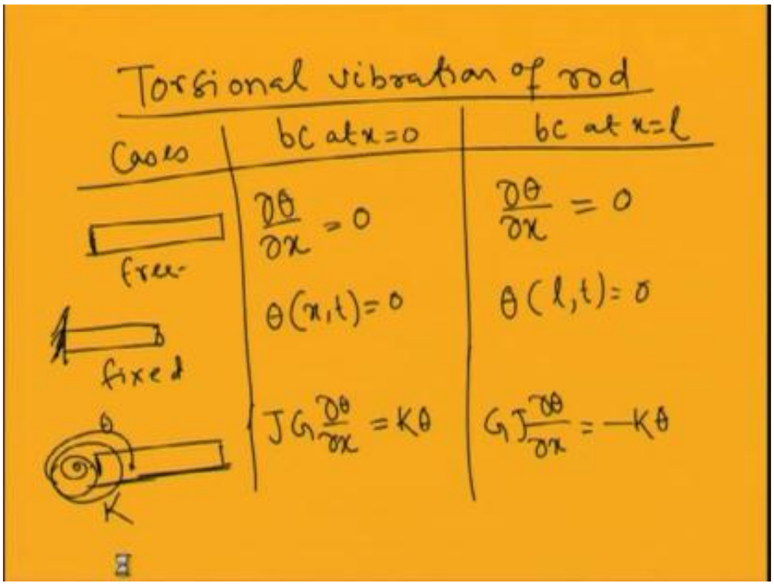

Similarly, we can write the boundary conditions for the torsional vibration of a rod torsional vibration in case of torsional vibration of a rod, so we can write similarly the cases then the boundary condition at the left end, so boundary conditions at x equal to 0. So, this is left end and boundary conditions at x equal to l that is the right end. So, in this case we can take the 3 free, so we can take a rod with free at both the ends. So, in these cases, so the example is the space craft or the aeroplane. So, you can there when it is flying in the air, so no ends are supported. So, it may I rotate about its own axis. So, in that case it can be considered as a as the rotation of the rod rotation of the free rod. So, in this case the as both ends are free.

So, we can write the boundary conditions like this. So, we will have the slope at the both the ends will be equal to 0 or del theta by del x equal to 0 at left end. Similarly del theta the rotation rate of rotation at these end both the ends will be equal to 0. So, if it is fixed. So, in case of a fixed rod, so if the rod is fixed. So, in that case, so if the rod is fixed, so 1 end, so let the rod is fixed at the left end. So, in that case the displacement will be zero that is the rotation will be zero when it is fixed at the left end the rotation will be zero. So, this rotation, so I can write theta x t equal to 0 in this case similarly at the right end if it is fixed, so I will write if the right end is fixed, so then it will be theta l t will be equal to 0. So, this is 3 free boundary condition and this is fixed boundary condition similarly I can have other boundary conditions.

Let me put some string, so a spring let me put a torsional. So, if I will put a torsional spring with stiffness K. So, in this case the force at this or torque at the left end will be balance by the torsional spring or the force exerted by these torsional spring. The force exerted by the torsional spring will be equal to K theta and the force at this end due to this rotation at due to this torque can be written. So, this will be equal to, so this torque expression already you know, so that is equal to GJ del theta by del x. So, GJ del theta by del x will be equal to k theta. Similarly, in the right end, so this will be replaced by this minus k theta. So, GJ del theta by del x will be equal to minus k theta.

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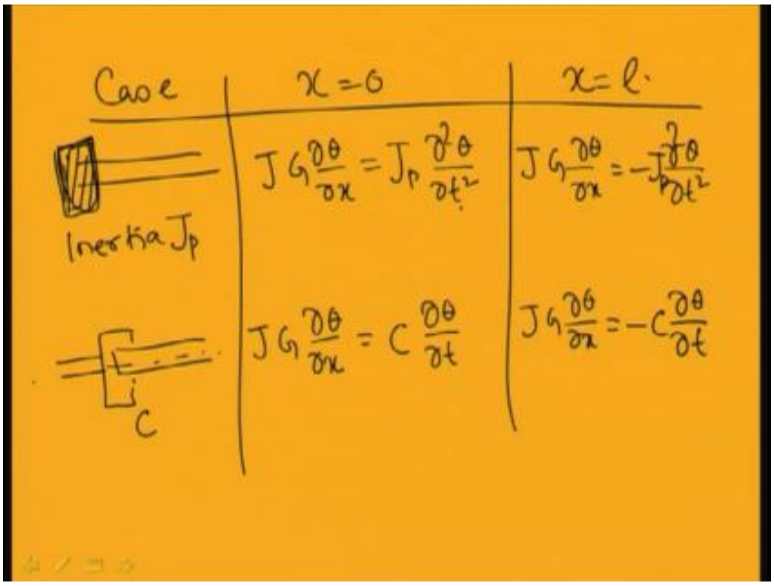

Let us see the other boundary conditions. So, the other boundary condition at x equal to 0 and x equal to l we have to see. So, in the case when it is subjected or when a mass is attached or in the rod let us attached 1 flywheel at the left end or at the right end. So, if an inertia if inertia is added, so inertia let the inertia Jp is added to the left end. So, if I am adding and inertia Jp at the left end. Then this torque at the left end that is equal to JG del theta by del x. So, that inertia, so this torque will be equal to this inertia torque, so inertia torque will be equal to Jp into del square theta by del t square. So, similarly in the right side it will be JG del theta by del x will be equal to minus Jp del square theta by del t square. Similarly, if we add 1 damper torsional damper at the left end or right end, so let this is a torsional damper.

So, in case of a torsional damper I can put a torsional damper like this. So, this is the rod when a torsional damper is attached here. So, in case of a torsional damper with damping coefficients C, so this will be equal to C del theta by dt. So, C del theta by dt will be the damping force. So, the force at the left end that is equal to JG del theta by del x. So, this will be equal to C del theta by del t. Similarly, in the right end it will be equal to JG del theta by del x equal to, so this will be equal to minus C del theta by del x del theta by del t. So, you can see the boundary conditions what you have found in the left end by putting 1 negative sign were getting the boundary condition at the right end. So, in this way by using these boundary conditions and the equation motion we will find the response of the system. So, next class we will study to determine the equation motion by using extended Hamilton’s principle and to study the transverse vibration of the beam.