Mechanical Vibrations
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Module - 8
Torsional Vibration
Lecture - 2
Multi-Degree of Freedom
Systems-Transfer Matrix Method,
Branched System

In previous lecture, we have analyze torsional vibration of some simple systems like single degree of freedom system or 2 degree of freedom system or 3 degree of freedom system. The problem in the methods which we covered till now is that if we want to analyze for larger systems like 4 degree of freedom system or 5 or 10 are may be 100 degree of freedom system. Then the problem comes whatever the algebra and it did to solve such problems are weigh cumbersome. And it is not we cannot able to solve them, but it is too cumbersome to solve, and it is too time consuming. But there are some other methods available which in which we can able to use the computer methods also.

Like we have two very strong methods which can able to handle any degree of freedom systems. One is the torsional that is the there is transfer matrix method, which will be studying next. And another method is there which is called finite element method that may be we will be studying subsequently. So, in that transfer matrix method generally what we do; we try to correlate the very various states of the various states of the stations. Here a state is representing that the angular displacement of the shaft or the torque which is acting at that particular location. So, will be relating these state vectors at various locations, with the help of either field matrix which is something like stiffness matrix. And, through point matrix which is nothing but something like mass matrix or polar mass moment of inertia effect will be there in that.

So, this particular method the advantage of the transfer matrix method is that even if the degree of freedom of the system is very large. Then, also there is the size of the matrix which we handle is remains same. There is another method that is called finite element method; in that there is the problem is, if the size of the matrices which we handle increases with the degree of freedom in the system. So, but there are some disadvantage of the transfer matrix method that the solution becomes non-linear. And generally we get the frequency equations by which we will be obtaining the natural frequency there polynomial form and the degree of polynomial increases as we increase the degree of freedom.

So, some kind of 4 root searching techniques will be using for solving those equations, but on the other hand in the finite element method; will be having large number of simultaneous equation. They will be linear and they can be solved using for full techniques which are available in the numerical analysis. So, let us see the multi degree of freedom system of the torsional vibration using the transfer matrix method.

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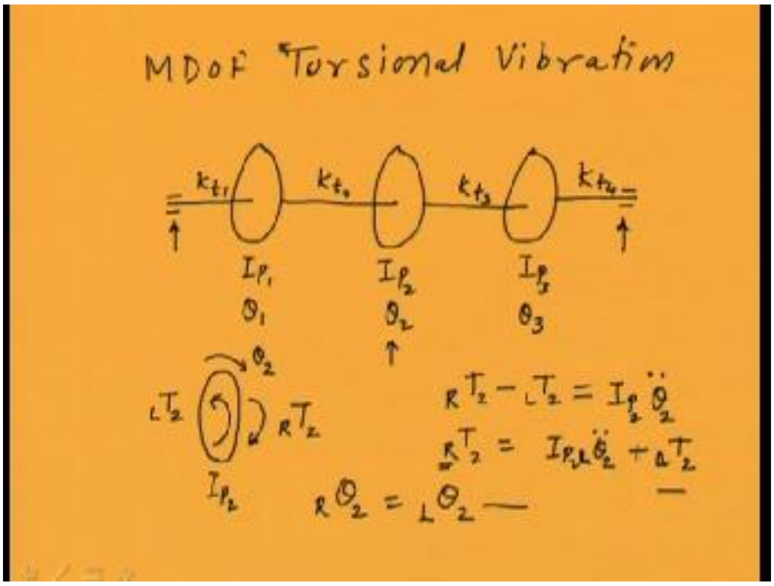

So, this is the multi degree of freedom torsional vibration, and this let us consider various numbers of discs which are connected by shaft here. We are assuming that shaft is flexible, but having no mass and discs are rigid, and having polar mass moment of inertias. We can able to consider any number of discs here, but for time being we are considering only 3 disc, but this method can be extended for any number of discs. These are frictionless bearing which is a supporting the rotor. So, that it is not providing any kind of frictional torque to the rotor. So, we are not considering the frictional torque on to their shaft they are just supporting it. So, that the frictionless motion between the shaft and bearing take place. This discs having angular displacements they are time dependent.

Again here we are analyzing the free vibration. So, we are not applying any force to the system. So, let us take the free body diagram of one of the disc less let us say disc 2, this is the disc 2. Let us say this having this particular oscillation. Now, because of this there will be torque on this side of the disc which will be coming from the shaft. Let us call it left of disc 2 torque left of disc 2. And, there will be another torque from the shaft which is right of 2. So, subscript R represents right, and that is back subscript R represent right and L represent the left of the disc and 2 represent the disc number.

Now, if we considered the equilibrium equations for this disc; I can able to write torque in the left of right of disc 2 minus left of 2, because their minus because series opposite to the displacement of the disc theta 2 should be equal to the inertia torque. So, this equation is can be written as theta 2 plus like this. Now, the angular displacement in the right and left of the discs are same, because we are considering discuss as a is point. So, these 2 equations if you see carefully, in the left hand side we have kept terms which having back subscript as right. And, in the right hand side we have kept the terms which are having subscript L. So, here because both are same, so we can add back subscript to the theta here that is L.

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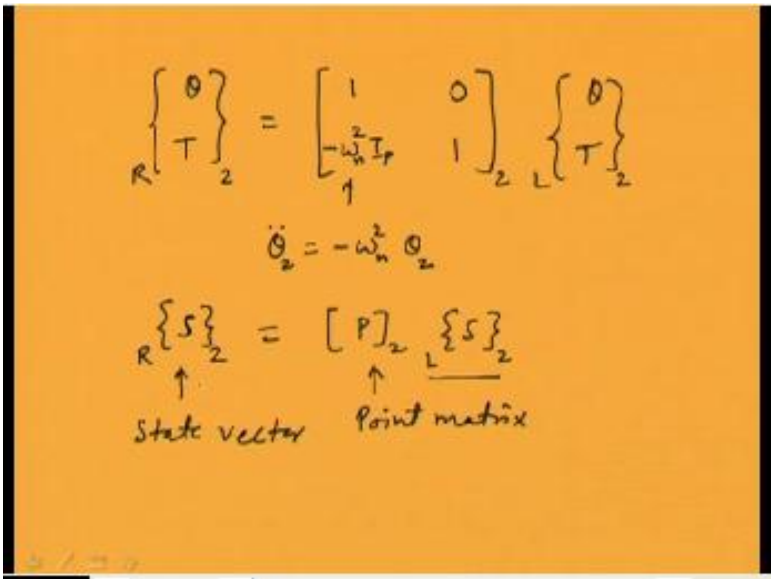

Now, these 2 equations can be combined in a matrix form like this. Right of 2 is equal to left of 2, and we will find that the first equation is representing theta in the right is equal to theta in the right left. And, second equation is this has come because for simple harmonic motion. We can able to write theta 2 like this. So, that is taken care. So, this particular matrix can be return in a more complex form as S which is or which represent the state vector is equal to P 2, which represent point matrix and state vector left of 2. So, a state vector you can see it contains angular displacement and the torque. So, at one particular station; we will be having 2 state 2 states once angular displacement and the torque. And, from this equation you can able to see that we are transferring we are the state vector which is there in the left of the disc using this point matrix to the state vector in the right of the disc. Now, we will consider the free body diagram of the shaft.

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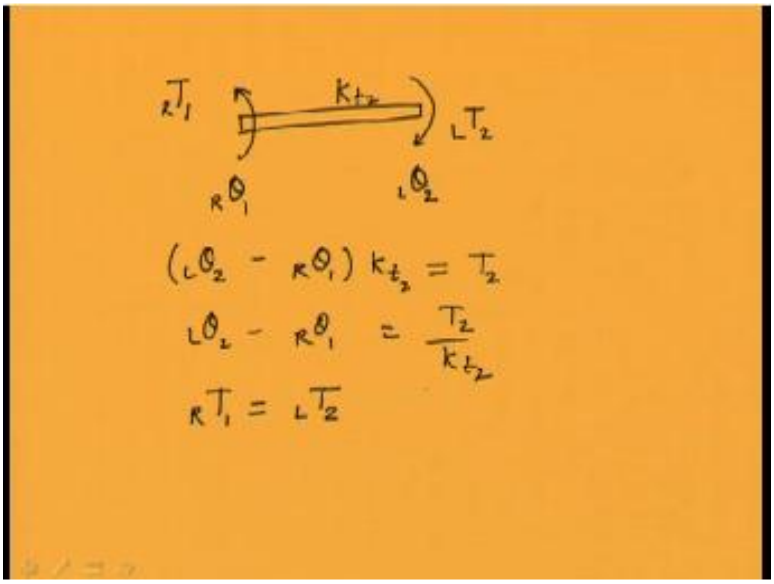

Let us say we have taken the second shaft segment. So, we have torque which is coming from the disc left of 2 and this is the torque which is coming from right of 1 disc. Now, if we want to obtain the equilibrium equation for this, we can able to write that equilibrium equation in that you can see that the displacement of the disc here is theta 1 and displacement of the disc here is theta 2, which is let us say left of 2. And this is write of 1, whatever the torque is being transmitted through this shaft can be written as the relative twist between the two ends of the shaft into torsional stiffness of the shaft is torque. This can be written as, now we can able to see that because there is no disc into the shaft in between. So, we will be having the torque in the right of 1equal to torque in the left of 2. Now, this particular equation can be combined, how we can able to combine?

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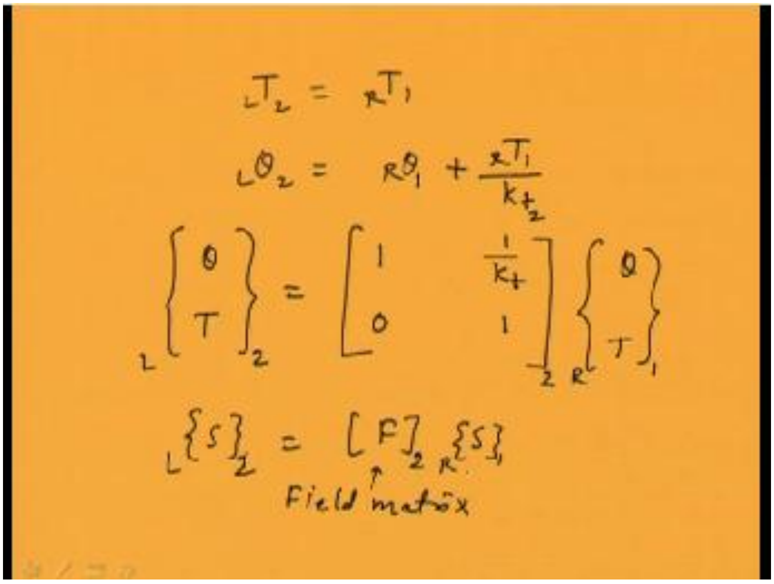

We can write these 2 equations in more convenient form that is left of 2 is equal to right of 1 and theta left of 2 is equal to theta right of 1 plus torque right of 1 divided by the stiffness of the shaft now. This particular equation again in as we did in the previous case. We can combine in the in matrix form. So, we have state vector here right of 1 which is relating the state vector in left of 2 and this will take 1 by k t 2 subscript we are keeping here and then 0 1. So, we can see that the last equation is representing the first equation of this and the first equation is representing this equation. So, this equation can be written in more complex form as F, F is the field matrix is. So, field matrix is relating the state vector which is there in the right of disc 1 to the left of disc 2.

Now, once we have developed equations that are a transfer equation as the method implies transfer matrix method using point matrix. We convert the state vector form left of the disc to the right of the disc and from the field matrix we transferred the state vector right of the disc 1 to the left of the disc 2. So, that means, we are transferring this state vector from one end of the shaft to the another end of the shaft. So, using this transfer what were the transfer matrix we have developed now we can able to relate the state vector from 1 end of the shaft to the another end of the shaft. Let us see how we can do this.

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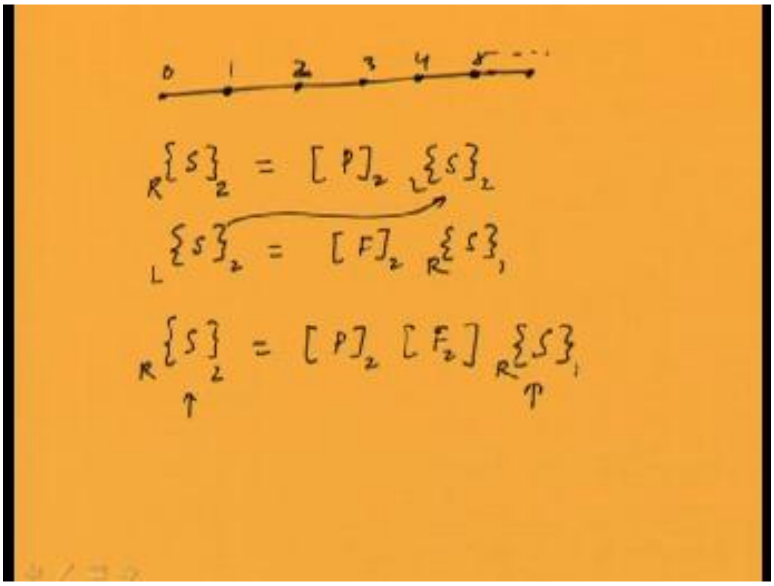

So, if we have a system like this in which various discs are here these points represent the discs. So, let us say 0 1 2 3 4 5; all these are the discs locations. So, we can able to write the, let us say the previous equations we I am, so writing here for disc 2. So, we develop state vector right of 2 is equal to point matrix 2 state vector left of 2. Also we develop this left of 2 is equal to field matrix state vector right of 1. So, you can see that we can able to substitute this here. And so we can able to write state vector right of 2 is equal to P 2 into F 2 and state vector right of 1. So, you can see that what we have done here? We are now transferring state vector from right of 1 of the disc to the right of disc 2. So, directly we are jumping to the shaft and then disc one shot and the same thing can be extended for the whole shaft. Let us see how we can do it.

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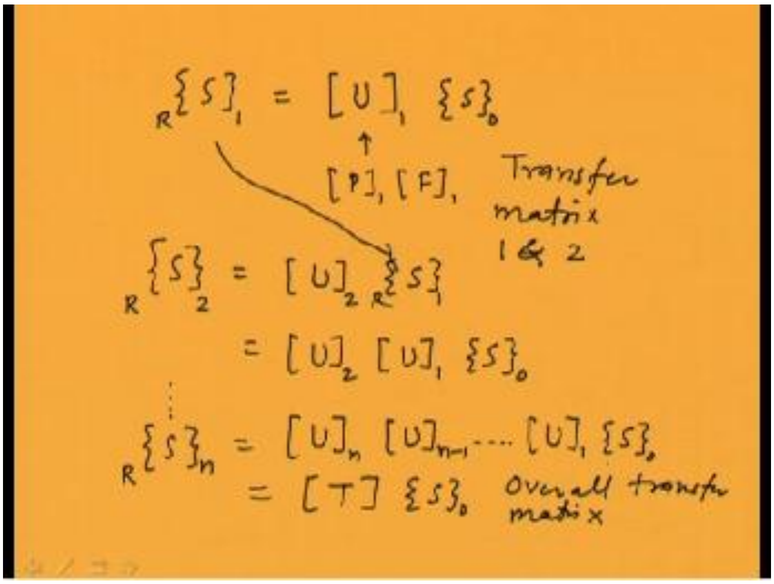

We are writing state vector right of 1 as a U matrix which is nothing but multiplication of the point matrix and the field matrix. So, U represent the transfer matrix relating to state vector 1 and 2, transfer matrix which relates station 1 and 2. So, the same thing we can able to write state vector 2 right of 2 S transfer matrix 2 state vector right of 1. And we can see that here we can able to substitute this. So, we will be having state vector 2 state vector 1 this. So, what we have done here, we are transferring the state vector of zeroth station to this state vector to the right of 2. So, this can be extended even if we have n number of discs. So, in that case we will be having state vector this transfer matrix n n minus 1 like this up to 1 state vector this.

So, you can see that once we have developed the point matrix and field matrix. So, we got the transfer matrix between 2 stations and it can be used to transfer the state vector from zeroth station to the last station. And, this can be simplified as T capital T and capital T is the overall transfer matrix is the overall transfer matrix and U was the transfer matrix between 2 neighboring stations. Now, we will see how we can able to obtain the natural frequency from the overall matrix which we have generated. So, what we did earlier will be obtained.

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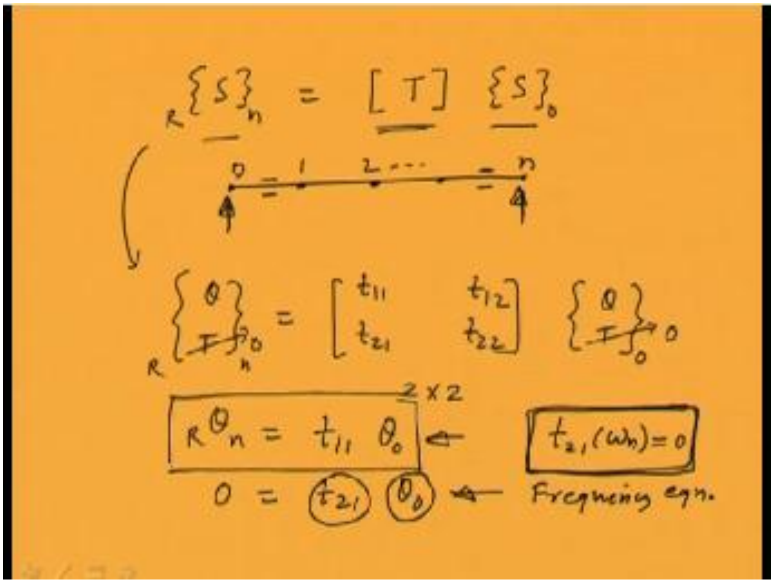

A matrix a equation like this which was a relating the state vector from beginning that is the extreme left of the shaft system to the right of the shaft system. So, if you see our previous figure in which we had 0 1 2 up to nth. So, this is the state vector at 0 locations and this is the state vector at the end station. Now, let us expand this equation, theta T we are writing in the expanded form and if you see this T matrix because this is multiplying various matrices, point matrices and field matrices and all are of having 2 into 2 size. So, final also we will be having same size, because we are multiplying the matrices. So, they will not increase in size they will remain in size. So, let us say whatever the multiplication final terms are like this and this theta T this is. So, 0 left is not there because this is the beginning of the station.

Now, we need to apply the boundary conditions. So, let us say we have for this particular case let us say we have simply supported ends. Now, to solve this particular equation we need to apply the boundary condition and if we see in this particular case we have supported this rotor over the frictionless bearings. So, they are not providing any torque in that so; that means, ends of the discs are free. So, here we have free free end conditions free free end condition means there will not be any torque at station 1 station 0 and there will not be any torque at the station n. So, what we can do we can equate this torque equal to 0 this equal to 0 this displacements will not be 0, because this free end it will be having some displacements.

So, once we have applied the boundary conditions for a specific problem in which we are considering the free free end condition. We can see this equation can be expanded as this is the first equation. Second equation will give, so you can see from the second equations especially because theta naught cannot be 0 because there is a displacement of the disc which is there at the station 0. So, it cannot be 0 because this is free end. So, this quantity has to be 0 so; that means, t 2 1 which is in fact, function of omega n equal to 0. So, this particular equation which will be a polynomial in omega n is the frequency equation, this can be solved if polynomial is what then fourth degree. Then obviously, we have to solve using some numerical technique up to third we can able to solve using closed form solutions. This second equation is that will be used to obtain the mode shape. So, once we have solved this particular frequency equation, will depending upon the number of degree of freedom in the system.

That means number of discs in this system we will be having that many number of natural frequency, like if we have n number of disc and the system rigid discs with mass less shafts will be having n number of natural frequency. And, so here those natural for finding the mode shapes corresponding to each of the natural frequency what we have to do? We have to come that to this particular equation in which let us go to the next slide.

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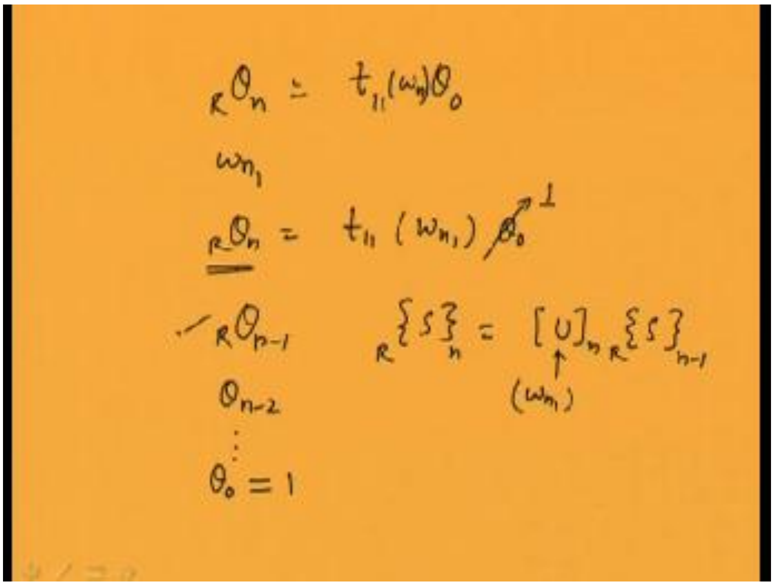

Because this t 1 1 is function of omega n. So, let us say if you want to obtain the mode shape corresponding to first natural frequency, we need to substitute this here and this can be taken as a reference let us say is having value one. So, if so we can get the what is the displacement in the nth station now we have relations earlier using transfer matrix method in which we are related the state vector of the nth to the state vector of the n minus 1. So, we can able to use those equations like state vector. So, these relations can be used and if you have this is function of omega n 1. So, a state vector at n minus 1 station can be obtained. Similarly, at n minus 2 and so on up to theta naught. So, these are the relative displacements of various discs and theta naught we have consider is 1.

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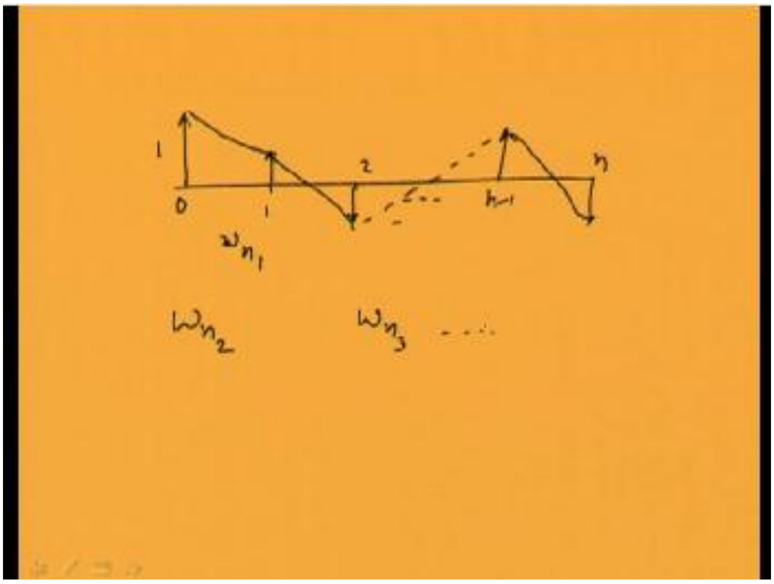

So, these displacements which we obtained for a particular speed, what we can able to do; we can able to plot them in this form this the 0 1 it will be having some value, 2 may be it will be some other value like this, up to nth this n minus 1 n. So, this is the mode shape for a multi degree of freedom system in mode shape is nothing but the relative displacements of the various discs. So, you can see that once we obtain the displacement of the shaft at one end, using transfer matrix we can transfer those information up to the other end. So, this corresponding to omega n 1.

Now, similar mode shapes we can able to obtain for other natural frequencies 2 or 3 just for 3 just what we have to do? We have to carry out the same transfer matrix equations we have to use to relate various displacements are corresponding to these natural frequencies. So, in a particular multi degree of freedom system; if we have n number of natural frequency; obviously, will be having corresponding n number of mode shapes.

Now, we have already seen how the transfer matrix method works. Let us see one example which we took earlier also. So, that the concepts regarding the transfer matrix method is more clear.

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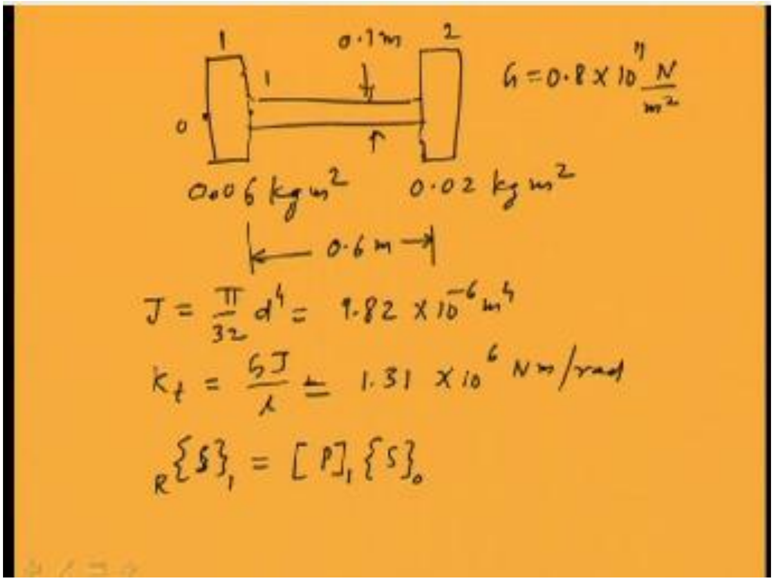

So, we are taking for simplicity two mass rotor, and polar mass moment of inertia of these are 0.06 and 0.02 kg meter per square the diameter of the shaft is 0.1 meter, and the length of the shaft is 0.6 meter. Now, the modulus of the rigidity is given as 0.8 into 10 raise to 11 Newton per meter square. With the help of diameter we can able to calculate polar moment of area that is meter 4. And, similarly the K t can be obtained as 1.31 into 10 raise to minus 10 raise to 6 Newton meter per radian. Now, let us come to the transfer matrix in which we have 2 stations; station 1 and 2 that is 2 discs are there and the state vector of right of 1 disc can be written as a state vector at zeroth mode. Zeroth station is here, station 1 is here, and this is related with only point matrix, because between 0 stations at 1 all the disc is there.

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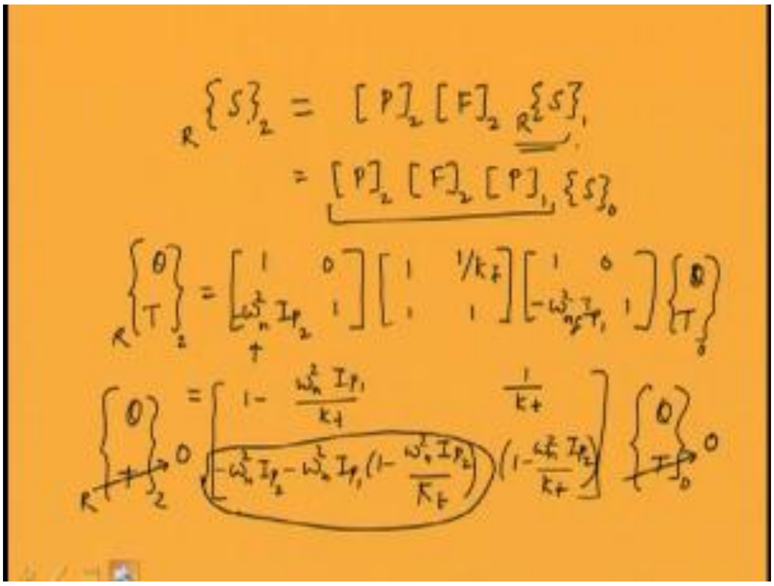

Then coming to the state vector and 2 that is right of 2 is given as; point matrix of disc 2 field matrix of shaft, and state vector right of disc 1. And, we already related this state vector with the state vector at zeroth station. So, we can able to expand this in this form. So, here overall transfer matrix is multiplication of 3 matrices P 1 F 2 and P 2, and for clarity I am writing these matrices. So, that the steps are more clear. This is the point matrix for disc 2 then; the field matrix for the shaft is this then the multiplication of the point matrix for disc 1 minus here, minus will come here also, in this place. Then the state vector theta and T at zeroth station. So, you can see that 3 matrices are there here to be multiplied, if we multiply this I am giving the final matrix.

So, that the steps are more clear to us this is the first element of the matrix then this is the second element, third element is having slightly bigger expressions, let us write that. This is K t and the last I p 2 by K t. So, this is the overall transfer matrix you can see that it contains omegas in various forms here then it will be having the terms like fourth degree omegas. So, now we can apply the boundary conditions. So, boundary condition is because if we go back to the previous thing this rotor is also supported on frictionless supports. So, both the ends are free for this case also. So, there is no torques at 0 stations and there is no torque at this second disc right side. So, both torques are 0, but displacements are not 0. So, you can equate this equal to 0 also this equal to 0. So, you can see that from the second equation or this term has to be 0 or let us expand this in the next slide.

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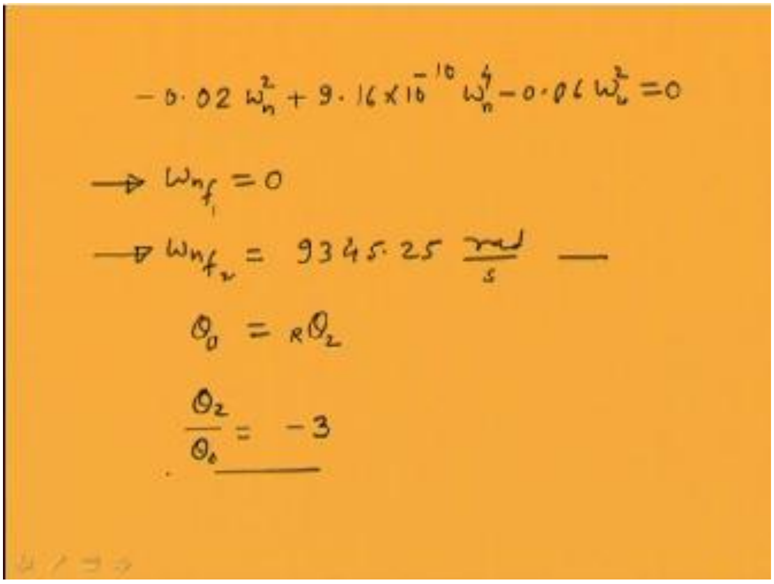

So, after substituting values of stiffness terms and mass polar mass moment of inertias we will be eating these terms is equal to 0. So, this the this particular term which we have equate it to 0, and we have substituted for various values. So, you can see there is a polynomial in omega n and it can be solved and of the natural frequency is 0, because omega n can common here. Another natural frequency will be obtaining is 9345. 25 and you can able to see that this is the similar example, which we are taken when previous lecture. And, the results are the same, but methods are different. There may be some difference due to the truncation error, numerical truncation errors.

Now, regarding the mode shape because natural frequencies are same, mode shape also remain the same. That means; for first one, first natural frequency this will come and this we can able to find out from the previous expression, the same expression we can substitute in this equation, the whole equation omega n is equal to 0. So, if we can see that theta which is right of 2 will come equal to theta naught. There is this is in the explicit form this form, and other intermediate displacements also can be checked through the transfer matrix method. The transfer matrices in between transfer matrices and you will see that all the displacements will be having same value, because corresponding to the zero natural frequency. We will be having all the displacement equal to 0 that is rigid body mode.

Similarly, if we substitute this in the previous equation, we will be getting relative displacement that is 2 by theta naught is equal to minus 3. Again we will be getting the similar as we got in the previous case, or now we will be starting another class of torsional vibration which is a part of the geared system which we have covered. In the geared system we have seen that the 2 shafts are having different speeds and we could able to convert that system to equivalent a single shaft system, but there are some application where we will find that the same gear one common gear gives power to 2 or 3 shafts.

Especially, in cotton industry we will find that there will be a common power source and various spindles will be given power from that, and because of that the power goes to various branches. And, that is why we call this particular system as a branch system, and this particular case will be doing will be analyzing for 2 branched 2 branches and the method can be extended for any number of branches. And, here we will be using transfer matrix method for this particular case.

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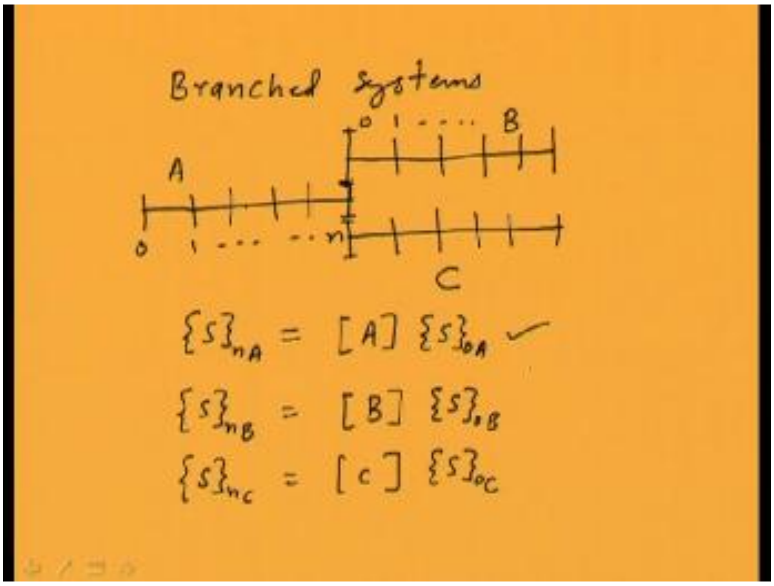

So, let us see what is branched system? So, we have one particular shaft it can have various discs and then this is the last one is gear, and it is transmitting power to another shaft. Let us say this is a branch A, this is branch B it can have 0 to n number of discs. Similarly, here also can have various numbers of discs. Then, there is another branch which is coming out from this, from the same gear that is let us say branch C. So, what is happening we have a branch A from where power is going to branch B also to branch B C. Now, our aim is to up to find out how we can able to get the natural frequency at the system.

So, that we will be knowing whether the resonance conditions, whether we can able to avoid this particular system. So, as I told we will be using transfer matrix method and we have already analyzed transfer matrix method for a particular branch. When there is single shaft we can able to write the state vector for that particular case like this. Let us say this is a branch we this state vector is corresponding to the n th disc here. And, let us say A is the corresponding overall transfer matrix for this. And, this is the state vector at the beginning of the branch A.

Similarly, we can able to write the state vector of n th disc of branch B here, with we can relate with the state vector of branch B to the at zeroth location. Let us say this is the overall transfer matrix for that branch B. Similarly, we have branch C. So, we can able to write this transfer overall transfer matrix relation for branch C. So, what we will be doing; we will be taking let us if the first one, first equation and will expand this because those 3 equation at present are independent. Now, what we have to do we have to find out the condition at the junction from where we are transferring power from one shaft another shaft. What are those conditions based on that will be linking this equations basically.

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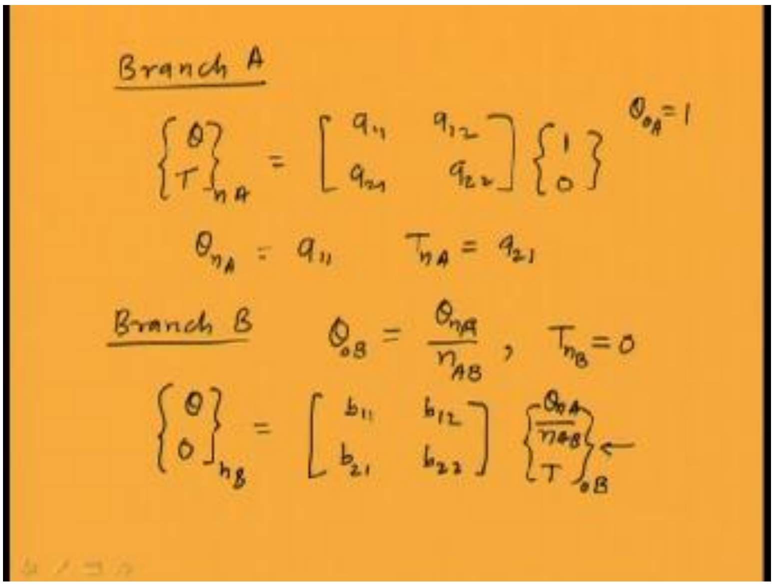

So, let us take branch A, and expanded form of that equation is let us say a 1 1 a 1 2 a 2 1 a 2 2 these are the matrices for corresponding to capital A. Then, here we have state vector at zero location, let us take 1 as this value. We are taking as a reference value that is theta naught A we are taking as reference value 1. And, because that particular and of the shaft is free. So, we are taking torque as 0, where is a free there will be no torque in this. So, from this we will get theta n A is equal to a 1 1 and from second equation we will get T n A is equal to a 2 1.

This is coming from these 2 equations; now, let us come to the branch B. In branch B how we can able to relate the branch A and B with the help of gear ratio we can able to relate the displacement at zeroth station of branch B, to the nth station of branch A by the gear ratio between branch A and B. This is the gear ratio. Now, of what is the other condition in branch B, because the right hand side of the branch B; that means, here this is the free end. So, we will be having torque in the nth station of branch B is equal to 0.

So, these are the 2 conditions we have which we need to apply in the second equation of this particular equation. So, let us expand this equation and if we expand it comes theta this is for the nth of B. So, this torque is 0 here, according to this should be equal to we are expanding for capital B these are the elements of the matrix. Then, here we have theta naught B. So, that can be written in terms of theta n A by gear ratio n A B may be and torque remains the same this.

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$$\theta_{nB} = b_{11}\frac{\theta_{nA}}{n_{AB}} + b_{12}\,\underline{\underline{T_{0B}}}$$

$$0 = b_{21}\frac{\theta_{nA}}{n_{AB}} + b_{22}\,T_{0B}$$

$$T_{0B} = -\frac{b_{21}}{b_{22}}\,\frac{\theta_{nA}}{n_{AB}}$$

Branch C

$$\theta_{0C} = \frac{\theta_{nA}}{n_{AC}} = \frac{a_{11}}{n_{AC}}$$

$$T_{nA} = \frac{T_{0B}}{n_{AB}} + \frac{T_{0C}}{n_{AC}}$$

Now, these equation can be expanded n B that is the first equation b 1 1 theta n A by gear ratio A B then we have b 1 2 and T B naught. And, second equation of the previous this expression will give us b 2 1 theta n A n A B then b 2 2 T B naught. Now, this second equation can be you can see that from this second equation you can obtain the torque zeroth station. So, this is b 2 1 divided by b 2 2 theta n A by gear ratio n A B. So, this expression we got which is the torque at zeroth station of branch B, okay. This and from the first equation you can see we can able to substitute this here. So, that we can an able to eliminate the theta naught B from the first equation here. So, let us if we do it then it will be containing only the displacement at branch A the last station of branch A here also.

So, you can able to see that this particular equation will be containing only the displacements. That will be using in future for obtaining the mode shapes. So, now let us come to the branch C. We already covered the branch A and B we have related them also. Now, from branch C what we can able to find out. So, at branch C we have similar relation as a branch B, first thing is with the gear ratio between A and C, and a part from this there is another condition which is coming from the how much strain energy is transferring from gear A to B and C. So, if we equate that will get a relationship of for torque which is transmitted through A and B and C is this torque C. So, these 2 conditions we have to satisfy are the branch for the branch C. So, these equations, if you see carefully can be written as theta n A we have already obtain earlier that is equal to a 1 1 by n A C.

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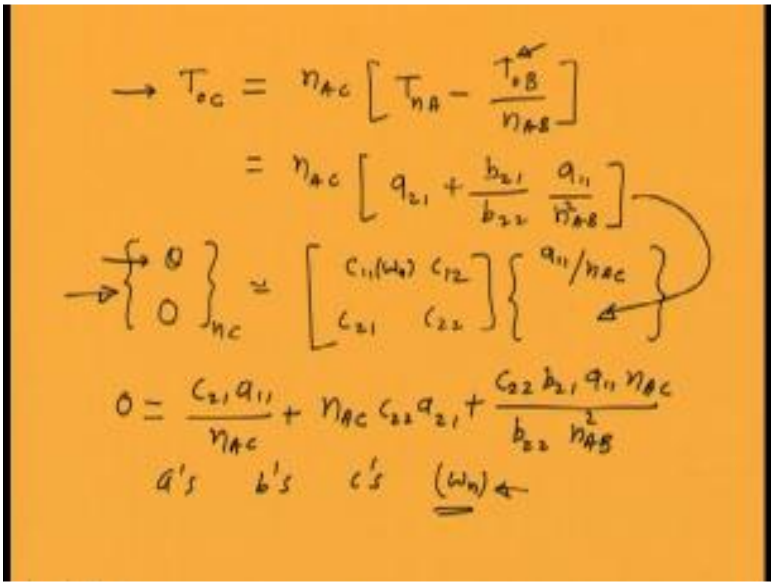

And, then the second equation we can able to write in this particular form in which we want to know the torque at station 0, in branch C equal to just we are transferring of the terms in other side. So, that we get the expression in terms of known quantities. So, here you can able to see that we have the expression for T naught B earlier. Let us go back to the that previous slide T naught B is this one, this can be written as b 2 1 theta n A is nothing but a 1 1 divided by b 2 2 n A B. So, this expression will be putting here, and if we substitute that we will find this particular form of the equation.

So, basically here we have eliminated all the torque terms, and now they are in terms of the overall transfer matrix for branch A B and C. You can see a 1 1 b 2 1 b 2 2 all these are related to the overall transfer matrix of the branch A and B. And, n A B is the this gear ratio between branch A and B. Now, we will be writing the equation for the expanded form of the equation for branch C here, theta is for n C because free this particular end of the shaft is free. So, torque is 0 is equal to these are the elements of overall transfer matrix branch for C then here are theta naught C is there which we have obtained like this.

So, we will be substituting this value directly there a 1 1 by n A C and we have theta naught C is there which we are all obtain expression. So, with this quantity we will be directly substituting here. So, once we are substituted here the from the second equation we will get a equation of this form which is nothing but the frequency equation of this branch system is a slightly bigger expression, but for completeness I am writing this. So, you can see this particular equation which is frequency equation; in this a s b s and c s all contained omega n. And, these here will be having something like polynomial in term of omega n. And, they can be solved if it is up to third degree you can able to solve close form.

Otherwise we have to go back for numerical techniques. So, for solving the polynomial and depending upon the number of discs we have in the system that many number of natural frequency will be getting that many roots of those polynomial will be getting. And, then once we got the natural frequency then again we have to go back to the these transfer matrix equations. So, that for a particular natural frequency we will be substituting various in various terms, like here all c s are function of omegas. So, we will be substituting here.

So, we will be getting the displacement at n th station of c and then using other transfer matrix method as earlier also it has been explained. We can able to obtain the relative displacement at various stations. So, that method is already explained in the previous discussions. So, I am not repeating that same thing here. Now, we will be taking one example of the branch systems. So, that our concepts are more clear, and especially I will be taking very simple examples, so that the more focus is towards the clearing the steps rather than complexity of the problem.

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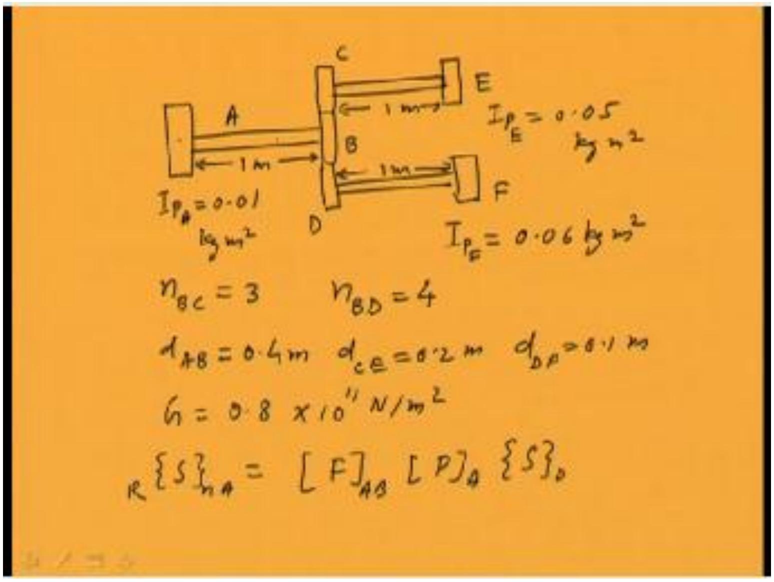

So, in this particular case, I am taking a very simple branch system in which this a branch A and this is the gear B and here 2 gears are connected one is C another is D. And, this particular shaft is connected with a disc E, and this particular shaft is connected with a disc F. So, here we are neglecting the mass moment of inertia of the gears C and D. This disc A is having mass moment of inertia polar mass moment of inertia. This disc is having polar mass moment of inertia, and this is also having polar mass moment of inertia. They are having the following values 0.01 kg meter square then is having 0.05 kg meter square, and the disc F is having 0.06 kg meter square.

The gear ratio between B and C is 3 and gear ratio between B and D is 4. So, these are the gear ratio given. And, various diameters and lengths of the shafts are like all the shafts are having 1 meter length for simplicity we have taken like this. All the shaft is having 1 meter length, and diameter of the A B shaft is 0.4 that is this particular shaft, and diameter of the second shaft that is here is 0.2 meter, they are all in meter. And, diameter of the third shaft is 0.1 meter. Now, the modulus of rigidity is 0.8 into 10 raise to 11 Newton per meter square. Now, what we can do; we can write the transfer matrix for the branch A that will be multiplication of the field matrix that is shaft A B and point matrix of shaft A. Field matrix and point matrix then that particular terms can be multiplied.

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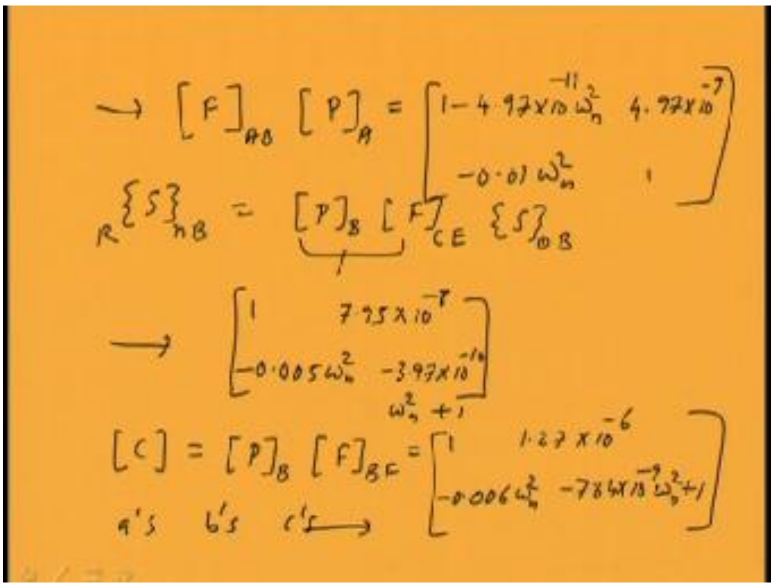

I am giving the final form of that matrix that is F A B and P A is given as; if you substitute the values here we will get this. So, this I am expanding it. So, that this method is more clear, I am expanding this because method should be more clear. And, for branch P similarly, we can able to write. Point matrix and field matrix of the shaft C E, and this particular if we substitute the values we will get here double 0, and here minus 3.97 10 raise to minus 10 omega n plus 1. Similarly, we can get the overall transfer matrix for C matrix as point matrix for B and field matrix for B F that is having this form and last term is plus 1. So, here we got all the 3 matrices for A this is the A then C. So, we now all the terms of a s b s and c s. Now, we can go to the frequency equation.

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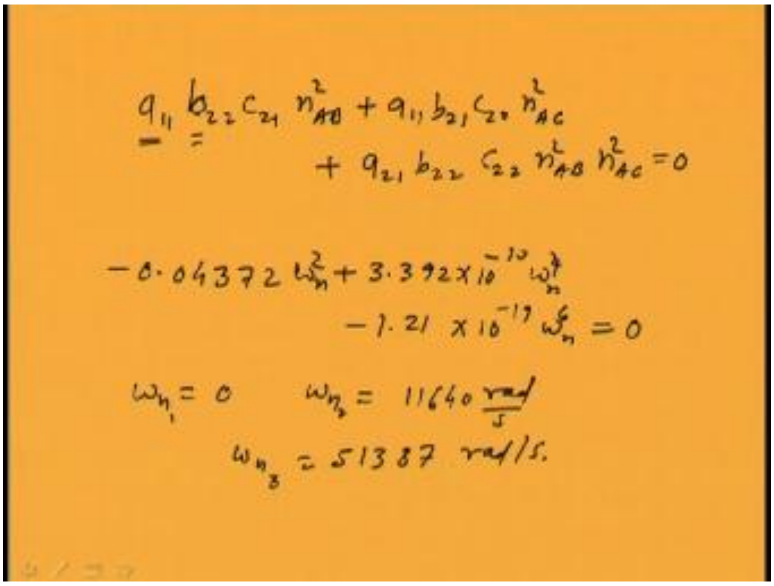

Frequency equations in the more simplified form, is of this form, this plus another term. So, you can see that this equation is nothing but a polynomial in terms of omega, because all these term contain omega they are the elements of the matrices. So, if we substitute various values from the previous matrices. We will get a polynomial of this form minus 1.21 10 raise to minus 19 omega 6 and is equal to 0. So, this can be solved for omega n. So, you can see that there will be 3 roots first will be this, second is then third is. You can see that the method is very clear now for finding the natural frequency. The mode shapes can be obtained we can to go back to the various transfer matrices. So, in this particular lecture we have covered the transfer matrix method initially and for the single of shaft ((Refer Time: 01:01:45)) system. Then, we extended the method for the branch system also. And through numerical example we try to clear the basic concepts especially the help of very simple examples.