Mechanical Vibrations

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Module - 7
Multi DOF
Lecture - 4
Modal Analysis: Damped

Today we are going to revise what we have studied for multi degree of freedom systems. In the previous classes, you have studied the free and force vibration of single degree of freedom system, 2 degrees of freedom system, and systems with more than 2 degrees of freedom. In case of multi degree of freedom systems which are either 2 degrees of freedom system or a degrees of freedom higher than 2, we have studied different methods to find the equation of motions, and then to solve those equations of motion to find the response of the system. So, we have used Newton's method, we have used Lagrange principle or extended Hamilton principle to derive the equation of motion.

And, also we have used the influence coefficient method to derive the equation of motion in case of influence coefficient method. So, when we are taking displacement influence coefficient we are getting the Eigen frequencies which correspond to the higher mode frequencies. The lower Eigen frequency is correspond to the higher Eigen frequencies and in case of stiffness matrix formulation we are getting the frequencies in the order of lower to higher. And, we have also studied the modal analysis method, and we have studied the free and force vibration of multi degrees of freedom system. So, today we are going to solve some problems considering this modal analysis method.

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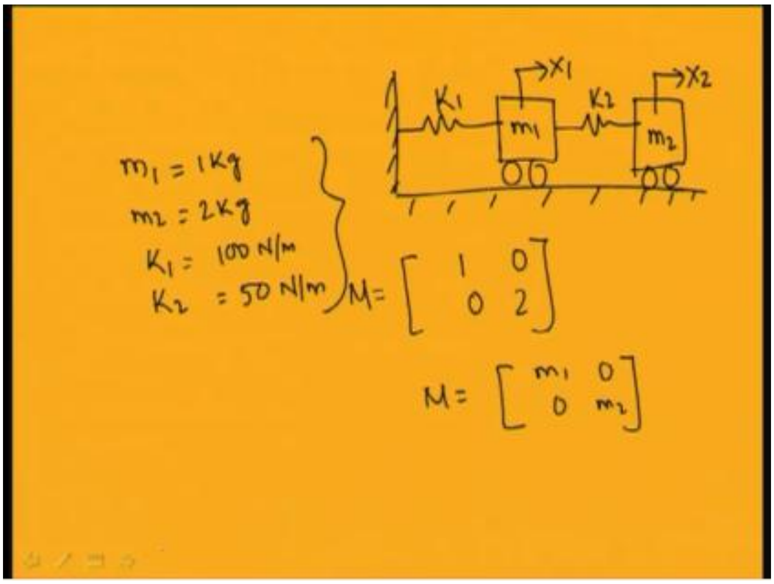

Let us first take one example in which I have taken a spring mass damper system. So, this is a spring and mass damper system. So, this is mass 1 and this is mass 2 and they are connected by the spring the spring K 1 this is spring K 2, and this is m 1 this is m 2. And, my aim is to find the principle coordinate system of the system. Here, I am taking m 1 let m 1 equal to 1 kg m 2 equal to 2 kg and then K 1 equal to 100 Newton per meter and K 2 equal to 50 Newton per meter. So, by taking these physical parameters let us derive this equation of motion, and let us find the principle coordinates of the system.

So, you know before principle coordinates are those coordinates which will yield both mass matrix and stiffness matrix uncoupled. Let us first take this coordinate system; generalized coordinate system this is X 1 and this is X 2. So, the mass matrix you can write it equal to mass matrix will be equal to 1 0 0 2. So, this is the mass matrix M which is equal to already you know this mass matrix equal to m 1 0, 0 m 2. So, I substituting this value. So, I can write this mass matrix equal to 1 0 0 2 to find the stiffness matrix I can use many different methods to find. So, let me use this influence coefficient method to find this stiffness matrix.

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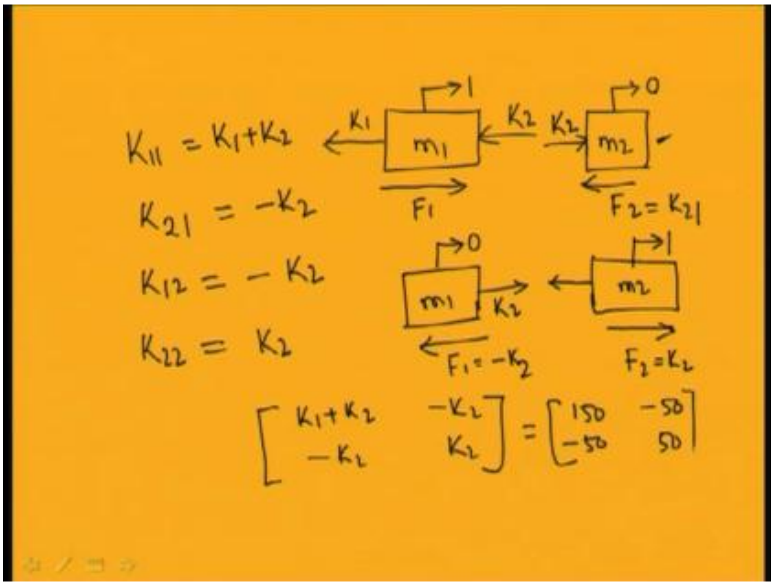

So, in this case I can draw the free body diagram. So, this is mass m 1 this is mass m 2. So, to draw the free body diagram when this mass m 1 has a displacement of 1 and these are the displacement 0. So, I can find this K 1 1. So, K 1 1 is nothing but the force required to have unit displacement at 1 and 0 displacement at 2. So, for this case I can find the free body diagram, so according to the free body diagram. So, this is the mass m. So, when it is moving a distance unity the spring will exert a force of K 1 in opposite direction. So, this is K 1 in opposite direction, and this side spring those right side springs that is K 2 will be, so this the spring K 2.

So, when it is moving a unit direction this towards right. So, and there is no displacement of X 2. So, the spring k 2 will be compressed by an amount unity. So, it will exert a force in opposite direction that is in this direction, so which is K 2. Similarly, for this mass 2 as it is stationary or it has X 2 equal to 0, for the time being. So, this m 2 will be subjected to a spring force K 2 which is acting on it in this direction. So, this is k 2. So, now for this case K 1 1 will be the force required at 1 to half unit displacement at 1 and displacement 0 at this.

So, you have unit displacement at 1. So, this mass is subjected to a force K 1 plus K 2. So, one has to apply a force K 1 plus K 2 to half this displacement. So, this is F 1 or K 1 1. So, this is equal to K 1 plus K 2. Similarly, K 2 1 displacement required at 2 to have unit displacement at 1 that is K 2 1 and K 1 2 is force required at 1 to half. Unit displacement at 2 K 2 1 K 2 1 is force required at 2 to have unit displacement at 1. So, already we have written that for unit displacement at 1, and 0 displacement at 2. This is the free body diagram. So, from this free body diagram we have found that in this case the mass m 2 is subjected to a force k 2.

So, to have 0 displacement here. So, it will be subjected to a force F 2 or one has to give apply a force F 2 which is equal to K 2 1. So, this force is applied in opposite direction. So, it will be equal to minus K 2, so K 2 1 equal to minus K 2. Now, k 1 2 I can draw the free body diagram. So, this is the free body diagram for this mass m 1 this is mass m 2. So, in this case as this mass m 1 has 0 displacement and m 2 has unit displacement then the spring k 1 spring will be not be displaced, or k 1 spring will be stationary. So, it will not exert any force on mass 1. But the middle, but the spring between m 1 and m 2 that is k 2 will be pulled by an amount unity.

So, this mass m 1 will be subjected to a force K 2 towards right to keep it in the equilibrium position. So, one has to apply a force that is F 1 that is equal to minus K 2 to keep it in the equilibrium position. So, K 1 2 will be equal to minus K 2. So, this is equal to minus K 2 and K 1 2 will be equal minus K 2. Now, to determine K 2 2 that is force required at 2 to have unit displacement at 2, and 0 displacement at 1. So, you can find from this free body diagram that this mass m 2 is subjected to a force K 2 in opposite direction. So, or to maintain this displacement one has to apply a force that is equal to F 2 which is equal to K 2 and so K 2 2 will be equal to K 2. So, the stiffness matrix by this method you can write it equal to. So, the stiffness matrix will be equal to K 1 plus K 2 minus K 2 and minus K 2 K 2. So, this will be equal to 150, and this is equal to minus 50 minus 50 and K 2 equal to 50.

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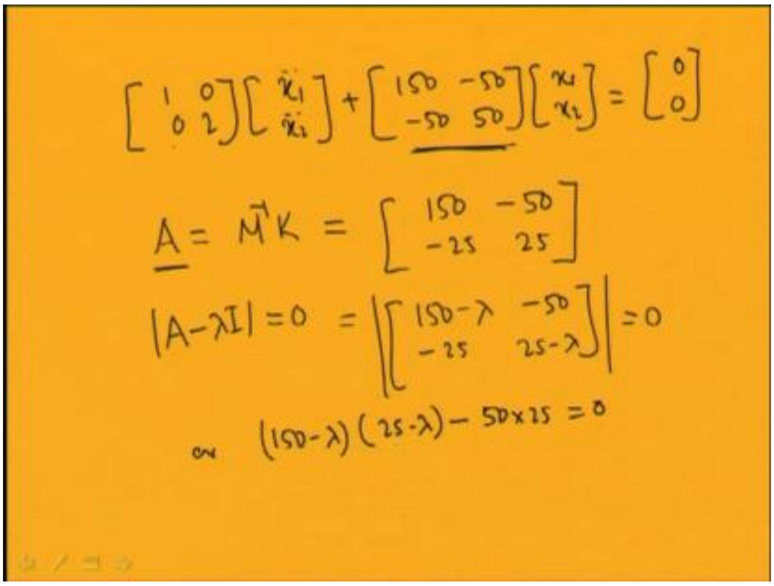

So, I can write the equation of motion in this form. So, the equation motion becomes 1 0 0 2 into x 1 double dot x 2 double dot plus 150 minus 50 minus 50 this is 50, and this is x 1 x 2 equal to 0 0. So, in this case you can observe that the system is dynamically uncoupled, but statically coupled. So, this is static coupling of the system. So, as it is statically coupled these x 1 x 2 are not the principle coordinate. So, we have to find the coordinate Y. So, that this both mass matrix and stiffness matrix will be uncoupled, to find that thing we should find the normal mode, and we can find from the normal mode, we can find the modal matrix and using that modal matrix we can find this principle coordinates.

So, to find this normal mode, so we can assume the solution x 2 be in the form of x sin omega t. So, I can write this modal matrix. So, already you know I can write this A matrix equal to M inverse K, I can find this M inverse K. So, M inverse K will be equal to 150 minus 50 this is minus 25 and 25. And, you know that the Eigen value of A will give the normal mode frequency and Eigen vector of A will give the modal matrix of the system.

So, to find these things if these Eigen modes and Eigen vectors, so you can find A minus lambda I determinant of A minus lambda I equal to 0. You can make determinant of A minus lambda I equal to 0, which will give you 150 minus lambda minus 50 minus 25 25 minus lambda. So, determinant of this equal to 0 or 150 minus lambda into 25 minus lambda minus into minus plus.

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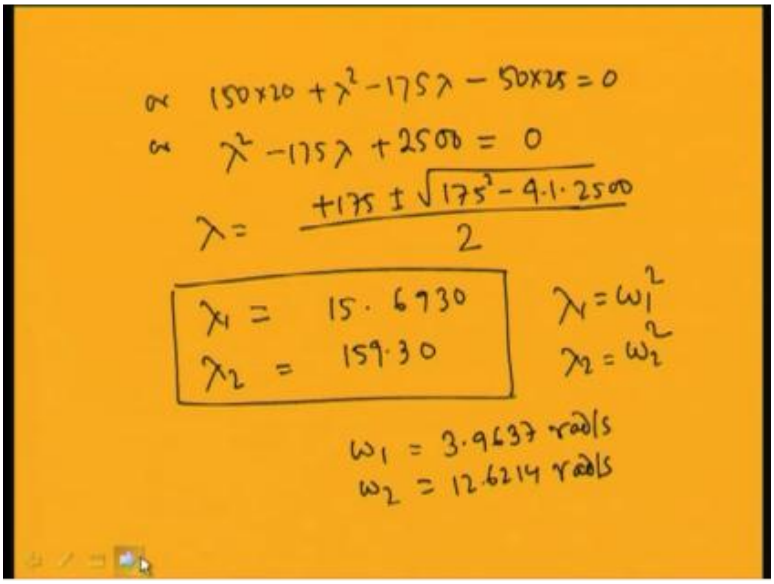

So, minus 50 into 25 equal to 0 or from this you can find this is 150 into 25 or 150 into 25 plus lambda square minus 175 lambda minus 50 into 25 this is equal to 0 or your equation is reduced to lambda square minus 175 lambda plus. So, 100 minus 50 150 minus 50 100. So, this is 2500 equal to 0 or lambda will be equal to minus b. So, minus minus plus 175 minus b root over minus b square minus 4 a c by 2 a minus 4 a equal to 1 and c equal to 2500 by 2. So, if you calculate these. So, you will get 2 values lambda. So, these values becomes, so the first value becomes 15 point. So, it becomes 15.6930 and lambda 2 becomes, so lambda 2 equal to 159.30.

So, these are the Eigen frequencies of the system and these correspond to the square of the natural frequency or normal mode frequency of the system. So, lambda 1 equal to omega 1 square and lambda 2 equal to omega 2 square. And from these you can find omega 1 equal to and omega 2 these 2 values you can find. Omega 1 will becomes equal to 3 point. So, this becomes 3.9637 and this becomes 12.6214. So, these are the natural frequencies or normal mode frequencies of the system. So, omega 1 corresponds to the first mode frequency, and omega 2 corresponds to the second mode frequencies. Now, to find the modal matrix one has to find this Eigen vector of the system to find this Eigen vector already you know.

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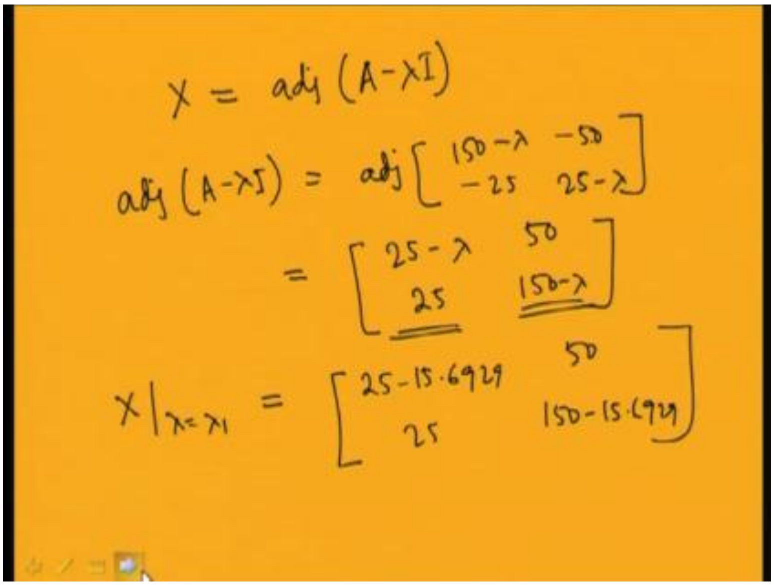

That you can find directly you can find by finding the Eigen vectors of the system or you can find this thing from these expression X equal to adjoint A minus lambda I. So, by finding the adjoint of A minus lambda I you can find this modal matrix or normal modes of the system or directly you can find this thing by finding the Eigen vectors of the system. So, now let us find in this way by finding the adjoint of A minus lambda I already you know A minus lambda I. So, adjoint of A minus lambda I equal to... So, I can write this as A already you know it is equal to. So, this is adjoint of 150 minus lambda this is minus 50 this is minus 25and 25 minus lambda.

So, the transpose of that will give it is equal to 25 minus lambda and this is 25 and 50 minus 150 minus lambda. So, X you can obtain at lambda equal to lambda 1 by substituting lambda equal to lambda 1 which is equal to 15.6929. So, you can find the first mode frequency. So, you just note that either you take this column or this column first column or second column the normalized value will be same. So, now to verify that thing now, you substitute lambda equal to lambda 1. So, you can find that first column becomes, so 25 minus 15.6929. So, 25 minus 15.6929 this is 25 and this is 50 by 150 minus 15.6929.

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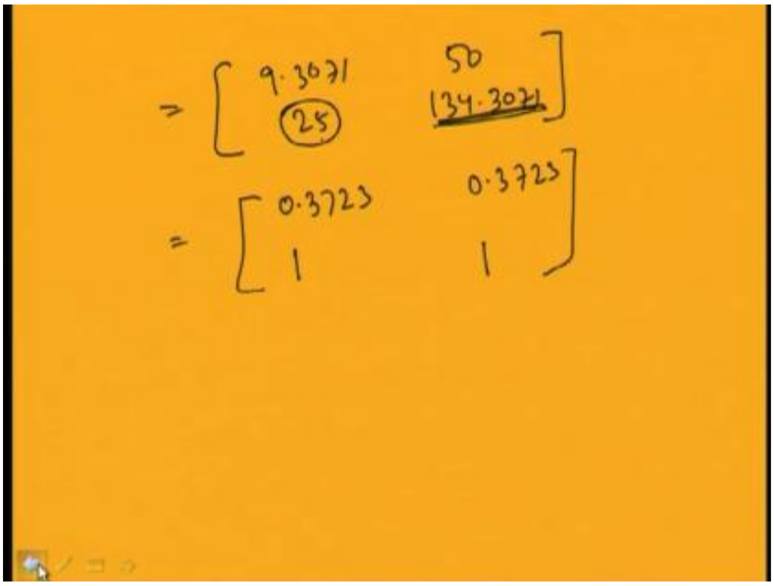

So, if you calculate, so this becomes 9.3071 this is 25 and this is 50 134.3071. So, you can find that normalized value of both the columns are same. Let me write this, so these let me make this displacement unity. So, this will becomes. So, if I will make it unity. So, I will divide 9.3071 by 25. So, that will becomes 0.3723 and in this column also I can make this column equal to this element equal to unity. So, I can divide this 50 by 134. 3025 3071. So, this becomes also 0.3723. So, you can check that both the columns yield the same value.

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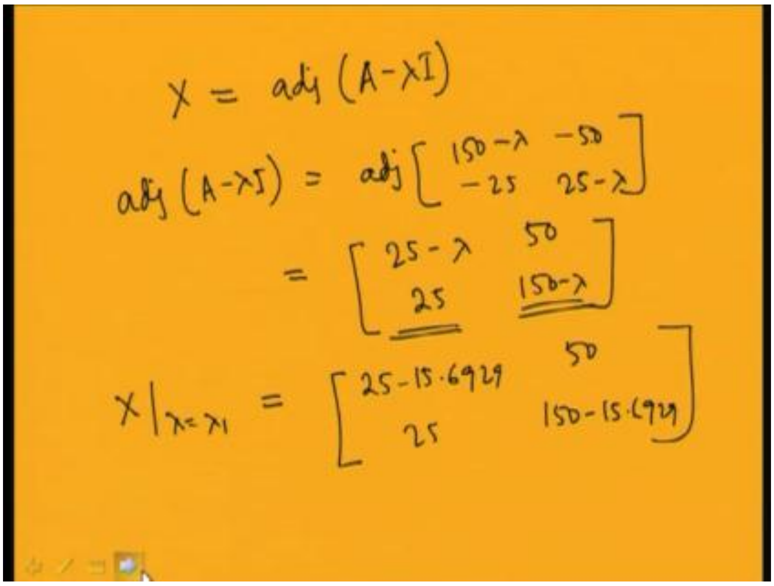

So, in this case without taking all the columns of this adjoint matrix you can take only a single column of that adjoint matrix to find this modal participation.

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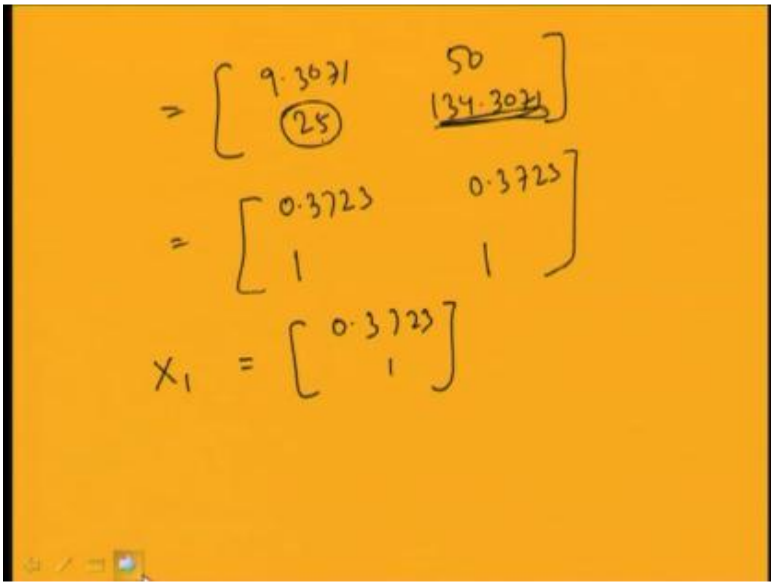

Or the mode normal modes of the system, so the normal mode X 1, now it becomes 0.37231.

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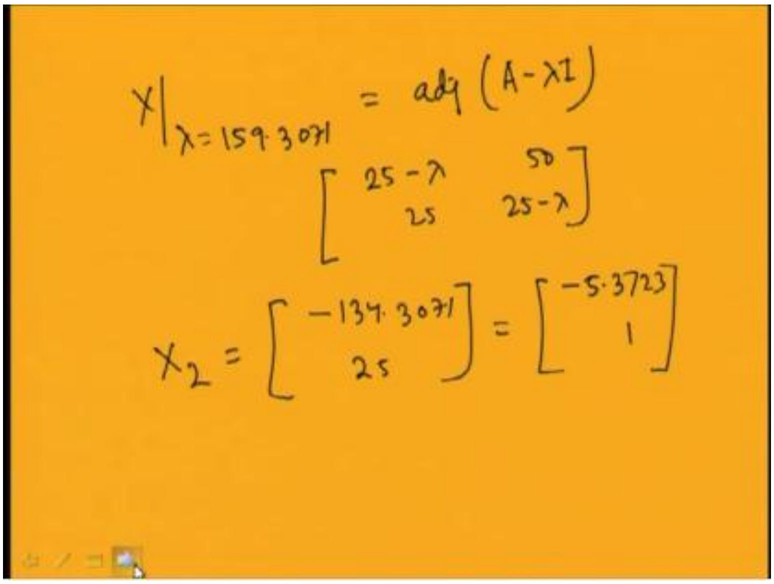

Similarly, you can find X 2. So, X 2 can be obtaining by. So, X at lambda equal to lambda 2 that is lambda equal to 159.3071 equal to adjoint A minus lambda I. And already you got adjoint A minus lambda I that is equal to. So, A minus lambda I adjoint equal to 25 minus lambda minus 50 minus 25 25 minus lambda. Now, you substituting this value of 159.3071 for the second mode you can find this X 2 you can take any column this. So, let me take this first column. So, 25 minus lambda 25 minus 159.159, so this adjoint equal to this is plus 25 this is plus 50, so 125 minus lambda becomes taking this first columns it becomes minus 134.3071 by 20 this 25. So, by normalizing this thing I can get this is equal to minus 5.3723 by 1. So, you can obtain the second mode equal minus 5.37231 physically it means.

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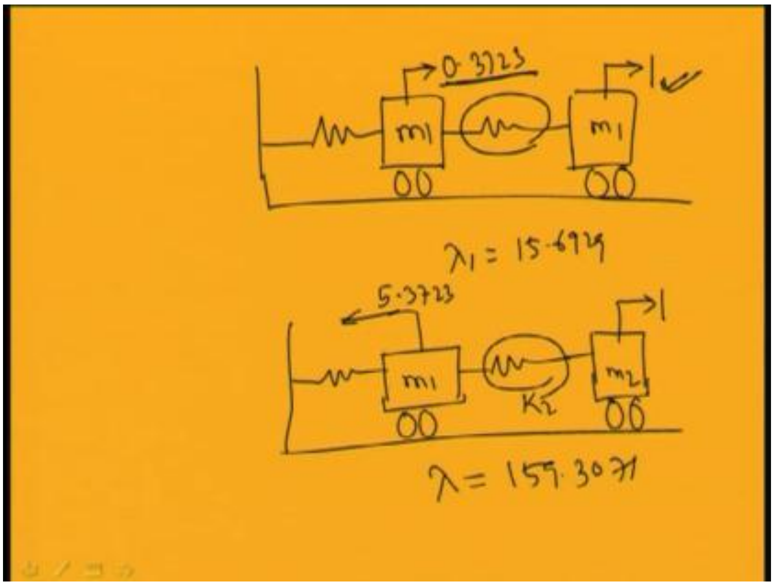

So, in the spring mass damper system, physically it means when the system or when this mass m 1 is moving with the or the when the system is moving with the first mode frequency that is lambda 1 equal to 15.6929. Then when this will, this mass m 2 will move a unit distance mass m 1 will move a distance of point. So, it will move a distance of 0.3723. So, this for first mode and the for the second mode. So, they are moving in the same direction and in case of second mode, so second mode the first mass when the first mass is moving in opposite direction. So, in this case lambda equal to 159.3071.

So, this is mass m 1 this is mass m 2. So, for unit displacement of mass m 1, m for unit displacement of mass m 2, m 1 will move in opposite direction with a magnitude of with a magnitude of 5.3723. So, in this case the spring k 2 will be under tension. And in this case as both the springs and as both of them are moving in the same direction. So, this is while it has a unit displacement m 1 has a displacement of 0.3723. So, by using these 2 modes I can write this modal matrix P.

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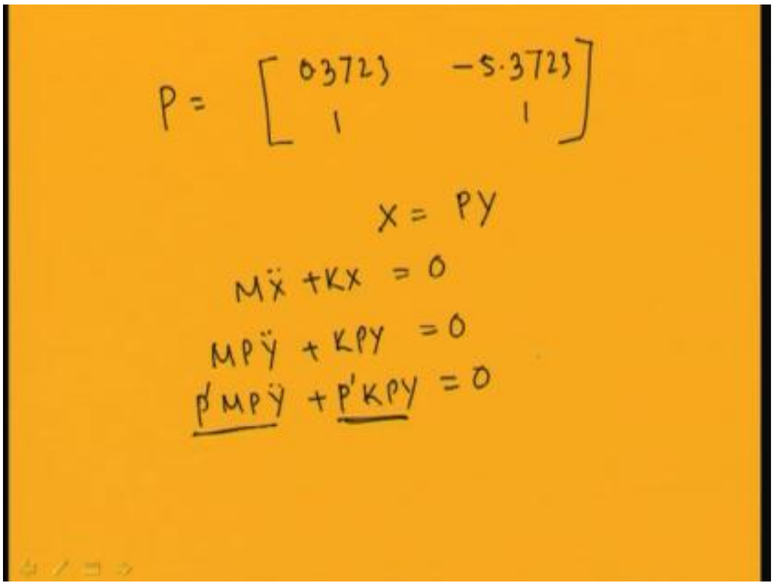

So, modal matrix P equal to 0.3723 1 and this is equal to minus 5.3723 1. So, now to find the principle coordinates I can write, let the principle coordinate be Y. So, I can write let X equal to P Y. So, I will substitute this equation in the original equation that is M X double dot mass matrix. So, that is M X double dot plus K X equal to 0 equations. So, this becomes M P Y double dot plus K P Y equal to 0. Now pre multiplying P transpose, I can write P transpose M P Y double plus P transpose K P Y will be equal to 0. Let us find this P transpose M P and P transpose K P. Already you have known from this orthogonality property of the modes that P dash M P and P dash K P are diagonal matrices.

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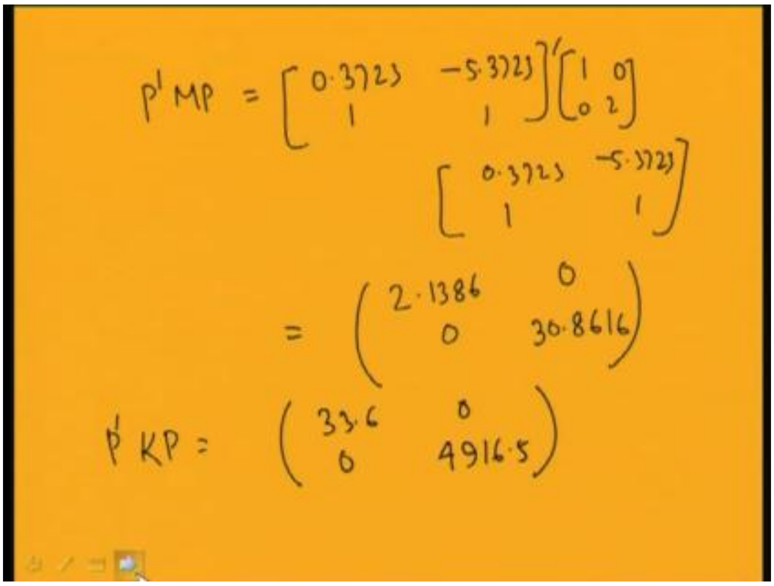

Now, let us find P dash M P, so P dash M P will becomes equal to 0.37231 and this is minus 5.37231. So, these transpose and M equal to 1 0 0 2 and multiplied by these 0.37231 minus 5.37231. So, if you multiplied this you can find this P dash M P equal to 2.1386 0 0 and 30.8616. Similarly, you can find this P transpose K P equal to 33.6 0 0 49 16.5. So, this is P dash K P and P dash M P equal to 2.1386 0 0 30.8616. So, you can find from this that both P dash M P and P dash K P are diagonal matrix.

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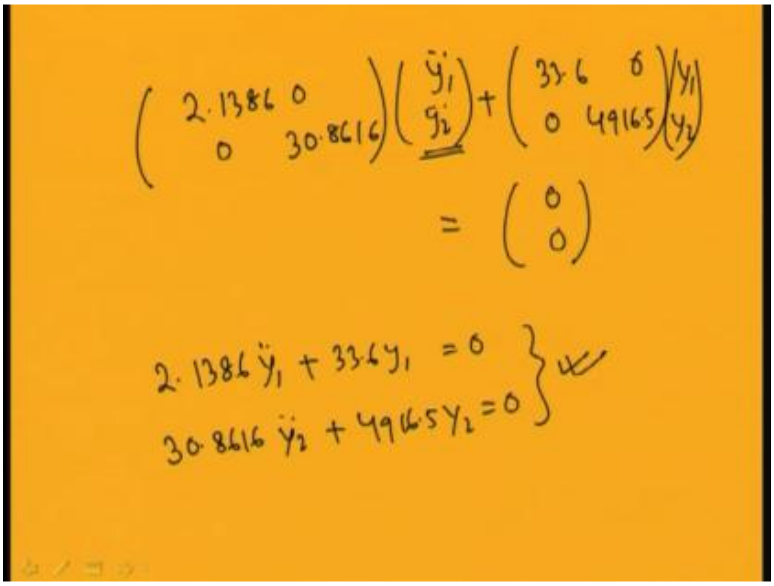

So, you can write this equation in this form, so this becomes 2.1386 0 0 and this becomes 30.8616 y 1 double dot y 2 double dot plus these matrix 33.6 0 and 0 4916.5 into y 1 y 2 this becomes 0 0. So, in this case you can see that by using this matrix or by using this coordinate y 1 y 2 both the mass matrix and stiffness matrix are uncoupled. That is they are the half diagonal terms are 0. So, you can write these equations in terms of 2 single degree of freedom system equation that is 2.1386 y 1 double dot Plus 33.6 y 1 equal to 0, this is the first equation and the second equation becomes 30.8616 y 2 double dot plus 4916.5 y 2 equal to 0. So, from these one can obtain this y 1 and y 2 by solving these two equations you can obtain y 1 and y 2. So, let us solve these first equation and second equation to find y 1 y 2 the same set of equation you can obtain by using the weighted modal matrix also. Before finding these solution of these let us use this weighted modal matrix method.

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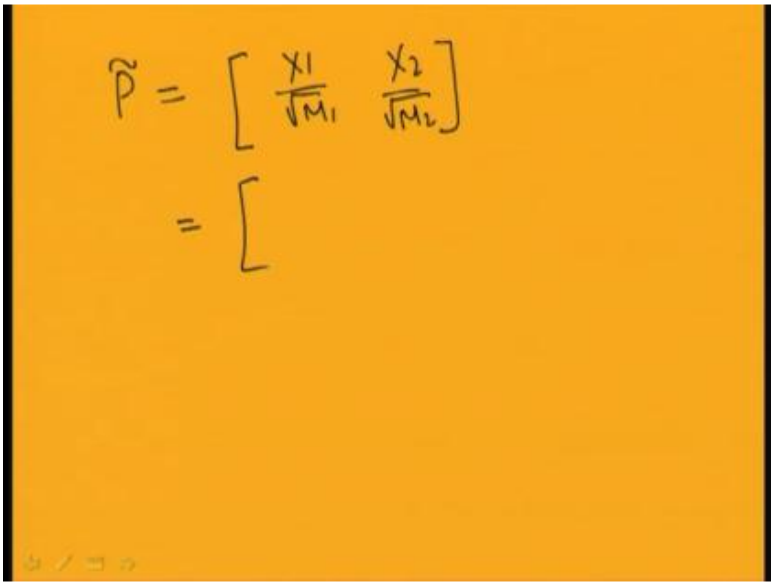

So, let us find this weighted modal matrix, already you know the weighted modal matrix is nothing but these X 1 by root over M 1 and this is X 2 by root over M 2. The weighted modal matrix can be obtained by dividing the square root of the first mode in the first column and the square root of M 2 in the second column. Already you know the P matrix and M 1 and M 2.

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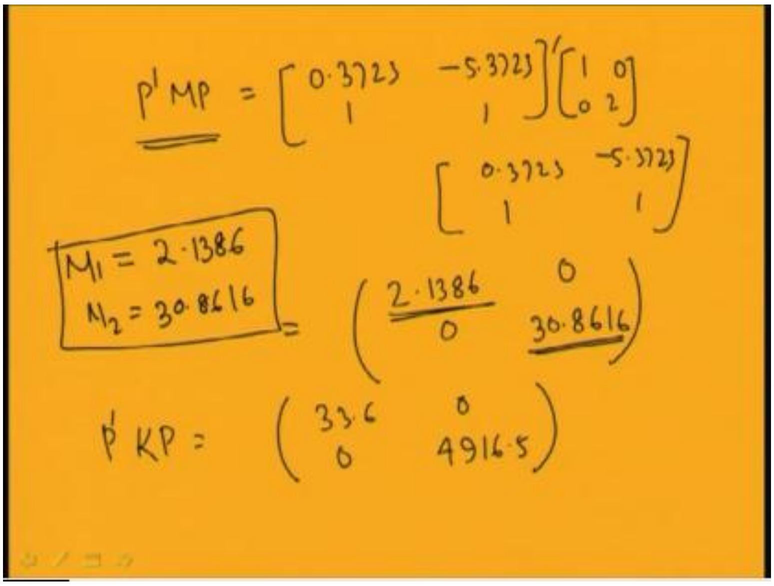

We have obtained previously, so by finding these P transpose M P this is M 1 and so P transpose M P. So, when we have drawn P transpose M P. So, M 1 is this 2.1386 and M 2 equal to 30.8616. So, M 1 is 2.1386. So, this is M 1, so I can write this generalized mass M 1 equal to 2.1386 and generalized mass M 2 equal to 30.8616. So, to find these weighted modal matrixes, the first column of the modal matrix is divided by root over M 1 and the second column is divided by root over M 2.

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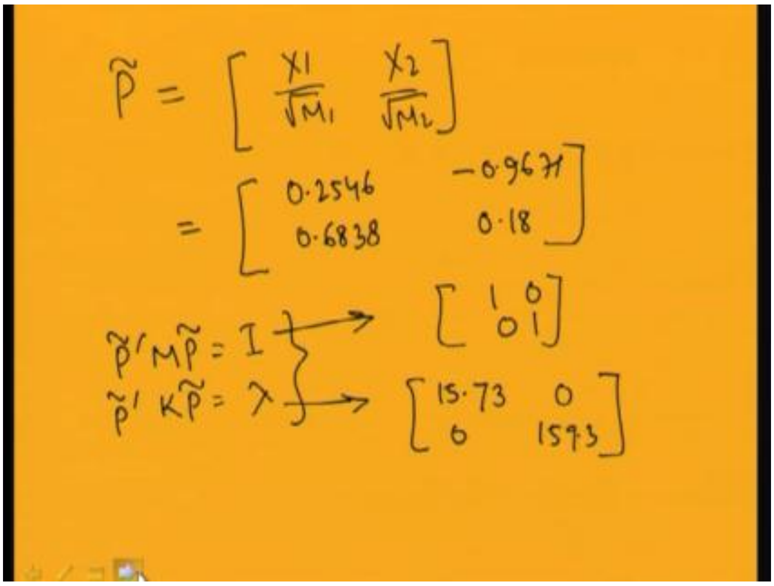

So, by doing that division, so P weighted modal matrix 1 can obtained equal to 0.25 46 this is 0.6838. And this becomes minus 0.9671 and this is 0.18. So, now you can verify this P weighted modal matrix transpose M P equal to I matrix that is, identity matrix and P weighted modal matrix K P is nothing, but the lambda matrix. So, that is P weighted modal matrix the first one becomes 1 0 0 1 and the second one becomes, so this is first lambda 1 that is 15.73 and 0 and 0 159.3. So, P weighted modal matrix K P weighted matrix equal to lambda that is 15.73 that is lambda 1. So, lambda 1 0 and this is 0 lambda 2.

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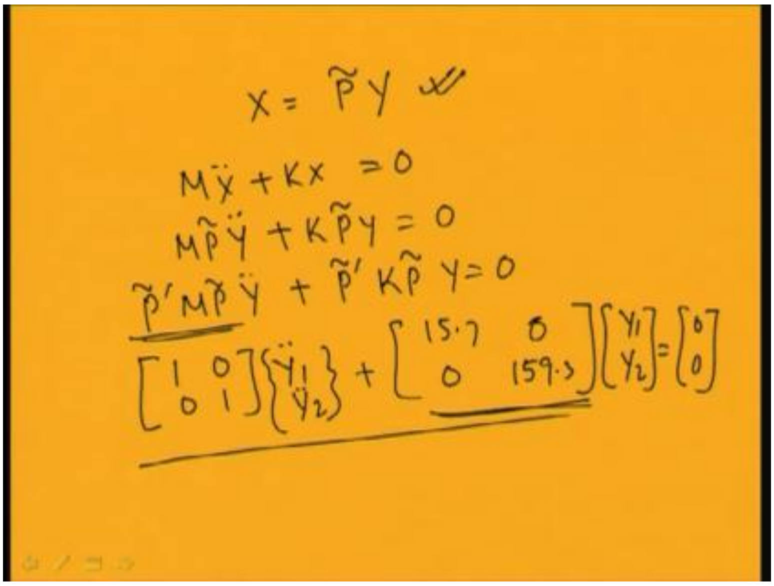

So, instead of substituting X equal to P Y, if I am substitute X equal to P weighted modal matrix Y, then the equation M X double dot plus K X equal to 0 will reduce to M P weighted modal matrix Y double dot plus K plus into P weighted modal matrix Y equal to 0. Now, re-multiplying by P weighted modal matrix transpose M P weighted modal matrix Y double dot plus P weighted modal matrix K P weighted modal matrix Y equal to 0. So, already you know that this is identity matrix that 1 0 0 1. So, Y 1 Y 2 double dot this is double dot plus. So, this becomes 15.7 this is 0 and this is 159.3 this is Y 1 Y 2 this becomes 0 0.

So, you need not have to calculate this P weighted modal matrix into M P weighted modal matrix and K weighted modal matrix K P K P weighted modal matrix transpose K P weighted modal matrix, because already you have found this lambda 1 lambda 2. So, directly you can write when you are substituting these weighted modal matrixes X equal to P weighted modal matrix Y you can directly write this equation form in this. So, after knowing this equation, so you can write, so you can see that these are also a set of uncoupled equation and you can find this Y Y 1 and Y 2 from this equation.

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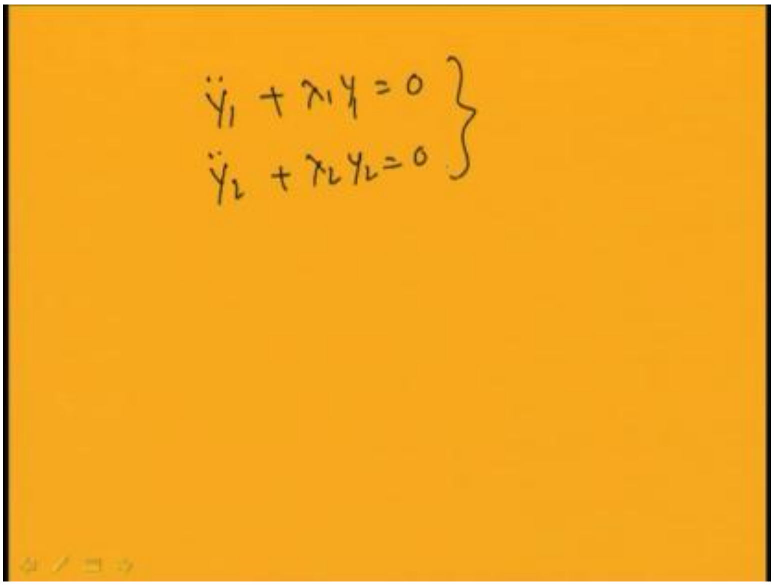

So, from this equation you can write this is, Y 1 double dot Y 1 double dot plus lambda 1 Y that is lambda 1 Y equal to 0 first equation, Y 1 equal to 0 and second equation becomes Y 2 double dot plus lambda 2 Y 2 equal to zero. So, you can verify that in the previous case also you have obtained the same equation that is Y 1 double dot.

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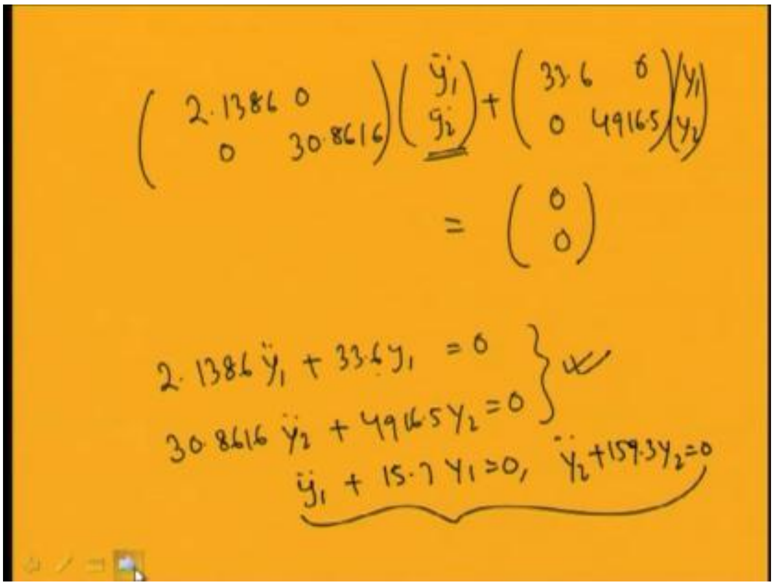

So, if you divide this 33.6 by 2.1386 this is becomes your lambda 1 and here Y 2 double dot plus. So, this 4916.5 Y 2 this becomes lambda two. So, this first equation becomes y 1 double dot plus 15.7 Y 1 equal to 0 and the second equation becomes Y 2 double dot plus 159.3 Y 2 equal to 0. So, using both the methods you are getting the same equation or same expressions for this Y 1 Y 2.

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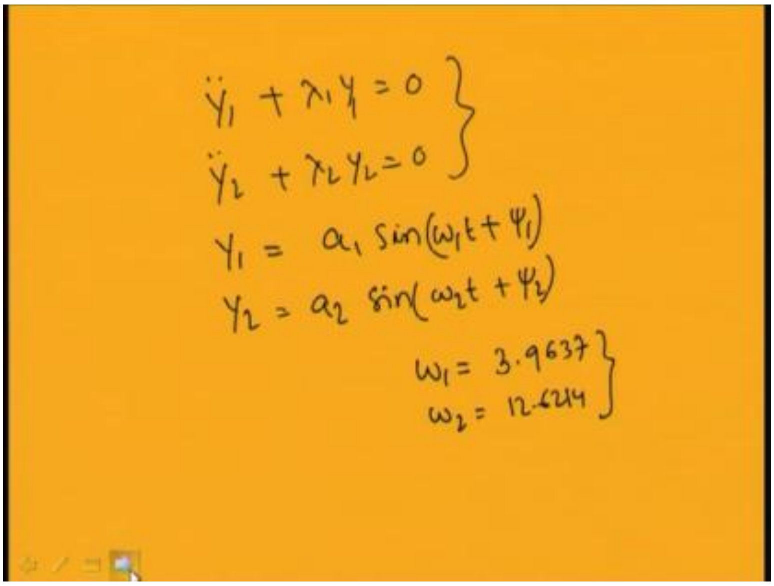

Now, you can write the solution of this, so the solution of the first equation that is Y 1 double dot plus lambda 1 Y 1 equal to 0 you can write. So, Y 1 equal to a 1 sin omega 1.

So, this is lambda 1 this is omega 1 square. So, this becomes omega 1 t plus psi 1 and Y 2 becomes a 2 sin omega 2 t plus psi two. So, in this case you can obtain this a 1 sin omega 1. So, this omega 1 already you know, so this becomes omega 1 is 3.9637 and this omega 2 equal to 12.6214.

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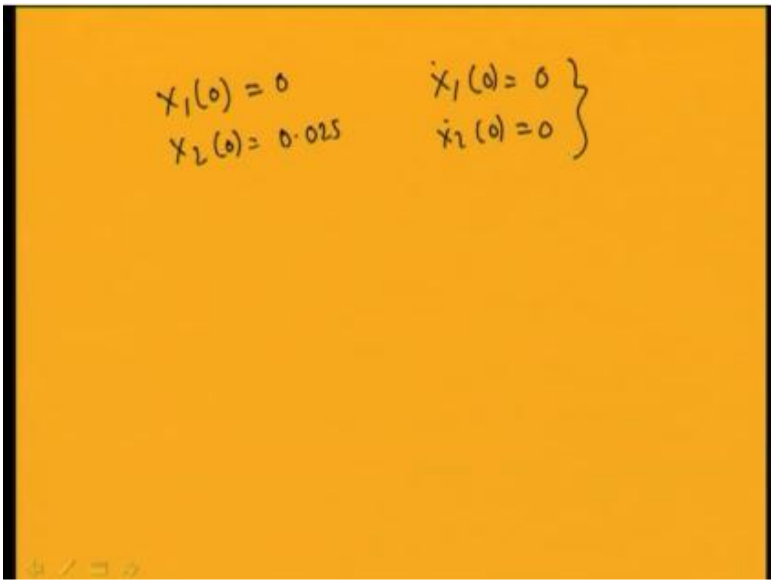

Let us take some initial condition. Let the initial condition is that X 1 0 at t equal to 0; X 1 equal to 0; X 2 equal to 0.0 25 and X 1 dot 0 equal to 0 and X 2 dot 0 equal to 0. So, we require four initial conditions, two initial conditions in displacement and two initial conditions in terms of velocity.

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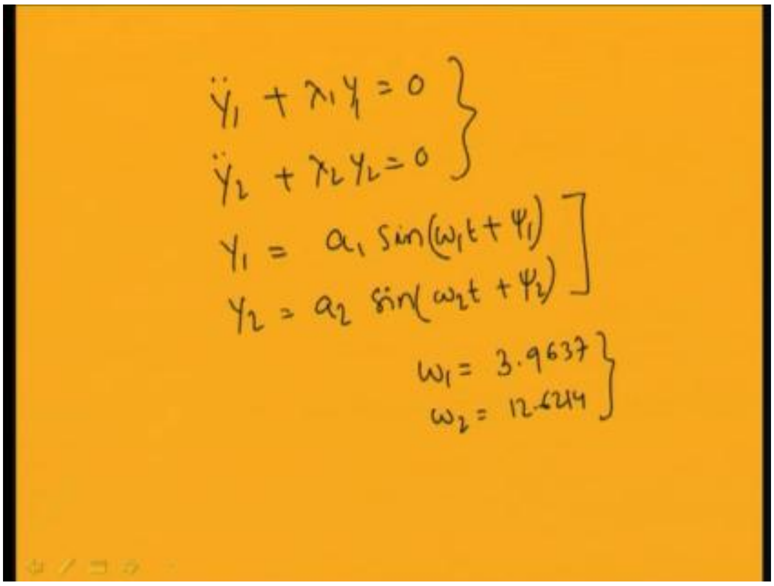

So, by up by taking these 4 initial conditions, so in the previous equation, we can substitute in this equation I can obtain Y 1 0 equal to 0.

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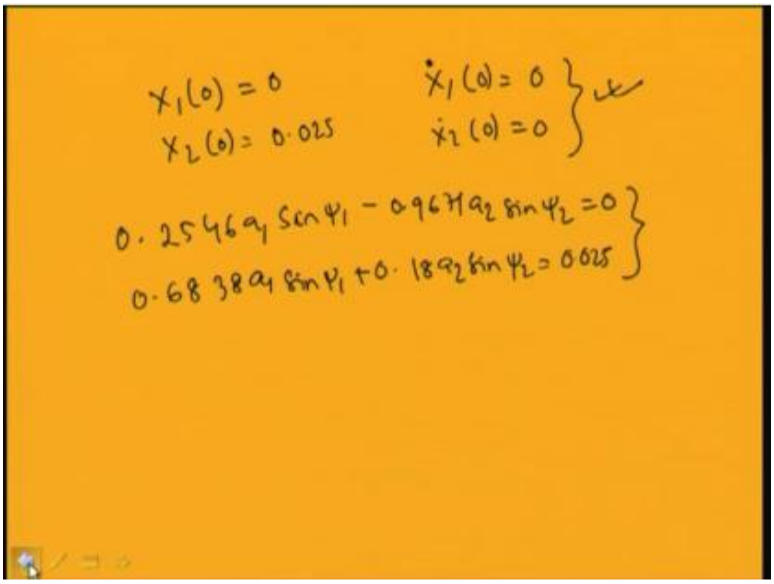

So, this as I have substituted this X 1 equal to P into Y, so if X 1 equal to 0, so Y 1 becomes 0 and when X 1 dot equal to 0, Y 1 dot also equal to 0. So, by substituting these things I can write the equations or I can find this coefficients a 1 a 2 and gamma 1 a 1 a 2 and psi 1 psi 2. So, by we substituting in the first expression you can obtain the equations in this form 0.2546 a 1 sin psi 1 minus 0.9671 a 2 sin psi 2 equal to 0. Second expression you can obtain it in this form. So, 0.6838 a 1 sin psi 1 plus 0.18 a 2 sin psi 2 equal to 0.025. So, before substituting these X 1 0 equal to 0, X 2 0 equal to 0 and this initial conditions once we would find the expression in terms of, so this Y 1 Y 2 in term of X 1 X 2. So, one can substitute X equal to P Y.

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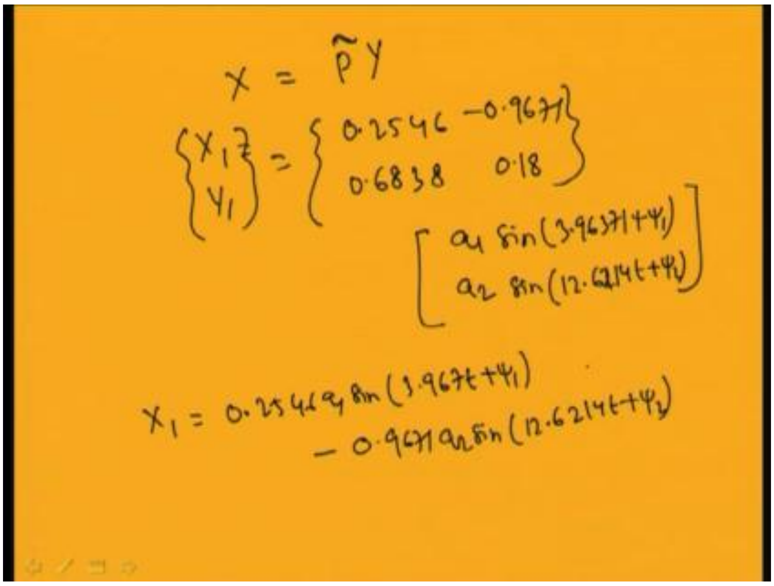

So, one can write X 1 equal to or X 1 Y 1 equal to P weighted transpose into Y either weighted transpose, weighted modal matrix into Y. So, that will give you 0.2546 minus 0.9671 this is 0.683 and this is 0.18 into Y 1 Y 2 and already Y 1 you got equal to a 1 sin 3.96371 plus psi 1 and this becomes a 2 sin 12.6214 t plus psi 2. So, from this you can obtain X 1 equal to 0.2546 a 1 sin 3.967 t plus psi 1 minus, so this becomes these into these minus these into these, so this becomes minus 0.967 1 into a 2 minus point Minus 0.9671 into a 2 sin a 2 sin 12.6 214 t plus psi 2. So, this is the expression for X 1.

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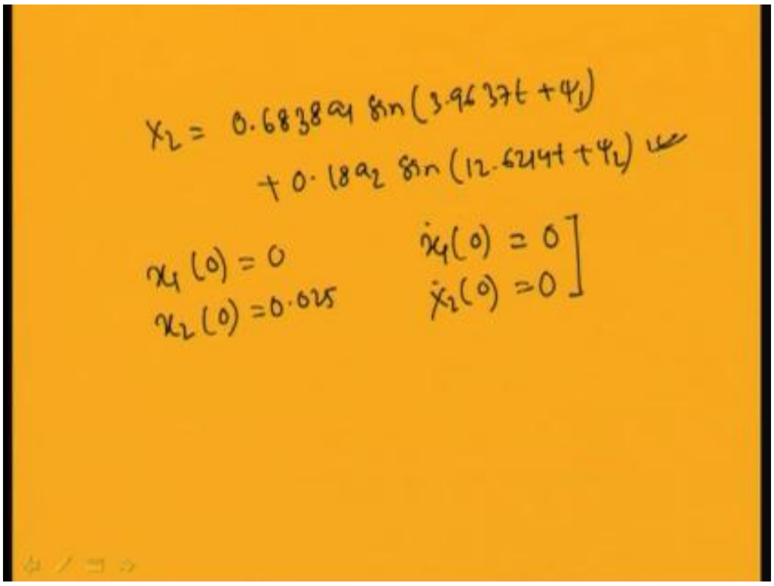

And, similarly X 2 equal to, X 2 can be obtain from this equation X one. So, this is X 1 X 2. So, X 2 will becomes X 2 equal to 0.6838 into a 1 sin 3.9637 plus psi 1 plus 0.18 a 2 into this, so this thing can be written. So, this is 0.6838 a 1 sin 3.9637 t plus psi 1 plus 0.18 a 2 sin 12.6214 t plus psi 2. As the initial conditions are given, x 1 at t equal to 0 equal to 0 and x 2 t equal to 0 equal to 0.025 and x 1 dot equal to 0 x 1 dot t equal to 0 equal to 0 and X 2 dot 0 equal to 0.

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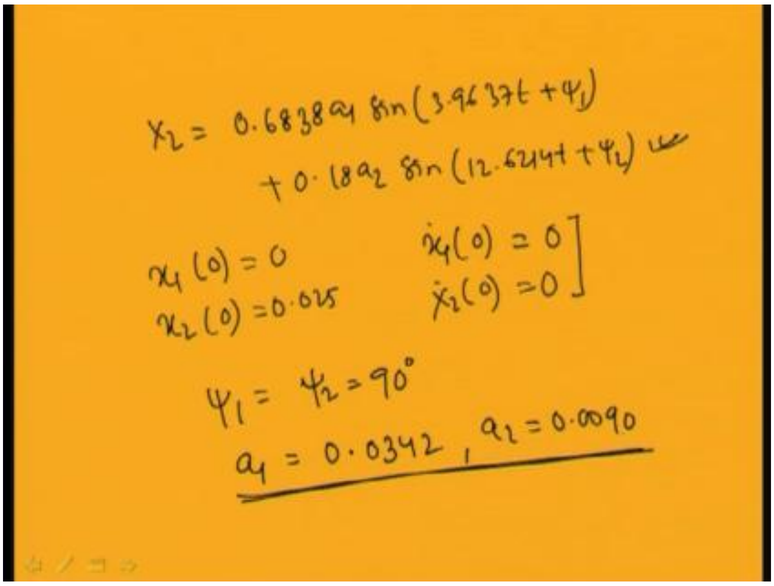

So, by substituting in these X 2 and previous X 1 expression one can obtain these constants a 1 a 2 psi 1 and psi 2. So, one obtain psi 1 equal to psi 2 equal to 90 degree and a 1 equal to 0.0342 and a 2 equal to 0.0090. So, one can write the expression for X 1 X 2 in terms of a 1 a 2 psi 1 psi 2. So, a 1 is given by 0.0342 and a 2 is given by 0.0090. So, in this way one can obtain the response of the system by using these modal analysis methods and one can obtain the principle coordinates which uncoupled the equation motion. So, let us see the case of a force vibration, let us take the same system when it is subjected to a force vibration let us find the response of the system.

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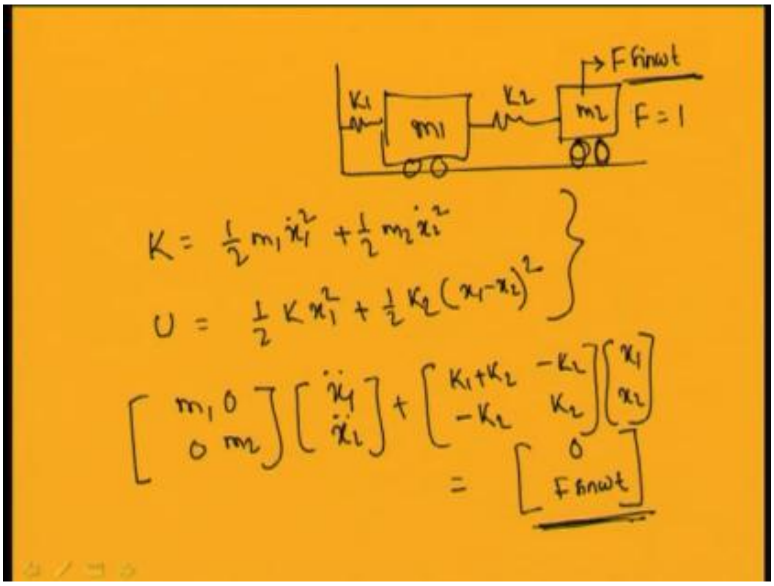

So, in the same system mass this is mass m 1 m 2 and this is spring k 1 and this is spring k 2. Let the mass m 2 is subjected to a force F sin omega t where, F equal to let me take F equal to 1 and I can take the same mass and stiffness property of the system. So, if I will take the same mass and stiffness property of the system I can write this equation motion by using the Lagrange principle. So, by using Lagrange principle I can write the kinetic energy of the system equal to half m 1 x 1 dot square plus half m 2 x 2 dot square and potential energy U equal to half k x 1 square plus half k 2 x 1 minus x 2 square.

Similarly, the work done I can write equal to the displacement into this force. So, by using the Lagrange principle we can find the equation motion which can be written in this form. So, this will be m 0 m 1 0 0 m 2 plus into x 1 double dot x 2 double dot plus k 1 plus k 2 minus k 2 minus k 2 this is k 2 into x 1 x 2. So, this will be equal to the force.

So, this becomes 0 and this becomes F sin omega t. So, by writing this equation motion you can see in the previous case the forcing term are 0, now the forcing term becomes 0 F sin omega t. So, previously I told you have to determine the response of the system. Now using this modal matrix method also we can determine that already we have found the modal matrix for the system. Modal matrix can be obtained from the free vibration response of the system which you have obtained before.

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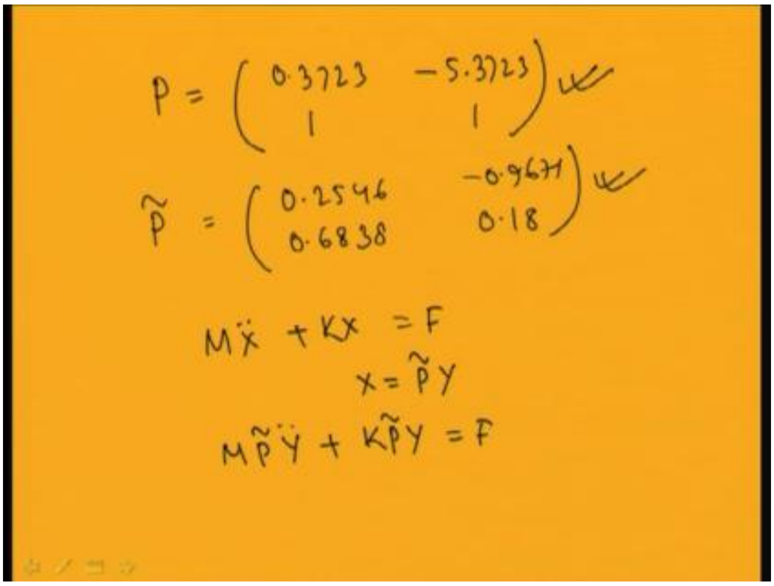

So, this is the modal matrix. So, by using the modal matrix or weighted modal matrix weighted modal matrix also you can take. And this modal matrix P you have obtained before that is equal to 0.3723 this is 1 and this becomes minus 5.3723 and this 1. So, this becomes, so already you have found the weighted modal matrix this is equal to 0.2546 and 0.6838 minus 0.9671 and this becomes 0.18. So, either you can use this modal matrix or this weighted modal matrix to reduce this equation.

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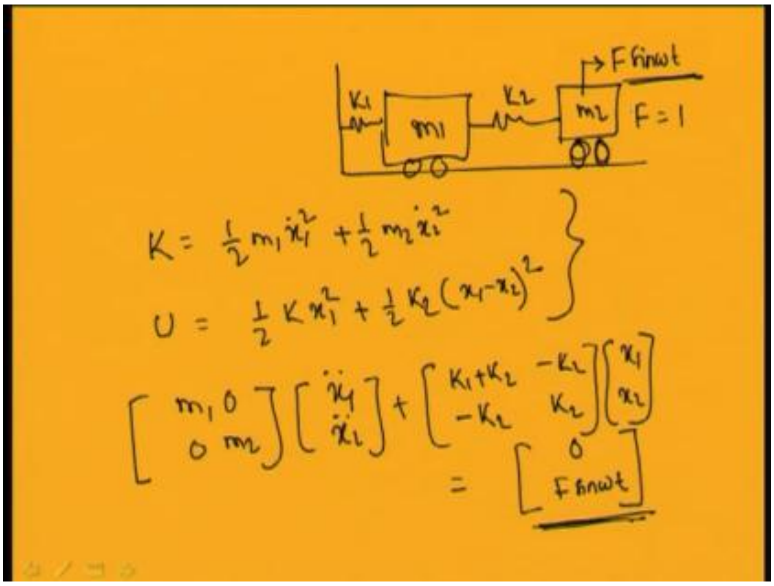

Reduce this previous equation to a set of uncoupled equation. Now using this weighted modal matrix I can write this equation M X double dot plus K X equal to F in this case. So, I can write substituting these X equal to P weighted modal matrix Y I can write this equation M P weighted modal matrix Y double dot plus K P weighted modal matrix Y equal to F.

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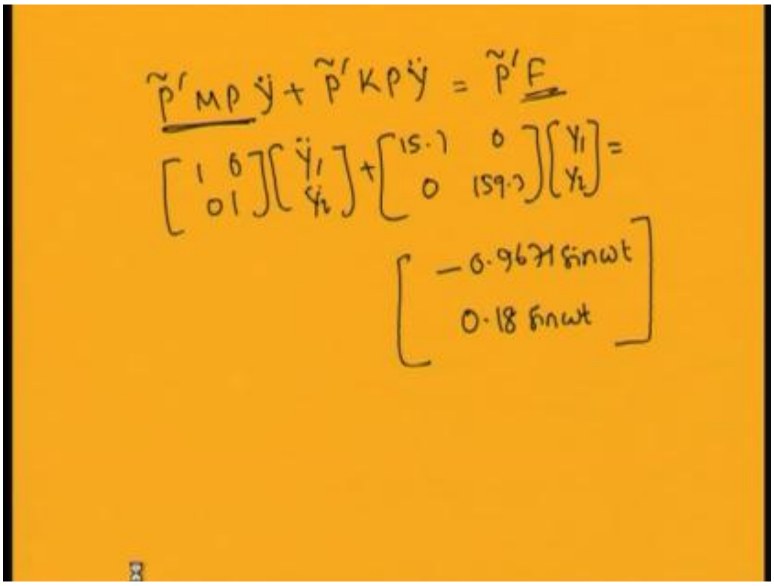

Now, pre-multiplying P weighted modal matrix transpose M P X double dot plus weighted modal matrix K P. So, this is y, so this y double dot x I am substituting by y.

So, this is y double dot. So, plus this is y double dot. So, this becomes P weighted modal matrix into F. So, already from the property of this weighted modal matrix you know this becomes unity matrix or identity matrix. So, this becomes 1 0 0 1 into Y 1 double dot Y 2 double dot plus this becomes lambda 1 lambda 115.7 and this is 0 0 and this is 159.3 Y 1 Y 2 Y 1 Y 2 this becomes P weighted transpose into F. So, if I will multiply this thing, so these becomes minus 0.9671 sin omega t I have taken F equal to 1. So, this becomes minus 0.9671, so F sin omega t. So, this is F matrix F vector this F the F vector. So, minus 0.9671 sin omega t and this becomes 0.18 sin omega t. So, you can obtain 2 uncoupled equations.

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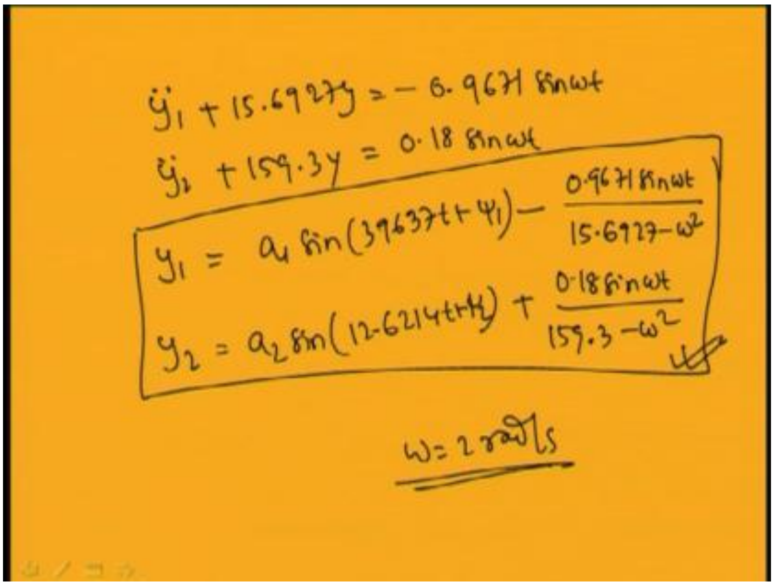

So, these uncoupled equations can be written in this form that is equal to y 1 double dot plus 15.6927 y equal to minus 0.9671 sin omega t. And the second equation becomes y 2 double dot plus 159.3 Y equal to 0.18 sin omega t. So, you can solve these 2 equations. So, for solving the first equation it has both complimentary part and the particular integral. So, you can write this y 1 equal to complimentary part becomes a 1 sin 3.9637 t plus psi 1 plus for the particular integral these becomes 0.9671 sin omega t by root over, so this becomes d square minus 15.69. So, d square for d square you can substitute this minus omega square. So, this becomes minus 0.9671 sin omega t by 15.6927 by omega square.

Similarly, y 2 becomes a 2 sin 12.6214 t plus psi 2 plus 0.18 sin omega t by 159.3 minus omega square. So, these are the expression for y 1 and y 2. Now for a particular value of omega, let omega equal to 2 radian per second. So, if omega equal to 2 radian per second then this particular integral part becomes 0.9671 by 15.6927 minus 4 that is 11.6927 and here in the second mode it becomes 154.3. So, 0.18 sin omega t sin 2 t by 154.3. So, in this way you can find the response of the force vibration system.

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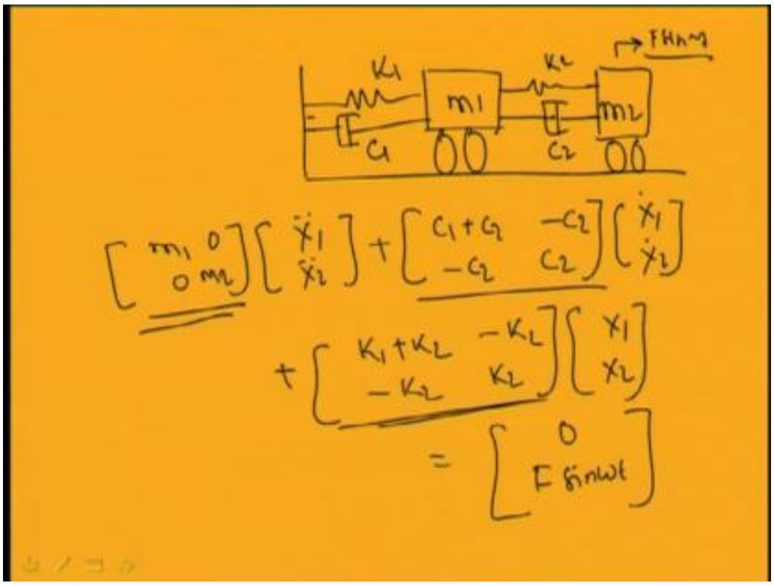

So, if you have a system with damping, let us take a system with damping same system with damping. So, in this case, so this is the spring and let me use the damper, this is the second mass and again I can put this force F 2 also. So, let the force is acting on mass m 2. So, in this case also you can write the equation motion in this form. So, this becomes m 1 0 0 m 2 X 1 double dot X 2 double dot plus C 1. So, let this is C 1 this is C 2 this is k 1 this is K 2. So, this becomes C 1 plus C 2 minus C 2 minus C 2 C 2 X 1 dot X 2 dot plus k 1 plus k 2 minus k 2 this is minus k 2 and this is k 2 into X 1 X 2 these becomes 0 no force is acting on mass 1. So, this force term becomes 0 and as a force F sin omega t is acting on mass two. So, this becomes F sin omega t.

So, in this case this stiffness matrix is k 1 plus k 2 minus k 2 this becomes minus k 2 minus k 2 stiffness matrix and this is the mass matrix and this is the damping matrix. So, this damping matrix if it is not symmetric in this case it is symmetric if it is not symmetric then you cannot find a coupled equation uncoupled equation you cannot find the uncoupled equation, but in this case you can find a find an uncoupled equation by using the modal matrix, so by using the modal matrix or weighted modal matrix.

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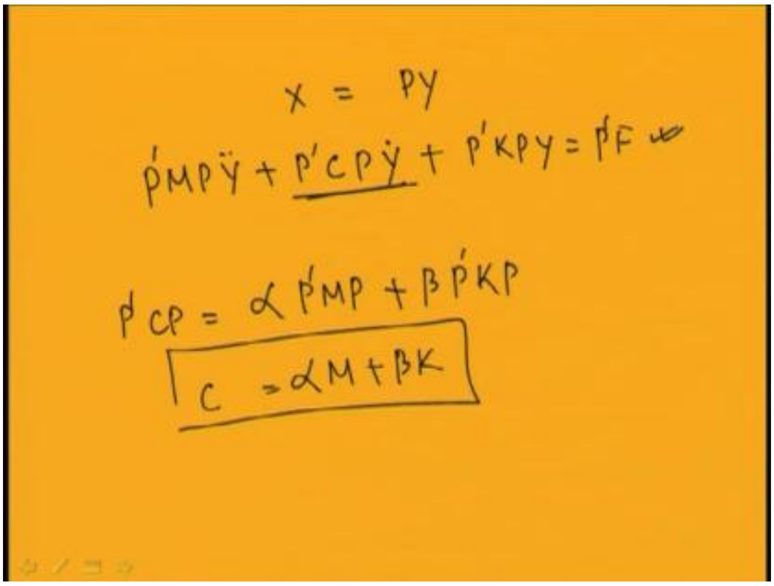

So, let you use the modal matrix X equal to P Y. So, if you substitute that in that expression then this equation will reduce to P transpose M P Y double dot plus P transpose C P Y dot plus P transpose K P Y equal to P transpose F. So, if this is symmetric matrix then it will reduce to that of a diagonal matrix. But if it is not symmetric matrix then you can you can write these matrix P transpose C P matrix as alpha of P transpose M P plus beta part of P transpose K P or the C matrix you can write alpha M plus beta K. So, by writing C equal to alpha M plus beta K you can reduce this equation to that of a diagonal form that of its diagonal form and you can find the response of the system.

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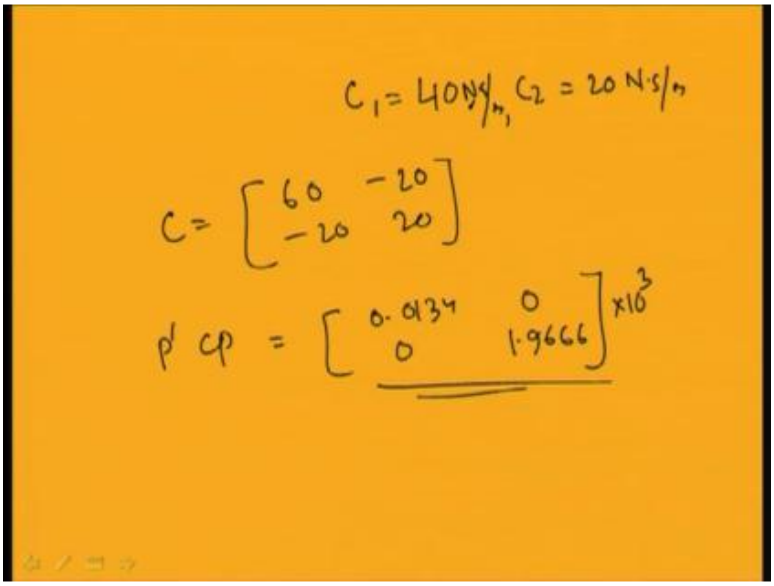

So, in this case let me take C 1 equal to C 1 equal to 40 and C 2 equal to 20. So, 20 Newton second per meter 40 Newton second per meter. So, then the C 1 C matrix will reduce to 60 minus 20 minus 20 20. So, in this case you can find this P dash C P will be equal to 0.0134 this becomes 0 and this becomes 1.966. So, into 10 to the power 3, so in this case you can see that this P dash C P is a diagonal matrix. So, you need not have to convert it to that proportional form and you can write this equation in this form. So, it will reduce to Y double dot plus 2 zeta omega n Y dot plus omega n square equal to F sin omega t.

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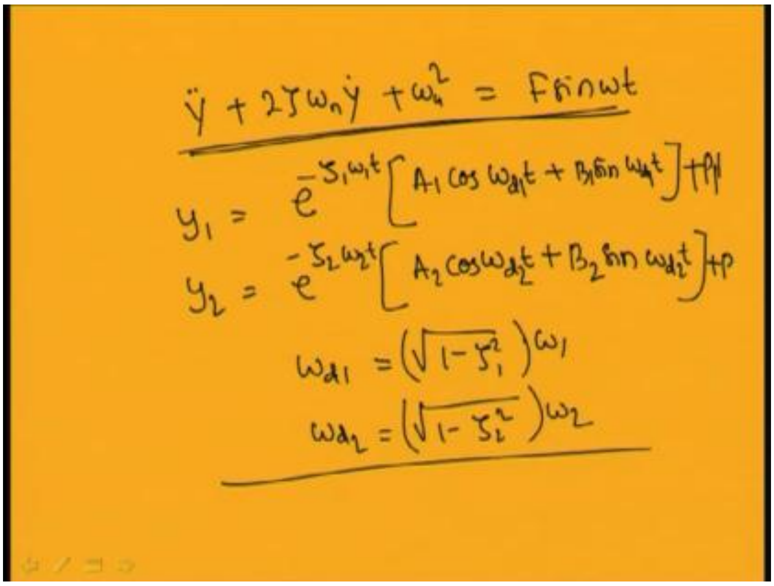

So, it will reduce to this form. So, as it is reducing to this form you can write this y 1 equal to e to the power minus zeta 1 omega 1 t to A 1 cos omega d t plus B 1 sin omega d t, this way you can write and similarly, y 2 you can write equal to 1 minus zeta 2 omega t into A 2 cos omega d t plus omega d t 1, so this is omega dt 2. So, this is omega d t 1 plus this becomes B 1 this becomes B 2 sin omega d t 2 t. So, where omega d 1 becomes 1 minus zeta 1 square and omega d t 2 becomes root over this is into omega 1 and this is 1 minus zeta 2 square into omega 2. So, in this way you can find the response of the system. So, this is free vibration response. So, this is the complimentary part of the response and the particular integral part. So, these y and y y 1 and y 2 are the complimentary part and you can find the particular integral part also P P 1 and P 2.

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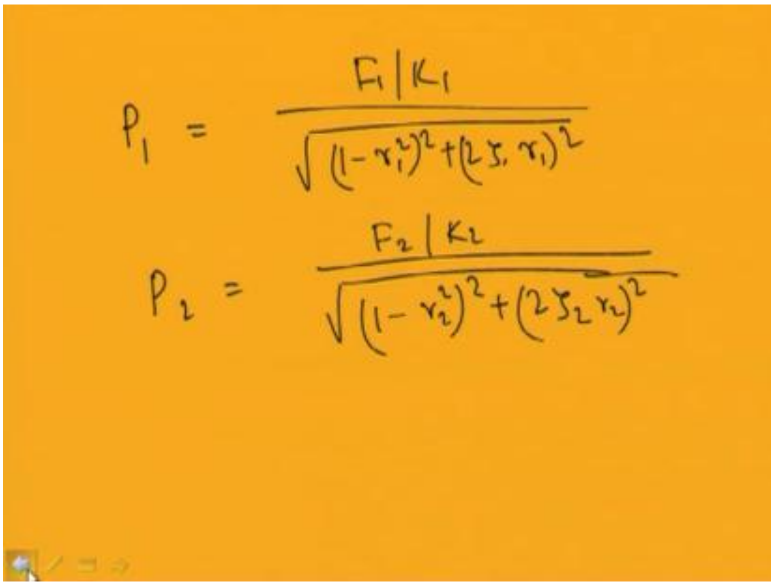

So, where P 1 already you know your P 1 will be equal to F 0 by k root over 1 minus r 1 square whole square plus 2 zeta 1 omega 1 2 zeta omega 2 zeta r whole square. And P 2 will becomes F by, so this be this becomes F 1 by K F 1 by k one. So, this becomes F 2 by k 2 root over 1 minus r 2 square whole square plus 2 zeta 2 r 2 whole square.

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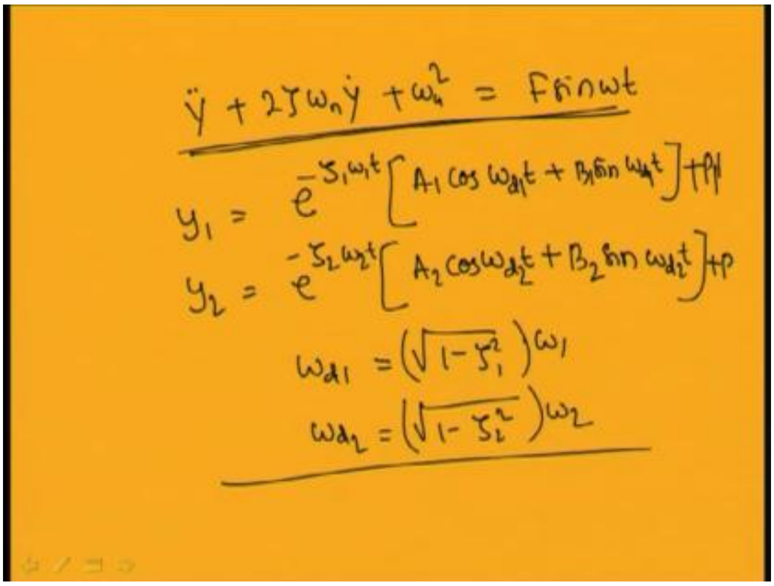

Where this k 1 k 1 and k 2 k 1 will becomes, so k 1 by M 1 equal to omega 1 square omega 1 square and k 2 by M 2 equal to omega 2 square. Where omega 1 square already you have found that is equal to lambda 1 that is equal to 15.7 and omega 2 square that is equal to 159 159.3.

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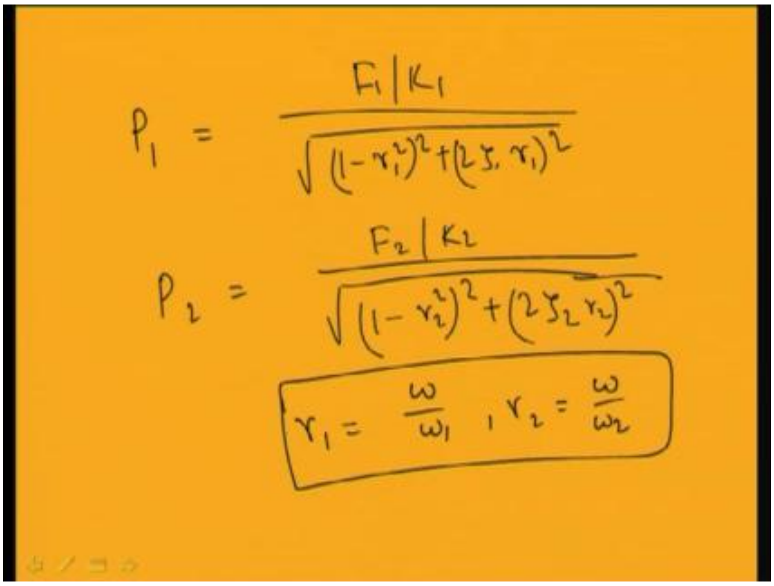

So, from this you can find this r 1 equal to omega by omega 1 and r 2 equal to omega by omega 2. So, in this way you can find the response of a forced vibration response.

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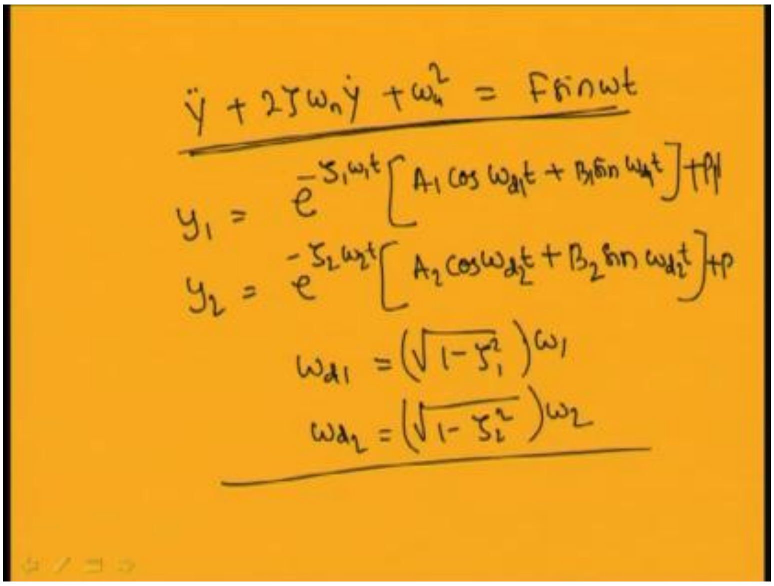

With a system with damping and without damping also you can find, for the system response when the damping is proportional and when it is not proportional also. So, when it is not proportional then the C matrix you can write equal to alpha M plus beta K and you can find the response of the system.

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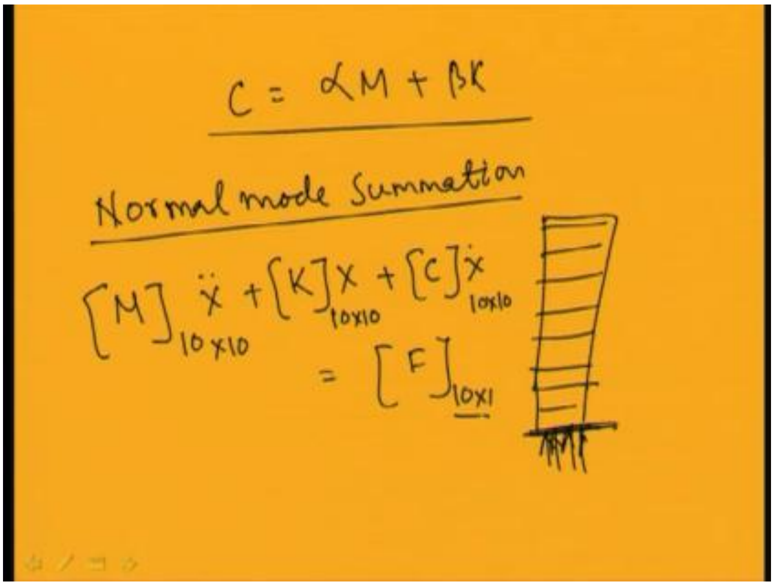

So, let us see one example for this normal mode summation method normal mode summation. So, already in the last class it is told that you can use this normal mode summation method when you do not require finding the response for all the modes of the system. So, an example as given that time. So, let us take a multi storage building. So, this is a multi storage building, let us take a 10 stair building when it is subjected to an earthquake or it is subjected to an excitation at base excitation a from the support there is some excitation. And these excitations are let us assume that it is limited to the first 3 mode frequencies only.

So, in this case one can write the equation motion using this mass matrix. So, this mass matrix if it is a 10 stair buildings then it is 10 is to 10. So, M X double dot plus K X equal to, so K X plus one can write the C matrix also C X dot equal to F matrix. So, in this case this M equal to 10 is to 10, K also equals to 10 is to 10 and see also is a 10 is to 10 matrixes and this K F that this forcing is a 10 is to 1 vector. So, one can find this modal matrix P. So, modal matrix P can be found from this Eigen value of Eigen vector of M inverse K.

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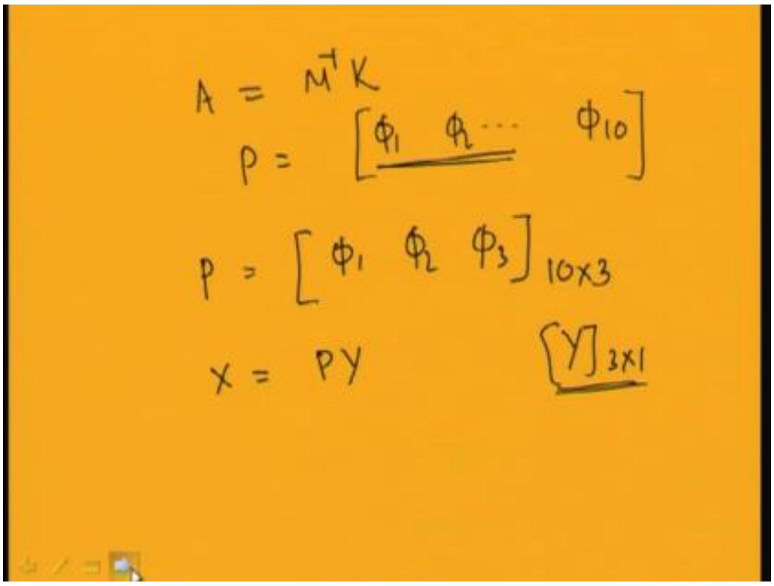

So, if A equal to M inverse K. So, one can obtain 10 Eigen values and corresponding Eigen vectors. So, those 10 vectors can be written as phi 1 phi 2 phi 1 phi 2 and phi 10.

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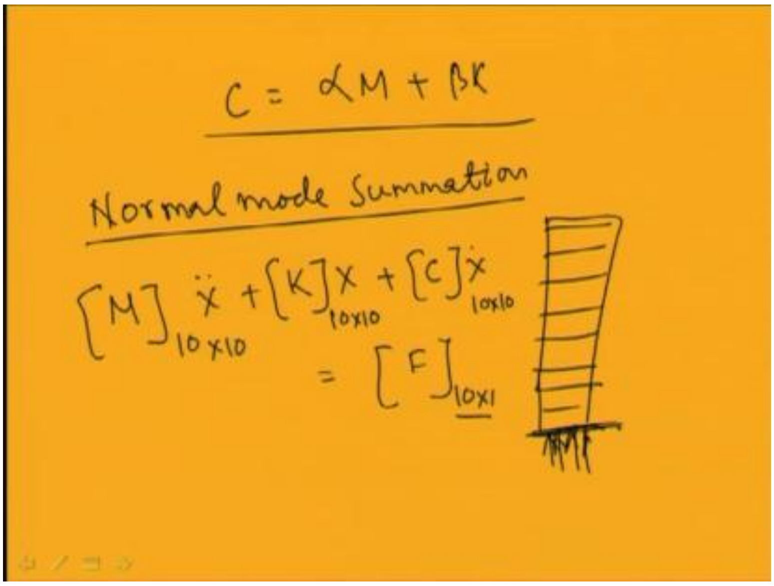

So, as it is known that the frequencies of these External frequencies of the systems are limited to first 3 modes. So, instead of taking these ten modes 1 can take the fast 3 modes and write this P matrix. So, this P matrix one can write equal to phi 1 phi 2 phi 3. So, these phi 1 phi 2 phi 3 are the first 3 mode frequency first 3 modal frequencies. Now by using X equal to P Y, so where Y will be.m. So, P equal to phi 1 phi 2 phi 3 P is a 10 is to 3 matrix, so 10 rows and 3 columns. So, one can take this Y equal to. So, Y will be a 3 is to 1 vector. So, only we can take the first 3 frequencies or first 3 modes and I can substitute these X equal to PY.

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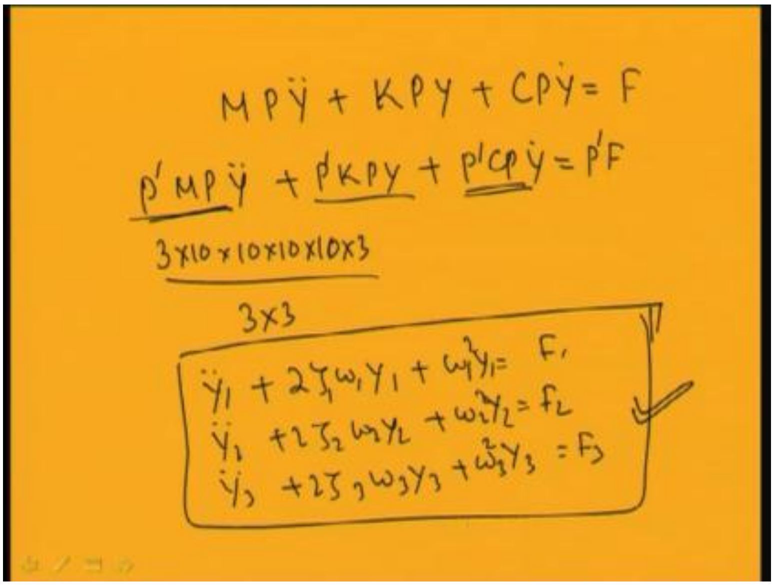

So, by substituting X equal to P Y in this expression M. So, M P Y double dot plus K P Y plus C P Y dot equal to F. Now I can write by pre-multiplying P transpose M P Y double dot plus P transpose K P Y plus P transpose C P Y dot equal to P transpose F. So, now you can check that this P transpose M P is a diagonal vector of 3 is to 3. So, P is 10 is to 3. So, P transpose will be 3 is to 10. So, this becomes 3 is to 10 and M is a 10 is to 10 and this P is 10 is to 10 is to 3. So, this becomes 3 is to 3 matrix. Similarly, P transpose K P is a 3 is to 3 and this also 3 is to 3. So, now you can obtain a set of equations set of equations which are diagonal.

So, these diagonal equations are it can be written Y 1 double dot plus 2 zeta omega 1 Y 1 plus omega 1 square equal to F 1. Similarly, Y 2 double dot plus 2 zeta 2 omega 2 Y 2 plus omega 2 square equal to F 2. And Y 3 double dot plus 2 zeta 3 omega 3 Y 3 plus omega 3 square, this is Y 1; this is; Y 2 and this is Y 3 equal to F 3. So, the solution, so you can solve these 3 equations easily to find the response of the system. So, instead of carrying out a solution of 10 is to 10 matrix, you can reduce that thing to a 3 is to 3 matrixes and you can solve this equations to find the response of the system.

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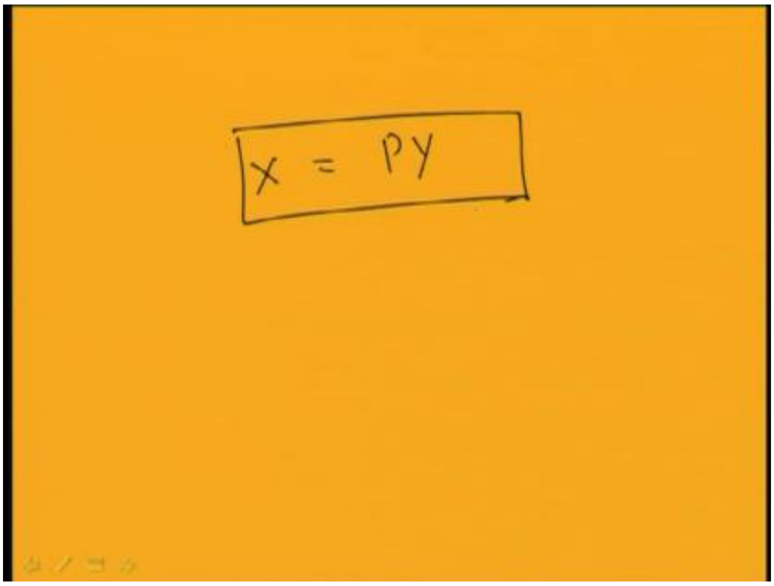

So, after finding this Y 1 you can obtain this X equal to P Y and you can obtain the response of the system. So, in this way you can solve the problems of a multi degree of freedom system by using these normal modes summation method. So, in this multi degree of freedom systems, we know have to find the equation motion of systems also have to find the response of the system by using modal matrix method. So, you have solved the model.

So, we have found the modal matrix or we have used this modal matrix method for free and force vibration of the system. And we have used this normal mode summation method to find the response of a system when a multi degree of freedom or when the system degrees of freedom is very high. So, by using these normal modes summation method, you can reduce a multi degree of freedom system or a very higher degree of freedom system to a very low order system and you can solve the response of the system very easily. So, in the next class we will see different approximate method to find the natural frequency of a multi degree of freedom system.