Mechanical Vibrations

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Module – 7
Multi DOF
Lecture - 3
Modal Analysis: Undamped

Last class we have studied about finding the normal modes of multi degree of freedom systems from the stiffness matrix, and from the flexibility matrix, and we have found the normal modes, and we have written the modal matrix of the system.

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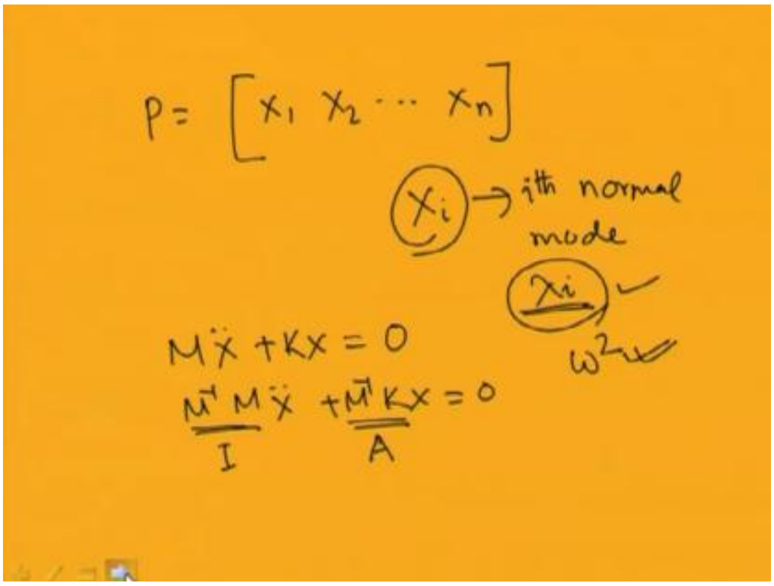

The modal matrix P can be written equal to the normal modes X 1 X 2 and X n. So, we are X 1 X 2 and X n are the normal modes, or X i is the i th normal mode of the system. So, this is the i th normal mode corresponding to lambda i that is the Eigen vector or the Eigen value lambda i. So, this is the Eigen vector of i th mode and this is the Eigen value corresponding to the square of the normal mode frequency. So, this Eigen value or this normal mode frequency can be obtained either by this flexibility matrix method, or from this stiffness matrix method formulation. So, when one do this stiffness matrix formulation method the equation motion can be written in this form; M X double dot plus K X equal to 0 for free vibration. And, you can write this M inverse M X double dot plus M inverse K X equal to 0. So, this M inverse K equal to A, and this is I.

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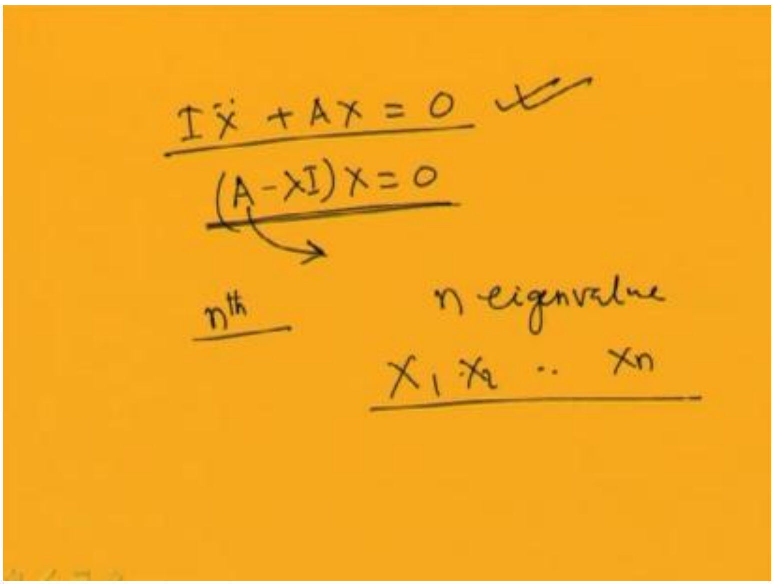

So, the equation reduces to I X double dot A X equals to 0. So, assuming the normal mode X equal to X sin omega t, so this equation reduces to this form A minus lambda I X equal to 0. So, this A minus lambda I X equals to 0 gives the Eigen value, and this Eigen vector corresponding to the Eigen values are the normal modes of the system. So, for a n th degree of a freedom system. So, you have n Eigen value and corresponding to n Eigen value you have n number of normal mode frequencies, and n number of modal. So, you will have this; X 1 X 2 and X n and using these normal modes you can write this modal matrix P.

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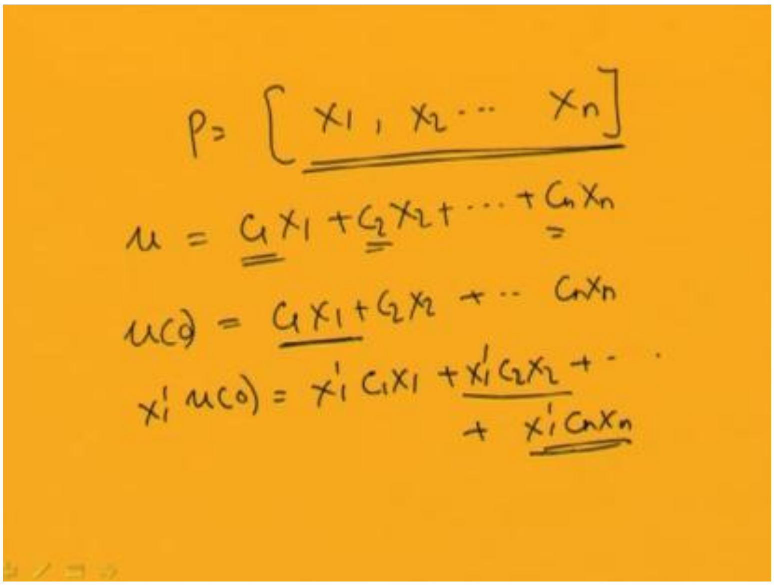

So, P equal to X 1 X 2 X n and using these normal modes 1 can find the free vibration of a system. So, the free vibration of a system u can be written as C 1 X 1 plus C 2 X 2 plus C n X n. Where, this C represents the modal participation of different modes in the resulting free vibration of the system. So, by using this orthogonality principle of normal modes we can find these C. So, let at initial condition u equal to 0. So, this resulting free vibration equal to C 1 X 1 C 2 X 2 and C n X n. Now, by if I want to find what is the value of C 1? I can pre-multiply this by X 1 transpose, so X 1 transpose u (0) equal to X 1 transpose C 1 X 1 plus X 1 transpose C 2 X 2 plus X 1 transpose C n X n. Previously we have studied the orthogonality property of this normal modes So, from this orthogonal property we know that this X 1 dash X 2 and X 1 dash X n when I naught equal to j or equal to 0. You can apply this orthogonality principle to this equation.

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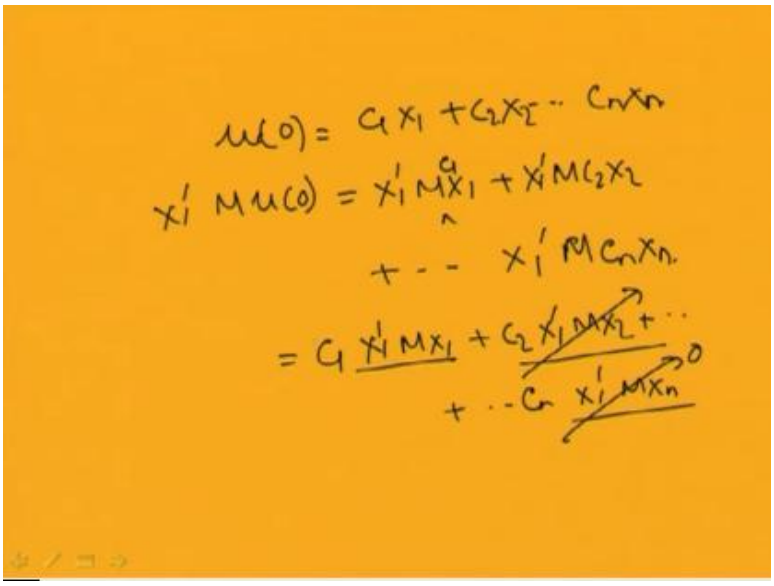

So, here I can write this u (0) equal to C 1 X 1 plus C 2 X 2 C n X n. Now by pre-multiplying this by X 1 dash M u I can write this equal to X 1 dash M X 1 plus X 1 dash C 2. So, there is C 1 X 1 dash C 1. So, X 1 dash M C 2 X 2 and X 1 dash M C n X n and from previously we know. So, this is equal to C 1 X 1 dash M X 1 plus C 2 X 1 dash M X 2 and C n X 1 dash M X n. So, from the orthogonality property; we know that this X 1 dash M X 1 dash is the generalized mass matrix M 1 and other terms when I not equal to j that is X 1 dash M X 2 X 1 dash M X n. So, all these are the terms are 0. So, these terms are 0.

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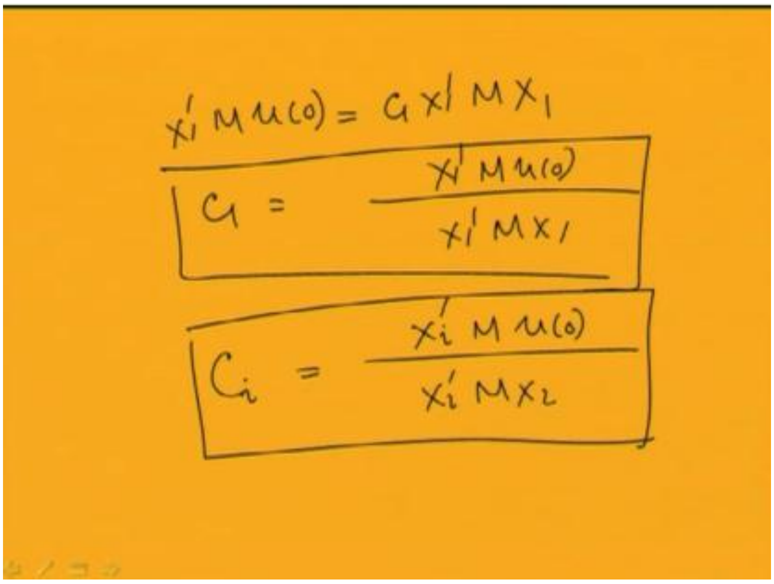

So, this reduces to X 1 dash M u (0) equal to C 1 X 1 dash M X 1. So, from this I can write the C 1 equal to X 1 dash M u (0) by X 1 dash M X 1. Similarly, I can find C 2 and C i for i th mode the i th modal participation can be written as; C i will be equal to X i dash M u (0) by X i dash M X i. Either I can use M matrix here or I can use. So, I can pre-multiply this X 1 dash M or I may pre-multiply X 1 dash k, and applying this orthogonality principle C i, I can write equal to X i dash M u 0 by X i dash M X 2.

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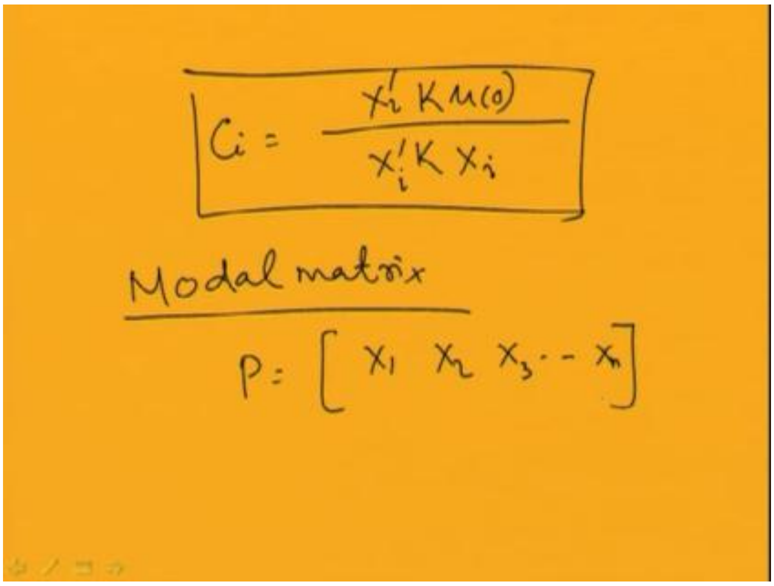

Or, I may write also this C i equal to X i dash K u (0) by Xi dash K X i. So, from the known stiffness matrix and mass matrix I can find the modal participation of different modes in the resulting free vibration of a system. So, now by writing, so we know this modal matrix of the system can be written in this form. So, P equal to X 1 X 2 X 3 or X 3 X n.

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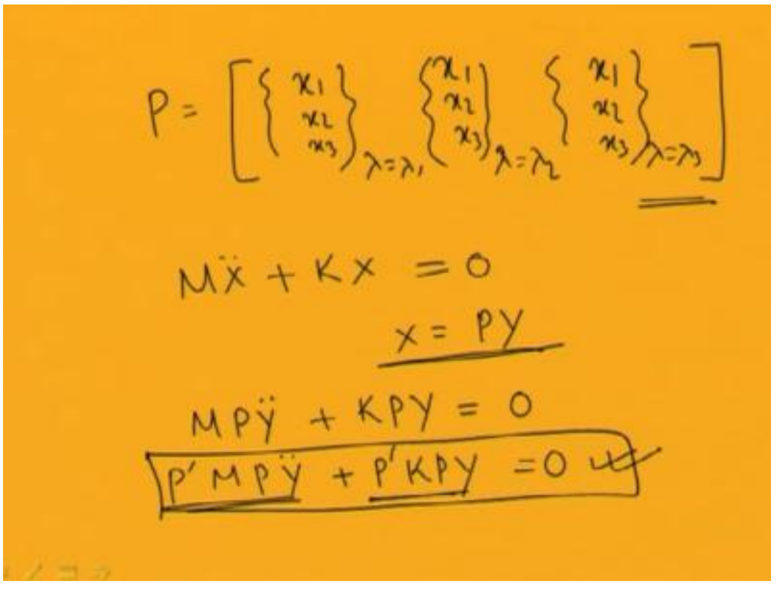

So, let us take the case of a 3 degree of freedom system. So, in that case the P will be written as. So, this will be equal to x 1 x 2 x 3 at lambda equal to lambda 1 then x 1 x 2 x 3 lambda equal to lambda 2 and x 1 x 2 x 3 at lambda equal to lambda 3. So, by using this modal matrix 1 can uncouple the equation motion. So, let us have an equation motion in this form; this is M X double dot plus K X equal to 0. Now, we can uncouple this equation by writing X equal to P Y. So, we can write this equation equal to M P Y double dot for X I have written it is equal to Y. So, M P Y double dot plus K P Y equal to 0.

So, I can pre-multiply this P transpose. I can write this P transpose M P Y double dot plus P transposes K P Y. So, this will be equal to 0 now, one can show that this P transpose M P and K P transpose K P are diagonal matrix. So, this equation is reduced to a diagonal matrix form in terms of another set of coordinate that is Y. So, in this way 1 can reduce a multi degree of freedom system to an equivalent single degree of freedom system by using this modal matrix.

So, this modal matrix by using this modal matrix one can reduce a multi degrees of freedom system to an equivalent single degree of freedom system. So, this method is known as the modal analysis method. So, by using modal analysis method one can reduce a multi degree of freedom system to a single degree of freedom system, and one know the solution of a single degree of freedom system. So, one can find the solution of a multi degree of freedom system from this resulting single degree of freedom system. So, this method of analysis by using this modal matrix is known as modal analysis method.

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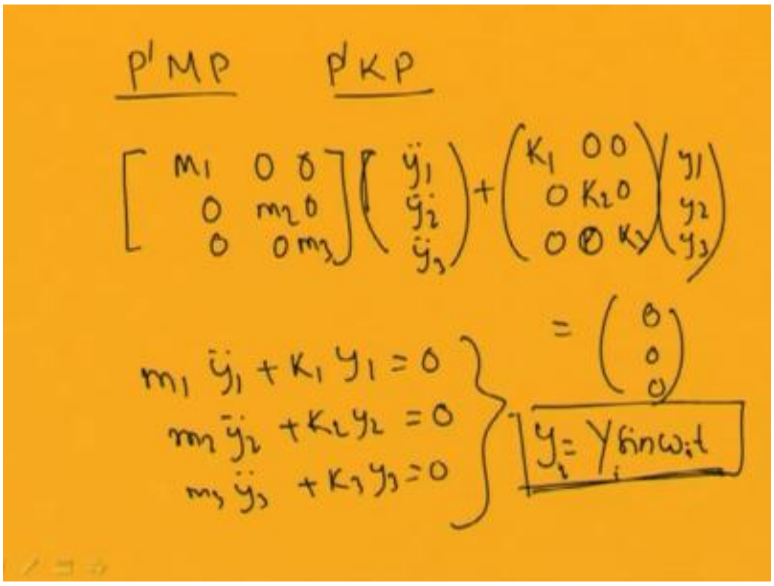

So, 1 can show that this P dash M P and P dash K P are orthogonal matrix or diagonal matrix, and by using this diagonal property. So, this equation is reduced to that of a set of single degree of freedom system. So, this equation will be written in this form. So, it will be M 1 0 0, 0 M 2 0, 0 0 M 3 into y 1 double dot y 2 double dot y 3 double dot plus K 1 0 0, 0 K 2 0, 0 0 K 3, y 1 y 2 y 3 equal to 0 0 0. So, this equation can be written in terms of 3 first order single degree of freedom system equation m 1 y 1 double dot plus k 1 y 1 equal to 0, plus k 2 y 2 equal to 0, and m 3 y 3 double dot plus k 3 y 3 equal to 0.

So, from these 1 can find the frequency of the system will be k 1 by m 1 root over, second mode frequency will be k 2 by m 2 root over, and third mode frequency will be equal to root over k 3 by m 3. And, y 1 y 2 y 3 or y can be written equal to Y sin omega or y i will be written Y i sin omega i t form. So, these are the normal mode, so after finding the single degree of freedom system solution.

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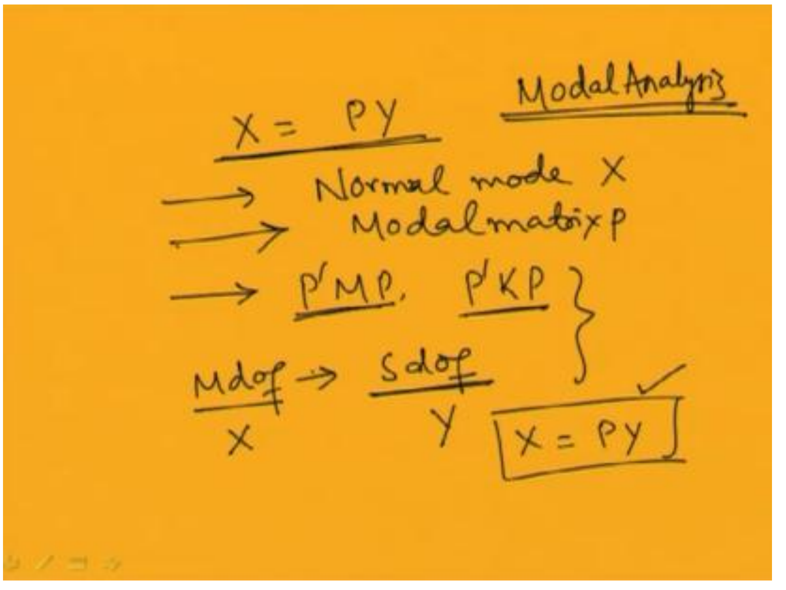

So, one can convert this Y by using that modal matrix to X. So, X will be equal to P Y. So, the modal analysis method involves first; find the normal mode of the system, find the modal matrix of the system then find m inverse or P inverse M P and M K. So, the steps are first find the normal mode of the system. So, after obtaining the normal mode then obtain the modal matrix, so from the modal matrix P. So, this is X this is modal matrix P, then find P dash M P and P dash K P. So, these are the diagonal matrix and using this diagonal matrix you can find the or you can convert this multi degree of freedom system to a set of first order or single degree of freedom system.

So, you can convert this multi degree of freedom system to a single degree of freedom system. So, here your vector is X and, here you can have the vector Y. So, where X equal to P Y. So, by using this transformation you can do the analysis, and this analysis is known as this modal analysis method. Also, you can simplify this thing by using another matrix that is known as weighted modal matrix. So, the weighted modal matrix can be obtained from the modal matrix.

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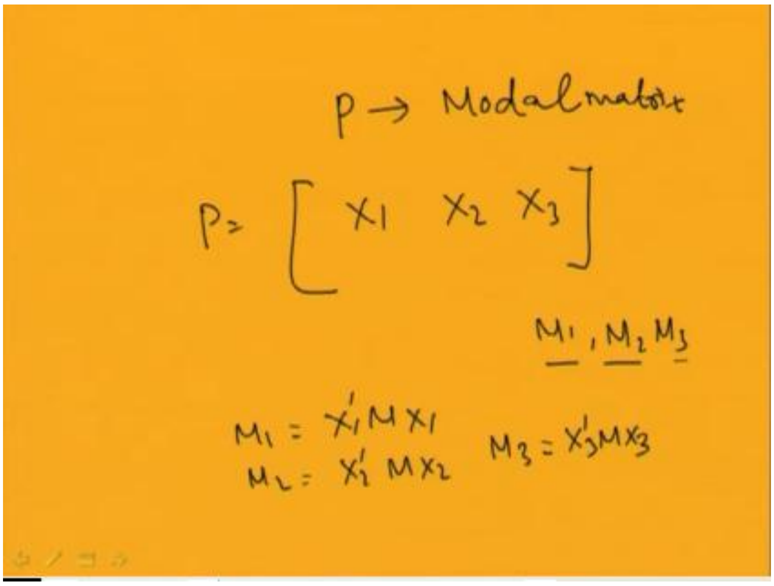

So, let modal matrix is P, so P is the modal matrix the weighted modal matrix can be obtained by dividing the. So, let P matrix let me write it equal to X 1 X 2 X 3 for a 3 degree of freedom system. And let M 1 M 2 M 3 are the generalized mass. So, M 1 can be obtained from these. So, M 1 equal to X 1 dash M X 1, and M 2 equal to X 2 dash M X 2 and similarly, M 3 equal to X 3 dash M X 3.

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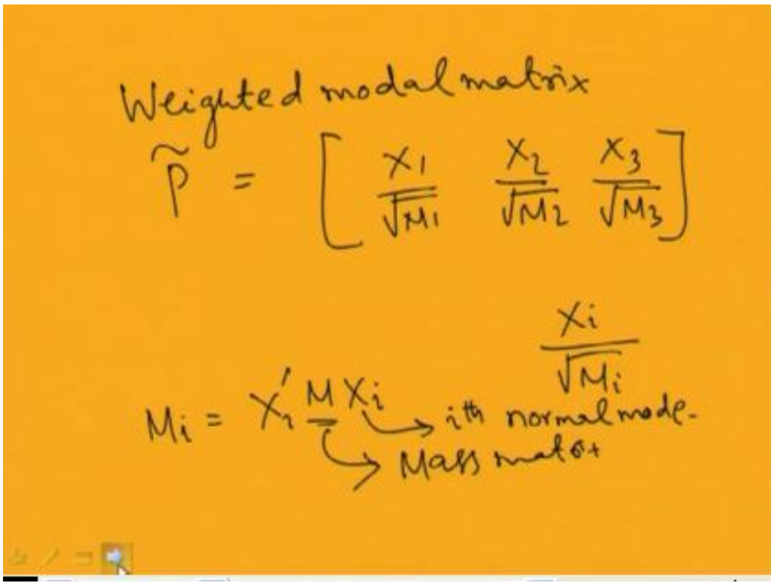

So, the weighted modal matrix weighted modal matrix, so which is written like this. So, it is equal to X 1 X 2 X 3. So, this is X 1 by root over M 1 X 2 by root over M 2 and X 3 by root over M 3. So, by dividing this square root of the generalized mass of the i th column. So, i th generalized mass in i th column. So, one can obtain this weighted modal matrix. So, this weighted modal matrix can be obtained by dividing this first column by root over M 1, second column by root over M 2, and third column by root of M 3 or i th column by root over M i.

So, where M i equal to X i transpose M X i. So, where M is the mass matrix and X i is the i th normal mode. So, in this way one can find the weighted modal matrix. So, after finding this weighted modal matrix using this weighted modal matrix and. So, 1 can find the principle coordinates of a system. Already in case of 2 degrees of freedom system you know the principle coordinate of a system is defined as that coordinate of the system which will yield both mass and stiffness matrix uncoupled. So, which will make the mass matrix and stiffness matrix uncoupled or using this principle coordinates one can get uncoupled mass matrix and stiffness matrix.

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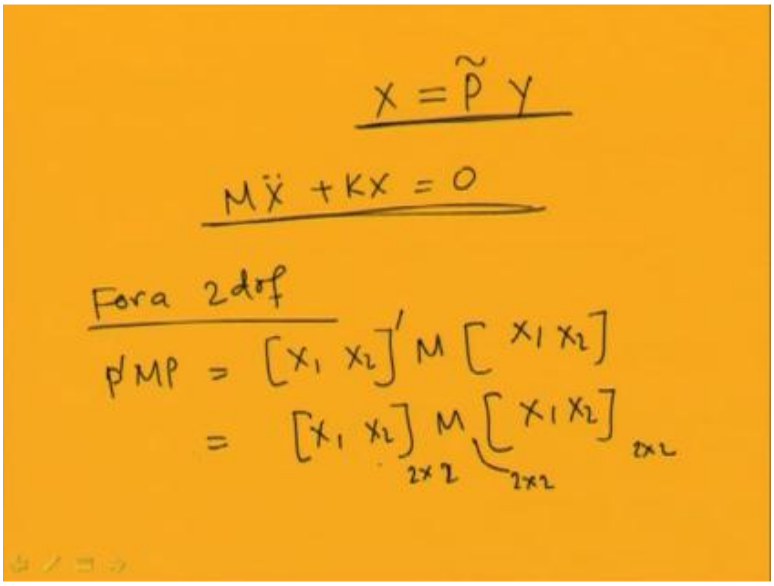

So, if I am assuming this uncoupled mass principle coordinate is Y. So, I can substitute this X given the coordinate system X Y can use this X equal to P weighted modal matrix Y. And, I can also decouple this equation motion M X double dot plus K X equal to 0. So, before decoupling this thing; let us see some of the properties of this weighted modal matrix and modal matrix. Let us see one example; so for a 2 degree of freedom system.

So, let us find for 2 degrees of freedom system. Let us find what is this P dash M P? So, P dash M P will be equal to let X 1 X 2 M P X 1 X 2 dash M and P, P equal to X 1 X 2.

So, this will give rise to. So, this will be equal to. So, if I will multiple these. So, this is X 1 X 2 M X 1 X 2. So, this thing I can write. So, this is 2 is to1. So, this is M equal to 2 is to 2, so this is 2 is to 2, and this is 1 is to 2. So, this becomes 2 row 1 column. So, each X, so this is X 1 X 2 transpose this X 1 is 2 row 1 column. So, this is X 2 is also 2 row 1 column. So, transpose of X 1 X 2. So, I can write this is transpose of X 1 X 2, X 1 X 2 transpose into M. So, this is 2 is to 2, and this is also 2 is to 2, and this is also 2 is to 2 matrix and by these I can find this P dash M P.

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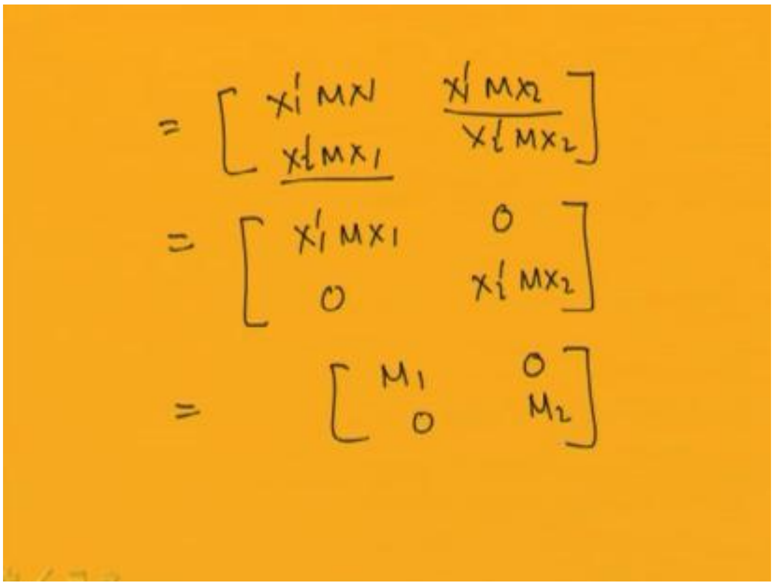

And, this P dash M P I can write equal to, so this will be equal to X 1 dash M X 1 X 2 dash M X 1 X 1 dash M X 2 and X 2 dash M X 2. So, now using this orthogonal property I can write this X 1 dash M X 2 equal to 0, and X 2 dash M X 1 equal to 0. So, this becomes X 1 dash M X 1 0 this is 0 and this is X 2 dash M X 2. So, this is equal to the generalized mass matrix M 1 0, 0 M 2.

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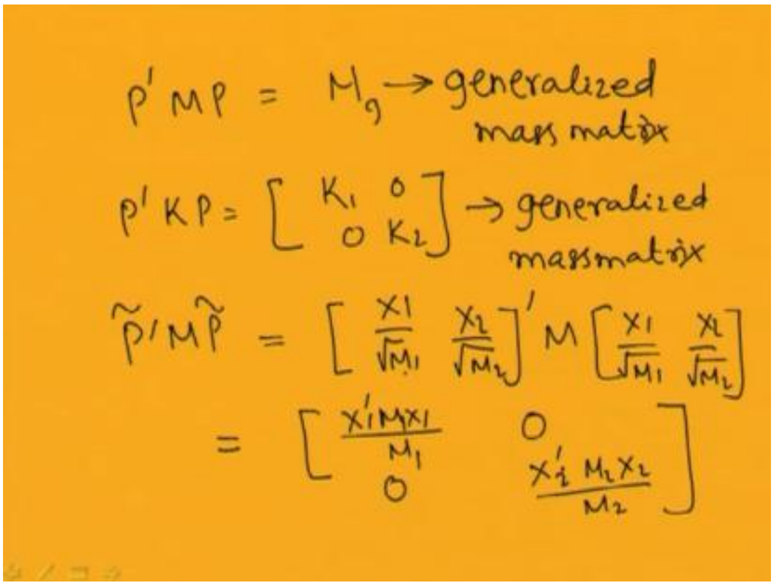

So, by using this P dash M P; I can get the generalized mass matrix M. So, this is the generalized mass matrix, and similarly; I can find this P dash K P will be equal to K 1 0, 0 K 2. So, this is the generalized mass matrix of the system. So, already I told you about the weighted modal matrix. So, let me find this P weighted modal matrix M P weighted modal matrix. So, this will give, so you can check that, so as this is equal to X 1 by root over M 1 X 2 by root over M 2, transpose M and then this is equal to X 1 by root over M 1 and X 2 by root over M 2. So, by multiplying these you can find. So, this will becomes X 1 X 1 transpose M X 1 by root over M 1 root over M 1 this is M, M 1. And, similarly you can find, so this is M 1. So, this is 0 and this is 0 and this is you can find X 1 X 2 dash M 2 X 2 by M 2.

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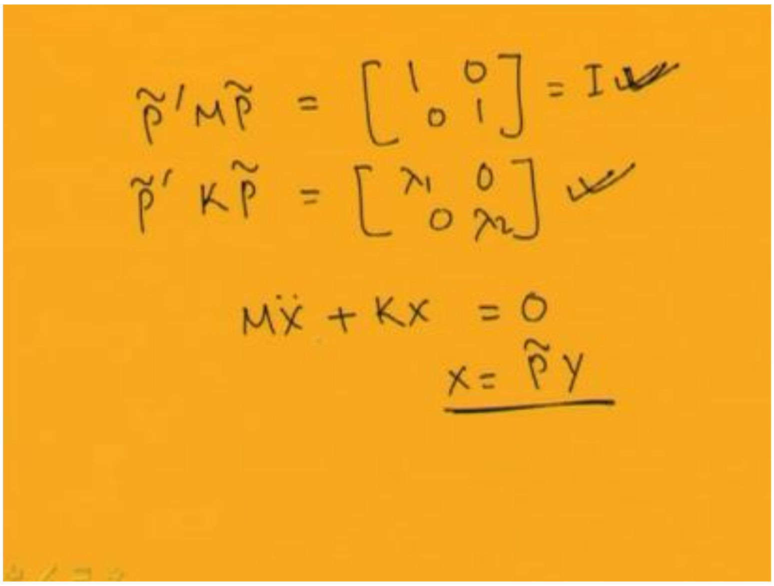

So, by dividing this thing you can see this is equal to this P weighted modal matrix transpose M P weighted modal matrix this becomes 1 0 0 1, so this becomes I matrix. And, similarly you can show that P weighted modal matrix transpose K P equal to lambda 1 and lambda 2 lambda 1 0 0 lambda 2. So, from the weighted modal matrix you can find this P weighted modal matrix M P weighted modal matrix equal to I, and P weighted modal matrix K P weighted modal matrix equal to lambda 1 . So, this gives a Eigen values, and this is a identity matrix. So, by using these 2 properties now; you can use this modal analysis method and reduce a multi degree of freedom system to its equivalents single degree of freedom system. So, let us take this equation. So, generalized equation M X double dot plus K X equal to 0, now substitute X equal to P weighted modal matrix Y.

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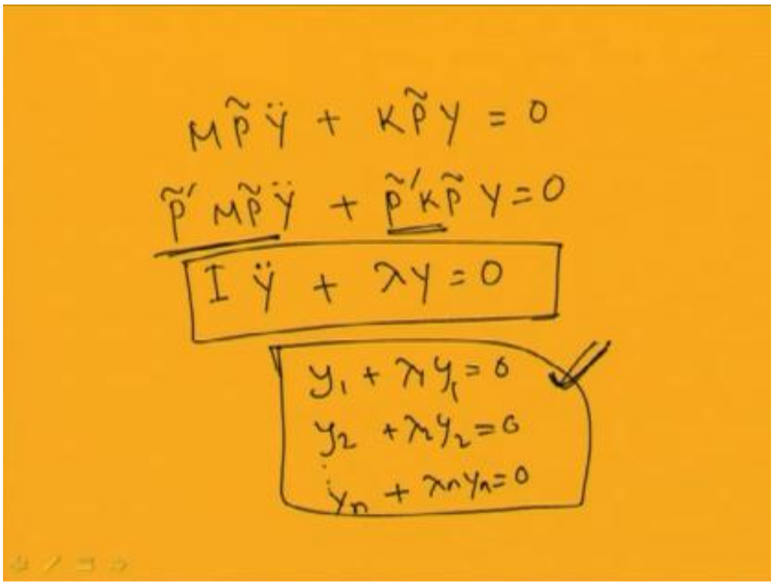

So, by substituting this equation will be reduced to M P weighted modal matrix Y double dot plus K P weighted modal matrix Y equal to 0. Now, pre-multiplying this by P weighted modal matrix M P weighted modal matrix Y double dot plus P weighted modal matrix K P weighted transpose this becomes Y equal to 0. Already, you know that this P weighted modal matrix M P weighted modal matrix is the identity matrix. So, I Y double dot plus and this is the matrix with lambda. So, lambda matrix lambda Y equal to 0.

So, directly you are getting this equation in the form y 1 plus lambda 1 y equal to 0, y 2 plus lambda 2 y equal to y 1 equal to 0. And, similarly for n th mode y n lambda n y n equal to 0. So, by using this weighted modal matrix directly you are getting these frequencies of the system. And you can you are converting the multi degree of freedom system to a you to an equivalent single degree of number of single degree of freedom systems, whose solution you know or you can find very easily.

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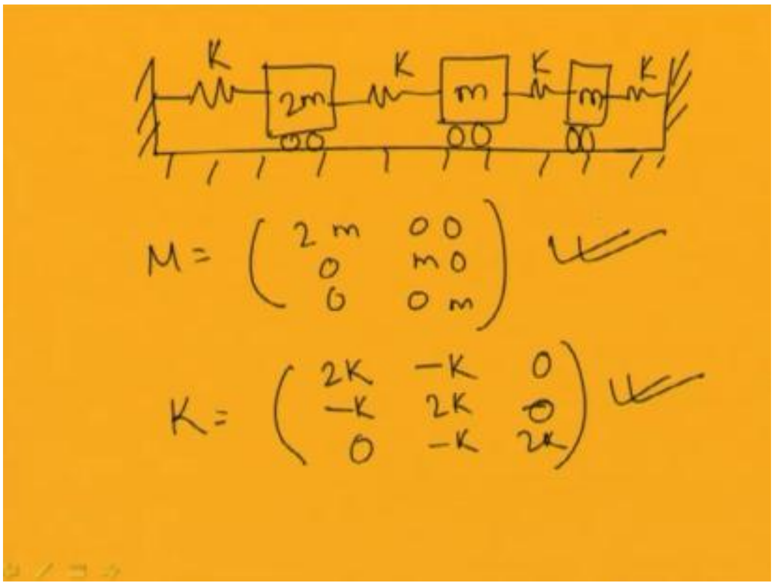

Let us take one example; so let us take this system. So, let us take the system what we have studied before. So, this is a spring mass system. So, let us take this 3 degree of freedom system. So, it is attached by 4 springs K 1 K 2 K 3 K 4. So, let these springs constant are same, let it is K and the first mass let it is 2 m, and the other masses are m and m. So, let us find the normal mode of the system. So, the mass matrix of the system already you know can be written in this form. So, mass matrix equal to 2 m 0 0, 0 m 0 and 0 0 m. And, K matrix already you have found by using the stiffness matrix method. So, from the stiffness matrix method you have found that this K matrix is K 1 plus K 2. So, this is equal to 2 k and minus K 2 minus K and 0 then… So, this is equal to minus K and this is K 2 plus K 3 that is 2 K and this is 0 0 minus K 2 K. So, if you are taking a system with mass matrix this and stiffness matrix this.

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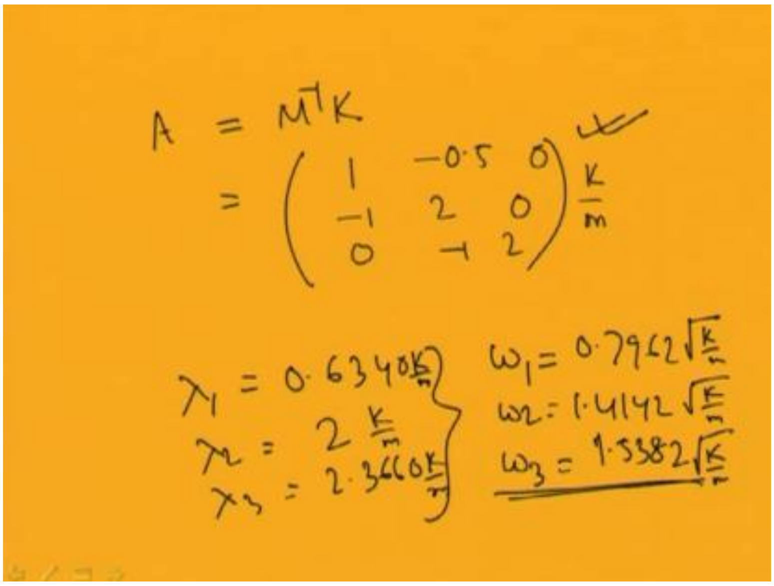

Then, you can find this A matrix either you can find this A matrix from this M inverse K. So, this M inverse K will be equal to. So, you can write, so this is equal to K by M. So, I can take this K by M outside. So, this becomes 1 minus 0.5 0, minus 1 2 0, 0 minus 1 2 K by M and you can find. So, from these you can find the Eigen value of this matrix. So, if you find this Eigen value of this matrix. So, you are getting your lambda 1 equal to 0.6340, lambda 2 equal to 2, and lambda 3 equal to 2.3660 are the corresponding omega 1 will be equal to 0.7962, omega 2 equal to 1.4142 and omega 3 equal to 1.5382.

So, root over k by M. So, in all these cases this is K by M and it is K by M. So, this is K by M root over K by M root over K by M and here also root over K by M. So, this problem you can solve now, you can find this phi by substituting A minus lambda i, you can find A minus lambda i and finding the adjoint of this A minus lambda i by substituting this lambda value you can get the modal matrix.

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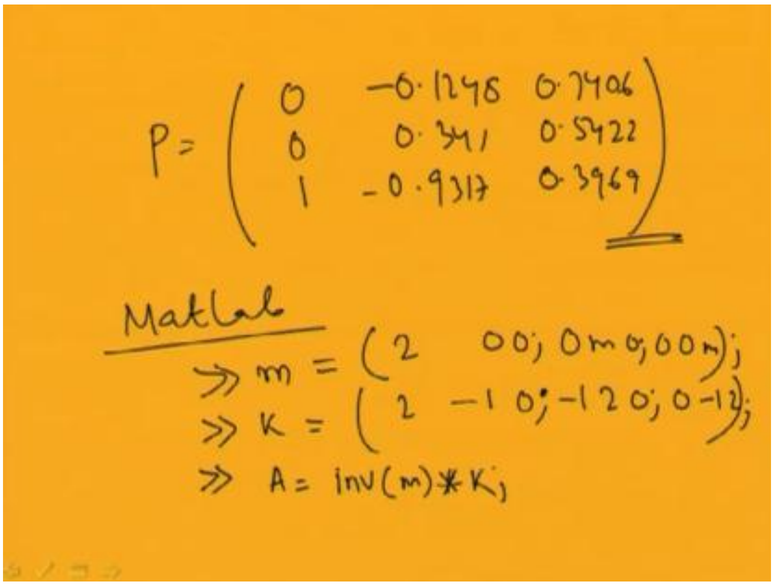

The modal matrix you can write this P modal matrix equal to 0 0 1, minus 0. 1248 0.341 minus 0.9317, so this is 0.7406 0.5422 0.3969. So, I have obtained this value by using this mat lab. So, using mat lab you can solve this problem very easily. So, here you just write A matrix. So, A matrix you can write, so in your mat lab problem you can write A equal to. So, this is let you are taking 2 0 0, then you can write 0 m 0, and then 0 0 m. So, this is the mass matrix m matrix and you can write the stiffness matrix K, K you can write equal to 2 minus 1 0; you can give here. So, this is minus 1 2 0 and then 0 minus 1 2. So, this will give the stiffness matrix now, you can write K inverse m. So, A equal to m inverse k. So, A equal to inverse inv (m) into K. So, as I have this mass matrix and stiffness matrix.

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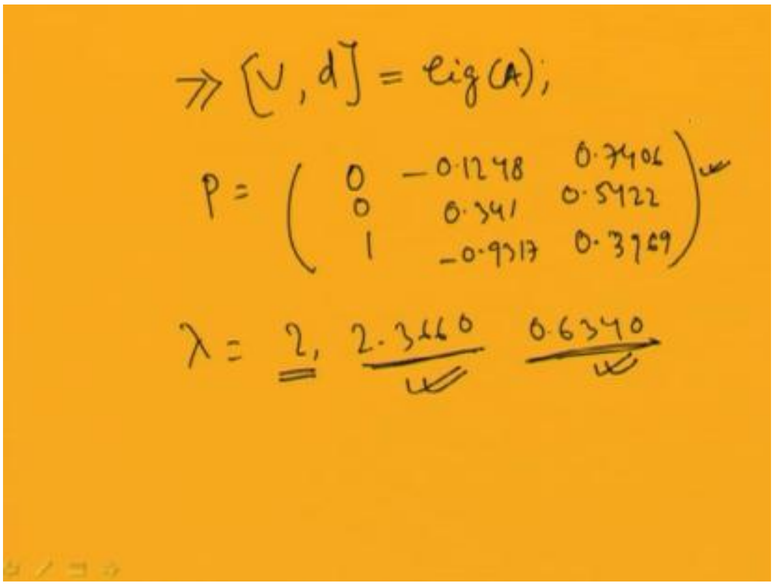

I can find A matrix and then by writing to find the Eigen value of this A matrix I can find the Eigen matrix and Eigen vector simultaneously. So, I can write V d equal to eig (A). So, this will give me the Eigen value and Eigen vector. So, I can get the phi matrix or the P matrix directly. So, this is the v is the P matrix. So, this P matrix is obtained are 0 0 1 minus 0.124 0.341 and 0.9317 and this becomes 0.7406 0.5422 and 0.3969, and this d matrix is a diagonal matrix. And, this diagonal matrix is given the Eigen values of the system. So, the Eigen of the systems are obtain lambda equal to 2 2.3660 and 0.6340.

So, from this you can find that; this is the minimum Eigen value. So, this corresponding to this corresponds to the first normal mode frequency, and this corresponds to the second normal mode frequency and this corresponds to the third normal mode frequency. So, in this way by using this mat lab also you can find the modal matrix of the system. So, now let us take a simple 2 degrees of freedom system. So, in these 2 degrees of freedom system let us find the modal and weighted modal matrix of the system.

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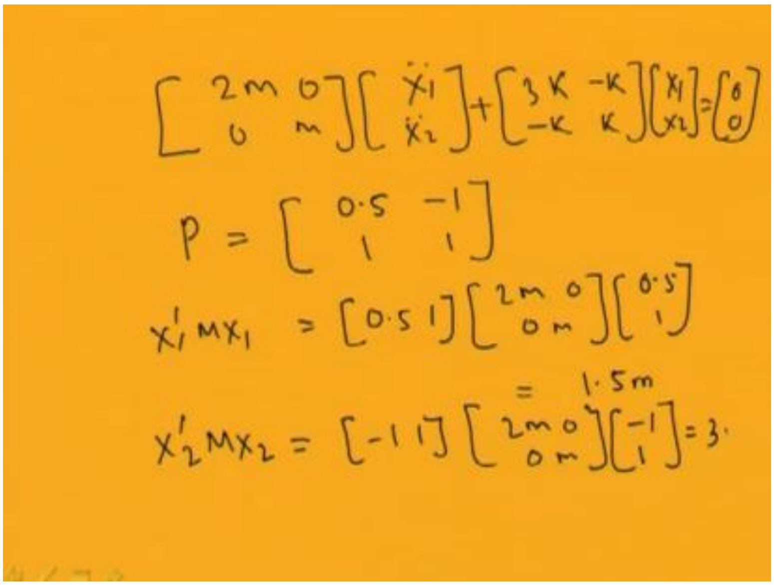

So, let the system previously we have considered is 2 m 0, 0 m X 1 double dot X 2 double dot plus 3 K minus K minus K K X 1 X 2 equal to 0 0. So, in this system already you have found that modal matrix P equal to the modal matrix we have found in this case. So, P you have obtained equal to 0.5 1 and minus 1 1. So, to find the weighted modal matrix we should find the generalized mass matrix and generalized stiffness matrix. So, to find generalized mass matrix I can find it in this way. So, I can find X 1 dash M X 1. So, this becomes 0.51 into 2 m 0 0 m into 0.5 1 already I have found it. So, this is equal to 1.5 m. Similarly, I can find the second M X 2. So, this becomes minus 1 1 into 2 2 m 0 0 m into minus 1 1. So, this becomes 3 m.

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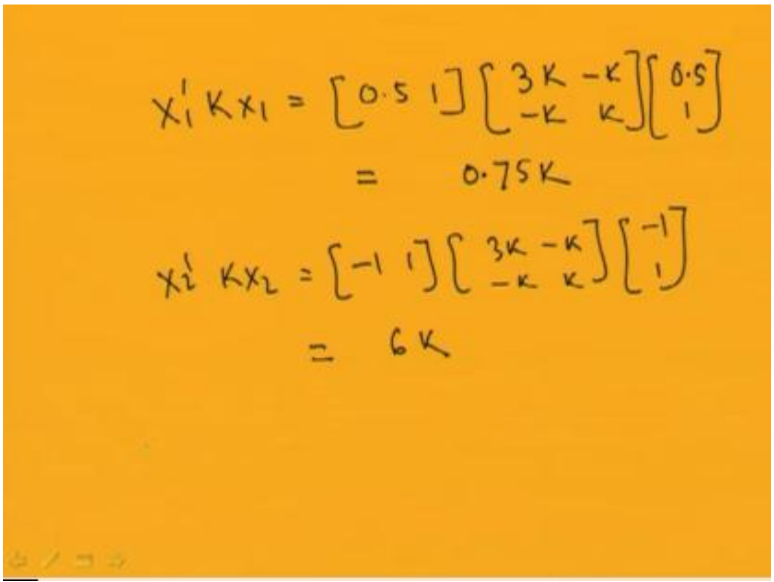

So, to find the generalized stiffness matrix I can write it in this form X 1 dash K X 1. So, this is equal to 0.5 1 into. So, the mass matrix stiffness matrix is 3 K minus K minus K K and I can multiple these with point, so this is equal to 0.5 1. So, if you multiply this. So, you are getting it equal to 0.75 K. Similarly, X 2 dash K X 2 equal to minus 1 1 K equal to 3 K minus K minus K K into minus 1 1, so if you multiple these is coming to be 6 K.

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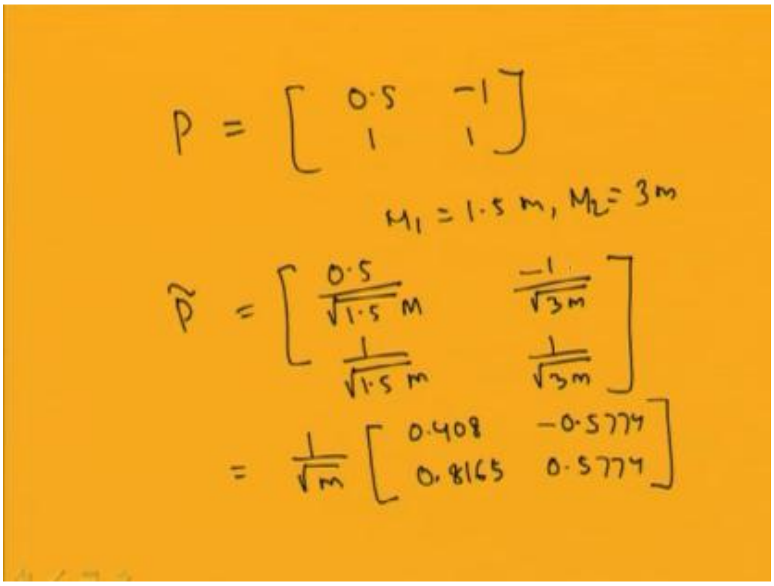

So, this P weighted modal matrix already you know the modal matrix P. So, P equal to 0.5 1 minus 1 1 and M 1 equal to 1.5 m and M 2 equal to 3 m. So, P weighted modal matrix will be equal to. So, the first column will be divided by square root of M 1 and second column will be divided by the square root of M 2. So, P weighted modal matrix will be 0.5 by root over 1.5 m. So, this is 1.5 m and this is 1 by root over 1.5 m and second column will be divided by 1 by root over 3 m and this is equal to 1 by root over 3 m. So, I can take this root m outside. So, this becomes 0. 408 minus 0.5774 0.8165 and this is 0.5774. So, in this way you can obtain the weighted modal matrix of a system.

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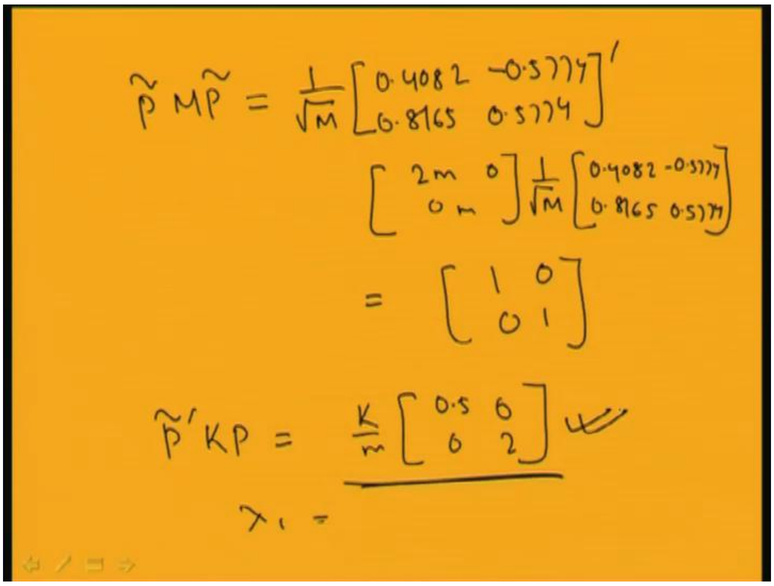

And, we can now you can check that P weighted modal matrix M P equal to I, M P weight modal matrix equal to I. So, P weighted modal matrix already we have obtained it equal to by 1 by root over M 0.4082. So, this 0.8165 minus 0.5774 and this is 0.5774. So, this transpose into m, m equal to 2 m 0 0 m and into root over 1 by M into 0.4082 minus 0.5774, 0.8165 and 0.5774. So, if you multiple this you can check. So, multiple and check that you are getting 1 0 0 1. Similarly, you can check that you are getting P transpose P weighted modal matrix transpose K P. So, this becomes equal to K by m 0.5 0 0 2.

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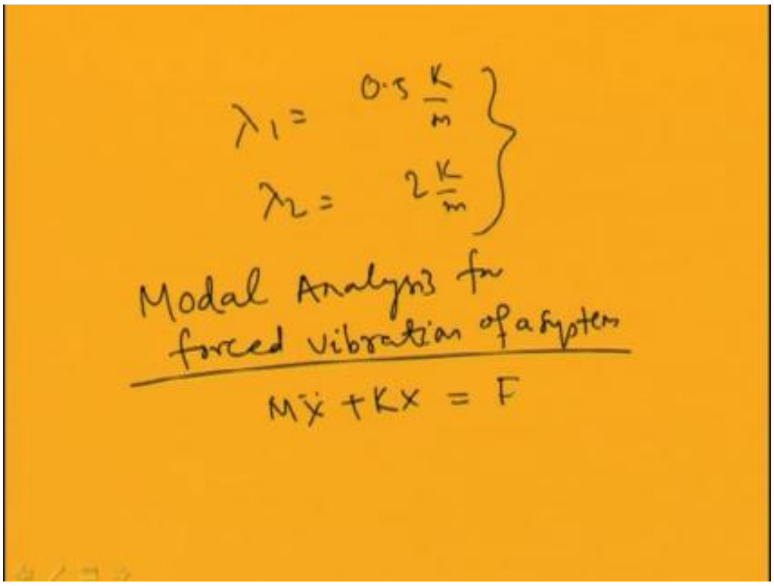

So, you have seen that lambda 1 equal to 0.5 K by m, and lambda 2 equal to 2 K by m. So, you can check that this is equal to your lambda 1 lambda 2 the diagonal terms are lambda 1 lambda 2, and in case of mass matrix P weighted modal matrix transpose M P equal to identity matrix that is 1 0 0 1 and this is equal to. So, this P weighted modal matrix into K P equal to the Eigen value the diagonal elements are the Eigen value of the system. So, already I told you how to decouple a set of equation multi degree of freedom system equation by using this modal matrix method. So, now I can tell to if you have a force vibration case. So, how to solve a force vibration case by using this modal analysis method? So, modal analysis method for force vibration of a system, so in case of force vibration the equation motion can be written in this form M X double dot plus K X equal to F.

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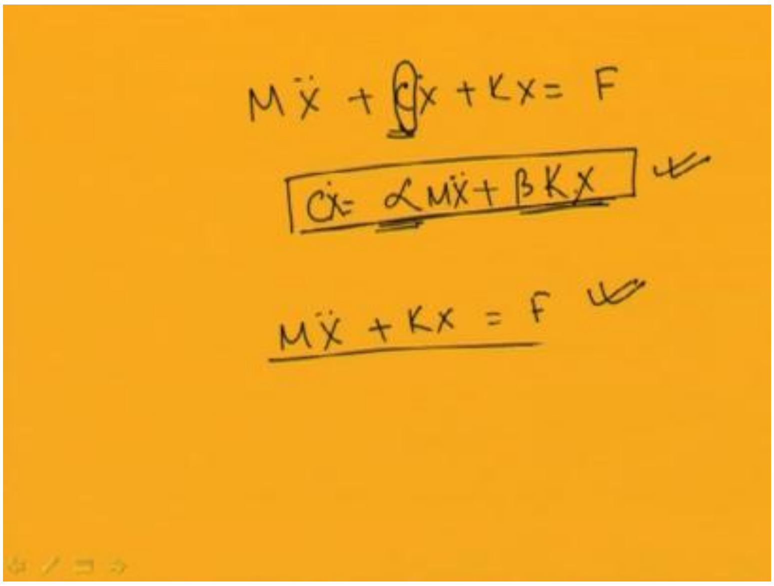

So, if this is undamped system and if damping is present in the system, in this case can be written in this form M X double dot plus C X dot plus K X equal to F. Generally in case of while solving this multi degree of freedom system the C matrix this damping matrix is written as alpha M plus beta K form. So, it is written in this form alpha. So, it is divided into 2 parts 1 is alpha and other is beta K. So, by dividing this into 2 parts; so this modal participation or this damping matrix is divided into 2. So, it is added to this mass matrix and stiffness matrix terms and equation is converted into the previous form that is M X double dot plus K X equal to F form.

So, this damping is known as relay damping or proportional damping of the system. These damping matrix is converted are the C X dot equal to. So, alpha M X double dot plus beta K X. So, this damping force is divided proportionally into this inertia force and stiffness force and, so by proportionally converting that thing by using this really damping. So, it can be converted to equivalent undamped system. So, this damped system is converted to an equivalent undamped system by using this relay damping. So, now this equation is converted to this form. Now, either using this modal matrix method or weighted modal matrix of the system. So, it can be converted to a set of uncoupled equations or by using this principle coordinate Y it can be converted to a set of uncoupled equation.

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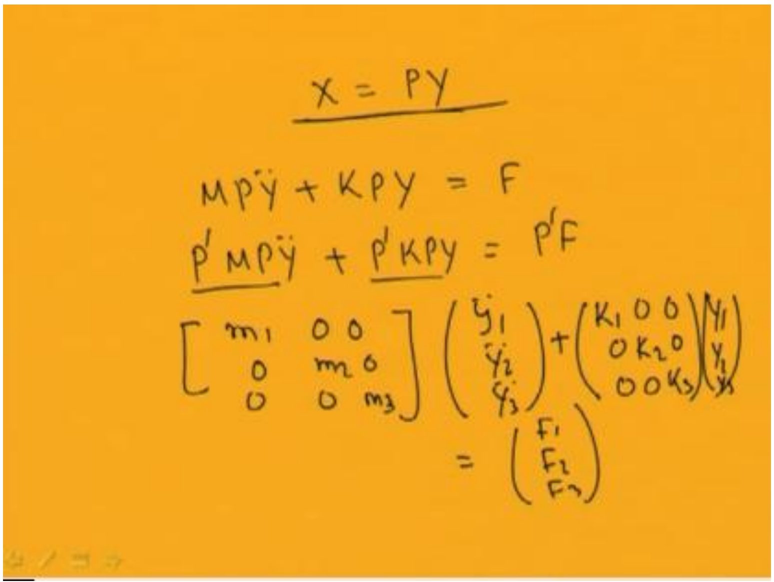

Let us see by using P So, let me write this X equal to P Y. So, if I am using this modal matrix P then I can write this X equal to P Y then I can write this M X double dot I can write P Y double dot plus K PY equal to F now I can pre-multiply by P transpose. So, P transpose M P Y double dot plus P transpose K P Y equal to P transpose F, and this is a diagonal matrix and this is a diagonal matrix also. So, in this case let for a 2 degree or 3 degree of freedom system, I can write this equal to m 1 0 0 0 m 2 0 0 0 m 3 into m 3 into y 1 y 1 double dot y 2 double dot y 3 double dot plus. So, this is plus. So, this is also a diagonal matrix with P dash K P Y. So, this is also I can write k 1 0 0, 0 k 2 0, 0 0 k 3 into Y 1 Y 2 Y 3 equal to P transpose F. So, I can write this is equal to F 1 F 2 F 3.

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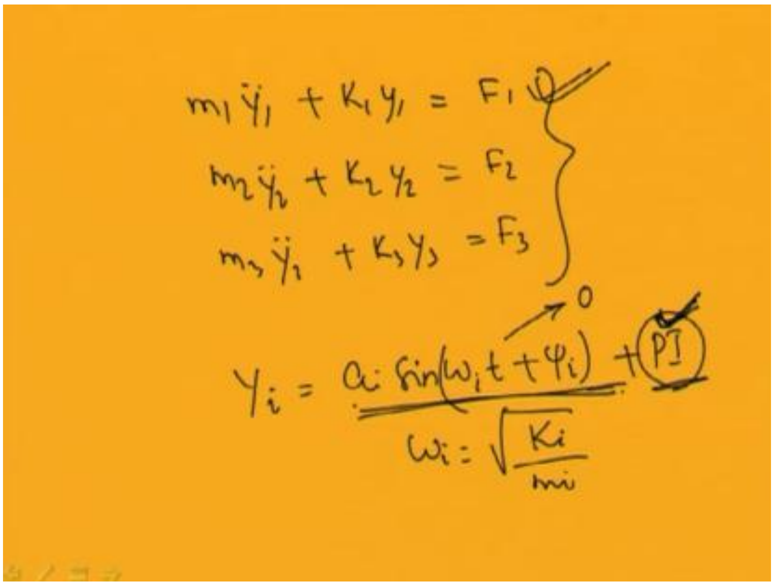

So, this multi degree of freedom system equation this equation now converted to a set of equation which are uncoupled equation and which can be written in this form m 1 Y 1 double dot plus K 1 Y 1 equal to F 1 and m 2 Y 2 double dot plus K 2 Y 2 equal to F 2 and m 3 Y 3 double dot plus K 3 Y 3 equal to F 3. So, in case of single degree of freedom systems see you know the solution of these equations this type of equation. So, you can take the first equation it solution will contain 2 parts; the first part is the complementary function, and the second part is the particular integral.

So, you can find the solution of this system by finding the complementary part of the system which is nothing, but Y i will be equal to a i sin omega i t. So, where omega i equal to K i root over omega i equal to root over K i by M I, and this a or sin omega i t plus psi i. So, this a i and psi i can be obtain from these initial conditions. So, this is the complementary part of the solution, and plus the particular integral. So, these particular integrals you can find are these forces. So, this will give rise to the steady state respond of the system and this is the transient part of the system. So, due to the present of damping, or when damping is present, so this part will be 0 and one can obtain the particular integral part of the system.

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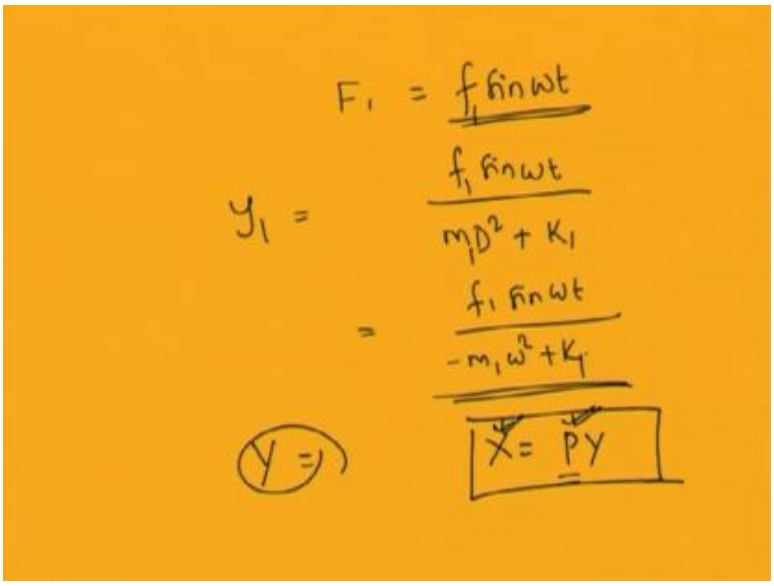

So, in this case the response will be. So, if one apply this or one can write F 1 equal to f sin omega t. So, it may be f sin omega t. So, in this case if it is f sin omega t then this y 1 will be equal to f sin or f 1 sin omega t by. So, I can write this particular integral. So, this is equal to m 1 m 1 D square I can write this is D square. Where, D I will substitute it by minus omega square M 1 D square plus K 1. So, this D square can be substituted by omega square. So, this becomes f 1 sin omega t by minus m 1 omega square plus K 1.

So, this is the particular integral part of the system and the complementary part is a i sin omega i t plus psi i. So, summation of these 2 terms will give you if Y 1 similarly, Y 2 and Y 3 can be obtain. So, after obtaining this y 1 y 2 y 3 or Y matrix or Y vector. So, y after obtaining this Y which is combination of y 1 y 2 y 3 you can obtained you can obtain X equal to P Y already, P weight the modal matrix is known to you. So, by substituting this Y in this equation you can obtain this X of the system also you may use this weighted modal matrix in this case.

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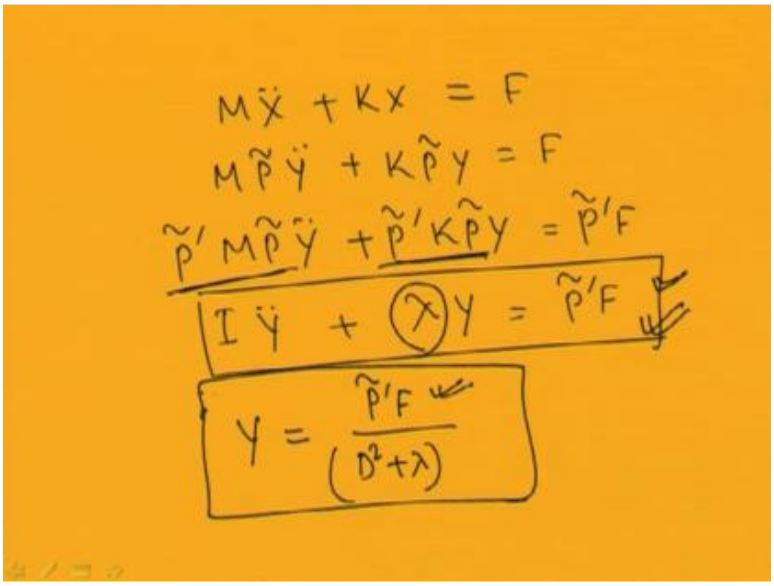

So, when you are using this weighted modal matrix. So, this equation M X double dot plus K X equal to F will reduce to M P weighted modal matrix Y double dot plus k weighted modal matrix Y equal to F. Now, pre multiplying transpose of this weighted modal matrix. So, you can obtain. So, P weighted modal matrix F. Now, this P weighted modal matrix MP weighted modal matrix is equal to this I, so I Y double dot. So, this becomes I Y double dot plus. So, this is lambda. So, this is lambda Y this equal to P weighted transpose F. So, in this case, so in this way you can get directly the equation I Y double dot plus lambda Y equal to P transpose F.

Already, you know the Eigen value of the system. So, if you know the weighted modal matrix of a system then by using that weighted modal matrix you can find the response of the system very easily. So, in this case just you have to multiply this P weighted modal matrix transpose with this F, and you can get the solution from this equation. So, this equation the solution can be written Y will be equal to. So, if we are finding this particular integral. So, this will be equal to P weighted modal matrix F by, so this is Y. So, this is D square plus lambda. So, by taking different function F by taking different function F, you can find this Y of the system.

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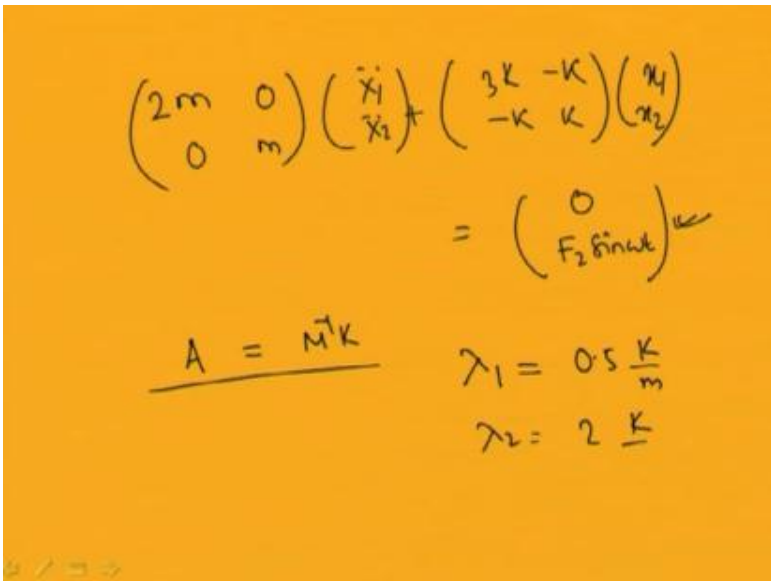

So, let us take this spring mass damper system and write this equation, let this is 2 m 0. So, this equation is 2 m 0 and 0 m X 1 double dot X 2 double dot plus 3 K minus K minus K K X 1 X 2. Let the second mass is subjected to a force F 2 sin omega t. So, this is equal to 0 F 2 sin omega t. So, in this case we have to find the response of the system. So, to find the force vibration response of the system first we have to find the modal matrix. Already, we obtained the modal matrix of the system from this A matrix, first we have found A matrix. So, A matrix is M inverse K from this A matrix we have obtained the normal mode of the Eigen values. First we have obtained the Eigen values of the system. So, this Eigen values we have obtained it is equal to lambda equal to 0.5 K by m and lambda 2 equal 2 K by m.

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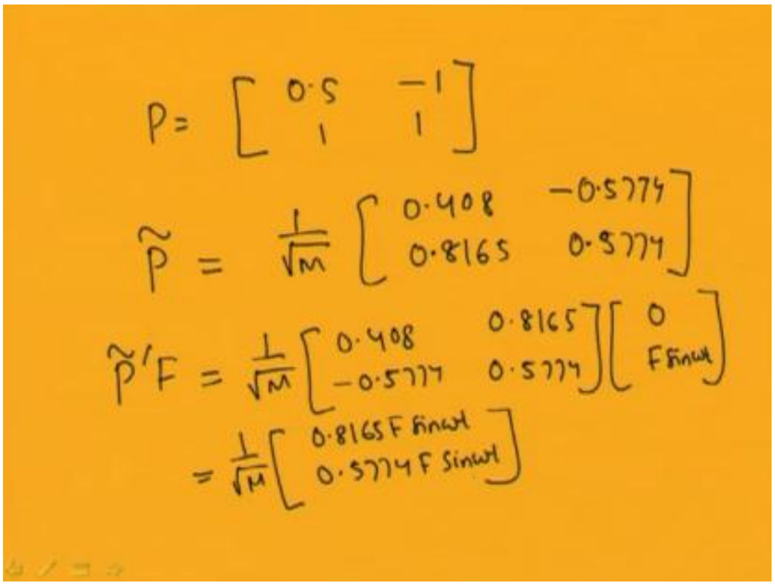

Also we have obtained the modal matrix previously. So, this modal matrix equal to 0.5 1 and this equal to minus 1 1. Also, we have obtained this weighted modal matrix by dividing this modal matrix by root over the generalized mass m 1 and m 2 and this P weighted modal matrix equal to. So, this becomes this weighted modal matrix already we have obtained. So, this equal to root over M 0.408 and this is 0.8165 this is minus 0.5774 and this is 0.5774. So, this is the weighted modal matrix. And, now to find the response of the system I should pre-multiply this P weighted modal matrix with F.

So, this becomes. So, this becomes 1 by root M 0.408 minus 0.5774 and then this is 0.8165 and this is 0.5774 into F. So, F equal to 0 F sin omega t. So, I can write this equal to. So, this will be equal to this multiply 0, so this into 0. So, this becomes 1 by root M. So, first equation first part becomes this into this. So, this becomes 0.8165 F sin omega t, and the second equation becomes this into 0 and this will be equal to 0.5774 F sin omega t.

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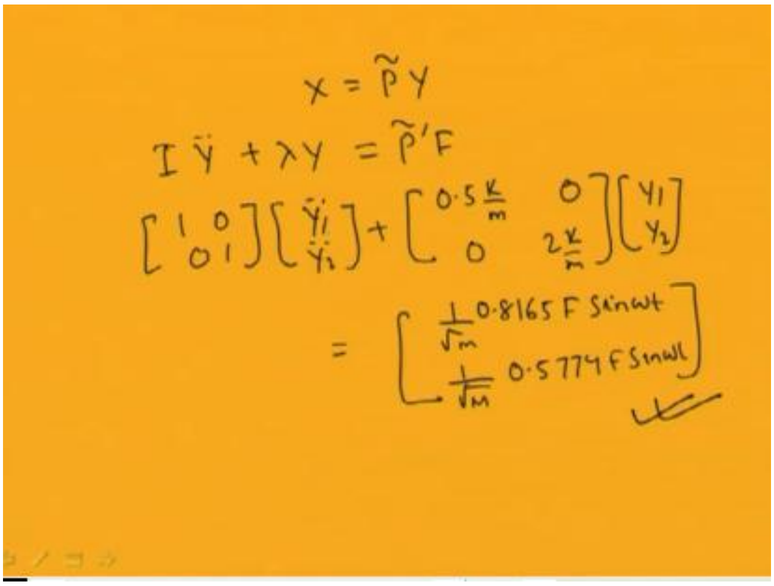

So, now the equation by using this X equal to P weighted modal matrix Y. So, it will reduce to this form I Y double dot plus lambda y will be equal to P weighted transpose F. So, in this case it reduces to 1 0, 0 1 Y 1 double dot Y 2 double dot plus lambda 1 already you know. So, this is equal to 0.5 K by m and this is equal to 0 and this is 0 this is 2 K by m Y. So, this is Y 1 Y 2 it is equal to already, we have obtained this expression. So, root over 1 by M 0.8165 1 by root over m 0.8165 F sin omega t, and 1 by root over m. So, this becomes 0.5774 F, 0.577 4 F sin omega t. So, you can observe that this uncoupled equation this coupled equation. So, here it is coupled in stiffness matrix. So, this coupled equation now converted to a set of uncoupled equation here. So, this is a set of uncoupled equation and you can find the solution of this equation very easily.

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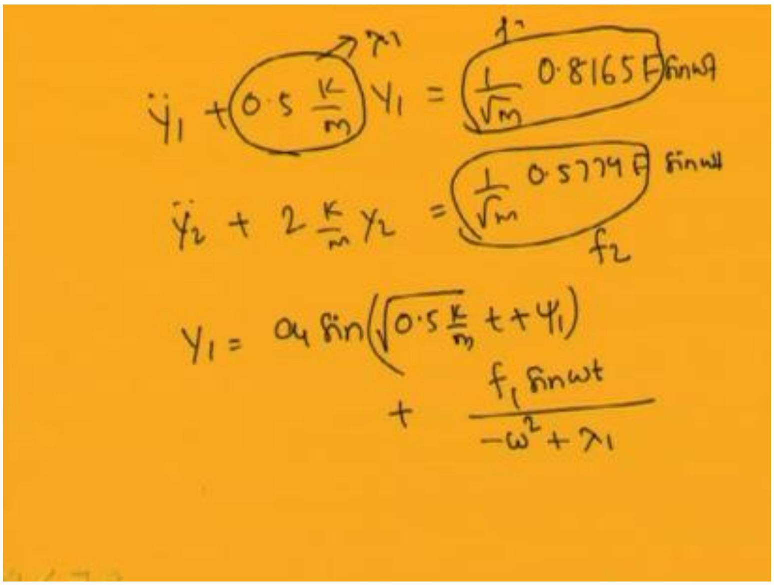

So, the equation will be Y 1 double dot plus. So, you can write this first equation Y 1 double dot plus 0.5 K by m Y 1 equal to. So, this equal to 1 by root m into 1 by root m into 0.8165, 0.8165 F sin omega t, and second equation becomes Y 2 double dot. So, Y 2 double dot plus point. So, this is 2 K by m. So, 2 K by m Y 2 equal to 1 root M. So, this becomes 0.5774 F, 0.5774 F sin omega t. So, let this part becomes F 1, so given a value of F. So, let this is F 1, and this is F 2 this part is F 2. So, the solution, so for this first equation the solution will become Y 1 equal to the complementary part will becomes Y 1 equal to the complementary part is a 1 sign omega n t.

So, this is omega n square this part is omega n square. So, this becomes omega n root over 0.5 k by m t plus psi 1 and for the particular integral this becomes F sin omega t. So, this becomes F sin omega t by. So, this is D Square. So, minus omega square plus let me write this as. So, this is lambda 1, so plus lambda 1. So, the solution become a 1 sin 0.5 K by m root over t plus psi 1 plus F sin F 1 sin omega t by minus omega square plus lambda 1.

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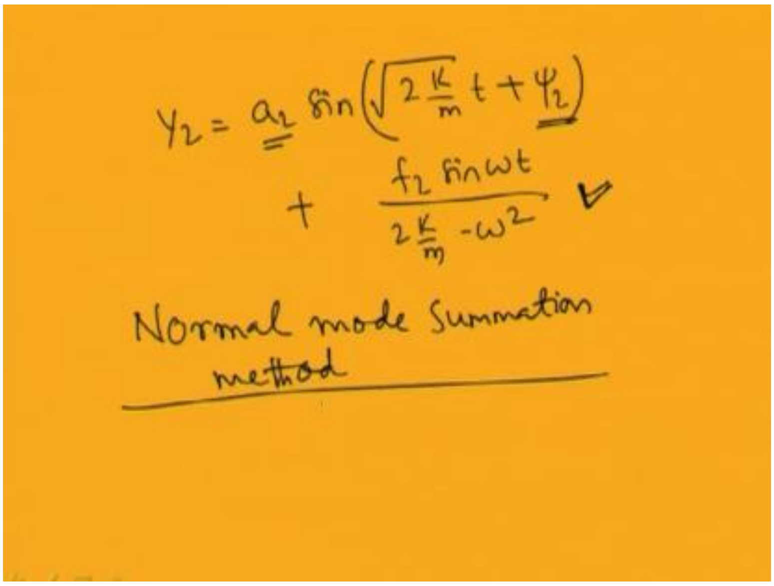

Similarly, Y 2 can be written as; Y 2 equal to a 2 sin this is 2 K by m root over 2 K by me t plus psi 2, and plus this particular integral will becomes F 2 sin omega t by lambda 2 minus omega square. And, lambda 2 is nothing but this is equal to 2 K by m and this lambda 1 is nothing but this is lambda 1 equal to 0.5 K by m. So, for given K by m one can find the solution of the system and here a 1 psi 1, and a 2 psi 2 are depends on the initial condition of the system. So, by knowing these initial conditions we can obtain the response of the system. So, now let us see the normal mode summation method; in many force vibration cases the frequencies are very high and we are. So, let us take 1 example of a multi storage building.

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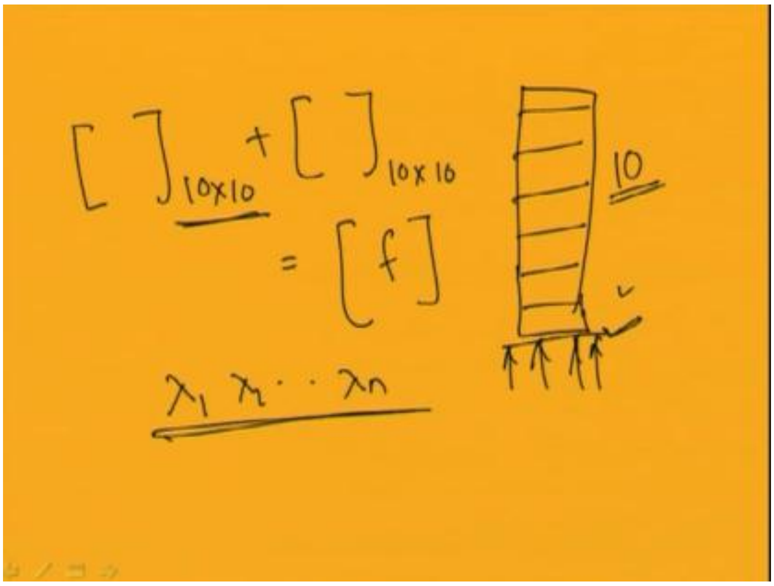

So, this multi storage building is subjected to an earthquake excitation. So, it is subjected to this earthquake excitation multi storage let us take a ten storage building. So, here this mass matrix and stiffness matrix can be written in this form. So, the mass matrix will be 10 is to 10 matrix and the stiffness matrix will be 10 is to 10 matrix. So, and it is subjected to some forcing; let this force vector I can write equal to f, but the frequency of this excitation and the natural frequency if you find from the modal analysis you can find the natural frequency or normal mode frequency of the system.

So, the let the normal mode frequencies are lambda 1 lambda 2 lambda n. So, you can observe that the excitation frequencies are very very low, and may be limited or may be within these fast few modes of the normal mode frequency. So, in those cases as we know that the resonance occur when the natural frequency is near the external excitation frequency. So, instead of taking the full modal matrix that is 10 is to 10 modal matrix we may take a fewer mode and do these analysis.

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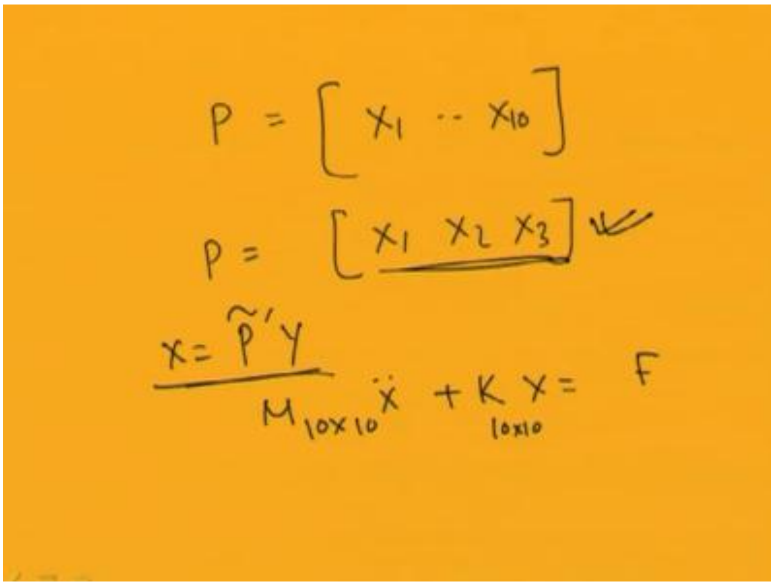

So, instead of taking this P; all the modes of the system X 1 to X 10. So, we may take only few mode of the system. So, let me take only the first 3 modes. Let me take X 1 X 2 X 3 only and now by taking X equal to P weighted modal matrix Y and here P only the first 3 modes is taken. Then, we can convert this equation which was the 10 is to 10 X double dot plus K X equal to F. So, this is 10 is to 10. So, now I can write this equal to P weight modal matrix Y.

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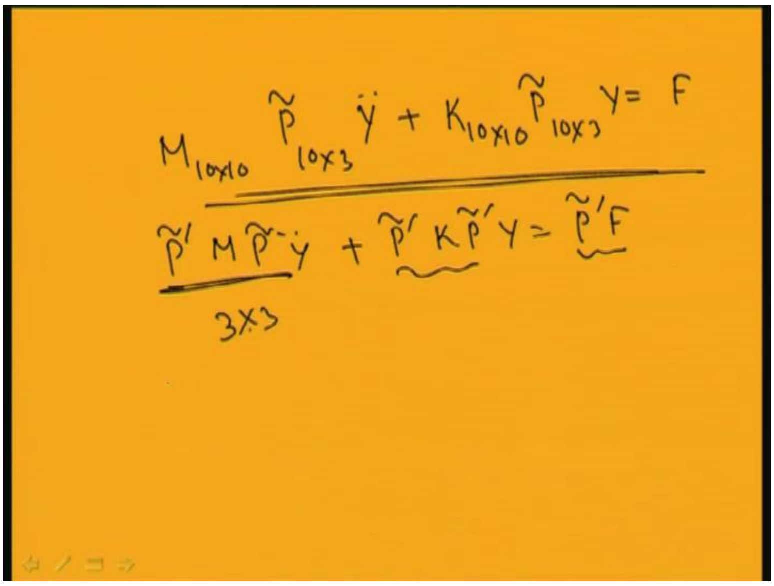

So, this M ten is to ten this P weighted modal matrix. So, this is this contain 10 row, but 3 column and this is Y double dot plus k. So, this is 10 is to 10. So, this is P weighted modal matrix. So, this is 10 is to 3 Y equal to F. So, now by pre-multiplying this P weighted modal matrix transpose. So, you can convert this equation into a 3 is to 3 matrix form. So, P weighted modal matrix M P weighted modal matrix Y double dot plus P weighted modal matrix K P weighted modal matrix Y equal to P weighted modal matrix transpose F.

So, you can see that this is the 3 is to 3 matrix, and this 3 is to 3 matrix, and this is a 3 is to 1 vector. So, in this way you can reduce a very high degrees of freedom system to that simpler degrees of freedom system. So, this method is known as the normal mode summation method. So, today class we have studied about the modal analysis method of pre and force vibration of a system also we have studied this normal mode vibration of the system. Next class we will solve some examples to see this pre and force vibration and normal summation of multi degrees of freedom system.