Multi DOF

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Lecture No. #01

Derivation of Equations of Motion, Influence Coefficient Method

An ITG person promises only what he can deliver, an ITG person delivers what he promises.

So, today we are going to study about multi degree of freedom systems, in the previous class is we have studied about the free and force vibration of single and two degrees of freedom systems. In case of single degree of freedom systems, we have studied how to find the response of the system and in case of two degree of freedom systems, we are also find the response of the system for free and force vibrations.

We have used different methods to find the equation motions, the equations motions are found by using either Newtons method or delamed principle or by using the energy principle which include Lagrange principle and Hamiltons principle. Either we have used Hamiltons principle or extended Hamilton principle depending on the systems so for conservative systems, we have used Hamilton principle and for non conservative system and can go for this extended Hamilton principle.

And by using Lagrange principle you can derive the equation motion for multi degree of freedom systems and by using this Hamilton principle though you can find the equation motion for any type of systems, but particularly it is useful for continuous systems.

Similarly, this Lagrange system Lagrange principle is particularly use for multi degree of freedom systems and it though number of coordinates or number of degrees of freedom or is the less than two are in the if it is a single degree of freedom or it is two degrees of freedom system either you can go for these energy based principle or you may give go for these Newtons or delamed principle, but for higher systems are for systems more than two you should go for Lagrange principle to derive the equation motion.

If the system is a continuous system or distributed mass system you should go for extended Hamilton principle and now will study about these multi degree of freedom systems.

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So, multi degree of freedom systems for one in which you require two or more coordinates to describe the motion of the system already you have seen.

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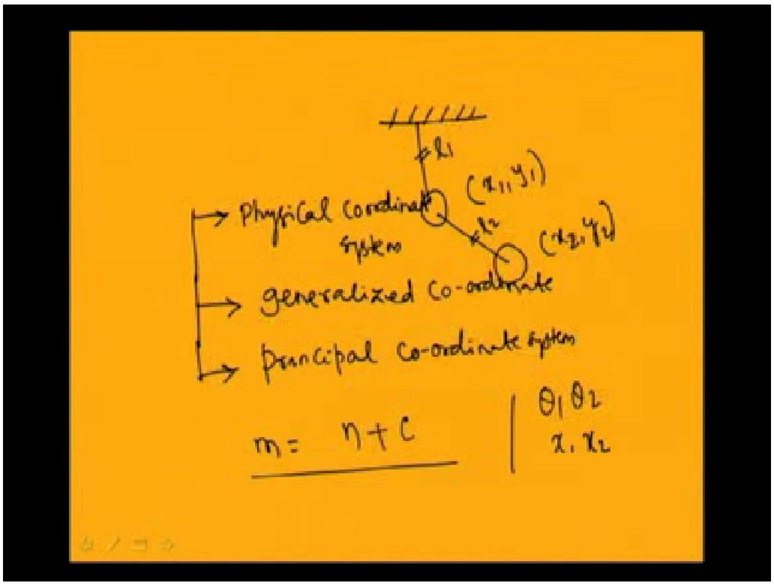

In the case of a double pendulum and if you take a double pendulum so it is a two degrees of freedom systems. So, for more than one degree of system it is a multi degree of freedom system. So, already you know how to write the equation motion. So, for that previously I told you have different types of coordinate systems, one is the physical coordinate system physical coordinate system and second one is the generalize coordinate system.

And third one is the principal coordinate systems coordinate system. So, in case of physical coordinate system, you can put any physical coordinates to describe the motion of the system and in case of generalize coordinate system, these are the minimum number of coordinates we require express the motion of the system.

So, the relation between these generalize coordinates and the physical coordinates you know by using the number of constants you can describe the let if n in the number of generalize coordinate and m in the number of physical coordinates. So, your m will be equal to n plus number of constants let c be the number of constant. So, m will be equal to n plus c, m is the number of generalize coordinate and c the number of constants.

So, in this case of this double pendulum you may write the equation motion by using this coordinate physical coordinate x 1, y 1 x 2, y 2. So, these are the physical coordinate you can use so are the generalize coordinates will be either you can write the x displacement of these mass 1 or these mass 2. So, either you can use these theta 1 theta 2 are the generalize coordinates or you may use x 1 and x 2 coordinates as the generalize coordinates in the space of double pendulum.

But you may use x 1, y 1 x 2, y 2 are the physical coordinates of the system. So, this length is constant. So, these length equal to x 1 square plus y 1 square equal to l 1 square and similarly these length l 2 equal to x 2 minus x 1 square plus y 2 minus y 1 square. So, these are the constant equation you have two constant equation and four physical parameters.

So, you have number of coordinates generalize coordinates equal to 2. So, these are the two degree of freedom system and you have two generalize coordinates to describe this motion of these system. Already, I told you by putting the coordinate system are different faces you can get different equation motion. So, when the equation motion in the equation motion you can write the equation in this path.

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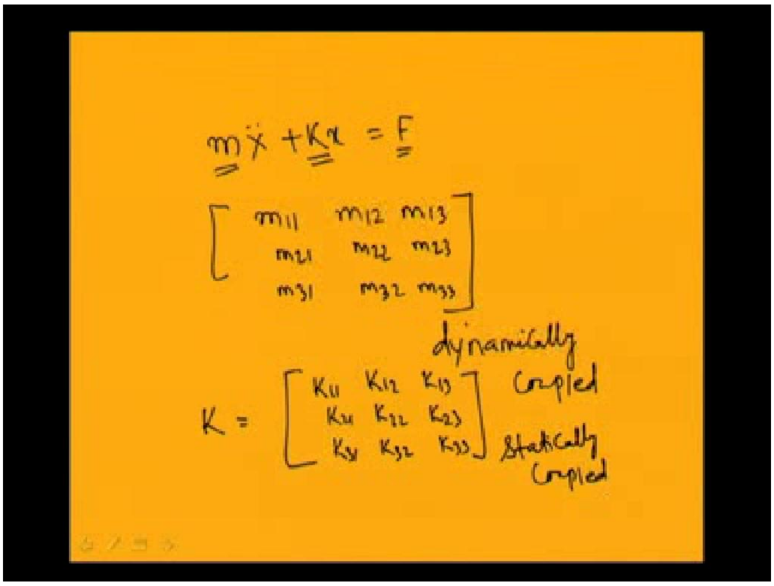

You can write in its path m x double dot plus k x equal to f. So, we are m is the mass matrix and k is the stiffness matrix and f is the force vector. So, these when these mass matrix so let be write for a three degree of freedom system. This mass matrix will look like this so m 1 1 m 1 2 it will be m 1 1 m 1 2 m 1 3 m 2 1 m 2 2 m 2 3 and m 3 1 m 3 2 m 3 3.

So, this is the mass matrix for a three degree of freedom system. So, if all the components of this mass matrix are present or if some of the half diagonal elements are present then this mass matrix is known to be coupled. So, this is couple mass matrix and the system is known to be dynamically coupled.

Similarly if the stiffness matrix if an write this stiffness matrix for these three degree of freedom system in this way, k 1 1 k 1 2 k 1 3 k 2 1 k 2 2 k 2 3 and k 3 1 k 3 2 k 3 3. So, by using or if these all the elements are the half diagonal elements are present in addition to the diagonal elements then this is known as statically coupled.

So, if by using some coordinates you can decouple this mass matrix unstiffness matrix those coordinates are known as the principal coordinate of the system. So, for the coordinate system for which we are getting a dynamically uncoupled and statically uncoupled system then those systems that coordinate system is known as principal coordinate system.

So, you have three different coordinate system, one is the physical coordinate system, second one is the generalize coordinate system and third one is the principal coordinate system. So, by using principal coordinate system you can reduce this equation motion to that of three for a let for a three degree of freedom system, you will get three different equation motion, uncoupled equation motion.

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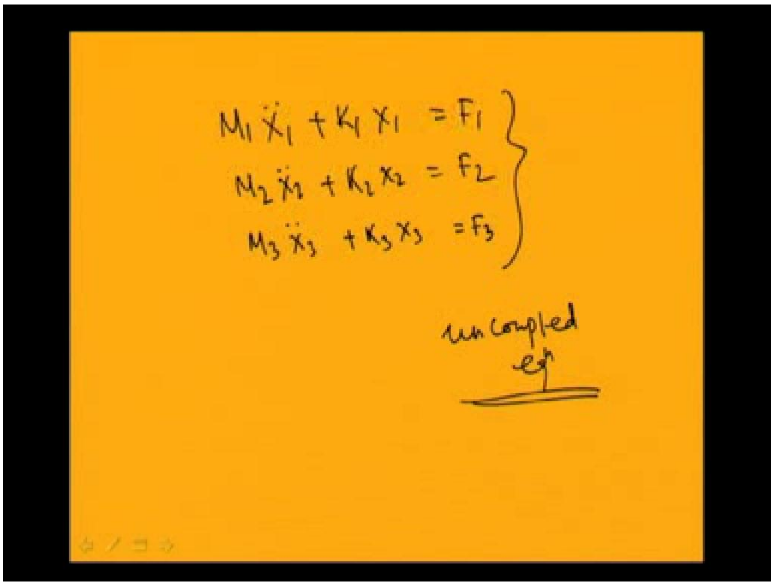

So, first equation will be m 1 x 1 double dot plus k 1 x 1 will be equal to f 1, second equation will be m 2 x 2 double dot plus k 2 x 2 equal to f 2 and third equation will be m 3 x 3 double dot plus k 3 x 3 equal to f 3.

So, if you have principal coordinate if you are using principal coordinate, then x 1 y 1 principal coordinate x 1 x 2 x 3 then you will get a set of uncoupled equation. So, this is uncoupled equation and these equations are same as the equation you have seen for a single degree of freedom system. So, as you are reducing these equation to that of single degree of freedom system. So, you can find the response of the system there individually.

But when these equations are uncoupled then also you can find the equation motion, they also you can find the response of the system by using different methods. So, for a two degree of freedom systems already we have seen how to find that response of that system.

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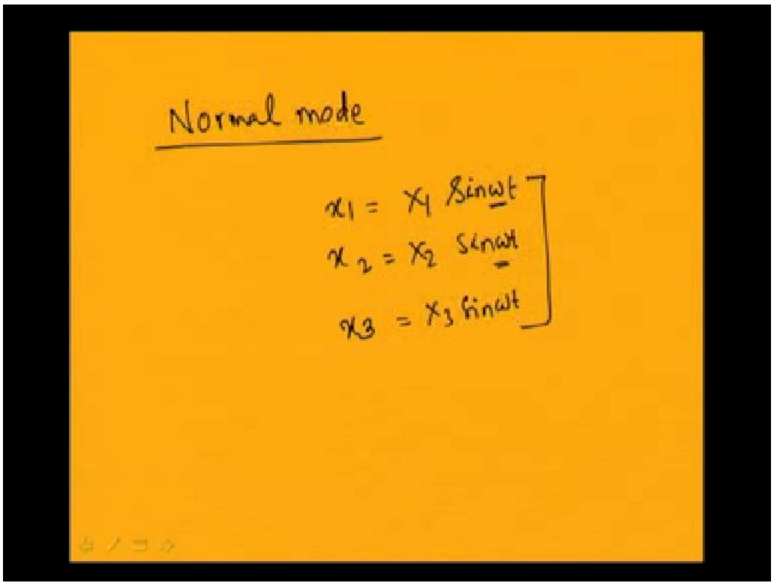

So in that case, we have used this normal mode method. So, we have defined the system response in terms of normal mode. So, normal mode is nothing but this is the free vibration response of the system when we are assuming all the message are passing through the equilibrium position at same time and they have the same frequency. So, we have assumed in case of two degree of freedom system, we have assumed the solution in this form.

So, we have assume x 1 equal to x 1 sin omega t and x 2 equal to x 2 sin omega t. So, we have assume these frequency of both the modes x 1 and x 2 to be same, but their amplitudes are different x 1 and x 2 and they have same face that means they are in case of normal mode, we are assuming that all the particles of the system are moving which same frequency and passing through the equilibrium position at the same time.

So, for multi degree of freedom system also we can extended this idea, from the second degree of freedom system and we can write the thermal mode x 3 or x 4, X 5 in that way we can write. So, x 3 will be equal to. So, small x 3 I can write equal to capital x 3 sin omega t. So, there are many systems which we can at module it as the single degree of freedom system or two degree of freedom system in that case you should module it as multi degree of freedom system.

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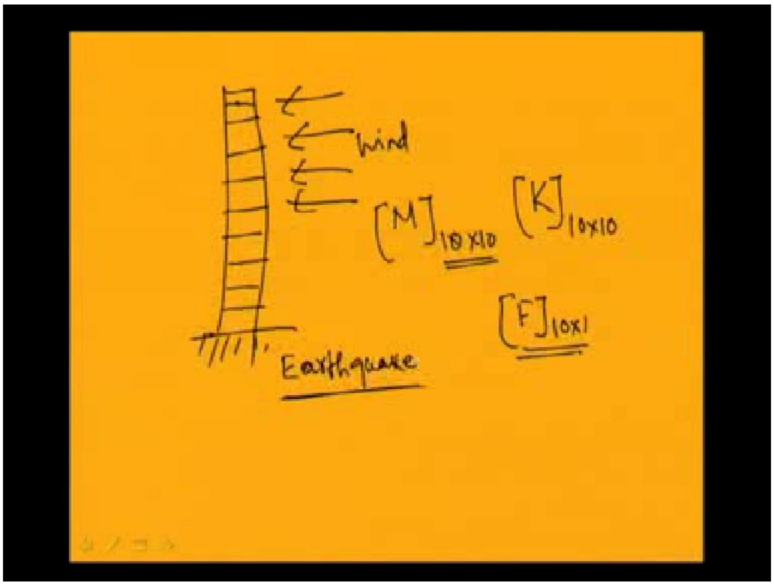

Let us, take the example of the terms to be building in these term terms to be buildings building subjected to this air force. So, wind force, if the subjected wind to force or it may be subjected to some earthquake. So, even to find the response of the system response of the building so what will happen to the building?

In case of this wind or when it is subjected to the earthquake. So, in these cases if you want to study the response of system, so you cannot model it as a single degree of freedom or two degree of freedom system. In that case you should go for multi degree of freedom system in which you may have a mass matrix for these term terms to a building you may have a mass matrix of 10 into 10, 10 is to 10 and a stiffness matrix of 10 is to 10 and this force vector depending on wire the force which acting you may have the force vector.

So, the force vector will be 10 is to 1 force vector you may get. Similarly, for other systems you may have a system with hundred degrees of freedom system or you may have thousand degrees of freedom system.

So, in those cases you cannot solve these equations by using this single or deducing it way or single or two digit of freedom system. So, that time is to apply other methods to solve these equations and you can extend the methods what you have studied in two degrees of freedom system in this case also.

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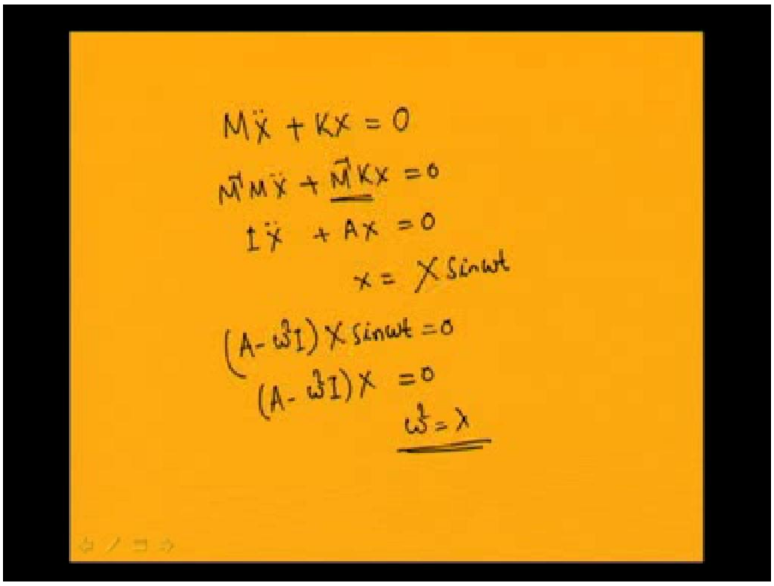

So, in this case you can write the equation motion in these one. So, you are writing equation motion m x double dot plus k x equal to f. So, f equal to 0 each for free vibration and when for taking f equal to some the forcing function the it will be a force vibration system. So, let us study has the free vibration response of the system.

So, m x double dot plus k x equal to 0 if the equation for this free vibration so in these case you can find or you can premultiply these m with m inverse. So, m inverse m x double dot plus m inverse k x equal to 0. You can write in this way this equation and these m inverse m is the identity matrix. So, in the i x double dot plus these m inverse k let be write equal to A. So, A x equal to 0 and now I can assume the normal mode in these for so, these x i can write x equal to capital x.

So, x equal to capital x sin omega t. I can take sin omega t or I may take also x equal to x A to the power omega t. So, I assuming x equal to x sin omega t and substituting these equation in the main equation in the previous equation I can write this equation in this form. So, it will be comes. So, x double dot will becomes minus omega square where x sin omega t or minus omega square x. So, these f of equation will deduce that form. So, it will become a minus omega square x a minus omega square.

So, A minus omega square i into x or x sin omega t and can write it is x sin omega t equal to 0. So, for non trivial solution that is when the response is non zero that obtain you can find this A minus omega square i a minus omega square i x equal to 0. So, this omega square let me put equal to lambda. So, these equations if the deduce to in these for these the known for of this Eigen value function.

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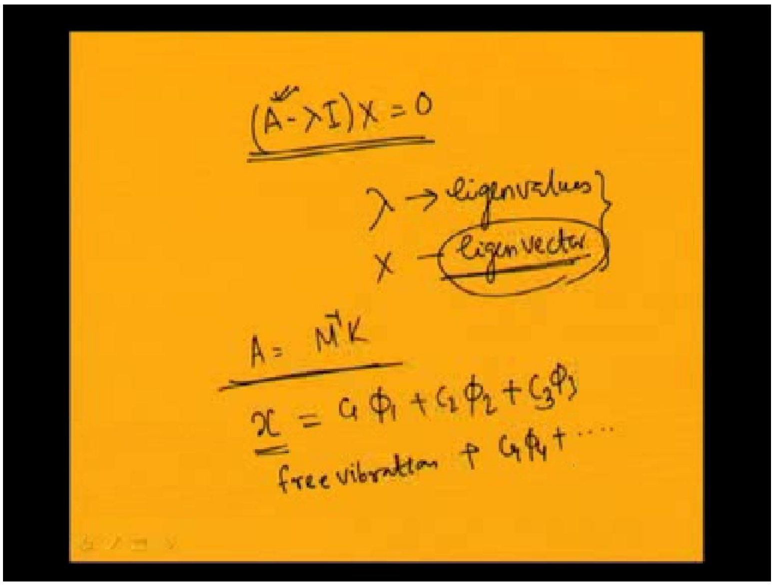

So, we have studied this equation in case of two degrees of freedom system also. So, in case of multi degree of freedom system also, these systems with reducing to this Eigen value problem why are you have to find the Eigen value of matrix a to find this lambda. So, lambda at the Eigen values.

So, lambda at a Eigen values and x will be the Eigen vector. So, you can find the Eigen values and Eigen vectors of this matrix A. So as is nothing but if the m inverse k matrix. So, first find A matrix and then find the Eigen values and Eigen vectors of this matrix to find the free vibration response of the system.

So, by using this Eigen vector this Eigen vectors corresponds to the normal mode of the system. So, by finding the normal mode of the systems you can find the free vibration response of the system if you known the initial conditions already, we have seen in case of a two degrees of freedom system the resulting free vibration x at any time t can be written can be written as c 1 phi 1 plus c 2 phi 2.

So, for a phi 1 phi 2 are the normal modes of the system and c 1 and c 2 are the modal participation in the resulting free vibrations with depends on the initial conditions. So, by knowing this initial conditions and you can find by knowing the initial conditions and the modal matrix modal matrix will come from these Eigen vector by knowing these Eigen vector you can find the modal matrix and by using this modal matrix phi 1 and phi 2.

So, if are taking only two modes phi 1 phi 2 in your taking multi modes then it will be phi 1 phi 2 c 3 phi 3 and c 4 phi 4 in this way it depends on the number of modes you are taking. So, the resulting free vibration will be resulting free vibration x such the free vibration response with the summation of different modal frequencies. So, where phi 1 phi 2 phi 3 at the Eigen vectors for the correspond to first mode, second mode and third mode respectively.

So, in this way you can analyze or you can find the free vibration of a system by using this normal mode method similar to the two degrees of freedom system, first you should find the A matrix is nothing but m inverse k has find m inverse k that is A.

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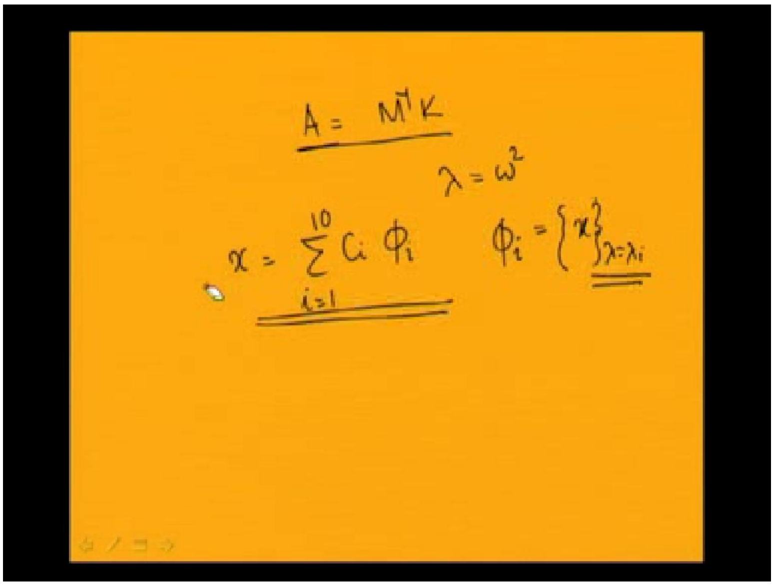

So, A equal to m inverse k. So, after writing the equation you will get the mass matrix m and stiffness matrix k by using m inverse k, find the A matrix and find the Eigen value of A matrix and Eigen vector of A matrix. So, Eigen value will give you omega square. So, from omega square you can find omega the frequency of the system. So, for a multi degree of freedom system, you will get number of frequencies.

If you have a three frequency of system you will get value of lambda. So, you will get three natural frequencies of the system or three normal frequency of the system. So, corresponding to these three normal mode frequency, you will get three modal amplitude or you will get three modal values or Eigen vector. So, corresponding these three Eigen vector or normal modes so you can find the free vibration response of the system.

So, for finding free vibration response you will the super position principle of the normal modes. So, let you have you ten modes. So, in that case you can write the free vibration x will be equal to summation i equal to 1 to 10 c i c i phi i. So, were phi i correspond to phi i equal to x i it is equal to x or lambda equal to lambda i.

So, in this way you can find the free vibration response of a system. So, you may use some other method to find this x that that is the normal mode, the normal mode you can find using in from the driven matrix of the system. So, you can write this A minus lambda i by using this adjoint matrix method are also you can find.

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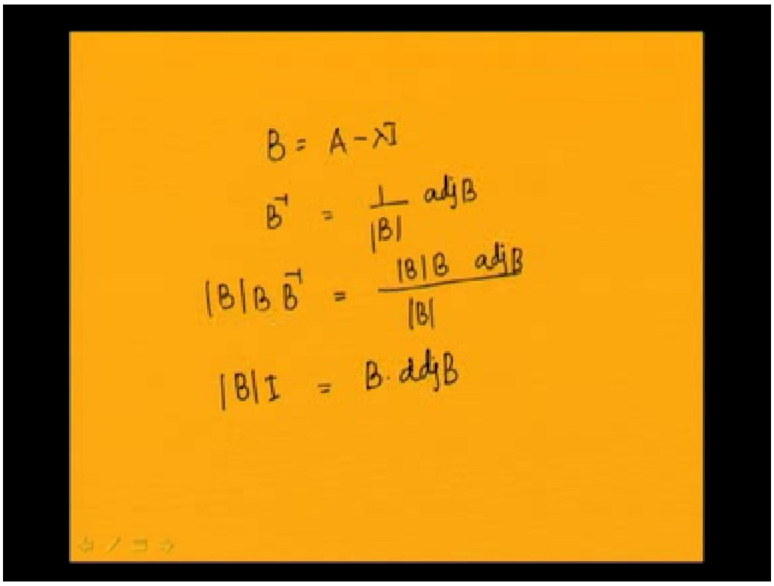

Let me write the b equal to A minus lambda i. So, if b equals to A minus lambda i then I can write this b inverse equal to 1 by b 1 by determinant b into adjoint b. b inverse equal to 1 by determinant b into adjoint b. So, if you pre multiply this determinant b into b in this determinant equation in this determinant b into b into b inverse.

So, this will be equal to determinant b b and this is determinant b and in adjoint b. So, already you know b into b inverse, if the i matrix your b into i. So, determinant b into i. So, this will be equal to. So, this cancel. So, this become b into adjoint b. So, b into adjoint b matrix already you know your b equal to you have written this b equal to A minus lambda i. So, A minus lambda i A minus lambda i determinant into i so this is I equal to in that way you can write.

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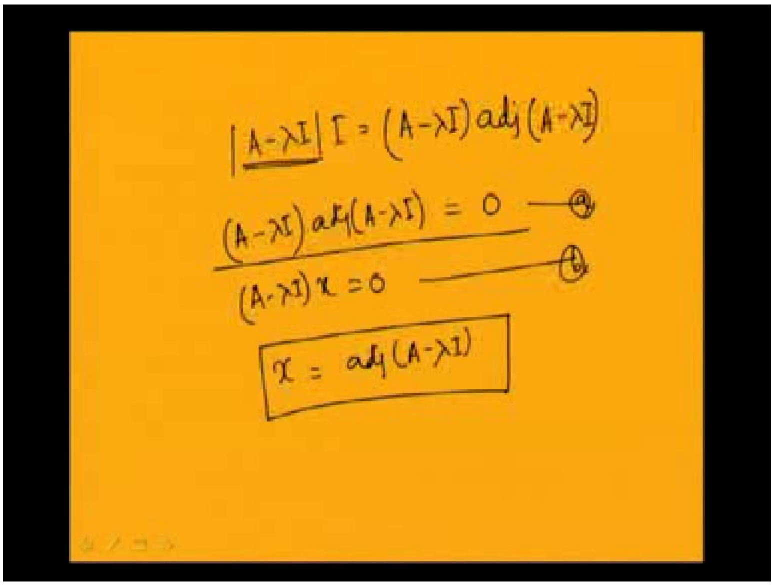

So, this will b equal to A minus lambda i A minus lambda i into adjoint A minus lambda i. But already you know that this A minus lambda i for lambda to be the Eigen value this A minus lambda i is determinant of A minus lambda A equal to 0. So, you can write this equation. So, A minus lambda i into adjoint A minus lambda i equal to 0, but you know this A minus lambda i into x equal to 0.

So, if you compare this two equation A and equation b if you compare, then you can write x equal to adjoint A minus lambda i. So, you can find this Eigen vector of the system by finding the adjoint of A minus lambda i matrix. So, easily you can find the Eigen vector by using this adjoint A minus lambda i by using mat lab also very easily you can find the Eigen value and Eigen vector of the system. So, in mat lab you have this function by using mat lab.

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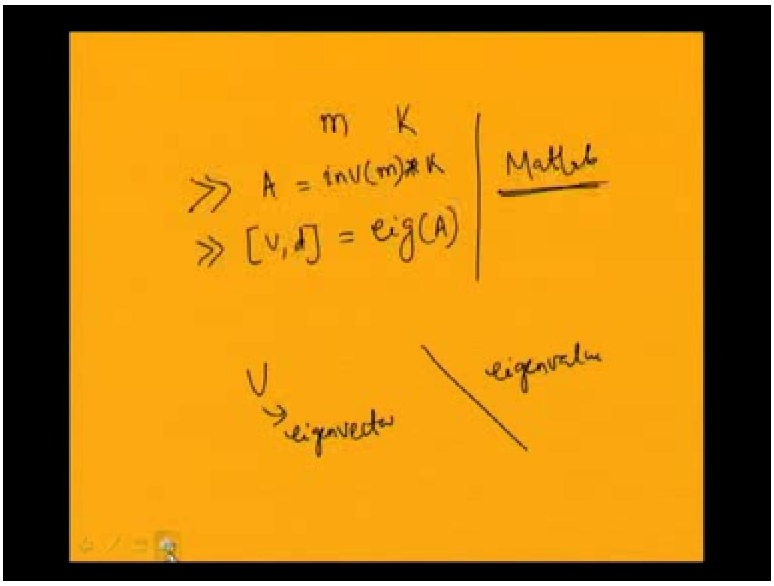

Let you have this matrix m, you have this matrix k, you may the stiffness matrix k in the m is the mass matrix and k in the if the stiffness matrix. So, you can write this code like this. So, it is find the A. So, A equal to m inverse. So, i and A m and multiplied by k. So, these will give you A matrix and you can find just write v d. So, by writing this v d equal to eig of A. So, this will give you this b matrix, so b will contain the Eigen values and d will contain a diagonal element.

So, these diagonal element are the Eigen values of the system. So, d will contain the Eigen diagonal element will contain the Eigen value and this v will contain the Eigen vector. So, by using this mat lab also you can find the Eigen value and Eigen vector. So, when we are doing if by hand or manually for a lesser degrees of freedom system, you may use this adjoint matrix method. So, adjoint A minus lambda i in this find to find these normal modes of the system.

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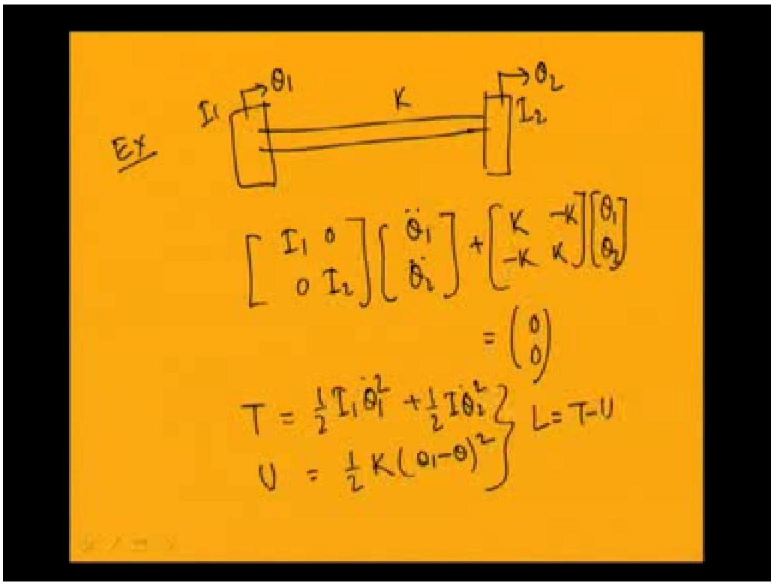

So, let us take one example. So, will take the example of a simple two degrees of freedom system. Let us take a two rotor system what we have studied before. So, already you know that this two rotor system.

It is a semi definite system. So, in case of semi definite system one of the natural frequencies equal to 0 and you has a rigid body motion. So, let us take the system for simplicity, let i 1 if the inertia of this rotor and i 2 is this inertia and k in this stiffness of this rod. So, you can write this is theta 1 generalizing coordinate theta 1 and theta 2 are sin 2 these to rotors now you can find the normal modes.

So, let us find the normal modes by using this adjoint matrix method for this case. So, in this example so the equation motion already you have upon by using different methods. So, the equation motion if will write it. So, it will be like this. So, it will be i 1 0 0 i 2 theta 1 double dot theta 2 double dot plus k minus k minus k plus k theta 1 theta 2 equal to 0 0.

So, this equation motion either you can find by using this Lagrange principle. So, in case of Lagrange principle, the kinetic energy t equal to half i 1 theta 1 dot square plus half i 2 theta 2 dot square and the potential energy u. So, if are using this stiffness. So, it will be equal to half k into theta 1 minus theta 2 square. So, by using this kinetic energy and potential energy, you can find lagrangian of the system that is equal to l is equal to t minus u and then by using the Lagrange principle that is d by d t of del l by del q k dot minus del l by del q k equal to 0.

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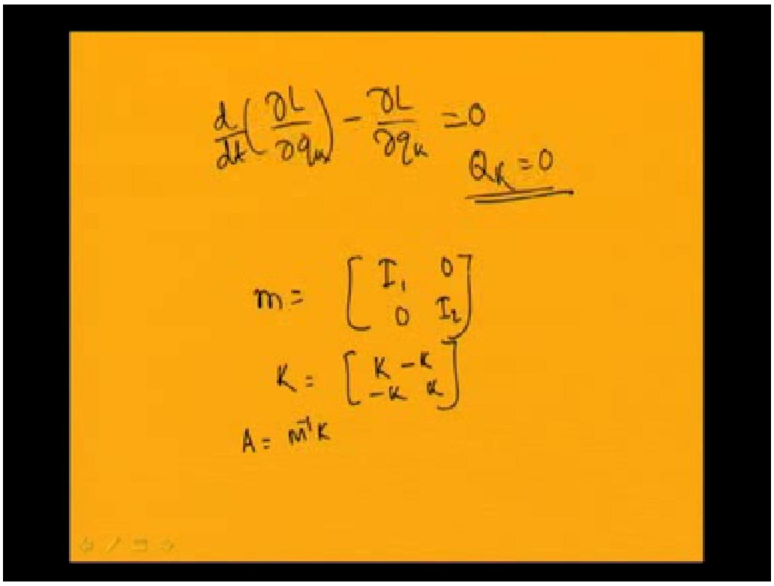

So, as there is no force acting on the system you can take this as a free vibration. So, this forcing part equal to 0 or this non conservative forcing q k q k equal to 0. So, these equations deduces to this one and by using this Lagrange principle, you will get this equation also you may use this Newtons method and in which by drawing the free body diagram of this mass or this inertia 1, rotor 1 and rotor 2 you can find the equation motion.

So, it will be it can be retaining this one. So, here your mass matrix mass matrix equal to i 1 0 0 i 2 and the stiffness matrix k equal to k minus k minus k k. So, after finding this mass matrix and stiffness matrix you can find the A matrix. So, A will be equal to m inverse k, you can find the inverse of this matrix and you can find this m inverse k and this m inverse k is nothing but.

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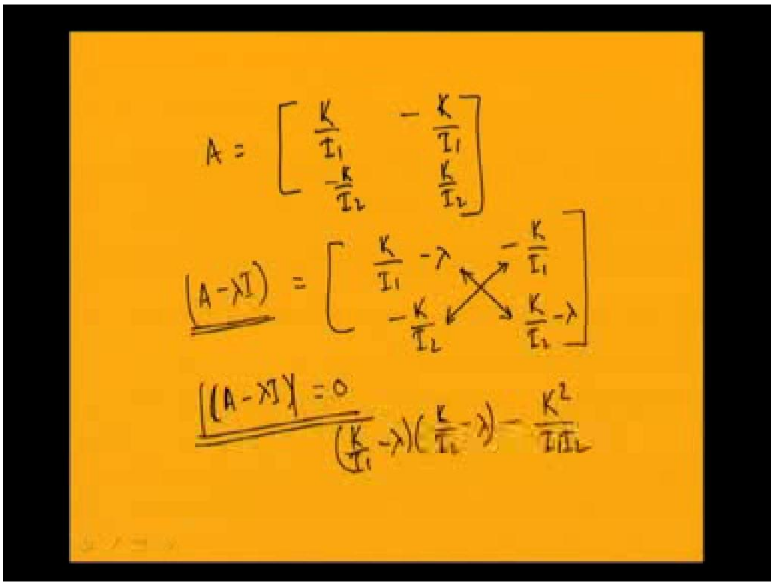

So, you can get it like this. So, it is equal to k by i 1 minus k by i 1. So, in this way you can write A minus lambda i matrix if you want to write for A matrix equal to. So, these will be equal to k by k 1 k by k 1 minus k by i 1. So, this is minus k by k 2 and k by i 2. So, if you want to write A minus lambda i matrix. So, A minus lambda i will be equal to k by i 1 minus lambda minus k by i 1 and minus k by i 2 this k by i 2 minus lambda. So, the diagonal elements are subtracted by this lambda and you got this A minus lambda i vector.

So, A minus lambda i matrix you got. So, after finding this A minus lambda i so, if you want to find the Eigen values. So, you can find that thing by finding the fast the characteristic equation. So, this characteristic equation is equal to A minus lambda i determinant of A minus lambda i equal to 0. So, you substitute determinant of A minus lambda i equal to 0 that will give you the value of lambda and this x that is the Eigen vectors you can find by finding the adjoint matrix of A minus lambda I.

So, by find this determinant of A minus lambda i equal to 0. So, this equation will becomes k by i 1 minus lambda into k by i 2 minus lambda minus these into these minus these into these. So, in this way you can find A minus lambda i determinant of m minus lambda i. You will give you, so you will get this equation k by i 1 minus lambda into k by i 2 minus lambda minus so minus minus and then 1 minus.

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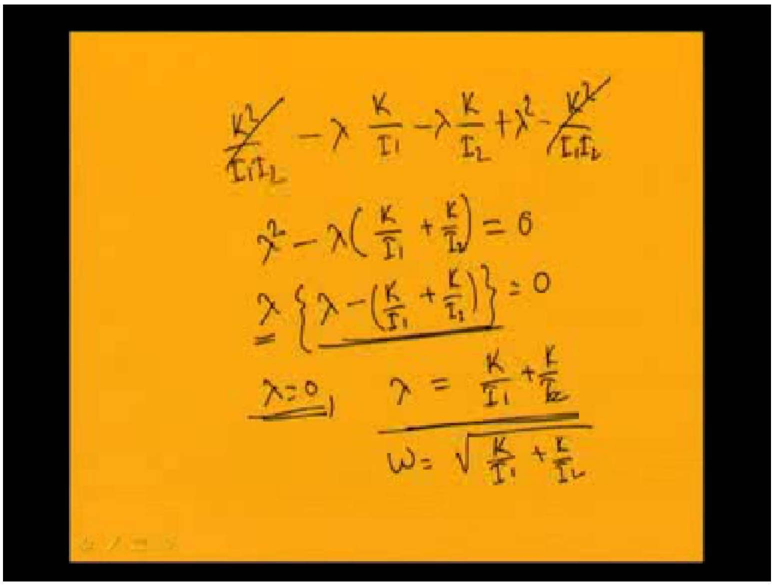

So, this becomes k square by i 1 i 2. So, you multiply this. So, this is k k square by i 1 i 2. So, this equation you can write in this way. So, it will becomes k square by i 1 i 2 in these look here this multiplied by this m minus lambda into k by i 1 minus lambda into k by i 1 again from these minus lambda into k by i 2 minus lambda into k by i 2 and then next are these lambda square. So, plus lambda square plus lambda square minus k square by i 1 i 2.

So, these cross and this minus cancel. So, you can take so this is minus lambda k by i 1. This equation becomes lambda square minus lambda equal to take come on this is k by i 1 plus k by i 2.

So, the characteristic equation reduces to lambda square minus lambda into k by i 1 plus k by i 2 equal to 0. So, for lambda into lambda if will take lambda come on. So, these comes k by i 1 plus k by i 2. So, lambda into lambda these equal to 0. So, lambda equal to 0 either lambda equal to 0 or this part equal to 0 or this part equal to 0.

So, either lambda equal to 0 or lambda equal to k by i 1 plus k by i 2 already you know in the previous class that this degenerate system in which one of the natural frequency equal to 0 which you have obtain here and the second natural frequency you are getting that is equal to k by i 1 plus k by i 2. So, this is lambda. So, frequency will be route over these. So, these will be equal to k by i 1 plus k by i 2.

So, your Eigen values are 0 and k by i 1 plus k by i 2 and now you have to find the Eigen vector. So, you can find Eigen vector in many different ways, but just now we have we know that we can find it by finding the adjoint of a minus lambda I.

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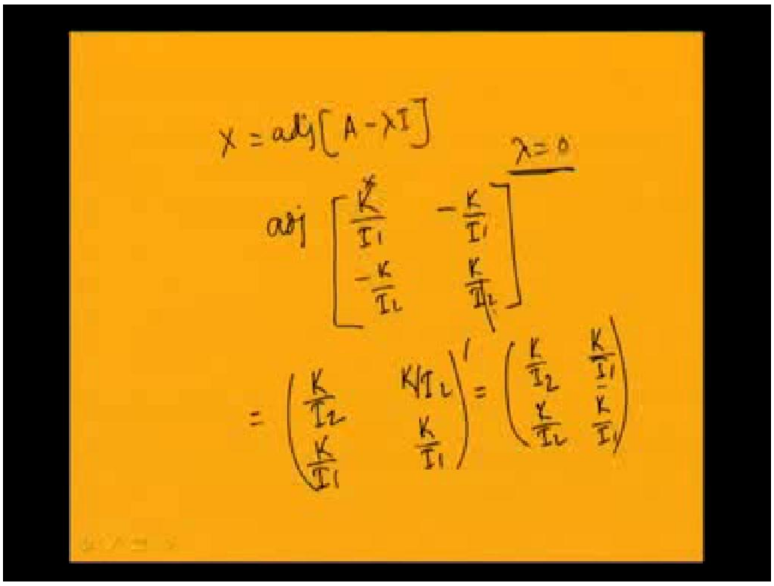

So, we know our x will be equal to adjoint A minus lambda i already we got the value of lambda. So, one value of lambda i equal to 0 by substituting this value. So, let us find it will be equal to when lambda equal to 0 adjoint a minus lambda l. So, I can write this equal to adjoint A minus lambda l already we know, but is A minus lambda i. So, this is equal to k by i 1 minus lambda. So, lambda i put equal to 0.

So, this is 0 and this is 0. So, it will be k by i 1 minus k by i 1. So, I can write in these ways. So, this will be minus. k by i 1 k by i 1 minus k by i 1 minus k by i 2 k by i 2. So, I have to find the adjoint. So, as lambda equal to 0 and to find the adjoint of these matrix. So, adjoint of this matrix equal to. So, I can write as the cofactor matrix and transpose of the cofactor matrix will be the adjoint of this matrix.

So, for finding the cofactor of this matrix for this position you can find the cofactor by suppressing these and these. So, it becomes k by i 2. So, it becomes k by i 2. So, for this position it will be negative of this row and this row you suppress. So, these becomes minus minus minus plus. So, this becomes k by i 2 and for these position for these position you can find.

So, these will become minus k by i 1. So, this becomes k by i 1 and for the last position it will be k by i 1. So, these will be the transpose of this matrix equal to transpose of these matrix. So, it will becomes k by i, k by i 2 these becomes k by i 2 k by i 1 and k by i 2 and k by i 1. So, you just note from these that the first column contains k by i 2 and k by i 2 and in the second column it contains k by i 1 and k by i 1. So, if I normalize or by making the second.

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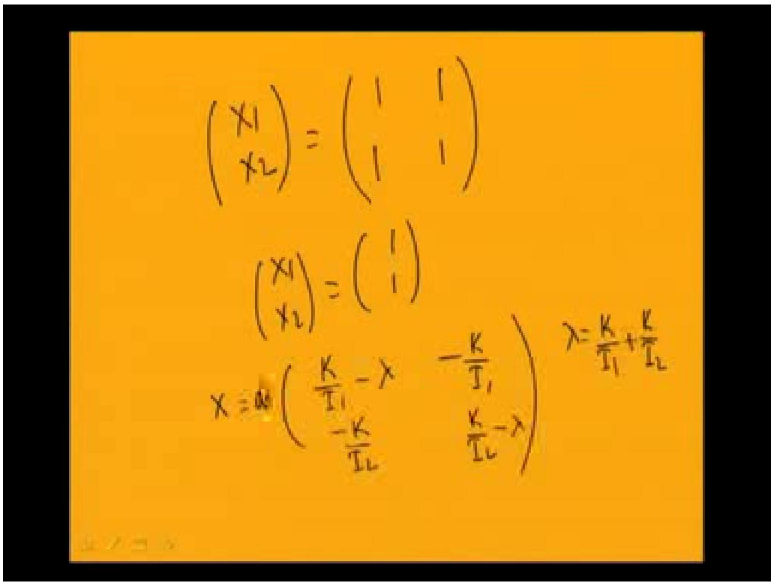

So, this x is nothing but you are theta 1 and theta 2 that is the amplitude of theta 1 and theta 2. So, this amplitude theta 1 and theta 2 or x 1 and x 2, I have written for that. So, this becomes x 1 and x 2. So, either you take the first column or second column, if you are taking the first column or second column you can absorb that by keeping this x 2 equal to one. So, this will deduce as to 1 1 1 1.

So, I am normalizing this with respect to x 2 or I am keeping this k 2 k by i 2 equal to 1. So, by keeping this I can write these adjoint A minus lambda i equal to 1 1. So, this is the adjoint A minus lambda i. So, your x 1 and x 2 you can obtain by taking any column of this adjoint matrix. So, you can absorb that both columns the normalize value of both columns as same.

So, you take any column to find at find the normal modes of the system or find the Eigen vector of the system. So, Eigen vector becomes 1 1 similarly you can find for the other case. So, for the other case also you can find the normal mode and in that case you can find the normal mode. So, it will be it will become your x will be equal to k by i 1 minus lambda minus k by i 1 minus k by i 2 and these becomes k by i 2 minus lambda. So, here I have to substitute lambda equal to. So, for the second mode.

So, you have taken that this lambda equal to k by i 1 plus k by i 2. So, I can write this k by i 1 plus k by i 2. So, in this case if I am substitute this thing. So, k by i 1 minus k by i 1 will cancel. So, this becomes minus k by i 2. So, this x becomes you have to find the adjoint of this matrix.

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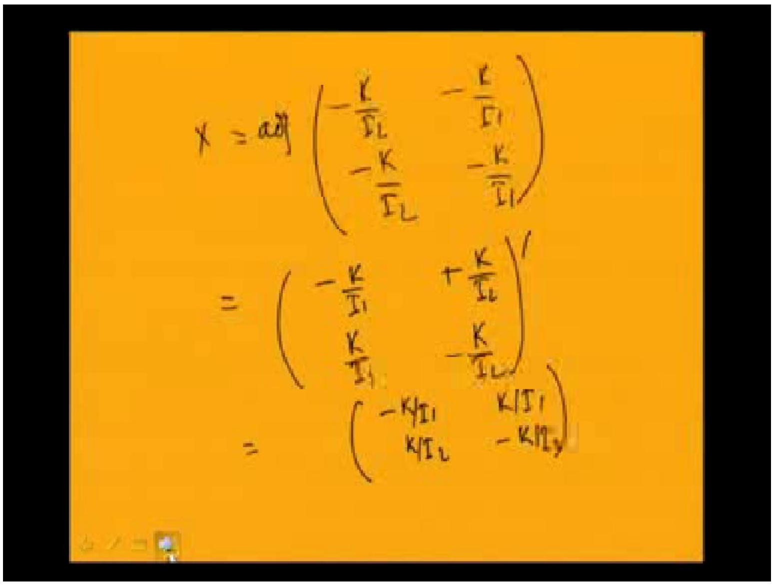

So, x equal to adjoint of minus k by i 2 and these becomes minus k by i 2 and this is minus k by i 1. So, this is minus k by i 1 and this position it becomes minus k by i 2 minus k by i 2 and in the last position these becomes k by i 2 minus lambda. So, lambda m putting k by i 1 plus k by i 2. So, this k by i 2 k by i 2 cancel minus k by i 1. So, this becomes minus k by i one.

So, you have to find the adjoint of this matrix. Already I told you have to find these adjoint of this matrix. So, at for finding adjoint of a matrix you have to find the cofactor matrix and transpose of that cofactor matrix will be to the adjoint of that matrix. So, by finding that thing you can write in it these one. So, it will become these will the cofactor matrix becomes cofactor matrix you just find the cofactor matrix.

So, that will become minus k by i 1 and for these position these becomes minus k by i 2. These becomes plus and for these for these position for these becomes k by i 1 plus and for these position it becomes k by minus k by i 2. So, minus k by i 2 transpose of this matrix.

So, transpose of these matrix will become so these will nothing,, but these is minus k by i 1 and these becomes k by i 2 and it will becomes k by i 1 and minus k by i 2. So, in this case also you just absorb that both columns if you normalize that thing will be same. So, you take any of the column and you normalize that thing.

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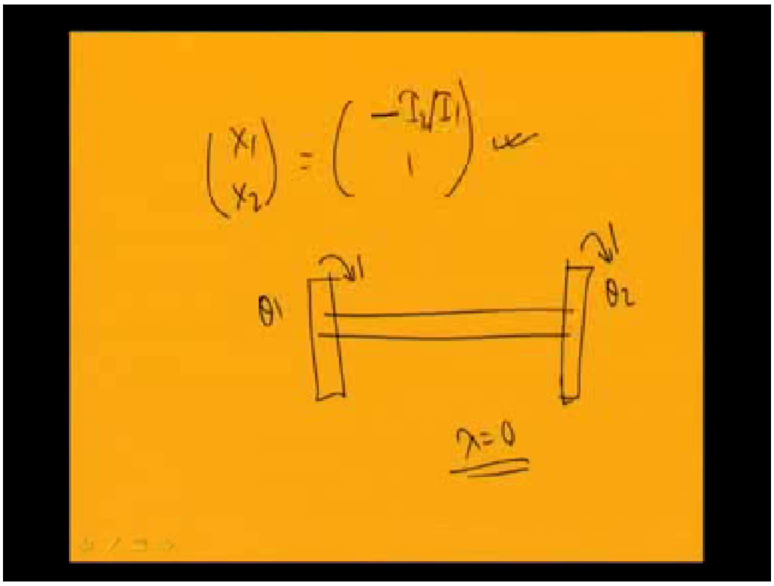

So, in this case you will get equal to your x 1 x 2 will become minus i 2 by i 1. So, in this case you must absorb that this amplitude ratio or inverse of the inertia ratio. So, if you low the inertia of the system in that case you can find the of the amplitude of the system by using this relation. So, from these two rotor system you have absorb that if have a two rotor system.

So, in the first mode that is the rigid body that lambda equal to 0. So, you have absorb that these x 1 x 2 equal to 1 1. So, in that case it becomes both have rotating with same amplitude. So, when it is rotating with type equal to 1. So, this side also rotate with same theta is equal to 1 and in the second mode and in the second mode when this the rotating in this direction in clock wise direction this will rotate the anti clock wise direction.

For that begin you this minus sign and this theta 1 by theta 2 will be theta 1 by theta 2 will becomes negative of these I 2 by I 1. So, in this way you can find by using the normal mode the mode sets of the system. So, by using this adjoint matrix method also you can find the Eigen vectors of those systems. So, let us see another method to determine the equation motion of the multi degrees of freedom system. So, that is the influence coefficient matrix method.

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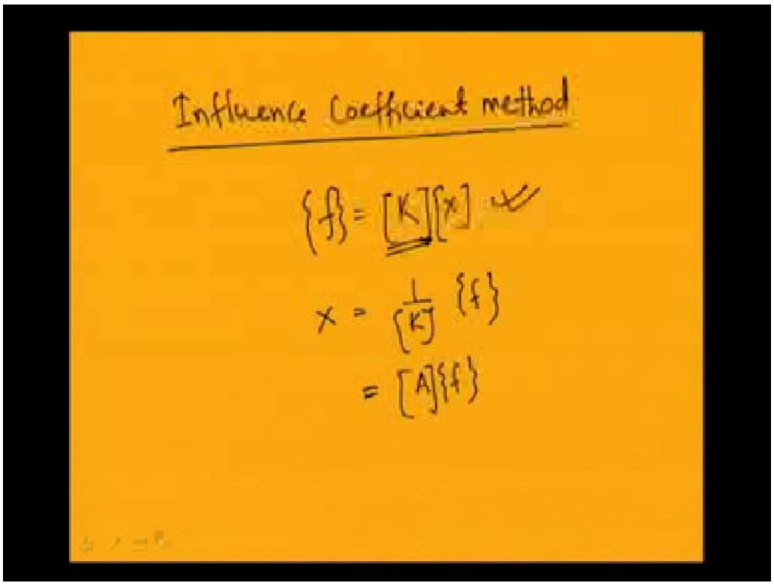

Influence coefficient method this is very powerful method to find the equation motion of multi degrees of freedom systems. So, before going for that we should see different definitions. So, what is the influence coefficient, displacement influence coefficient or stiffness influence coefficient that in first will see and then you will this influence coefficient to derive the equation motion of a system.

So, you know the force of a system if you may writing this force equal to if are neglecting the inertia of the system you can write the equation motion in this for f will equal to the force vector will be equal to your stiffness matrix into these displacement vector. So, if are writing in this place these stiffness formulation method or you may write this equation x equal to 1 by k 1 by k into f.

So, here 1 by k I can write equal to A. So, these will be equal to A f. So, this A matrix is known as the displacement influence matrix of the system and the coefficient of this matrix are known as the coefficients of the influence coefficient matrix, coefficient of the displacement influence coefficient matrix. Similarly the coefficient of this k matrix is known as the influence coefficient of the stiffness matrix. So, first let us find this is defined this influence flexibility coefficient or the elements of this matrix A.

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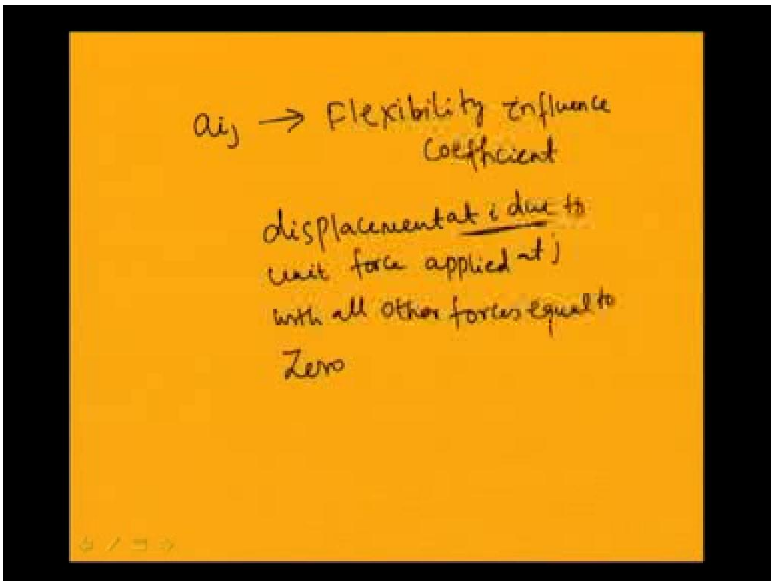

So, A i j that is flexibility influence coefficient, this is known as flexibility influence coefficient. So, A i j is known as the flexibility influence coefficient and it is defined as it is the displacement at i. So, it is defined as the displacement at i due to unit force applied at applied at j with all other forces equal to 0.

This flexibility influence coefficient A i j is defined as the displacement at i due to unit force applied at j by other force are 0. So, we can take a simple example of a spring.

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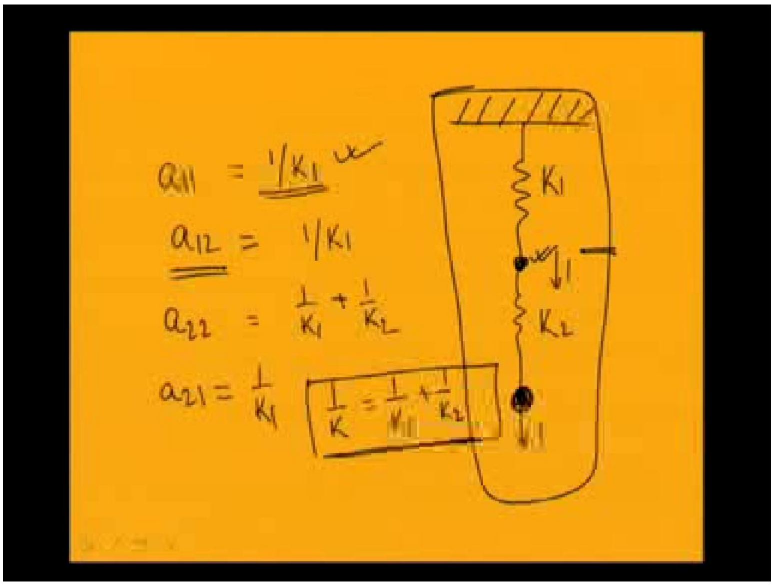

So, let us take two springs. This is one spring and this is the point square it is connected to another spring. So, we have let us find the influence coefficient of the system. So, let k 1 if the stiffness of these first spring and k 2 is the stiffness of the second spring. So, in these case the influence coefficient A 1 1 that is displacement at 1 due to a unit force applied at 1 when the other force is are 0, that means when a unit force is applied at this position, but there is no force at this position.

So, what is the displacement at this point? So, in that case this is a simple spring we are unit force is acting on the string. The displacement will be equal to force by the stiffness of the system. So, by this A 1 1 will becomes 1 by k 1 now, A 1 2 equal to displacement at 1 when we are applying a unit force are 2. So, a unit force is applied here. So, if I am applying a unit force at this position though I have to find the displacement at this position.

So, these will give if this A 1 2. When if unit force is applied at this position these two springs are in these two springs are in series then the equivalence stiffness equal to 1 by equal to 1 by k 1 plus 1 by k 2. So, the displacement here at this position will becomes.

So, displacement at this position we have to find these is the displacement at this position. So, this is you A 2 2 equal to 1 by this is equivalence stiffness of the system. So, A 2 2 becomes 1 by k 1 plus 1 by k 2 and we have to find A 1 2 and A 2 1.

So, when a unit force is applied at this position the displacement of these equal to 1 by k 1 and the displacement as your not applying any force at this position. So, this point will have a rigid body motion. So, this rigid body motion this will be equal to the displacement at this position. So, A 2 1 that is displacement are to with a unit force at 1 will becomes same as A 1 1 that is equal to 1 by k 1.

Similarly, when we are applying a force to find these A 1 2 we have to find, but the displacement at this position on due to the force at this position. So, when we are applying a force at this position to the displacement will becomes equal to 1 by K 1.

So, the same force will act at this position. So, at the same force is acting at this position as they are in series. So, this displacement will be equal to 1 by k 1. So, this A 1 2 becomes 1 by k 1. So, let me explain it again. So, let us take the system. So, in the system I am taking these two springs.

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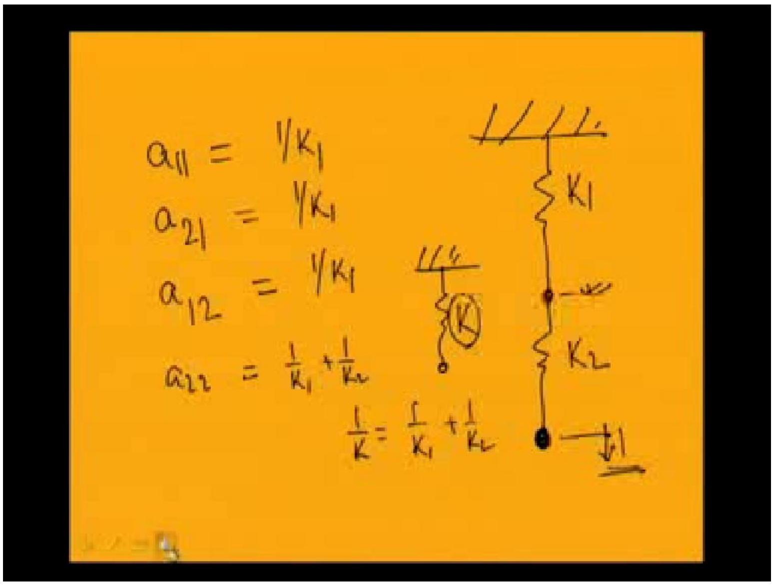

So, in the system I am taking these two springs in these two springs, the spring constant is k 1 here, spring is k 2 here. To find A 1 1 you have apply a unit force, here apply unit force at this position. So, if are when you are applying a unit force at this position, then the displacement of these will be equal to force by this stiffness is equal to 1 by k 1 and due to this displacement as there is no force acting on these position 2, the position 2 also will have the same displacement.

So, you can write a displacement at 2 due to force at 1. So, A 2 1 is equal to 1 by k 1 now let me applied force at this position. So, this force unit force I have applied at this position. So, when I am applying a unit force at this position, displacement at this will we equal to the force applied at this by stiffness at this position or the two springs have in series same force will have at this position also. So, a unit force 1 by k 1 will be the displacement at this position due to a force unit force at position two.

So, these becomes 1 by k 1 and A 2 2 will become when the force is acting at this position. So, this will be equivalent to a single spring with single spring with equivalence stiffness k. So, this equivalence stiffness you know it is equal to 1 by k equal to 1 by k 1 plus 1 by k 2. So, you can find this k and these 1 by k if the displacement at this position.

So, this equal to 1 by k 1 plus 1 by k 2 in this way, you can find the influence coefficient or flexibility influence coefficient of the system. So, the flexibility influence coefficient matrix becomes so 1 by k 1 1 by k 1 then 1 by k 1 k 1 plus k 2 by k 1 k 2. So, I have added this two when you are adding these two 1 by k 1 plus 1 by k 2.

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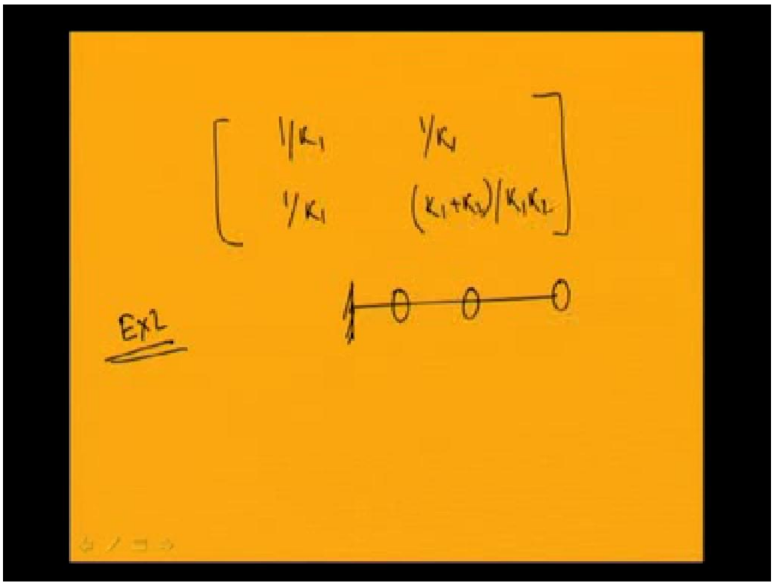

These become k 1 plus k 2 by k 1 k 2. So, in this way you can find this influence coefficient of the system simple system. So, let me take another example to find the influence coefficient. So, let me take a simple contileaver beam. So, in these contileaver beam let 3 masses attach to this contileaver beam. So, in this case I have to find the influence coefficient.

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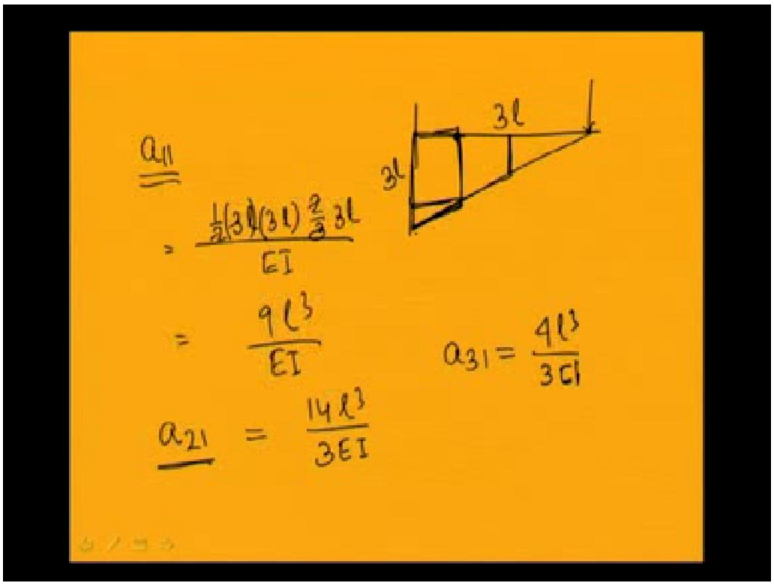

So, influence coefficient A 1 1 let me find A 1 1 to find A 1 1. So, I have to put let me position this this is position 1, this is position 2 and this is position 3,

So, in if have to find this A 1 1 1, have to apply unit force at 1 and find the displacement at one if will apply a unit force at one. So, to find the displacement at ,1 I mean draw this vending venn diagram. So, from the vending venn diagram area of the vending venn diagram I can find the displacement though the area of vendingvenn diagram let been draw.

So, the vending venn moment will becomes the vending venn this becomes vending venn diagram the unit force is acting on the system. So, the length equal to let me take equal length. So, this l l and l. So, this length becomes 3 l. So, this is 3 l as I am applying a unit force this vending moment here the position also becomes 3 l. So, this vending moment is 3 l and length is also 3 l and I have to find the displacement at these by using this area moment method.

So, by area moment method I can find the displacement at this position. So, this is equal to the moment of the area of this by e i. So, moment of the area this area become. So, this becomes half. So, this becomes half 3 l into 3 l. So, this is 3 l into 3 l is the area and I to take the moment. So, the moment the center of this at position here it is a two-third of this.

So, I have to write this becomes A 1 1 becomes two-third its length equal to two-third of it is at distance two-third of 3 l by e i. So, this becomes 2 2 cancel 3 3 cancel. So, this becomes 3 into 3 nine. So, these becomes nine l l l l nine l cube by nine l cube by e i.

Similarly, you can find A 2 1 so A 2 1 is nothing, but this the displacement at position 2 due to a unit force acting at 1 when other forces are 0 . So, in that case the vending moment area diagram will be these we have applied a force here and this is the vending moment area diagram. So, from these you can find the A 2 1.

So, A 2 1 becomes the you can find it is equal to 14 l cube by 14 l cube by 3 e i. So, similarly you can find A 3 1 for finding A 3 1. So, you have finding the displacement at this position.

So, these is the position 3 are taking. So, you finding the displacement at this position and for this position these let me write these. So, for this area you for this area you can find the area of this you can find in many different ways you can find. So, let you draw. So, this is a rectangle for this triangle if an take and in this way you can find A 3 1 you can find similarly it will becomes 4 l cube by 3 e i 3 e i.

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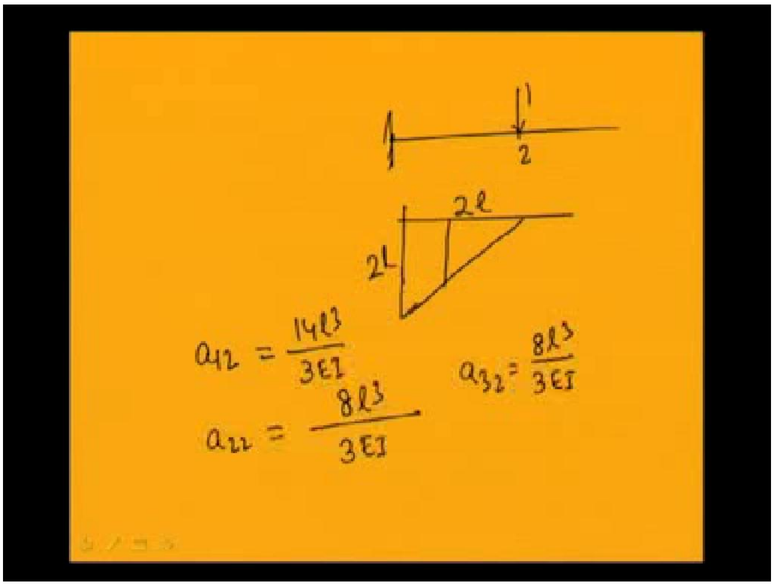

Similarly, you can find when where applying a load at position to you can apply a load at position, to you can draw the vending moment diagram. So, in this case the vending moment diagram will be this will be the vending moment diagram.

So, this vending venn diagram. So, this length is 2 l. So, this becomes 2 l find the area. At different position when you are finding a different position if can finding these from these area moment diagram. So, in this case you can find A 1 2 these is displacement at 1 due to load at two. So, at two position you are applying at two position you are applied a unit force.

So, you can find A 1 2. So, a 1 2 you can find equal to 14 l cube by 3 e i. Similarly, you can find A 2 2, in this case A 2 2 will give these area and by e i moment of this area by e i. So, that thing you can find it equal to A 8 l cube by 3 e i and similarly you can find A 3 2. So, these is equal to 8 l cube by 3 e i.

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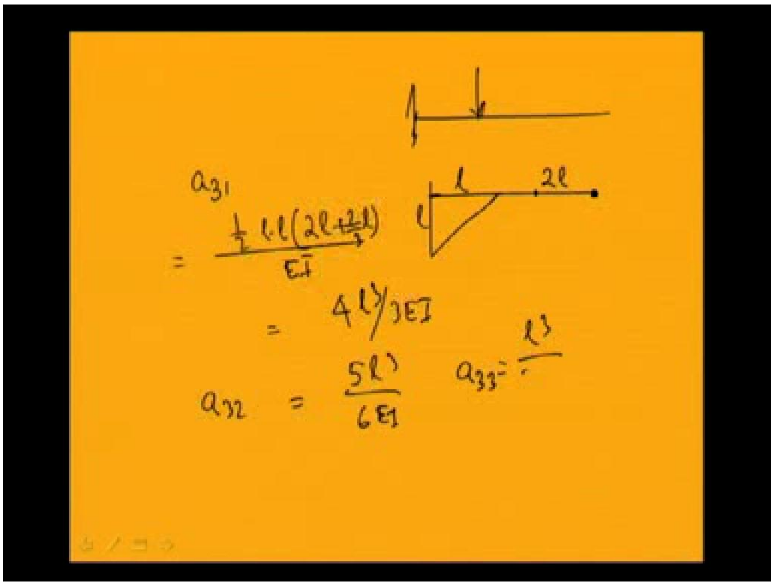

Now, for the last case you can apply unit force at position 1, position 3 and you can find A 3 1. So, if you draw the vending moment diagram. So, A 3 1 will be comes equal to half also this triangle. So, this is l vending moment will be l. So, this becomes l l into 2 l plus this length is 2 l. So, these 2 l plus it will the moment.

So, you have to take the moment as you are finding from these position. So, you will take it is equal to 2 l plus 2 by 3 l and by e i. So, these becomes 4 l cube by 3 i and similarly you can find A 3 2. So, from this triangle you can find for this position.

So, these will become 5 l cube by 6 e i and finally you can find A 3 3 this is equal to l cube by 3 e i. So, in this way can determine this influence coefficient matrix A. So, this will be equal to l cube by 3 e i.

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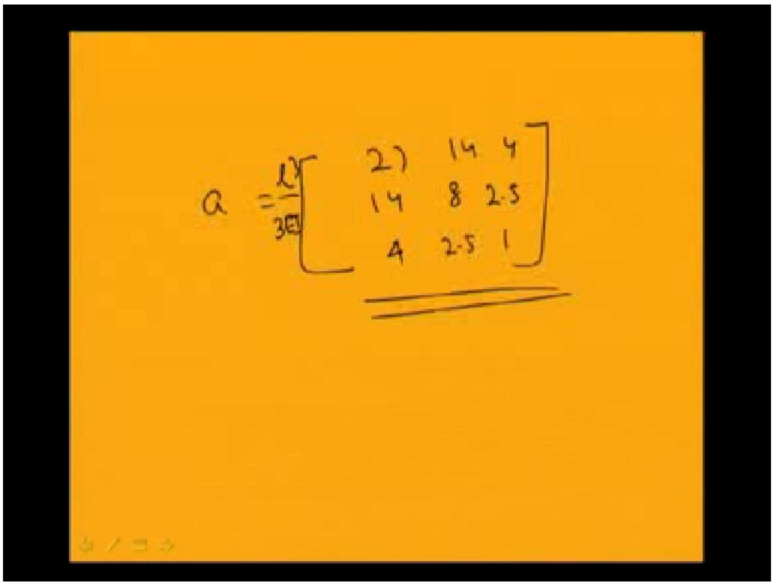

So, these becomes 27 14 4 14 8 2.5 and 4 2.5 1. So, in this way you can find the flexibility influence coefficient matrix of a system. So, today class we have studied about three degrees of freedom system, we have found the Eigen value and Eigen vector I told that is Eigen vector can also be determine from the adjoint matrix. So, for a given system m k you can find first the A matrix.

So, A matrix will be equal to m inverse k. So, have to finding this a matrix you can find the Eigen value and Eigen vector and you can find this Eigen vector by using in this adjoint of A minus lambda i and you can find these Eigen value by finding the determinant A minus lambda i equal to 0.

From this characteristic equation, you can find the Eigen value and after getting this Eigen value and Eigen vector you may find the free vibration response if the initial conditions are known and then I told you I have another method of determining the equation motion. So, that this influence coefficient method. So, in this influence coefficient method putting only we have found are we know how to determine this flexibility influence coefficient.

Next class we will see this reciprocity theorem and will find these influence coefficient for the stiffness matrix and will determine the determine the response of the system free vibration response of the system using this influence coefficient method also will study about the modal matrix method to determinant the response of the system.

So, will determine the free and force response of these multi degrees of freedom system by using this modal analysis method and then will see some numerical methods are some approximate methods to determine the natural frequency of a system when it is more than when the coordinates, the minimum coordinates is more than two or for a multi degrees of freedom systems. Later, will see numerical approximate methods to find the natural frequency.