Mechanical Vibrations
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Module – 6
Vibration Absorber
Lecture - 2
Tuned and Damped Absorber,
Untuned Viscous Damper

Last class we have studied about this tune vibration absorber in which I told you about this primary system is subjected to some vibration. So, if the primary system is subjected to a forcing of F sign omega t.

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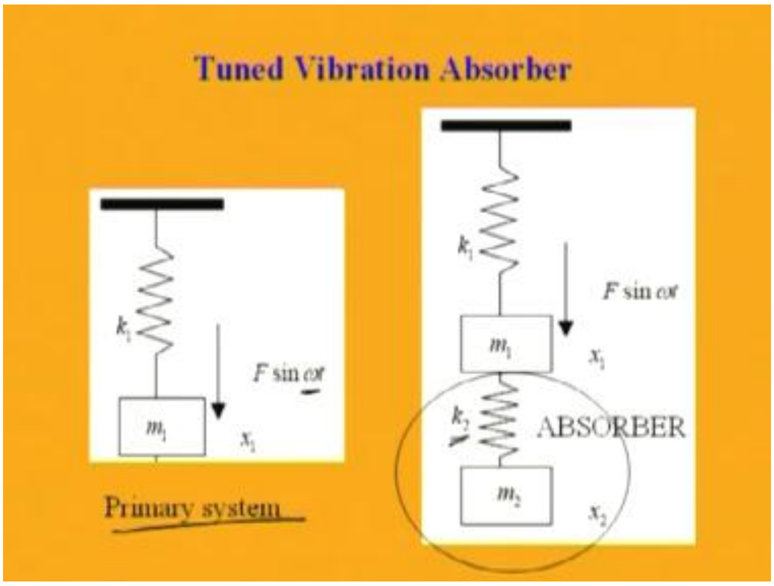

Then, you can absorb that vibration by appending another secondary system to the primary system. So, the secondary system has a stiffness of k 2 spring stiffness of k 2 and mass m 2, and you can choose this k 2 and m 2 in such way that this omega, the frequency of external excitation will be equal to root over k 2 by m 2. So, in that way you can eliminate the vibration of the primary system completely. But this will be applicable only for a particular frequency omega. So, when you are changing the frequency of the system or when the disturbing frequency of the system primary system is changed. So, you have to change another spring and mass or another absorber to absorb that vibration. So, this type of vibration absorber is applicable for a particular frequency that is why it is known as tuned vibration absorber. Already you know that in case of IC engine in automobiles or some rotating systems.

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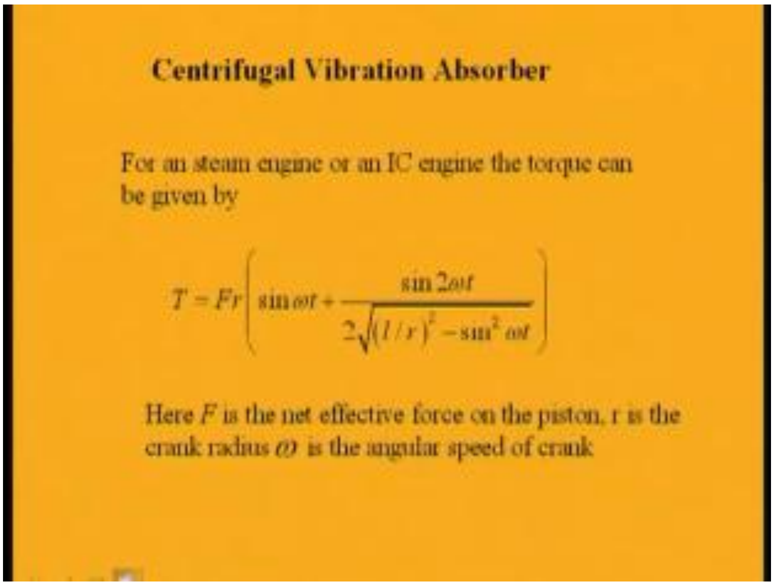

So, in those systems there are inherent unbalance forces in the system.

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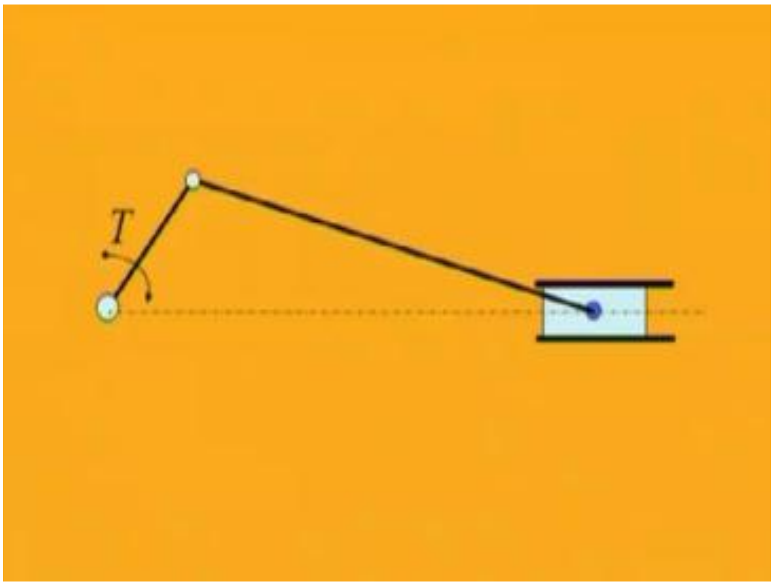

So, this is the schematic diagram of a slider crank mechanism usually used IC engine or reciprocating engines. So, in that case due to the unbalance forces or due to the forces of the piston. So, there will be an unbalance force always there is a unbalance force which cause the shaking moment, or shaking force which is also known as shaking force at the main bearing or at the crank shaft. So, due this force or this unbalance force which is a function of the rotating speed of this. So, this unbalance force is a function of the rotating speed. So, to avoid that frequency to avoid that resonance condition or to avoid that vibration of this crank shaft or rotational vibration of this crank shaft you require n number of spring and damper to eliminate that vibration. So, to avoid that thing centrifugal type of vibration absorber can be used.

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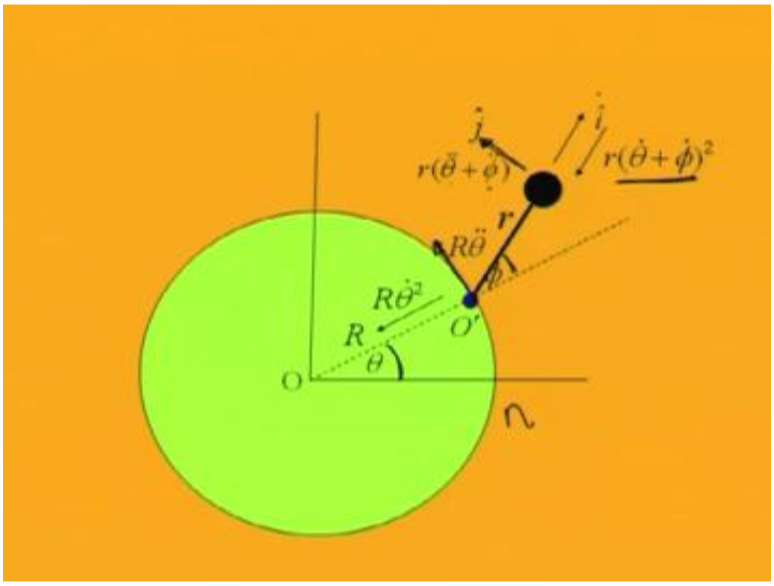

So, in case of a centrifugal vibration absorber; so this is the figure of a centrifugal vibration absorber. So, the disc which is mounted on the crank shaft on that you can attach its pendulum. So, this pendulum when this disc is rotating this pendulum will be subjected to a centrifugal force. So, it can show later that; this is the frequency of oscillation or the natural frequency of the centrifugal. This centrifugal pendulum can be written in the form of the, or will be a function of the natural frequency of this disc.

So, let n is the frequency of the oscillation frequency of oscillation of the disc. So, in this case the acceleration of point; so point on this mass can be written in terms of 2 components. So, one will be the centripetal component, other will be the radial component. So, the centripetal component will be r theta dot plus phi dot square and it can be noted that this theta. Let at time t the rotation of the disc is represented by theta. So, this pendulum has rotated by an angle phi. So, the centrifugal or the acceleration of this point is the acceleration of point O dash plus the acceleration relative acceleration of this point with respect to O dash. The acceleration of point O dash as it is on this rotating disc can be written by 2 components. One is R theta dot square which is towards the center of this and another component is R theta double dot. So, this is R theta double dot component and this is this component towards the centre is R theta dot square.

Similarly, the relative acceleration of this point that is the centre of the pendulum to this O dash point at which it is hinged can be written in 2 components. So, one component will be the will be towards the center of this. So, these components equal to r theta dot plus phi dot Whole Square and another component which is radial to it or which in the tangential direction. So, this is the tangential direction component. So, that thing will be equal to r theta double dot plus phi double dot. It can be noted that as this is rotating with theta and this is rotating with phi. So, this point will have an angular velocity of theta dot plus phi dot and hence the tangent the centrifugal component or centripetal component is R theta dot plus phi dot whole square that is r into angular velocity square and the radial at the tangential component.

So, the tangential component is r into alpha that is the angular acceleration. Here the angular acceleration at this point equal to theta double dot plus phi double dot. So, the tangential component equal to R theta double dot plus phi double dot and the centripetal acceleration term is equal to r into theta dot plus phi double dot whole square. Similarly, for point O dash, we have already found that the centripetal component or this normal component is r theta dot square and this tangential component is equal to r theta double dot. So, now the total acceleration at this point that is the centre of this pendulum will be equal to the acceleration at O dash plus the acceleration of this point with respect to this point.

So, we can divide this or we can choose a coordinate system in such way that. So, let this is the i direction that is along the length of the pendulum. So, this is i direction and perpendicular to this is j direction. So, we can write all the acceleration in terms of i component and j component. So, i component acceleration will be. So, this i component acceleration and the acceleration at this point can be divided into 2 component that is along this direction that is i direction and j direction. So, already we know along this direction the acceleration is R theta dot square. So, R theta dot square can be divided into this along this direction i direction and j direction as it makes an angle theta. So, this component will be R theta dot square cos phi and the other component will be R theta dot square sin phi. Similarly, this R theta double dot can be divided into 2 components.

So, you may note that this R theta double dot makes an angle 90 minus phi with this length of the pendulum. Or along the i direction it makes an angle of 90 minus phi with the i direction that is why the component along. So, R theta double dot component along i direction will be equal to R theta double dot cos 90 minus phi or it will be equal to R theta double dot sin phi. Similarly, its component along the j direction will be equal to R theta double dot sin nine. So, it will be sin 90 minus phi j. So, total acceleration can be written in I direction and j direction as this.

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$$a_m = \left[-R\dot{\theta}^2\cos\phi + R\ddot{\theta}\sin\phi - r(\dot{\theta}+\dot{\phi})^2\right]\hat{i} + \left[R\dot{\theta}^2\sin\phi + R\ddot{\theta}\cos\phi + r(\ddot{\theta}+\ddot{\phi})\right]\hat{j}$$

So, this will be equal to a m is the total acceleration of the mass of the pendulum. So, that can be written as minus R theta dot square cos phi plus R phi R theta double dot sin phi minus r theta dot plus phi dot square. So, this is the i component and the j component will be R theta dot square sin phi plus R theta double dot cos phi plus r theta double dot plus phi double dot. So, this is along the j direction.

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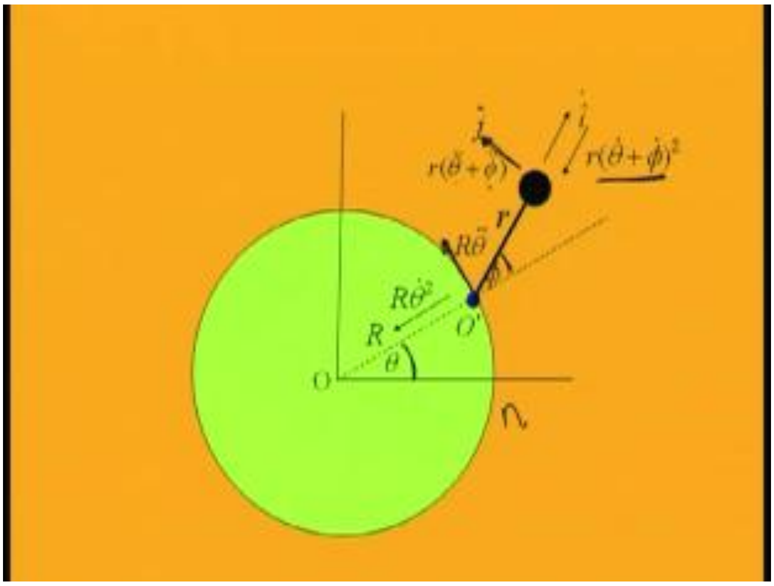

So, again you can visualize that thing. So, only we have to divide this R theta double dot acceleration along the along this i direction and j direction. So, you know that this R theta double dot acceleration makes an angle 90 minus phi with respect to this i direction that is why you can have this component. So, you can have R theta double dot.

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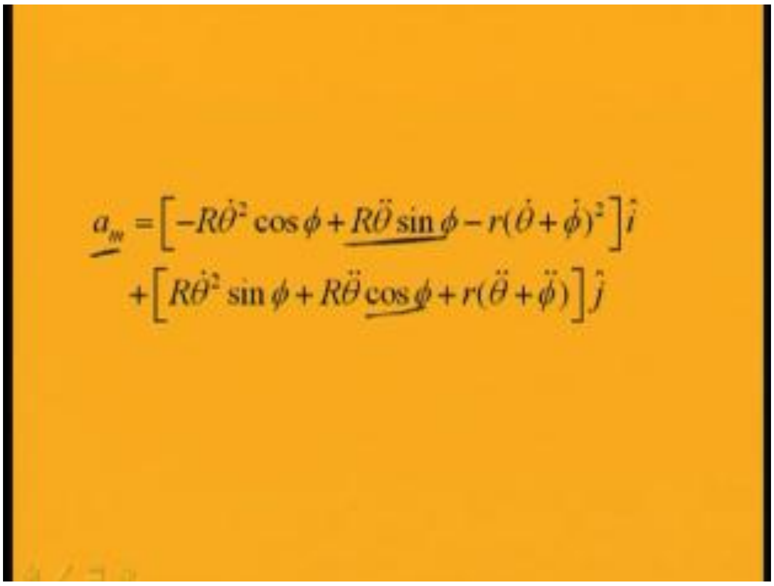

So, R theta, R theta double dot cos phi along j direction that is sin 90 minus phi that is cos phi and similarly, this R theta double dot sin phi along i direction. And, you have to divide this R theta dot square, so this along i and j direction. So, this is R theta this is the direction of R theta dot square. So, it makes an angle phi with respect to this i. So, this component will be R theta dot square cos phi along i direction and sin phi along j direction. So, this is minus R theta dot square cos phi and this is R theta dot square sin phi. So, in this way you can find acceleration of the mass of the simple pendulum.

So, in this pendulum we have assumed that length of the pendulum equal to R and this radius of the disc on which the pendulum is attached is capital R. And the disc is rotating with an angular speed of n and after getting this angular acceleration at this point. So, as you know that this point this pendulum is hinged at this point. So, we can find what is the moment acting on this pendulum or the torque exerted by this pendulum on this disc. So, before that the force we can calculate the inertia force of this pendulum will be equal to mass into acceleration of the pendulum. Already we have calculated this acceleration. So, mass into acceleration will give the inertia force or the force acting by this pendulum on this disc.

So, the force will be, so this force as it hinged at this point the as it is hinged at this point the i th component will be or the if I am taking this moment about this point. So, R into R into the j th component of the force will be equal to 0, because this i th component passes through this point, passes through this hinge point. The i th component of the force that is mass into acceleration is the force and its i th component as it passes through this point O dash. So, if I will write M O the moment about this point equal to 0. So, then the j th component of acceleration into mass into this R will be equal 0. So, I can as this point it is hinged. So, M O dash equal to 0.

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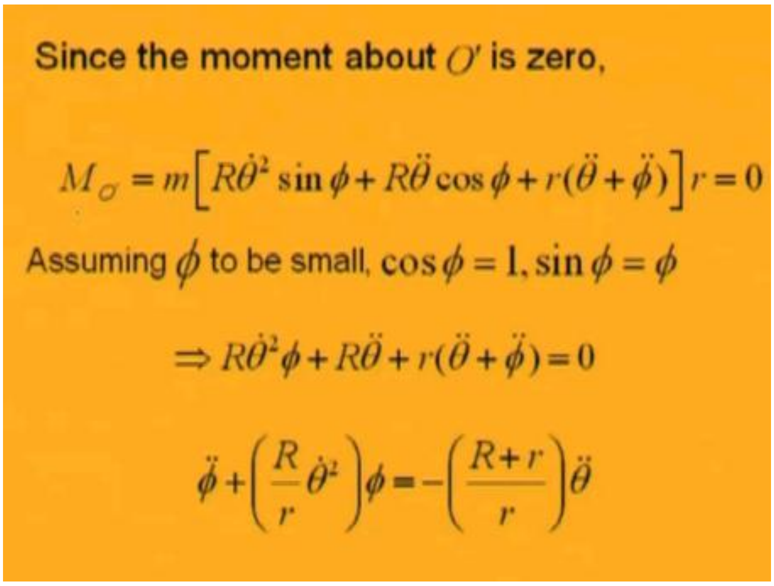

So, I can write mass into the j th component of the acceleration equal to 0 and I can assume this assume this displacement is very small displacement of this pendulum to be very small that is despite to be small. So, as phi is small then I can assume this cos phi equal to 1 and sin phi equal to phi.

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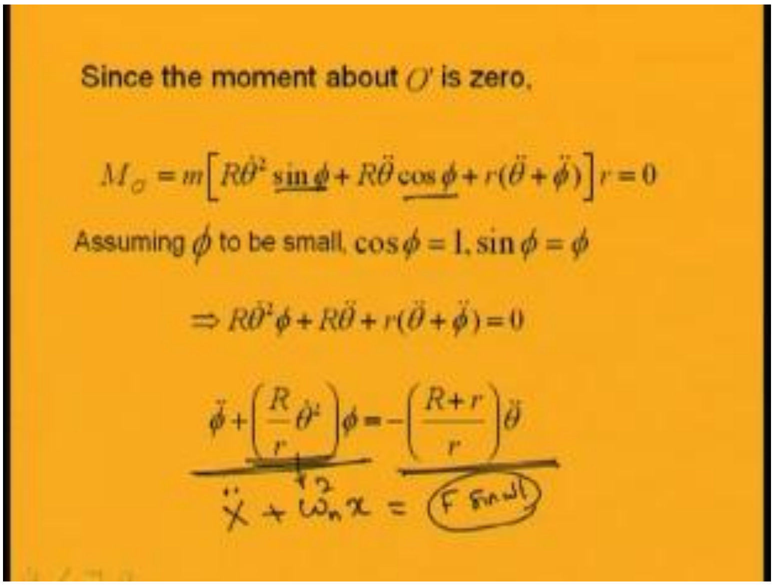

So, substituting this cos phi equal to 1 and sin phi equal to phi. So, in this expression that is M O dash equal to m R theta dot square sin phi plus R theta double dot cos phi plus r theta double dot plus phi double dot into r equal to 0. So, I can substitute this cos phi equal to 1 and sin phi equal to phi and it will leads to this R theta dot R theta dot square this sin phi equal to phi and here cos phi equal to 1. So, I can write this is equal to R theta dot square phi plus R theta double dot plus r theta double dot plus phi double dot equal to zero. So, if I will arrange this expression I can write this phi double dot plus R by r theta dot square phi equal to minus R plus r by r theta double dot.

So, these expressions you just note that this is the equation of a simple harmonic it is the equation in which. So, this expression you can write in this form. So, this expression you can write the previously written equations which you are familiar with. So, it can be in this form, X double dot plus omega n square x equal to F sin omega t or some forcing. So, this is the forcing part. So, this right side is the forcing part and this left side equivalent to phi double dot plus R by r theta dot square phi. So, this term this R by r theta dot square is the is equivalent to omega n square of the system. So, the natural frequency of the pendulum you can write in the form of R by r theta dot square. So, I can write the natural frequency of the system as R by r theta dot square.

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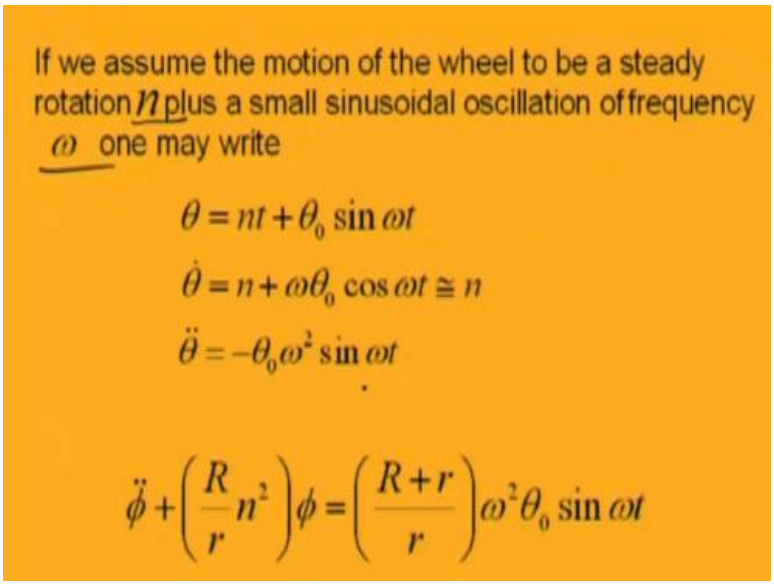

So, here I can assume this theta the rotation of the disc as a rotation steady rotation n plus the small sinusoidal oscillation with frequency omega and I can write this theta equal to. So, n t plus theta 0 sin omega t. So, theta dot will be equal to I can differentiate this expression, and I can write this theta dot equal to n plus omega theta 0 cos omega n t cos omega t. And as this product of these 2 small terms I can neglect this term and I can write this theta dot nearly equal to n. And theta double dot if I will differentiate this expression. So, I can write this theta double dot equal to minus theta 0 omega square cos theta. So, it will be sin omega t. So, theta double dot I can write in terms of minus theta 0 omega square sin omega t and theta equal to n t plus theta 0 sin omega t.

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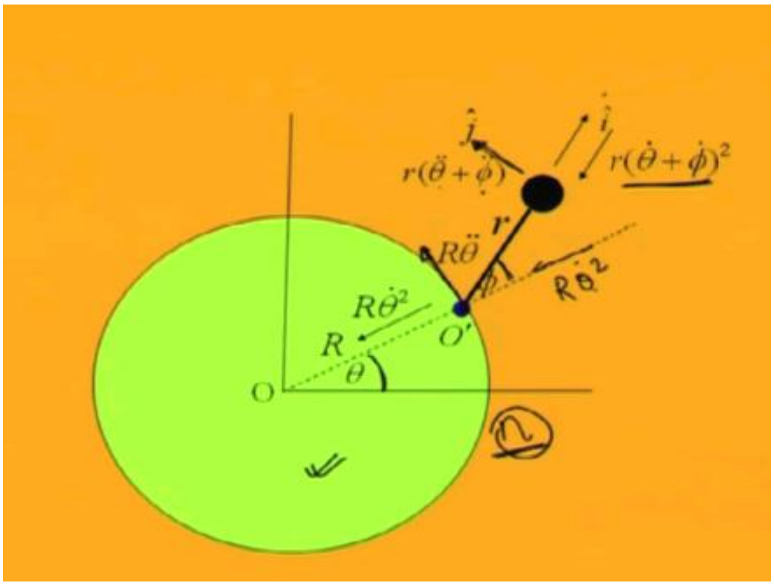

So, I am assuming the rotation of this wheel as a steady rotation n plus some small disturbance. So, that small disturbance I am writing with a frequency omega after writing that thing.

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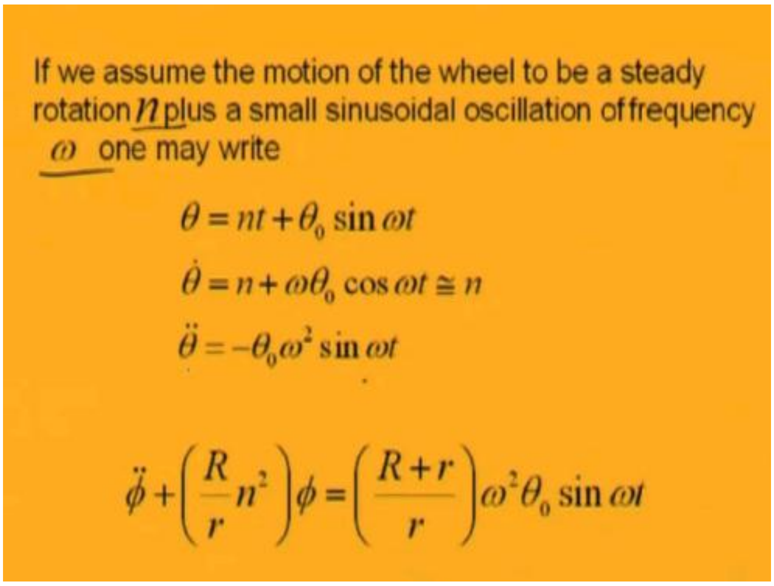

So, I can write this theta equal to n t plus theta 0 sin omega t. So, theta dot will be equal to n plus omega theta 0 cos omega t which is nearly equal to n and theta double dot I can write it equal minus theta 0 omega square sin omega t. So, if I will substitute this theta double dot expression in the previous equation.

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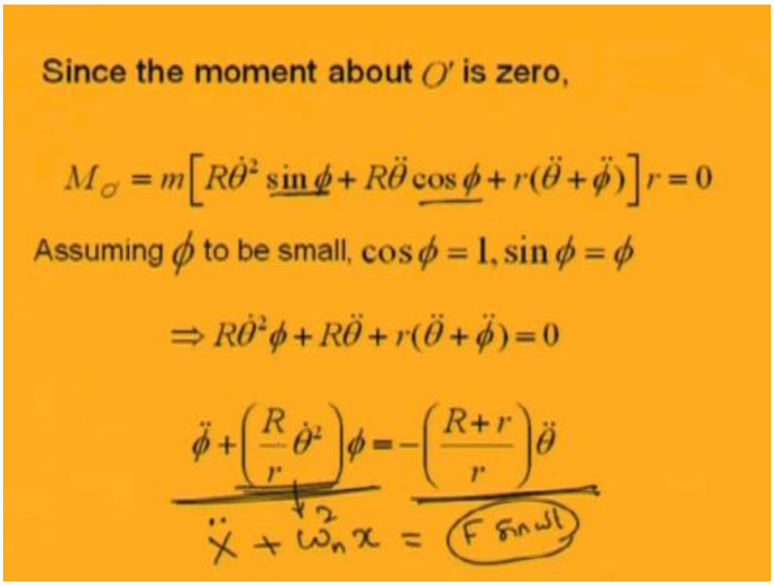

So, this equation will reduce to phi double dot plus R by r. So, this theta dot square I can replace it by n square. So, theta double dot plus R by r n square phi equal to Theta double dot plus R by r n square phi equal to R plus r by r omega square theta 0 sin omega t. So, this is the expression of an undamped or the system without damping of a single degree of freedom system without damping, which you have studied in your vibration of single degree of freedom system.

So, in this case this R by r n square is the natural frequency of the pendulum. So, here you can note this omega n square that is the natural frequency of the pendulum is directly propositional to the rotation of the disc that is n. So, omega n square equal to R by r n square. So, the natural frequency of the secondary system you are attaching to the disc is proportional to the rotational speed of the disc. So, as you can vary the rotational speed of the disc. So, the natural frequency of the pendulum automatically changes and that will take care the vibration of the disc. So, in that way you can eliminate the vibration of the disc.

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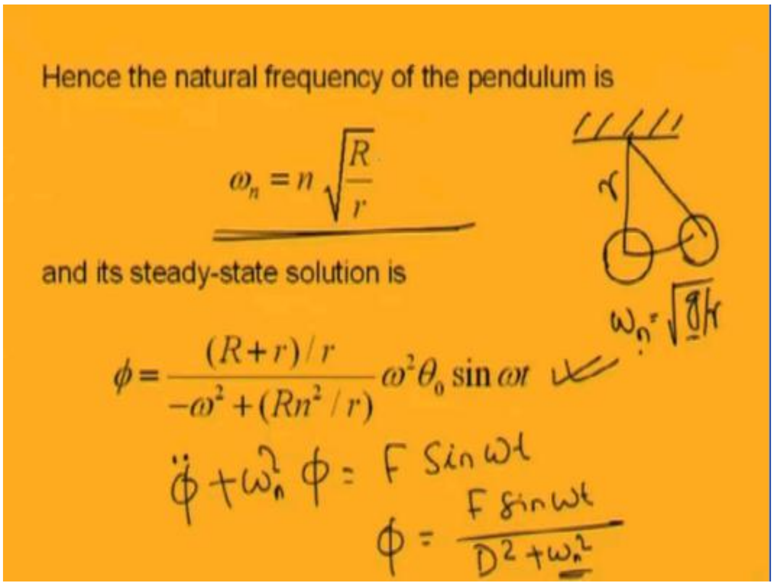

So, in this system we have seen that this omega n equal to, so if I will simplify this expression or if I will write this omega n square equal to R by r n square, I can write this omega n equal to n root over R by r. So, the steady state solution for this system, so this equation is phi double dot plus omega n square phi equal to R by R plus r by r. So, this is equal to your F force, I can write this is equal to this whole term equal to force. So, this is F sin omega t. So, your equation is phi double dot plus omega n square phi. So, if the equation is phi double dot plus omega n square phi equal to F sin omega t. So, the particular solution or the steady state response will be, so it will be F sin F sin omega t by. So, this thing I will replace it by D square plus omega n square.

So, phi will be equal to F sin omega t by D square plus omega n square, here in place of D square you have to substitute this minus omega square. So, this F equal to R plus r by r minus omega. So, I am substituting this D square equal to minus omega square, so this omega n square equal to R n square by r. So, this expression for the steady state response becomes this, so phi equal to R plus r by r minus omega square plus R n square by r omega square theta 0 sin omega t. So, this is the steady state solution of the system. So, you can find the steady state solution for any particular value of omega from this expression.

Now, you may note that as this omega n equal to n R by r or omega n square equal to R n square by r. So, we know for a simple pendulum in gravity field, if we compare this thing this pendulum with that of a pendulum in gravity field. We note that when the simple pendulum moves through this position, we can write the natural frequency of the simple pendulum omega n equal to root over g by l or g by if r is the length of the simple pendulum then g by r. So, if we will compare this omega n equal to R n square by r root over equal to root over g by r you may note that here the gravity field is replaced by this centrifugal field. So, the centrifugal field is equal to n square R and the gravity field is g. So, in case of the simple pendulum it is the gravity field for this natural, for finding this natural frequency and in case of this simple in case of this centrifugal pendulum which is used for vibration absorber. So, this frequency equal to n square R.

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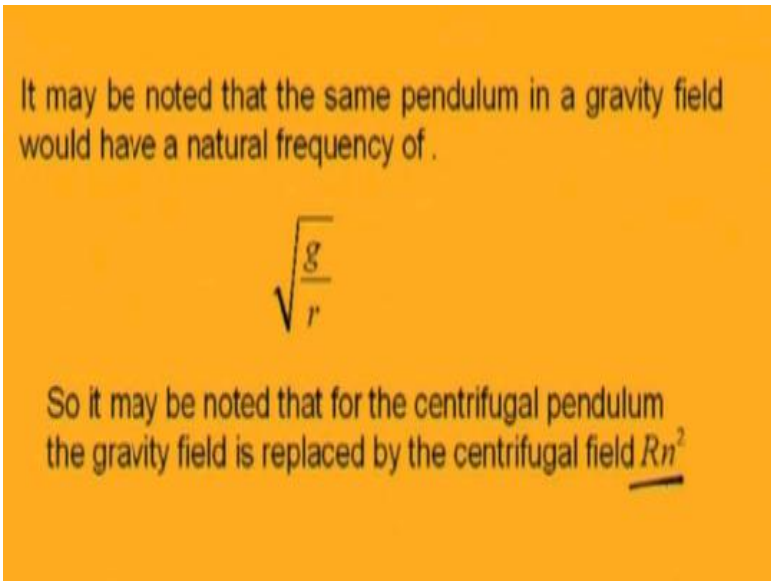

So, you can replace it by, so g by n square R. So, you have to note that the centrifugal in centrifugal pendulum the gravity field is replaced by the centrifugal field that is R n square.

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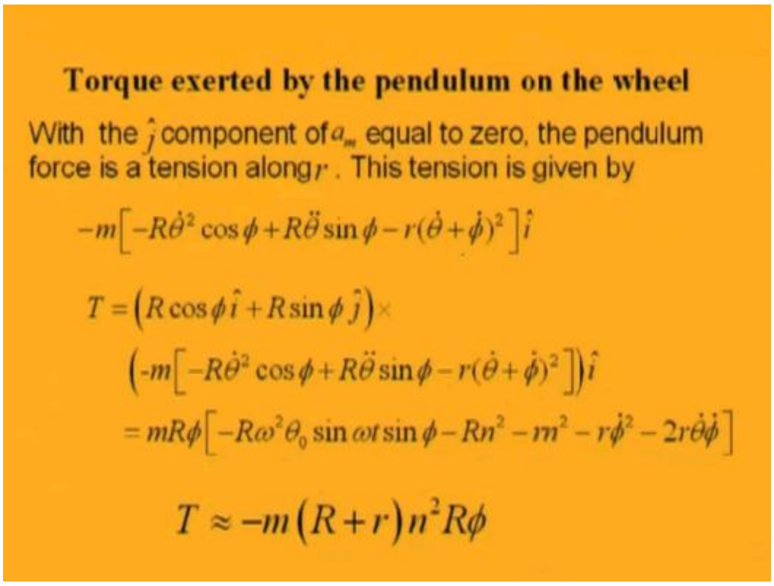

Now let us find the torque exerted by this pendulum on the disc. So, the torque exerted by the pendulum we can find from the force acting on this. So, this force already we have found.

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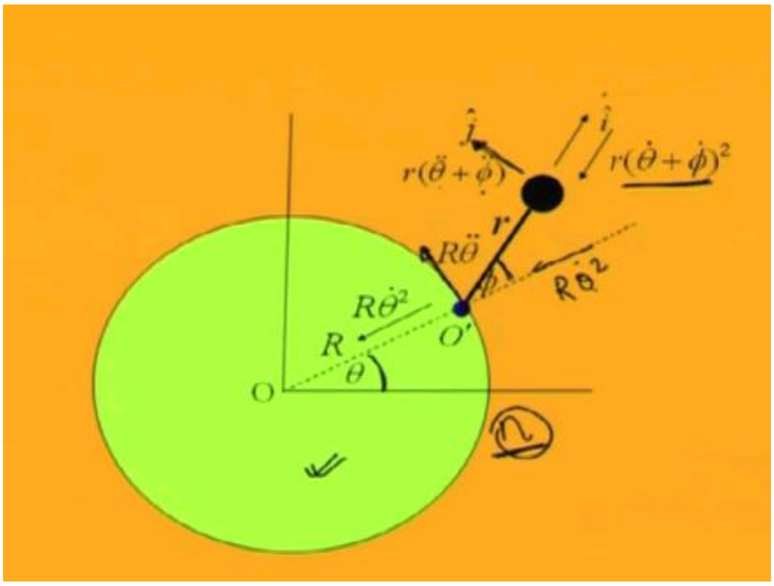

This is equal to mass into acceleration and, we know that mass into acceleration component along j direction equal to mass into acceleration component along j direction when multiplied by this R that equal to 0. So, already we know that torque equal to, so torque will be equal to R cross F. So, we can find using this expression this R cross F that is the torque exerted by this pendulum on this wheel.

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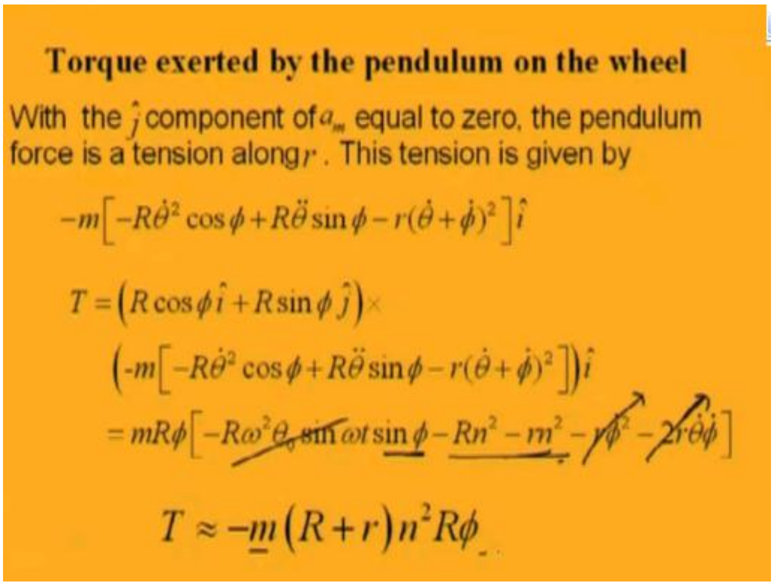

So, we know as that component j component equal to 0. So, the total force acting on this pendulum will be the i component. So, this will act as a tension in the tension on the wheel. So, this tension can be written as mass into this i component of the force. So, mass into I component of the acceleration that is the force or tension by this pendulum. So, the torque will be R into, so the torque equal to R. So, R equal to I can write this R equal to R cos phi I plus R sin phi j and this is the force that is equal to minus m minus R theta dot square cos phi plus r theta double dot sin phi minus r theta dot plus phi dot square i. So, if I multiplied this thing. So, you know the i component into i component cross product equal to 0 and i component cross product j component will be equal to minus k.

I am writing only the magnitude here. So, it will becomes m R phi into minus R omega square theta 0 sin omega t sin phi minus R n square minus r small n square small r n square minus r small r phi dot square minus 2 r phi dot theta dot. So, you can see that there are many small terms or there are terms like this phi dot and theta dot and phi dot square. So, sin phi, so sin phi equal to phi already you know. So, these small terms you can neglect in comparison to the terms which are bigger terms like this R n square and minus R n square minus r small r n square. So, you can take or you can write this t is nearly equal to minus m R phi. So, this is equal to minus m R phi into this R plus r n square. So, I am neglected these terms.

So, this term I have neglected and this term these 2 terms also, I have neglected in comparison to these 2 terms, that is equal to capital R plus small r that is the radius of the disc plus the radius of the pendulum or the length of the pendulum into this square of the disc velocity. So, this is a larger term in comparison to this other terms, so I have neglected these terms. So, the torque is equivalent to or torque is equal to minus m R plus r n square r phi. So, this is equal to the effective inertia of the pendulum into this theta double dot. So, if you equate this 2 expression you can write J effective will be equal to minus m R plus r.

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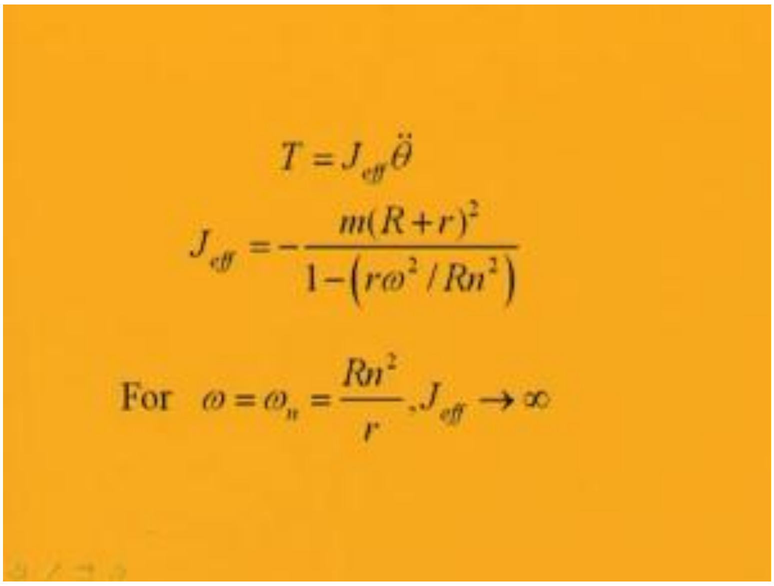

So, the previous expression minus R plus r. So, this n square term I will replace it by, so this n square R term I know. So, this thing is written, so J effective will be equal to m R plus r theta double dot expression already we know. So, if you substitute that expression here. So, this becomes m R plus r whole square by 1 minus r omega square by R n whole square. So, for omega equal to omega n, so this is your omega n that is R n square by r. So, this term becomes 1 and this is J effective. So, effective inertia of the pendulum will tend to infinity. So, when omega equal to natural frequency of the system then you can see that the pendulum has a, or it is required to a have very high inertia as J effective is infinity. So, you required to have a very high inertia to damp the vibration.

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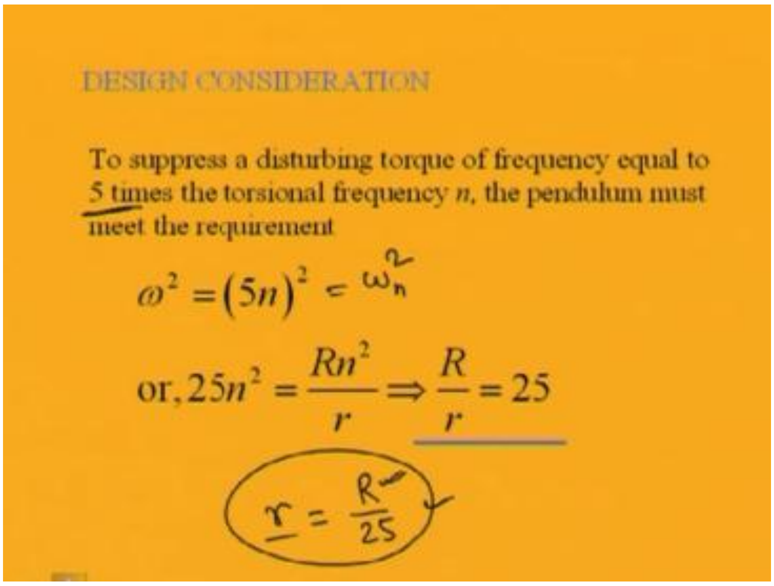

So, let us see the disadvantage of this type of system, led to suppress the disturbing torque at a frequency equal to phi times the torsion frequency n the pendulum must meet the following requirement. So, the requirement will be this omega square equal to phi n square. So, when this is equal to phi n square. So, this is equal to omega n square. So, already we know that omega n square equal R n square by r. So, this 25 n square should be equal to R n square r or this R by r should be equal to 25. So, in this case you can observe that we have to make this r. So, this small r that is the length of the pendulum equal to R by 25.

So, this is impossible to make for a physical system. So, as we go on increasing the frequency for the, as we go on increasing to suppress the frequency or higher harmonic frequency, it will be difficult or it to be very difficult to manufacture this length of the pendulum which is which will be very, very less. So, when you are suppressing a vibration which is where the frequency is 5 times the in frequency of the rotation, then we required the length of the pendulum to be 25 times less than the length of the or radius of the disc.

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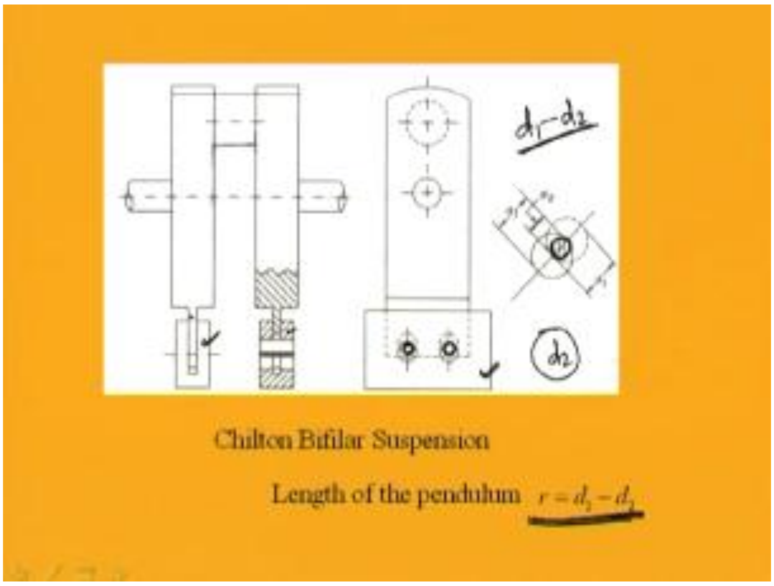

To avoid that thing, we may use another type of centrifugal absorber that is known as a Chilton Bifilar suspension or a bifilar suspension. So, in this case, so this is the crank. So, in this crank you just see this is the crank wave. So, in this crank wave we have attached a u shaped, counter mass is attached. So, this is the u shape counter mass this is also the u shape counter mass. So, this u shape counter mass is attached to this crank at 2 position. So, it is attached by to this crank by using 2 pins. So, this pin has a diameter of d two. So, this is the diameter of the pin d 2. So, you can see this is the diameter of the pin. So, this is also the diameter of the pin which is d 2. So, d 2 is the diameter of the pin and, let d 1 is the diameter of the hole make made on this u or counter mass and also made on the crank.

So, the counter mass will role on this pin which in turn roles on this crank. So, the total oscillation or the oscillation of this mass or the counter weight will be, so it will move in a circular path of radius d 2 minus d 1. So, where so it will move in a circular path of radius d 1 minus d 2. So, d 1 minus d 2 will be the radius of the path. So, this is the length of the simple pendulum or length of the pendulum we are using. So, this will be equivalent to the length of the pendulum we are using to suppress the vibration of the crank shaft. So, in this case by reducing this d 1 minus d 2 that is the gap between this, the gap between the gap between the pin and the hole we made on this counter weight or the hole we made on this crank. If you reduce this gap and then we can reduce this length of this pendulum.

So, in that way we can reduce the length of pendulum to any smaller extent and we can achieve the suppression of the vibration. So, in this case, and so this is the way we can suppress the vibration by using this absorber. So, already we have seen 2 types of absorber, in one case we have absorbed the vibration by using this spring. So, we have observed the vibration by using secondary spring and in the other case. So, in the case when we have absorb the vibration by using the secondary spring

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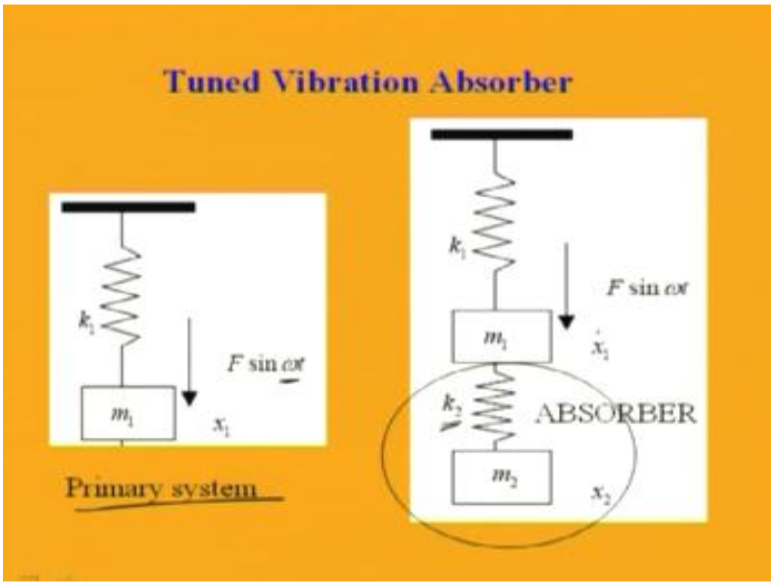

We have taken the frequency omega of the primary system equal to the frequency k 2 by m 2 of the secondary system. So, in that way we have absorb the vibration of the primary system and the secondary system is subjected to a very huge vibration. And, by properly choosing this k 2 and m 2, we have found the we have suppressed the vibration. But when this omega is not constant and when the frequency of the disturbing forces is proportional to this speed of the system or like in case of these automobiles. So, in that case we can use a centrifugal Pendulum absorber.

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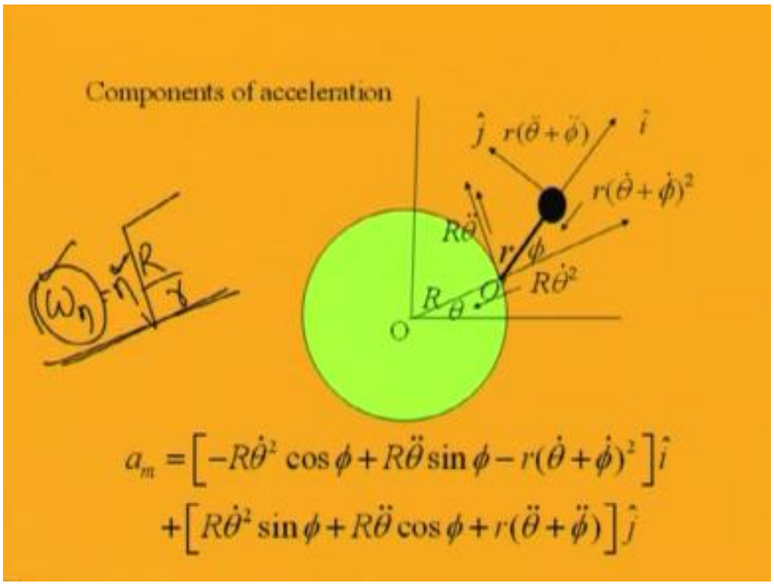

And, by using this centrifugal pendulum absorber, we have seen that the natural frequency of this centrifugal vibration absorber or centrifugal pendulum that is equal to R n square by r. So, this omega n square we have found it equal to R n square by r or this omega n equal to root over, so n into root over R by r. So, this natural frequency is proportional to this speed of the disc. So, as we can increase or when the speed of the disc varies automatically the natural frequency of the system or the absorber or the secondary system will increase. And in that way it will also become a tuned vibration absorber. So, in both the cases, when the natural frequency of the secondary system becomes equal to the frequency of excitation of the primary system the vibration get, vibration of the primary system get absorbed.

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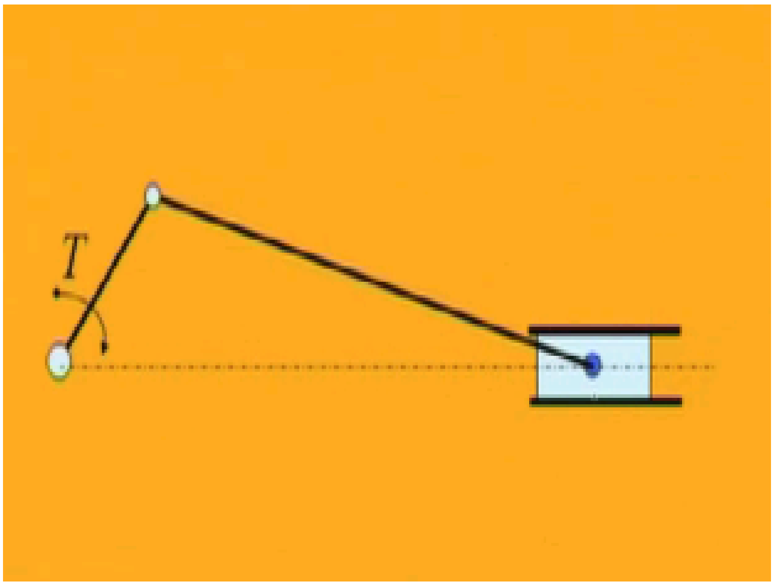

But in case of the automobiles, Already we know that there may be several harmonics present in this system or there may in the torque component unbalanced torque component there may be several harmonics present. So, to suppress each harmonic, so we may require a number of centrifugal vibration absorber. To avoid that thing 1 may use the damper to suppress this vibration. So, instead of using vibration absorber 1 may use vibration damper to reduce this vibration. So, in case of vibration damper so will see about 2 different types of vibration damper.

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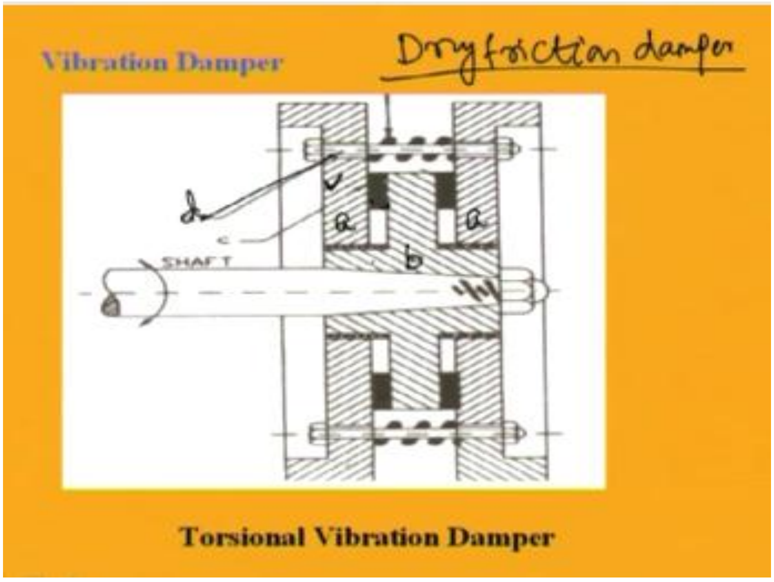

So, one is this dry friction damper. So, this is dry friction damper, so in case of dry friction damper, so our aim is to damp out the vibration not to absorb the vibration. In the previous case, in case of vibration absorber the secondary mass absorb the energy of the oscillation and it get excited itself. But in case of vibration damper, the energy of oscillations will be damped out by using some damper. So, we can use viscous damper and in this case we are using this dry friction damper. So, to absorb the or to damp out the vibration of a torsional system generally this type of vibration damper are used. And so you can see this damper consist of 2 fly wheels. So, this is a fly wheel a. So, these 2 are the fly wheel. So, these 2 fly wheels are loosely connected or loosely mounted on the wave which is mounted on the shaft. This is mounted on this wave b you may note that this is loosely connected to this wave b which is rigidly connected to this shaft.

So, this hub is rotating with same speed as that of the shaft and this fly wheel is loosely mounted on this hub. So, there is a friction, so there is a friction plate attached to this or this fly wheel is driven by this friction plate which is mounted on this wave hub. So, it is mounted on this hub and you may note that we can apply this force by this clamping or spring mounted bolt. So, this is a spring mounted bolt. So, this d is the spring mounted bolt. So, you can apply any pressure on this fly wheel by tightening this bolt and you can drive this fly wheel. So, when it is loosely connected or when this friction, we are assuming this friction to be zero. So, when the shaft is rotating. So, no energy will be transferred to this fly wheel.

Also, when the friction is very high then there will be a rigid bond between this hub and this fly wheel, and it will act as a single degree of freedom system or single system, and it will increase the inertia of the system. So, the natural frequency of the system will be changed. But we can control the vibration or we can damp out the vibration by suitably choosing the damping which we can make by suitably adjusting this pressure on this fly wheel. So, when the shaft is rotating with a lower speed at the time this due to the inertia very huge inertia of the fly wheel it will not follow the speed of the shaft. So, there will be some relative motion between this fly wheel and this hub and due to the presence of a friction plate between these two, there will be relative probing of the surface which will damp out some of the energy and there by this disturbing forcing which cause this vibration of the system will be reduced.

So, in this way by using this dry friction one may damp out the vibration of the system. So, I may repeat this process or in this case on the shaft. So, on the shaft this hub is mounted and the fly wheel is loosely mounted on this hub and between this fly wheel and this hub there is a friction plate, and the fly wheel is driven by this friction plate by applying the pressure by this bolt which is. So, there is a spring also the spring mounted bolt. So, by applying suitably the pressure by this bolt the pressure on this friction plate can be adjusted. So, when it is rotating at a lower speed, when it is rotating at a lower speed at that time there will be relative motion between this hub and the fly wheel. As, the fly wheel has a very huge inertia it will not follow this hub. So, there will be relative motion between this fly wheel and this hub.

Due to this relative motion there will be rubbing of this surface on this friction plate. So, due to this rubbing, so some energy will be transferred. So, the disturbing energy of the shaft will be will be dissipated by this friction damper. So, in this way some part of the disturbing energy will be dissipated and the system will be the system vibration will be damped out. So, instead of using a dry friction damp damper one may use viscous friction damper also.

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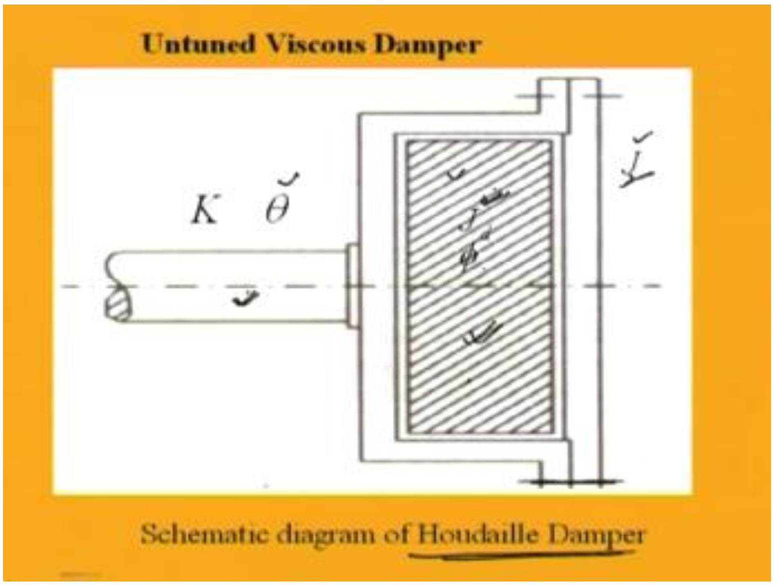

In this case, so this is the shaft and you can put this viscous liquid viscous liquid in a cylindrical container and it is kept in between or loosely mounted in between a cylindrical cavity. So, this is the cylindrical cavity. So, this side of this is mounted or attached to the shaft which has which is vibrating. So, this is the vibrating shaft. Let the stiffness of the vibrating shaft is K, let it is rotating with theta. So, if this rotating with theta and. So, let it is mounted on this on a mass, so this is a cavity in which you are putting this liquid. So, in this case, so we can write the inertia of this mass equal to J d and the inertia of this equal to J.

So, this is a pulley inside which there is a cavity cylindrical cavity. So, in that cylindrical cavity there is a mass hollow mass in which you are putting the viscous fluid. So, this is the system and this system is known as this schematic diagram of a Houdaille damper. So, when it is rotating, so this mass will be subjected to a torque. So, due to free rotation of this, so as it is loosely mounted on this. So, there will be free rotation of this mass with inertia J d with respect to this primary system which has inertia J.

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$$M_0 e^{i\omega t}$$

$$J\ddot{\theta} + K\theta + c(\dot{\theta} - \dot{\phi}) = M_0 e^{i\omega t}$$

$$J_d\ddot{\phi} - c(\dot{\theta} - \dot{\phi}) = 0$$

$$\theta = \theta_0 e^{i\omega t}$$

$$\phi = \phi_1 e^{i\omega t}$$

So, let it is subjected to a torque of M 0 e to the power i omega t. So, let this mass is subjected to a torque of M 0 e to the power i omega t. So, I can write the equation motion. So, let phi is the relative rotation of this mass and theta is the rotation of this shaft. So, in this case I can write the equation motion which will contain the inertia of the system inertia will be equal to J theta double dot and the stiffness of the shaft is k. So, it will resist with a force of K theta and the damping force will be equal to C into this relative displacement between the shaft and this mass. So, that is equal to theta dot minus phi dot. So, I can write this equation motion in this form. So, it will be J theta double dot plus K theta plus C theta dot minus phi dot equal to M 0 e to the power i omega t.

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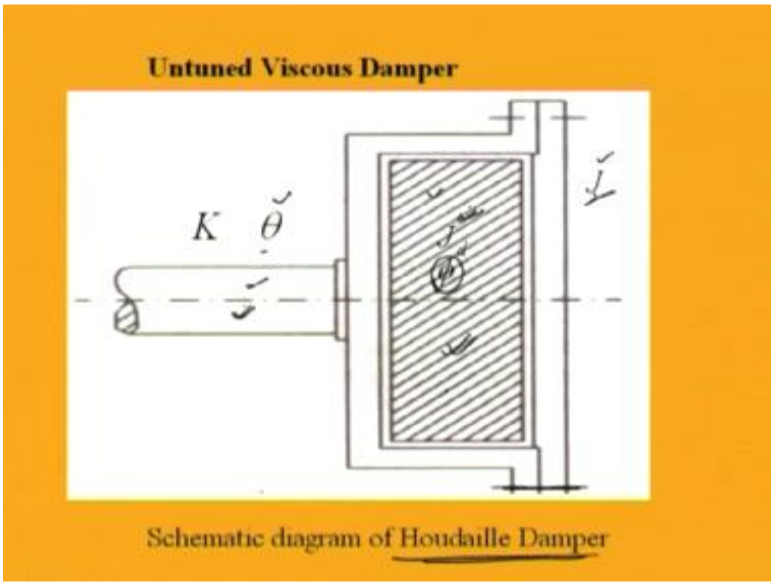

Similarly, I can write J or this mass for the rotating mass inside rotating mass I can write as there is no force or there is no torque on this. So, I can write that is equal to J d phi double dot. So, J d phi double dot minus C theta dot minus theta dot minus phi dot equal to 0. So, the disturbing force is acting on this that is equal to M 0 e to the power i omega t. There is no disturbing force on this and so this will be equal to, so the force on this inertia force of this will be equal to the damping force on this. So, in this case I can write J d phi double minus C theta dot minus phi dot equal to 0. So, I can assume this solution in these two cases like this.

So, I can assume theta equal to theta 0 e to the power i omega t and phi equal to phi 0 e to the power i omega t. So, by assuming these 2, so I can substitute this to in these two equations. So, this equation will become J theta 0 omega square. So, minus term is there J minus J theta 0 omega square e to the power i omega t plus K into theta 0 e to the power i omega t plus C into for theta dot. So, it will be equal to i theta 0 omega e to the power i omega t and phi dot it will be equal to i phi 0 omega e to the power i omega t.

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$$\left[\left(\frac{K}{J}-\omega^2\right)+i\,\frac{C\omega}{J}\right]\theta_0-\frac{iC\omega}{J}\,\phi_0=\frac{M_0}{J}$$

$$\left(-\omega^2+i\,\frac{C\omega}{J_d}\right)\phi_0=\frac{iC\omega}{J_d}\,\theta_0$$

So, that will be equal to M 0 e to the power i omega t, substituting those expression there. I can write K by J minus omega square plus i C omega by J theta 0 minus i C omega by J phi 0. So, this will be equal to M 0 by J. So, this is the first expression and the second expression I can write in this form. So, that will be equal to minus omega square plus i C omega by J d into phi 0. So, this will be equal to this will be equal to i C omega by J d theta 0. So, from these 2 expressions I can write; I can eliminate this phi 0 terms. So, I can substitute this expression in this expression or I can substitute this phi 0 equal to i C omega by J d theta 0 by minus omega square plus i C omega by J d and I can write this expression in this form. So, I will eliminate this phi 0 from this two expression and I can write theta 0. So, I can write theta 0 by M 0 terms. So, I can write this theta 0 by M 0.

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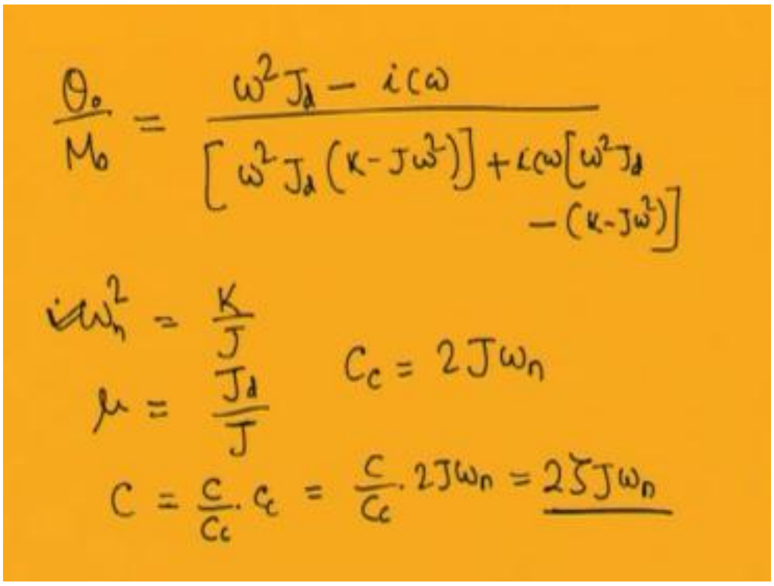

So, theta 0 by M 0 will be equal to, so this expression will be equal to omega square J d minus i C omega by omega square J d into K minus J omega square plus i C omega into omega square minus J d omega square. So, it is omega square into i C omega into omega square into J d minus K minus J omega square. So, this is the term, so now I can use some non dimensional terms like, I can substitute this omega n square equal to K by J. So, this is the torsional stiffness K and J is the inertia of the system. So, omega n square equal to K by J, also I can write this mass ratio mu equal to J d by J. So, this is the inertia ratio inertia of the rotating mass inside this cavity and the primary mass of the system. So, this is J d by J that is the mass ratio and the critical damping already you know the critical damping C c.

So, I can write this critical damping C c equal to 2 J omega n. So, I can write C equal to, so you know C equal to C by C c into C c for. So, this will be equal to C by C c into 2 J omega n and C by C c is nothing, but the damping ratio. So, I can write the C equal to 2 zeta J omega n. So, C that is the damping equal to 2 zeta J omega n. So, substituting these terms that is omega n square equal to K by J mu equal to J d by J and C equal to 2 zeta J omega n in this expression , I can write this theta 0 by M 0 equal to, so I can take K common here.

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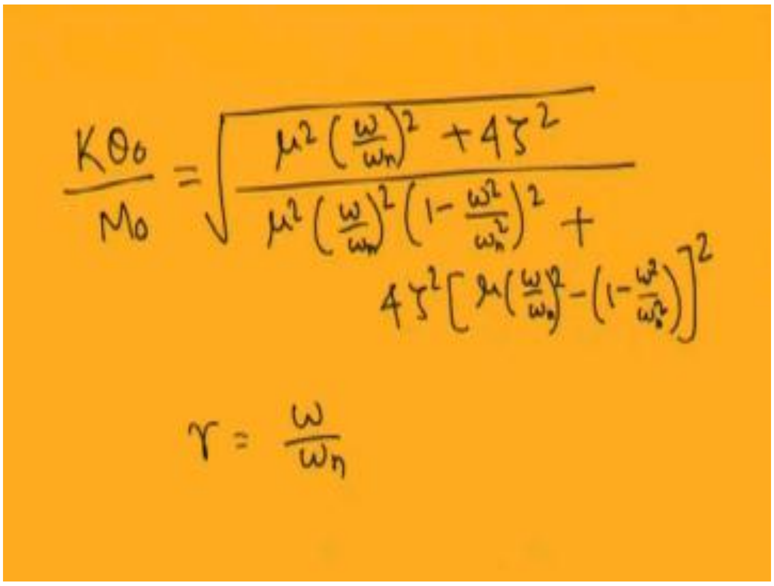

So, I can write this expression as K theta 0 by M 0. So, this is nothing, but the magnification factor in case of a single degree of freedom system. So, this is K theta 0 by M 0 equal to root over mu square into omega by omega n whole square plus 4 zeta square by mu square into omega by omega n whole square into 1 minus omega square by omega n whole square Plus 4 zeta square into mu into omega by omega n whole square minus 1 minus omega square by omega n square whole square. So, this k theta 0 by M 0 is the magnification factor; M is the mass ratio or the inertia ratio of the inertia and this I can take this frequency ratios, I can substitute r equal to omega by omega n. So, by substituting this r equal to omega by omega n. I can write this expression in this way.

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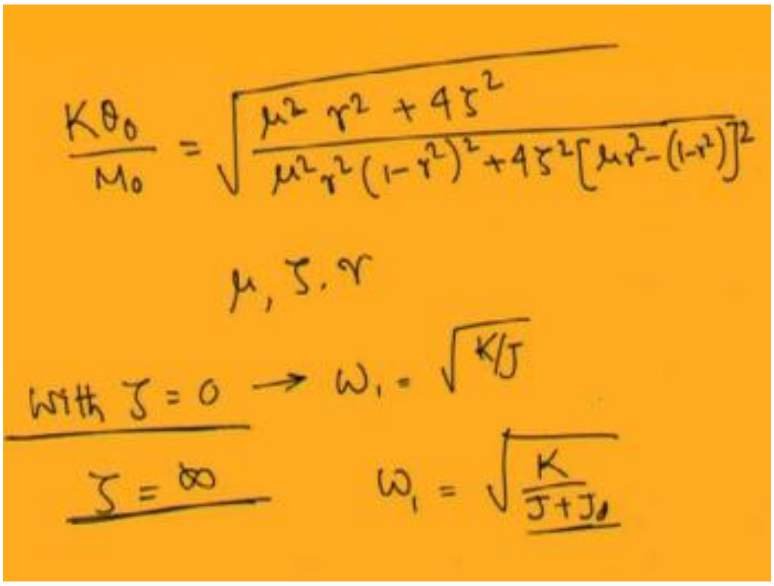

So, this is K theta 0 by M 0. So, this will be equal to root over; so this is equal to mu square into r square plus 4 zeta square by mu square r square into 1 minus r square whole square plus 4 zeta square into mu r square minus 1 minus r square 1 minus r square whole square. So, you can note that this magnification factor is a function of three parameters. So, the parameters are mu. So, it is a function of mu then it is a function of zeta and it is a function of r. So, when zeta equal to 0, so you can note that when zeta equal to 0 this omega 1 equal to root over by…

So, when zeta equal to 0, so omega 1 equal to root over K by J. So, we consider 2 systems when there is no damping that is zeta equal to 0, that time the natural frequency equal to root over K by J and we will have a undamped single degree of freedom system. So, there is as there is no damping. So, the system will be equivalent to an undamped single degree of freedom system. And, when we have J zeta equal to infinity that is there is huge damping is there. So, that time we can assume that the damper mass and the wheel will move together as a single mass. And again we have an undamped single degree of freedom system with a lower frequency and in that case that frequency will be equal to, so in the frequency will be equal to root over K by J plus J d.

So, when zeta equal to 0, we have an undamped system with frequency omega n or this is omega 1 that is equal to K by J. And, when we have zeta equal to infinity that time that time the damper mass and the wheel mass will move together. So, the effective mass will be equal to J plus J d and the frequency or natural frequency of the system will change to root over K by K by J plus J d root over. So, if one plot this K theta 0 by M 0 that is magnification factor verses this r. So, one can see that the plot will look like this.

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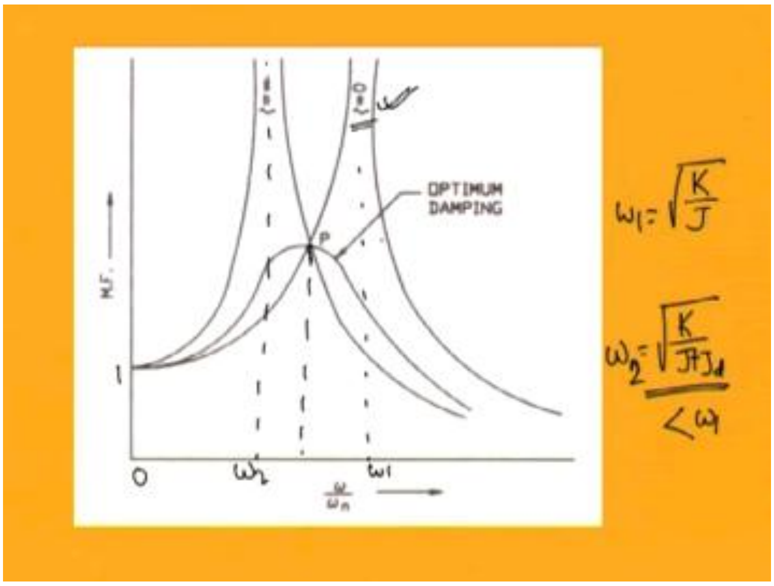

So, this is the magnification factor verses r that is omega by omega n. So, when zeta equal to 0, we have the response of a single degree of freedom system. So, already we have studied about the single degree of freedom system, magnification factor verses omega by omega n. So, when zeta equals to 0, so this is the curve for that. So, this is magnification factor 1 it is starts from. So, when omega by omega n equal to 0 it starts from 1 and when this omega equal to omega one, so in this case this is omega 1. So, this omega 1 is nothing, but root over K by J.

So, omega 1 equal to root over K by J and it is equal to, so that time you will have an infinity magnification factor when omega equal to omega n. But when zeta equal to infinity, so in that case when zeta equal to infinity. So, your omega n is reduced to or the natural frequency is reduced to K by J plus J d so K by J plus J d. And, in this case it will be less than, so it will be less than omega 1. So, as it is less than omega1. So, you have this plot, this also behaves as a single degree of freedom system. So, you can see that there is a point P which is common to both the damping factor. So, for damping equal to 0, you have this point P and also when damping ratio is infinity also this is point P.

So, you can see that irrespective of damping all curves passes through this point P. So, to have an optimum value of magnification factor you can have the peak at this point P. So, to find this point P, I can write this frequency. So, this frequency is omega 2. So, this omega 2 equal to root over K by J plus J d and this omega 1 equal to root over K by J. To find this frequency at which this peak occurs or to find the optimum value, I can equate the magnification factor at zeta equal to 0 and magnification factor at zeta equal to infinity. So, by equating that, so when magnification factor equal to, so you can see when magnification factor equal to 0.

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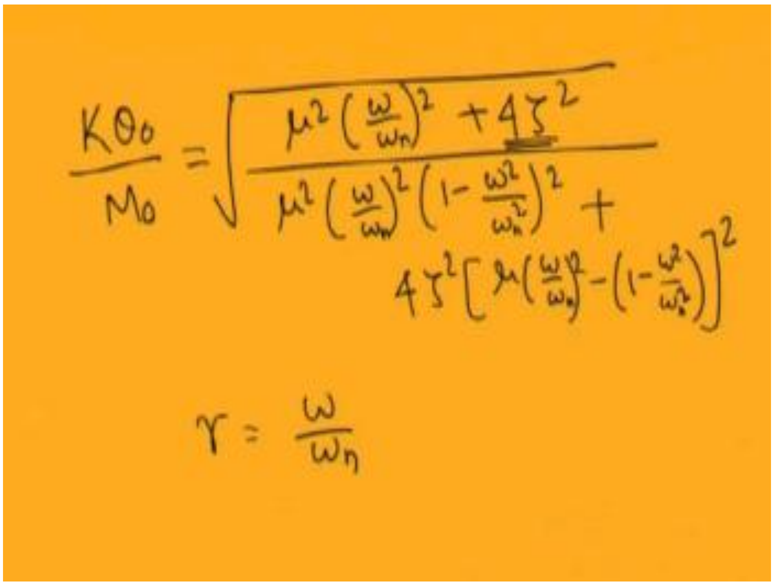

When damping equal to 0. So, you can you can eliminate this term and also you can eliminate this terms.

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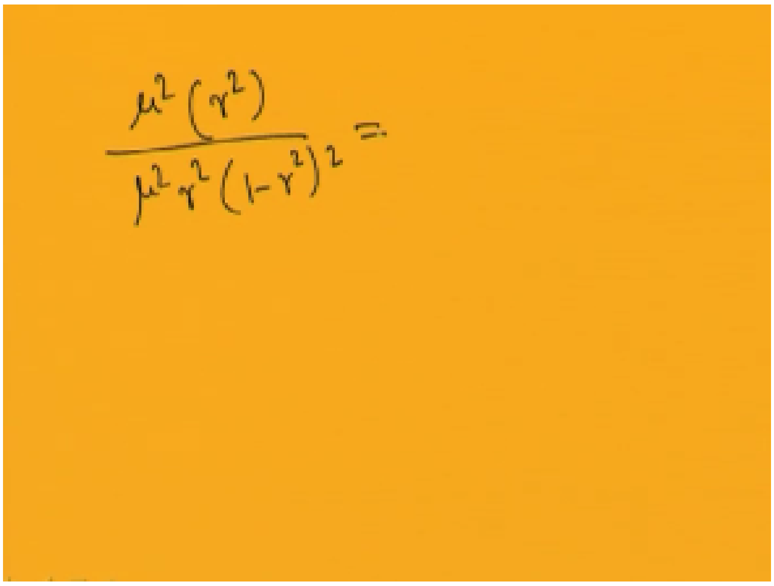

So, your K theta 0 by M 0 becomes, mu square by mu square into r square by r square into mu square into r square mu square into r square into 1 minus r square whole square and when it is equal to infinity.

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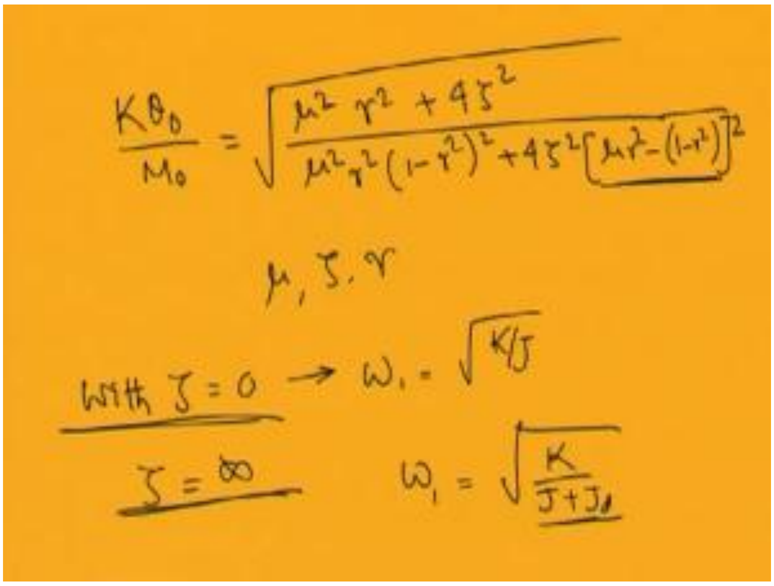

So, when it is equal to zeta equal to infinity, so you divide this term. So, you can divide this term. So, if you divide this term then on the top it will be 1 and in the bottom the terms remaining will be this term only.

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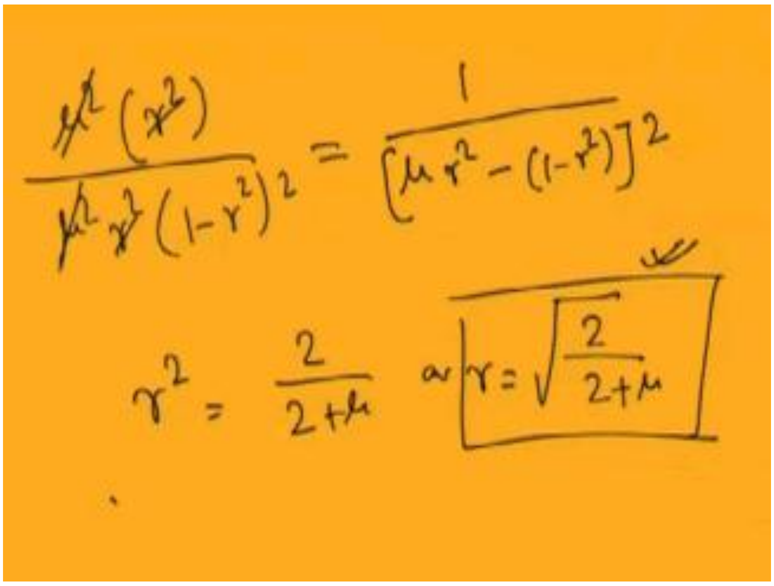

So, you can write that term and square it. So, you can find this is equal to 1 by mu r square minus 1 minus r square whole square. So, by equating the magnification factor at zeta equal to 0 and magnification factor at zeta equal to infinity I can get. So, I can delete this r square r square mu square mu square. So, this is 1 by 1 minus r square whole square equal to 1 minus mu r square minus 1 minus r square whole square or by simplifying this thing, I can write r square I can cross I can write this r square equal to 2 by 2 plus mu or equal to r equal to r equal to root over 2 by 2 plus mu. So, this is the optimum frequency or this is the frequency at which this optimum damping will occur. So, you can find the optimum damping value by. So, from this expression for r you can find this optimum value of damping.

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$$\zeta_0 = \frac{\mu}{\sqrt{2(1+\mu)(2+\mu)}}$$

So, this optimum value of damping will be equal to zeta 0 equal to mu by root over 2 into 1 plus mu into 2 plus mu. So, today class we have studied about the centrifugal vibration absorber and different types of vibration damper. So, in case of vibration damper we have studied about this dry friction damper and about the viscous friction damper. So, by using this damper and absorber one may eliminate or one may damp out the vibration of the system. So, by using the vibration absorber one can absorb the vibration. So, in that case the secondary system will be vibrating with very high amplitude. And in case of vibration damper we can damp out some of the vibration of the system. So, we have studied both tuned and untuned type of viscous absorber and damper.