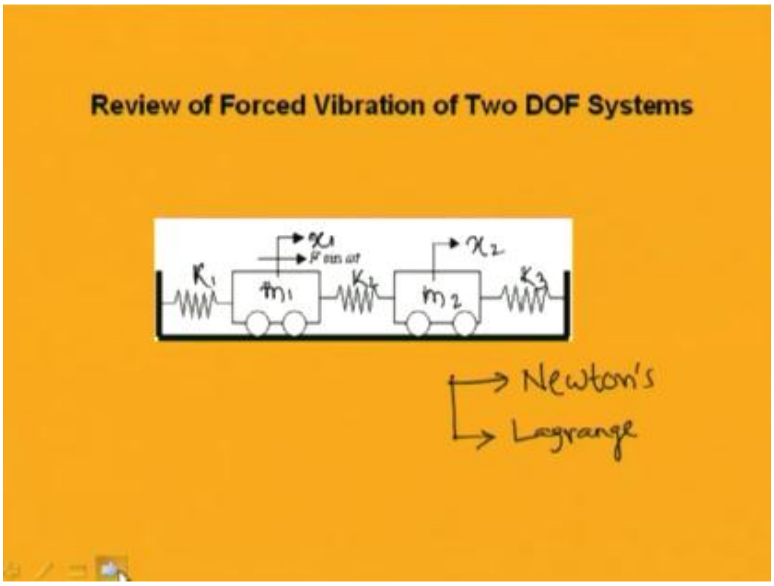
Mechanical Vibrations

Prof. S. K. Dwivedy
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 6
Vibration Absorber
Lecture - 1
Tuned Absorber, Determination of Mass Ratio

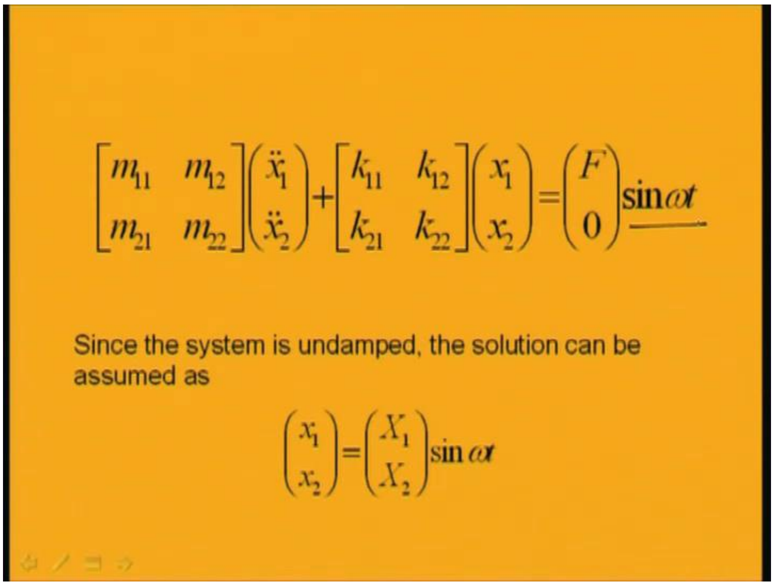
Welcome, to this class of vibration engineering. In the previous classes, we have studied about the free and forced response of single degree of freedom systems and 2 degrees of freedom system. Today, we are going to study about these vibration absorbers. So, before going to study about this vibration absorber; let us first revive about the force response of 2 degrees of freedom system, so in case of 2 degrees of freedom system.

(Refer Slide Time: 01:26)



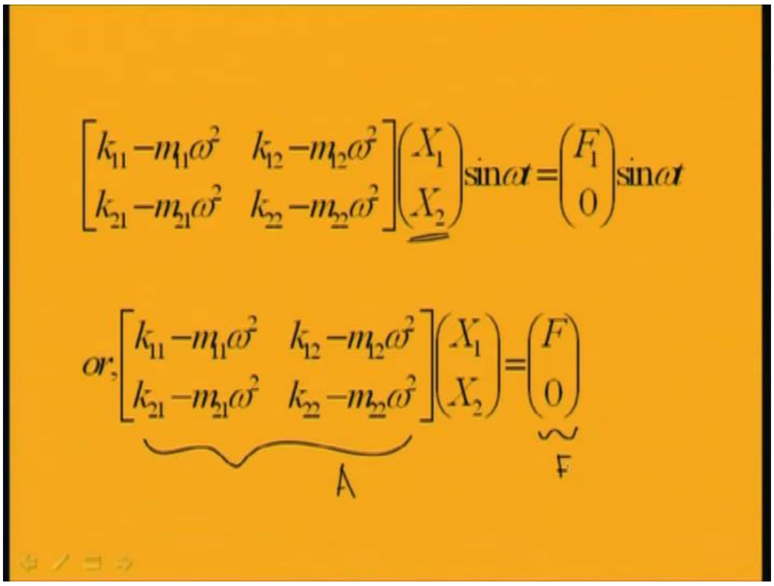
A spring mass system is shown here. So, m 1 and m 2 are the mass of this system. And k 1, k 2, k 3 are the stiffness of the system. Let this mass m 1 is subjected to a harmonic force of F sin omega t. So, in this case already you know to derive this equation motion either you can go for this Newtons method by using the Newtons second law; you can derive the equation motion or you may use the Lagrange principle or the Hamilton extended, Hamilton principle to derive this equation motion.

(Refer Slide Time: 02:08)



So, in this case already you have derive this equation which can be written in this form this mass matrix into x double dot plus stiffness matrix; into x will be equal to do this force vector. And as I am taking a harmonic forcing then this can be written has F sin omega t. Here the first mass is subject to a force F sin omega t that is why you have a force in this form F 0 sin omega t. Here m 11, m 12, m 21, m 22 are the element of the mass matrix k 11, k 12, k 21, k 22 are the elements of the stiffness matrix. So, already you know from the single degree of freedom system that when you are applying a sinusoidal force; then the response also will have similar frequency. So, the frequency of the response will be in the form of sin omega t. So, I can assume the response in this form x 1, x 2 equal to X 1, X 2 sin omega t for this system.

(Refer Slide Time: 03:21)



So, if I substitute this solution in this equation then it will reduce to this form; so k 11 minus m 11 omega square. Because this x 1 double dot will becomes x 1 into omega square, sin omega t and with a negative sign so it becomes minus m 11 omega square x 1; similarly, this m 12, x 2 double dot will becomes. So, when I will substitute this equation it will becomes m minus m 12 omega square. So, I can substituting this equation in the previous equation 1 can write it in this form k 11 minus m 11 omega square, k 12 minus m 12 omega square, k 21 minus m 21 omega square, and k 22 minus m 22 omega square; X 1, X 2 these are the modes normal modes.

So, X 1 and X 2 sin omega t; so this is can be written in terms of F 1 0 sin omega t. So, the sin omega t and sin omega t you can cancel. And so this will reduce to this form that is k minus m 11 omega square, k 12 minus m 12 omega square, k 21 minus m 21 omega square and k 22 minus m 22 omega square into X 1, X 2 equal to F 0. So, this contains 2 algebraic equations with unknown X 1 and X 2. So, you can solve these 2 equation algebraic equation to find X 1 and X 2, there are several methods to find this thing; either you may go for this Crammers rule to find if the equation motion will be of higher order. And as it is only 2 degree freedom system and you have only 2 unknowns then you can simply solve these equations to find the solution.

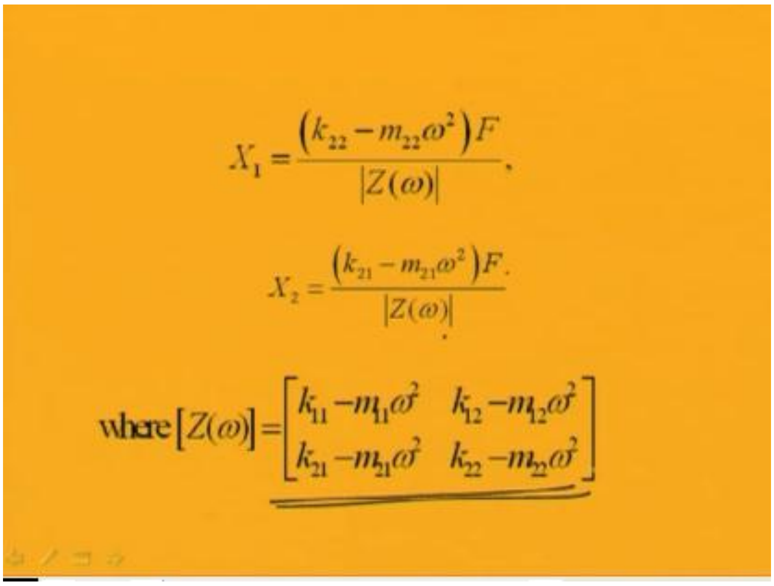
So, to find the solution I can also use this inverse method. So, if I am writing these as A matrix; then I can write this X 1, X 2 will becomes A inverse into if this is F. So, it will become A inverse F.

(Refer Slide Time: 05:32)

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \frac{\begin{bmatrix} k_{22} - m_{22}\omega^2 & -k_{12} + m_{12}\omega^2 \\ -k_{21} + m_{21}\omega^2 & k_{11} - m_{11}\omega^2 \end{bmatrix} \begin{pmatrix} F \\ 0 \end{pmatrix}}{\underbrace{\begin{vmatrix} k_{11} - m_{11}\omega^2 & k_{12} - m_{12}\omega^2 \\ k_{21} - m_{21}\omega^2 & k_{22} - m_{22}\omega^2 \end{vmatrix}}_{Z(\omega)}}$$

So, using these 2 so I can write this X 1, X 2 equal to A inverse F; so this inverse equal to adjoint of A by determinant of A. So, this adjoint matrix can be written in this form. So, this is equal to k 22 minus m 22 omega square, minus k 12, m 12 omega square, minus k 21, plus m 21 omega square, k 11 minus m 11 omega square and the determinant of matrix A and then this is F 0.

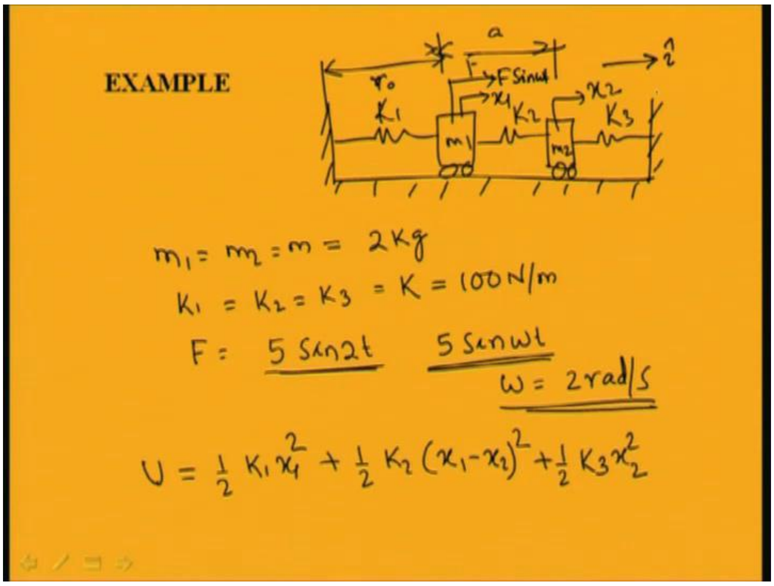
(Refer Slide Time: 06:07)



So, this thing can be simplify to this form. So, one can write this X 1 equal to k 22 minus m 22 omega square into F by Z omega. So, I have written this as Z omega this is a function of omega. So, I am writing this lower matrix or Z omega matrix; so this X 1 will become then k 22. So, you can see this thing by multiplying these. So, the X 1 will becomes k 22 minus m 22 omega square into F and this part is multiplied with 0. So, this becomes k 22 minus m 22 omega square F by determinant of these Z omega. So, that is written here k 22 minus m 22 omega square F by determinant of Z omega and this X 2 from these equation you can see X 2 will becomes.

So, here you are multiplying minus k 21 plus m 21 omega square F and these part is multiplied with 0; so this into this by the determinate of Z omega is X 2. So, this X 2 equal to k 21 minus m 21 omega square by Z omega F. So, where Z omega I have already told you. So, this is the K 11 minus m 11omega square, K 12 minus m 12 omega square, k 21 minus m 21 omega square and k 22 minus m 22 omega square. So, from this you can find these X 1 equal to k 22. So, given the value of stiffness; so you can find k 22, m 22 and this omega and this X 1, X 2 the response you can find.

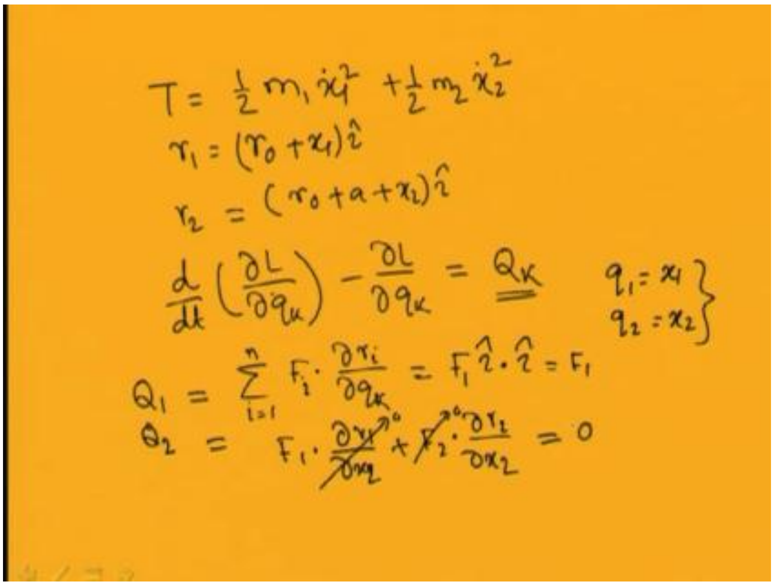
(Refer Slide Time: 08:00)



So, let us take a simple example, let us take the system, let us take this spring mass system; so let us take 3 springs and 2 mass it is supported by some roller. And this is k 1, k 2 and k 3 this is m 1 and m 2; for simplicity let us take this m 1 equal to m 2 equal to m. So, this becomes this is I am taking as 2 kg and then k 1 equal to k 2 equal to k 3. Let us take and this is equal to k; and let us takes this equal to 100, Newton per meter. And it is subjected to this first mass is subjected to a force; so this force F equal to let us to take it is equal to 5 sin 2 t. Initially, let us take this force equal to 5 sin omega t. And then let us find for the special case when omega equal to 2 radian per second.

So, already we have derived the equation motion for this case. So, the equation motion can also be derive by using this Lagrange principle. So, if you use the Lagrange principle then the kinetic energy can be written as half m x dot square, x 1 dot square plus half m 2, x 2 dot square and the potential energy can be written like this. So, the spring k 1 is subjected to a displacement of X 1; this k 2 is subjected to a relative displacement of x 1 minus x 2 and this spring k 3 is subjected to a displacement of X 2 only. So, this potential energy I can write U equal to half k 1, x 1 square plus half k 2, x 1 minus x 2 square plus half k 3, x 3 square. So, here I am taking as no x 3. So, this is x 2 square this k 3 is subjected to a displacement of x 2 only. And this is subjected to a displacement of x 1; so this is x 2 square.

(Refer Slide Time: 10:29)



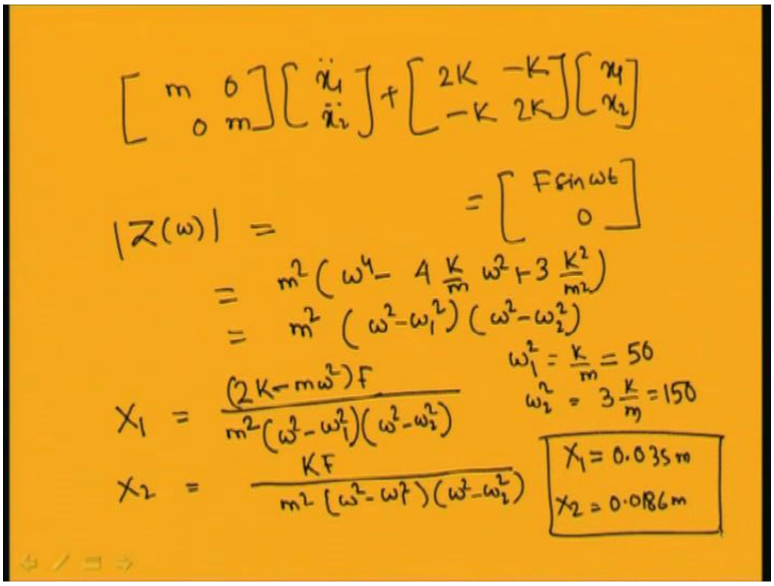
And, the kinetic energy is equal to half m, m 1, x 1 dot square plus half m 2, x 2 dot square and the force vector Q k generalized. So, I can take the generalized coordinate in this case; if I am using this Lagrange principle as x 1 and x 2 at the generalized coordinates. I can take a physical coordinate from this base; I can take a physical coordinate. Let the physical coordinate I am taking from this position let this is r 0; r 0 is the distance from this fixed end to this equilibrium position so that is r 0. And so the displacement of first mass r 1 I can write in this form. So, r 1 I can write it is equal to so r 1 equal to r 1 vector I can write equal to r 0 plus x 1 i. So, the horizontal direction I can take x 0. So, I can write this as r 0 plus x 1 i.

Similarly, I can take at the equilibrium position from this to this; let me put these distance equal to A. So, then this becomes displacement of 2; so this r 2 will become r 0 plus a, plus x 2 i. So, now to find the equation motion I can use this Lagrange principle which tells d by d t of Del l by Del q k dot minus Del l by Del q k equal to Q k. So, this capital Q k is the generalized force. So, this generalized force can be obtained by. So, Q1 to find Q 1, I can write this expression in this form. So, Q 1 will be equal to summation I equal to 1 2 n; so this becomes F dot Del r i by Del q k. So, in this case we have 2 only 1 force; this force equal to F sin omega t is acting here this force is F sin omega t. So, it is in the positive this direction.

So, I am taking this direction as i with a unit vector I; so this force equal to F sin omega Ti or I can write this force so only 1 force is acting. So, I can write this is equal to F or simply I can write this as F 1 equal to F sin omega t; so I can write this is F 1 i dot. So, this Del r i by Del q k; so q 1 equal to x 1 and q 2 equal to x 2. So, using this so q 1 when i differentiate with respect to q 1 or x 1; so this r 1 gives only i. So, this becomes F 1 i dot i so this becomes F 1. So, the q 1 generalized for q 1 equal to F 1 that is F sin omega t; similarly, one can find q 2. So, in this case you can find that there is no force, as there is no force is acting on mass 2. So, this q 2 becomes; so this is F i so F 1 into Del r 1 by Del q k, q k is x 2.

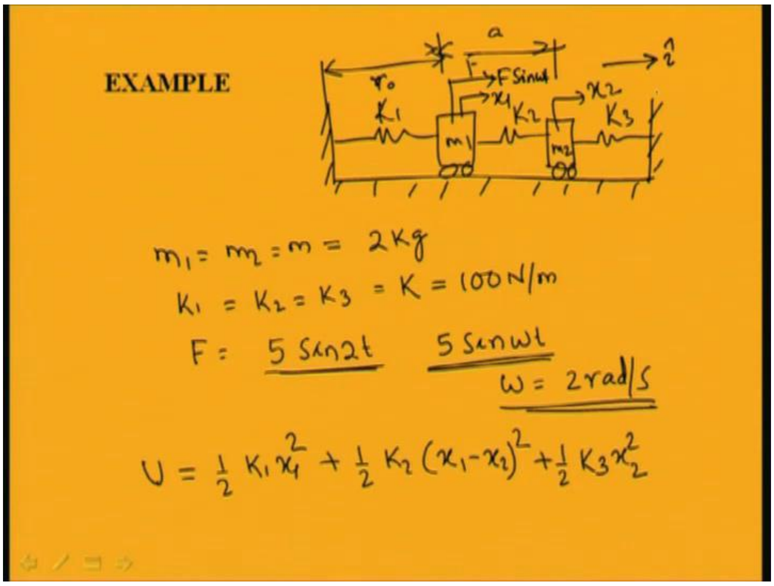
So, now, k equal to 2; so that q 2 equal to x 2. So, if you differentiate with respect to r 1 this becomes 0; so F1 dot 0 plus F 2 dot so as F 2 equal to 0 so this becomes also 0. So, both the terms that is F 1 so this thing will be equal to F 1 dot del r 1 by del x 1, plus F 2 dot del r 2 by del x 2; so as del r 1 by x 2 becomes 0 and F 2 becomes 0 so this is 0 and this is 0, so the whole term q 2 becomes 0.

(Refer Slide Time: 15:06)



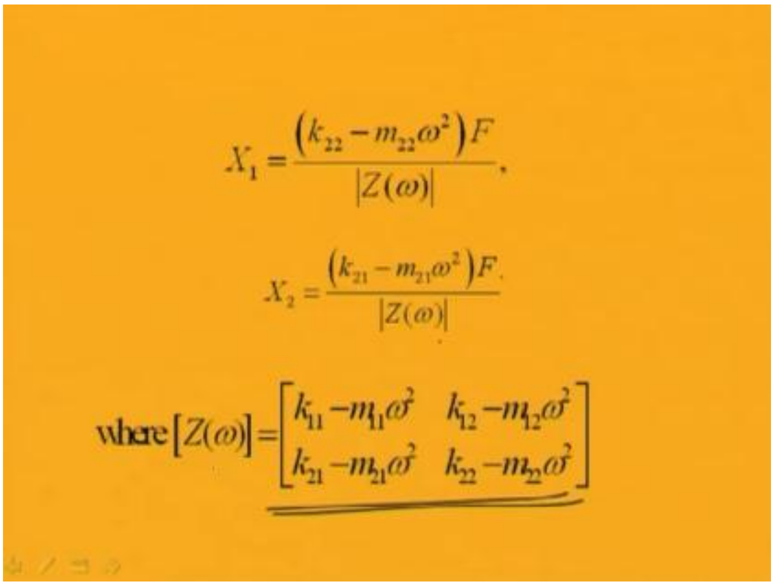
So, one can write the equation motion in the simple form. So, that is m 1 0 or I have taken m 1 equal to m 2; so I can write this is simple m. So, m 0, 0 m, x 1 double dot x 2 double dot plus this is k 1 plus k 2 minus k 2. So, this becomes so k 1 plus k 2 becomes so this becomes 2 k. So, this is 2 k minus k and this becomes minus k and this becomes k x 1, x 2, x 1, x 2 it becomes.

(Refer Slide Time: 15:49)



So, I have 2 mass here, I have 2 stiffness; so this becomes k 2 plus k 3. So, this becomes 2 k; so this equal to this thing I can write it equal to F sin omega t 0. So, now comparing this equation with the general equation I have derived before.

(Refer Slide Time: 16:14)



So, this equation I can write this Z omega; so this Z omega can be written in this form. So, the Z omega will give so I can find this Z omega; and Z omega in this case determinant of Z omega becomes m square into omega 4th minus 4 by minus. So, this becomes so I can write this equal to Z omega equal to m square into omega 4th minus 4 k by m omega square plus 3 k square by m square. So, this thing can be simplified and can be written in this form m square into omega square minus omega 1 square into omega square minus omega 2 square. And I can write X 1, X 1 in this form X 1 will becomes 2 k minus m omega square into F by Z omega; determinant of Z omega that thing can be written as m square into omega square minus omega 1 square into omega square minus omega 2 square.

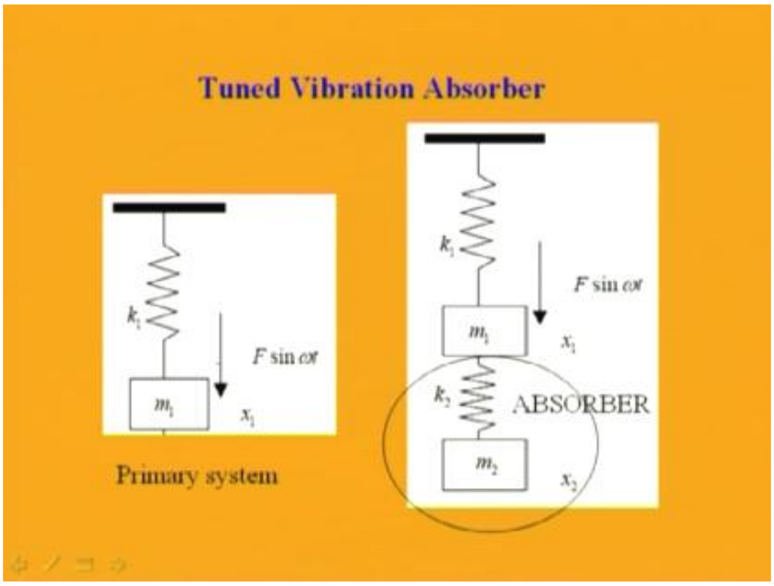
So, here omega 1 square equal to k by m and omega 2 square equal to 3 k by m. So, substituting the value you can find this omega 1 square equal to 50 and omega 2 square equal to 150. Similarly, X 2 can be written in this form; so X 2 becomes X 2 becomes K F by m square into omega square minus omega 1 square into omega square minus omega 2 square. Now, one can substitute the value of this omega 1 square that is equal to 50, omega 2 square equal 150; if one want to find the value at omega equal to 2. So, you can obtain this X 1 equal to X 1 equal to 0.035 meter and X 2 equal to 0.0 186 meter; so for omega equal to 2 so this are the value. And one can note from this expression that when this k by m.

So, this omega becomes this omega square becomes this 2 k by m; then this X 1 equal to 0 or also 1 can see that when this omega square equal to omega 1 square or omega square equal to omega 2 square X 1 and X 2 tends to infinity. So, X 1 becomes 0 when 2 k equal to m omega square or omega square equal to 2 k by m. And X 1 and X 2 tends to infinity when omega square equal to omega 1 square or omega square equal to omega 2 square; here, omega 1 and omega 2 are the normal mode frequency of the system. So, these are the normal mode frequency that thing can be written by k by m omega 1 square equal to k by m and omega 2 square equal to 2 k by m.

So, in this way one can find the response of a forced vibration of 2 degrees of freedom system. So, let us know study about the vibration absorber of the system. So, in this problem, in this particular probably we have seen that for some value of omega we are getting this X 1 equal to 0. So, if we have a primary system or primary system with mass m1 and stiffness k1 we can which is subjected to a force of F sin omega t; harmonic force then we can make it is steady state amplitude X 1 equal to 0 at some frequency. So, here we have seen that this frequency omega becomes minus this omega becomes 2 k by m.

So, we can completely observe the vibration of a system by systematically designing this system or by systematically adding another spring and mass to the system; and by suitably arranging the frequency of the system we can find or we can make this response of the primary system equal to 0. So, let us so this is the principle of vibration absorber, which we are going to study in detail now.

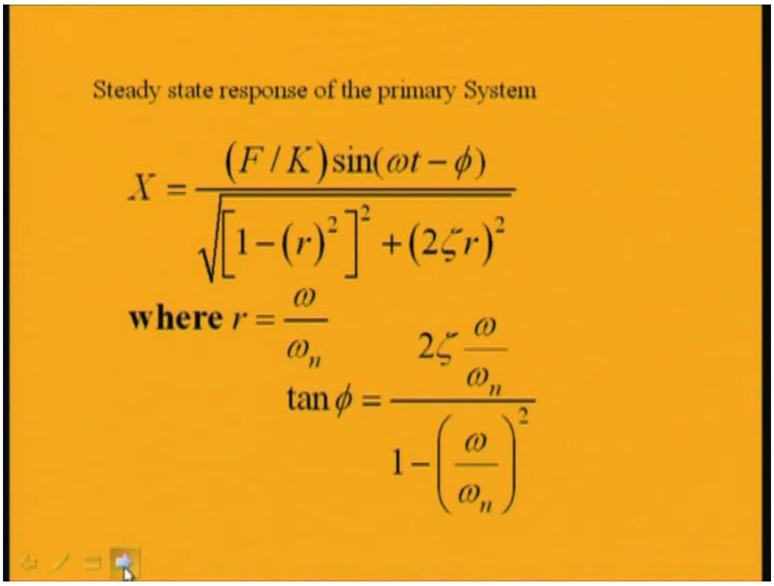
(Refer Slide Time: 21:11)



So, this is a primary system; so we have many machineries which can be modeled as a single degree of freedom system. So, this is which can be modeled as spring and mass system or spring and mass damper system so this is the primary system. Now, this primary system let it is subjected to a harmonic force of F sin omega t; to absorb this vibration we have seen in the previous example we have taken that as a 2 degrees of freedom system where are we have added or we have 2 spring and 2 mass.

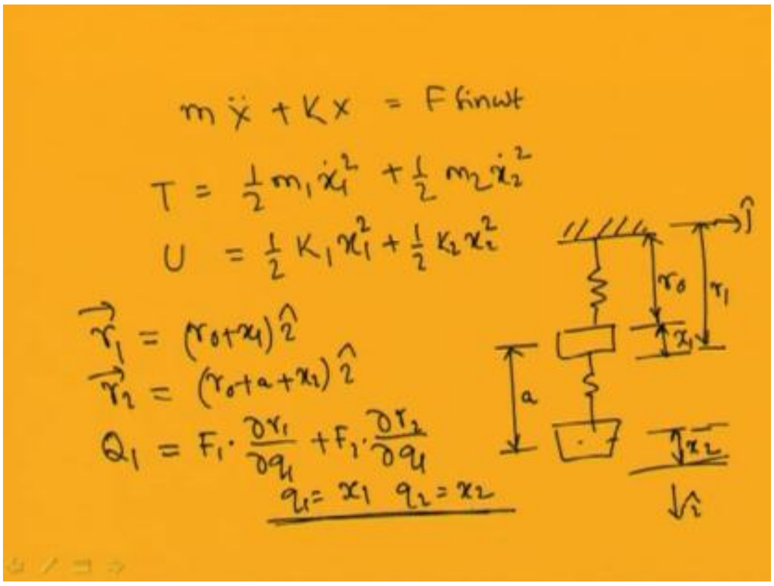
So, from this example we have seen that we can absorb that vibration by adding some spring and mass. So, here in this primary system; so I have added another spring and mass and I will find the general expression for this vibration absorber. So, let k 1 with this spring concerned of this primary system and even in the mass of these and the additional absorber has it is spring constraint or stiffness of k 2 and mass of m 2; this primary system is subjected to a force of F sin omega t.

(Refer Slide Time: 22:28)



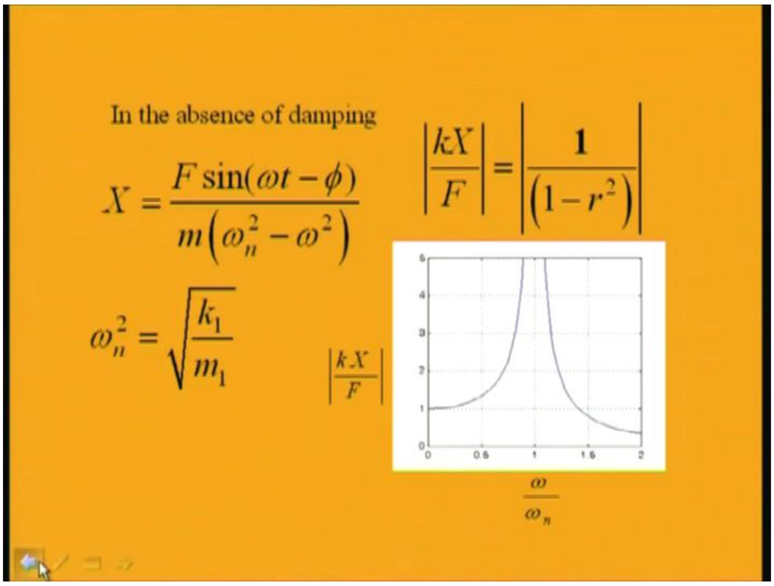
So, the equation motion of the system can be written using this Lagrange principle or Newtons method and that thing can be written.

(Refer Slide Time: 22:37)



So, that thing can be written in this form M x double dot plus k x equal to F sin omega t which will see in a few minutes after. So, in this case of primary system already we know that the response of the system when we subjected to a force F sin omega t can be written in this form. So, if there is some damping present the steady state response of the system can be written in this form X equal to F by k sin omega t minus pi by root of r 1 minus r square whole square plus 2 zeta r. So, where phi is the phase, difference between these response and the forcing frequency or forcing term. And this r equal to the frequency ratio that is the external frequency and the natural frequency of the system. And this phi can be given tan phi equal to 2 zeta omega by omega n 1 minus omega y 1 minus omega y omega n whole square; but as we are studying an undammed system or system without damping.

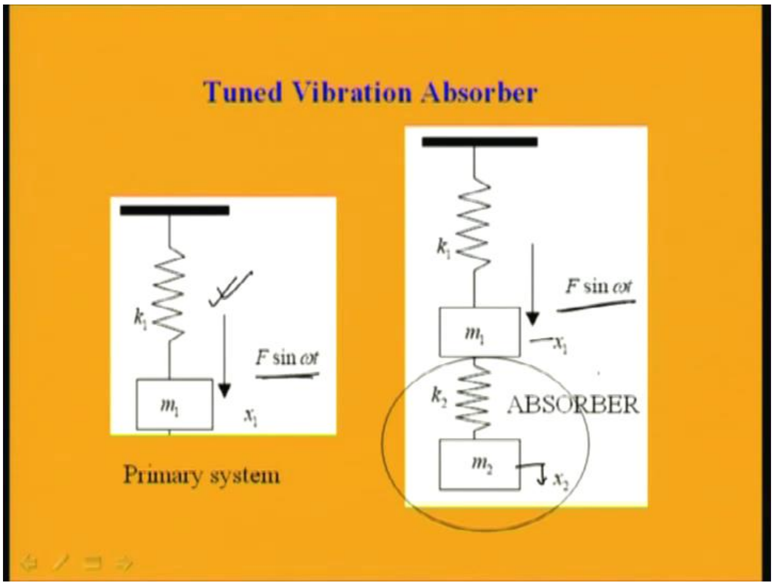
(Refer Slide Time: 23:50)



So, in this case I can put this zeta equal to 0 and I can write this X in this form. So, X becomes F sin omega t minus phi by m into omega n square minus omega square or I can write this expression in this form. So, I can divide this expression by K so I can divide k here and here; so this becomes F by k and m by k equal to omega n square. So, I can write this F by k equal to 1 by 1 minus r square; this already we known this X by F by k, is the static deflection already you know so that is equal to X 0. So, this X by X 0, 1 can plot with respect to this omega by omega n. And one can see that at omega equal to omega n that is omega by omega n when it becomes 0; the response tends to infinity. The system will have very large amplitude of vibration at omega equal to omega n.

So, when the system natural when the excitation frequency equal to the natural frequency of the system; in the absence of damping the system will regenerate and it will tends to the response tends to infinity. So, to absorb this vibration 1 we should add this absorber and when we are adding that absorber; so we can write the equation motion to find that equation motion we can use different method. So, let us use this Lagrange method to find that equation motion. So, in this case the kinetic energy of the system can be written as half m1, x1 dot square plus half m 2 x 2 dot square. And the potential energy I can write it equal to U equal to half k into… So I will write K1.

(Refer Slide Time: 25:51)

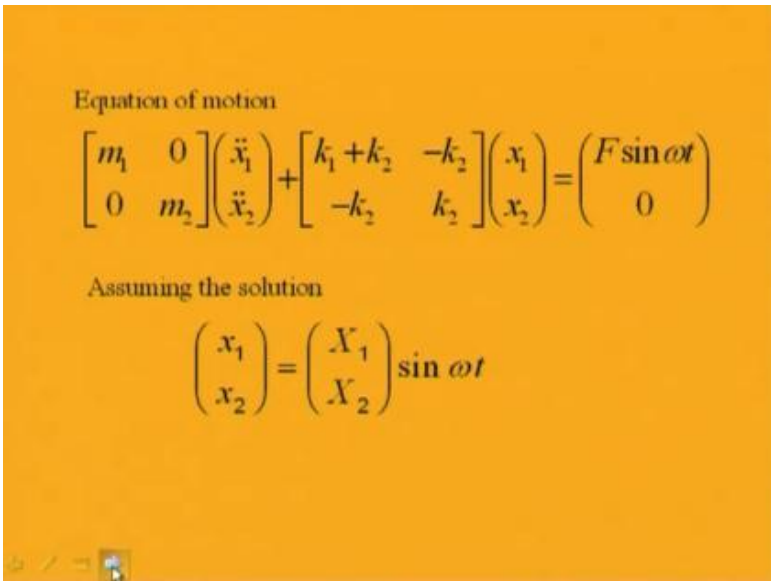


So, this k 1 so in this figure this has the displacement of x 1 and the second mass as a displacement of x 2. So, this spring k 2 is subjected to a displacement which is relative displacement that is x 1 minus x 2 and the spring k 1 has a displacement of x 1. So, the potential energy becomes half k 1 into x 1 square and for potential energy of this becomes half k 2 into x 1 minus x 2 whole square; you may note that this x 1 minus x 2 whole square equal to x 2 minus x 1 whole square. So, you need not have to bother about the sign whether the x 1 is greater than x 2 or x 2 is greater than x 1 when you are applying this Lagrange principle.

So, in this case you can write the potential energy equal to half k x 1 square plus half k 2, x 1 minus x 2 whole square. So, you can write this k 1, x 1 square plus half k 2, x 2 square. Now, to derive the force, forcing function or generalized force so this is the system; so in this system like previous case also I can take this physical coordinate r 0. So, it has a displacement later it has to come to this position after displacement so this is x 1; similarly, this is the equilibrium position of this and it has come to this position this is x 2. And previously like previous case I can write this as a; so I can write this physical coordinate as r 1.

So, these 2, these as r 1 so these physical coordinate I can write. So, r 1 becomes r 0 plus x 1 so I can take this coordinate so this direction as I and this direction or J. So, this becomes r 1 plus r 0 plus x 1i and r 2 becomes r 2 vector, r 0 plus a plus x 2 i. So, I can find this Q K; that is the generalized force in this case similar to the previous case. And you can find this Q 1 equal to f 1 dot Del r 1 by Del q1 plus F 2 into Del r 2 by Del q 1. So, here q 1 equal to in this case similar to the previous case here x q 1 equal to x 1 and q 2 equal to x 2.

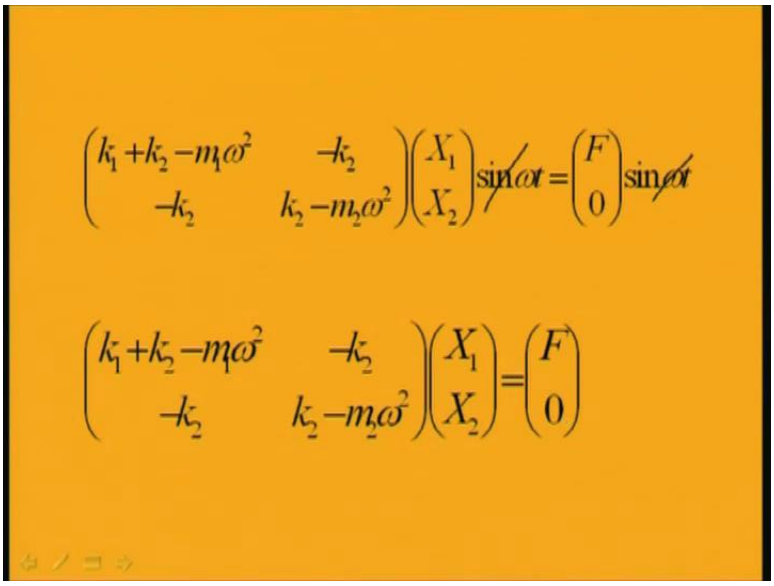
(Refer Slide Time: 28:45)



So, by applying the Lagrange principle you can find the equation motion which can be written in this 1. So, this equation motion reduce to m 1, 0 0, m 2, x 1 double dot x 2 double dot plus k 1 plus k 2 minus k 2, minus k2 here; k 2 and x 1, x 2 equal to x sign omega t 0. And similar to the previous case I can assume that this is the force vibration; so the response frequency will be same as these forcing frequencies. So, I can assume a solution this x 1, x 2 equal to X 1, X 2 sin omega t. So, the frequency of the response equal to the frequency of the forcing function.

So, you may note the forcing function has a frequency omega and here the solution also you have assumed with a frequency omega. So, this x 1, x 2 can be written as X 1, X 2 sin omega t. So, by substituting this equation in this equation I can write k 1 plus k 2 minus m 1 omega square minus k 2, minus k 2; and k 2 minus m 2 omega square into X 1, X 2 sin omega t will becomes F sin omega t 0.

(Refer Slide Time: 30:05)



And, I can delete this omega t terms; so it will reduce to this form. So, the equation becomes k 1 plus k 2 minus m 1 omega square minus k 2 minus k 2, k 2 minus m omega square X 1, X 2 sin omega t this becomes F 0 sin omega t. So, this sin omega t term can be deleted; so this cancels. So, this equation becomes k 1 plus k 2 minus m 1 omega square minus k 2, minus k 2; k 2 minus m 2 omega square X 1, X 2 equal to F 0. So, previously I use this inverse method to find X 1, X 2. Now, I may use this Cramers rule to find this X 1, X 2. So, you know while finding using Cramers rule this X 1 when you are finding X 1 you can replace this column this first column by F 0. And when you are from this matrix; so you just replace when finding X 1 you replace the first column by F 0 and when you are finding X 2 replace the second column by F 0.

(Refer Slide Time: 31:17)

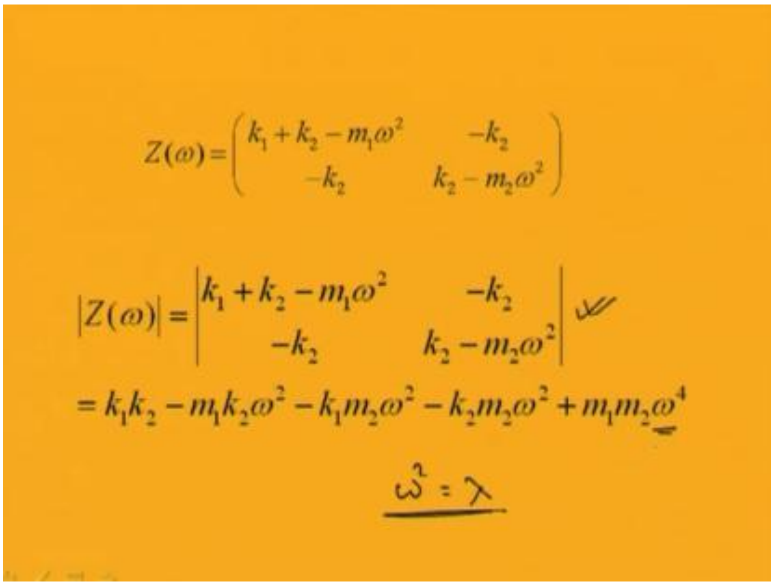
$$X_1 = \frac{\begin{vmatrix} \begin{pmatrix} F & -k_2 \\ 0 & k_2 - m_2\omega^2 \end{pmatrix} \end{vmatrix}}{\begin{vmatrix} \begin{pmatrix} k_1 + k_2 - m_1\omega^2 & -k_2 \\ -k_2 & k_2 - m_2\omega^2 \end{pmatrix} \end{vmatrix}} = \frac{(k_2 - m_2\omega^2)F}{|Z(\omega)|}$$

$$X_2 = \frac{\begin{vmatrix} \begin{pmatrix} k_1 + k_2 - m_1\omega^2 & F \\ -k_2 & 0 \end{pmatrix} \end{vmatrix}}{\begin{vmatrix} \begin{pmatrix} k_1 + k_2 - m_1\omega^2 & -k_2 \\ -k_2 & k_2 - m_2\omega^2 \end{pmatrix} \end{vmatrix}} = \frac{-k_2 F}{|Z(\omega)|}$$

So, using Cramers I can find X 1 equal to F 0 minus k 2, k 2 minus m 2 omega square by determinant of this matrix. So, while finding X 1 you just recall that using Cramers rule you have to replace this first column by F 0 and take the determinant of that then divide it by in the determinant of this Z omega; so this thing I am taking as Z omega. So, you can write X 1 equal to F 0 minus k 2; this is k 2 minus m 2 omega square by determinant of k 1 plus k 2 minus m 1 omega square minus k 2 this is minus k 2, k 2 minus into omega square or this thing becomes. So, determinant of this becomes k 2 minus m 2 omega square into f minus 0; so this becomes k 2 minus m 2 omega square F by determinant Z omega.

And, X 2 can be obtained from this also so when you are getting X 2; so just replace this second column by F 0. So, this becomes k 1 plus k 2 minus 1 omega square minus k 2, F 0. So, this is the thing by the determinant of this so this as it is multiplied by 0; so this first term becomes 0 and second terms equal to minus k 2 F. So, this becomes minus, minus; so plus k, k 2, k 2, F by Z omega. So, determinant of Z omega so you can see that this X 1. So, from this expression you can see that X 1 becomes 0 when k 2 minus m 2 omega square equal to 0; k 2 minus m 2 omega square when becomes 0, X 1 equal to 0.

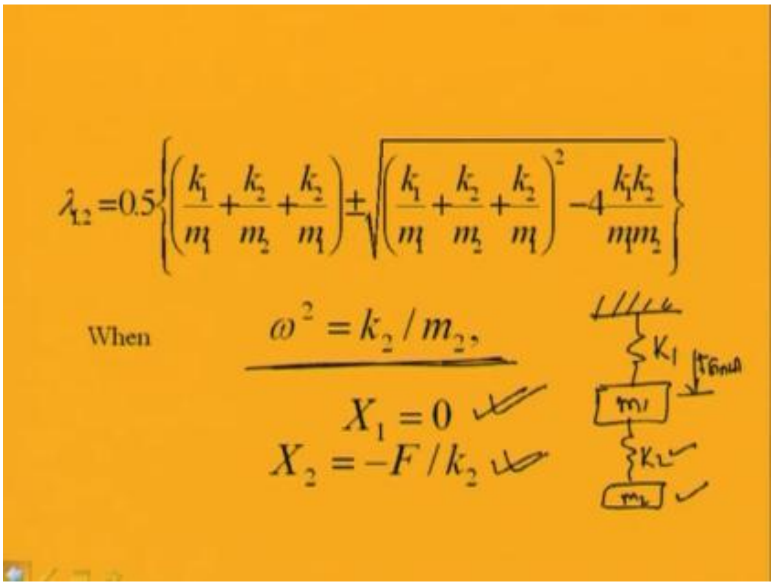
(Refer Slide Time: 33:23)



So, from these we can write or we can find the principle of vibration absorber; before that let us find the determinant of the Z omega. So, Z omega so this is the Z omega matrix to determinant of Z omega can be written in this form. So, this becomes k 1, k 2 minus omega m 1, k 2 omega square minus k 1, m 2 omega square minus k 2, m 2 omega square plus m 1, m 2 omega 4^{th}. You may recall that this Z omega matrix is same as the matrix; you have taken for the free vibration analysis of 2 degrees of freedom system. So, this matrix or the determinant of this also can be written in terms of the normal modes or normal mode frequency of the system.

So, either the normal frequency of the system is omega 1 or omega 2. So, this expression you can write in terms of omega square minus omega 1 square. So, this thing you can write so if you take this; so you can write this in the form of omega square minus 1 square into omega 2 minus omega 2 square multiplied by some constant.

(Refer Slide Time: 34:31)

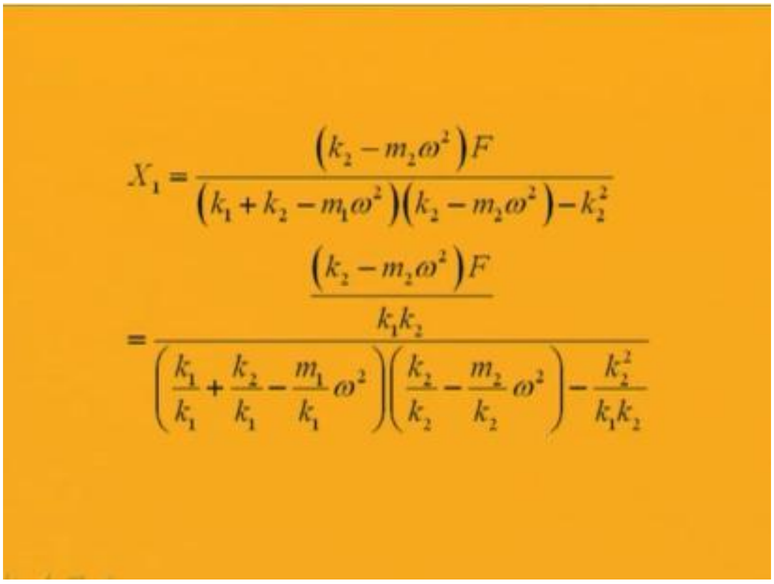


So, now I can write this Z 1; so this Z 1 to find this normal mode I can find the I can solve this equation and find the frequencies. So, this frequencies can be obtained from this equation I can substitute these omega square equal to lambda these are the Eigen values. So, by substituting this omega in omega square equal to lambda, this reduces to a quadratic equation. So, in this quadratic equations or solving these quadratic equation you can find the normal mode frequencies of the system. So, the normal mode frequencies of the systems are lambda 1 to equal to 2.5 k 1 by m 1 plus k 2 by m 2 plus k 2 by m 1 plus minus root k 1 by m 1 plus k 2 by m 2 plus k 2 by m 1 whole square minus 4 k 1 k 2 by m 1, m 2.

So, from the previous equation here already I told you that X 1 becomes 0 when you are taking omega square equal to k 2 by m 2. So, when omega square equal to k 2 by m 2; X 1 becomes 0 and X 2 becomes minus F by k 2. So, this is the principle of vibration absorber that is when you are adding a additional mass and stiffness to the original system; let the you have a original system this is the primary system. So, let this is the primary system with m 1 and k 1 I am adding another system secondary system with mass m 2 and k 2. So, if I add this m 2 and k 2 in such a way that so this is subjected to a force F sin omega t. So, if I will add a mass and stiffness; in such way that this omega square that is the frequency of external excitation becomes square of the frequency of external excitation becomes k 2 by m 2.

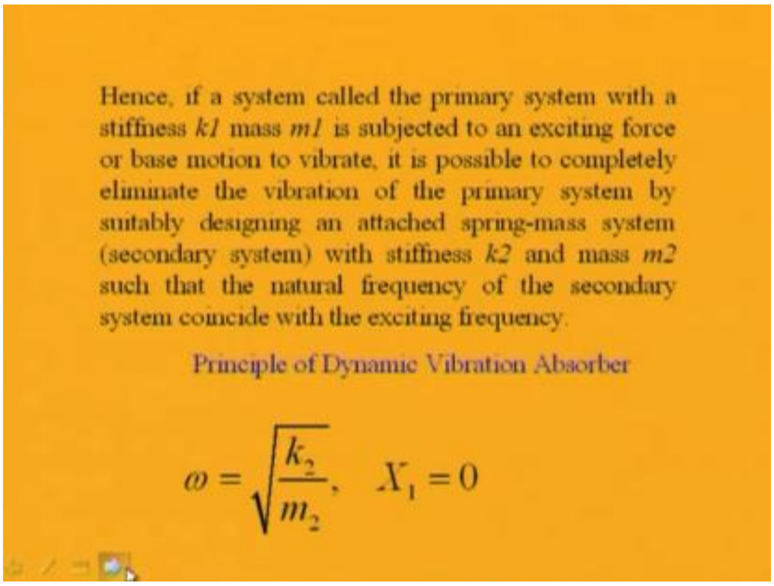
Then, these vibrations of the first mass will be completely absorbed; that is X 1 equal to 0. And in that case you can see that F 2 when you are substituting omega square equal to k 2 by m 2 that time you may check that this becomes this expression Z omega; you can find and you can see that this X 2 becomes minus F by k 2. So, the displacement of the second mass is limited by, this is limited by the stiffness of the second spring k 2. And it shows that this force; whatever force we have applied to the first mass is completely absorbed by k X 2. So, F becomes k X 2 and it shows that it is completely absorbed by the second secondary system that is k 2, m 2 what we have added to the primary system.

(Refer Slide Time: 37:52)



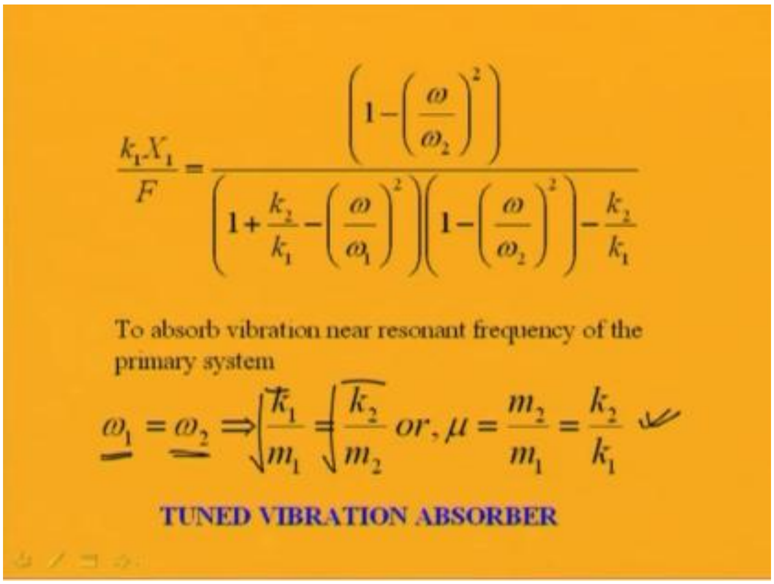
So, already we have written this X 1 equal to k 2 minus m 2 omega square F by the Z omega; can be written as k 1 plus k 2 minus m 1 omega square into k 2 minus m 2 omega square minus k 2 square or if I will divide by k 1, k 2.

(Refer Slide Time: 38:19)



I can divide it and I can simplify this expression in this form and already I told you about the principle of vibration absorber. So, to absorb the vibration of a primary system; we have to add a secondary system in such way that the external frequency square will becomes k 2 by m 2, where k 2 and m 2 are the stiffness and mass of the secondary system.

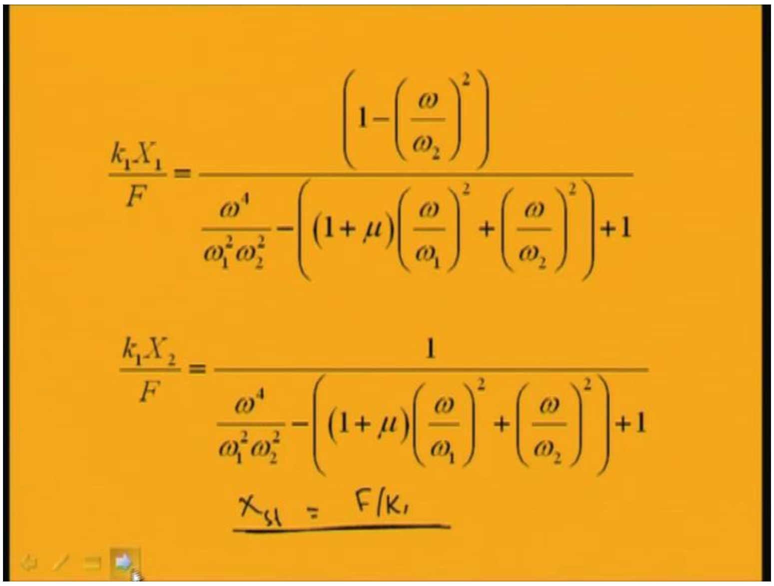
(Refer Slide Time: 38:44)



So, as you have seen before that we are interested at the resonant frequency that is when the system the natural frequency of the primary system becomes equal to the external frequency. So, that time resonance occurs and we want to suppress that vibration at resonant frequency to suppress the vibration at resonant frequency. Hence, we should write the frequency equation in this form that is omega 1. Already, we know that omega 1 equal to root over k 1 by m 1 and should be equal to omega 2 that is root over k 2 by m 2. So, to absorb the vibration near the resonant frequency we can write this omega 1 that is natural frequency of the primary system equal to the natural frequency of the secondary system; that is root over k 2 by m 2 equal to the frequency of the external excitation.

And, let me put these mu as the mass receive that is mass of the secondary system by the mass of the primary system. And as k 2 root over k 1 by m 1 equal root over k 2 by m 2 or k 1 by m 1 equal to k 2 by m 2. So, I can write this mass received equal to m 2 by m 1 equal to k 2 by k 1. So, by substituting this equation in the previous equation that is k 1, X 1 by F.

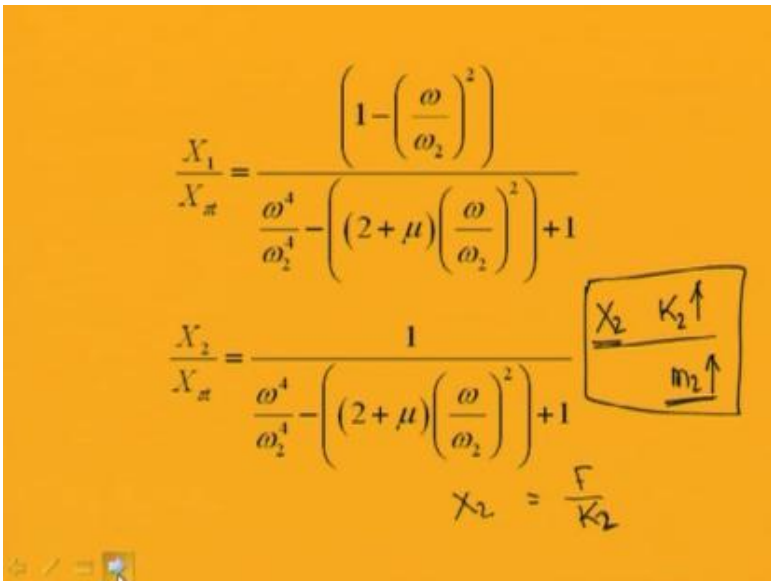
(Refer Slide Time: 40:21)



So, by substituting this equation in the previous equation that is k 1, X 1 by F. I can write this expression k 1, X 1 by F equal to 1 minus omega by omega 2 squares by omega 4th by omega 1 square, omega 2 squares minus 1 plus mu into omega by omega 1 whole square plus omega by omega 2 whole square plus 1. You can do, you can find this from this already I have written this thing k 1, X 1 by F equal to 1 minus omega by omega 2 squares. So, this thing can be obtained by deriving the previous equation by k 1, k 2. So, if you divide this expression by k 1, k 2; so this k 2, k 2 cancel and if you rearrange this terms. So, you can write this expression in this form.

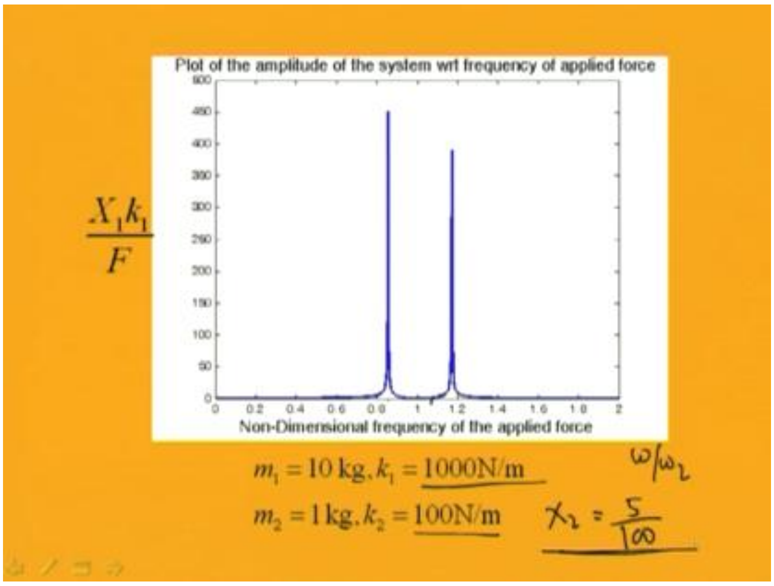
So, it can be written in this form, we should write in this form k 1, X 1 by a F equal to 1 minus omega by omega 2 whole square by 1 plus k 2 by k 1, k 2 by k 1; already I told you this is can be written in terms of mu. So, this becomes 1 plus mu minus omega by omega 1 square into 1 minus omega by omega 2 square minus this k 2 by k 1 is written in terms of mu. So, you can write this expression in this 1 and k 2, X 2 by F can be written by 1 by omega 4th by omega 1 square, omega 2 squares minus 1 plus mu omega by omega 1 square plus omega by omega 2 squares plus 1. So, this frequency receive this magnification factor that is k 1, X 1 by F is written in terms of the natural frequency of the primary system that is omega 1 and the natural frequency of the secondary system that is omega 2 and the mass received mu.

(Refer Slide Time: 42:14)



So, by using this I can write X 1 by X s t; X s t is nothing but F by X 1 that is the static deflection of this primary spring or the primary system. So, X 1 by X s t equal to 1 minus omega by omega square whole square by omega 4th minus omega 2 square minus 2 plus mu into; so 2 plus mu into omega by omega 2 whole squares plus 1. Similarly, X 2 by X s t in X s t also it is equal to F by k. So, this becomes X 2 by F by k. So, this X s t is nothing but F by k 1 that is the initial or the static deflection of the spring k 1. So, from this expression this X 2 by X s t can be written by 1 by the lower expression. So, this becomes X 2 by X s t equal to 1 by omega 4th by omega 2 squares minus 2 plus mu into omega by omega 2 whole squares plus 1.

(Refer Slide Time: 43:32)



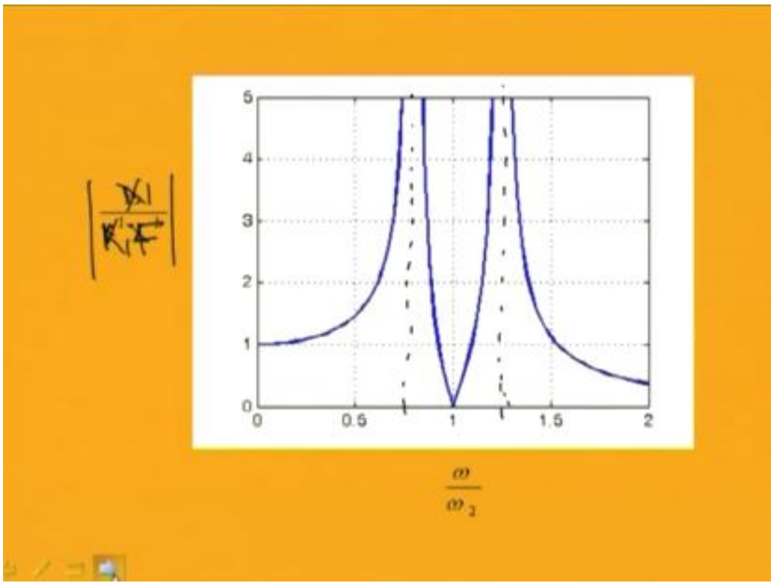
And, using this expression 1 can plot; so it is plotted. So, we have taken 1 example where m 1 equal to 10 kg, k 1 equal to 1000 Newton meter, Newton per meter m 2 equal to 1 kg and k 2 equal to 100 Newton per meter. So, in this case it is assumed that; so m 2 by k 2 use a see m 2 by k 2 becomes100 and this k 1 by m by k 1 also it becomes 100. So, in both the cases I have taken this omega 1 equal to omega 2 and when this. So, here I have plotted this with respect to omega by omega 2; so this is omega 2 also equal to omega 1. So, when so you can see that previously you have seen that when omega becomes omega 1 that is omega by omega 1 equal to 1; you had seen the system had a the system had a huge response. The system had this; the response of the system became infinite in that case.

So, this dotted line show the response of the system, primary system when omega equal to omega 1. But in this case you have seen that this becomes 0, this becomes completely equal to 0; this response becomes completely equal to 0 at omega equal to 1.So, in this way we have absorbed the vibration at the resonant frequency of the system. But while absorbing this vibration you can absorb that this resonant frequency are this resonant frequency is shifted to 1 is shifted to left side and right side. So, now the resonant frequency becomes 0.8 around 0.8 and 1.2; so instead of hugging 1 resonant frequency at omega equal to 1. So, now we have 2 resonant frequencies that is 1 at 0.8 around 0.8 and other at 1.2.

As we are absorbing this vibration at this particular frequency omega equal to omega 1; this type of absorber are known as tuned vibration absorber. So, in case of tune vibration absorber it absorb the vibration at a particular frequency; also it depends on the mass receive of the system. I have already written this expression of this X 1 k 1 by F in terms of mass receive also you have absorbed that this X 2 expression for X 2 that is X 2 becomes equal to F by k 2. So, at resonant frequency that is when omega equal to omega 1 equal to omega 2 though X 1 become 0, X 2 becomes; so X 2 becomes F by k 2.

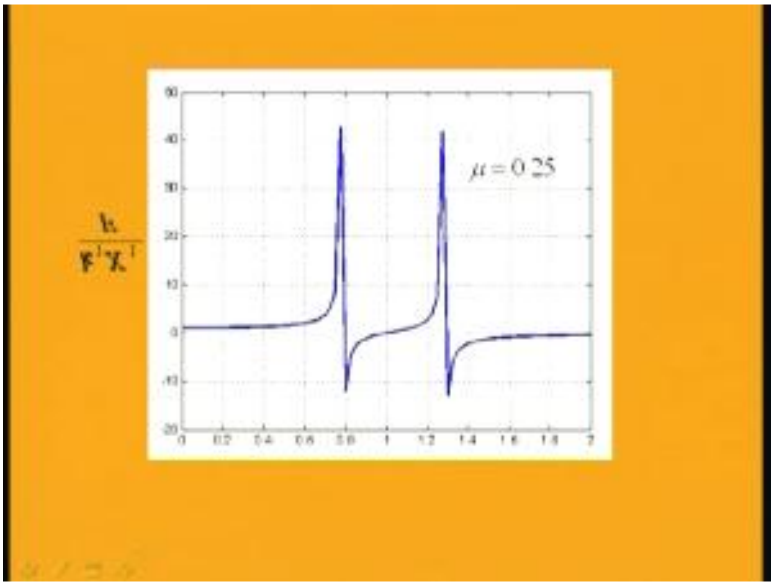
So, in this case if we are taking is stiffness of 100 Newton meter; if you are applying a force let we are applying a force of 5 Newton then will have this X 2 will becomes 5 by 100. So, 5 by 100 meter; so it will have a huge the response, the second mass or the secondary mass will undergo a very large undergo a large deflection. So, to avoid that large deflection we have to increase the value of k 2. So, if we are increasing the value of k 2 without changing this value of m 2; so that time it will not be a absorber. So, we have to increase this value of m 2 also to keep omega 2 constant. So, to reduce k 2 we have to; so to reduce X 2 to reduce X 2 we have to increase k 2. So, have to increase k 2; so when we are increasing k 2 simultaneously we have to increase m 2.So, if we are increasing this m 2 also then we may need a large secondary mass to absorb the vibration. So, this is the limitation of this tuned vibration absorber.

(Refer Slide Time: 48:17)



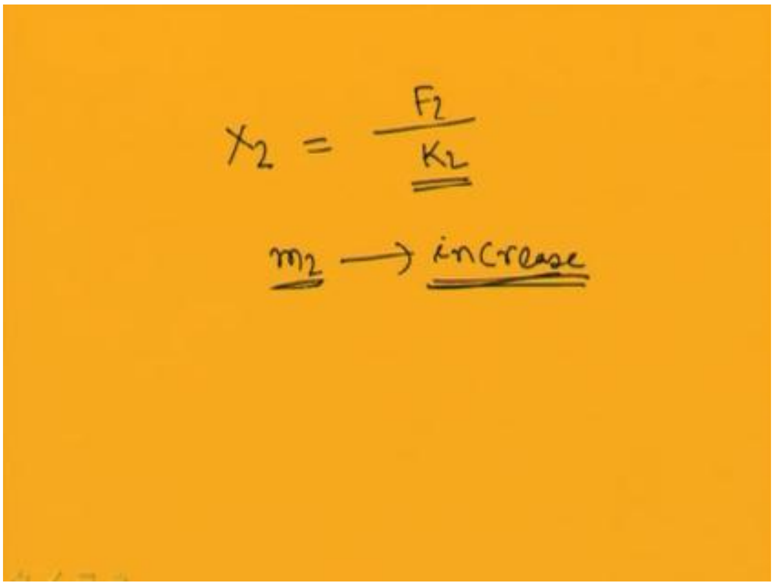
So, if you plot this response already we have plotted this and if we see this thing again if you write this is equal to. So, this becomes X 1, k 1 by F that X 1, k 1 by F. So, this expression X 1, k 1 by F this is omega by omega 2. So, here you have not clearly seen this response. So, in a clearer, clear form it is plotted here; so you can see this response. So, at omega equal to omega 2 equal to 1 you can absorb that this response becomes 0; and here the response. But the resonant frequency is shifted to this. So, in this case the absolute value of the response is plotted that is X 1 by k 1 F.

(Refer Slide Time: 49:19)



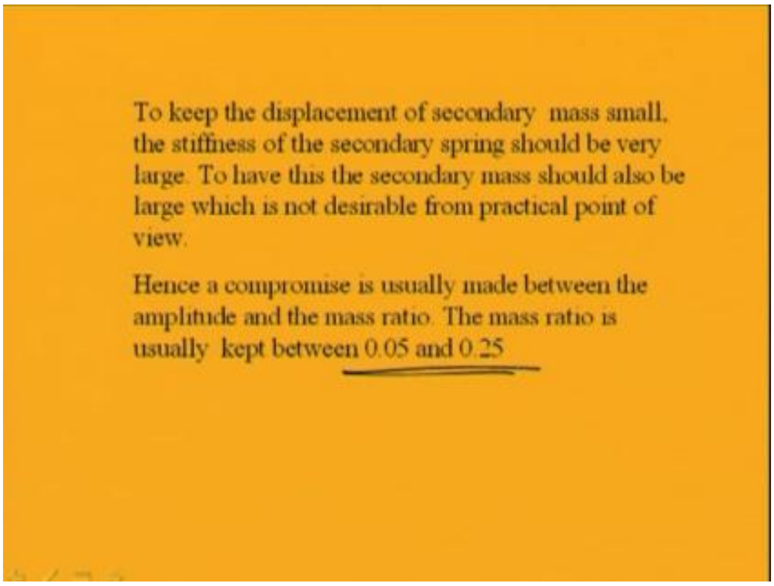
So, if are not plotting the absolute value; then the response curve will look like this. So, this will be the response curve for a particular value of mu. So, this is plotted for mu equal to 0.25.

(Refer Slide Time: 49:33)



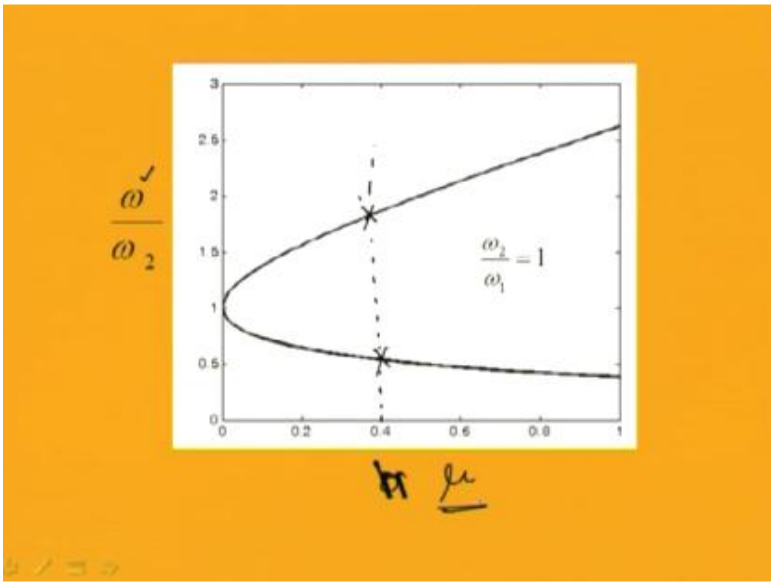
So, already I told you to half less value of X 2; so which is equal to F 2 by k 2. So, you have to increase this stiffness. So, to increase this stiffness you have to increase this mass received mass m 2. So, to increase mass, to increase the secondary mass 1 should have the secondary system will have a very huge mass and that will be the limitation of the vibration absorber. So, 1 should have to compromise this value of this mass m 2.

(Refer Slide Time: 50:12)



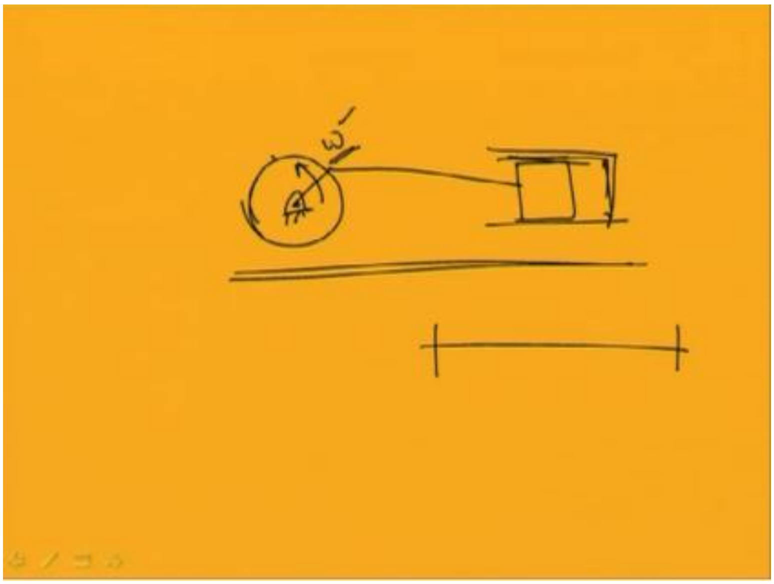
So, the 1 should take the mass receive in such way that are 1 can compromise this thing usually and keep this receives usually kept between 0.0 5, 0.2 5. So, by keeping this mass received between 0.0 5 and 0.2 5; X 2 can be limited a certain extend and X 1 can be made to 0.

(Refer Slide Time: 50:36)



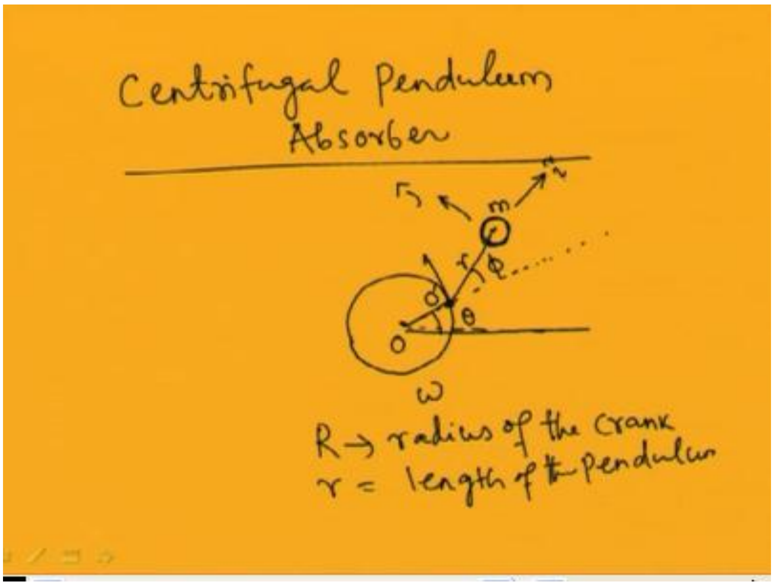
So, if 1 plot this mass received, these omega by omega 2 verses mass received 1 can obtain this curve or this omega to by omega 1 equal to 1. So far tuned absorber can absorb that you have 2 frequencies that is this frequencies omega at 2 points. So, the resonant will occur at this 2 frequencies corresponding to this frequency and this frequency for a particular value of mu. So, by using a tuned vibration absorber you can absorb the vibration of the system at a particular frequency. But in many machines like this IC engines the resonance occur resonance frequency, resonant frequency is a function of the frequency of the excitation of the system.

(Refer Slide Time: 51:44)



So, in case of a IC engine. So, let me draw for IC engine; so you have a crank and this is the piston cylinder arrangement. So, in this piston cylinder arrangement; so due to the unbalance force of this reciprocating mass the system has a unbalanced frequency, unbalanced force with multiple frequency which are proportional to this rotation of this crank. So, if the crank is rotating with frequency omega and you are changing the frequency of the crank; then one can find the unbalanced force at a wide range of frequency. So, in this case to avoid this frequency or avoid this resonance, resonant conditions or to avoid this unbalanced force. So, one will not be able to use this tuned vibration absorber or the tuned vibration absorber is applicable for a particular frequency. So, it cannot be applicable for a system in which the unbalance forcing frequency is a function of this rotating frequency.

(Refer Slide Time: 53:09)



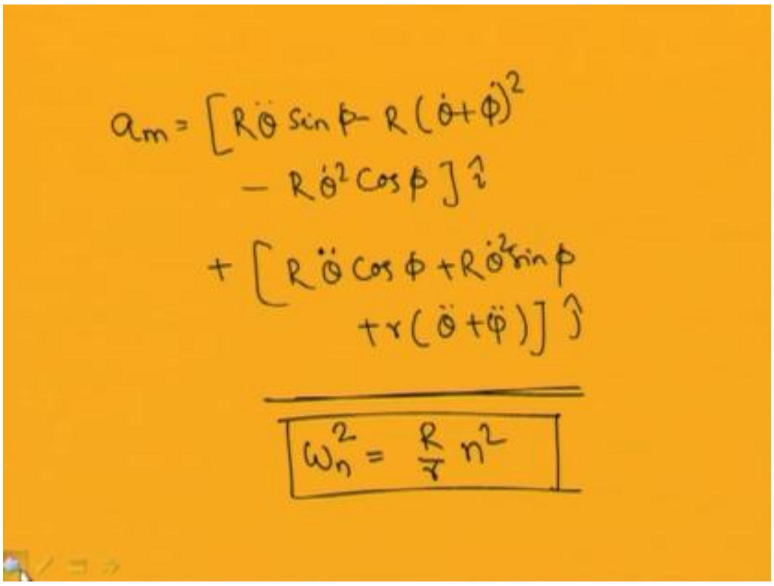
So, in that case 1 should go for a pendulum centrifugal pendulum absorber. So, this is the disk or this is the crank rotating with frequency omega. So, let it is rotating with frequency omega and we have to add a pendulum at any point. So, on this let us add this pendulum; so this is a simple pendulum with mass m and it is it has a length r. So, in this case I can write, so let me take; so let theta is the rotation or time t, theta is the rotation of the crank or the disk and phi is the rotation of this pendulum. So, original in case of phi very large system will be a non-linear system; but we limit our analysis to a linear system by assuming this phi is the very small.

So, in this case I can write this pendulum has a mass m and it is rotating with phi. So, I can find the force of this pendulum. So, the force of the pendulum can be obtained by finding the acceleration of the pendulum. So, the acceleration of the pendulum can be written in this form. So, the acceleration of this pendulum equal to the acceleration of point let this is origin O and this point is O dash; so O dash is the point at who is this pendulum is attached. So, the acceleration of point m or mass m can be written as the acceleration of point O dash plus acceleration of m with respect to O dash. So, this acceleration can be divided in to 2 parts.

So, let me put a coordinate system this is along this friction and perpendicular to this j. And so I can write this acceleration of this mass m with respect to this origin O dash and it can be written in this form. So, let this R is the radius of the crank. So, if r is the radius of crank and small r is the length of the pendulum. So, point o dash will have the acceleration. So, velocity, first the velocity of this point equal to angular velocity equal to theta dot and angular velocity of this pendulum becomes theta dot plus phi dot. So, the angular the acceleration of this point if it is rotating with uniform velocity; then this angular acceleration becomes 0. But if it is not rotating with uniform velocity we can write this angular acceleration equal to theta double dot.

So, I will have 2 component of this acceleration here. So, this becomes R theta double dot and another component equal to R theta dot square in this direction. Similarly, for this pendulum I will have the acceleration 2 components of the acceleration; one this centripetal component and other will be the radial component.

(Refer Slide Time: 57:09)



And, by using this acceleration I can write this acceleration a m equal to R theta double dot sin phi minus R into theta dot plus phi dot square minus R theta dot square cos phi i plus R theta dot square theta double dot cos phi plus R theta dot square sin phi plus r into theta double dot plus phi double dot j; so this is the acceleration term. So, using this acceleration I can find this force and as this all the pendulum is attach to this point the moment of this pendulum about this point equal to 0. So, I will have only the j component and from the j component I can find the natural frequency of the system.

So, by deriving in that way I can find the natural frequency of the system; will becomes or in the next class I will I am going to show you that the natural frequency of the system becomes this equal to R by r into n square by n; where, n is the rotational speed of this crank. So, in this way you can find; I have told you about 2 different types of absorber, 1 is the tuned vibration absorber and other one is the centrifugal pendulum absorber. In the next class, I will tell about this centrifugal by pendulum absorber in detail. And I will tell how these absorb the vibration at different frequency or at a wild range of frequency.