Mechanical Vibrations

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Module – 5
2 DOF Free Vibrations
Lecture - 4
Forced Harmonic Vibration

Last three classes, we were studying about the 2 degree of freedom systems and we have studied the free vibration response of 2 degree of freedom system. And in this case of 2 degree freedom system we were finding 2 natural frequency and 2 normal modes of the system. Unlike the single degree of freedom system; here you have two natural frequency and already I told you how to derive this equation motion. Equation motion can be derived using Newton's method or D'Alembert principle or by using this Lagrange method or extended Hamilton principle. So, after deriving this equation motion you can find the free vibration response of the system by assuming normal modes of the system. And by assuming this normal mode you can find the natural frequencies of the system.

So, after finding the natural frequencies then you can find the normal mode of the system or you can study the free vibration response of the system. In case of free vibration you can assume that this free vibration of a system is the summation of the normal mode of the systems. So, in this case of 2 degree of freedom system already you have seen for a double pendulum; how to determine the normal modes of the system. And from the normal modes also how to determine the free vibration response of the system; also we have studied the damped free vibration of 2 degree of freedom system; last class we have studied about this.

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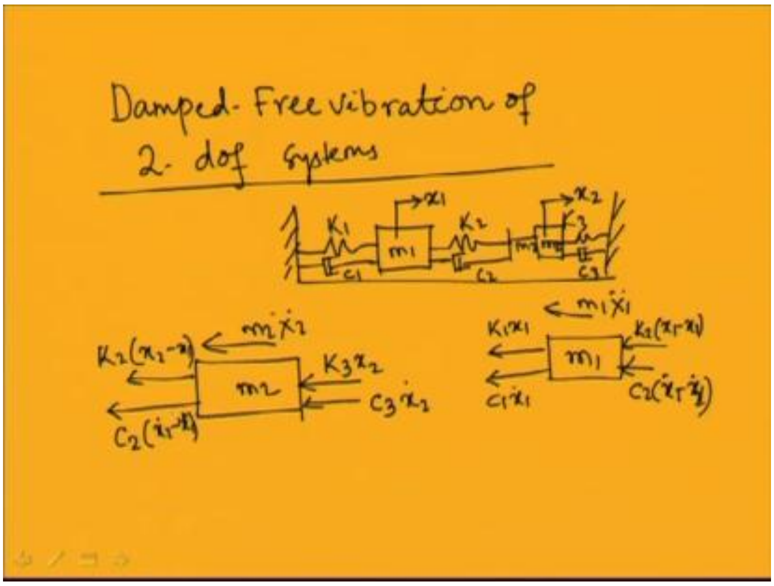

And, I can draw a spring mass damper system with damping. And today class we will study about this will see a example of this damped free vibration of 2 degree of freedom system. And also will study semi definite systems and force vibration of 2 degree of freedom system. So, to start with this damped free vibration of 2 degree freedom system; let us see the example we have taken in the last class. So, I am drawing a spring mass damper system with 3 springs and 2 mass. So, this is mass 1 this is mass 2 and already you have seen. So, this is mass 1, this is mass 2, this is k 1, k 2, k 3 and also we have put some damper here also; so this is damper 1.

So, I have put damper 2 and this is the third damper. So, let me draw it clearly; so this the second mass and spring and I will put a damper also; so this is damper. So, this is c 1 c 2 and c 3. So, in this case you have found the equation motion by assuming the general generalized coordinate x 1 and x 2; x 1 is the generalized coordinate for mass m 1 and x 2 is the generalized coordinate for mass m 2. So, by taking this you can write the equation motion. So, already you have seen that equation motion I have derived using either Lagrange principle or Newton's method. So, you can use the Newton's method for simplicity here. So, you can draw the free body diagram of the system. So, for this mass m 1; if you draw the free body diagram so it will have inertia force of m 1 x 1 double dot; then this is the spring force k 1 x 1 and damping force c 1 x 1 dot. And similarly, for this k 2 and c 2 elements so it will be k 2. So, this spring will be will have a relative motion of x 1 minus x 2; so it will be k 2 x 1 minus x 2 and this is c 2 x 2 dot minus x 1 dot minus x 2 dot. So, in this way you can find the equation motion for the first mass.

So, this will becomes m 1 x 1 double dot plus k 1 plus k 2 x 1 minus k 2 x 2 plus c 1 x 1 c 1 plus c 2 x 1 dot minus c 2 x 2 dot. Similarly, for mass 2 also you can draw the free body diagram. So, for mass two, if will draw the free body diagram. So, this is mass m 2; so the free body diagram. So, this is x 2. So, inertia force m 2 x 2 acting in this direction and this spring force I can write it equal to k 2 into x 2 minus x 1 and damping force equal to c 2 into x 2 dot minus x 1 dot. And this side you have spring force of k 3 x 3 k 3 x 2 k 3 x 2 and damping force of c 3 into x 2 dot. So, your equation motion will becomes m 2 x 2 double dot plus k 2 plus k 3 x 2 minus k 2 x 1 plus c 2 plus c 3 x 2 dot minus c 2 x 1 dot will be equal to 0. So, you have already written the equation motion.

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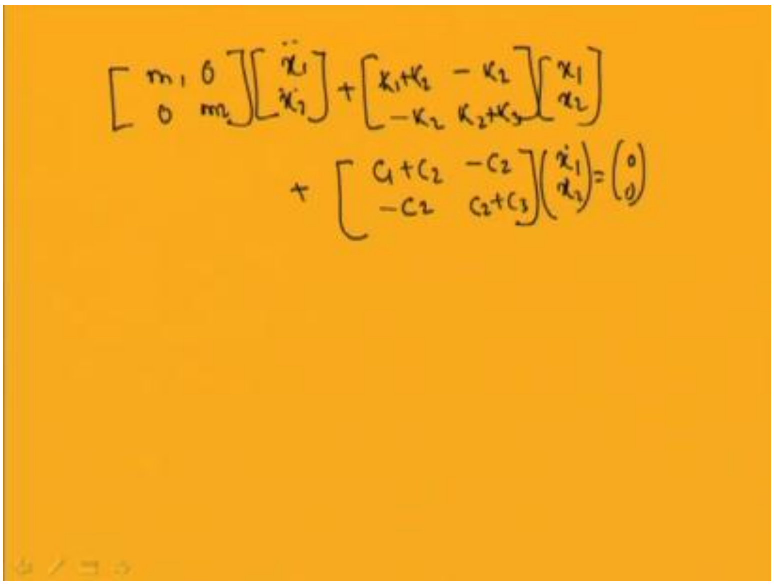

And, you know that equation motion is nothing but so this equation becomes m 1 x 1 m 1 0. So, this becomes m 1 0 0 m 2 x 1 double dot x 2 double dot plus k 1 plus k 2 minus k 2 minus k 2 k 2 plus k 3 x 1 x 2 plus C 1 plus C 2 minus C 2 minus C 2 C2 plus C 3 into x 1 dot x 2 dot equal to 0 0.

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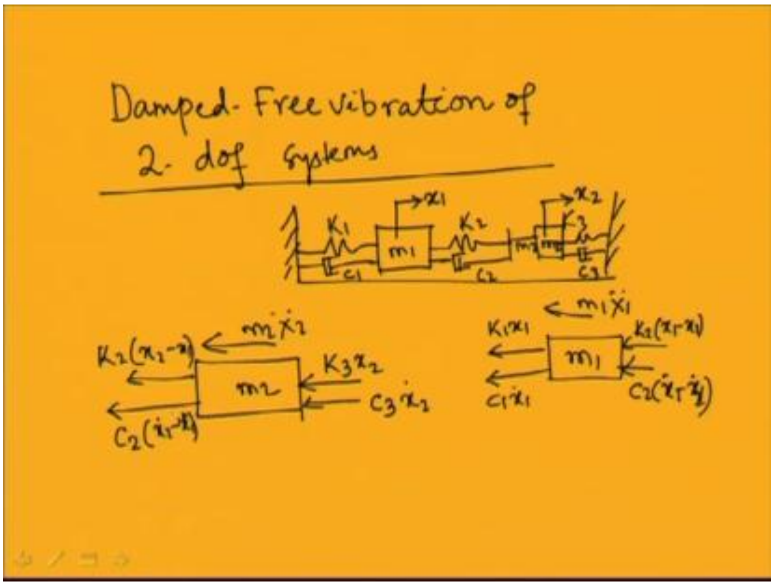

So, from this free body diagram of the system we have determined the equation motion by applying the Newton's method here; also you may apply Lagrange principle to find the this equation motion in that case you can write the kinetic energy of the system. So, kinetic energy of the system equal to half m 1 x 1 dot square and kinetic energy of the second mass equal to half m 2 x 2 dot square. So, total kinetic energy of the system becomes half m 1 x 1 dot square plus half m 2 x 2 dot square and potential energy of the system is due to the springs. So, it becomes half k 1 x 1 square plus half k 2 into x 1 minus x 2 square plus half k 3 into x 2 square.

And, the dissipation energy you can write equal to half c 1 x dot square plus half c 2 x 2 dot minus x 1 dot whole square plus half c 3 x 2 dot square. So, by writing this 3 energy that is the kinetic energy, potential energy and dissipation energy and then applying the Lagrange principle also you will get the same equation of motion.

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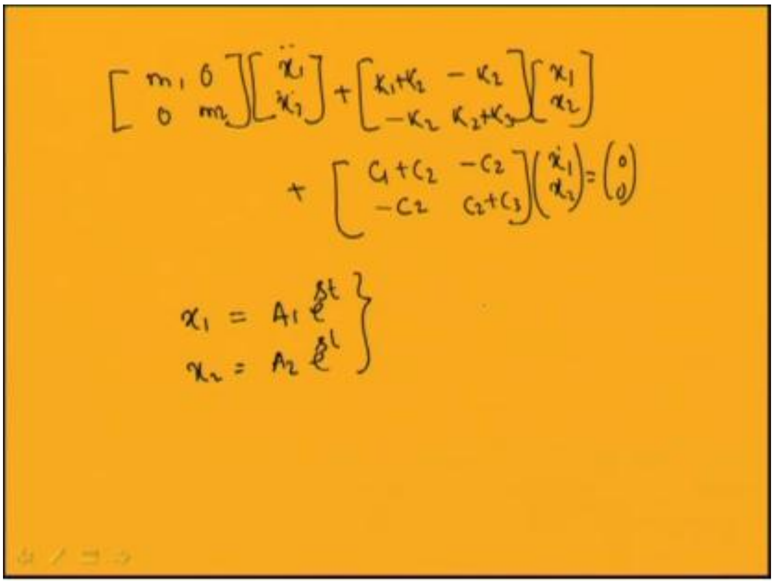

So, after getting this equation motion; so you can assume a solution in this form. So, you can assume a x 1 equal to… So you can write x 1 equal to A 1 e to the power s t and x 2 equal to A 2 e to the power s t. And last class we have seen by applying this equation in this equation in this original equation motion you can reduce this equation to this form.

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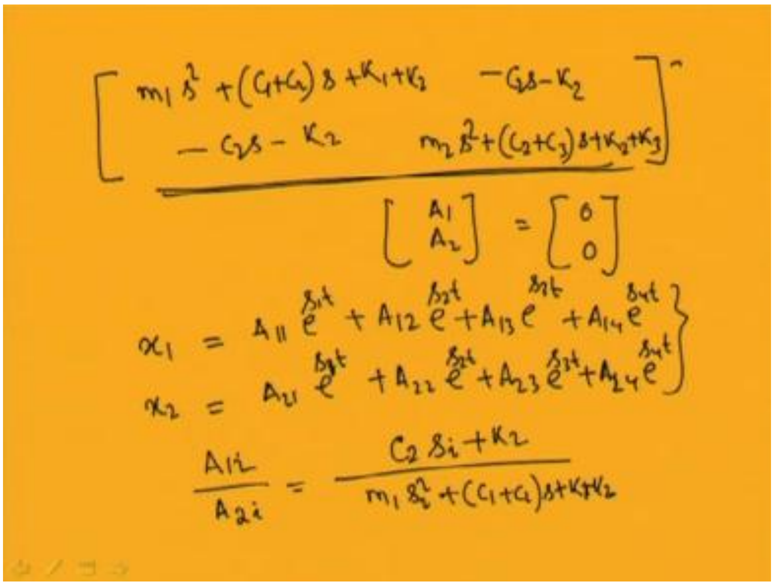

So, this becomes m 1 s square plus C 1 plus C 2 s plus k 1 plus k 2 then minus C 2 s minus k 2 then minus C 2 s minus k 2. And this becomes m 2 s square plus C 2 plus C 3 into s plus k 2 plus k 3 into A 1 A 2 equal to 0 0. So, for non trivial solution A 1, A 2 you can write or you can find the determinant of this matrix will be equal to 0. And from this you can find 4 value of s and after finding this roots you can write the expression for x 1 and x 2. So, x 1 and x 2 will become; so x 1 will becomes A 1 1 e to the power s 1 t plus A 1 2 e to the power s 2 t plus A 1 3 e to the power s 3 t plus A 1 4 e to the power s 4 t. Similarly, you can write x 2 will be equal to A 2 1 A to the power s 2 t e to the power s 1 t plus A 2 2 A to the power s 2 t plus A 2 3 e to the power s 3 t and plus A 2 4 e to the power s 4 t.

So, you can obtain the solution of this damped free vibration system by after finding these roots of this characteristic equation. So, this is the characteristic equation which you got by finding the determinant of this matrix. So, after finding this determinant you can find 4 roots of this and by substituting these roots you can find the equation in this form. So, you can observe that out of this 4 roots some roots may be real or some may be complex also or complex conjugates you can find these roots. So, after finding these roots you can find x 1 and x 2; you may note that this A 1 i and A 2 i are not independent So, they are dependent on each other or linearly dependent. So, after finding this 4 roots from this equation you can write that A 1 I; so if you substitute this thing.

So, this into so for half set of roots. So, you can write A 1 i by A 2 i. So, this into A 1 i into this into A 2 i will becomes 0. So, from this you can write A 1 i by A 2 i is nothing but this is equal to C 2 s I plus k 2 by m i s i square or m 1 m 1 s I square. So, this is m one. So, this is m 1 s m 1 s i square plus C 1 plus C 2 s plus k 1 plus k 2. So, for a stable system all the 4 roots must be either real negative number or complex number with negative real parts. So, you know for a stable system.

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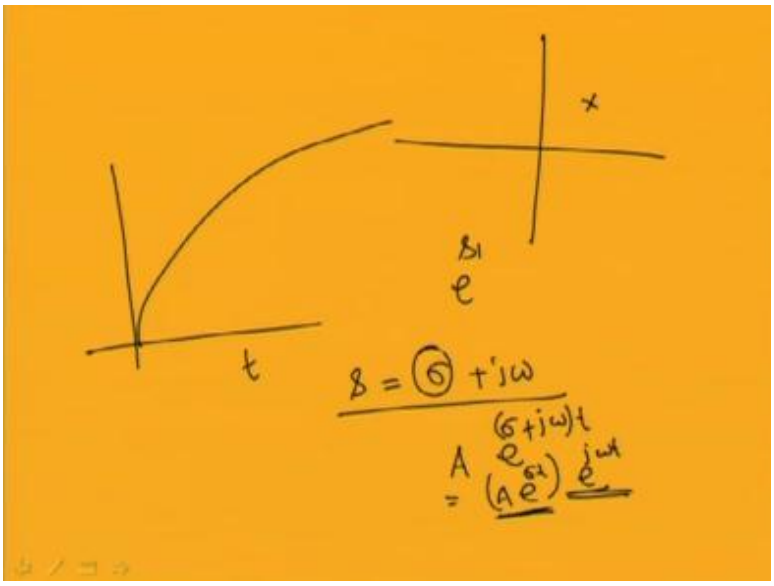

So, for a stable system all roots should be all the roots should have a negative real part; if they have a positive real part then the solution will contain e to the power s 1 t. So, this s let me write this s equal to sigma 1 plus j omega for so as s is a complex number you can write in this form sigma plus j omega. So, if I am writing this s equal to sigma plus j omega if I have a root here so the solution will be in this form. So, it will contain A into e to the power sigma plus j omega t.

So, this will becomes A into e to the power sigma t into e to the power j omega t. So, this part will give the oscillatory motion but the part containing A into e to the power sigma t if sigma is positive will exponentially grow with time t. So, with exponentially it grow; so this part will grow exponential. So, the system will have a unbounded solution. So, for a stable system this sigma should be sigma should be negative. So, for a stable system the root should have negative real parts or a complex conjugate yes a complex conjugate with negative real part or a real number with or a negative real number. So, after finding this roots. So, you can find the solutions.

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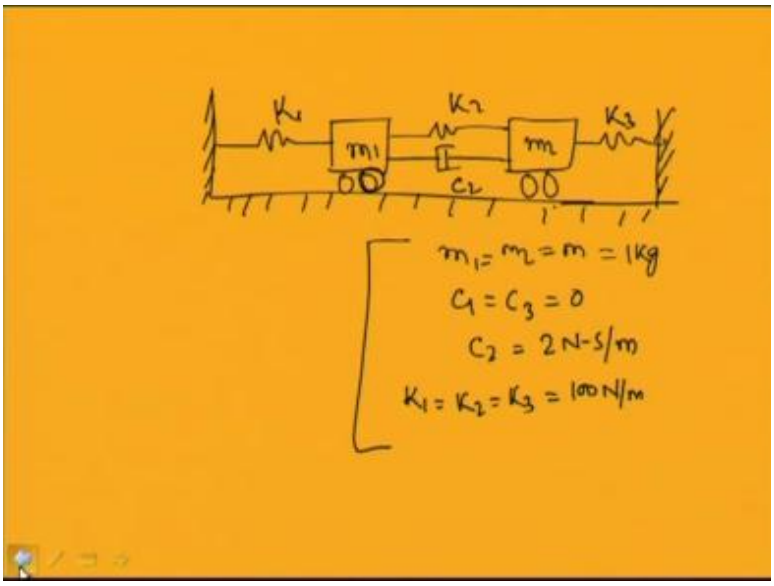

And, let us take another physical example to find the response of a system. So, let us take a similar system similar spring mass damper system but here I will reduce the number of damper. So, let me put a damper only in the middle to simplify the calculation I can do it. And so this is the system; let us take the system. So, this is m 1, m 2 let me take this in this case. So, let us study the free vibration of the system. Here, m 1 equal to m 2 equal to m; let me take it equal to 1 kg and then let me this k 1. So, this is k 1, this is k 2, this is C 2, this is k 3.

So, I am taking this C 1 equal to C 3 equal to 0 in the previous example; here I am taking C 2, C 2 equal to 2 Newton's second per meter let me take. And this k 1 equal to k 2 equal to k 3 equal to 100 Newton per meter. So, for simplicity this data I have taken. So, you may take any other data to find the 3 vibration response of the system. So, in this case already you know the equation motion which can be written in this form m 1 s square or the equation motion is written in this form m 1 0 0 m 2 x 1 double dot x 2 double dot plus k 1 plus k 2 minus k 2 minus k 2 k 2 plus k 3 x 1 x 2, and for damping this. So, this equation is reduced to; so I can write this equation is this form.

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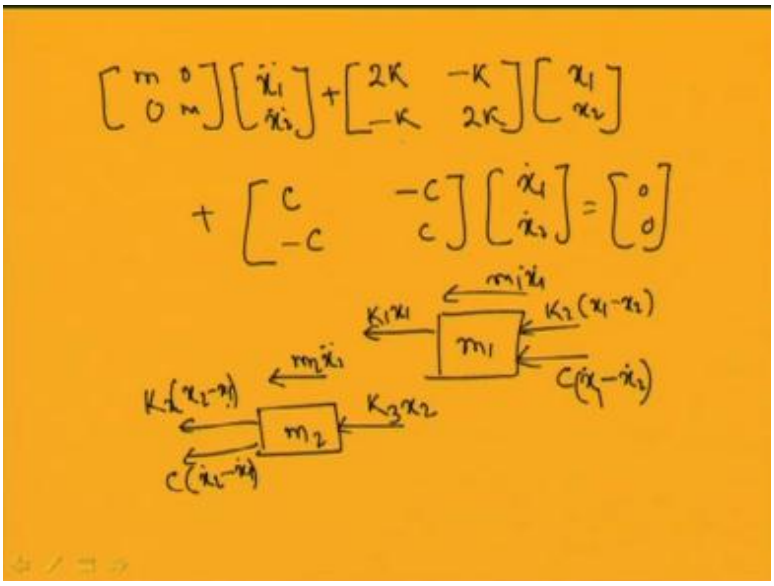

So, for this case I can write the equation; so this is equal to m 0 0 m. So, x 1 double dot x 2 double dot plus; so this becomes k 1 plus k 2. So, k 1 plus k 2 this becomes 2 k and this is minus k 2. So, minus k and this is minus k and k 2 plus k 3 becomes 2 k and so this becomes x 1 x 2 and for damping case. So, this becomes C 1 plus C 2. So, as C 1 equal to 0. So, this becomes C I can write it equal to C. And this becomes minus C and this is minus C and this is C and this becomes x 1 dot and x 2 dot equal to 0 0. So, the equation motion for the system you can write in this form. So, without referring the previous case also you can directly write the equation motion by drawing the free body diagram of the system. So, if you draw the free body diagram for this mass m 1. So, the inertia force is m 1 x 1 double dot. So, spring force equal to k 1 x 1 damping force there is no damping force this side and this side you have the spring force k 2 x 1 minus x 2.

And, the damping force equal to C x dot so C x 2. So, this becomes C. So, it will have a displacement of x 1 minus x 2. So, velocity difference will be x 1 dot minus x 2 dot. So, the damping force equal to C into x 1 minus x 2 dot. Similarly, for this mass m 2. So, second mass you can draw the free body diagram and you can write the equation motion. So, this is m 2 x 2 double dot and you have a spring force k 2 into x 2 minus x 1 and damping force equal to C into x 2 dot minus x 1 dot and this side you have a spring force of k 3 into x 2. So, using these forces you can draw using these forces you can write the equation motion of the system.

So, first equation becomes as m 1 equal to m. So, m x 1 double dot plus k 1 plus; so here k 1 x 1 here k 2 x 1. So, k 1 plus k 2 as k 1 equal to k 2. So, you can write 2 k 2 k into x 1 and minus k 2 minus k 2 k 2 equal to k. So, minus k 2 into x 2 then for damping you can write this is equal to C 1 minus x 1 dot. So, into C into x 1 dot minus C into x 2 dot equal to 0. So, this is the first equation. And the second equation becomes m 2. So, m 2 x 2 double dot. So, m into x 2 double dot plus this k 2 x 2 this also k 3 x 2. So, this becomes k 2 plus k 3 x 2 and for k 1 minus k x 1. So, this becomes minus k. So, this becomes minus k x 1. So, this becomes minus k x 1 and k 2 plus k 3 becomes 2 k x 2. And for damping this becomes C minus C x 1 dot. So, minus C x 1 dot plus C x 2 dot. So, in this way you can write the equation motion.

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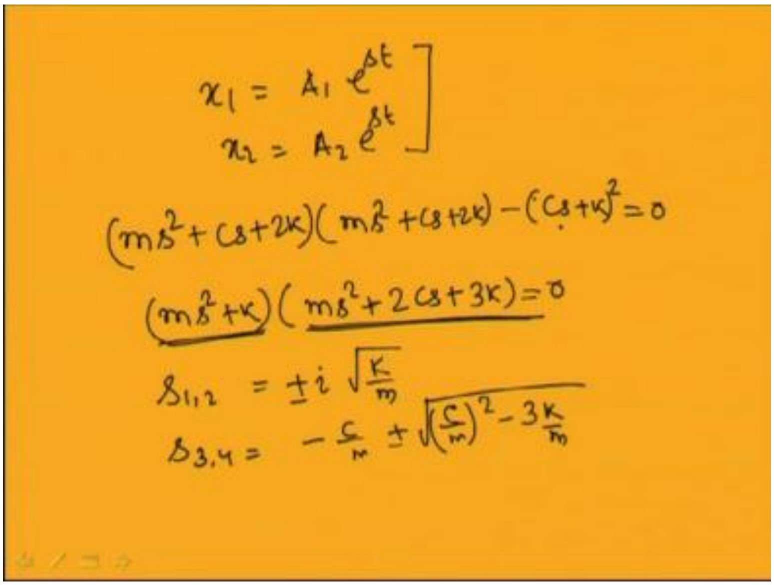

Now, you can substitute x 1 equal to A 1 e to the power s t and x 2 equal to A 2 e to the power s t. So, if you substitute this equation in the equation motion your equation motion will reduce to this form. So, it becomes m s square plus C s plus 2 k into m s square plus C s plus 2 k minus C s plus k whole square equal to 0. So, you will get this characteristic equation. So, now you have to find the solution of this characteristic equation to find all the 4 roots. So, this equation you can write in this form. So, if you multiply this thing and you can write this equation in this form. So, it will reduce to m s square plus k to simplify this equation So, you will get m s square plus k into m s square plus 2 C s plus 3 k equal to 0. So, you multiply this write this equation and then you can write the same equation in this form.

So, this becomes so you can do this thing by following 2, 3 steps. So, you just first multiply this equation with this equation and minus C s plus k square you expand this terms. And after expanding this thing you can rewrite this equation in this form. So, from this you can find at s 1, 2 from this. So, this will be equal to 0 when this first part either this first part equal to 0 or this second part equal to 0. So, s 1, 2 you can find will be equal to plus minus i root over K by m and from this you can see that s 3, 4 will be equal to so this will becomes minus C by m plus minus root over C by m whole square minus 3 K m. So, in the present case by substituting this value, so I can write this s 1, 2.

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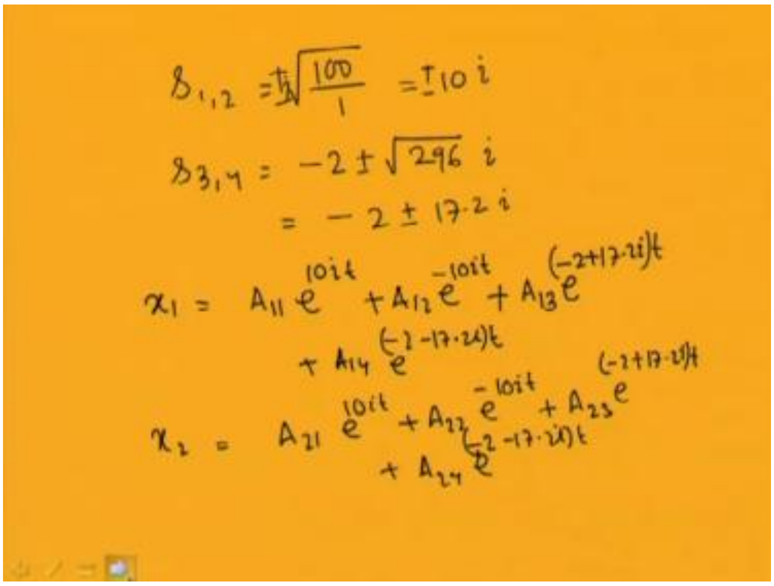

So, s 1, 2 becomes root over k by m k I have taken equal to 100 Newton per meter and m equal to 1; so this becomes 10. So, plus minus 10 I am getting 2 roots, those are plus minus 10 plus minus i into 10.

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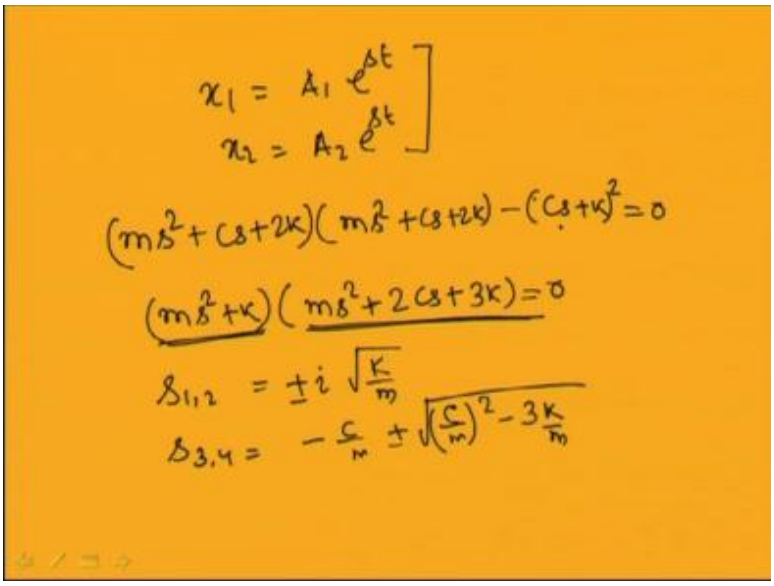

So, this is plus minus 10 i and previously I have written s plus I into root over K by m. So, because this s square equal to minus K by m. So, root over minus root over K by m equal to plus minus i into root over K by m. So, s 1, 2 you can find this and s 3, 4 becomes; so if I substitute those values. So, it becomes minus 2 plus minus root over 296 i. And this equal to minus 2 plus minus 17.2 i. So, after getting this 4 roots now you got 4 roots this s 1 equal to plus 10 i, s 2 equal to minus 10 i, s 3 equal to minus 2 plus 17.2 i. And s 4 becomes minus 2 minus 17.2 i. So, here you just note that you got 4 complex numbers out of these 2 are imaginary, purely imaginary and other 2 are complex conjugates. So, this imaginary roots will give oscillatory motions and you can write your x 1 in this form already you know you can write x 1 A 1 1 e to the power 10 i t plus A 1 2 minus 10 i t plus A 1 3 e to the power minus 2 plus 17.2. So, this becomes 17.2 i into t plus A 1 4 e to the power minus 2 minus 17.2 i t.

Similarly, x 2 i can write in this form A 2 1 e to the power 10 it plus A 2 2 e to the power minus 10 i t plus A 2 3 e to the power minus 2 plus 17.2 t plus A 2 4 e to the power minus 2 minus 17.2 i t. So, here you can observe that the system have 2 frequency; one frequency is 10 and the other frequency is 17.2. So, this 2 degree of freedom system will have 2 frequencies you obtain those 2 frequencies; and you can simplify these 2 expression to write the expression for x 1 and x 2. Also you may note that this x A 1 1, A 1 2, A 1 3, A 1 4 and A 2 1, A 2 2, A 2 3, A 2 4 can be obtained from the initial conditions.

So, you know you have 4 initial conditions these are the displacement and velocity of mass 1 and 2; so but you have 8 unknowns. So, out of this 8 unknowns and you have 4 equations and also you know that A 1 i and A 2 i are related. So, if you find some of this A either A 1 i or A 2 i then you can find the other 4 unknowns.

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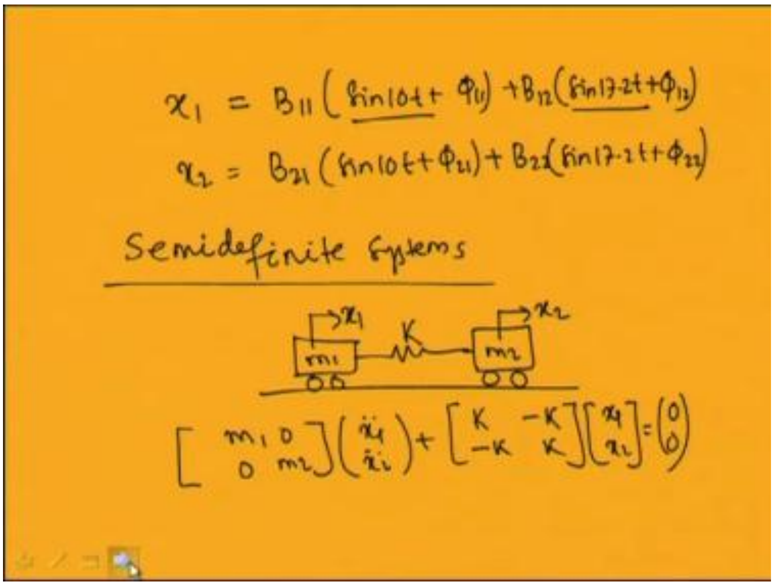

So, you can simplify this equations and you can write this equation also in this form. So, by using this property e to the power i theta equal to cos theta plus i sin theta if you use that property then you can write this equation in this form. So, you can write this x 1 equal to B 1 1 sin 10 t plus phi 1 1 plus B 1 2 sin 17.2 t plus phi 1 2. And x 2 also you can write equal to B 2 1 sin 10 t plus phi 2 1 plus B 2 2 into sin 17.2 t plus phi 2 2. So, in this case you have this B 1 1 phi 1 1 and B 1 2 phi 1 2. So, B 1 1 this is the magnitude and phi 1 1 is the phase difference. So, this is the first mode and this is the second mode. So, already you know by using this normal modes how to write this equation motion. So, these are the normal modes. So, sin 10 t and sin 17.2 t. So, 17.2 is the second frequency and 10 is the first frequency of the system.

So, in this way you can find the response of a 2 degree of freedom system with damping. So, let us see some other system, which are known as semi definite system; semi definite system or degenerate system; there are class of system for which out of this 2 natural frequency you can find that 1 natural frequency equal to 0. So, the system for which at least 1 natural frequency is 0 are known as the degenerate system or semi definite systems. So, let us take the system simple spring mass damper system in which the sides are not constant. So, this is m 1 and m 2; so you have 2 systems or 2 bogies they are connected by the spring k. So, in this case you can derive this equation motion. So, for this case the equation motion you can derive by using Lagrange principle. So, by using Lagrange principle you can write the equation motion; so this is x 1 and this is generalized coordinate x 2. So, kinetic energy equal to half m 1 x 1 dot square plus half m 2 x 2 dot square and potential energy equal to half k into x 2 minus x 1 whole square. So, you can find the equation motion of the system this form. So, it will leads to m 1 0 0 m 2 into x 1 double dot x 2 double dot plus k minus k minus k k into x 1 x 2 equal to 0 0.

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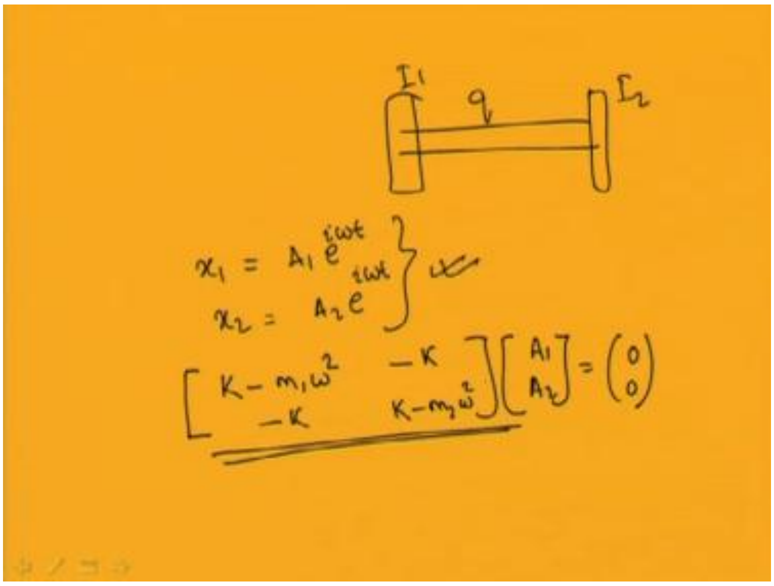

So, you can take a 2 rotor system also which will have similar equation motion. So, this is a 2 rotor system with inertia i 1 and i 2 and let q is the stiffness of the system. So, in this case also you can have a similar equation motion. So, for this case I can write now this x 1 equal to I can assume a solution x 1 equal to A 1 e to the power i omega t and x 2 equal to A 2 e to the power i omega t. So, I am taking the normal modes. So, already you know in case of normal modes all the masses will vibrate with same frequency. So, I have taken the frequency to be same in both the cases; in case of x 1 I have taken it omega and in case of x 2 also I have taken it omega but the amplitudes are different. In case of normal modes the systems or the masses of the system are vibrating with same frequency and passing through the equilibrium position at the same time. So, I can assume the solution like this and after assuming the solution I can find and substituting in this equation motion I can write this in this form. So, the equation motion will reduce to k minus m 1 omega square minus k minus k k minus m 2 omega square into A 1 A 2 equal to 0 0. So, for non trivial response; so the determinant will be equal to 0 to find the determinant. So, determinant will be k minus m 1 omega square into k minus m 2 omega square minus k square equal to 0.

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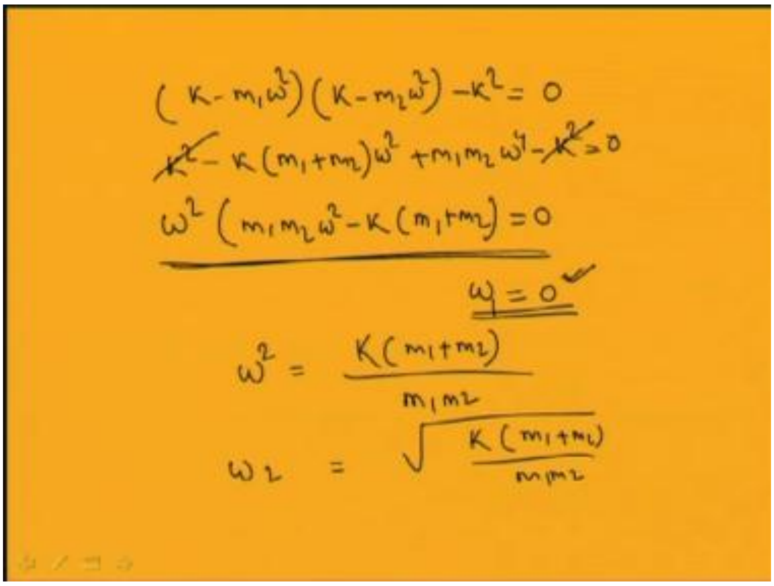

So, I can write the determinant in this form. So, the determinant becomes k minus m 1 omega square into k minus m 2 omega square minus k square equal to 0. So, I will multiply this; so I can write this equation in this form k into k. So, this becomes k square minus k into m 1 plus m 2 omega square plus m 1 m 2 omega fourth minus k square equal to 0 or I can write this equation in this form omega square. So, I can take omega square common from these 2. So, this becomes omega square into m 1 m 2 omega square minus k into m 1 plus m 2 equal to 0. So, this as this k square k square cancel. So, this equation becomes minus k into m 1 plus m 2 omega square plus m 1 m 1 into m 2 omega fourth equal to 0. So, I can take omega square and this becomes m 1 m 2 omega square minus k into m 1 plus m 2 equal to 0. So, from this equation you can observe that either this omega equal to 0 or this part will be equal to 0.

So, you are getting a frequency omega equal to 0. So, the other frequency can be obtained from this. So, this will gives omega square equal to K into m 1 plus m 2 by m 1 m 2 or you can write. So, this is your omega 1 omega 1 equal to 0 and omega 2 you can write equal to root over K into m 1 plus m 2 by m 1 m 2. So, you have seen you have 2 frequency out of which one frequency equal to 0. So, this is A degenerate system and for which you have this omega 1 equal to 0. So, if I substitute this omega 1 equal to 0 in this equation.

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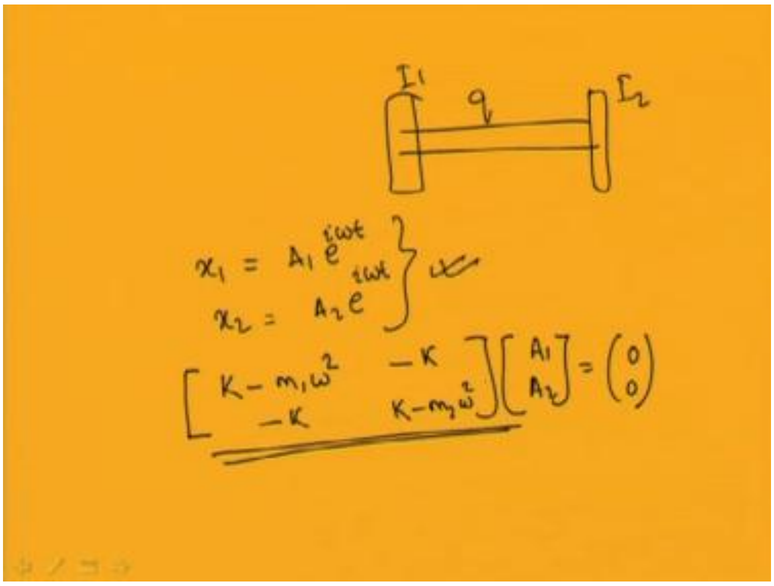

So, let omega equal to 0. So, if omega equal to 0; so you can see that K into A 1 minus K into A 2 equal to 0 that means A 1 equal to 0 A 1 equal to A 2.

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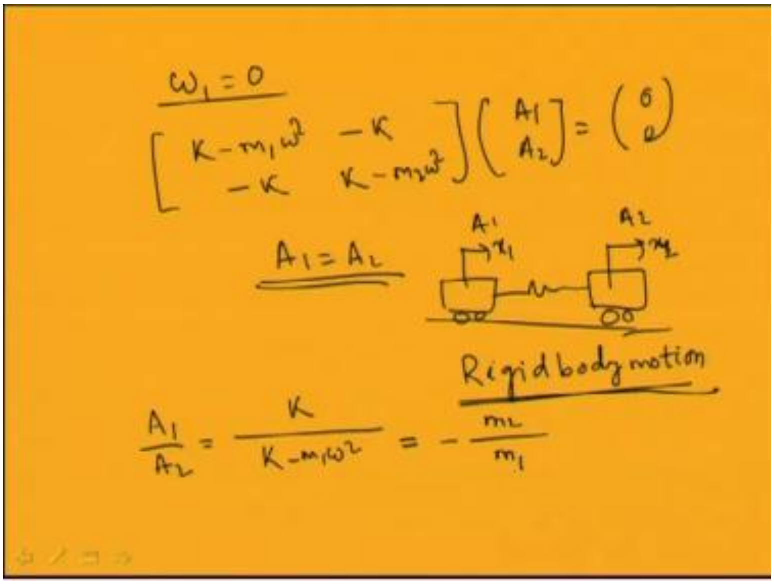

So, for omega 1 equal to 0 you have observed that from this equation. So, your equation is m 1 omega square minus K minus K K minus m 2 omega square A 1 A 2 equal to 0 0. So, from this equation if you substitute omega equal to 0 omega equal to omega 1 equal to 0. So, you are getting A 1 equal to A 2. So, A 1 equal to A 2 means both the springs or both the mass; so this is x 1 this is x 2 x this is x 1 this is x 2. So, amplitude A 1 equal to A 2 that means both the mass will move in the same direction with same amplitude. That means, there will be a rigid body motion in the system. So, the body may have a rigid body motion in this case of degenerate system and in the second case it will have a frequency. So, you can you can find in case of second mode.

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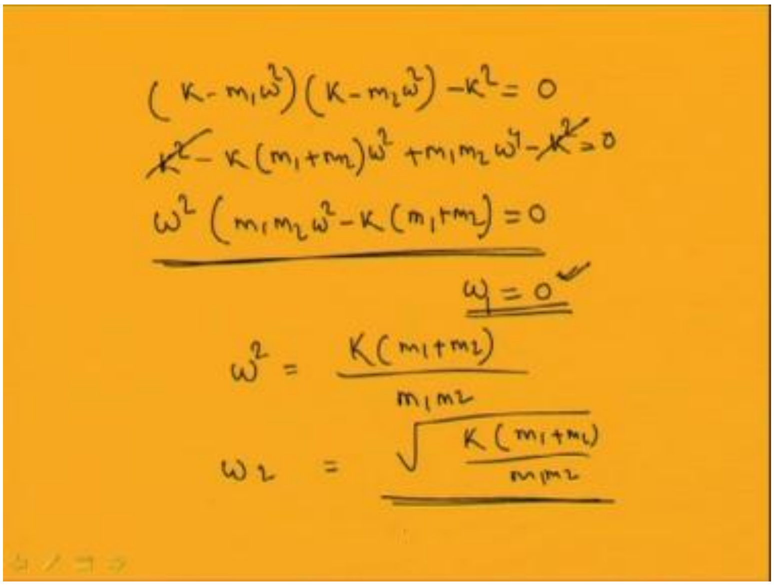

So, this A 1 by A 2 if you substitute A 1 by A 2 if you substitute this value of omega 2, then you can find to omega square will be equal to K into m 1 plus m 2 by m 1 m 2; if you substitute in this equation A 1 by A 2 equal to… So this becomes k by so this equation gives k minus m 1 omega square A 1 equal to k into A 2. So, A 1 by A 2 becomes K by K minus K minus m 1 omega square you substitute the value of omega square that is equal to k into m 1 plus m 2 by m 1 m 2. So, if I will substitute it here in this equation. So, this reduces to minus m 2 by m 1. So, this becomes minus m 2 by m 1. So, in this case you can observe that this amplitude ratio is inversely proportional to the mass ratio of the system A 1 by A 2 equal to m 2 by m 1.

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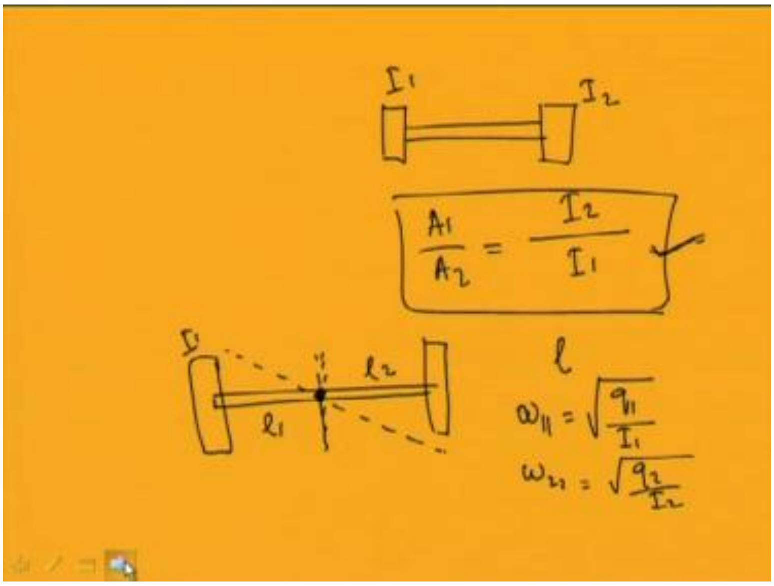

So, in case of a 2 rotor system similarly you can write. So, this is a 2 rotor system. So, in this case of 2 rotator system also you can write if the inertia is I 1 and this inertia I 2. So, you can write this A 1 by A 2 that is the amplitude of rotation of this mass by amplitude of rotation of this mass will be equal to I 2 by I 1. So, it will inversely proportional to the inertia ratio in case of this 2 mass or 2 rotor system you can proceed in this way also in another way also. So, in this case I can write when they are moving with same frequency I, let me find the same relation by another method. So, this is the system; so in this system when it is rotating with frequency. So, already you know that 1 frequency equal to 0 the other frequency you have to find.

So, let we have given that this system will have a length of this rod equal to l and when it is rotating with the other frequency; so there will be node point here; this node point node point means this point will have no vibration. So, you can assume that this 2 rotor system is equivalent to a cantilever beam or rotating cantilever beam. So, in which it is fixed at this point. So, at this point it is fixed. So, let this length is l 1 this length is l 2. So, this natural frequency of this part you can write equal to; so omega 1 or omega 1 1 you can write natural frequency of this part it is equal to root over stiffness of this part by. So, let stiffness is q 1.

So, stiffness of this part by inertia of this part q 1 by I 1. Similarly, you can write this omega 2 2 you can write or this part you can write it is equal to q 2 by I 2.

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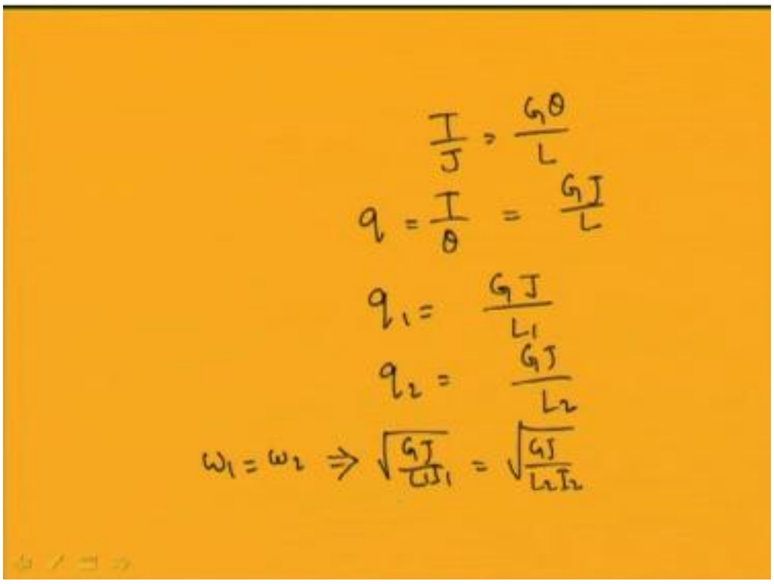

But this q 1 is nothing but so q 1. So, already you know this formula for torsional vibration of this system T by J equal to G theta by L you know this formula. So, from this you know T by theta equal to G J by L. So, T by theta is nothing but the stiffness of the system so q. So, you can have q 1 equal to G 1 or as I am taking the same system or same rod. So, I can write G J by L 1; similarly, I can write q 2 equal to G J by L 2. So, the as the natural frequency is same for both the parts I can write omega 1 equal to omega 2. So, this implies that root over q 1 by I 1; that means G J by L 1 I 1 will be equal to root over G J by L 2 I 2.

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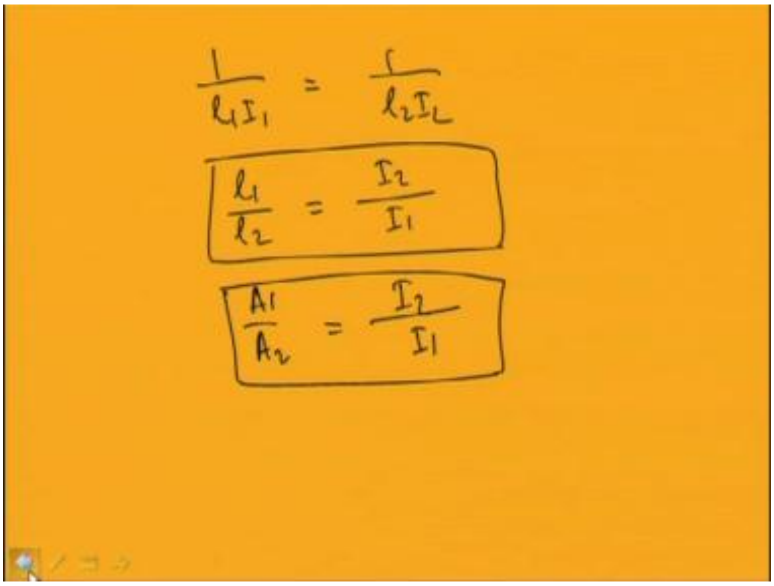

So, from this you can find that 1 by L 1 I 1 equal to 1 by L 2 I 2. So, from this you can see that this L 1 by L 2 L 1 by L 1 by L 2 equal to I 2 I 1. And from this you can see from this figure so the amplitude ratio is amplitude ratio will proportional to this L 1 L 2. So, this amplitude ratio A 1 by A 2 will be equal to so amplitude ratio will be equal to I 2 by I 1. So, in this way you can show also that this amplitude ratio is inversely proportional to the inertia ratio.

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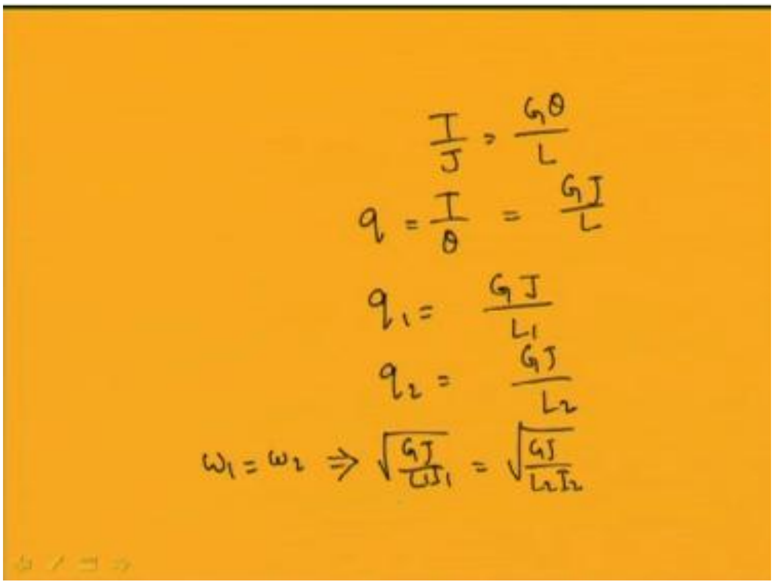

And, you can find for a given system parameters G J and length of the system. So, you can find the natural frequency omega also. So, one natural frequency in this 2 rotor system equal to 0 and the other method frequency will be equal to G J by L 1 I 1. And this node point you can find by using this relation. So, you know the inertia of the system. So, by using that inertia ratio you can find L 1, L 2 and also you know L equal to L 1 plus L 2. So, by using that you can find the node point. So, after finding this node point you can find this amplitude ratio. So, in this way you can find the response of a degenerated system or semi definite system. So, till now we have studied about the free vibration response of 2 degrees of freedom system.

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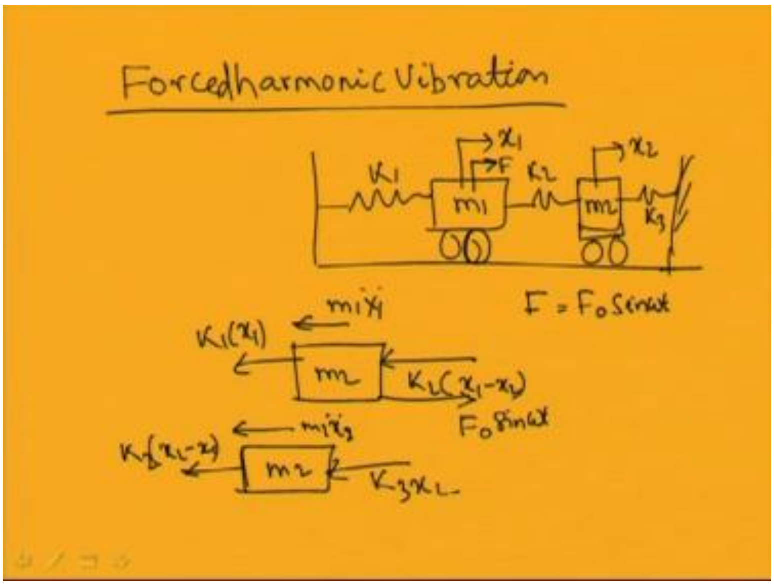

So, let us see the forced harmonic vibration of a 2 degree of freedom system, forced force harmonic vibration. So, in case of force harmonic vibration the system is subjected to a harmonic force. So, for simplicity we can take the same system what we have studied before and we can write the equation motion. So, let us take the system. So, this is a spring mass damper system. So, this K 1 this is K 2 and I will take another spring also this is K 3. So, this is mass m 1 this is mass m 2 and so this mass m 1 is moving with displacement x 1. So, this x 1 and x 2 are the generalized coordinate of the system. So, in this case you know that the equation motion equal to m 1 0 0 m 2. So, for free vibration case you have already studied this equation motion. So, let me give a force or let me assume that this mass m 1 is subjected to a force.

So, this is F and this force F is nothing but this is equal to F 0 sin or F 0 sin omega t. So, I can write the equation motion by drawing the free body diagram of the system. So, by drawing the free body diagram of the system you can see. So, let me draw the free body diagram of this mass m 1. So, in this case it will have a inertia force in this direction that is equal to m 1 x 1 double dot and spring force k 1 into x 1 and this side we have a spring force k 2 into x 1 minus x 2. And in addition to this it is subjected to an external force that is equal to F and this external force equal to F 0 sin omega t for this mass m 2. So, the free body diagram of this mass m 2 is same as before. So, this is equal to m 2 x 2 double dot then this is equal to k 2 into x 2 minus x 1 and this side it is equal to k 3 into x 2.

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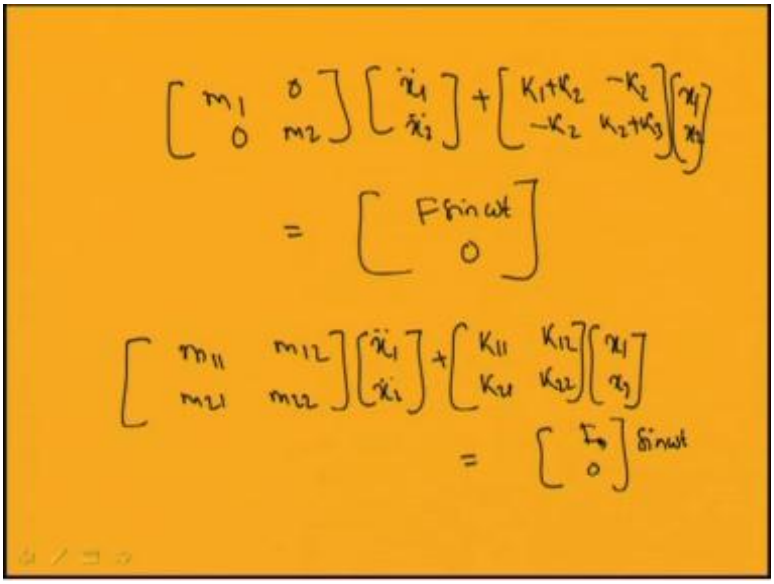

So, the equation motion you can write in this form. So, this is equal to m 1 0 0 m 2 x 1 double dot x 2 double dot plus K 1 plus K 2 minus K 2 this is minus K 2, this is K 2 plus K 3 into x 1 x 2 will be equal to F sin omega t 0. So, in this way you can write the equation motion of the system. So, let us first take a general system. So, in a general system your mass matrix may be coupled and stiffness matrix may be coupled. So, for a general system I can write this equation in this form. So, it will becomes m 1 1 m 1 2 m 2 1 m 2 2 x 1 double dot x 2 double dot plus k 1 1 k 1 2 k 2 1 k 2 2 x 1 x 2. So, this is equal to F F 0 0 sin omega t.

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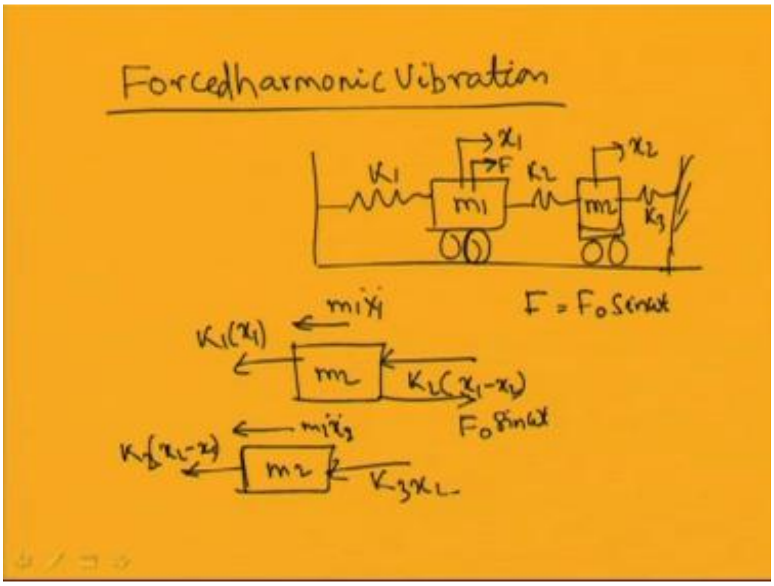

So, instead of applying a force in first mass if I will apply a force in second mass I can write this equation in this form. So, it becomes 0 and this becomes F sin omega t; already you know in case of a single degree of freedom system when a system is subjected to a harmonic force of F sin omega t the system will oscillate with the same frequency. But with and different with different magnitude and with a phase difference. So, you can assume for this case that both x 1 and x 2 are moving or oscillating with same frequency omega. So, you can assume a solution in this form x 1 x 2.

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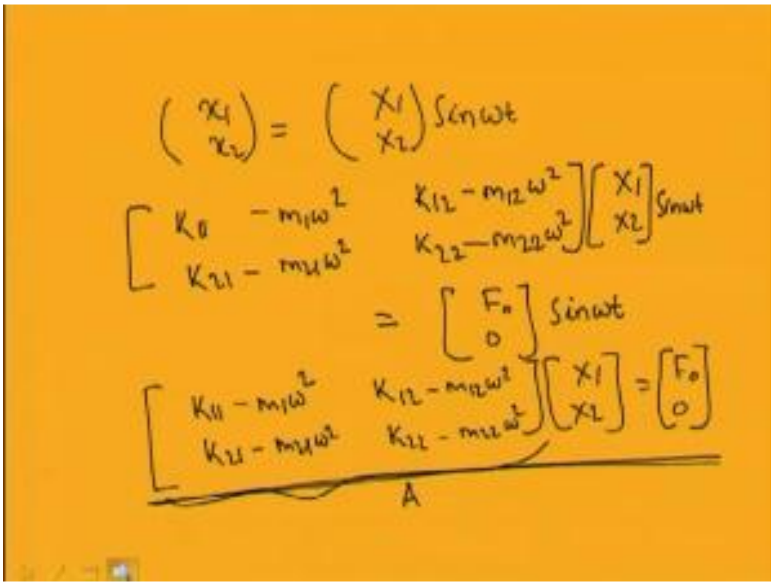

So, for steady state; so this for steady state. So, you can assume x 1 and x 2 equal to X 1 X 2 sin omega t. So, you can substitute this equation in this equation motion then it will reduce to this form. So, it will become K 1 1 minus m 1 omega square K 2 1 minus m 2 1 omega square K 1 2 minus m 1 2 omega square. And this becomes K 2 2 minus m 2 2 omega square into X 1 X 2 sin omega t equal to F 0 0 sin omega t or I can write this equation in this form. So, this will becomes K 1 1 minus m 1 omega square K 2 1 minus m 2 1 omega square k 1 2 minus m 1 2 omega square k 2 2 minus m 2 2 omega square into X 1 X 2 equal to F 0 0. So, all this is a 2 degree of freedom system or you have only 2 equations you can solve this equation to find this X 1 and X 2 by using a number of methods.

So, directly you can solve this equation like this K 1 1 minus m 1 omega square into X 1 plus K 1 2 minus m 1 2 omega square into X 2 equal to F 0. And the second equation you can write K 2 1 minus m 2 1 omega square X 1 plus K 2 2 minus m 2 2 omega square X 2 equal to 0. So, you can solve these 2 equations to find this X 1 and X 2. So, these equations will be written in terms of K 1 1 m 1 m 1 m 2 and omega or you may use the Cramer's rule to find the equation motion or you can write this X 1 X 2 if I write this matrix. So, let this matrix I am writing it as A matrix.

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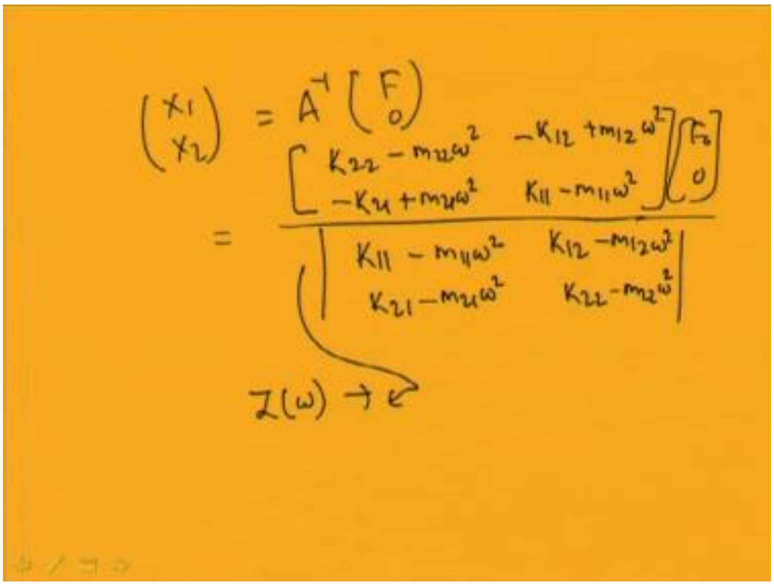

So, X 1 X 2 I can write also in this form. So, X 1 X 2 will be equal to A inverse into F 1 0. So, this way also you can write. So, in this case; so you can write this equal to so it will be equal to so if you find the inverse of this matrix. So, inverse of this matrix can be obtained by finding the adjoint matrix and divided it by the determinant of this matrix. So, if you write in this way; so you can find this will becomes K 2 2 minus m 2 2 omega square minus K 2 1 plus m 2 1 omega square minus K 1 2 plus m 1 2 omega square; this becomes K 1 1 minus m 1 1omega square into F 0 0 by determinant of this. So, determinant equal to K 1 1 minus m 1 1 omega square K 2 1 minus m 2 1 omega square this side it is K 1 2 minus m 1 2 omega square and this K 2 2 minus m 2 2 omega square. So, if this determinant part I can write it equal to Z omega determinant of this.

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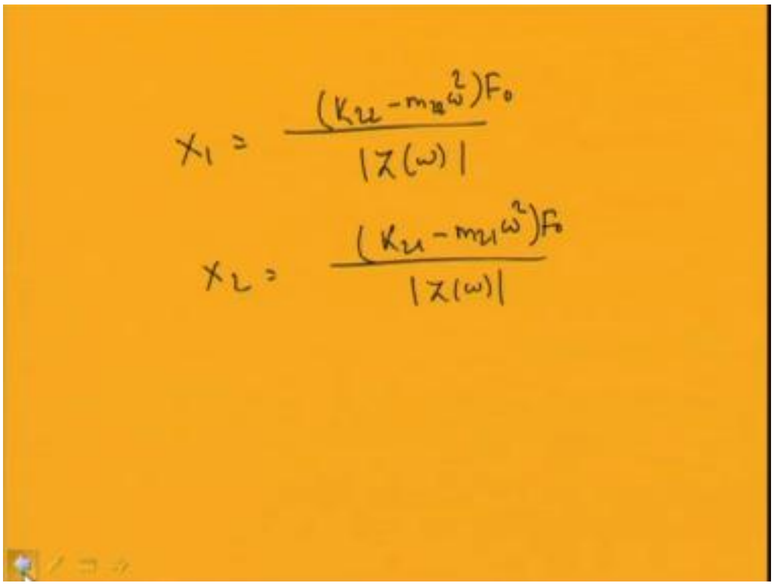

So, if this equal to Z omega; so I can write this X 1 equal to K 2 2 minus m 2 2 omega square by determinant Z omega into F 0 and X 2 equal to k 2 1 minus m 2 1 omega square F 0 by determinant Z omega. So, in this way you can find this X 1 X 2 or in this or you may find this by using Cramer's rule also in this equation.

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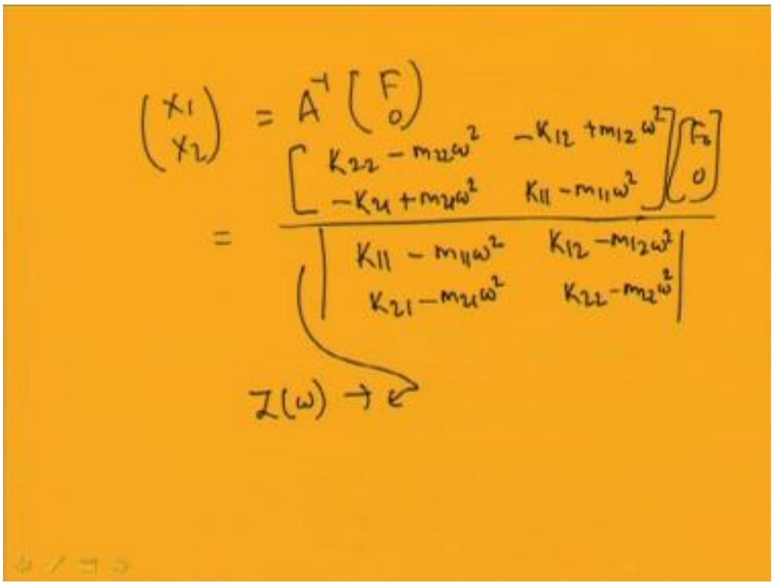

So, when you are using Cramer's rule then this x 1 will be will becomes. So, in this place you just replace it by F 0 0. So, it will becomes determinant of F 0 0 then this 2 terms will remain same by the determinant of A. And similarly X 2 will becomes K 1 1 minus m 1 omega square K 2 1 minus m 2 1 omega square; this part this part you may replace by F 0 by 0 F 0 0. So, in that way by using this Cramer's rule also you can find the solution X 1 and X 2 of the system.

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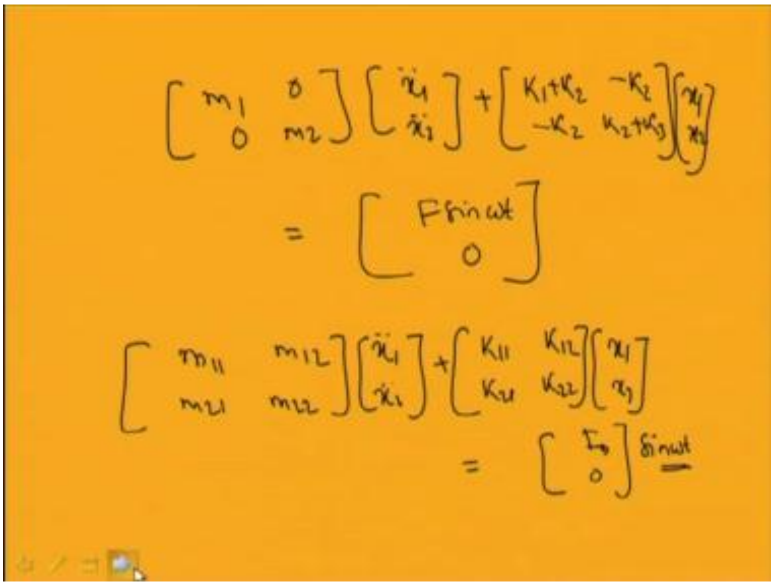

So, for the system we have taken before. So, in this case this mass matrix becomes m 1 0 zero m 2 and the stiffness matrix equal to K 1 K 2 minus K 2 minus K 2 K 2 plus K 3.

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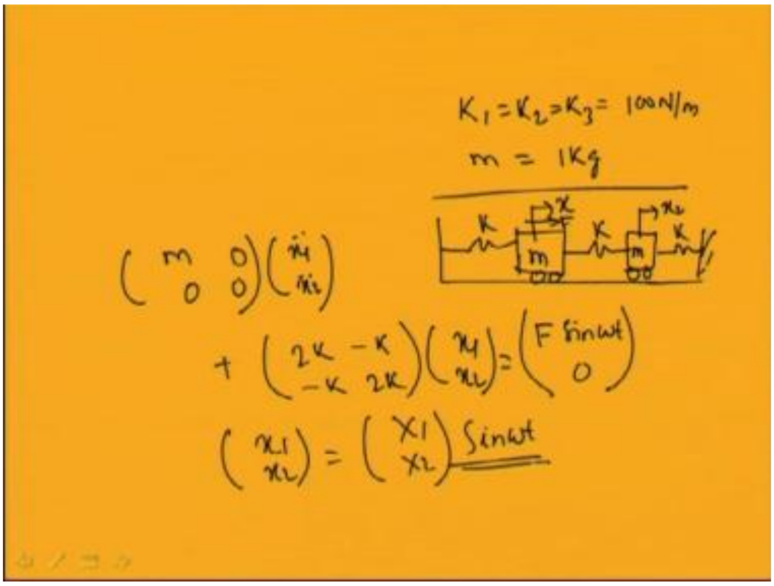

So, let me take this example same example in which I will take this K 1 equal to K 2 equal to K 3. So, let this becomes 100 Newton per meter and then let me take m equal to

1 kg. So, in this case I can write already you have seen that the equation motion can be written in this form. So, m 1 0; so I am taking the system. So, I have taken the system for which already I have derived the equation motion. So, I have taken this is K, this is K, this is K and I have taken this m 1 equal to m 2 equal to m. And I have to find this is x 1 x 2 and this mass m 1 is subjected to a force F. So, in this case I have already derived this equation motion m 1 0 or m 0 I can write m 0 0 m x 1 double dot x 2 double dot plus. So, this becomes 2 K minus K minus K 2 K x 1 x 2 equal to F sin omega t 0. So, now I will assume a solution in this form x 1 x 2 equal to X 1 X 2 sin omega t I can assume and find the steady state response.

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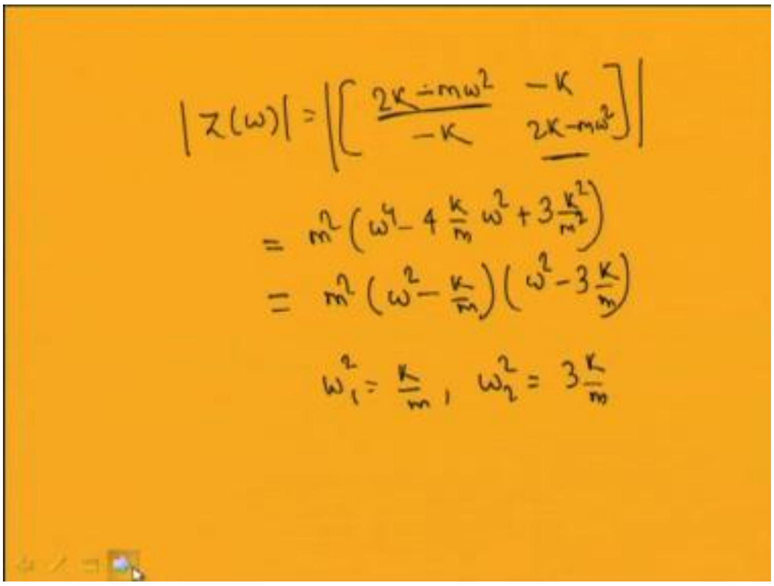

So, by assuming this thing so I can first find this Z omega. So, Z omega is nothing but this becomes 2 K determinant of this matrix 2 K minus m omega square minus K minus K. So, this becomes 2 K minus m omega square; so determinant of this matrix. So, the determinant of this matrix you can expand this. So, this multiplied with this minus this multiplied with this. So, you can find the determinant becomes equal to m square into omega fourth minus 4 K by m omega square plus 3 K square by m square. So, you can simplify this thing. So, this becomes m square into omega square minus K by m into omega square minus 3 K by m. Let me write this omega 1 equal to omega 1 square equal to K by m and omega 2 square equal to 3 K by m.

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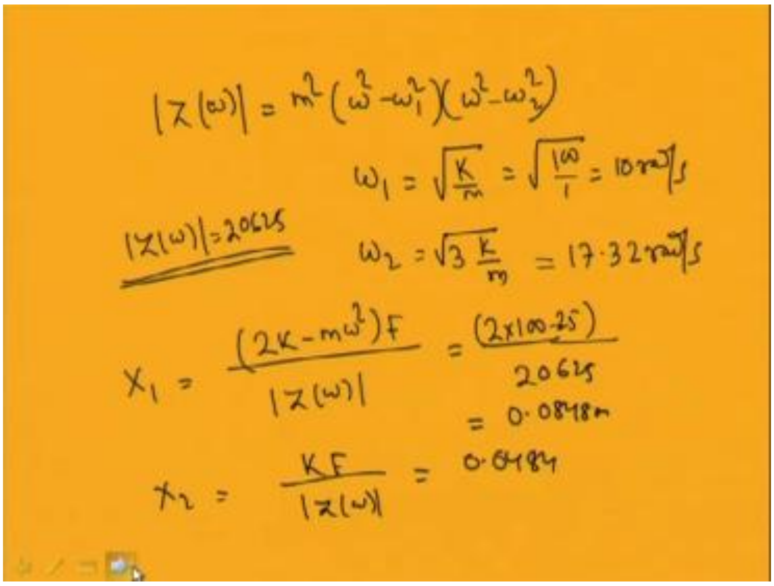

So, this equation becomes; so this determinant of Z omega becomes m square into omega square minus omega 1 square into omega square minus omega 2 square. So, here you can find your omega 1 equal to root over k by m. So, this becomes root over hundred by one. So, this becomes 10 radian per second. And similarly omega 2 becomes root 3 into K by m or so this becomes 17.32 radian per second. So, you got omega 1 and omega 2 now you can find this X 1. So, X 1 equal to from this expression you can find. So, by substituting this equation in this equation you can find X 1, X 2 already you have found that expression. So, this becomes 2 k minus m omega square into F by this determinant of this determinant Z omega. So, this determinant of Z omega you can find this equal to; so determinant of Z omega I have found it equal to 20625. So, I can substitute it is in this expression. So, this becomes 2 into 100 minus 25.

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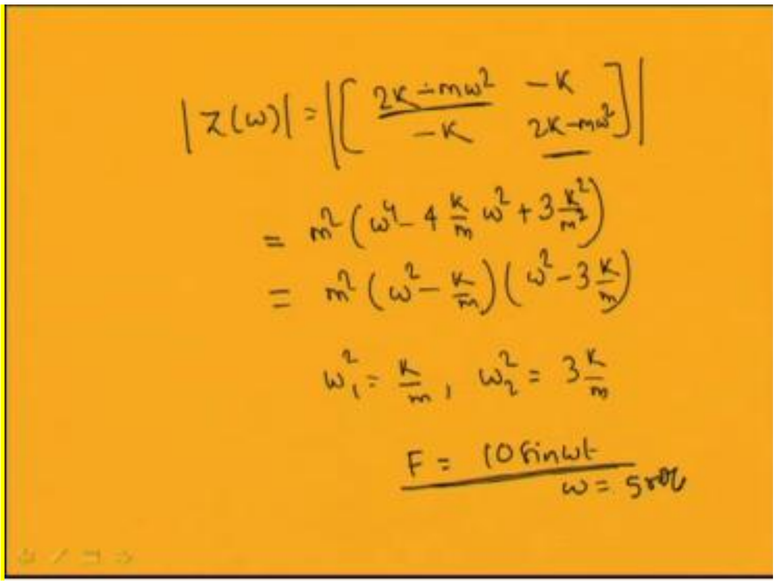

So, let me take a force F equal to 10 sin omega t; this omega also I can assume. So, let I am assuming this omega equal to 5 radian per second. So, if I am assuming this equal to 5 radian per second. So, in this case I can write this omega square equal to 25 and this becomes 20625. So, this becomes 0.0848 meter. And similarly you can find X 2 equal to K F by Z omega. So, this gives rise to 0.0484.

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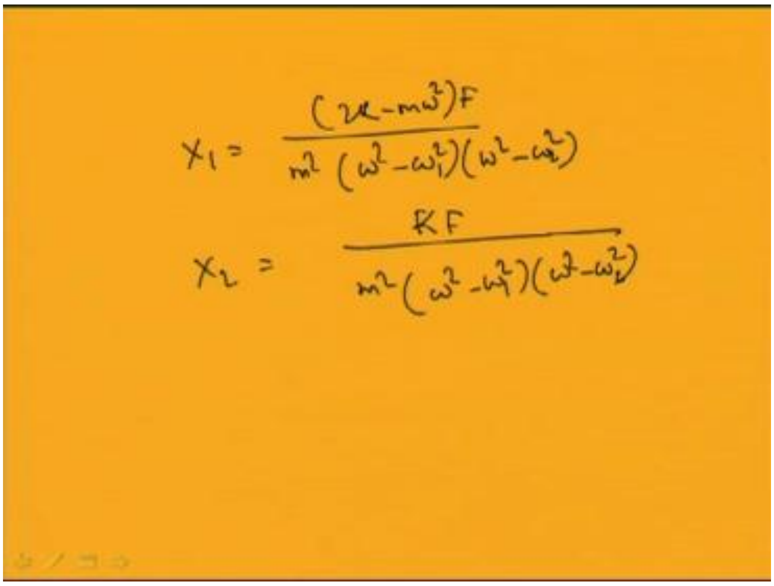

So, from this expression you can see all right I can write this X 1 equal to 2 K minus m omega square F by m square into omega square minus omega 1 square into omega square minus omega 2 square. And x 2 equal to K F by m square into omega 1 square omega square minus omega 2 square. So, you can observe that when omega equal to omega 1 or when omega equal to omega 2 this X 1 or X 2 tends to infinity. And when omega square equal to 2 K by m omega square equal to 2 K by m X 1 becomes 0. So, in this class we have studied or in this module we have studied the free and forced vibration of damped and undamped system. And we have studied this semi definite system; we have derived the equation motion by using different principles. And today class we have studied about the force vibration of damped system. Next class will study about using this force vibration will study the vibration absorber or principle of vibration absorber. And will apply this vibration absorber principle to suppress the vibration or absorb the vibration for different mechanical systems.