Mechanical Vibrations
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Module – 5
2 DOF Free Vibrations
Lecture - 3
Coordinate Coupling

So, in the last two classes we are studying about 2 degrees of freedom system. So, in these cases we have studied about the physical coordinate system, generalized coordinate systems and principle coordinate system. In case of physical coordinate system you are taking a set of coordinate systems and finding the physical parameter or coordinates of the points of which you are finding the equation motion. In case of the generalized coordinates so you are these are these set of minimum number of coordinates equal to express the motion of the system. And in case of principle coordinate system you are taking a coordinate system in which your mass matrix and stiffness matrix are uncoupled.

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So, you have studied about this physical coordinate system, generalized coordinate system and principal coordinate systems.

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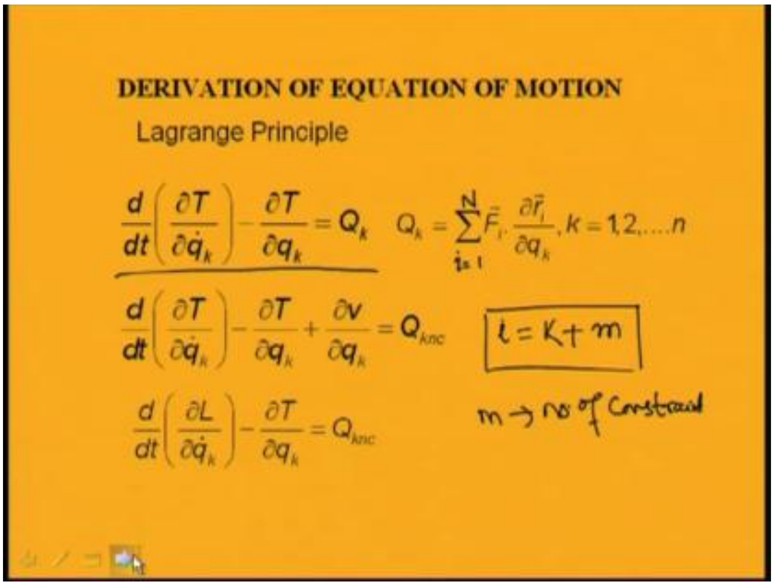

Also I told you about the derivation of equation motion using Lagrange principle. So, in case of Lagrange principle you are finding the kinetic energy of the system, the potential energy of the system and you are using this formula that is d by d t of del T by del q k dot minus del T by del q k equal to this Q k; this Q k is the generalized coordinates. So, were this generalized coordinates equal to i equal to 1, 2; 1 to n. So, if we have n number of or your system contains n particle; so it will be I equal to 1, 2, n. So, your Q k equal to i equal 1, 2 n, if i dot del r I, del r i by del q k.

So, where this r i is the physical coordinates of the system and q k in the generalized coordinate of the system. So, you know that this if you have i number of physical coordinates and k number of generalize coordinates; then this i and k can be related like this. So, I will be equal to so number of physical coordinates will be number of generalized coordinates for plus the number of constants. So, let constant I am writing m. So, m in the number of constants of this system, then m, if the number of constant. So, using this number of constant and the generalized coordinates, so this is the physical coordinates. For example, in case of a; so is the state the case of a.

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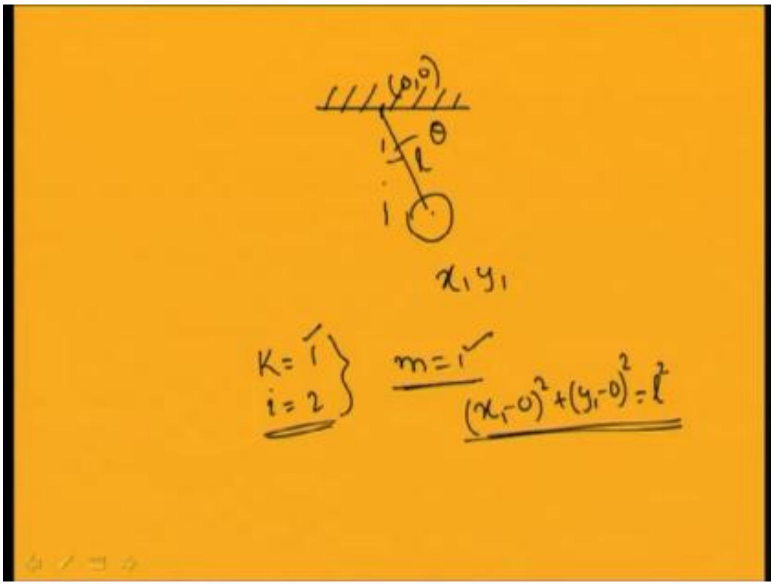

Double pendulum, for a single pendulum in this case you just take. So, in case of a single pendulum the physical coordinates are x 1, y 1 and your generalized coordinate is theta. So, this theta. So, your K equal to 1 and here physical coordinates your i equal to 2 and number of constant that is equal to m equal to 1. So, this constant is nothing but your x 1 square. So, the length is constant. So, this x 1 minus 0; so this point has a coordinate of 0, 0 you can take. So, x 1 minus 0 square plus y 1 minus 0 square will be equal to this length square. So, or your l will be equal to x 1 square plus y 1 square root over. So, you have 1 constant that is the length of this pendulum; so that is the constant. So, you have m equal to 1, K equal to 1 and i equal to 2. So, you can have the relation between this number of physical coordinates and the generalized coordinates by using the number of constants.

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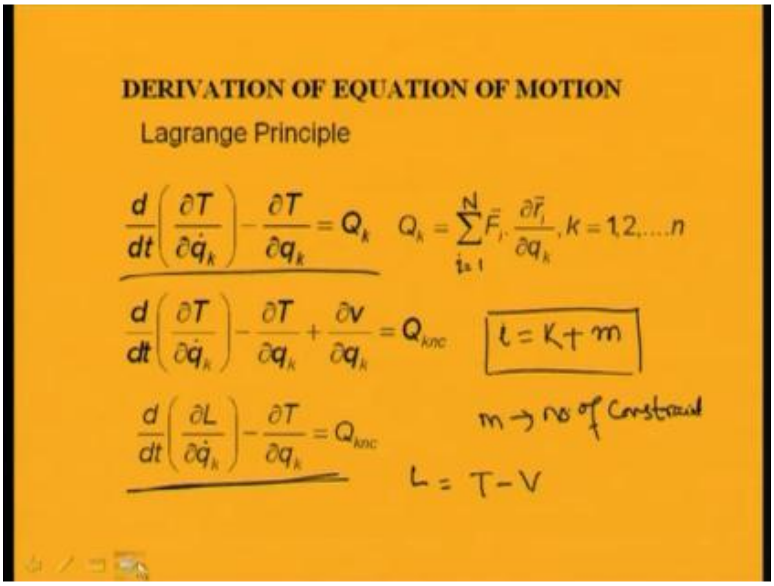

So, previously I told you about the Lagrange principle; using Lagrange principle you can find the equation motion either you can use this from this kinetic energy. And this generalized force you can find the equation motion or you can use the Lagrangian on the system which is equal to L equal to T minus V; T is the kinetic energy, V is the potential energy. So, you can use this expression to find the equation motion.

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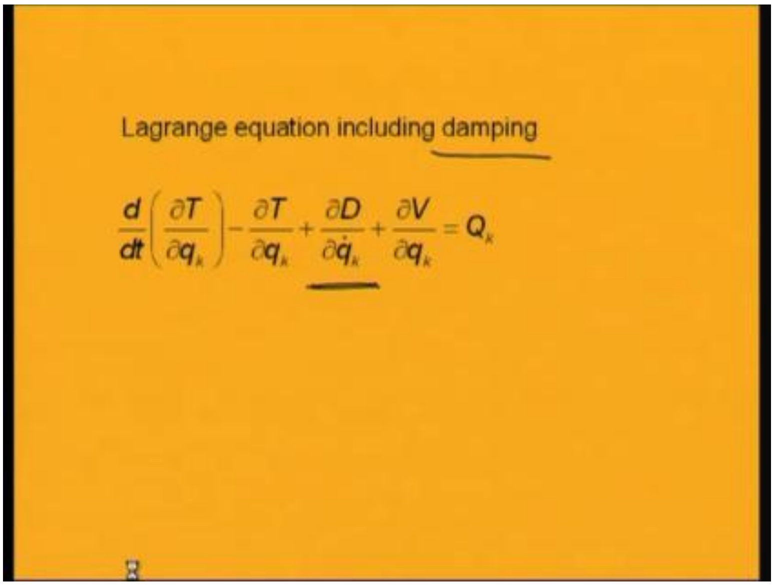

For a system with damping, you can use this dissipation energy to find the equation motion.

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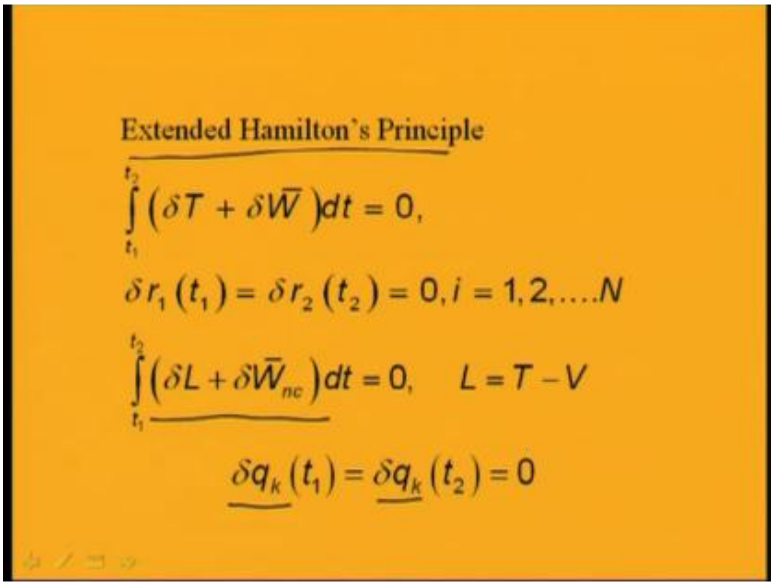

Also you can find the equation motion by using this extended Hamilton principle. In case of extended Hamilton principle you can use this expression that is del L plus integral t 1 to t 2 del l plus del W n c d t equal to 0, where these del q k at t 1 equal to del q k at t 2 equal to 0; for del q k is the virtual displacement of the system. So, virtual displacement of this generalize coordinates equal to 0. So, you may note that this generalized coordinate may or may not have any physical meaning; but they are the minimum number of coordinates used to define the motion of the system.

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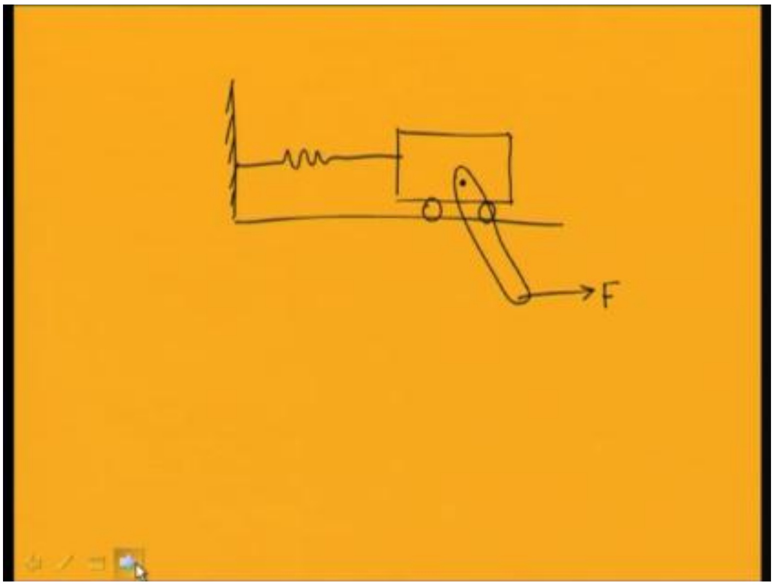

Also we have studied about the normal mode of vibration of a system. And already I have found the equation motion for the systems, I have taken a system this simple spring mass system in which another rod is hanging. And I have found the equation motion using Hamilton principle; also you may use the Lagrange principle to find this equation motion.

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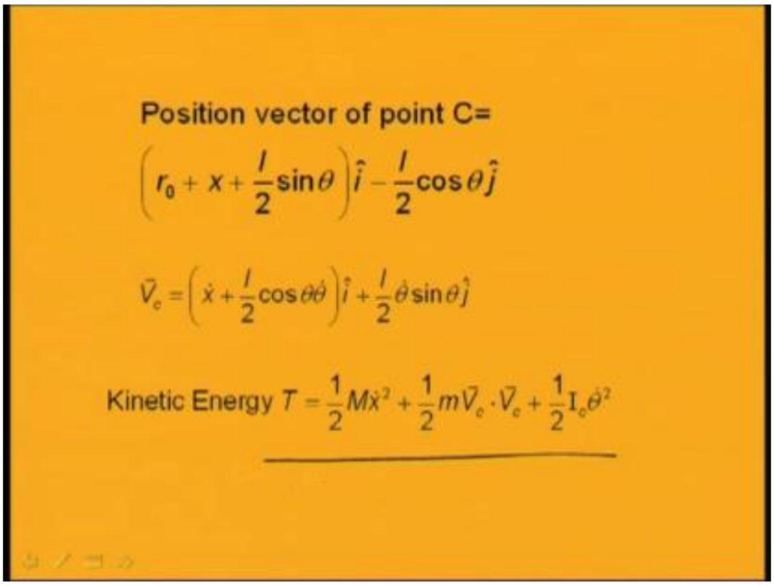

Already you have found for these same system you have found the kinetic energy of the system equal to this half M x dot square plus half m V c into V c; c is the mass center of these rod plus I c theta dot square; last class we have studied this equation.

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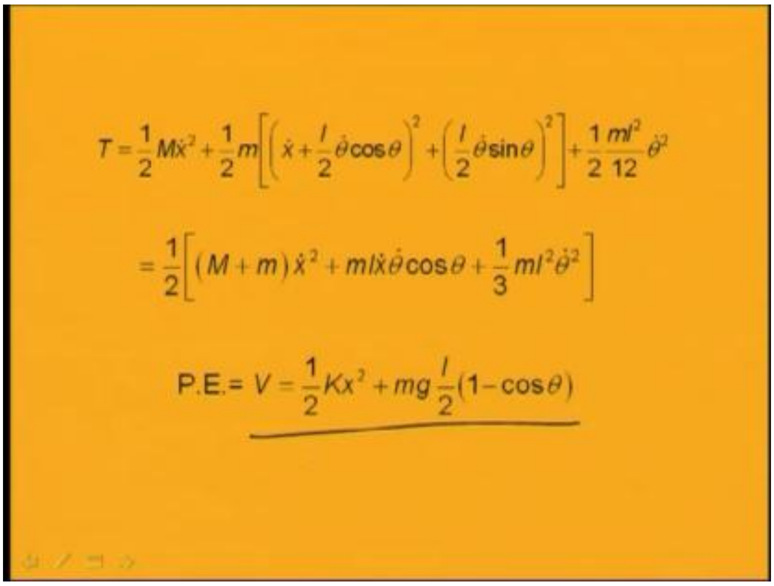

So, already you know the potential energy of this system; potential energy is the potential energy of the spring plus the potential energy due the change in height of this rod.

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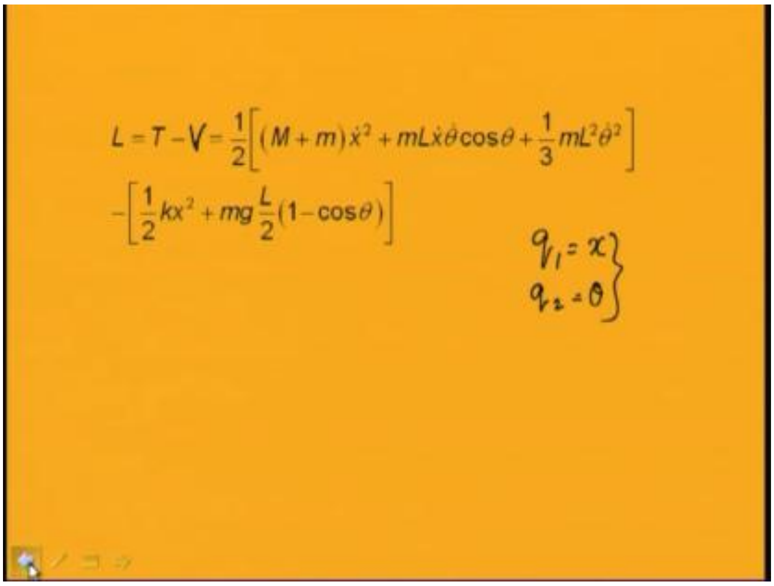

So, after knowing this kinetic energy and potential energy you can write the Lagrangian of the system equal to T minus V. Now, to find the q k, n c that is q 1 and q 2. So, you can find this Q; q 1 and q 2. So, in this case you can take your q 1 equal to x, q 1 and this it is generalized coordinate q 1 equal to x and q 2 equal to theta; and so your q 1 n c. So, in this case already you know in the first was there is no force acting on the system.

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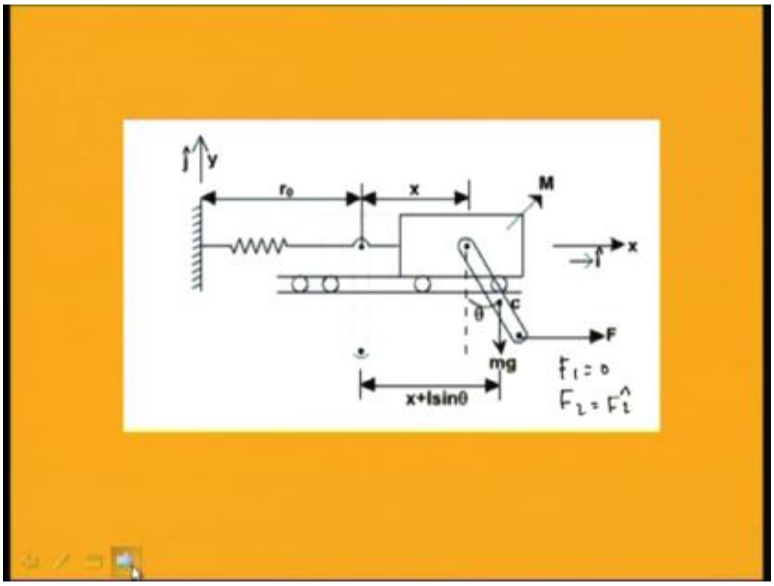

So, in this first mass there is no force acting and here you have a horizontal force acting. So, your F 1 equal to 0 and F 2 equal to this F i. So, I am taking a unit vector in this x direction as i. So, F 2 equal to F i and F 1 equal to 0. And the position vector of this point already you know. So, this is equal to r 0 plus X; so r 0 plus x plus l; so this is l sin theta. So, plus l sin theta i.

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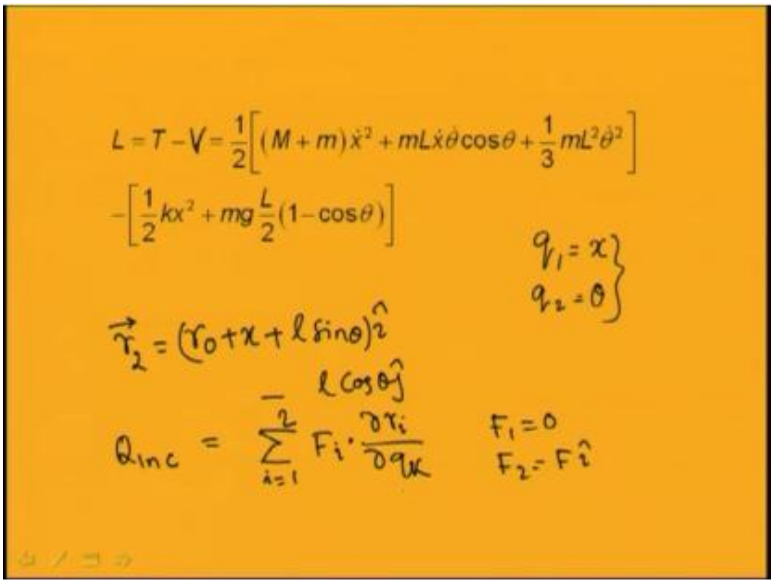

So, you can write. So, you can write your r equal to; so r already you know. So, this is your r 2. So, r 2 will be equal to r 0 plus x plus l sin theta i minus l cos theta j. So, this is the position vector of. So, position vector of that point where the force is acting.

So, you can find this Q 1 n c; so q k n c. So, to find the non conservative forces of the system. So, Q k n c will be equal to. So, if you apply the formula; so this equal to i equal to 1 to 2; this is F i dot del r i by del q k. So, you can. So, as a F 1 equal to 0 and F 2 equal to; so your F 2 equal to F i. So, you can substitute it in this equation. So, your r 1. So, this is expression for r 1 and as there is no force acting on the mass 1; so F 1 into del r i del r 1 by del q 1 will be equal to 0 and plus F 2 into del r 2 by del q k will be equal to.

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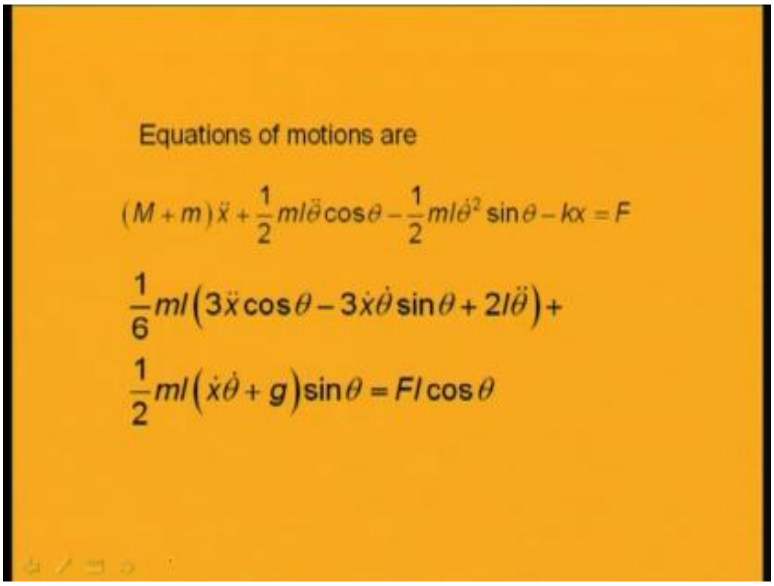

So, you can find that expression.

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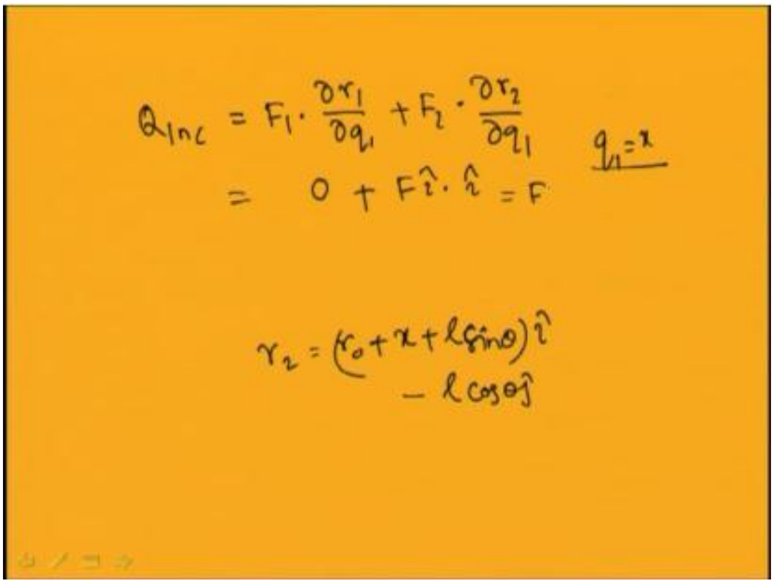

So, you can find this Q 1 n c will be equal to summation i equal to 1 to 2 or directly you can write this way. So, you can write this is equal to F 1 dot del r 1 by del q 1 plus F 2 dot del r 2 by del q 1. So, this will be equal to. So, this as F 1 equal to 0 this first part to 0 and the second part; so already you know the expression for r 2. So, where r 2 equal to r 0 plus x plus l sin theta i plus or minus; so it is equal to minus l cos theta j. So, if you differentiate with respect to this q 1 equal to x. So, q 1 equal to x. So, you differentiate; so this becomes 1; so this becomes i. So, this is equal to F i dot I; so this becomes F. So, already you know that your first force. So, the force equal to Q 1 n c equal to F.

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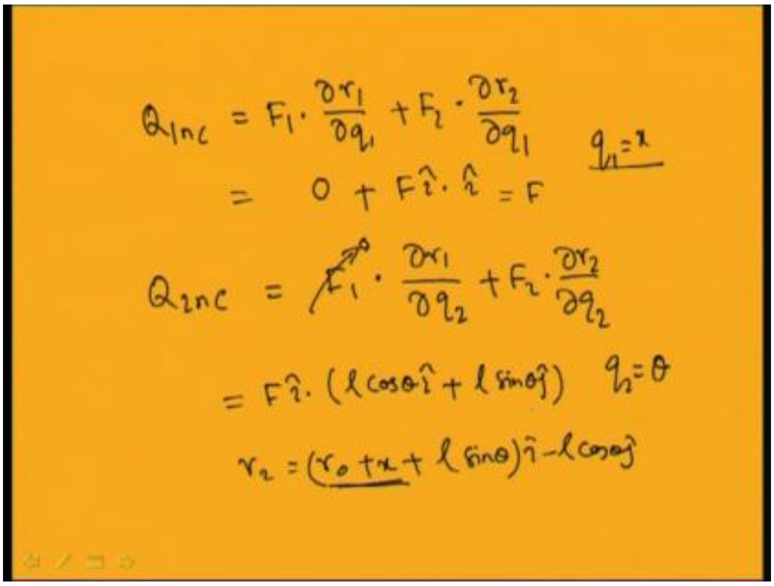

Similarly, your Q 2 n c you can find. So, this will becomes F 1 dot del r 1 by del q 2 plus F 2 dot del r 2 by del q 2. So, already you know your q 2 equal to theta F 1 equal to zero. So, as F 1 equal to this first term equal to 0. And second terms will give you. So, this is equal to F i dot del r i by del q y if you find. So, already you know your r 2 equal to. So, as r 2 equal to r 0 plus x plus l sin theta i plus or minus l cos theta j. So, differentiation of these with respect to theta. So, these 2 terms 0 and this will give you; so this is l sin theta. So, differentiation of this equal to l cos theta; so this is into l cos theta i minus; so minus minus plus. So, l sin theta j. So, from this you can find. So, from this you can write. So, this becomes equal to. So, this j i dot j equal to 0 and i dot I you can take.

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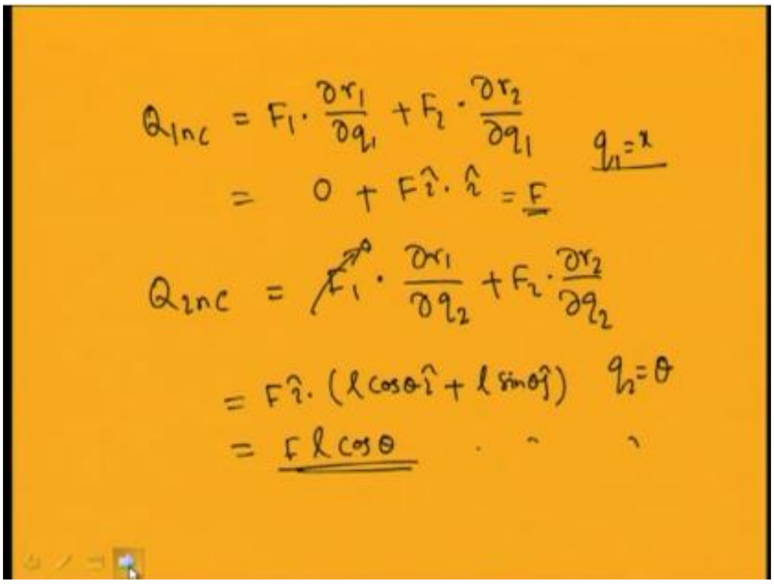

So, these becomes F l cos theta. So, the second force, second q k n c equal to F l cos theta and the first 1 equal to F. So, you can write the equation motion by using the Lagrange principle.

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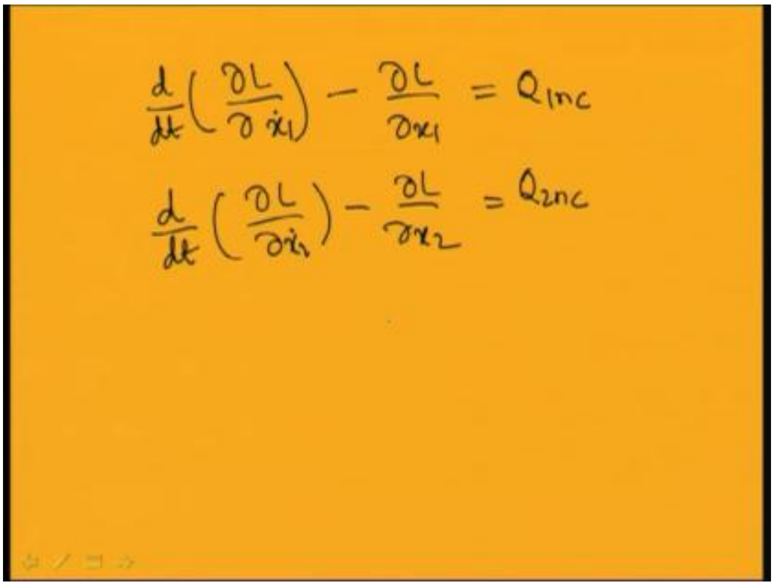
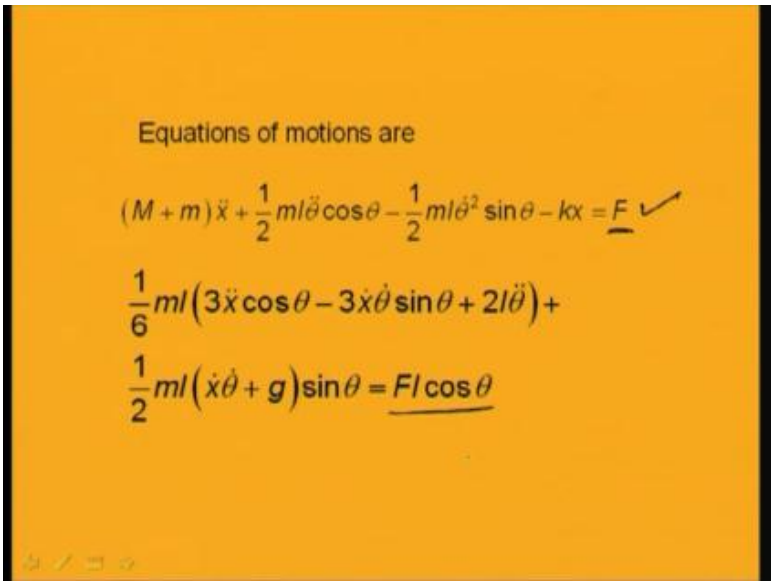


That is d by d t of del L by del. So, for the first equation you can write del L by del x 1 dot minus del L by del x 1. So, this will be equal to your Q 1 n c, and second equation will becomes d by d t of del L by del x 2 dot minus del L by del x 2 equal to Q 2 n c. (Refer Slide Time: 12:52)



So, using these 2 expression; so you can find the equation motion is this. So, the first equation motion is reduced to M plus m x 1 double dot plus half m l theta double dot cos theta minus half m l theta dot square sin theta minus k x equal to F. And the second equation becomes 1 by 6 m l 3 x double dot cos theta minus 3 x dot theta dot sin theta plus 2 l theta double dot plus half m l x dot theta dot plus g sin theta equal to F l cos theta.

So, you have seen; so these are the generalized coordinates we have obtain. So, these are the same equations for it have obtain using these extended Hamilton principle also; also last class I told you about the normal mode of vibration. So, normal mode refers to the motion of the system in which all the particles of the system have are moving with same frequency; and passing the equilibrium position at the same time.

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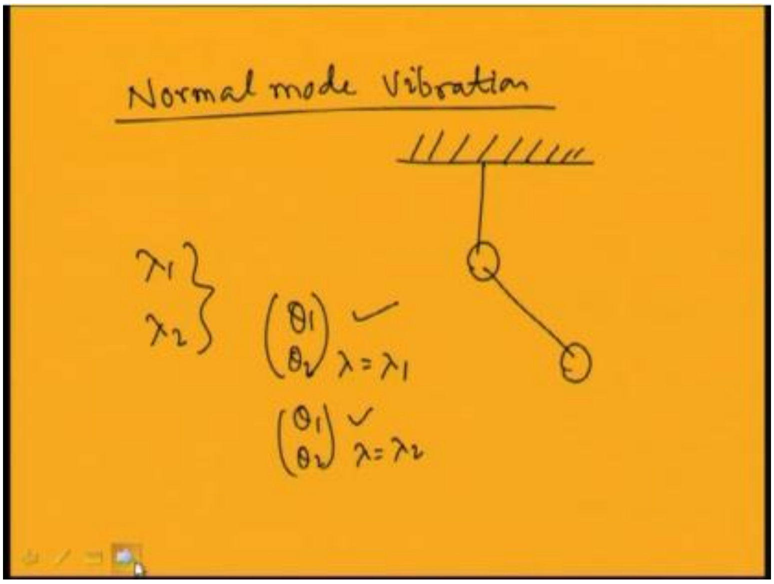

So, using these normal modes last class we have also find; we have also found the free vibration response of a system and normal mode vibration. So, using the normal mode vibration; so for this case we have taken the double pendulum. So, in case of double pendulum we have derived the equation motion using Lagrange principle. And from that we have found the normal modes of the system. So, in case of normal. So, we have found 2 normal mode vibration; so one we got lambda 1 and lambda 2.

So, 2 frequency we obtained and taking those 2 frequency we have obtain the normal modes of the system that is theta 1, theta 2. So, you obtain theta 1, theta 2 at lambda equal to lambda 1 and theta 1 theta 2 at lambda equal to lambda 2. So, these are the normal mode vibration of the system.

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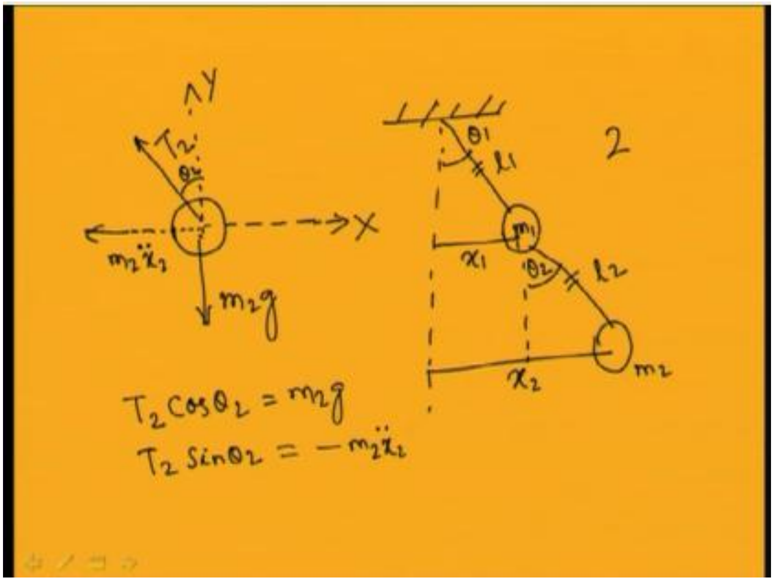

So, in the first case we have seen; so in the first normal mode we have seen that 2 the 2 masses are in phase and in the second case the 2 masses are out of phase. So, now let us derive the same expression or derive for the same simple double pendulum. So, let us take this double pendulum and derive the equation using another coordinate system. So, that is if will take the coordinate system as x 1, x 2 instead of these theta 1, theta 2. So, I will use the other method that is the Newton’s method to derive this equation motion.

So, in this case let me take the generalized coordinate as x 1 and this generalized coordinate as x 2. So, as you know these x 1, y 1, x 2, y 2 and this theta 1 and theta 2 they are related or if I am taking these theta 1, theta 2 are the generalize coordinates as this lengths are constant. So, I will have 2 constants equation that is length of this is constant and length of this is also constant. So, I can have 2 constant equation. So, this x and y; x 1 and y 1 are also related and this x 2 and y 2 are also related.

So, I can take this x 1 and x 2 as the generalized coordinate systems also in this case. And by taking these generalized coordinate system instead of taking this theta 1, theta 2; I can also derive these equation motion. So, let us derive this equation motion using the Newton’s method. So, this is theta 1 and this is theta 2. So, this is mass m 1, this is mass m 2, this length is l 1, this length is l 2. So, let me draw the free body diagram to derive this equation motion. So, derive this equation motion using Newton’s method.

So, I have to draw the free body diagram of this mass and this mass. So, for this mass m 1. So, let me draw for mass m 2. So, this is mass m 2. So, the forces acting on this mass m 2 are this weight; weight is acting downwards. So, this weight is m 2 g and the order force what is acting on the system is its tension; tension T 2. So, I can take this m 2 g and m 2 g and this tension to find this equation motion for the system. So, I can take a coordinate system x, y coordinate system; this is x direction I can take. So, this is the x direction, this is the y direction and as this second link make an angle theta 2 with this vertical line.

So, this angle will be equal to theta 2. So, this angle is theta 2 and so I can write the resolving these forces I can write the equation motion. So, the other force acting on the system is the inertia force that is as it has moved a distance x 2. So, the inertia force will be acting in opposite direction. So, that magnitude will be m m 2 into x 2 double dot but it will act in a direction opposite to the direction of acceleration.

So, this becomes m 2, x 2 double dot. So, now I can resolve the forces in vertical and horizontal direction and I can write the equation motion. So, the equation motion in this case or in this case if I write the force balance I will do the force balance. Then, we can see that this force the component of this force in this vertical direction; that this T 2 T 2 cos theta 2 will be equal to m 2 g and T 2 sin theta 2 T 2 sin theta 2 plus m 2 x 2 double dot will be equal to 0 or I can write T 2 sin theta 2 equal to minus m 2 x 2 double dot. Now, I can draw the free body diagram of this mass m 1.

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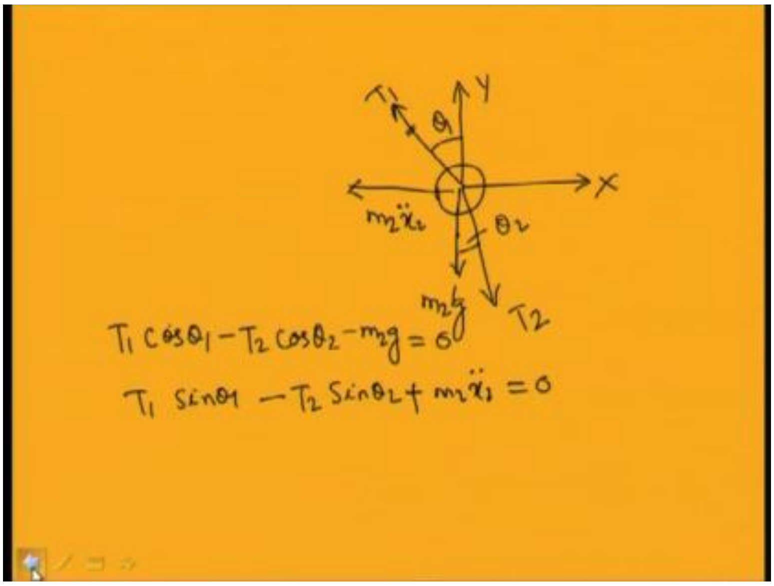

So, for this mass m 1 the forces acting. So, this is the vertical line, this is the horizontal line I can. So, this is the X direction, this is the Y direction and this is the tension. So, this is theta 1. So, this angle is theta 1 and I can write; so this is the theta 1 and the other 1 will be theta 2. So, I can write another force that is. So, this is this is tension T 1 and I can write this is tension T 2. So, this T 2 makes an angle theta 2 with vertical and T 1 make some angle theta 1 with the vertical.

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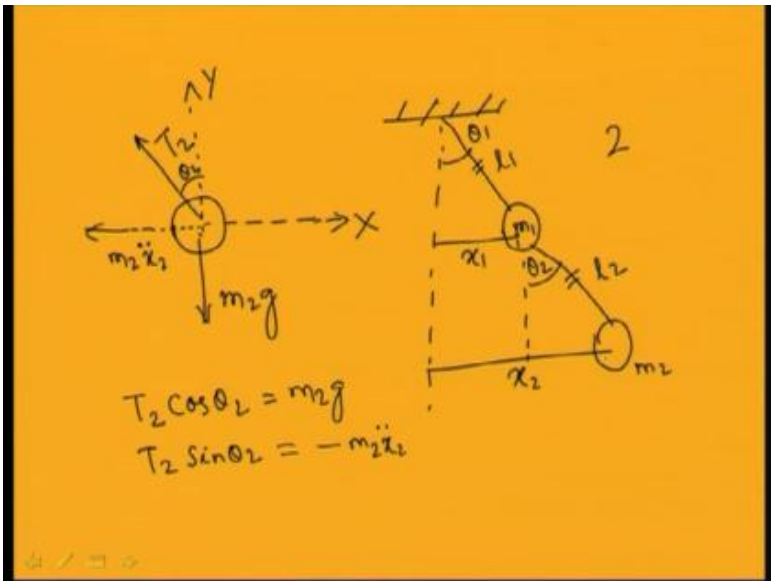

So, as I have taken x 2 as the generalized coordinate for this mass m 2. So, I can write the inertia force which is acting opposite to the direction of acceleration as m 2 x 2 double dot. So, in this case if I will do the force balance then so these are the forces acting on the system, another force that is the weight of the system that is m 2 g is also acting on this system. So, by doing the force balance; now this T 1; this force this T 1 sin T 1 cos theta 1 minus T 2 cos theta 2 minus mg will be equal to 0; if I am writing the forces in the vertical direction. So, I can write this T 1 sin T 1 cos theta 1. So, T 1 cos T 1 cos theta 1 minus T 2 cos theta 2 minus m 2 g will be equal to 0. So, this is one equation. And the other equation in this horizontal direction will be T 1 sin theta 1 T 1 sin theta 1 and minus so this becomes T 2 sin theta 2.

So, T 2 sin theta 2 I am taking; so this direction I have taken positive. So, or I can write this direction positive. So, I can write this plus this minus and minus m 2 x 2 double dot equal to 0 or in the other what I can write T 1 T 1 sin theta 1. So, if I have take a negative sign this becomes negative and the will becomes positive. So, T 1 sin theta 1 minus T 2 sin theta 2 plus m 2 x 2 double dot equal to 0. So, if I will assume some small motion that is theta 1 equal to theta 2 tends to 0 of the are very small. So, in that case I can half this cos theta 1 equal to 1, cos theta 2 also will be equal to 1. And this sin theta 1 will be equal to theta 1 and sin theta 2 I can write it equal to theta 2 or I can write.

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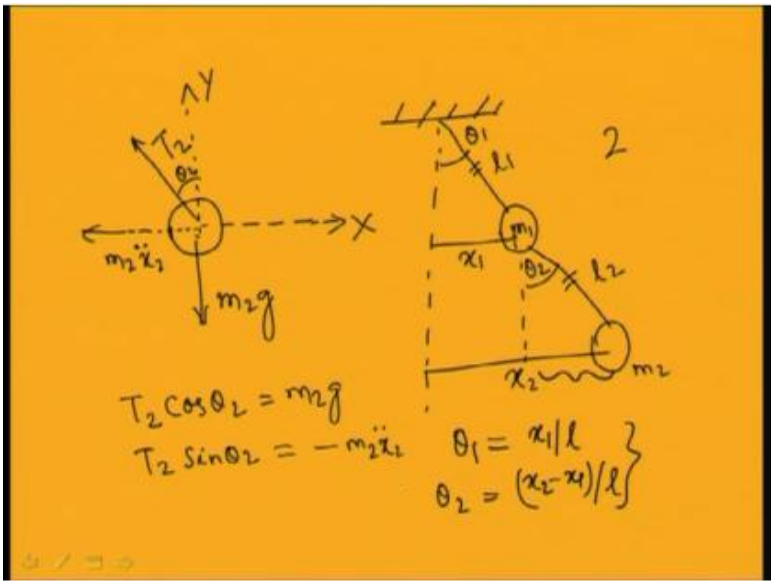

So, you can see from this theta 1; so this is sin theta 1 or equal to tan theta 1 sin theta 1 will be equal to tan theta 1. So, this will be equal to or this theta 1 will be approximately will be equal to x 1 by l and theta 2 will be equal to; so this theta 2. So, this will be equal to this distance by this l 2 sin theta 2 will be equal to x 2 minus. So, this is x 1; so this is this becomes x 2 minus x 1 by l. So, substituting this so from these equation you can see that these T 2 will be equal to m 2 g.

So, as theta 2 is very small then T 2 equal to m 2 g and this T 2 theta 2 will be equal to minus m 2 x 2 double dot or I can take it this side. So, I can write this m 2 x 2 double dot and already I know that T 2 equal to m 2 g and theta 1 equal to x by l. So, using these expressions I can write; so from these 4 equations.

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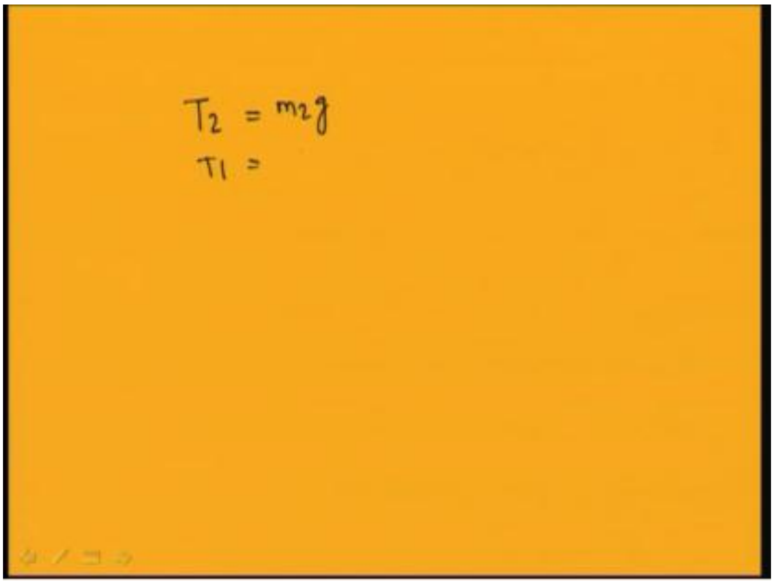

So, you can write that this T 2 equal to. So, T 2 already you have got equal to m 2 g and T 1 will be equal to so from this expression you can see that T 1. So, T 1 cos theta 1. So, this is equal to 1. So, this will be equal to m 2 g plus T 2. So, T 2 cos theta 2 equal to 1. So, already you know your T 2 equal to; so this is for the first mass equation for the first mass, this is not for the second mass. So, in this case it is equal to m 1 x 1 double dot m 1 x 1 double dot.

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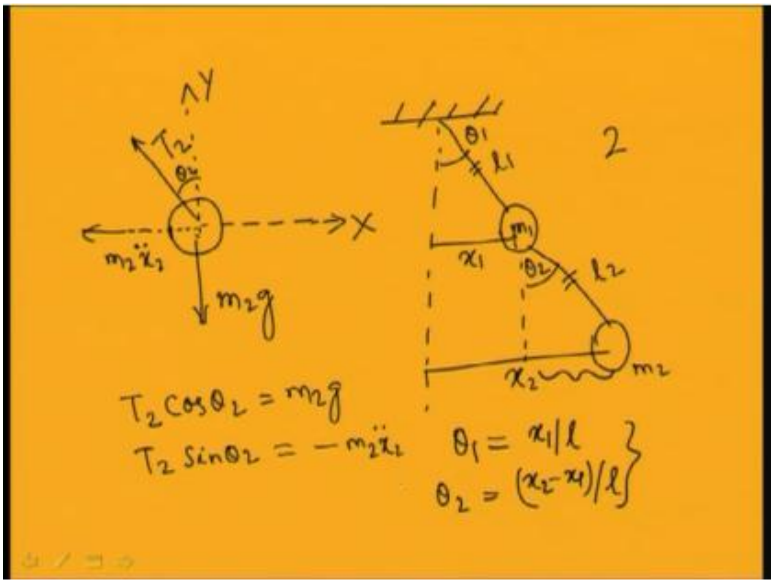

Previous case you have m 2 x 2 double dot. So, for this first for the first mass it is equal to m 1 m 1 x 1 double dot and so this is equal to T 1 cos theta 1 minus T 2 cos theta 2 this is equal to minus m 1 g. So, this is equal to this weight equal to m 1 g. So, this becomes as T 2 equal to m 2 g. So, this becomes T 1 becomes m 1 g plus m 2 g.

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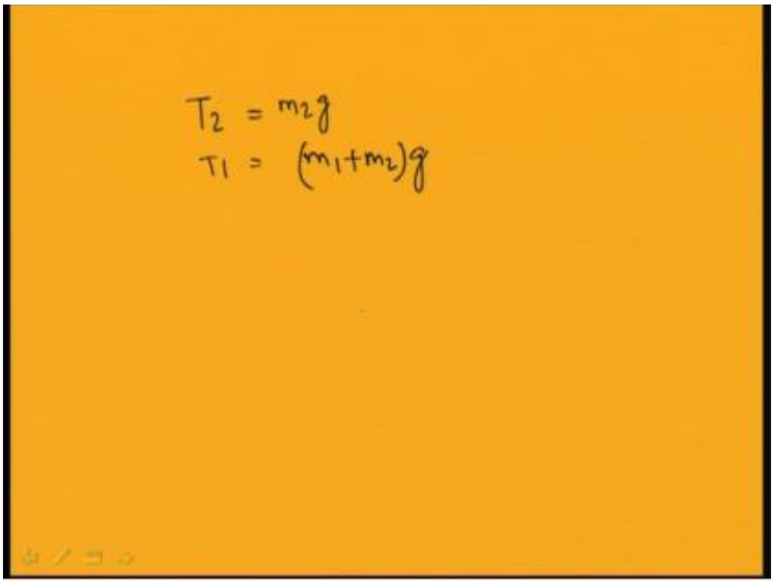

So, this is equal to m 1 plus m 2 g, m 1 plus m 2 g and already you have got T 2 equal to m 2 g.

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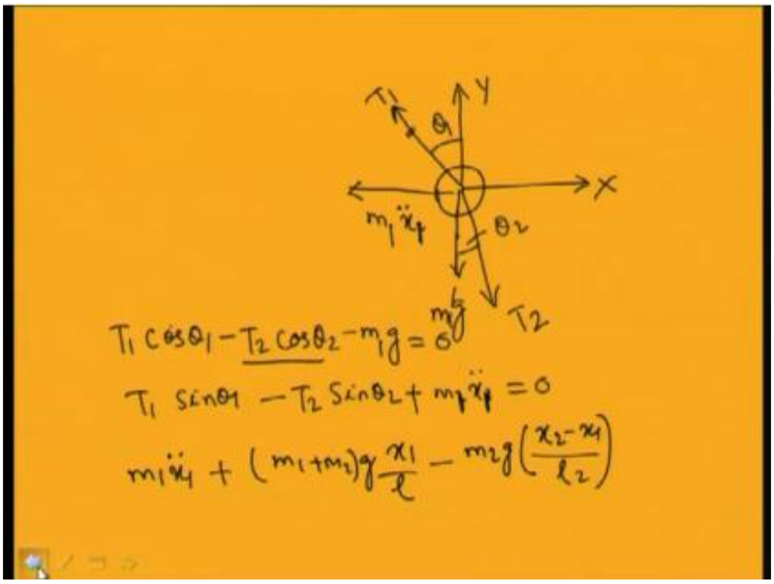

And, from the previous equation also you can find this m 1 x 1 double dot. So, now substitute all the expression here. So, this becomes m 1 x 1 double dot; for T 1 already you know the expression equal to m 1 plus m 2 g and sin theta 1 sin theta 1 equal to x 1 by l and minus for T 2, you can write it is equal to m 2 g. And this sin theta 2 already you have know equal to theta 2 that T equal to x 2 minus x 1 by x 2 minus x 1 by l 2; so into m 2 g.

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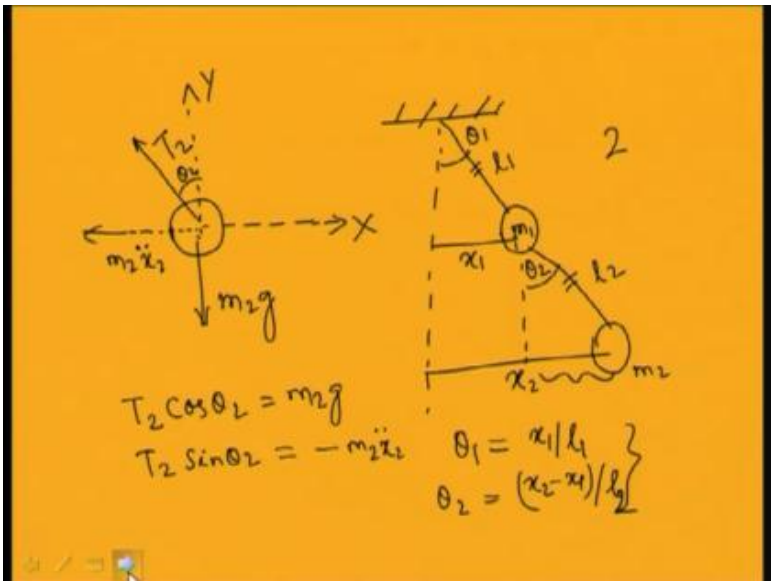

So, this is x 2 minus x 1 by this length I have taken l 1. So, this become x 1 by l 1 this theta 2 becomes this by this. So, this becomes l 2.

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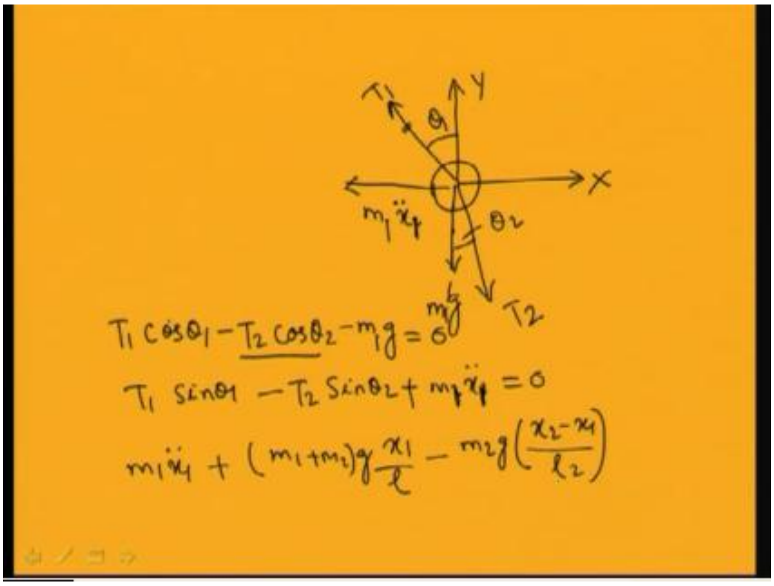

So, in these way; so you can got the expression for this. So, this is equal to m 1 x 1 double dot plus m 1 plus m 2 g x 1 by l minus m 2 g x 2 minus x 1 by l 2.

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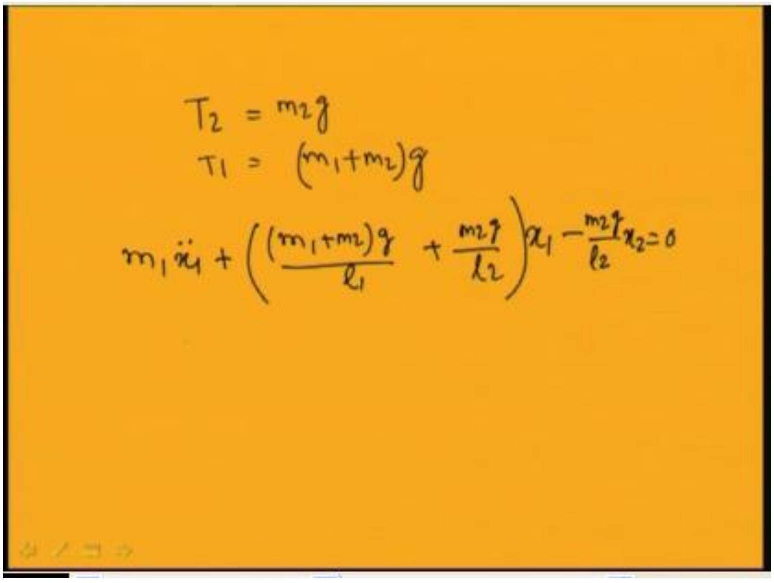

So, by rearranging this equation; so you can write the equation in this form. So, this equal to m 1 x 1 double dot plus m 1 plus m 2 g by l 1 plus m 2 g by l 2 x 1 minus m 2 g by l 2 x 2 equal to 0. And the second expression from this equation you can write the second expression; that is m 2 x 2 double dot plus for T 2 you can write equal to m 2 g into this expression.

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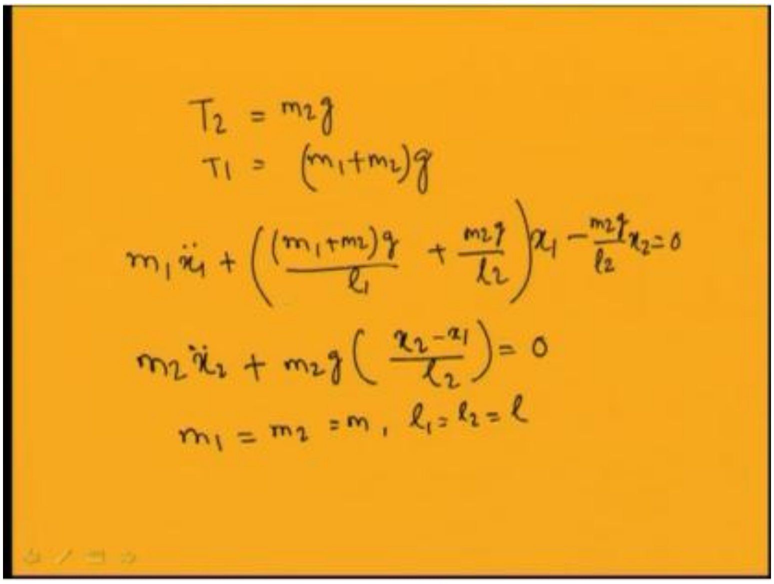

So, for the second equation you can write m 2 x 2 double dot plus m 2 g x 2 minus x 1 by l 2 equal to 0. So, you can express these 2 equation in terms of matrix formal also. So, for simplicity let us take this m 1 equal to m 2 equal to m and this l 1 equal to l 2 equal to l. So, I can write these 2 equation in terms of the matrix form.

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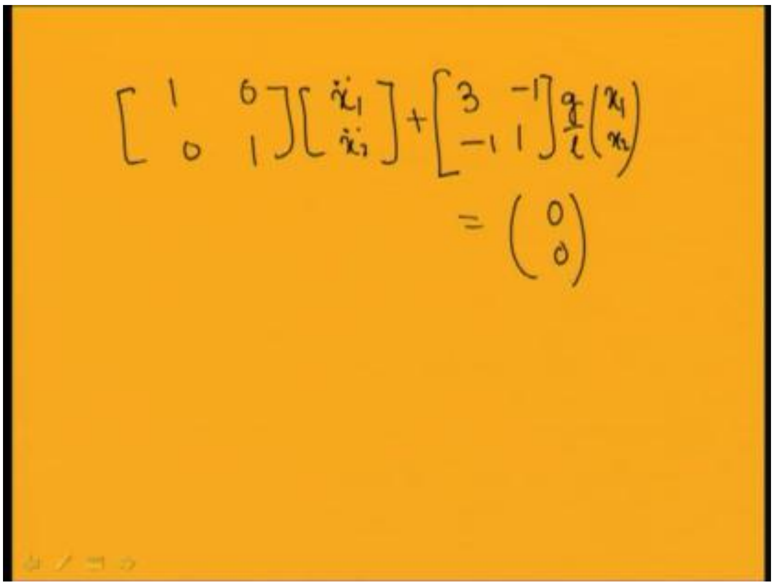

So, this matrix forms will give like this. So, this becomes m 1 0 0 m 2 x 1 double x 2 double dot plus or I can write in the matrix form by substituting that expression. So, it will becomes 1 0 0 0 1 x 1 double dot x 2 double dot. So, this becomes 3 minus 1 minus 1 one into g by l into g by l into x 1 x 2 equal to 0 0. So, in these way you can get the equation motion of the system. Now, proceeding in the previous way you can find the normal mode vibration of the system.

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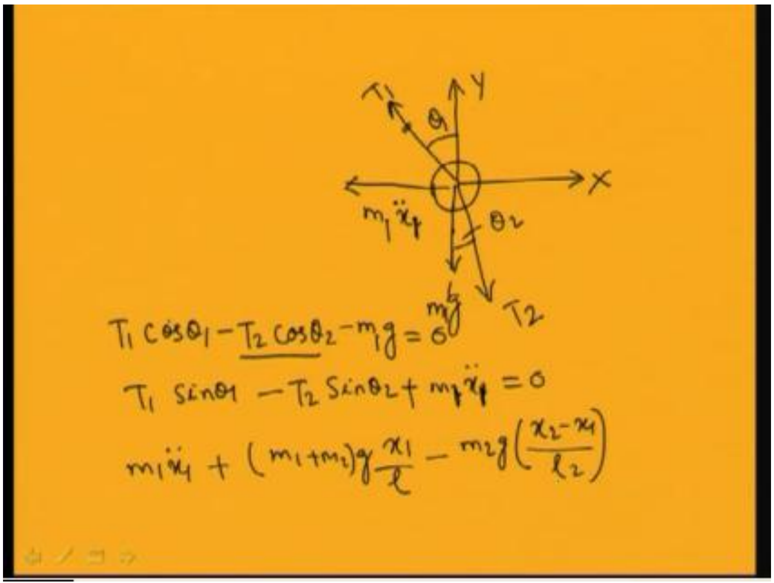

In case of normal mode vibration both the masses; this m 1 and m 2 will move with same frequency and will pass the equilibrium position at the same time. So, this is this is the normal mode we are assuming, and the actual vibration or actual free vibration will depend on the initial conditions that thing already we have seen.

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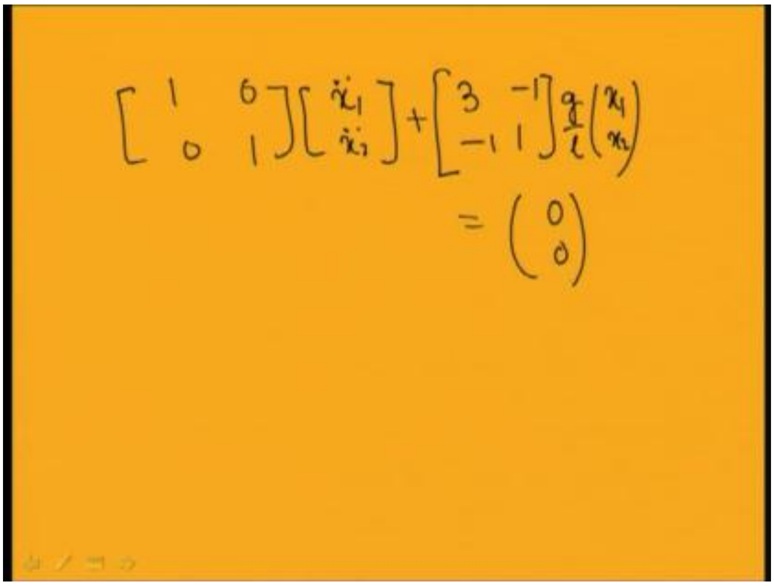

So, now I will tell you another method to determine this free vibration response of the system. So, let us take it general or we can determine the normal modes from the Eigen values of the system. So, let us take a general 2 degree of freedom system.

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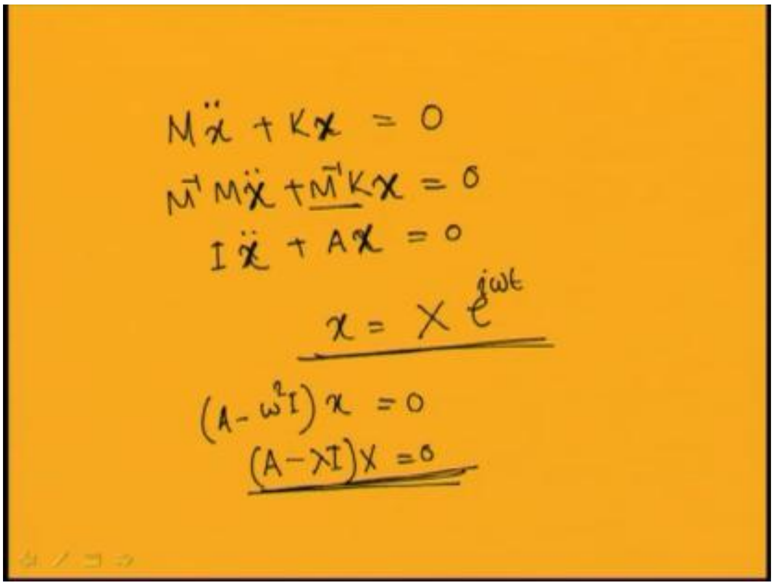

So, you can write the general 2 degree of freedom system in this form in matrix form that is x double dot plus K X equal to 0; where m is the mass matrix and K is the stiffness matrix. So, you just take the M inverse multiply pre multiply M inverse. So, M inverse M X double dot plus M inverse K X will be equal to 0. And this M inverse M is nothing but I X double dot plus. So, M inverse K let me write it call to A. So, A X will be equal to 0. So, if I will substitute this X. So, this is x I can write; so this make that x and so this is K x and this is x these are. So, I can write in this form. So, the x I can assume it equal to X e to the power i omega t. So, if will assume in these form i omega t or if we are if I am assuming the normal mode vibration that is all the masses are moving with same frequency omega.

So, if I will substitute this in this expression. So, this x double dot will radius to so x double dot will radius to minus omega square X x. So, these equation will radians to A minus omega square. So, A minus omega square I into x will be equal to 0 or if I will substitute this omega square equal to lambda. So, I can write this equation as A minus lambda Ix equal to 0. So, A minus lambda x equal to 0; this x I can write X e to the power I omega t. So, this becomes A minus lambda I x equal to 0. So, already you know this is a familiar equation of Eigen value problem.

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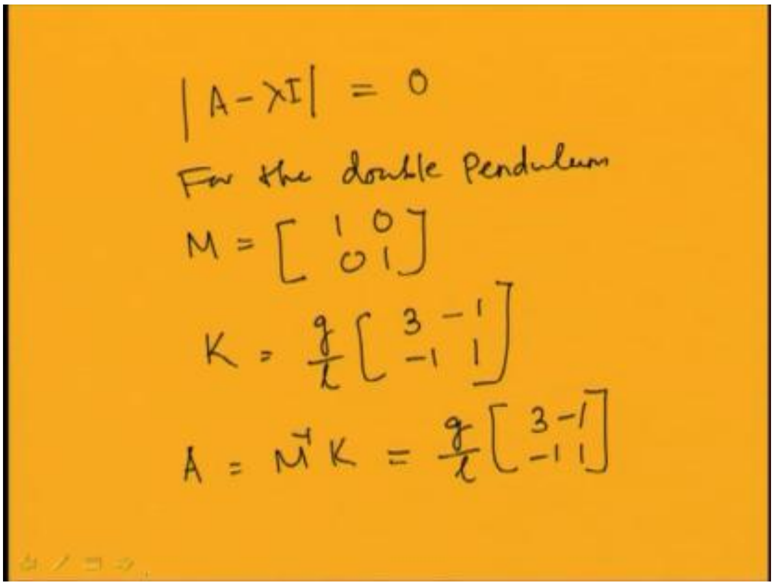

So, in this case you can obtain this lambda by finding the Eigen value of this matrix A or you can find the Eigen values or you can find the frequencies from by finding these A minus lambda I determinant of A minus lambda equal to 0. For these double pendulum already we know that the mass matrix equal to 1 0 0 1. So, for this double pendulum I can write this for the double pendulum. So, I can write this mass matrix M equal to 1 0 0 1 and the stiffness matrix K equal to g by l 3 minus 1 minus 1 1. So, I can find these A matrix. So, A is nothing but M inverse K. So, you just find the inverse of M matrix. So, inverse of this M matrix is also you can find it same.

So, M inverse K you can find. So, this M inverse K you can find it equal to g by l. So, this will be equal to 3 minus 1 minus 1 1, because this M inverse is the unit matrix. So, you multiply with these K matrix. So, it will remain. So, these becomes g by l 3 minus 1 minus 1 1. And so you can find A minus lambda I.

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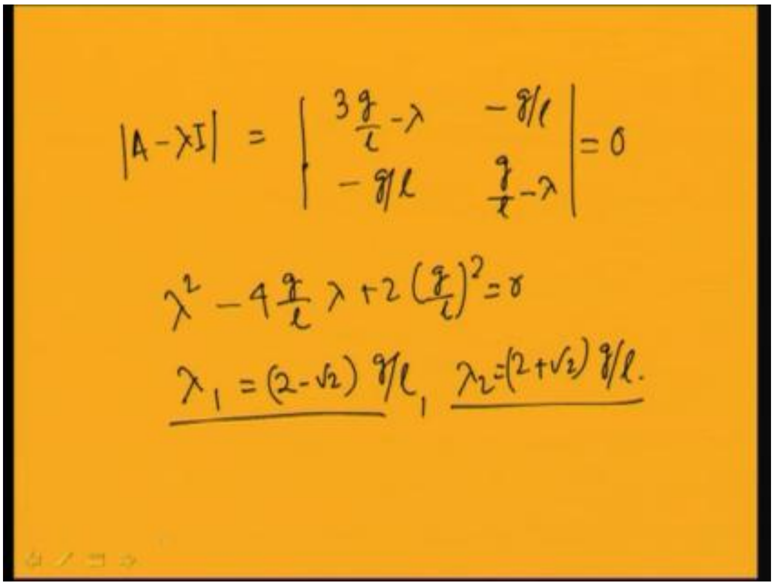

So, A minus lambda I will become; so A minus lambda I you can write will be equal to 3 g by l minus lambda. So, minus g by l and this is minus g by l and this is minus g by this is g by l minus lambda. So, the determinant of this will be equal to 0. So, you have a quadratic equation. So, this equation becomes lambda square this multiplied this minus this into this. So, these will give raised 2 lambda square minus 4 g by l lambda plus 2 into g by l square equal to 0. So, from this you can get lambda 1 equal to 2 minus root 2 g by l and lambda 2 equal to 2 plus root 2 g by l. You note that this lambda 1 and lambda 2 what you have obtain using these generalized coordinate theta 1 and theta 2; and using this coordinate system X 1 and X 2 are same.

So, the generalized; so whether you are using this X 1, X 2 as generalized coordinates are theta 1 and theta 2 are generalize coordinates they will lead to the same natural frequency; because the natural frequency of the systems are same. So, you can find now the normal mode. So, the normal modes can be obtained this is X 1 X 2.

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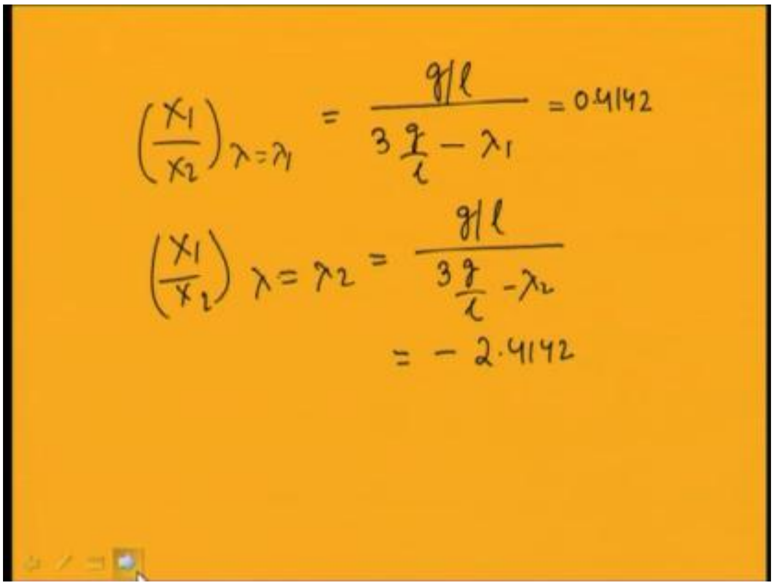

So, you can find this X 1 by X 2 at lambda equal to lambda 1. So, from this first expression; so from these expressions you can write. So, already you know this A minus lambda I X 1 X 2 equal to 0. So,3 g by l minus lambda into X 1 minus g by l X 2 equal to 0. So, this X 1 by X 2 will be equal to g by l by so these terms. So, the these g by l by 3g by l minus lambda. So, you can write this X 1 by X 2 is nothing but g by l. So, this equal to g by l by 3 g by l minus lambda. So, for lambda equal to lambda 1 you can substitute equal to this lambda 1.

So, this is reduced to or you can find it equal to 4 1 4 2. Similarly, X 1 X 2 lambda 1 lambda equal to lambda 2 will becomes g by l 3 g by l minus lambda 2; already you know your lambda 2 equal to 2 plus root 2 g by l. So, these becomes minus 2.4142. So, from these you know that when X 1 equal to 1 or when X 2 equal to 1 your X 1 becomes 2.4142. And when in the first mode when X 2 equal to 1 X 1 equal to 0.4142. So, you can draw these modes.

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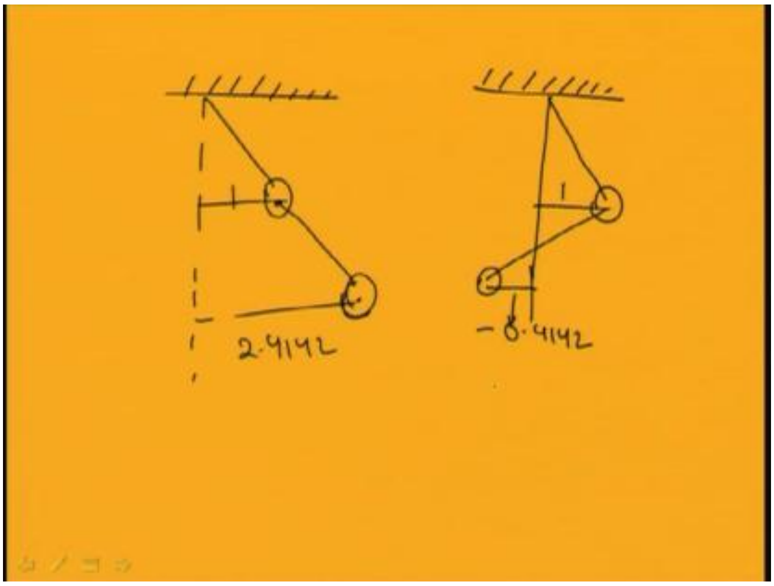

So, if we draw these modes of the first case you can see that; the first case the opposite way also you can do. So, when X 1 becomes 1; so this X 2 will becomes. So, when X 1 equal to 1, X 2 equal to 2; so this X 2 equal to 2.4142 that is 1 by 0.4 1 4 2 equal to 2.4142. And for the second mode you can see that so when this is equal to 1. So, this becomes you can find this; so you can see that this becomes minus0.4142. So, X 2 becomes minus 0.4142.

So, you can see that inverse of these 2.4142 equal to 0.4142. So, in the first case they are in phase and in the second case they are out of phase. So, in these way you can find the normal mode of a system by using the Eigen value method also. So, first; so the steps are you write the equation and write the mass matrix, write the stiffness matrix. So, now you find the A matrix. So, A matrix equal to M inverse K. So, after finding A matrix. So, you just find the determinant of A minus lambda I. So, from that you can find the value of lambda. So, in this case you will get 2 value of lambda I lambda is nothing but square of omega 1 omega square. So, from that you can get the normal mode frequency of the system. So, to find the free vibration response of the system; so you can use the super position theory or you can assume that the free vibration response of a system is the summation of the normal modes.

So, taking those summations and using the initial conditions you can find the free vibration response of the system. Now, I will tell you about the coordinate coupling of a system; already I told you the static coupling and dynamic coupling of the system. So, let me take some example to show you that how by changing this coordinate system, you can find or you can make the system uncoupled or coupled or you can make the system statically and dynamically coupled or uncoupled.

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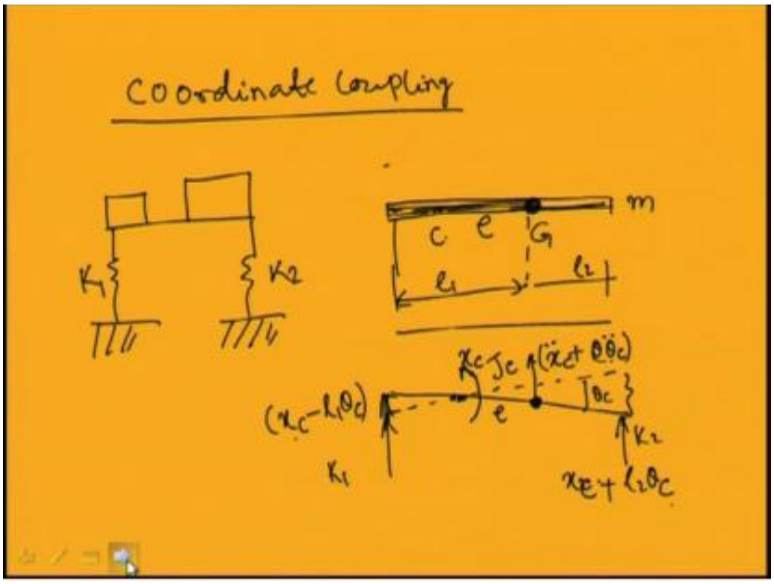

So, let us see these coordinate coupling of the 2 degree of freedom system. So, coordinate coupling; so let me take the simple example of that lathe machine previously I have told. So, in that case I can assume that lathe machine consisting of a head stock and tail stock as a single mass I can or I can assume this as a rod with mass concentrated are some position. So, let I am assuming that the mass is concentrated at this position G. So, this is the position G where the mass is concentrated; I can assume this the bed of this lathe machine. So, this is the lathe machine you can take; so with a head stock and tail stock. So, this is the head stock, this is the tail stock I am taking. So, the mass I am assuming that the mass is concentrated towards this head stock. So, this is the point G.

So, I can take different coordinate system now to find the equation motion of the system. And I will show you by taking different coordinate systems you can make the system coupled or uncoupled or you can make the system statically and dynamically coupled and uncoupled. So, let me take a point C here and I will define. So, this is the reference line. So, I can take a reference line; for initially let it is at this position Now, due to this vibration so it has come to this position. So, this is the point C which I am taking the where I am taking the coordinate frame. So, this point C will have a displacement x c and rotation theta c. So, it will rotate by an amount theta C. So, let these distance between C and e is e; that is eccentricity. So, I can take these as e. So, as I am assuming this theta C to be very small. So, these are the so I can take the spring or the spring force to be vertical. So, this is K 1 I am taking the spring constant is K 2.

So, this distance the displacement of this point and displacement of this point I can take; and I can find the equation motion or so the displacement at this point; so this is theta. So, displacement of this point let us find. So, displacement of the different points will find; and let us draw it clearly and do it again. Let me take the line like this. So, this is x c and I will draw a line parallel to this and this is the forces acting on this points. So, this is the G point. So, the inertia force will act at this point. So, this point will have a acceleration of so this point has a acceleration the C point has a acceleration of x c double dot.

So, these point which is at a distance e from these and this and this angle I am taking it is theta C. So, this point will have a displacement relative to these equal to e theta C. So, acceleration will be e theta C double dot. So, this point will have a acceleration of x c double dot plus theta e theta c double dot. So, if I am taking M as the mass of this bed; then I can write the inertia force equal to M into x c double dot plus e into theta c double dot. So, this point will have this is k 1, so this is k 1. So, this distance equal to so this distance equal to x c. So, this is the final position of this; so the as this is the final position ok. So, this is the final position the force will act here and here. So, this distance equal to so this is x c minus. So, let me take the distance between these and these equal to. So, this is l 1 and this is l 2. So, I can take this is l 1 this is l 2. So, this will be equal to x c minus. So, this total distance is x c so this will be so this distance equal to l 1 theta c theta c. So, the spring will be have a displacement of x c minus. So, this will have a displacement of x c minus l 1 theta c and this spring we have a displacement of x c. So, this is x c. So, this is the additional displacement it will have.

So, this is x c plus l 2 theta c. So, the force here will be equal to k 2 into this displacement that is x c into l 2 theta c and the force here equal to k 1 into x 1 minus l 1 theta c. And the inertia force acting at point G will be equal to mass into x c double plus e into theta c double dot. So, in addition to this at the point; here let me assume that the inertia, rotational inertia equal to J c. So, I can write J c. So, this point will have a rotational inertia force equal to J c into theta c double dot. So, I can write the equation motion by doing the force balance.

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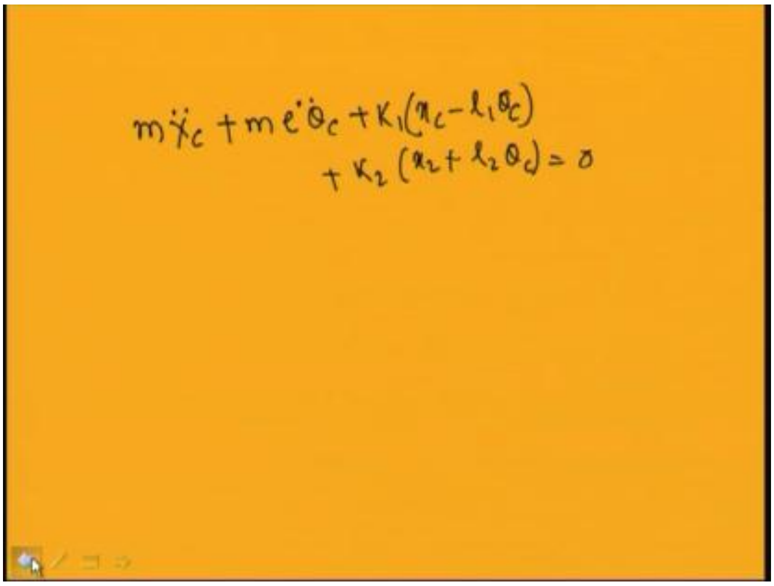

So, by doing the force balance I can write m into X c double dot plus m e theta c double dot plus k 1 into x c minus l 1 theta c plus k 2 into x 2 plus l 2 theta c equal to 0.

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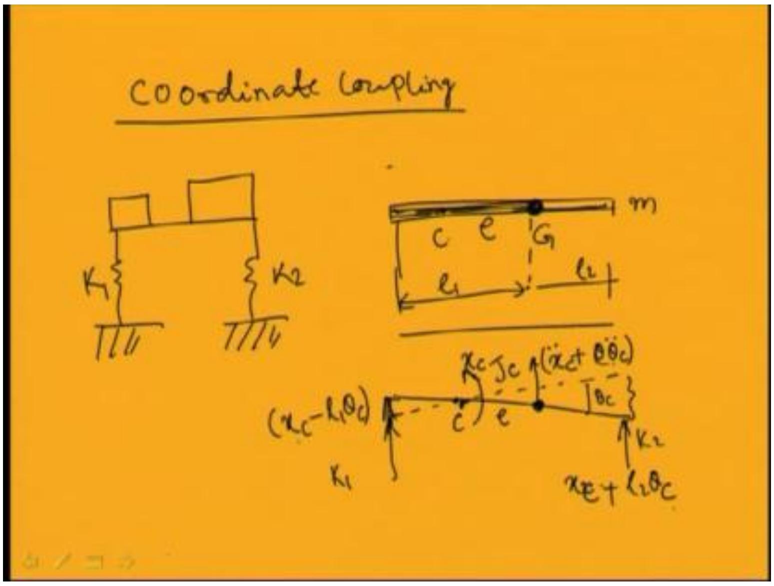

And, taking the moment above this point c; I can take the moment about this point c. So, moment about this point will be equal to 0. So, moment of all the forces about this point I can take so this will be equal to J c into theta c double theta c double dot plus this mass into this inertia force into this distance e. And then this force into this distance and this force into this distance.

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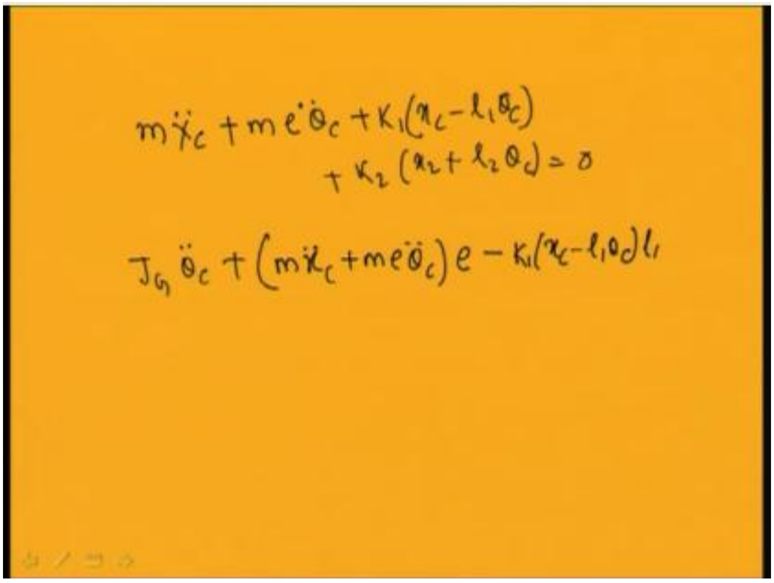

So, the equation I can write in this form. So, it will be equal to J G theta c double dot; inertia force will act at the mass entered that is at G. So, J G I can write; so J G theta c double dot plus. So, plus I can write it equal to m x c double dot plus me theta c double dot. So, these into distance e minus this k 1 into x c minus l 1 theta c into…

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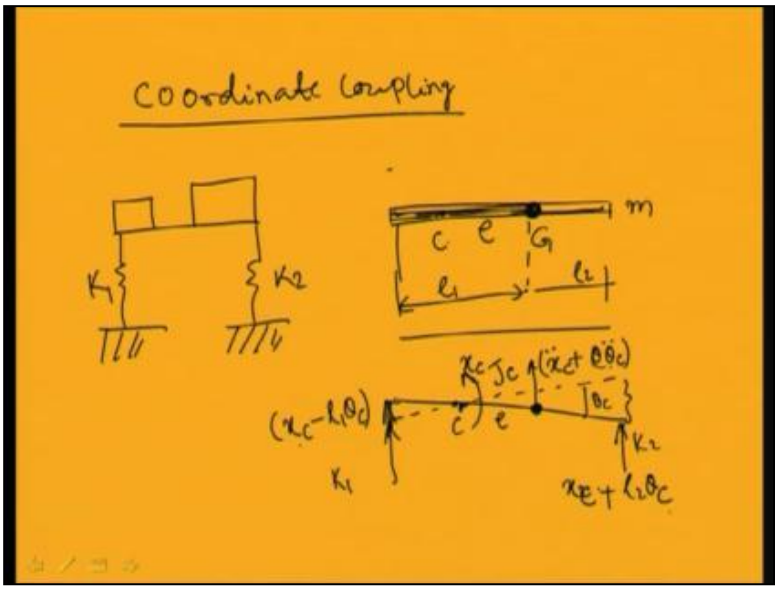

So, this is at a distance x 1; as I am assuming this theta 1 to be small. So, this distance I can take it as l 1.

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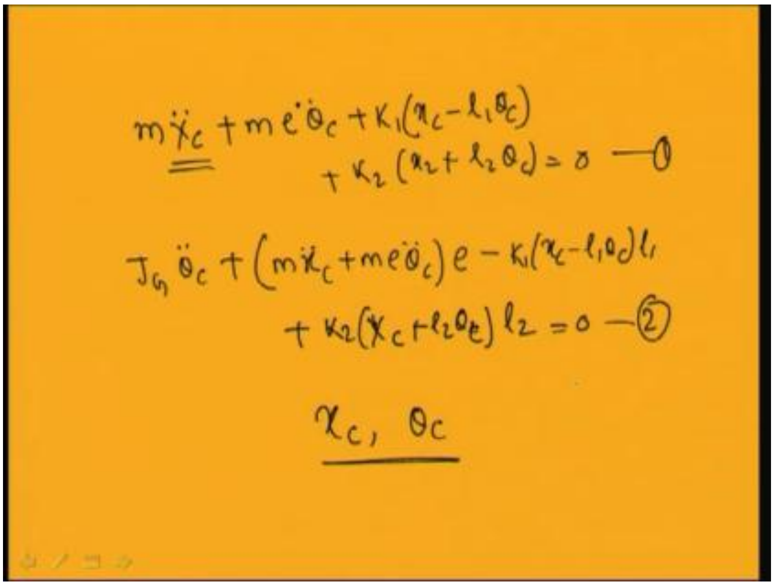

Similarly, that other support distance I can take it as l 2. So, plus k 2 into X c plus l 2 theta l 2 theta c into l 2 equal to 0. So, these are the 2 equations I obtain for this; let me seen I can write I have express the motion in terms of X c and theta c. So, the generalized coordinates I have taken are x c and theta c are the generalized coordinates I have taken. So, I can express these 2 equation in terms of the matrix form also.

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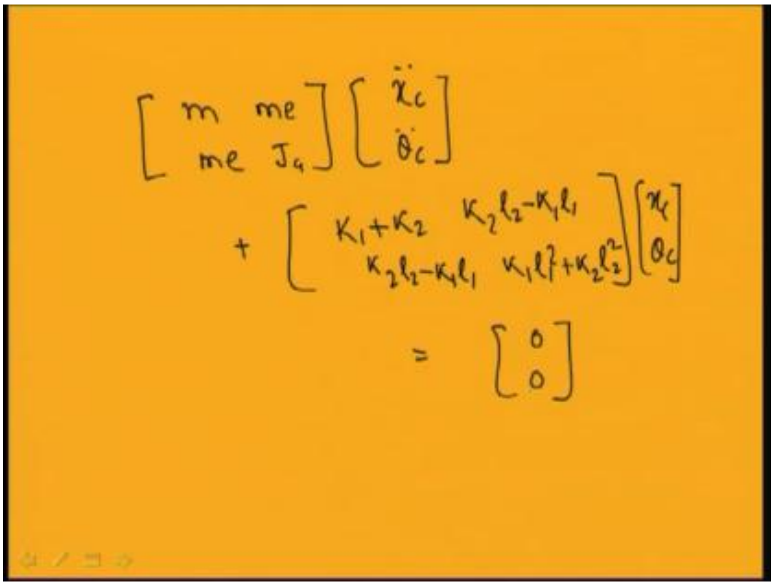

So, in matrix form I can write it like this. So, it becomes m m e m e J into x c double dot theta c double dot plus k 1 k 2 k 1 plus k 2 k 2. Then, so this is k 2 l 2 minus k 1 l 1. So, this is k 2 l 2 minus k 1 l 1 this is k 1 l 1 square plus k 2 l 2 square into x c theta c So, this becomes 0, 0.

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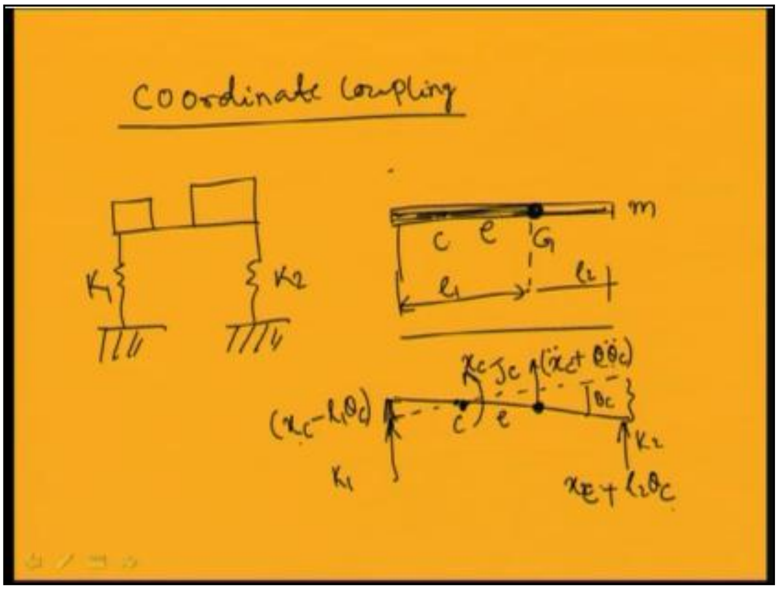

So, in this case you just observe that by taking a coordinate system at position C. So, I am taking a coordinate point at or I am describing the motion of the system by taking a coordinate system here; that is the displacement translational displacement x c and rotational displacement theta c.

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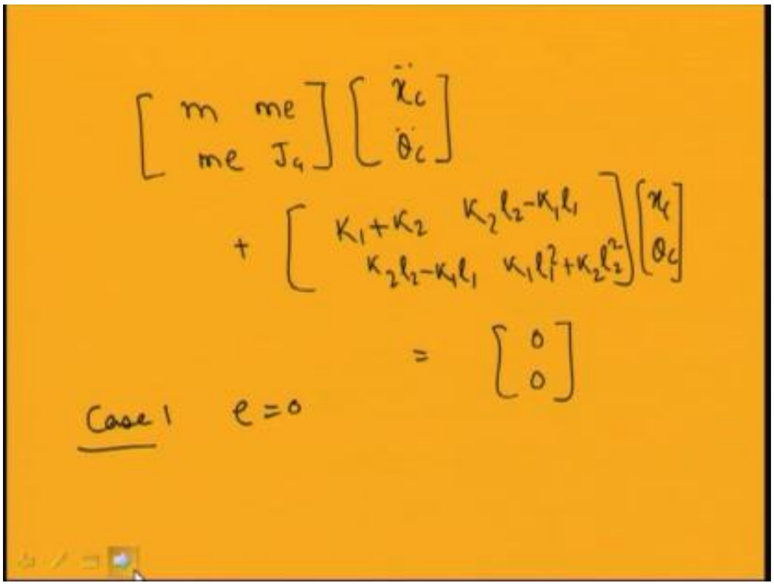

So, I am obtaining a equation motion where both the mass matrix and stiffness matrix are coupled. So, the system is both dynamically and statically coupled. So, I can make the system uncoupled by taking the coordinate system at other places also. So, let me take the different conditions. So, let for case 1 let us take e equal to 0.

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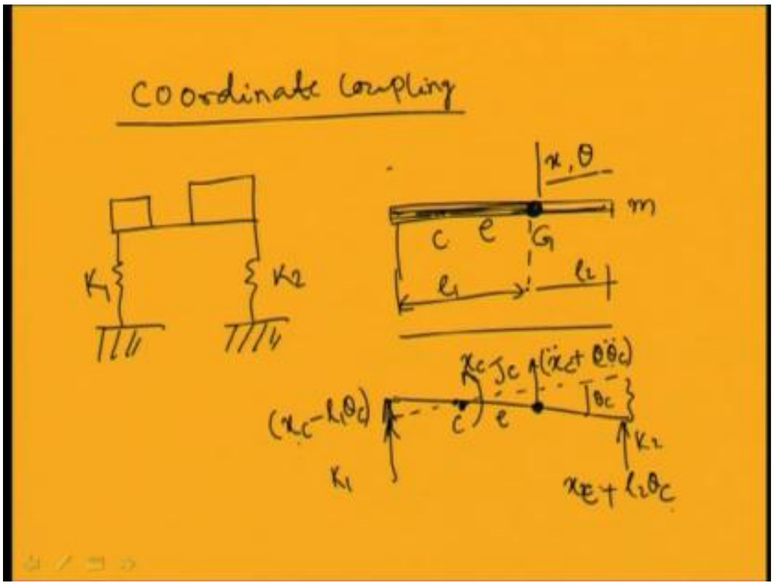

That means, if I take the coordinate system at point G instead of taking at point C. So, if I will take the coordinate position here, and I will describe this motion in terms of x; and theta rotation displacement x here and rotation theta here.

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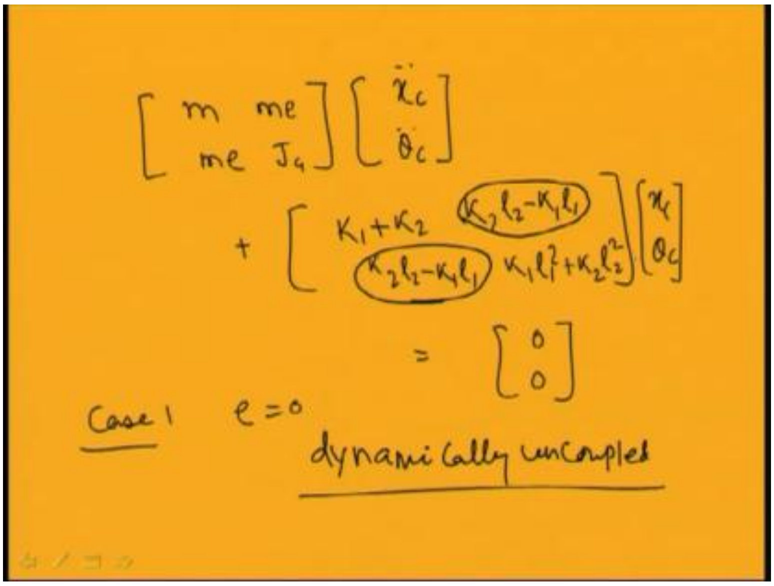

So, in this case this equation will reduce to so this e equal to 0. So, this mass matrix become m 0 and 0 J G but the stiffness matrix will be unchanged. So, in this case the system will reduce to or system will have a mass matrix uncoupled mass matrix. So, the system will be dynamically uncoupled. So, you will get a dynamically uncoupled system. Now, you see that if I will take this k 2 l 2 equal to k 1 l 1. So, if I will take k 2 l 2 equal to k 1 l 1. So, these part equal to 0. So, this term will becomes 0 and this term also will become 0. So, in this case we have a stiffness matrix where k 1; so stiffness matrix will be k 1 plus k 2 0 and 0 k 1 l 1 square plus k 2 l 2 square. So, this is also statically uncoupled; this becomes statically uncoupled if I am taking so if I am taking this k 2 l 2 equal to k 1 l 1.

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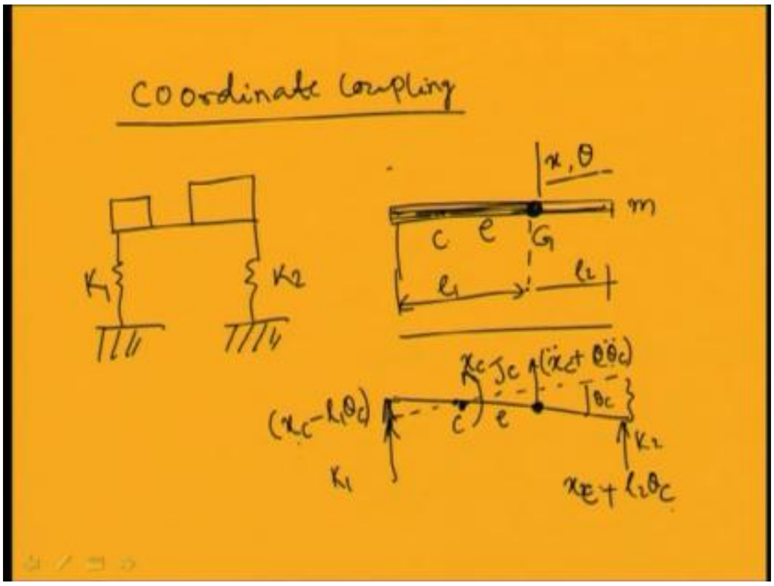

So, if I will take l 1 equal to 0 that means point C point coincide with the left hand. Then, the equation motion will become so you can see that l 1 if I will take at 0. So, in that case this equation will becomes like this.

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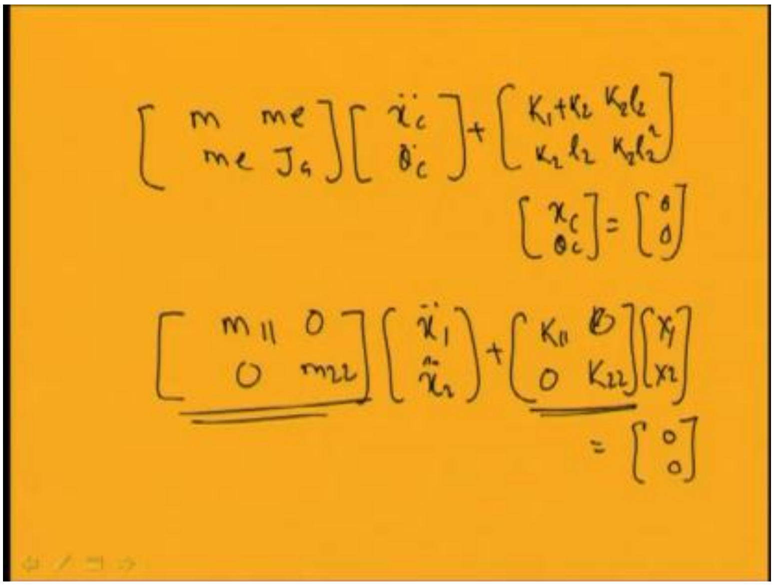

So, it becomes m m e then this me J x c double dot and this then theta c double dot plus k 1 k k 1 plus k 2 k 2 l 2 k 2 l 2 k 2 l 2 square theta into x c theta c equal to 0 0. So, this case also the system is both dynamically and statically uncoupled. So, by changing this position of the coordinate system you can make the system a coupled one or uncoupled one or you can get statically and dynamically coupled or uncoupled system. Already I told you the coordinate system for which you are getting a system which will have both dynamic uncoupled, dynamically uncoupled and statically uncoupled equations or that is mass matrix is an uncoupled and stiffness matrix is uncoupled; that coordinate system is known as the principle coordinate system. So, by changing this position of this coordinate system you can get a coordinate system or you can get the equation motion; where it will be reduced to a set of coordinate system which will looks like this. So, m 1 1 0 0 m 2 2 x 1 double dot x 2 double dot plus k 1 1 0 0 k 2 2 x 1 x 2 equal to 0 0; so these 2 equations. So, if they are uncoupled dynamically uncoupled and this is statically uncoupled. So, I can write these equation in this form.

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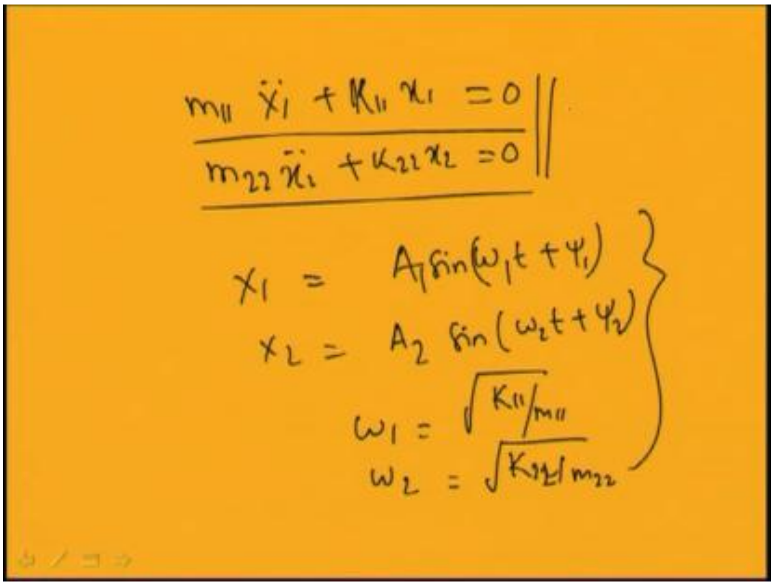

So, this will becomes m 1 1 X 1 double dot plus k 11 x 1 k 1 1 x 1 equal to 0. And second equation becomes m 2 2 x 2 double dot plus k 2 2 x 2 equal to 0. So, you can see that they have reduced or there reduce to a set of 2 equation which are the first order equations, which are the equations you have already studied for the single degree of freedom system. So, you can solve these equations very easily by writing X 1 equal to so X 1 will be equal to. So, this X 1 I can write equal to A sin omega 1 t plus psi 1 and this X 2 will be equal to A 2 sin omega 2 t plus psi 2; where this omega 1 is nothing but root over k 1 1 by m 1 1 and omega 2 is root over k 2 2 by m 2 2. And the psi 1 psi 2 and A 1 A 2 can be obtain from this initial conditions. So, if you are able to reduce the equation motion to that of a equations where both the mass matrix and stiffness matrix are uncoupled.

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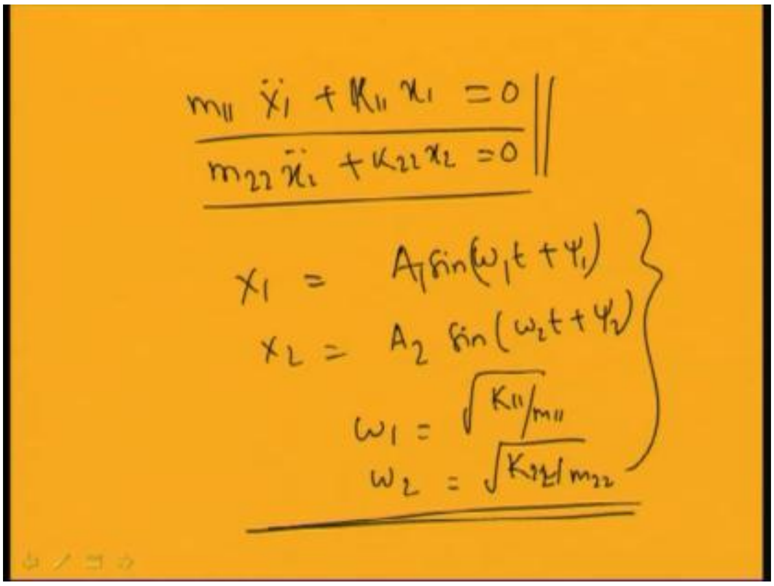

Then, you can convert this equation to a set of single degree of freedom system equation. And already for single degree of freedom system you know the solutions and you can find the solution of the system or you can find the normal mode of the systems very easily. So, now let us study about a system with damping.

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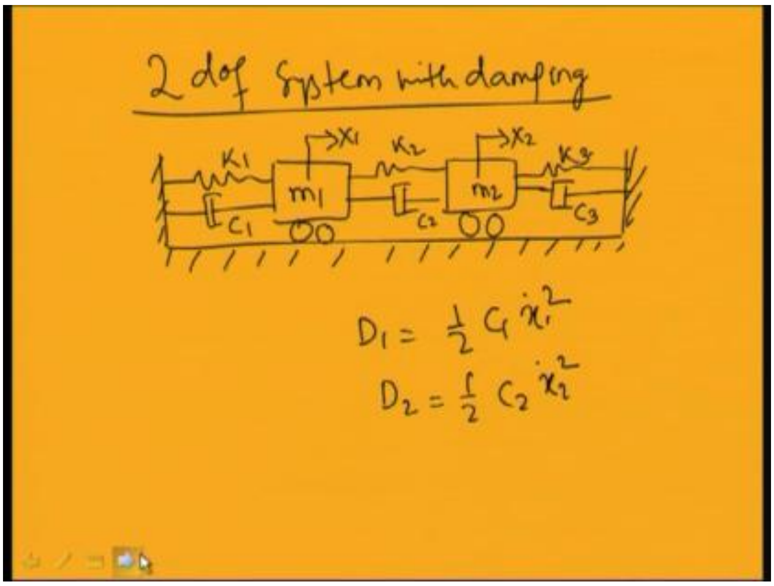

So, 2 degree of freedom system with damping. So, previously we have studied the system without damping. Now, let us take a system with damping. So, in this case I can draw the system like this. So, let me take this same system what I have taken before but here I will introduce some damping in the system. So, this is the mass, first mass. So, let me take a damper here. So, again another damper I can take it here. And so this is the third damper I have taken. So, it is connected to this constant one. So, now you can derive this equation motion for the system.

So, this is I will take it is k 1, this is mass m 1, this is mass m 2 and this is c 1, this is c 2, this is stiffness is k 2, this is k 3 and this is c 3. So, you can take the displacement of mass m 1 as x 1 or I can take a generalized coordinate x 1 and x 2. So, x 1 is a displacement of mass m 1 and the x 2 is the displacement of mass m 2; either you can use Lagrangian principle or you may use this Newton’s second law to derive this equation motion; while using this Lagrange principle you can take this dissipation energy. So, dissipation energy in the first case it will be equal to half c 1 x dot square x 1 dot square and in a second case it becomes half c 2 into… So in the first case your D 1 equal to half C 1 x 1 dot square and your D 2 equal to half C 2 x 2 dot square. So, the equation motion if we are using this Lagrange principle. So, the equation motion you can write in this form.

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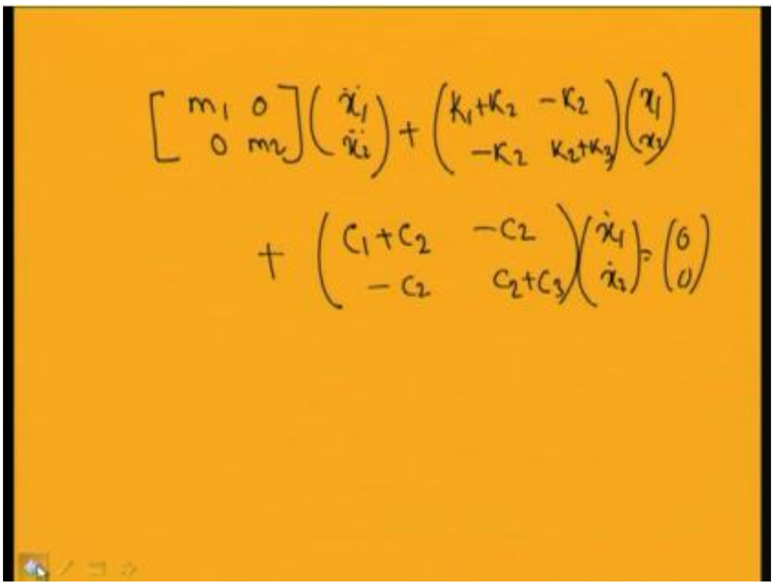

So, your equation motion will reduce to your equation motion is reducing to m 1 0 0 m 2 x 1 double dot x 2 double dot plus k 1 plus k 2 minus k 2 minus k 2, this is k 2 plus k 3 x 1 x 2. Already you got these expressions, and if your adding damping now your equation is reduced to this. So, this becomes C 1 plus C 2 minus C 2 this is minus C 2 then this become C 2 plus C 3 into x 1 dot x 2 dot equal to 0 0.

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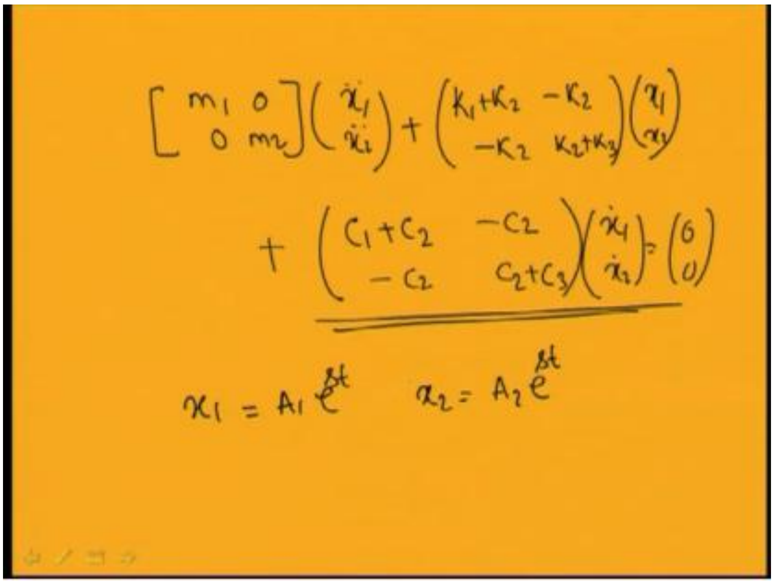

So, by adding this damper in the system your equation motion it just changed, just you have added this term to your previous equations what you have obtained before. So, now you can assume a solution normal mode solution in this one. So, you can assume your x 1 equal to A 1 e to the power s t and x 2 also you can assume it equal to A 2 e 2 the power s t. So, you just see that in case of normal mode vibration we are assuming the system to be vibrating with same frequency. So, I have taken this s; s is complex number So, you can take the solution in this form A 1 e to the power s t and x 2 equal to A 2 e to the power s t. So, if you substitute this in this equation; so you can write this equation in this form.

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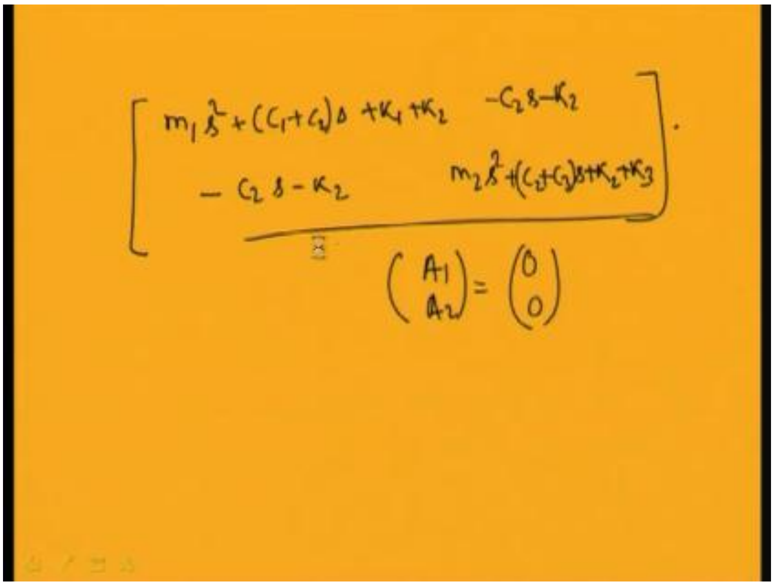

So, it becomes m 1 s square plus C 1 plus C 2 s plus k 1 plus k 2 then minus C 2 s minus k 2. And this becomes minus C 2 s minus k 2 and this becomes m 2 s square plus C 2 plus C 3 s plus k 2 plus k 3. So, these multiplied by so this into A 1 A 2. So, this into A 1 A 2 equal to 0 0. So, as this multiplied A 1 A 2 equal to 0 0. So, for non trivial solution the determinant of this the determinant of this will be equal to 0. So, if you find the determinant of this; so this will reduce to A forth order equation.

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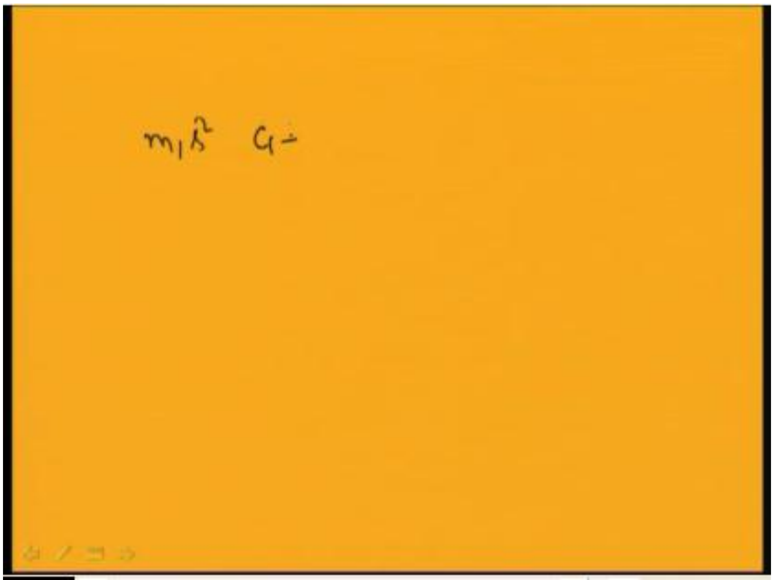

So, this equation you can write in this form. So, this equation becomes m 1 s square. So, if you multiplied that thing this becomes m 1 s square plus C 1 plus or you can find.

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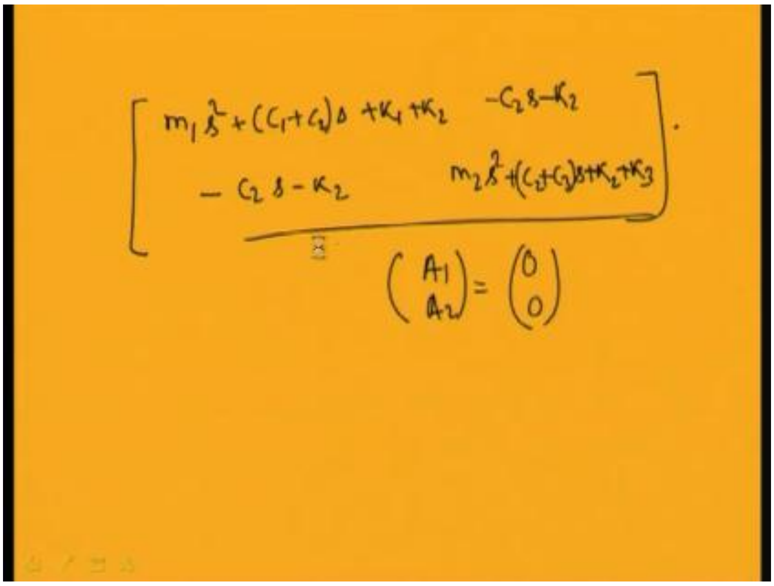

So, this multiplication this minus this minus this into this equal to 0. So, you will have 4 roots; so from these 4 roots you can find the solution of the system. So, using those 4 roots you can find 4 frequency and using those frequency, we can write the solution of the system.

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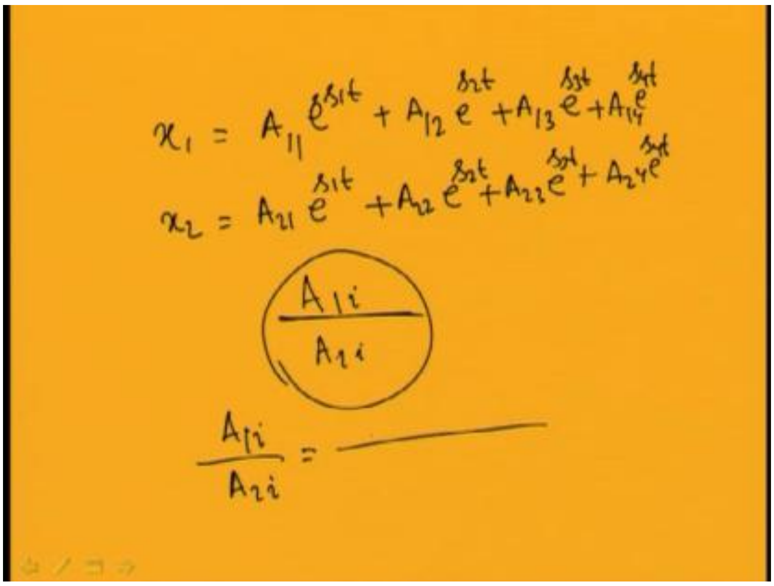

So, you can write your x 1 equal to A 1 e to the power. So you can write x 1 equal to A 1 e to the power s 1 t plus or A 1 1 s 1 t is A 1 2 e to the power s 2 t plus A 1 3 e to the power s 3 t plus A 1 4 e to the power s 4 t. And similarly you can write x 2 equal to A 2 1 e to the power s 1 t A 2 2 e to the power s 2 t plus A 2 3 e 2 the power s 3 t plus A 2 4e to the power s 4 t. So, you can see that this A 1 A 11 or A 1 i and A 2 i are arbitrary. And using these initial conditions either you can find A i or A 2 i as A 2 i and A 1 i are related. So, you finding this first A 1 i you can find A 2 i as you know that this A 1 i by A 2 i nothing but this is equal to so from these equation you can find.

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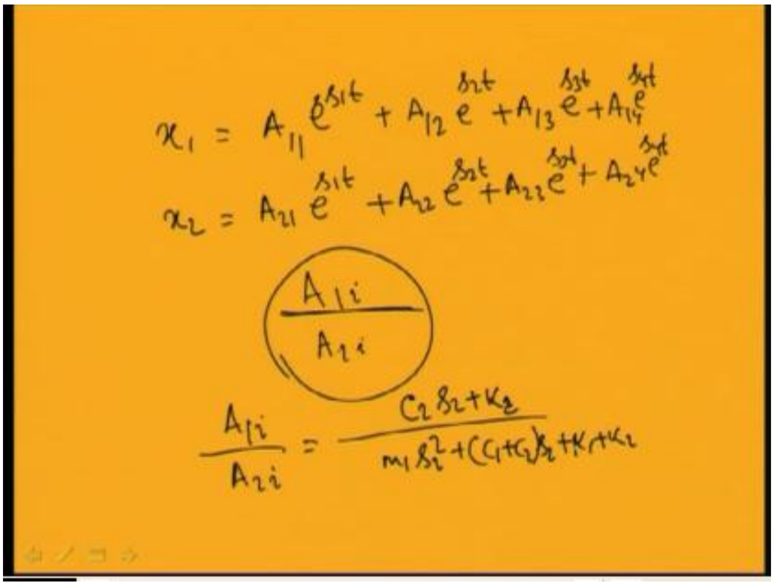

So, from these equation you can write A 1 i by A 2 i is equal to c 2 s i plus k i k 2 by this m 1 s i square plus the C 1 plus C 2 s i plus k 1 plus k 2. So, in this way you can determine the response of a system with damping. So, next class we will study, we will take one example solve this damp system, and also will study about the force vibration of 2 degree of freedom system.