Mechanical Vibrations
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Module - 5
Two DOF Free Vibrations
Lecture - 2
Lagrange's equation

In the last class, we have studied about the two degree of freedom system. There we have found the equation motion by using the Newton's principle or d'Alembert principle; also you have used Lagrange's principle and extended Hamilton principle to derive the equation motion. So, in case of two degrees of freedom system, we required minimum two coordinates to define the motion of the system. And in this case, I have already told you about three different coordinate systems.

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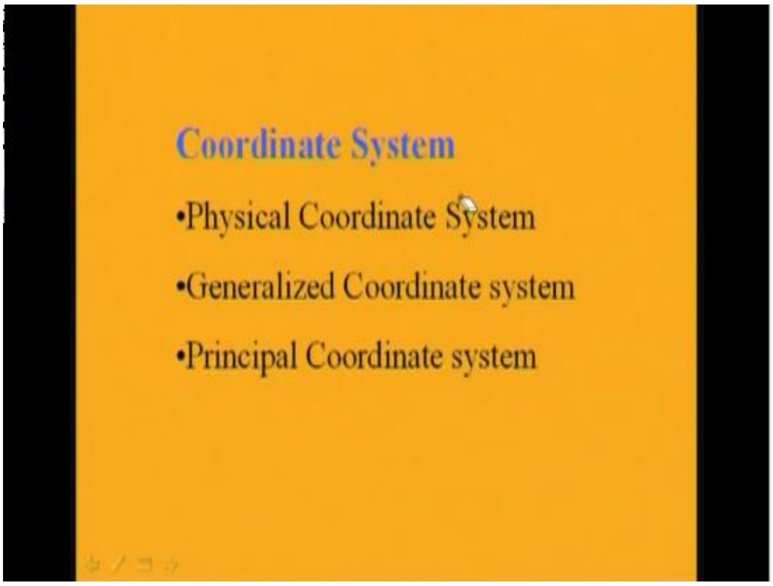

One is the physical coordinate system, second one is the generalized coordinate system, and the third one is the principle coordinate system. In case of a physical coordinate system, you can fix a physical coordinate or you can fix a reference frame. And from this reference frame, you just take the coordinates of this system or coordinates of the point where you are finding or using which you are finding the equation motion. In case of generalized coordinates, so these are the coordinates or minimum number of coordinates required to express the motion of the system completely.

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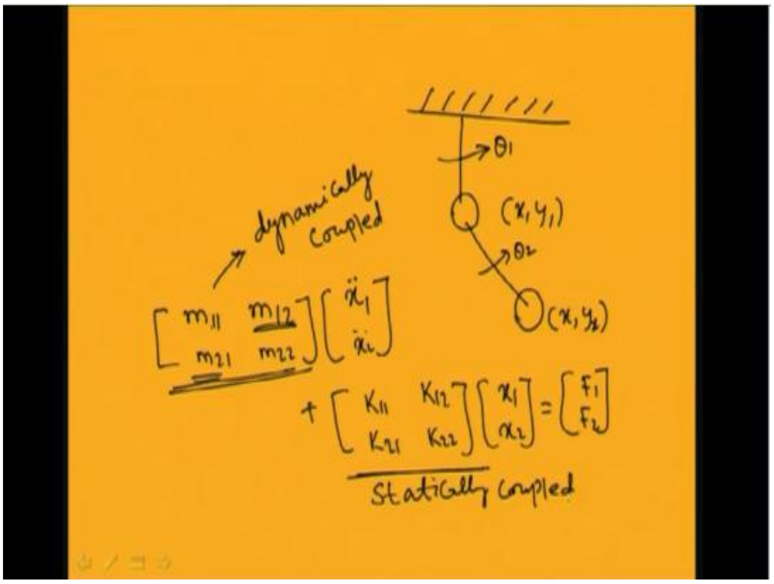

So, already you have seen in the case of a double pendulum. So, this is the double pendulum I have already told. So, in this case of a double pendulum, you have used x 1 y 1 x 2 y 2 as the x 1 y 1 x 2 y 2 as the physical coordinates and this theta 1 rotation of this theta 1 and rotation of this theta 2 as the generalized coordinates. And already I told you by using these coordinates, you can write the equation motion in case of a two degree of a freedom system and you can write it in the matrix form.

So, in the matrix form you can write it like this m 11 m 12 m 21 and m 22 x 1 or in this case you can write it equal to theta 1 double dot or you can write in terms of x 1 x 2. So, you can write it is equal to x 1 double dot x 2 double dot plus k 11 k 12 k 21 k 22 x 1 x 2 equal to f 1 f 2. So, in case of free vibration this force term the right hand side terms are zero and I told you about the dynamic and static coupling. So, if this mass matrix is coupled that is all the terms are present are half diagonal; some of the half diagonal terms are present, then the system is said to be dynamically coupled, so dynamically coupled.

So, in this case some of the terms of diagonal terms that is this is the diagonal terms; this m 11 and m 22 are the diagonal terms. So, this one and these two are the half diagonal terms. If these half diagonal terms are not zero, then the system is still to be dynamically coupled. Similarly, in case of the stiffness matrix if the half diagonal terms are not zero or if their present some half diagonal terms, then the system is said to be statically coupled. So, already you have seen the case of statically coupled and dynamically coupled system.

So, in this case to make the system dynamically and statically uncoupled you can take another set of coordinate system. So, those coordinate systems using which you can write the equation motion in the uncoupled form, or the system will reduce to a dynamically uncoupled and statically uncoupled form are known as the principle coordinate system. So, the principle coordinate systems are the generalized coordinate system by using which you can write the equation motion in an uncoupled way or in that case the dynamic matrix and static matrix will be uncoupled. So, in that case it will reduce to two single degree of free system equation and using these two equations you can find the solution of the system.

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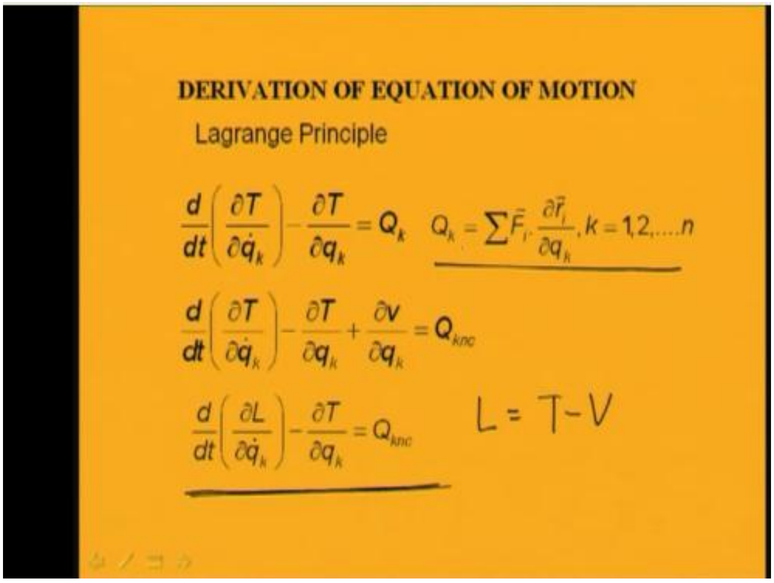

So, already I told you about the extended Hamilton principle and Lagrange's Principle. In case of Lagrange principle I told you, you can use this equation d by d t of del l by del q k dot minus del t by del q k equal to Q k n C. So, here l is the Lagrangian of the system that is equal to T minus. So, this Largrangian L equal to T minus V, T is the kinetic energy, V is the potential energy of the system, and this Q k is the generalized force which you can find from this. So, in this equation F i is the force acting at i th station, and r i is the position vector of the i th point on this system.

So, small q k is the generalized coordinates you are taking. So, in these two degrees of freedom system k will be equal to 2. So, it will be either 1 or 2 and using this formula you can find the generalized coordinates.

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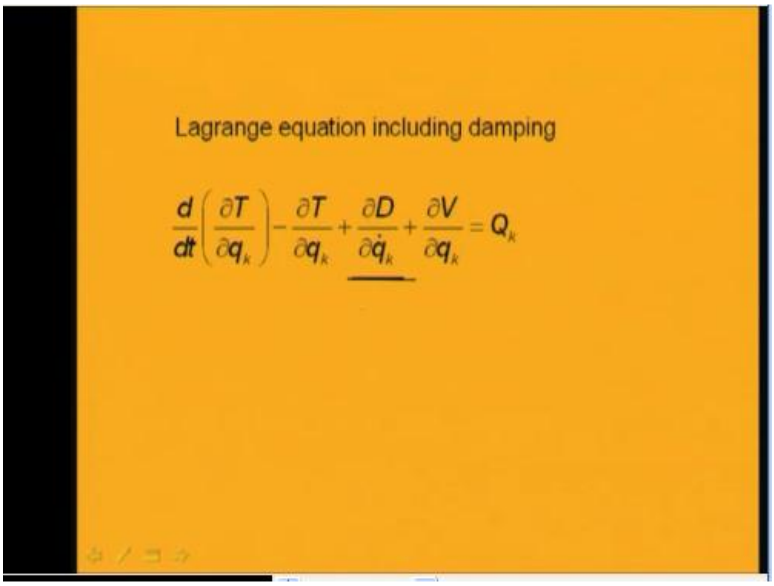

And for a system with damping, you can use this dissipation energy; this d is the dissipation energy. And using this dissipation energy you can find the equation motion.

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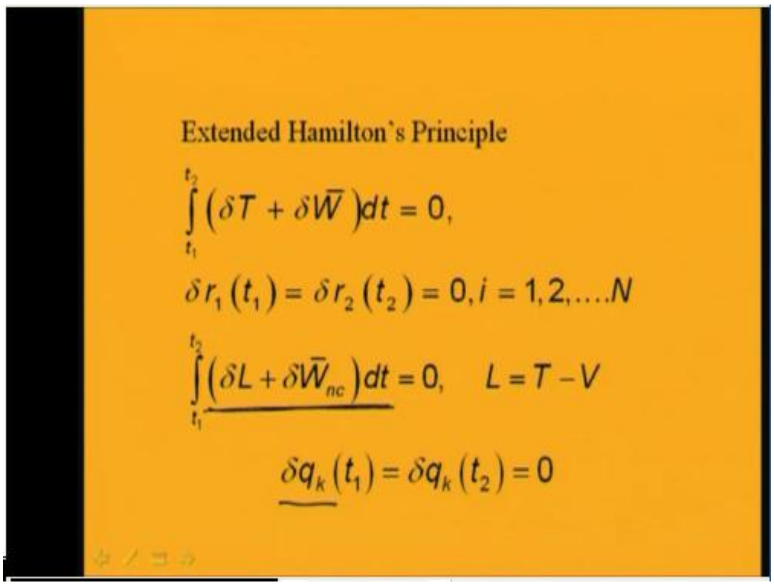

So, already also I told you about the extended Hamilton Principle. In extended Hamilton principle, you can write the equation motion by using this equation. So, it is equal to del

L plus del W n C d t equal to zero, where this virtual displacement del q k at t 1 equal to virtual displacement and del q k t 2 equal to 0.

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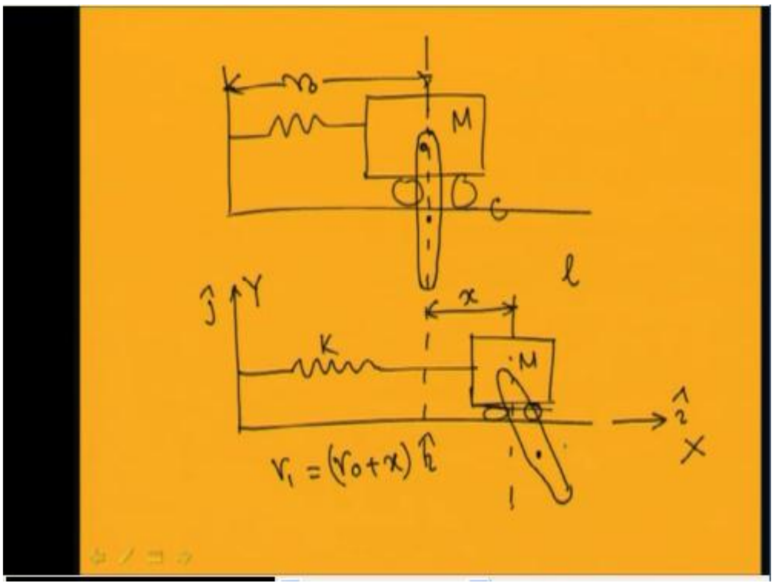

So, previously we have derived the equation motion of a system. So, these systems we have taken; to derive the equation motion we have take a spring and a mass. So, these spring mass system we have taken. So, in this spring mass system one rod is hanging from this cart. So, this is the mass. So, from this mass, a rod is hanging, and I have retained this point C. So, let C is the position of the mass centre of this rod. So, for simplicity I have assumed this mass centre is situated at l by2 distances from this l by two distance. Let l is the length of the rod and at l by 2 distance this mass centre is situated.

So, when this body is not moving or when it is not undergoing any vibration. So, this is the undeformed position and the deformed position if I will plot, so in the deformed position. So, this is the undeformed position, and you can draw the deformed position. So, in the deformed position the mass is at this position. So, the rod position is like this. So, this is the rod. So, this is the mass centre. So, you can have. So, this is the spring and the spring you can draw it. So, spring is traced, and this is the spring.

So, if I will take this undeformed position as r 0 and I can take this displacement as x. So, I can take a coordinate system this direction this is x, and position of this unit vector is I, and this is y direction, and unit vector I can take this. So, position vector of this mass m can be written as r 0 plus. So, r 1 I can write equal to r 0 plus x i and the position vector of point C already I told you that position vector of…

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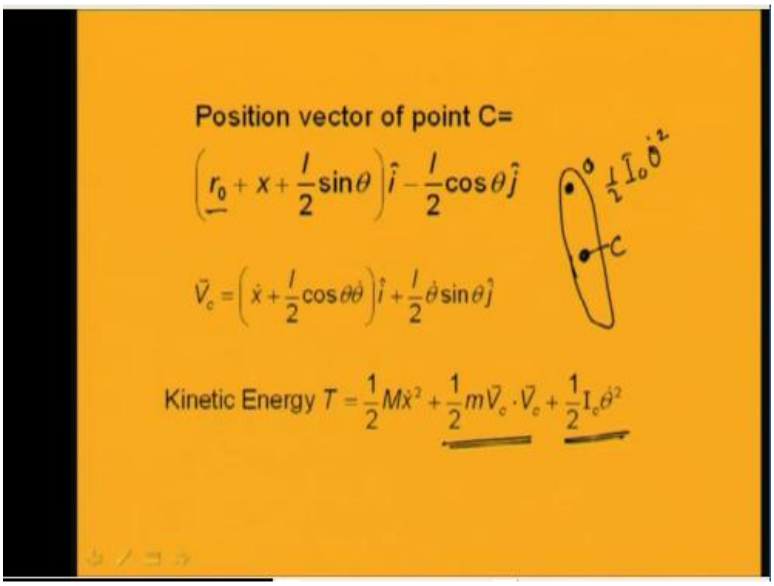

So, point C you can write. So, position vector of point C equal to. So, position vector of point C is given by r 0 plus x plus l by 2 sin theta i minus l by 2 cos theta j. So, from this by differentiating this position vector, you can find the velocity of that mass centre. So, V c will be equal to. So, if you differentiate this thing, this part is constant. So, it will become x dot plus l by 2 cos theta theta dot i plus l by 2 theta dot sin theta j. And kinetic energy already I told you this is equal to half M x dot square plus half m V c dot V c plus I c theta dot square.

So, if you are taking in these. So, this mass centre either you can take the body or the kinetic energy of this body by taking the purely rotation about this point or you can take the translation and rotation about the mass centre. So, both will yield the same thing. So, if you are taking the mass centre. So, it will have a translational kinetic energy that is half m V c dot this V c and plus rotational kinetic energy, or if we are taking the rotation about these then it will be if I will take this point i. So, it will be half i 0 theta dot square. So, this i 0 is nothing but it will be equal to m l square by 3. So, if you add these two terms, it will give the same expression.

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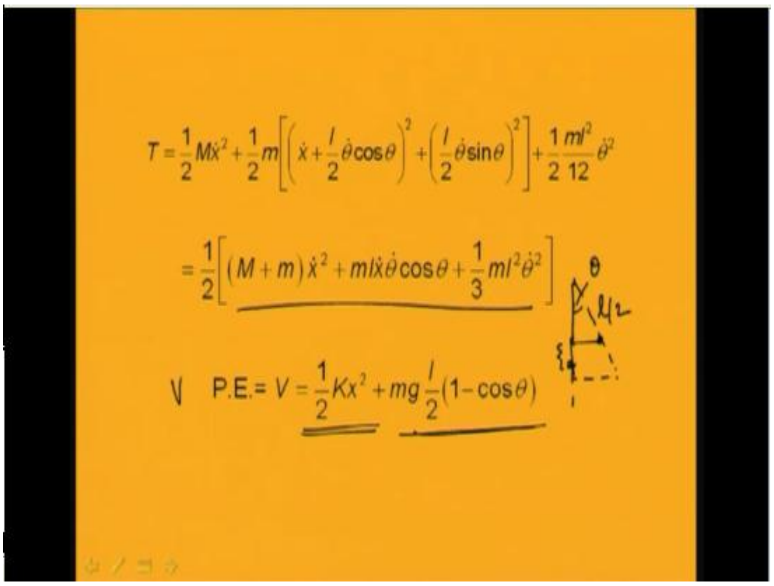

So, in this case you already know that this kinetic energy is given by this. So, this is the expression. So, one-third m l square theta dot square already you are getting, and this is due to the translation of this, and this is the couple term. So, now you can find the potential energy of the system. So, potential energy due to the spring; so that is equal to K x square and due to change in position of the rod. So, initially the rod was straight; now it has come to this position.

So, this change in position; so this is the mass centre at a distance l by 1. So, this angle you have taken theta. So, change in position will be this to this. So, this length initially it is l by 2 at a distance l by 2. Initially the mass centre is here; now it has come to this position. So, this is the change in the position of the mass. So, potential energy equal to m g into change in position of the mass. So, this is equal to mg in to l by 2 1 minus cos theta. So, this is the potential energy V and already you know the kinetic energy.

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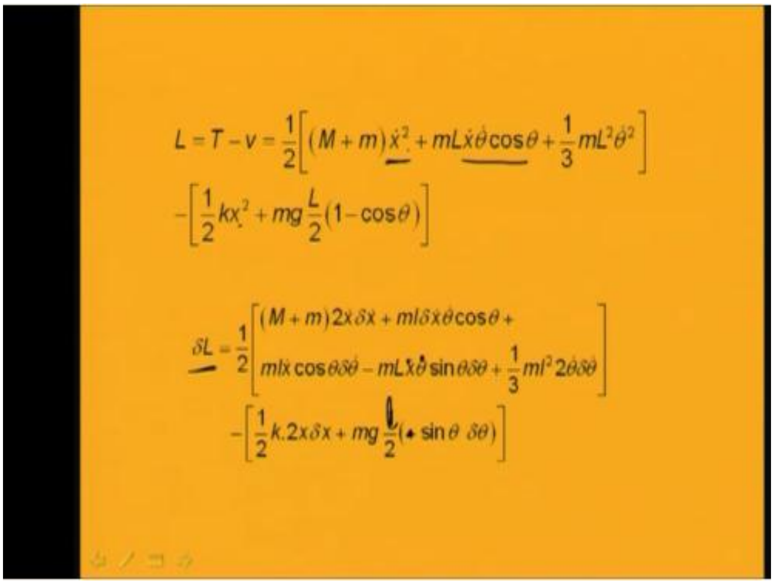

So, the Lagrangian of the system equal to t minus V equal to this. So, now you just take this del l operator. So, this del l operator will give you. So, x dot square if you take the del of this. So, it becomes 2 x dot in to delta x dot. Similarly, for this you have product of three terms here. So, you can write this as m L delta x dot into theta dot cos theta plus now you just take m L x dot into delta theta dot into cos theta, and the third term will be differentiation of cos. So, it becomes minus.

So, minus m L x dot theta dot m L x dot theta dot sin theta delta theta, and for this it becomes one-third m L square. So, this becomes 2 theta dot delta theta dot and for these terms. So, for these terms this becomes half half k into 2 into x into delta x plus m g into half. So, this term you can write. So, this is equal to l by t2. So, this is l by 2, and this one differentiation one is zero and minus cos theta equal to plus sin theta and theta differentiation with del. So, this becomes del theta. So, this term becomes del theta. So, now you can take these terms.

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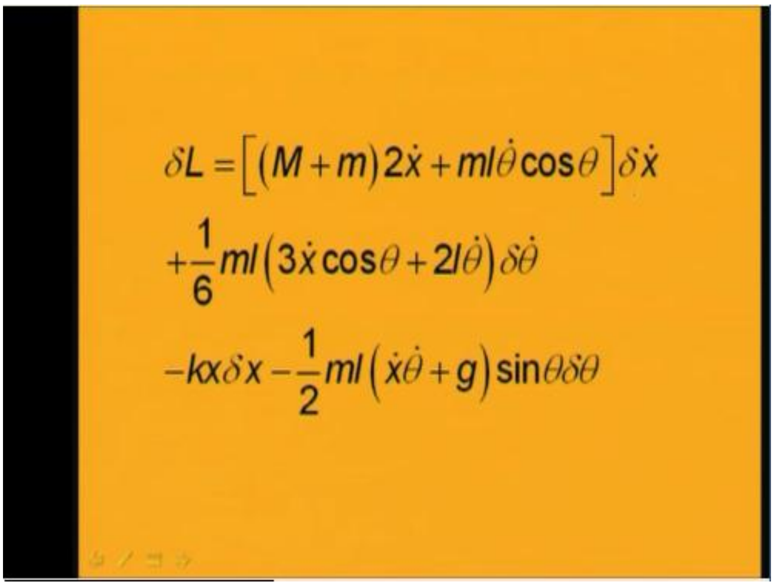

So, this delta L you now know. So, you can write this del L equal to m plus m 2 x dot plus ml theta dot cos theta in to delta x dot. Now you have to integrate this whole thing from t 1 to t 2.

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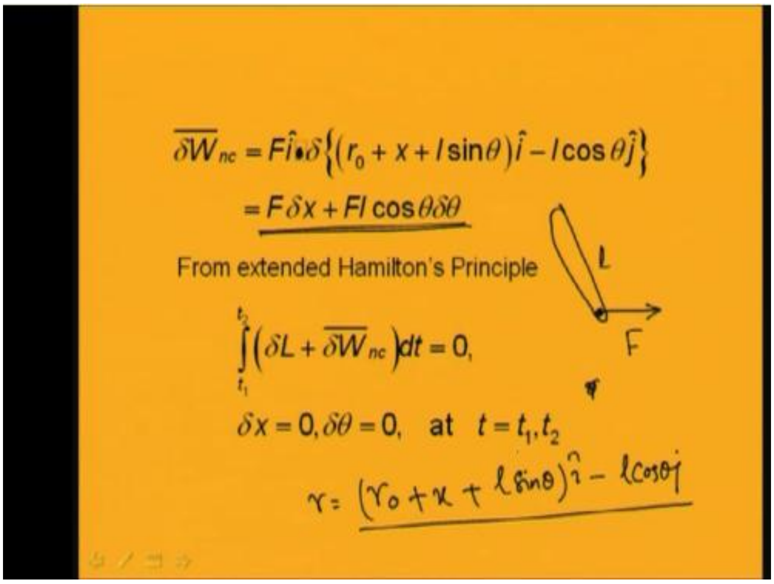

So, while integrating this thing from t 1 to t 2, you have to take these following or you have to do in the following way. And before that you should find the work done by this non-conservative force and already you know the position vector of the mass. So, this is the rod where a horizontal force F is acting. So, this horizontal force as it is acting in i direction. So, it is written F i. So, work done will be F i dot virtual displacement. So, this is the position vector; you just find the position vector of this point. So, this point position vector will be equal to.

So, this is equal to position vector of this point equal to r 0 x plus x plus l sin theta i minus l cos theta j. So, this is the position vector of this point. So, you have to find delta. So, you have find delta r. So, F i dot delta r. So, this is delta r, and you can find this is equal to F in to delta x plus F l cos theta delta theta. So, using this extended Hamilton principle delta l plus delta W n c d t equal to 0. So, here delta x would be equal to 0, and delta theta will be equal to 0 at t equal to t 1 and t 2.

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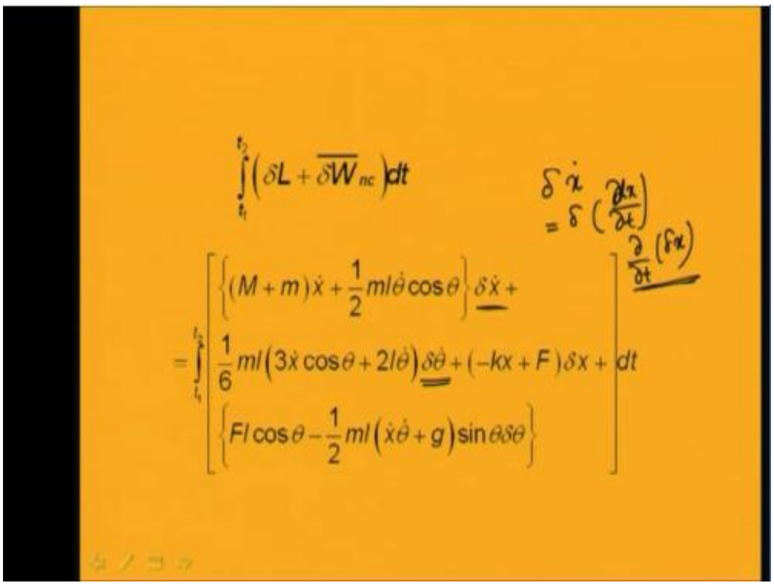

So, using this expression now you have to expand these terms. So, while expanding this term you just note that this delta x dot and this is d t. So, while integrating this delta x dot you can change. So, operators see this x dot equal to. So, delta x dot you can write it equal to del x by del t. So, you can exchange this and you can write it is equal to del by del t of del x. So, this way you can write this delta x term and delta theta term. So, delta theta also you can write del by del t of del theta and you are integrating it with respect to d t. So, it will be the integration.

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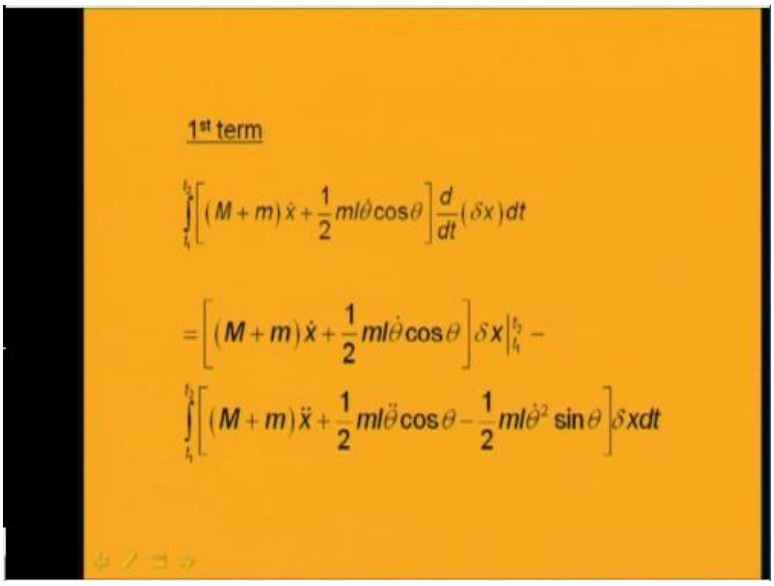

So, the integration will be like this. So, this term I have expressed in this form. So, the first term I have exchanged it d by d t of del x d t and in the second terms also I have written in the same way.

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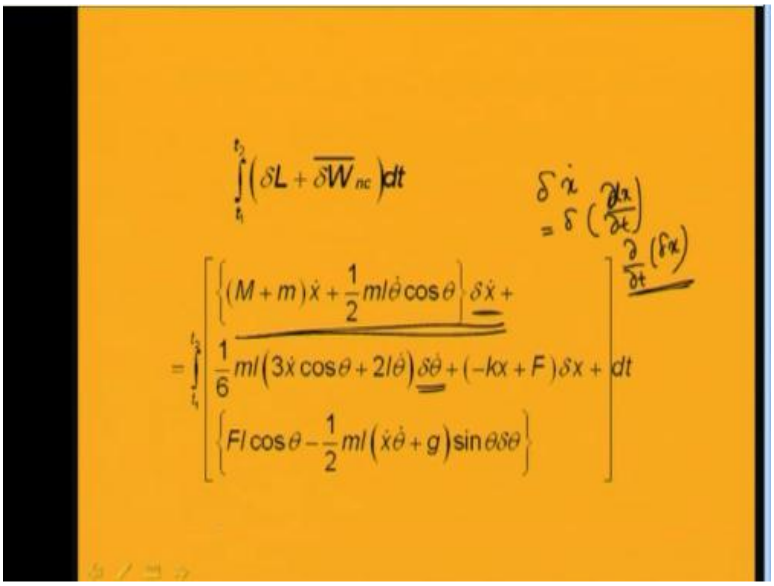

So, if I integrate this first term. So, this is the first term I am taking. So, this first term if you integrate it.

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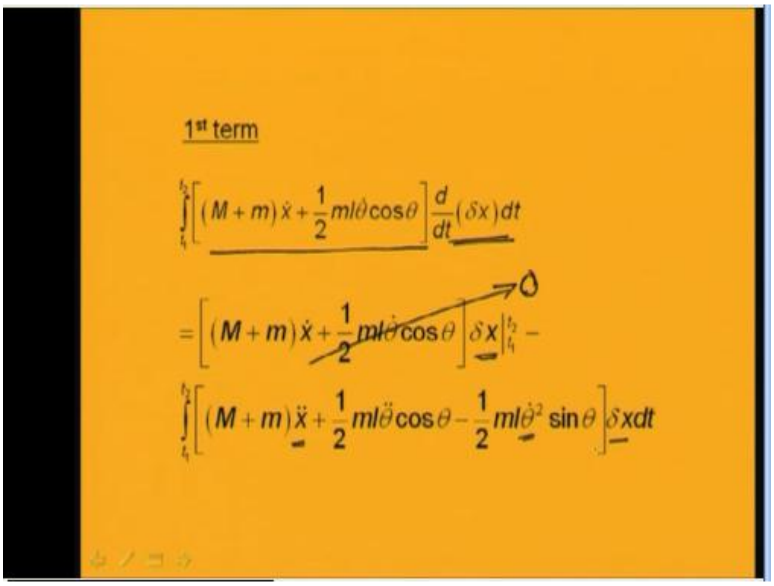

So, you can take the first. So, M plus m x dot plus half m l theta dot cos theta into d by d t of del x d t. So, this will be equal to. So, you just take this as the first function, one as the first function and other as the second function. So, this is the first function you take, and this is the second function you take. So, you just integrate it by parts. So, integrating it by parts; so first function remain as it is and integration of the second function. So, d by d t of del x. So, when you are integrating this thing. So, it becomes del x only from t 1 to t 2.

Already you know that this del x at t 1 and t 2 are 0. So, this term will tend to 0, and the remaining term will be minus integration t 1 to t 2. So, this is the first function remain as it is integration of the second. So, now minus integration of the second; so this del x and differentiation of the first. So, differentiation of first will give you. So, x dot differentiation is giving x double dot, and these differentiation is giving half m l theta dot differentiation theta double dot cos theta minus. So, this contain two terms. So, you will have two terms here. So, minus ml theta dot into cos theta differentiation is sin theta.

So, this becomes minus sin theta into theta dot. So, theta dot into theta dot giving rise to theta dot square. So, this is equal to half m l theta dot square sin theta t x d t. So, in this way you should integrate it by parts. So, using integration by parts, this term you have seen that it is equal to 0, and the remaining term is this.

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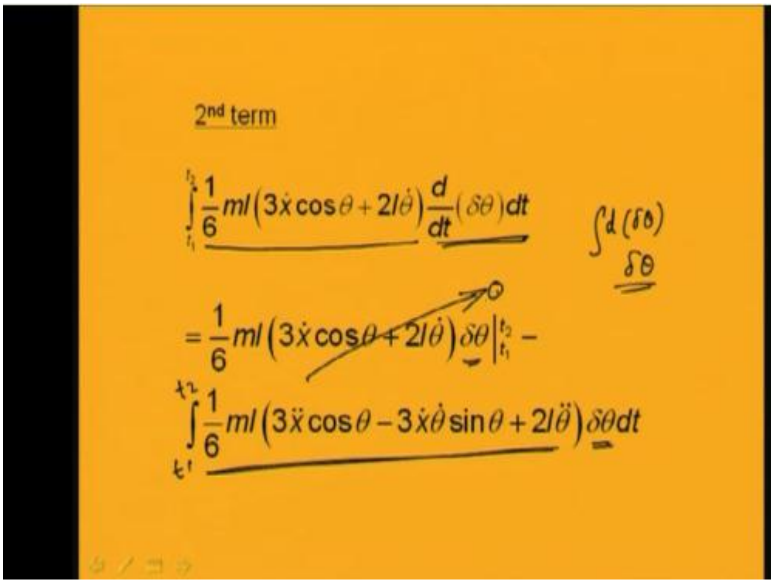

Similarly, you can carry out the second terms. So, it is equal to 1 by 6 m l 3 x dot cos theta plus 2 l theta dot d by d t delta theta d t. So, this is the first function you can also take. So, up to this you can take the first function. So, this is the first function, and this is the second function. So, integration of the second function d by d t del theta d t. So, this is equal to integration d of del theta. So, this becomes del theta.

So, integration of this equal to del theta, and so first function into del theta t 1 2 and then del theta into differentiation of this. So, differentiation of this is this. So, this becomes this. So, this is t 1 to t 2. So, this is. So, this part already you know that this virtual displacement delta theta at t 1 equal to t 2 equal to 0. So, this part becomes 0.

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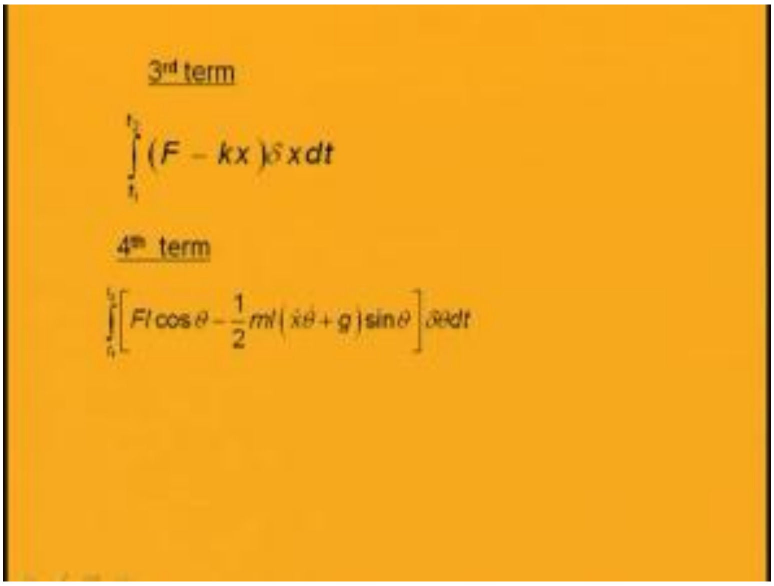

Similarly, you can go for the third term. Third term is F minus k x delta x d x. So, you just note that you need not have to go further or integrate it further; you just keep it, keep the term in this form. So, something into del x to d t. Similarly, the fourth term becomes this. So, now you combine the terms with del x d t or del theta d t.

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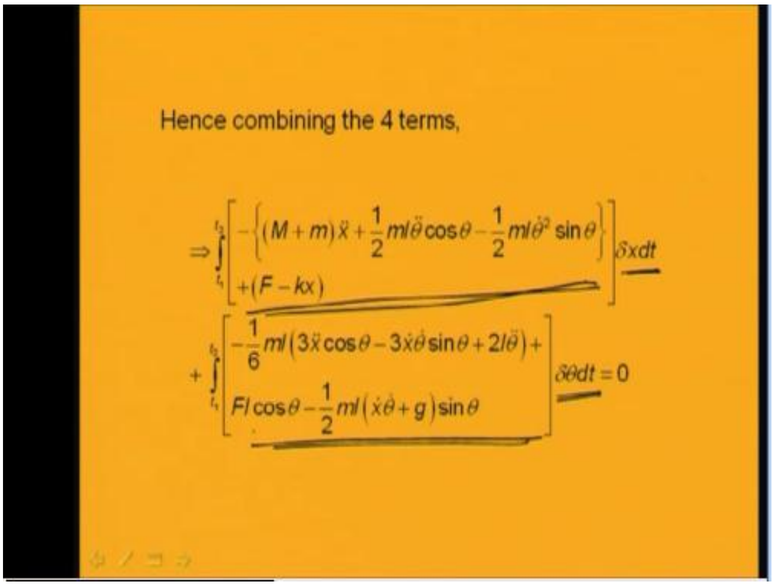

And combining that thing you just write this. So, this becomes this into del x d t plus integration this into del theta d t. Already you know that this del x and del theta are arbitrary as these are the virtual displacement. So, it can take any value. So, this integration to be equal to zero; in that case the coefficients of this del x d t or del theta d t should be equal to zero. So, as the coefficients of these two are zero, you can find the equation motion from this. So, this part will be equal to 0, and this part will be equal to zero. So, these are the equation motion of this system.

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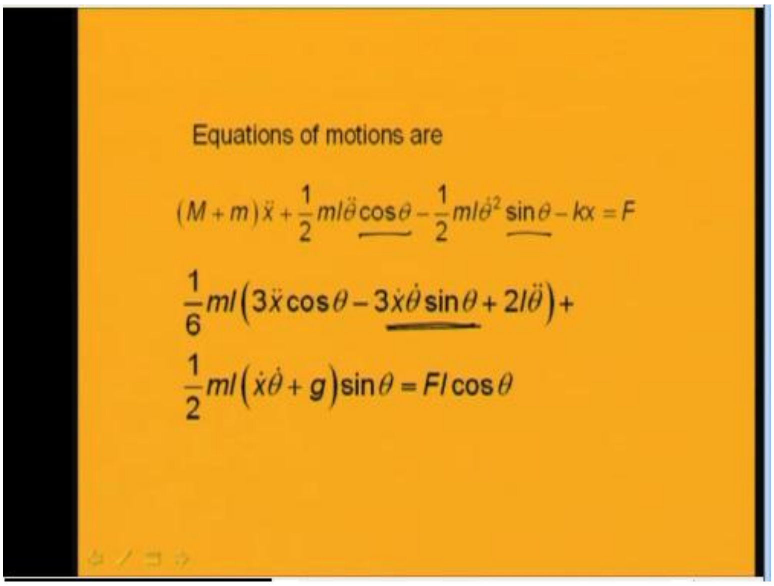

So, in case of this Hamilton using the extended Hamilton principle, so you got the equation motion. So, this is the first equation motion this equal to zero, and this is the second equation motion that is equal to zero. So, you are equating the coefficients of del x d t equal to zero to get the first equation. And equating del x del theta d t equal to zero to get the second equation. So, in this way you can get the two equations. So, in this case you just see that for this in this case you are getting the equation motion to be in this form.

So, this is M plus m x double dot plus half m l theta double dot cos theta minus half m l theta dot square sin theta minus k x equal to F and 1 by 6 m l. So, this m l 3 x double dot cos theta minus 3 x dot theta dot sin theta plus 2 l theta double dot plus half m l x dot theta dot plus g sin theta equal to F l cos theta. So, in this way you can find these two equations. So, these equations you just observe that these are non-linear equations as it contain the product of different terms.

So, this non-linear equation you may make linearization to find the linearized equation. So, you can substitute this for theta to be small. So, you can substitute cos theta equal to

1 sin theta equal to theta. And you may neglect the product of the higher order terms, and you can write the linearized equation motion. So, similarly you can find the same equation motion by using the Lagrange principle also.

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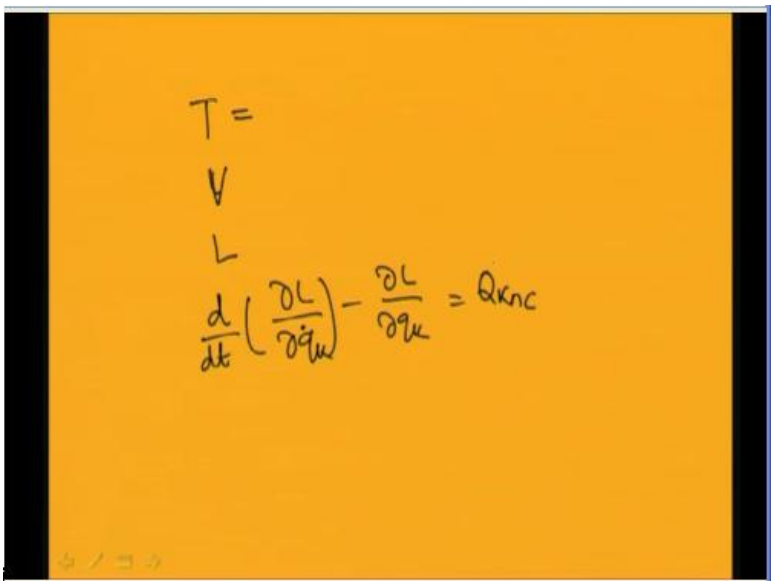

So, by using the Lagrange principle already you have found the kinetic energy of the system, already you have found the potential energy of the system. So, you know the Lagrangian of the system. So, after knowing the Lagrangian of the system, then you just use this equation. So, d by d t of del l by del q k dot minus del l by del q k equal to Q k n c. So, by using this equation you can find the equation in this case. So, you will get the same equation whatever you have got here by using the extended Hamilton principle.

So, till now you know the four different methods to find the equation motion of the system. First you know the Newton's second law, second by using this d'Alembert principle, third the Lagrange principle, and fourth the extended Hamilton principle. Generally, this Lagrange principle is used for multi degree of freedom systems, and Hamilton principle is used for continuous or distributed mass systems. So, most of the cases for this multi degree of freedom system or two degree of freedom system, you should use Lagrange principle or the Newton's or d'Alembert principle instead of going for this extended Hamilton principle.

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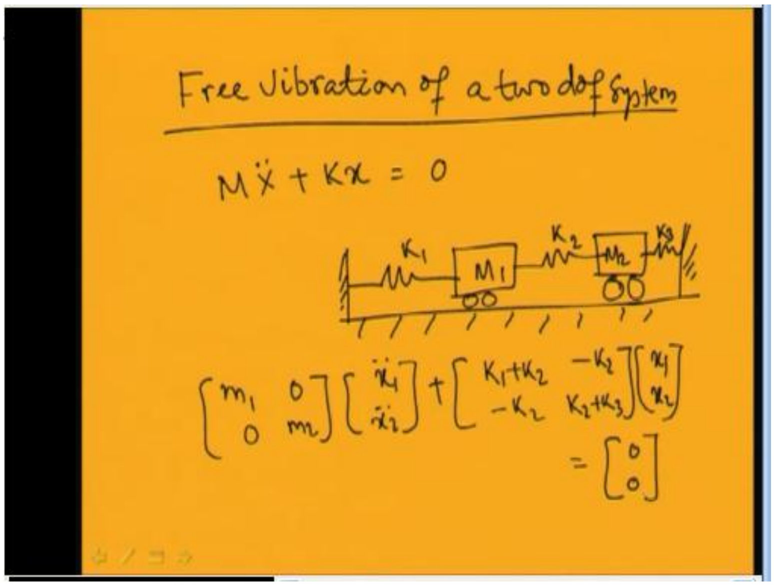

So, now we will see the free vibration of a two degree of freedom system. So, free vibration of a two degree of freedom system. So, already you have derived the equation motion, and you have written the equation motion in this form. So, mass matrix plus. So, mass matrix or in matrix form we have written the equation motion in this form that is M x double dot plus k x M x double dot plus k x equal to 0. So, let me take this simple system. So, this is the spring mass damper system.

So, with mass M 1 and already I have derived the equation motion for the system. So, this is the system I have taken. This is k; I have taken this is k 1; this is k 2; this is k 3, and I have derived the equation motion for the system. And already you know the equation motion for this system equal to m 1 0 0 m 2 x 1 double dot x 2 double dot plus k 1 plus k 2 minus k 2 minus k 2. This is k 2 plus k 3 x 1 x 2 equal to 0. Now I want to study the free vibration response of the system. So, in case of a single degree of freedom system you have seen the free vibration response can be written in this form.

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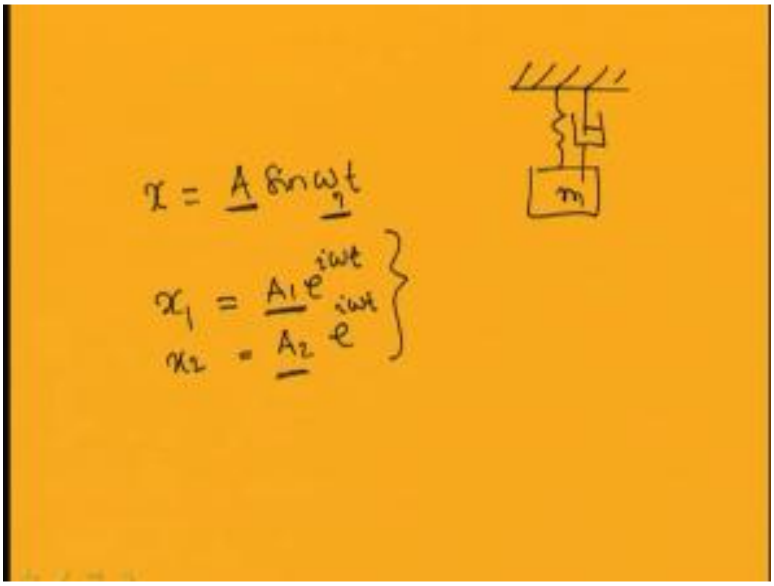

So, in case of a single degree of spring mass damper system, you have seen this is the mass. So, you have seen the solution to be written in this form. So, your x you have written in this form A sin omega n t. So, you have written where omega n is the natural frequency of the system, and A is the amplitude of the response. So, you have written the free vibration response of the system in this way.

Similarly, in this case also I can assume the solution to be in this form x 1 equal to A e to the power i omega 1 t or I can assume it e to the power i omega t. And similarly, x 2 I can assume it equal to A 2 e to the power i omega t. So, I can assume this type of solution in which both the frequency I am assumed to be same. So, in this case you can find this response amplitude A 1 and A 2.

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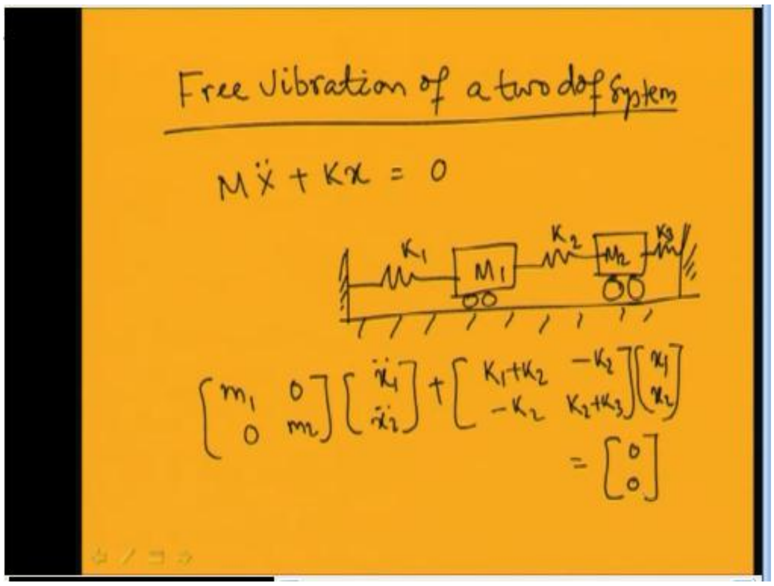

So, now substituting this expression in the previous equation, so you can write the equation motion in this form.

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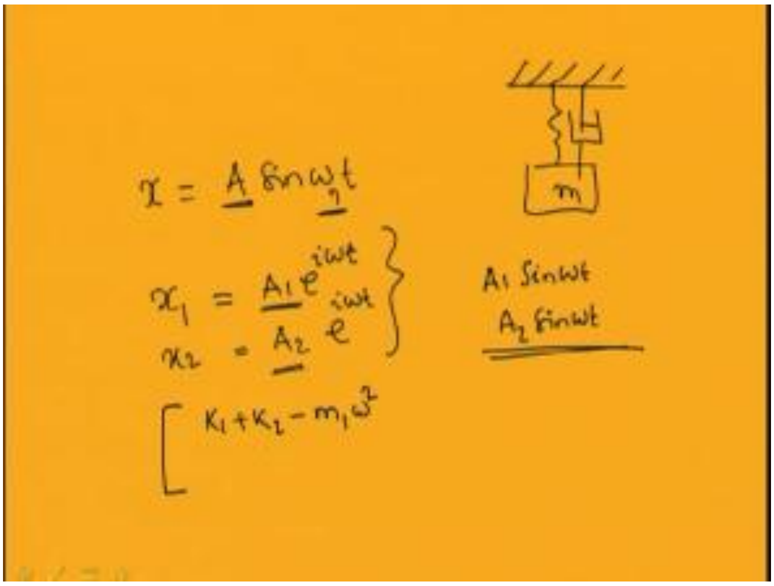

So, you can write. So, instead of assuming e to the power of i omega t also you can take this equal to A sin omega t and A 2 sin omega t also. So, either you take A 1 sin omega t A 2 sin omega t equal to x 1 and x 2, or you just take x 1 equal to A 1 e to the power of i omega t, x 2 equal to A 2 e to the power i omega t. Then you substitute this expression in this equation motion. So, if you substitute in equation motion, then you can obtain the equation in this form. So, you can write it in this from. So, it will become k 1 plus k 2 minus m 1 omega square.

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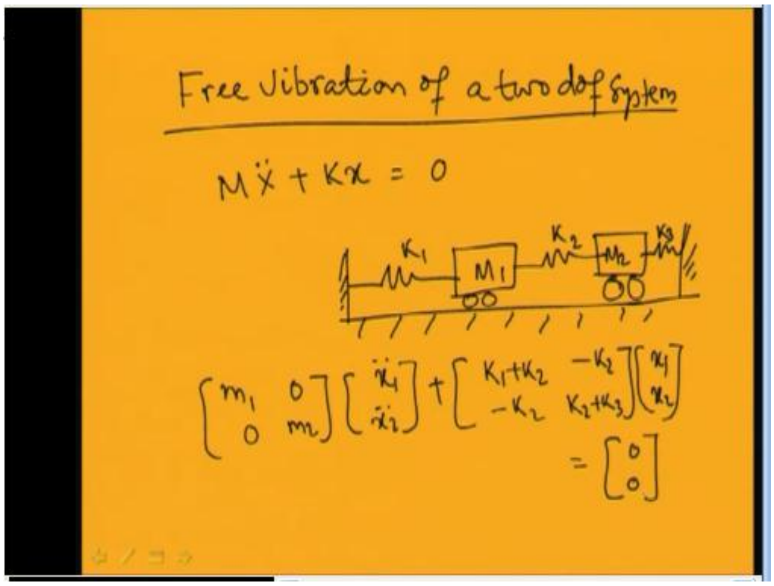

So, this m 1 x 1 double dot will become A 1 omega square e to the power i omega.

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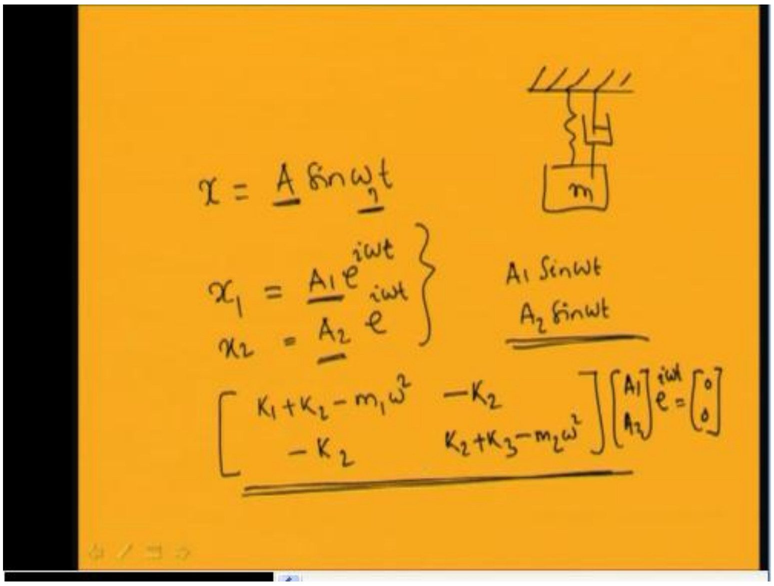

So, I can write and this is minus k 2. So, minus k 2 and then this is minus k 2 and this becomes k 2 plus k 3 minus m 2 omega square and x 1 or A 1 A 2, I have written it A 1 A 2. So, you can put it A 1 A 2 e to the power of i omega t; e to the power of i omega t will be equal to 0 0. So, from this equation you can observe that as e to the power i omega t is not equal to zero, and you are you are interested for a non-trivial solution where A 1 and A 2 are not 0. So, the determinant of this matrix should be equal to 0. So, you have to find the determinant of this matrix and equate to 0.

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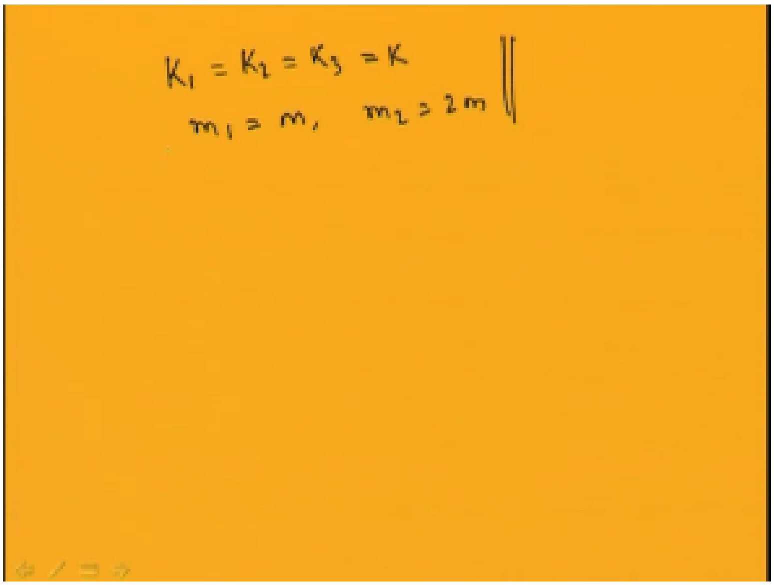

So, let us take a simple case in which this K 1 equal to k 2 equal to k 3 equal to k, and this m 1 equal m and m 2 equal to 2 m. So, in this case let us assume that this is the case we are considering.

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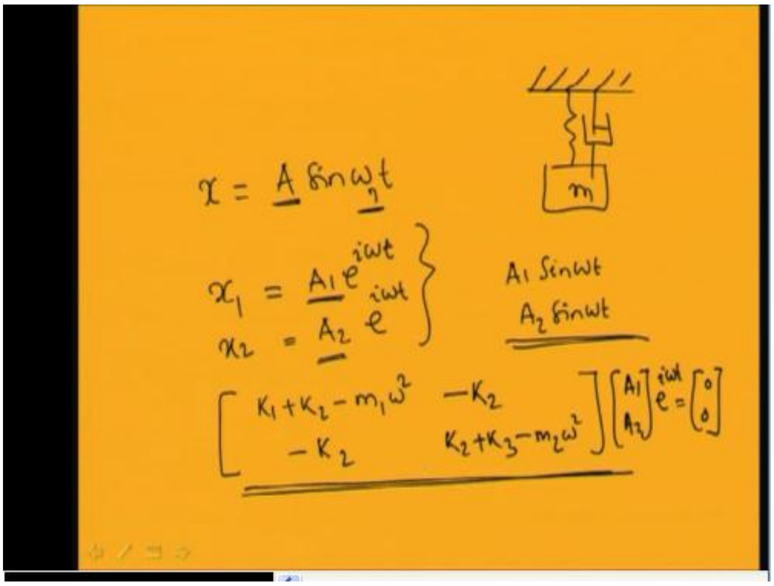

So, in that case this equation will reduce to. So, this becomes 2 k minus m omega square.

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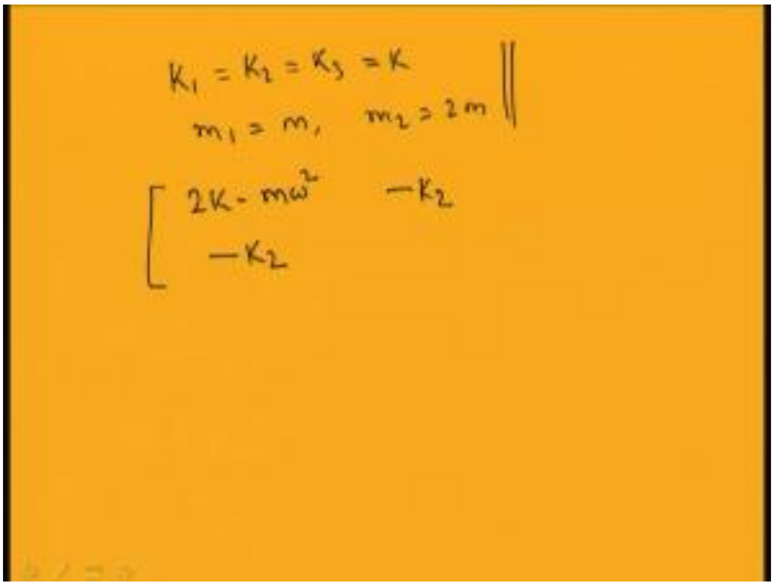

So, this becomes 2 k minus m omega square, and this becomes minus k 2; this becomes minus k 2 and this is equal to…

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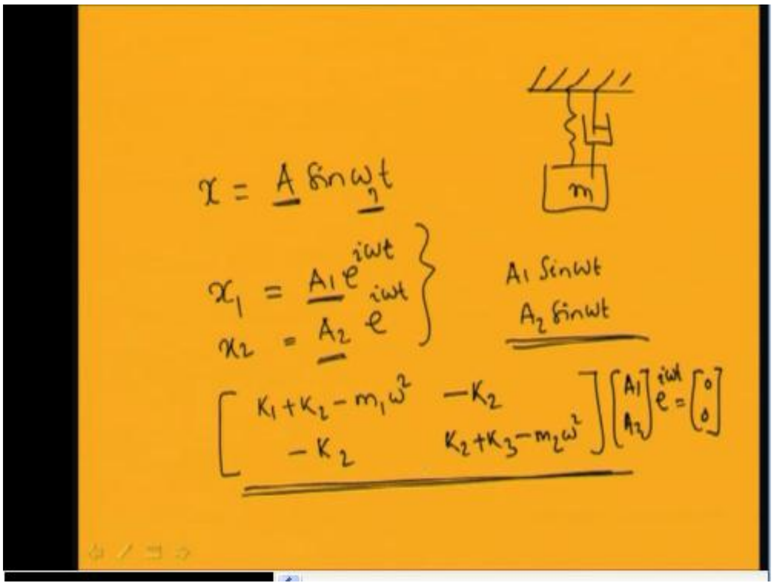

So, this is also 2 k minus 2 m.

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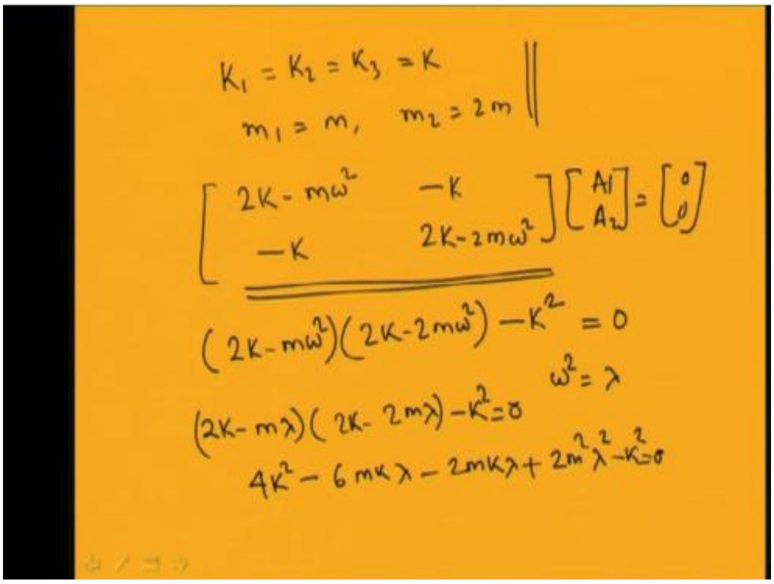

So, this becomes 2 k minus 2 m omega square into A 1 A 2. So, A 1 A 2 will be equal to 0 0. So, in this case I have to find the determinant of these to find the non-trivial solution. So, that is where A 1 and A 2 either A 1 or A 2 not equal to 0. So, in case of trivial solutions A 1 and A 2 are zero, but as we are interested to find the non-trivial solution; so this determinant part will be equal to zero. So, the determinant of this equation will become. So, this is equal to 2 k minus m omega square into 2 k minus 2 m square.

So, minus k 2 square, so this becomes minus k 2. So, already I have taken k 2 equal to k. So, this becomes k square. So, this becomes k square will be equal to 0. So, this is the determinant. So, let me substitute this omega square equal to lambda. So, this equation will become. So, this is 2 k minus m lambda into 2 k minus 2 m lambda minus k square equal to 0, or I can write this equation. So, this equation I can write equal to 4 k square. So, this is 4 k square minus 6 m k lambda. So, this becomes minus 2 m k lambda, then minus minus plus. So, this becomes 2 m square lambda square minus k square equal to 0. So, from this I can write the equation becomes.

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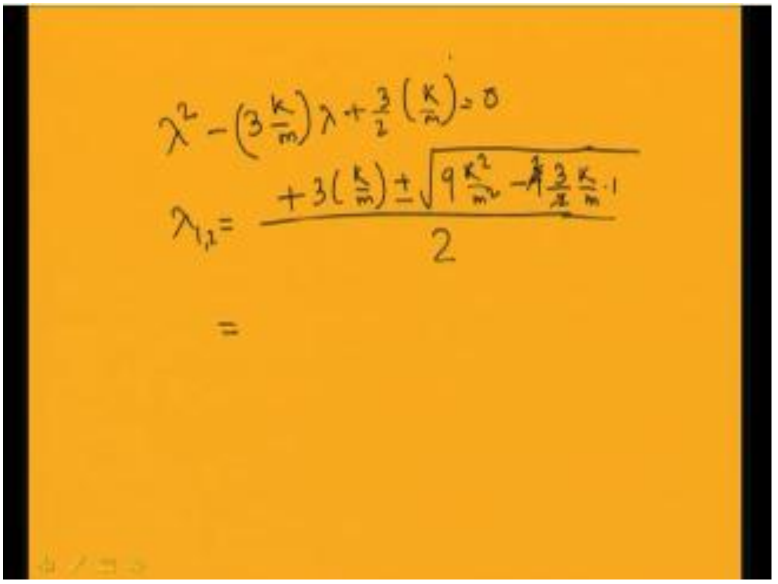

So, from this I can write lambda square. The equation reduces to lambda square minus 3 k by m lambda plus 3 by 2 k by m equal to 0. So, from this I can find this. This is the quadratic equation, and solution of this quadratic equation will give me the frequency equation. So, I can have two value of lambda. So, the lambda 1, I will have this lambda 1 equal to. So, this becomes. So, this is lambda 1 2 you can find minus b, so minus minus plus. So, 3 k by m minus b plus minus root over b square minus 4 a c by 2 a b square. This becomes 9 k square by m square and minus b square minus 4 a c. So, this is equal to 3 by 2 k by m into 1 by 2. So, this becomes. So, this is 3 k by m. So, this is 2, so this becomes. So, this is 2, so 2 into 3 is 6. So, 9 k square by m square and this becomes.

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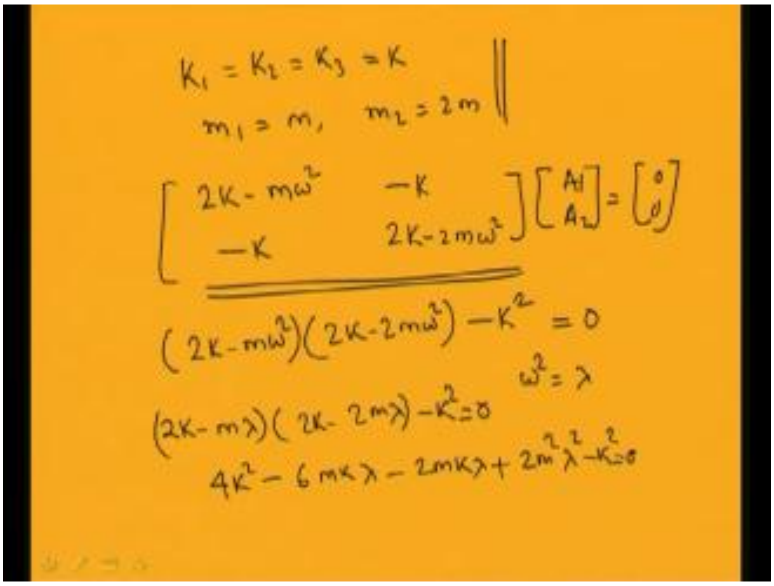

So, from this expression you can see that this is equal to.

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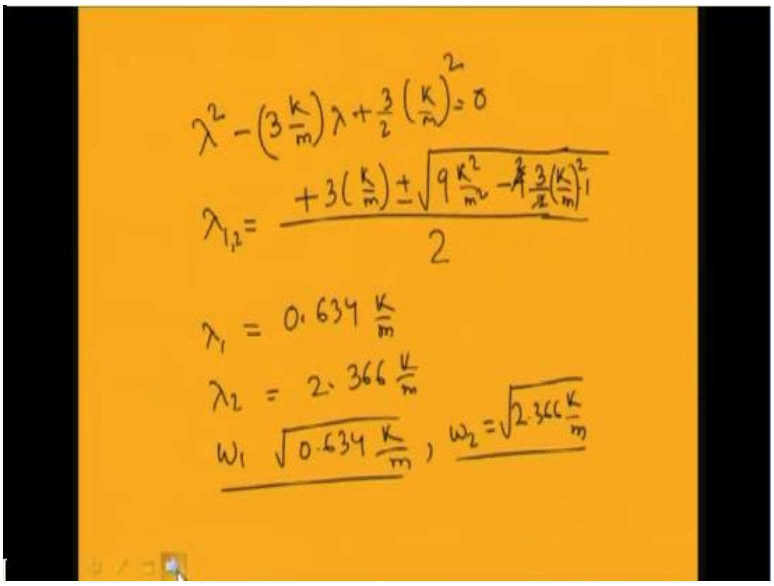

The square term here 3 by 2 k by m square. So, this is k by m whole square into 1. So, this becomes 0.634 k by m, and this is lambda 1, and lambda 2 equal to 2.366 k by m. So, I got two value of the natural frequency; one is lambda 1 that is 0.634 k by m, and lambda 2 equal to 2.366 k by m. So, from this I can find omega 1 that is the first natural frequency that omega 1 will be root over. So, root over 0.634 k by m and omega 2 will become two root over 2.366 k by m.

So, in this case I am getting two natural frequencies that is omega 1 and omega 2. In case of single degree of freedom system, you can recall that you got only single natural frequency that is equal to root over k by m. So, in this case you are getting two natural frequency.

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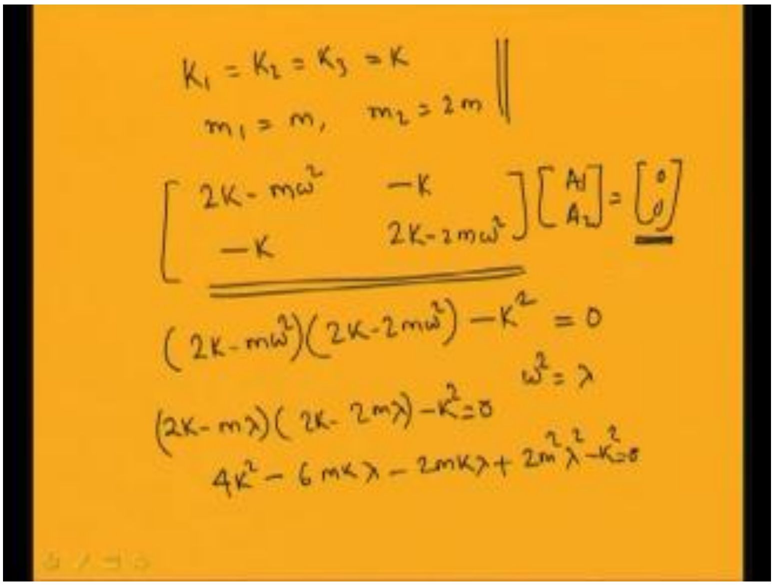

And substituting these two natural frequency in this equation 2 k minus m omega square minus k minus k 2 k minus 2 m square A 1 A 2 equal to 0 0. So, you can find the expression for A 1 and A 2 as you can see this right hand side equal to 0. So, you cannot get a unique solution for A 1 and A 2. So, taking any of this equation you can find the relation between A and A 1 and A 2. So, taking this first expression you can write this 2 k minus m omega square A 1 minus k A 2 equal to 0 or you can write this 2 k.

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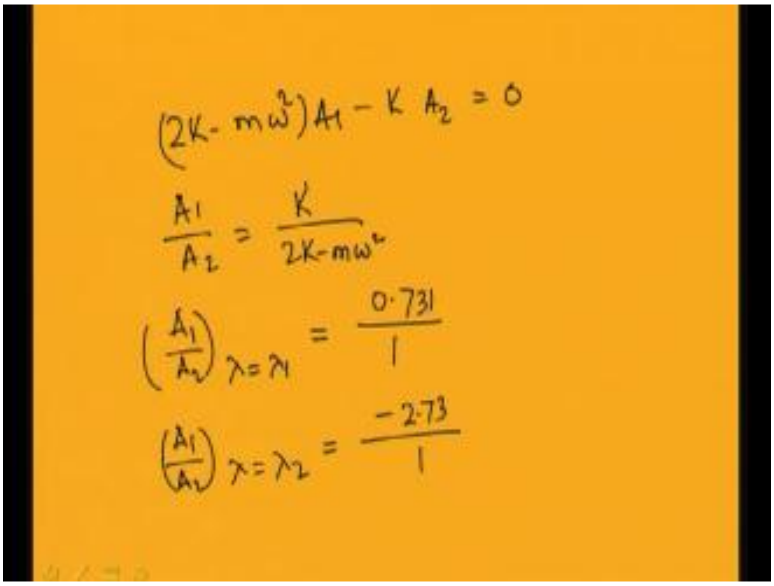

Two k minus m omega square A 1 minus k 2 A 2 equal to 0 or A 1 by A 2 you can write in this form. So, A 1 by A 2 equal to. So, this k 2 already I have taken it equal to k. So, this becomes k by 2 k minus m omega square and already I found the two values of omega or two values of omega square or two values of lambda. So, by substituting those two values, you can get. So, by substituting the first value, so A 1 by A 2 you can get. So, when lambda equal to lambda 1. So, you are getting this is equal to 0.731 is to 1.

Similarly, when you are substituting lambda equal to lambda 2, you are getting two value of lambda. So, lambda 2; so this becomes minus 2.731 1. So, in this case you can observe that in the first case when the natural frequency or lambda 1 equal to 0.634 root over 0.634 k by m.

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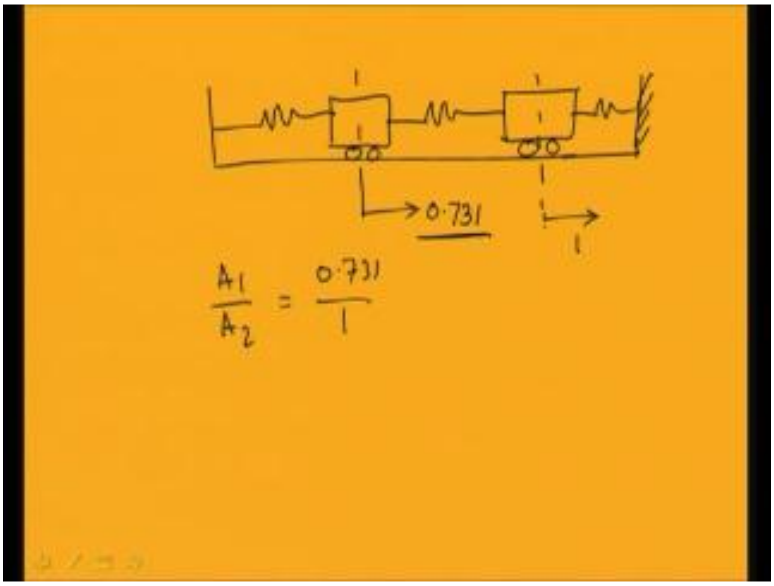

So, in that case you can observe or your system will be like this. So, this is the system. So, this is the original system. Now when it will have a frequency lambda equal to lambda 1, you have observed that when this is moving. So, this A 1 by A 2 you have seen it equal to 0.731 by 1; that means when this x 2 moves a distance one unit in distance. So, this will move a distance of 0.731. So, when x 2 is moving a distance or the second mass is moving a distance of one the first mass is moving a distance of 0.731.

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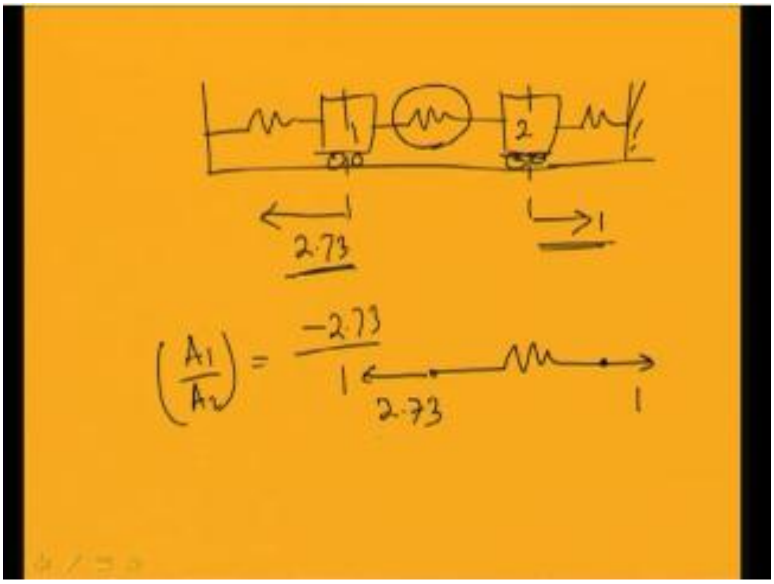

Similarly, in the second case you can show that or you can draw it like this. So, when the first mass is moving. So, you have these are the two masses. So, in the second case you have seen that this is x 1, this is x 2. So, you have seen A 1 by A 2 equal to minus 2.73 by 1. So, in this case, when this is moving a distance; so this negative sign indicates that it is moving towards left. So, this is 2.73; that means when it is moving a distance of one in this direction. So, when A 1 or second mass is moving a distance. So, this is the second mass; this is the first mass.

So, when the second mass is moving a distance of one towards right, the first mass is moving a distance of 2.73 towards left. So, in this case this spring the second spring will be under tension. So, this spring in this way it is moving a distance one and in this way it is moving a distance 2.73.

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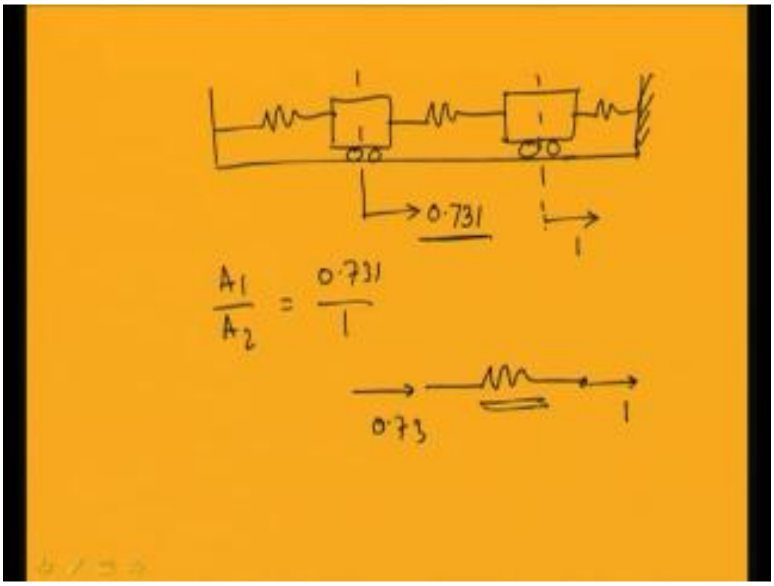

But in the previous case you can observe that this spring both of them are moving in the same direction when it is moving one. So, this is moving 0.73. So, the spring is compressed. So, in this case the spring is compressed.

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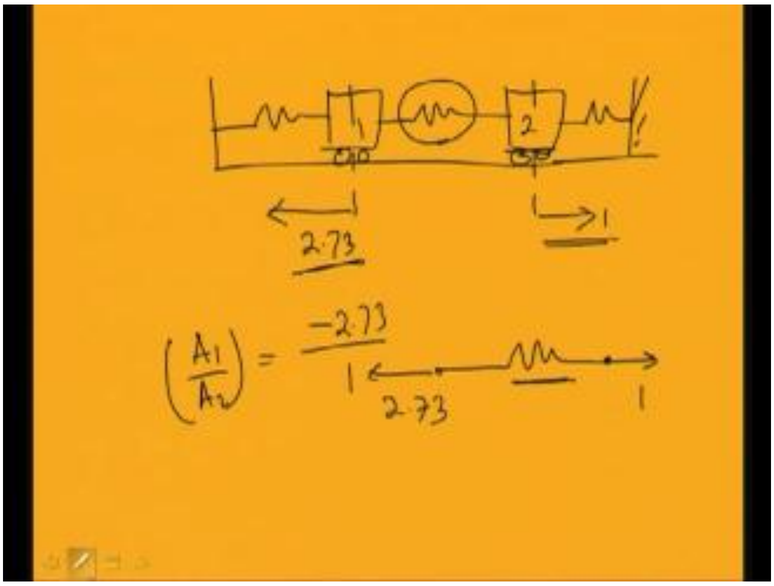

And in this case the spring is in tension.

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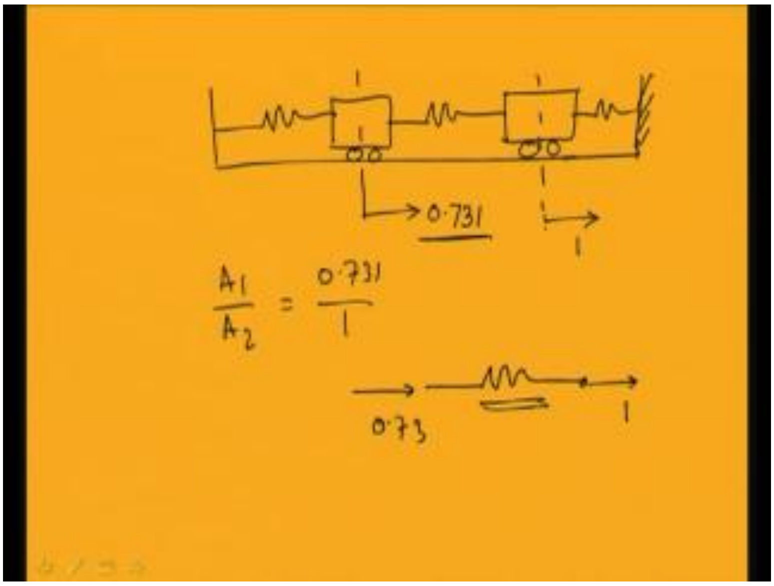

So, in the first case you can tell that both the masses are in phase or they are moving in the same direction, but in the second case you can tell that both the masses are out of phase.

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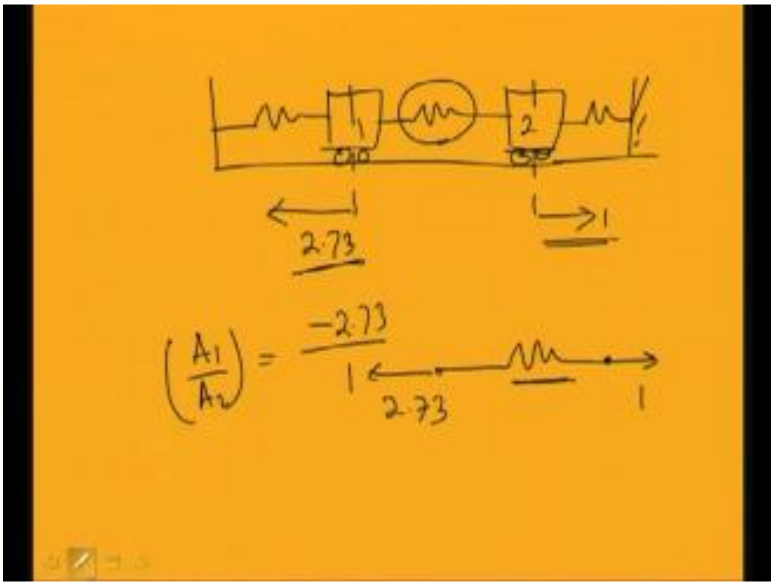

That is when one is moving towards right, the other one is moving towards left. So, this is the normal mode. So, when you are normalizing this thing. So, in this case in both the cases we have made this A 2 equal to 1, and we have found the value of A 1. So, in both the cases we are normalizing the displacement of the second mass, and in both the cases we have assumed that both the masses are moving with the same frequency and crossing the equilibrium position at the same time.

That means we are taking that they are moving with a particular frequency. So, these types of motion in which we are considering the motion is taking place at the same frequency are known as normal modes. So, we have two normal modes in this case. So, the first normal mode equal to…

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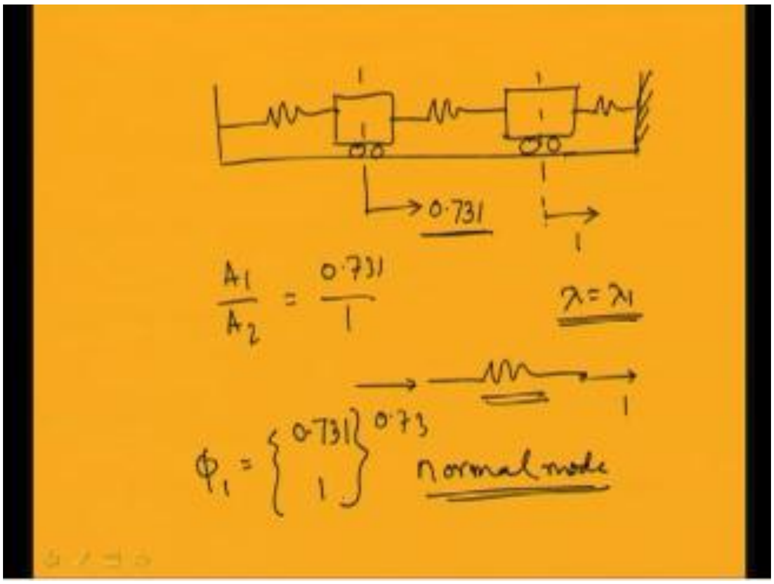

So, this is the first normal mode, I can write phi n. So, this phi 1 or I can write this first normal mode phi 1 equal to. So, this indicates the motion of these masses when it is moving with the first frequency lambda equal to lambda 1. So, when it is moving with lambda equal to lambda o1. So, these two masses will have displacement 731 and 1. So, this phi 1 this is the normal mode. So, the normal mode I can write for the first mass. So, the first normal mode equal to 0.731 1.

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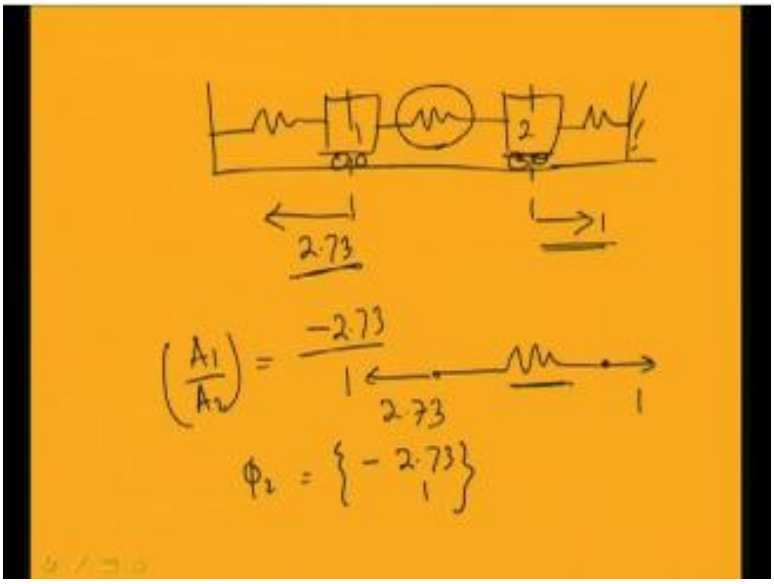

Similarly the second normal mode phi 2 is equal to minus 2.731 1.

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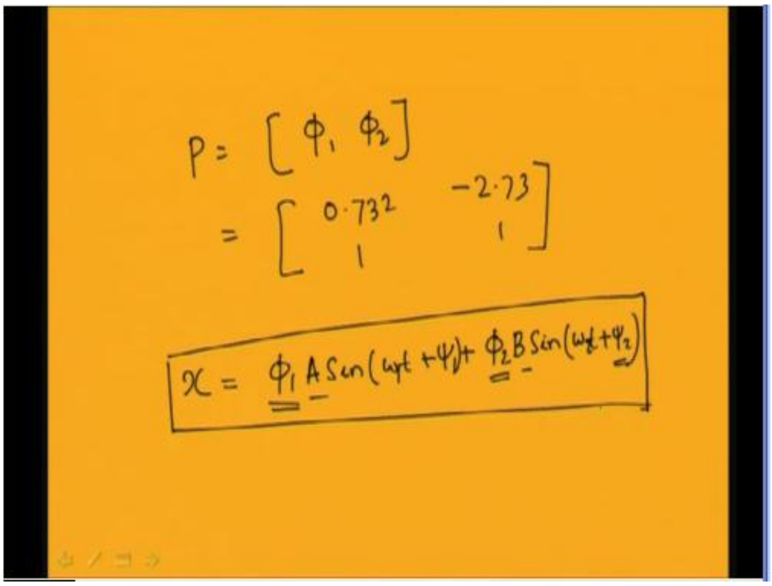

So, I can write the modal matrix P equal to phi 1 phi 2, and this thing can be written in this form. So, it will be equal to 0.732 1 and minus 2.73 1. So, in this way you can determine the normal mode of a system. So, to determine the free vibration response of the system, you can use these normal modes to find free vibration and at a particular time the free vibration can be considered to be the summation of the normal modes. So, the free vibration you can assume to the summation of the normal modes.

So, at a particular time the free vibration x you can write will be equal to. So, x will be equal to phi 1 A sin omega 1 t plus psi 1 plus phi 2 B sin omega 2 t plus psi 2. So, in this case already you know this phi 1 and phi 2. So, these are the normal modes and A B and this psi 1 psi 2 can be determined from the initial conditions. So, given the initial conditions of this mass one and two, so initial conditions represent the displacement and velocity. So, for each mass you have the displacement and velocity.

So, you have four known quantities, so two displacement quantities and two velocity quantities; so four known quantities. So, you have here four unknowns that is A B and psi 1 psi 2. So, you have four unknowns and four equations for displacement and velocity. So, by solving those equations you can find this A psi 1 and B psi 2, and you can find the response of the system. So, the free vibration response of the system you can find by using this normal modes. So, let us take the same problem.

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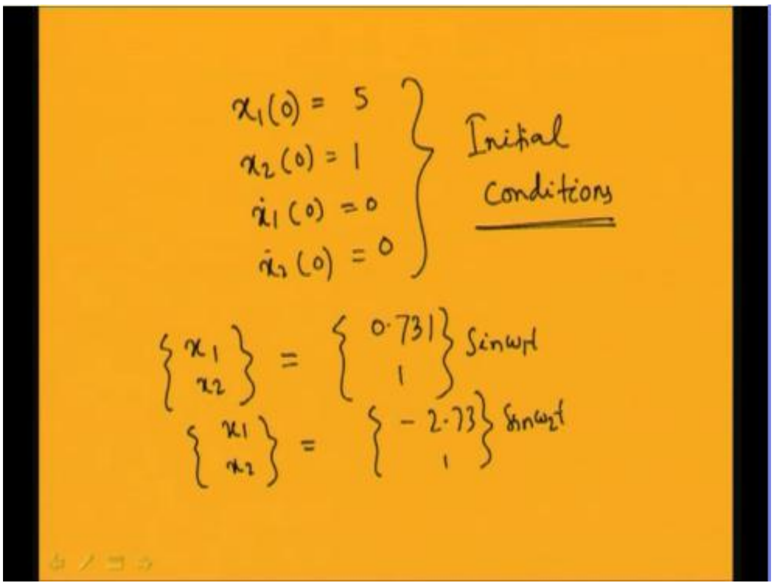

So, in this problem let me assume that this x 1, so this x 1 equal to. So, let x 1 0 equal to 5 and x 2 0 equal to 1 and this velocity terms are. So, x 1 dot 0 equal to 0 and x 2 dot 0 equal to 0. So, these are the initial conditions for the system. So, taking these initial conditions I have to determine the response of the system. So, already I know the response of the system x 1 and x 2 can be written in this form. So, x 1 x 2 you can write. So, already I know that x 1 x 2. So, for the first mode I can write. So, the first mode this is equal to x 1 x 2 equal to 0.731 1 sin omega 1 t and for the second mode x 1 and x 2 already you know this is equal to minus 2.731 1 sin omega 2 t.

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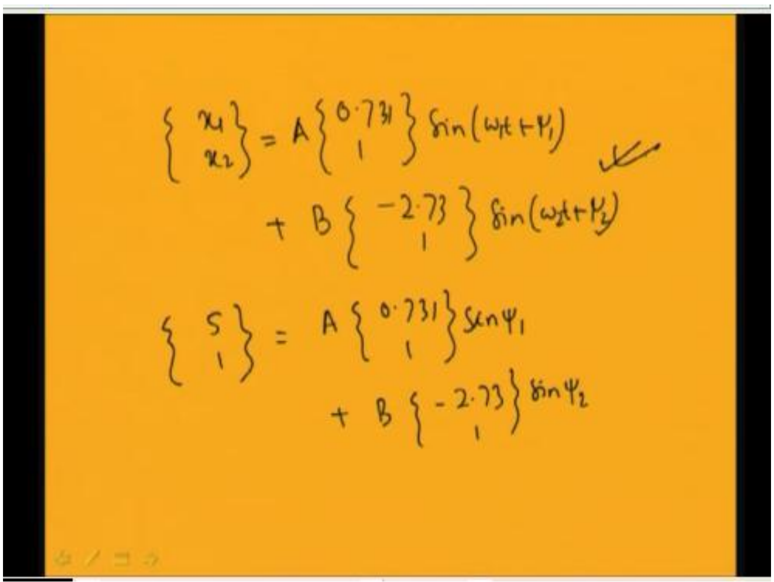

So, the general solution x 1 x 2 at any particular time t can be written as A phi 1. So, A phi 1 equal to 0.731 sin omega 1 t plus psi 1 plus B into minus 2.73 1 sin omega 2 t plus psi 2. So, here we have to find these A B and psi 1 psi 2. So, from this initial conditions so this x 1 0. So, when t equal to zero, I can substitute this in that expression. So, I can find x 1 0 and x 2 0. So, this 5 1 I can write. So, taking t equal to 0, I can write this equal to. So, 5 1 will be equal to A into 0.731 1 sin. So, t equal to 0. So, sin 0 plus psi 1. So, this is equal to sin psi 1 plus B into minus 2.731 1 sin psi 2. Now I can differentiate this equation to get the velocity terms, and from this I can substitute this velocity at initial velocity 0 0.

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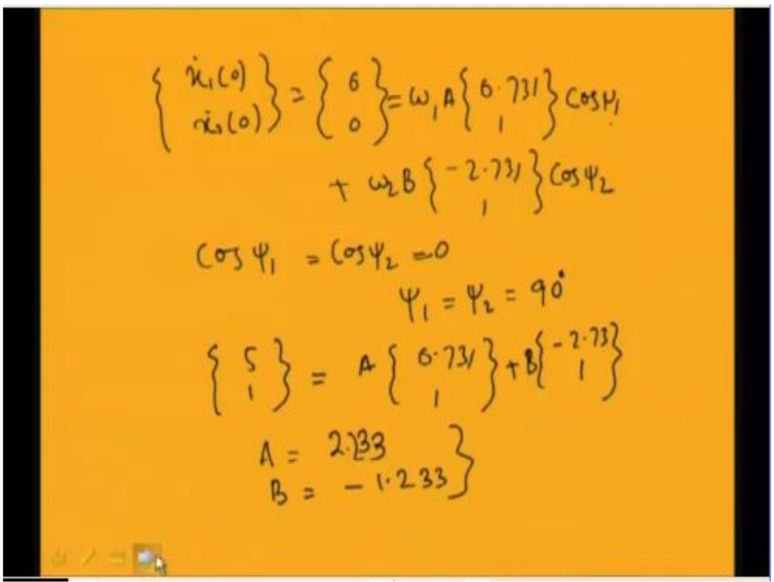

So, I will get this expression. So, x 1 dot 0 x 2 dot 0. So, these are already known to be 0 0. So, this becomes this will be equal to omega 1 A into 0.731 1, differentiation sin is cos. So, this becomes cos psi 1 plus omega 2 B into 2.731 1 cos psi 2. So, from these expression you can see that omega 1 A into this into cos psi 1 equal to 0. So, omega 1 A into 0.731 cos psi 1 plus omega 2 B into minus 2.731 1 cos psi 2 equal to 0, so from these expression you can tell or you can find that this cos psi 1 equal to cos psi 2 equal to 0. So, this expression is valid.

So, when cos psi 1 equal to cos psi 2 equal to 0 or psi 1 equal to psi 2 equal to 90 degree; so you can substitute this expression for psi 1 psi 2 in the previous equations and you can get this A 1. So, this equation you will get. So, this is equal to 5 1 will be equal to A into

0.731 1 plus B into minus 2.731 1. So, now you have two expressions. So, 0.731 1 A plus minus 2.73 1 B equal to 5 and A plus B equal to 1. So, you have two equations. So, while solving these two equations you can find A equal to 2.233 and B equal to minus 1.233.

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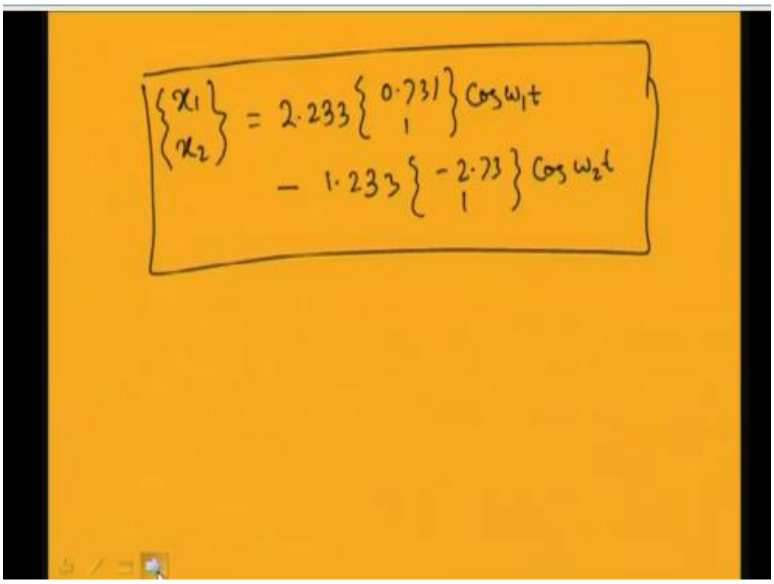

So, your equation is reduced to this form. So, the resulting free vibration response of the system can be written as x 1 x 2 equal to 2.233 0.731 1 cos. So, sin omega t plus 90 degree is cos omega 1 t. So, this becomes cos omega 1 t minus 1.233 into 2.73 1 cos omega 2 t. So, this is the required expression for this free vibration of the system. So, you know how to determine the normal mode of a system, and using that normal mode you can find the free vibration response of the system. So, let us take some other systems to find the normal mode of that system.

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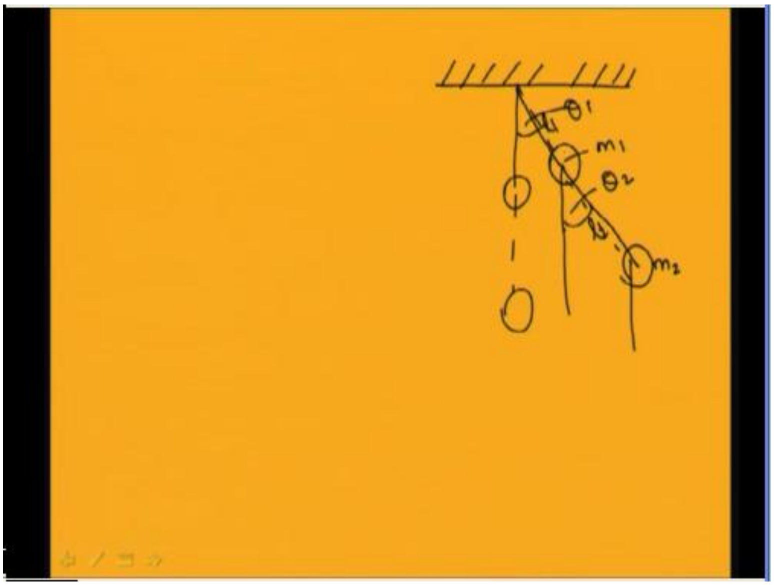

Let us take a double pendulum and for that double pendulum let us determine the normal mode of that system. So, for a double pendulum, this is the initial position of a double pendulum, now due to vibration. So, let it has come to this position; this mass has come to this position, and this mass the second mass has come to this position. So, you can express the motion of these two masses with this theta 1 and theta 2. So, this is theta 1 and theta 2. So, this is mass m 2 and this is mass m 1. So, let us first determine the equation motion using Lagrange principle and later we will find the normal mode of the system. So, in this case to find the kinetic energy of the system we should find the velocity of this mass m 1 and velocity of this mass m 2. So, let this length is l 1, this length is l 2.

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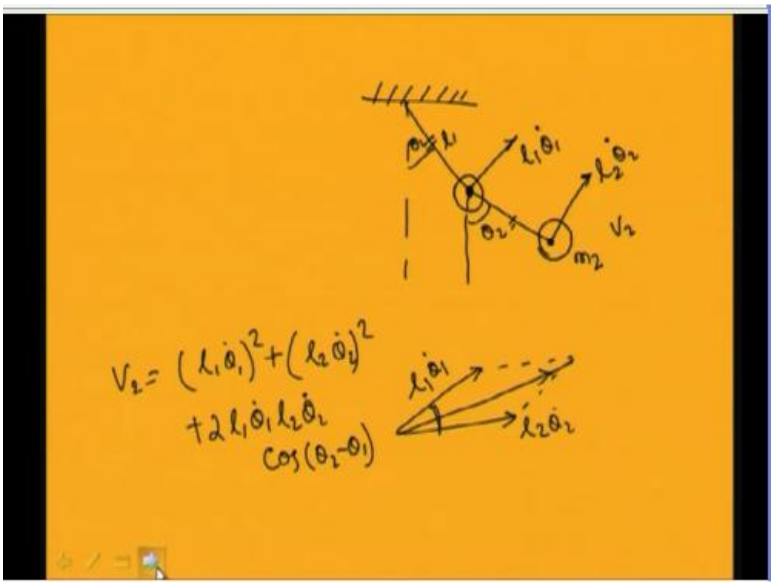

So, I can draw the velocity. So, for this case the velocity is perpendicular to this line. So, it will be perpendicular to this line, and this length is l 1, and this is theta 1. So, this is equal to l 1 theta 1 dot. And for the second mass, so this is for the second mass. So, the second mass makes an angle theta 2 with the vertical. So, it will have a velocity relative to this position which is perpendicular to this line. So, it will be perpendicular to this line and will have a magnitude theta l 2 theta 2 dot.

So, the velocity of this position or velocity of mass m 2 will be the velocity of mass m 1 plus the relative velocity of mass m 2 with respect to mass m 1. So, it will be the summation of. So, this is l 1 theta 1 dot, and this is l 2 theta 2 dot. So, the V 2 or the velocity of this mass m 2 will be equal to the vector sum of this l 1 theta 1 dot and l 2 theta 2 dot. So, you may use this parallelogram laws to find the velocity and already you know this velocity V 2 can be written as l 1 theta 1 dot square plus l 2 theta 2 dot square plus 2 l 1 theta 1 dot l 2 theta 2 dot into cos.

So, this angle between these two becomes theta 2 minus theta 1. So, this becomes theta 2 minus theta 1. So, this is the expression for velocity of this mass two. So, the kinetic energy of the system will be equal to half m 1 in to V 1 square that is equal to l 1 theta dot square and plus half m 2 V 2 square. So, the total kinetic energy system I can write.

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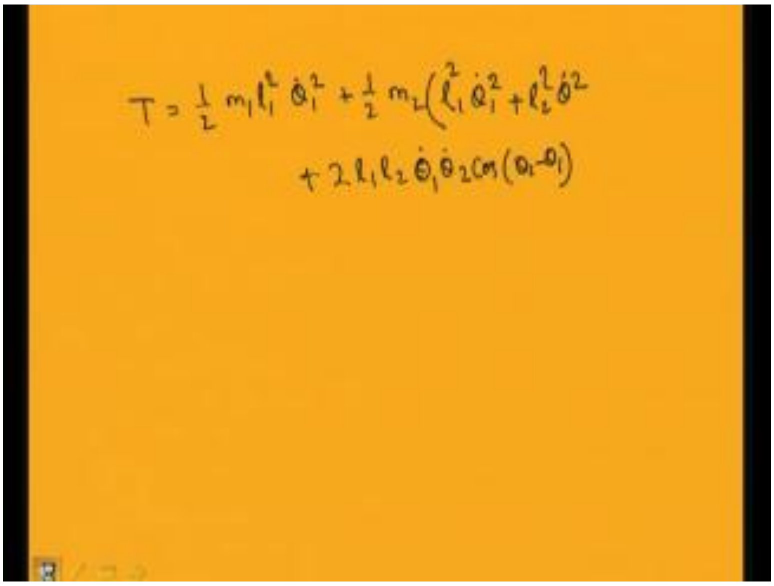

So, T will be equal to half m 1 l 1 square theta 1 dot square plus half m 2; already I have written the expression for V 2. So, V 2 dot V 2 that will be the velocity square of this mass m 2, and this will be written as l 1 square theta 1 dot square plus l 2 square theta 2 dot square plus 2 l 1 l 2 theta 1 theta 2 cos theta 2 minus theta 1. So, this is theta 1 dot, this is theta 2 dot.

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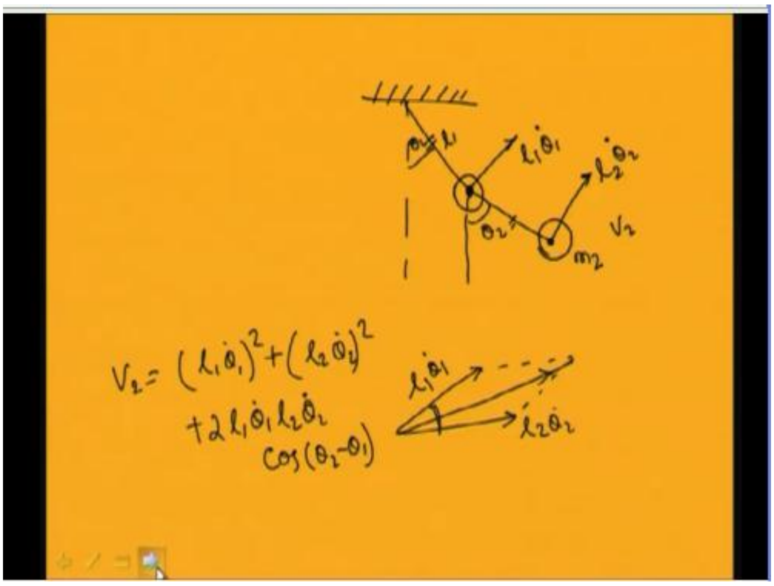

So, this is the expression 2 l 1 theta 1 dot l 2 theta 2 dot cos theta 2 minus theta 1.

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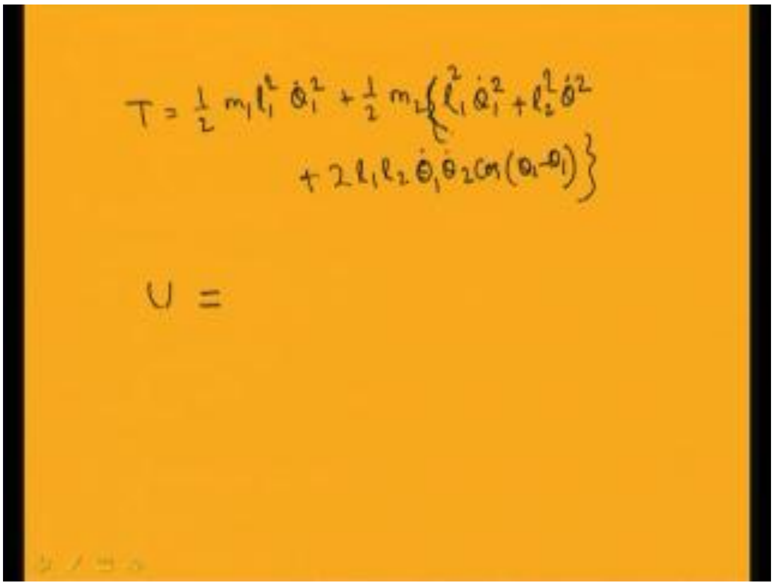

So, this is the expression for the kinetic energy of the system. Similarly, you can have the expression for the potential energy of the term. So, the potential energy of the system can be written as.

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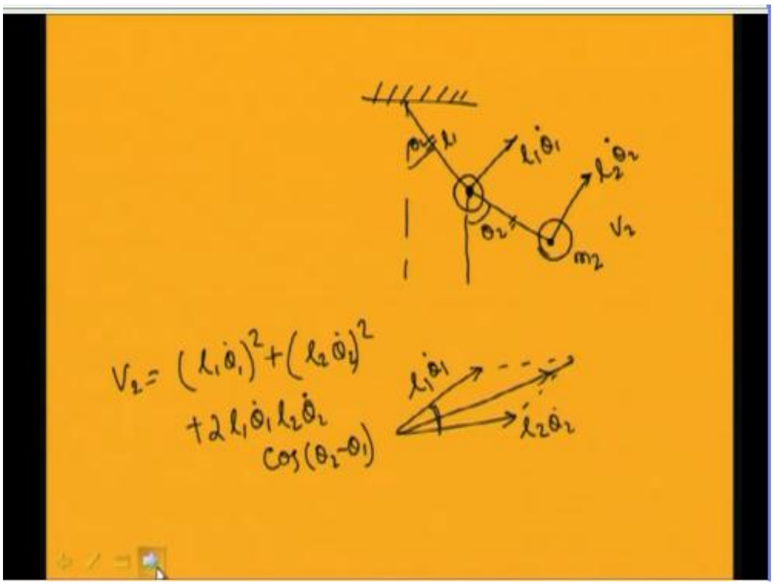

So, potential energy due to change in position of this first mass; so the change in position of the first mass, the first mass initially has a length of l. So, now it has come to this position. So, change in height. So, this is the change in height of this first mass. So, this becomes l 1 into 1 minus l 1 into. So, this length equal to l 1 cos l 1 into cos theta 1. So, this becomes l 1 minus l 1 cos theta 1.

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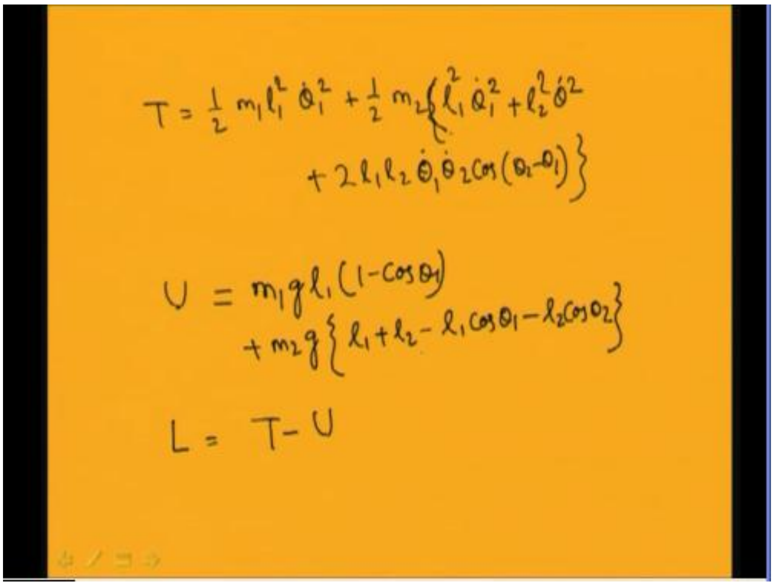

So, change in height. So, this is m 1 into g into l 1 into 1 minus cos theta 1. Similarly, the potential energy for mass two you can find. So, the change in height will be equal to l 1 plus l 2 minus l 1 cos theta 1 minus l 2 cos theta 2. So, this will be equal to m 2 g into change in height of that. So, this becomes l 1 plus l 2 minus l 1 cos theta 1 minus l 2 cos theta 2. So, in this way you can find this T and U. So, using this T and U, now you can write this L equal to T minus U. So, L equal to T minus U you can write.

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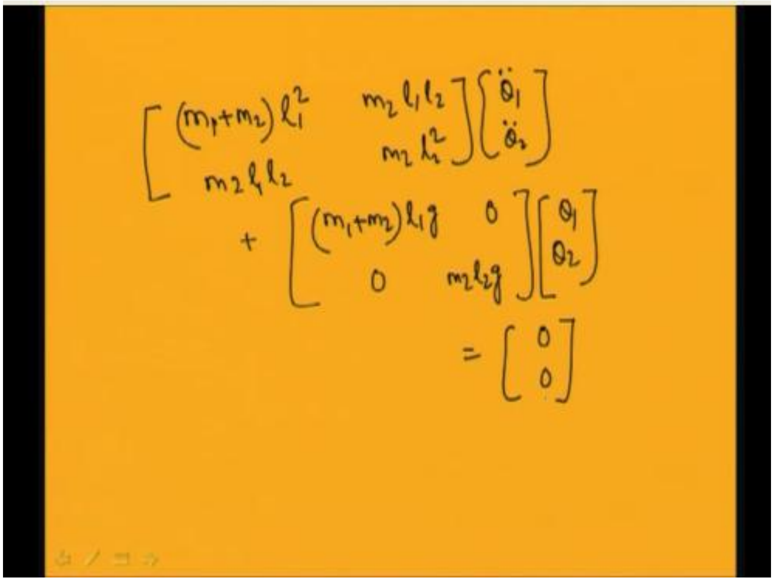

And now applying the Lagrange principle you can write the equation motion which can be given by this expression. So, you find that equation motion. So, this becomes m 1 plus m 2 l 1 square m 2 l 1 l 2. So, this is m 2 l 1 l 2 m 2 l 2 square into theta 1 double dot theta 2 double dot plus m 1 m 2 m 1 plus m 2 into l 1 g. So, this is 0. So, this is 0, and this is m 2 l 2 g into theta 1 theta 2. So, this becomes 0 0.

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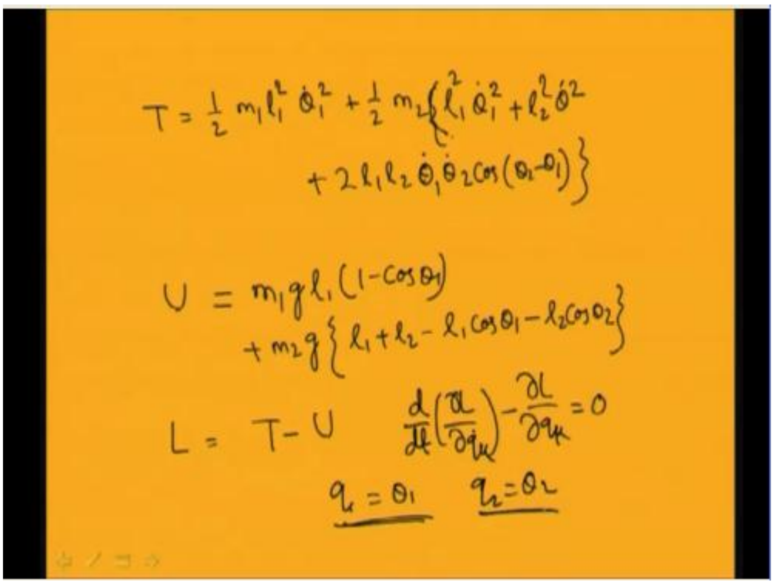

So, by using the Lagrange principle that is d by d t of del l by del q k dot. So, minus del l by del q k equal to 0. So, you can find the equation. So, here q 1 I have taken equal to theta 1, and q 2 equal to theta 2. So, by taking this is theta 1, this is theta 2.

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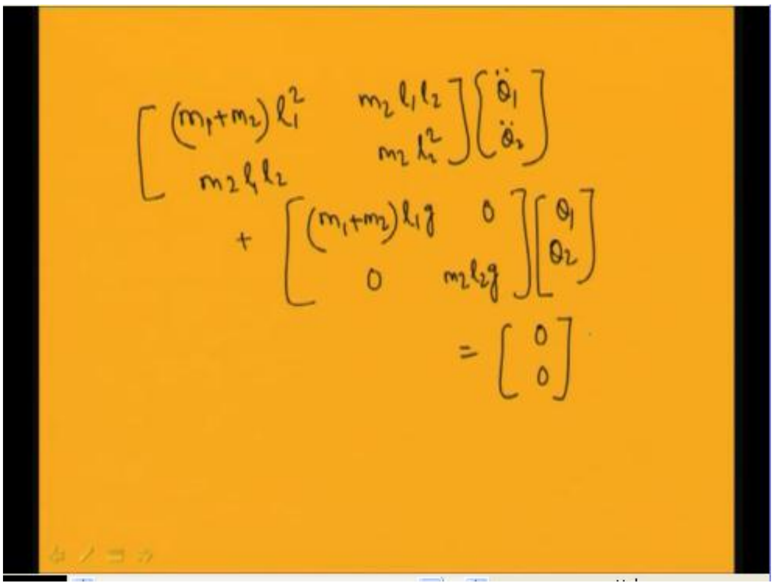

So, you can write the equation motion in this way.

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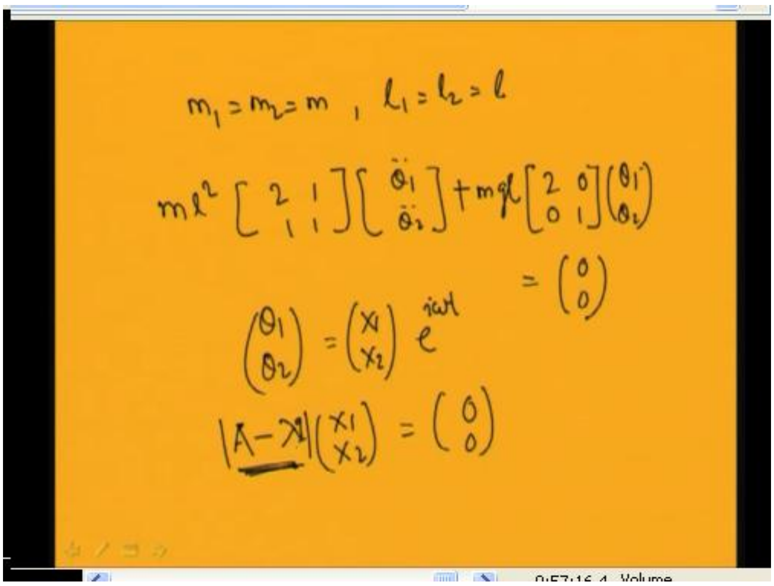

So, now let us take a simple case in which this mass M 1 equal to m 2 equal to m and this l 1 equal to l 2 equal to l. So, this equation will reduce to this form. So, this becomes m l square 2 1 1 1 theta 1 double dot theta 2 double dot plus m g l 2 0 0 1 theta 1 theta 2 equal to 0 0. So, proceeding in the previous way I can assume that this theta 1 theta 2 equal to A 1 A 2 e to the power i omega t. I can substitute it in this equation, and I can write the equation in this form.

So, I can write this equation or I can assume this equal to X 1 X 2 I can write let me write X 1 X 2 instead of A 1 A 2. So, I can write this equal to X 1 X 2 e to the power i omega t. So, you can substitute this equation in this form and get a equation A minus lambda into X 1 X 2 will be equal to 0 0. So, now finding the determinant of this A minus lambda i. So, A minus lambda i into x 1 x 2 equal to 0 0. So, this reduces to an Eigen value problem. So, either you just find the Eigen value of this matrix A or you just find the determinant of A to find lambda. So, either you can find the Eigen value or the determinant.

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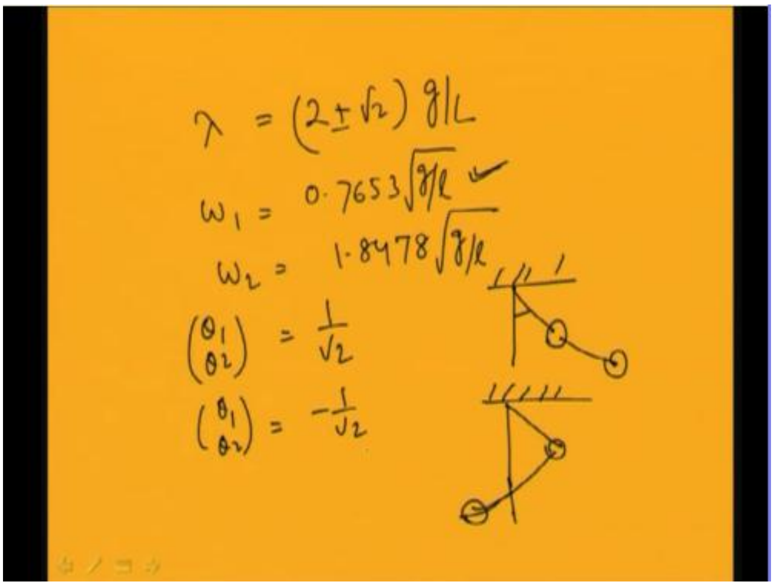

So, if you find. So, this lambda you will get will be equal to 2 plus minus root 2 g by L. So, your omega 1 will be equal to 0.7653 g by l. And omega 2 you can find equal to 1.8478 root over g by l. So, root over g by l you will get. By substituting the first one, you can find theta 1 theta 2. So, theta 1 theta 2 for the first one you can find will be equal to 1 by root 2 and the second one if you substitute then this theta 1 theta 2, you will find equal to minus 1 by root 2. So, in this first case both the masses are moving in same direction if you plot.

So, both the masses are moving in the same direction. So, when this theta 1 it is rotating at angle 1. So, this will rotate root 2, and in the second case when this will rotate at an angle theta 1, this will rotate in opposite direction that is minus root 2. Next class we will study about the coupling or coordinate coupling of the system. Already I told about the coordinate coupling where static and dynamic of coupling system I told. I will solve some problems on this dynamic and static coupling. Next class also I will tell you about the force vibration of two degree of freedom system.