Mechanical Vibrations
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Module - 4
Single DOF Forced Vibrations
Lecture - 5
Sharpness of Resonance, Vibration Measuring Instruments

So, in this module, we have studied about this single degree of freedom system in which we have studied the force vibration of this single degree of freedom system.

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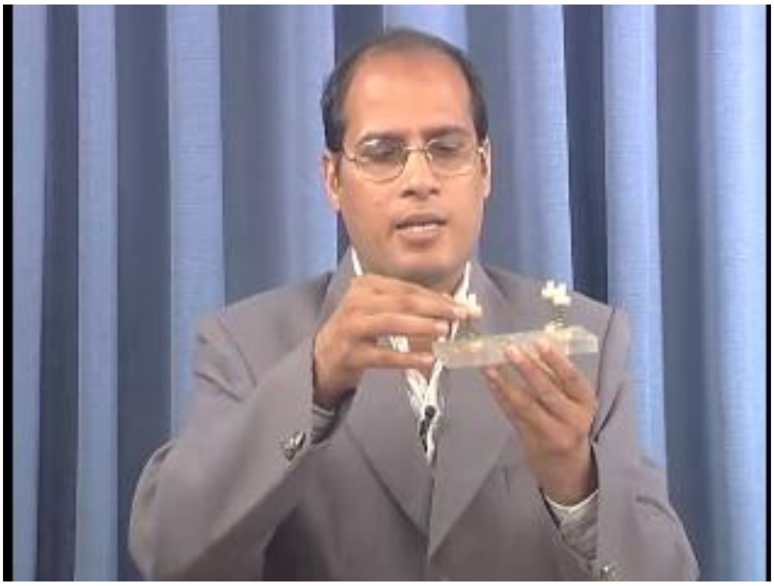

So, in case of a single degree of freedom system you have represented this by a spring and mass system, spring mass damper system you may take.

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So, this is a spring mass damper system. So, when this system is subjected to a force F sin omega t; that is forcing amplitude equal to F and frequency equal to omega. So, this will vibrate with a frequency of omega. So, we have studied about the rotating unbalance in this case, and also we have studied the whirling of shaft and the support motion.

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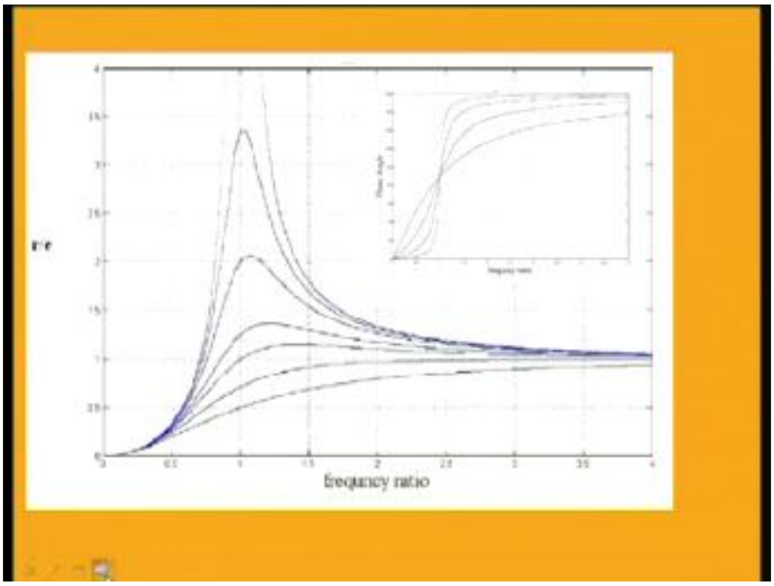

And also we have studied about the transmissibility in this case. So, in case of support motion we have seen the vibration from the support it is transmitted to this mass, and we have found the relation between the vibration of this amplitude of vibration of this mass and the amplitude of this support. Also we have found the amount of force transmitted to this support when it is subjected to a motion here. So, in both the cases we have seen that this X by Y equal to F t by F 0; F t is the force transmitted to the support, and F 0 is the amplitude of the excitation. So, this F t by F 0 we have seen equal to this X by Y; X is the amplitude of vibration of this mass, and Y is the support motion.

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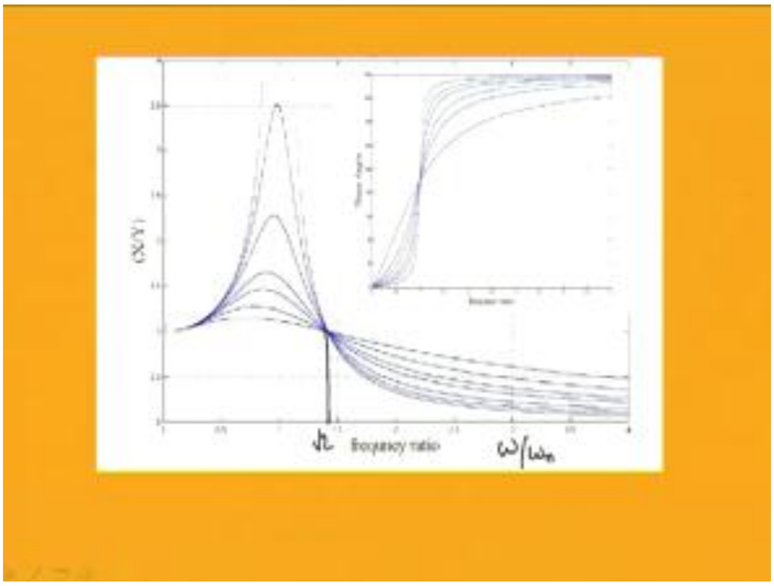

In case of support motion, we have seen that at omega equal to omega root 2 times omega n. So, this is root 2. So, when omega equal to root 2 times omega n irrespective of damping we have found that X equal to Y. That is the amplitude of motion of the support equal to the amplitude of the vibration of this mass, and we have seen that to isolate the vibration, you should always operate the system at a frequency more than root 2 times the natural frequency of the system. So, today class we are going to study about this sharpness of resonance, also about the vibration measuring instrument, and we will solve some problems on the single degree of freedom systems.

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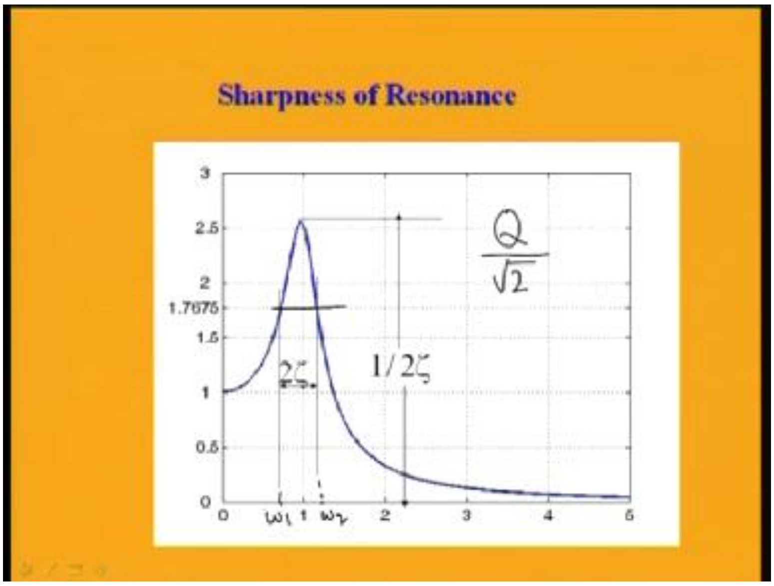

So, already you know that in case of a single degree of freedom system when it is subjected to a force F sin omega t. So, at resonance the amplitude of vibration equal to 1 by 2 zeta.

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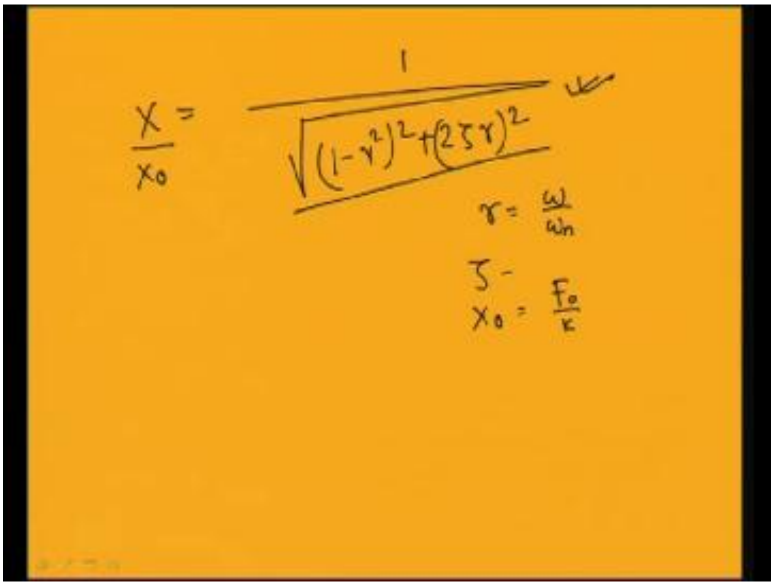

So, you have obtained this thing from this expression that is your X equal to F 0 by k, or I can write this X by X 0 equal to 1 by root over 1 minus r square whole square plus 2 zeta r whole square, where r is equal to omega by omega n. This zeta is the damping ratio that is C by C c. So, X 0 you know that is equal to F 0 by k.

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So, in the spring to this is a spring; initially you give a force F. So, due to that the displacement will be equal to F 0 by k. So, this F 0 by k will be the static deflection of this spring. So, you know the maximum amplitude of excitation X by X 0 will be equal to 1 by root over 1 minus r square whole square plus 2 zeta r whole square root over. So, in this case you know that when to find the maximum value you can differentiate this or you can know that it will be maximum when this denominator part is minimum.

So, to find the minimum of this you can differentiate this, and by differentiating you can find the frequency at which this maximum occur. And from that you can find the maximum amplitude of excitation or maximum amplitude of response amplitude equal to 1 by 2 zeta. So, this occurs at a frequency slightly left to this omega equal to omega n. So, we should know that we should not operate the system at this resonance condition. So, up to what level or up to what frequency should we operate? To know that thing we should know about this sharpness of resonance.

So, it is recommended that we should not run this system at a frequency when this amplitude of excitation is 1 or 1 by root 2 times the maximum amplitude of response. So, if I am taking this maximum amplitude equal to Q. So, let Q is the maximum amplitude. So, we should not operate the system in these ranges. So, when this is equal to Q by root 2. So, when the amplitude becomes Q by root 2, above that we should not operate the system and you can see that it is also known as the half power point.

So, at Q by root 2 if I will plot; so in this case I have taken zeta equal to 0.2, and I have plotted the response curve for a particular system. So, in this system if I will take Q by root 2; so it will be coming to be 1.7675. So, at that amplitude let me draw this line. So, you can see that it is cutting at two points. So, if I will draw. So, I have to find this magnitude or these frequencies. So, at what frequency this is occurring. So, let this frequency will be equal to omega 2; this frequency is omega 1. So, let us find this frequency at which or the range at which we should not operate this system. So, in that case, I have to find this omega 2 and omega 1.

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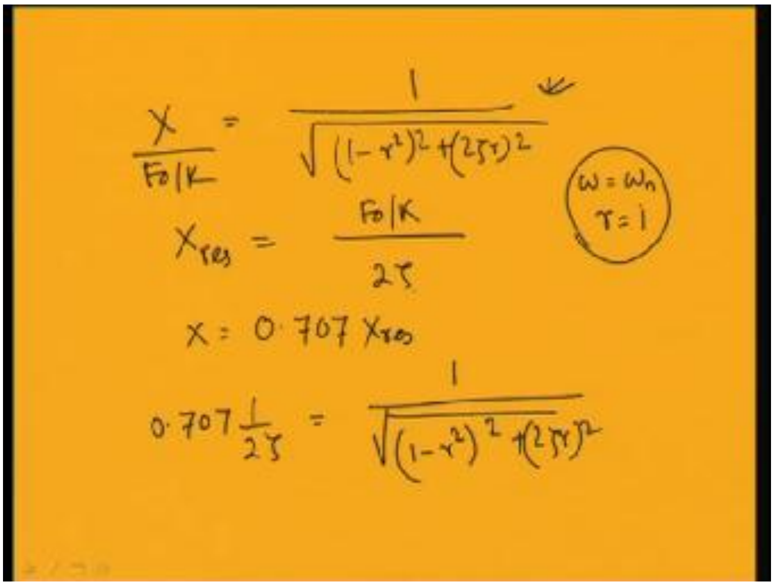

So, I know that this X equal to; so X equal to F 0 by k root over 1 minus r square whole square plus 2 zeta r whole square. So, at resonance I can write. So, as resonance occur nearly r equal to r 1 for lower value of damping r equal to 1; that is equal to omega equal to omega n or r equal to 1. So, let us consider this value. So, r equal to 1. So, this value becomes. So, this will be equal to F 0 by k. So, this is equal to 0. So, this becomes 2 zeta r. So, r equal to 1. So, this becomes 2 zeta. So, F 0 by k by 2 zeta.

So, at omega equal to omega n, you obtain this X equal to F 0 by k by 2 zeta. So, I can consider the amplitude when this amplitude equal to Q by root 2 or this X by root 2 or 0.707 times this resonance frequency. So, I can write this as X resonance. So, this is X resonance equal to F 0 by k by 2 zeta. So, I can take this X value at a 0.707 times X resonance. So, by taking that thing you can get two frequencies, and these frequencies are known as half power point.

So, I can take this X as 0.707 times this X and I can write this already you know this expression. So, substituting these in this expression, I can write 0.707 X resonance equal to 1 by 2 zeta will be equal to. So, this X resonance by F 0 by k equal to 1 by 2 zeta. So, I can take that thing. So, this X by F 0 by k I can write equal to 1 by. So, this becomes 1 by root over this. So, in this I will substitute this X resonance by F 0 by k equal to 1 by 2 zeta.

So, by substituting this X equal to 0.707 X resonance by F 0 by k; so I can get it equal to 1 by root over 1 minus r square whole square plus 2 zeta r whole square. So, from this I will get this r for which this side band occurs. So, now, I can square both the sides. So, by squaring both the sides, you can get it. So, this is half, and this is 1 by 4 zeta square, and this side it becomes 1 by 1 minus r whole square plus 2 zeta r whole square.

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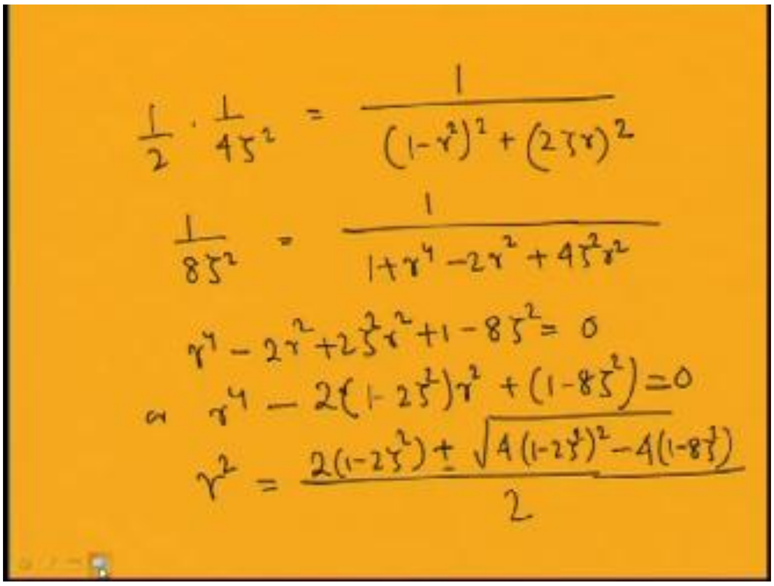

So, I can write this as 1 by 2 into 1 by 4 zeta square equal to 1 by 1 minus r square whole square plus 2 zeta r whole square, or this becomes 1 by 8 zeta square equal to 1 by. So, I can expand this thing. So, then this becomes 1 plus r fourth minus 2 r square plus 4 zeta square r square, or I can take it this side. So, this becomes 1 plus r fourth plus. So, I can write this way r fourth plus or minus 2 r square plus 2 zeta square r square plus 1 minus 8 zeta square equal to 0, or I can write this equation in this form r fourth.

So, this becomes. So, plus I can take minus common. So, minus 2 r r square I will take common. So, this becomes 1 minus 2 zeta square r square plus 1 minus 8 zeta square equal to 0. So, this is a quadratic equation in terms of r square. So, now I can find the solution for this r square. So, r will be equal to. So, minus b plus minus b square minus 4 a c by 2 a. So, in that formula if I will put. So, then it will becomes. So, this is equal to minus b.

So, it will be 2 into 1 minus 2 zeta square plus minus root over b square; that means it will becomes 4 into 1 minus 2 zeta square whole square. So, minus 4 into 1 minus 8 zeta square by 2. So, this becomes r square. So, this is r square as we are finding this is r fourth, then this is r square equal to this. So, if you expand this thing then you can write.

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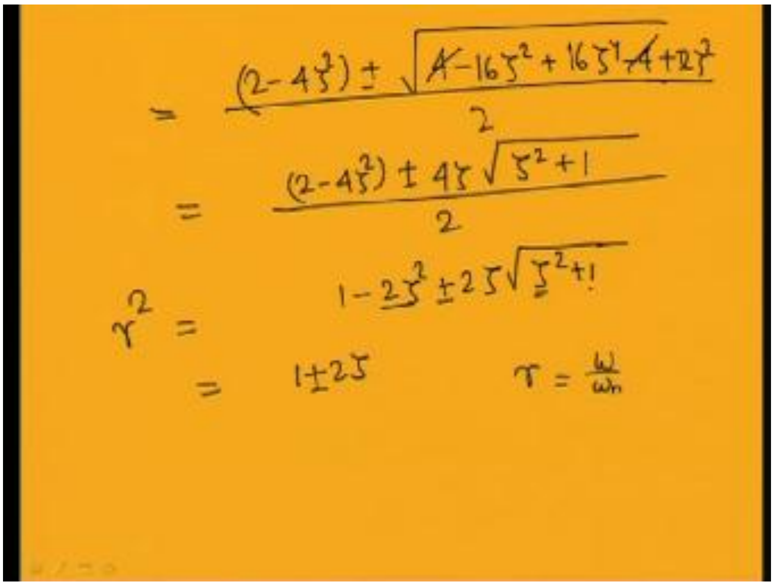

So, by expanding this you can write this equal to 2 minus 4 zeta square plus minus. So, by expanding that part you can write this will be equal to 4 minus 16 zeta square plus 16 zeta fourth minus 4 plus 32 zeta square by 2. So, this will become. So, now this plus 4 minus 4 you can cancel, and this is minus 16 zeta square and plus 16 zeta fourth plus 32 zeta square. So, this becomes 16 zeta fourth plus 16 zeta square. So, you can take this 16 zeta square common. So, then it will become 2 minus 4 zeta square plus minus let you take 16 zeta square common. So, that will give you 4. So, that will give you 4 zeta root over. So, here you will have zeta square plus 1.

So, this is zeta square plus 1 by 2. So, now as you are taking zeta very, very small; so when zeta is very, very small, then only the resonance will occur at omega equal to omega n. So, you are considering that condition. So, when you are taking this zeta equal to small, then this zeta square term can be neglected. So, neglecting this zeta square terms, you can write this r square will be equal to. So, this r square will be equal to you can write in this way. So, by expanding this thing you can write this is 1 minus 2 zeta square. So, plus minus 2 zeta.

So, this thing you can write equal to 1 minus 2 zeta square plus 2 zeta root over zeta square plus 1 by dividing this two and by neglecting this zeta square term. So, you can write this r square equal to 1 plus 2 zeta. So, you are neglecting this term, and you are neglecting this term with respect to this. So, this becomes 1. So, it becomes 1 plus minus 2 zeta. So, you will have two values. So, you are getting r square equal to 1 plus minus 2 zeta. So, this r equal to omega by omega n. So, this gives two frequencies and at those frequencies one corresponds to omega 1 and other corresponds to omega 2.

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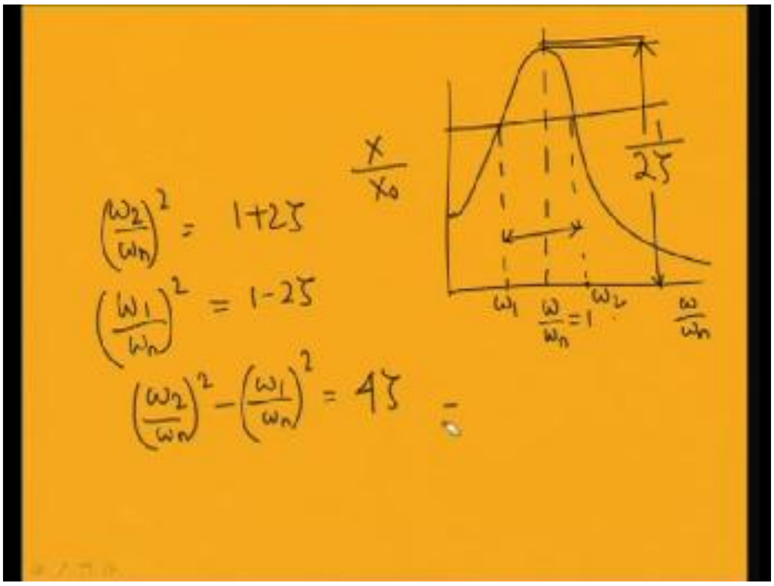

So, at these two frequencies you can write that is. So, this is if you are plotting these. So, it starts at 1 and at omega equal to omega n near omega equal to omega n. So, this is omega by omega n equal to 1. So, you are plotting omega by omega n versus X by X 0. So, this maximum value that is you have at omega equal to omega n, you have already seen this is equal to 1 by 2 zeta. And we are finding the side band or half power frequency the frequency at which or the range of the frequency we are finding now.

So, now you got two frequencies; one correspond to this 1 plus 2 zeta, other correspond to this 1 minus 2 zeta. So, this omega 1 square that is omega; so I can take this as omega 2 by omega n square. So, this is equal to 1 plus 2 zeta, and other one that is omega 1 by omega n square this is equal to 1 minus 2 zeta. So, this point corresponds to this omega 2. So, this is omega 2 and this is omega 1. So, at omega 2 by omega 1 you are getting this equal to 1 plus 2 zeta and omega 1 by omega n square equal to 1 minus 2 zeta.

So, now if you subtract this from this, then you can find this will be equal to that is omega 2 by omega n square minus omega 1 by omega n square. So, this is equal to 1 plus 2 zeta minus 1 minus 2 zeta. So, it becomes 4 zeta. So, this thing you can expand. So, by expanding this thing you can write this as...

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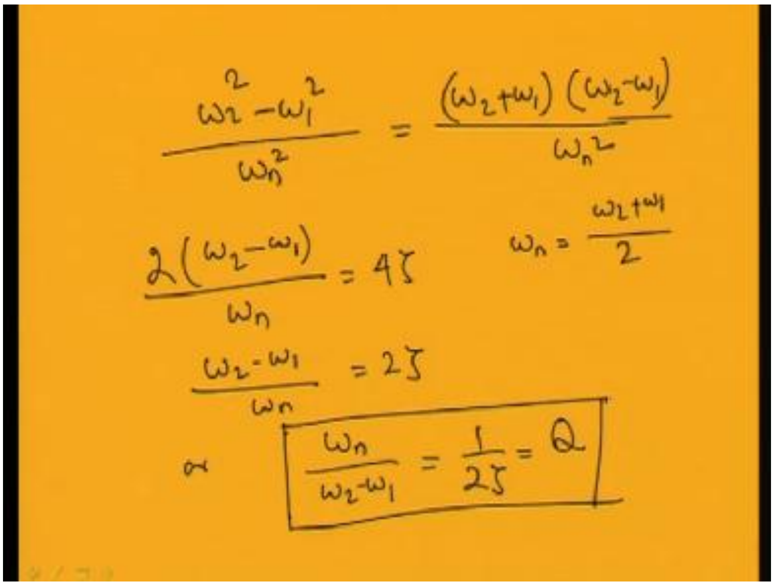

So, this will be equal to omega n square you can take. So, omega n square, so this becomes omega 2 square minus omega 1 square. So, this thing you can write it as omega 2 plus omega 1 into omega 2 minus omega 1 by omega n square. So, this as we are taking the side band in the left and right of this omega equal to omega n or omega by omega n equal to 1. So, you can have this omega 2 plus omega 1 by 2 equal to omega n. So, omega n will be equal to omega 2 plus omega 1 by 2.

So, in this case already you have this. So, I can write this omega 1 plus omega 2 by 2 equal to omega n. So, this 1 omega n will get cancelled and you can multiply 2. So, this becomes 2 into omega 2 minus omega 1 by omega n equal to 4 zeta or omega 2 minus omega 1 by omega n. So, this becomes 2 zeta, or you can write this omega n by omega 2 minus omega 1 equal to 1 by 2 zeta, and this 1 by 2 zeta already you have seen. So, this is equal to the maximum amplitude at resonance.

So, this factor this omega by omega 2 minus omega 1 is known as the quality factor of the system, also it shows the sharpness of the resonance of a system. So, this quality factor or sharpness of resonance is a function of damping only; it does not depend on other system parameter. So, by knowing the damping of a system, you can decide about these half power points, and you can operate your system in a safe range. So, you should not operate yours system in this range when it is approaching towards these resonance conditions.

So, from this point the resonance will be sharper, and you should not operate the system in this half power band. So, either you should operate the system in this range or when it is away from this omega 2. So, now we will study about the effect of this complex stiffness in a system. So, already we have seen. So, we are taking this stiffness as a linear parameter in this system.

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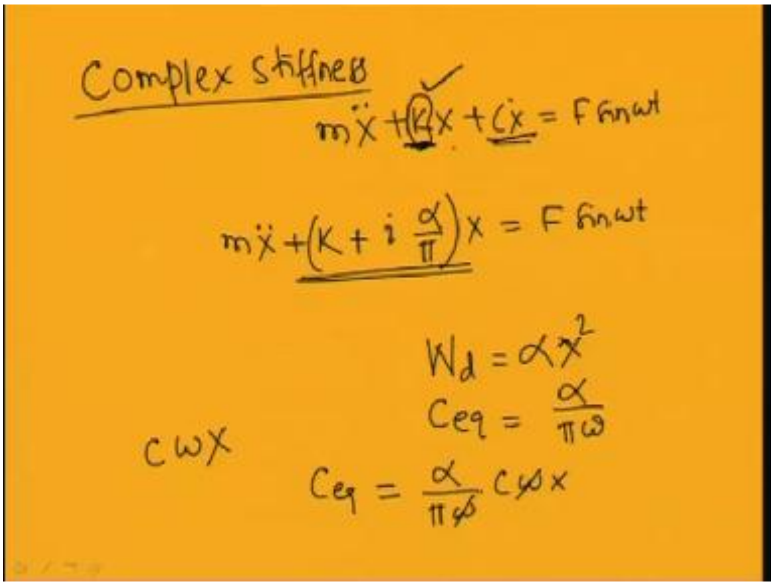

So, in all our systems you have taken this as m X double dot plus k x plus c x dot equal to F sin omega t, but in many aerospace structure the stiffness is. So, for many aerospace structures this stiffness parameter depends on the system parameters, and in that case we can write this equation in this form. So, you can write this equation as m X double dot plus. So, when the system is not real but it is complex, then we can write this equation in this form m X double dot plus k plus.

So, I can write this as alpha by pi. So, this alpha by pi is the equivalent structural damping of the system. So, alpha by pi I can write equal to this. So, i alpha by pi X plus or will be equal to F sin omega t. So, in many aerospace structures this damping will be the structural damping type of thing. So, in case of structural damping you have seen this energy loss due to damping equal to alpha X square where. So, it does not depend on the frequency. So, in that case you have seen this C equivalent you can find; so C equivalent in this case equal to alpha by pi omega.

So, as C equivalent equal to alpha by pi omega, then the C x dot term will be equal to this is C omega x. So, as it is equal to C omega x then substituting the C equivalent you can write this equal to. So, C equivalent in case of structural damping will be equal to alpha by pi omega into C omega X. So, this omega omega got cancelled, and in this case this damping term will be retained in this form. So, it will be alpha by pi C into X. So, I can write this as k plus i into alpha by pi into X. So, by taking this structural damping into account, you can have a stiffness which is complex in nature.

In other systems, we have seen that the stiffness we are separating the stiffness like k and C, and they are real numbers we have taken, but in most of the structure when we are considering the structural damping. So, when there is no viscous damping present in the system. So, we can write the equivalent viscous damping for the system, and in that case we can write the stiffness term like this k plus i alpha by pi X. So, we can find for this case of complex stiffness. So, when we are considering this complex stiffness of the system. So, we can find the response of the system.

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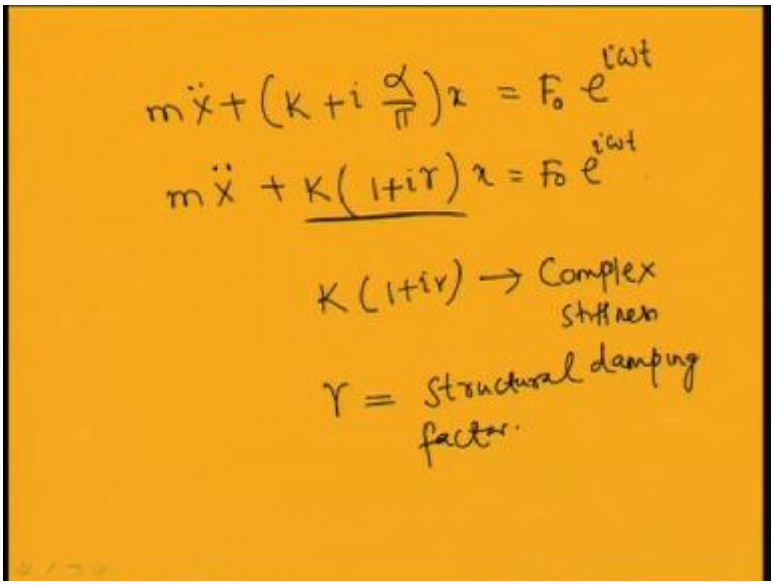

So, in this case the equation motion is retained in this form. So, that is m X double dot plus k plus i alpha by pi x equal to. So, let me take this forcing as e to the power i omega t. So, I can take this as e to the power i omega t. So, in this case this equation also you can write in this form that is m X double dot, you can take k common. So, it will be 1 plus i gamma you can put this gamma x equal to F 0 sin e to the power i omega t. So, in this case this term that is k into 1 plus i gamma is called the complex stiffness, and gamma is the structural damping factor. So, this is the complex stiffness of the system.

So, this is known as the complex stiffness, and this gamma is known as the structural damping factor. So, taking these two we can find the response of the system. So, in this case the response of the system I can find in this form.

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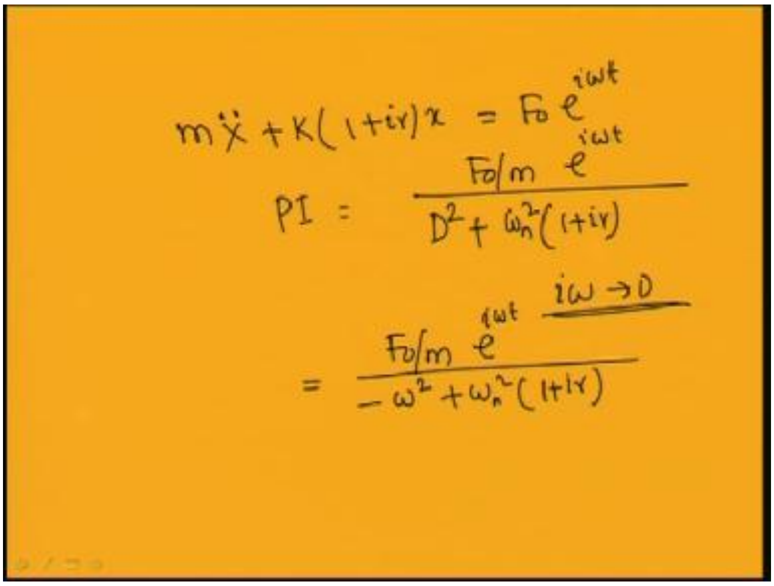

So, I know the particular integral for the system can be written in this form. So, as the equation is M X double dot plus k into 1 plus i gamma x equal to F 0 e to the power i omega t. So, the particular integral will be F 0 by m e to the power i omega t. I can write this equal to by dividing this m I can write. So, this will be D square plus. So, in this case I can write this is k by m that is will be equal to omega n square plus 1 plus i gamma. So, omega n square into 1 plus i gamma I can write and now will substitute i omega for D.

So, in this case I can substitute i omega for D. So, if I will substitute I omega for D. So, this will be in this form. So, this will be F 0 by m e to the power i omega t. So, this is minus omega square. So, this plus omega n square 1 plus i gamma. So, if you rearrange this thing.

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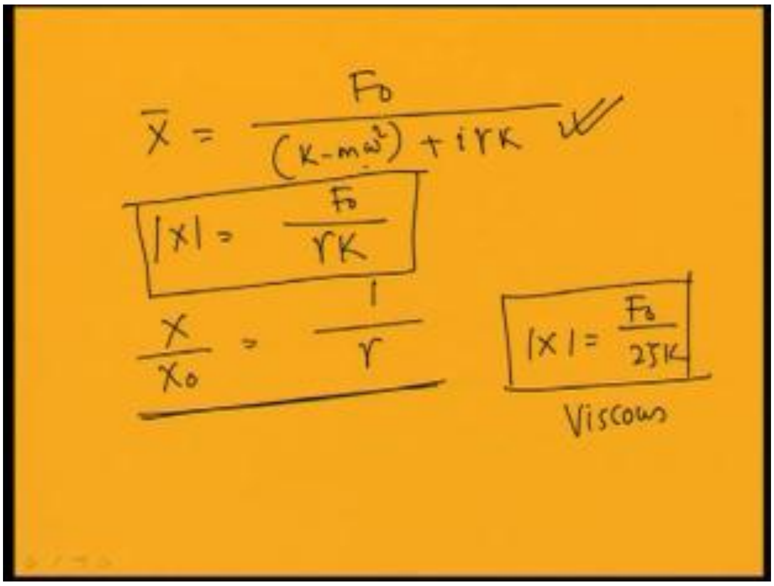

So, this particular integral that is the steady state response of the system you can write in this form. So, this will be equal to F 0 by k minus m omega square plus i gamma k. So, the amplitude at resonance you can find. So, the amplitude will be mod X of this you can find. So, this will be equal to. So, at resonance you can find this amplitude will be. So, this part will be equal to 0, and it will be equal to F 0 by gamma k. So, this is equal to F 0 by gamma k.

So, at resonance you can see that this is equal to F 0 by gamma k. So, now, you can write this X by X 0 in this case equal to 1 by gamma. So, the sharpness of resonance in this case you can write will be equal to 1 by gamma. So, in case of viscous damping you have seen that the sharpness of resonance was 1 by 2 zeta, and here it is equal to 1 by gamma. So, you can compare the resonant frequency of that of a viscous damping. So, in that case we have seen that this small x equal to F 0 by 2 zeta k. So, this is in case of a viscous damping and this is the case of a structural damping.

So, in this case of structural damping, you have seen you can write this stiffness as a complex stiffness, and by writing this stiffness as a complex stiffness you can find the response of the system. So, the response of the system can be written in this form that is F 0 by k minus m omega square plus i gamma k and at resonance you are finding this equal to F 0 by gamma k.

So, in this way you can find the response of the system also, and you can see that if you want to plot the frequency response with the structural damping then you can see. So, you can obtain two components of this response that is one imaginary part and other the real part and combining this thing you can get or you can see that the response curve is a circle. So, now we can study about this vibration measuring instrument. So, already I have shown you the vibration can be measured by using different types of vibrometer. So, you may measure the displacement of the vibration, you may measure the velocity of the vibration or you may measure the acceleration of the vibration.

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So, when you are measuring the displacement. So, you can use the seismometer and when you are using this velocity you can use Velometer and in case of acceleration you can use this accelerometer. So, this is an accelerometer; the basic principle of accelerometer is this vibration measuring instrument we will study now. So, this accelerometer, you can observe that there is a screw. So, either it can be screwed inside this structure or you can use a magnetic base. So, this is a magnetic base. So, you can see this is a magnet attached to this.

So, you can attach this with this and you can put it under structure which is vibrating. So, this thing you can put on the structure, so then that structure will contain a magnetic particle or magnetic. So, let we are taking this steel. So, in this case it is attached to this structure. So, when this structure is vibrating you can get the response or you can take the response or you can take the vibration of this by using this wire.

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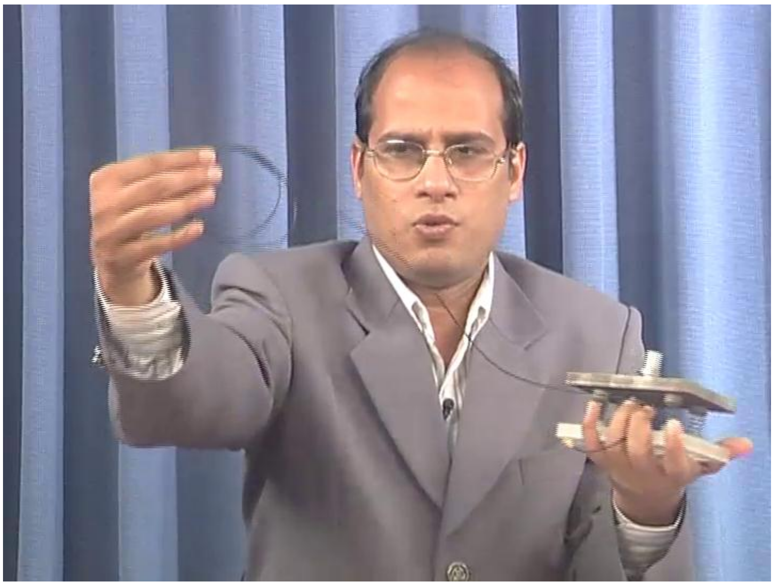

So, it can be taken to data acquisition system where this accelerometer data will be converted to the required acceleration form. So, you can plot or you can find the acceleration and by integrating this acceleration you can find the velocity and by integrating that thing you can get the displacement of the system. So, let us see the basic concept of this vibration measuring instrument. So, in this vibration measuring instrument you can see. So, I will show you the vibration measuring instrument.

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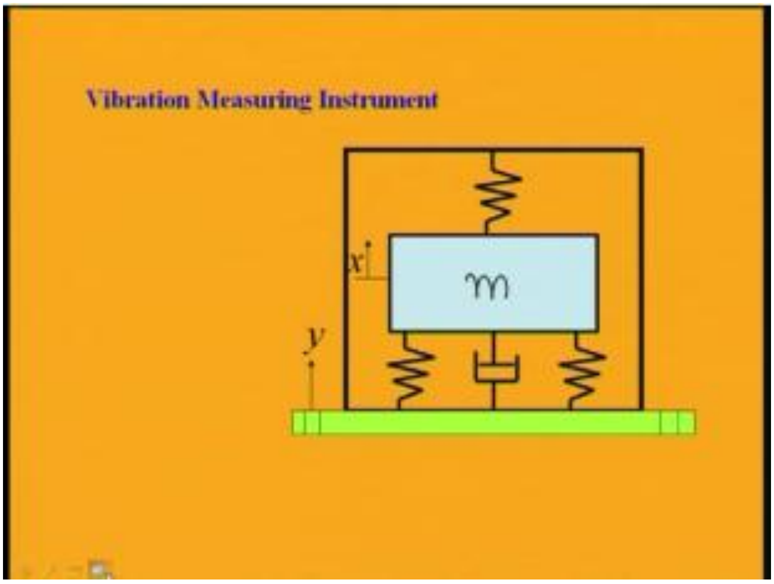

The basic principle of this vibration measuring instrument is that. So, this is a small mass or seismic mass which is put inside a box. So, in this box which is vibrating. So, this box is attached to the support which is vibrating; let the support is vibrating with y. Then in this box you can see. So, this is a spring or you have this spring and mass damper, and when there is vibration of this support as this box is fixed to the support, then this box will vibrate.

So, when it is vibrating there will be motion of this mass. So, there will be a relative motion between y and x, and that relative motion we can take to measure this vibration of the system. So, already you have studied about the support motion. So, in that case you have found the equation motion. So, the support motion is the basic principle of this vibration measuring instrument. So, let us derive this vibration measuring equation for this vibration measuring instrument. So, the motion of this we have to find. So, this mass is let x is the displacement of this mass when the support if moving with y. So, when the support is moving with y I can write or I can find.

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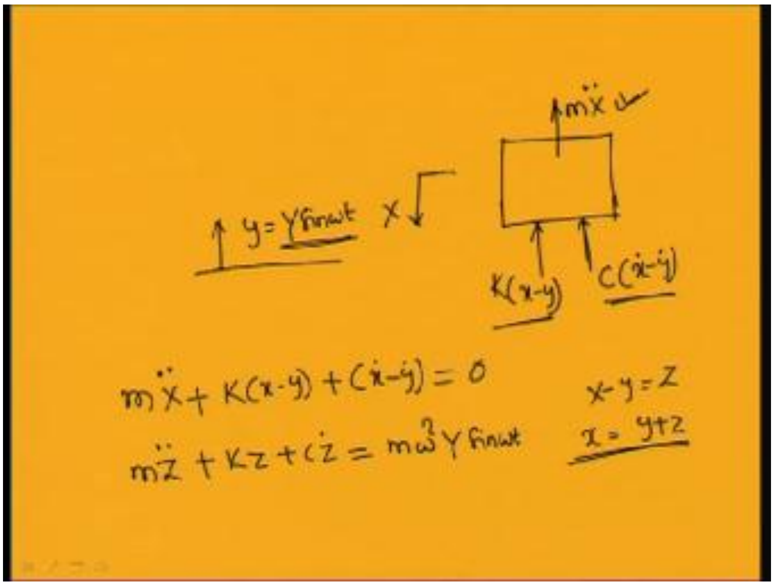

So, in this case when the support is moving; so I can draw the free body diagram of this. So, by drawing this free body diagram, the springs are subjected to forces equal to k x minus y, and the damper is subjected to a force c x dot minus y dot, and the mass is vibrating with m X double dot. So, it is subjected to an inertia force of m X double dot. So, when the mass is moving downward. So, this is X, I have taken. So, in that case if you draw the free body diagram of this system, then m X double dot is the inertia force k x minus y is the spring force, and c x dot minus y dot is the damping force in the system.

So, in this case, you can find the equation motion will be in this form. So, that is m X double dot plus k into x minus y plus c into x dot minus y dot equal to 0, or I can write this equation in this form that is equal to m X double dot. So, this x dot I can write as x minus y equal to z or x will be equal to y plus z. So, as x equal to y plus z, I can write. So, this I can write in this form. So, this will be equal to I will replace this x double dot by z double dot plus y double dot. So, it will be m z double dot plus k z plus c z dot then plus m y double dot

So, this m y double dot I will take to the right hand side, or I am assuming that the support is vibrating with y equal to y sin omega t that is it is vibrating in a harmonic manner. So, when it is vibrating in a harmonic manner, then I will take this y double dot will be equal to minus omega square y sin omega t. So, this m y double dot plus k z plus c z dot will be equal to. So, in this case it will be equal to m omega square Y sin omega t. So, this z will be equal to. So, z you can write.

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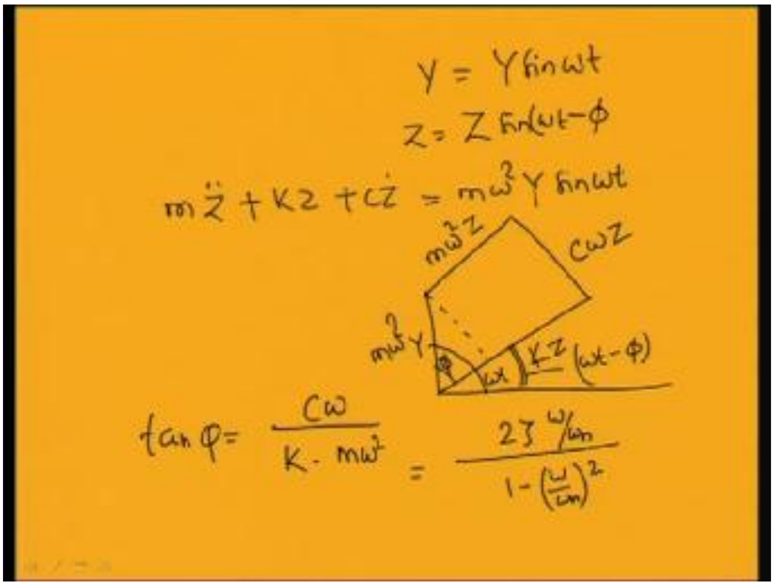

So, I can assume this z equal to Z sin omega t minus phi; already we have taken y equal to Y sin omega t, then z equal to Z sin omega t minus phi. So, the solution of this equation that is m z double dot plus k z plus c z dot equal to m omega square Y. So, this is m omega square Y sin omega t. So, you know the solution of this system; you can draw a reference line. Then this is the spring force, this is the damping force, and this is the inertia force.

So, spring force equal to k z, damping force equal to c omega z, and this inertia force equal to m omega square z, and this will be equal to the resulting force that is equal to m omega square Y. So, this m omega square Y. So, when this is equal to m omega square Y. So, from this you can find this angle phi and you know this angle equal to omega t, this is phi. So, this angle equal to omega t minus phi. So, from this you can obtain this z. So, z will be equal to. So, from this k z minus m omega square z whole square will be plus c omega z whole square will be equal to this m omega square Y. So, this is equal to m omega square Y.

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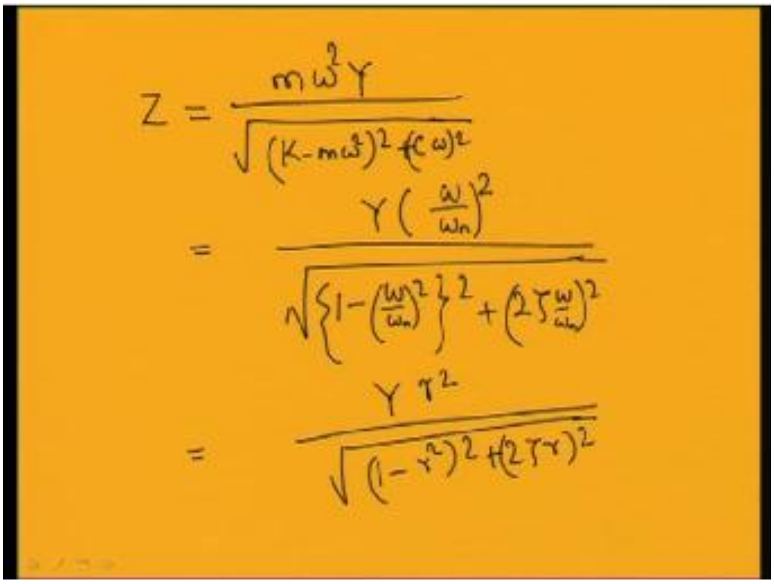

So, by adding the things you can write this Z will be equal to or already we have seen similar cases. So, Z will be equal to. So, this is equal to m omega square Y root over what k minus m omega square whole square plus c omega whole square. So, this will be equal to you can simplify this. So, you can write this is equal to Y into. So, let me divide this by omega m. So, if I am dividing this by m. So, then it will become. So, I will take this m down and this will be m square.

So, k by m will be equal to omega n square. So, you can simplify this, and you can write this equal to omega by omega n whole square into y by root over. So, this part will be equal to. So, this is 1 minus omega by omega n whole square, then this whole square plus 2 zeta omega by omega n whole square. You can note that in all places this denominator is same. So, in this case you have this Z equal to Y into omega by omega n whole square root over 1 minus omega by omega n whole square whole square 2 zeta omega by omega n square.

So, in terms of r if you want to write; so it will be Y r square by root over 1 minus r square whole square plus 2 zeta r whole square. So, this is the expression for z, and you can find this tan phi also from this. So, from this you can find tan phi. So, tan phi equal to omega c omega by k minus m omega square. So, this will be equal to 2 zeta omega by omega n 1 minus omega by omega n whole square.

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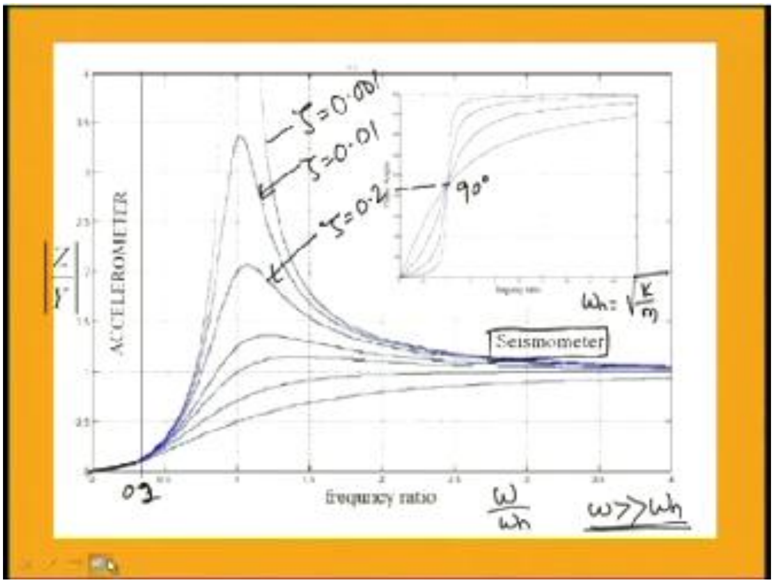

So, this is the plot between Z by Y and omega by omega n. So, Z by Y by omega by omega n if you plot. So, you can see that this is similar to the case what you have studied in case of rotating unbalance also in case of whirling of shaft. So, in those cases the curves are similar. So, in this case this is plotted for different value of zeta. So, this is for zeta very, very less 0.001. So, this is zeta equal to point let it is 0.01, and this is zeta equal to 0.02. So, in this way you have plotted, and this is the phase diagram of this. And you can observe that omega equal to omega n; this phase angle equal to 90 degree.

And when it is operating at a very higher frequency, then this angle tends to 180 degree and for lower value of omega. So, this angle is below this 90 degree. So, you can observe that up to this 0.5 or 0.3 let me take it 0.2 or 0.3; so omega by omega n equal to 0.3. So, you can assume that this curve is linear. So, this curve be linear that is Z by Y proportional to or Z by Y equal to omega square. So, from that equation you can see that when omega is very, very less than omega n. So, then this becomes equal to Z by Y become omega square.

So, Z will be equal to omega square Y and omega square Y is the acceleration term. So, if you have an instrument with very higher omega n value that is the omega n, omega n you can write equal to root over k by m. So, the system you are taking for the measuring the vibration.

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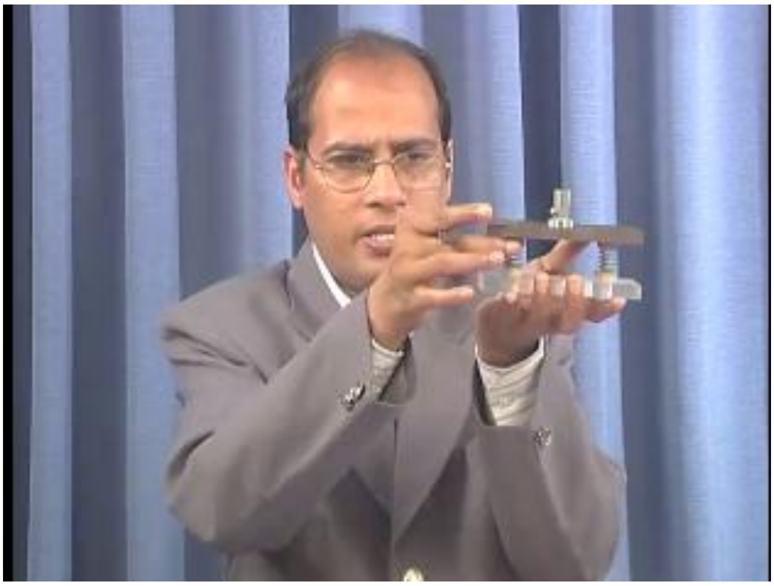

Let you are taking the system for measuring this vibration or inside that vibration measuring instrument you have a mass. So, that mass and the stiffness what you are using, so if root over k by m is very large in comparison to the frequency you are going to measure, then that instrument can be used as an accelerometer. That means when you are using an instrument with very higher value of natural frequency, then that instrument will be an accelerometer. And when you are operating it at a higher value of omega that is when omega is very, very greater than this omega n or you are using an instrument which has low value of this omega n low natural frequency, then in that case this Z by Y will be equal to. So, when Z by Y equal to 1, Z will be equal to Y.

So, in that case this relative displacement Z will be equal to Y that is the displacement of or the support motion. So, you can measure the support motion or the amplitude of motion of the support that is Y by this relative change in this motion. So, Z will be equal to Y. So, in that case you can use this as a seismometer to measure the displacement of the system, and when the system has a very high natural frequency, then you can use this as an accelerometer.

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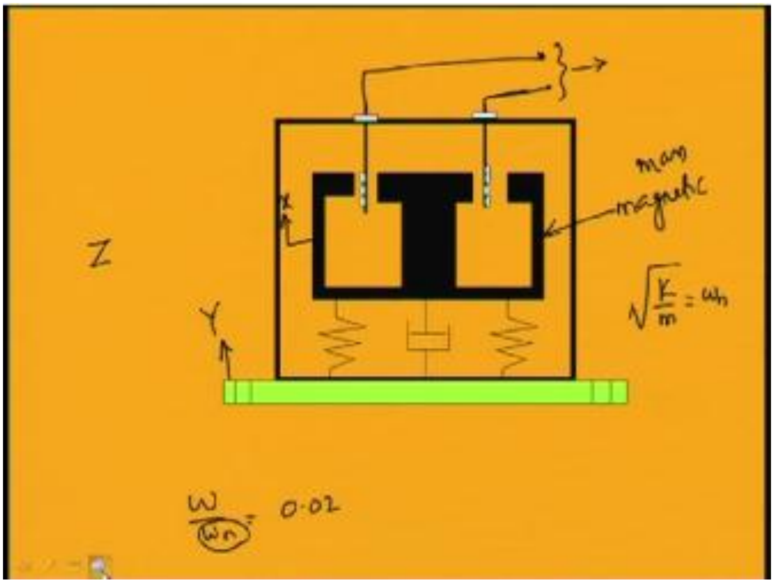

So, to measure this relative displacement, so in actual practice or in actual accelerometer, inside of that accelerometer the mass is a magnetic mass. So, this is a magnetic nature; the mass is of the magnetic nature, and this is the stiffness are the springs and damper attached to this, and there are some coil attached from the top of this box. So, this box is attached to this support. So, this support of this the box is attached to this support which is vibrating with Y. So, this is vibrating with Y, and this mass is vibrating with x.

So, due to this there is a relative motion Z which is proportional to the change in the flux in this coil. So, as the coils are hanging when this mass is vibrating up and down. So, there will be change in flux in this coil. So, due to the change in flux in this coil, you can tap out this as voltage. So, you can tap this voltage and you can measure this relative displacement. So, this voltage will be proportional to the relative displacement Z, and from this we can get the Z or the relative displacement of the system.

So, by measuring this relative displacement you can find either the acceleration when this is used as an accelerometer and when it is used as a seismometer you can measure the displacement of the system. So, in case of accelerometer, you have seen that this omega by omega n is very, very less than 1 or omega n will be very. So, omega by omega n, so let omega by omega n let me take it equal to 0.02. So, in this case this omega n will have a very high value, as omega n is very high value this root over k by m. So, this is equal to omega n.

So, as root over k by m equal to omega n; so you should have the system with very high stiffness. So, this should have very high stiffness and low mass. So, this low mass with very high stiffness spring, when it is vibrating then this magnetic flux will create the current in this coils and that coil you can take out this; you can measure the relative displacement, and you can find the acceleration of the system. So, when this omega by omega n or you are using this as a seismogram or seismometer; that time the system will have a very low natural frequency.

If the system has a very low natural frequency, then this omega by omega n will be greater than 1. So, when this omega by omega n is greater than 1, that time the system has a low natural frequency. If the system has a low natural frequency; that means this root over k by m will be low. That means you can use the spring with lower stiffness to use this as a seismometer. So, in this way you can. So, you can design the vibration measuring instruments to measure either acceleration of the system or you can measure the displacement of the system.

In all the cases you can find when you are measuring this displacement of the system, by differentiating you can find the velocity and again differentiating you can find the acceleration, or when you are finding this acceleration you can go on integrating to find the displacement of the system. So, now we will study or we will take some examples to study what we have studied in this single degree of freedom system.

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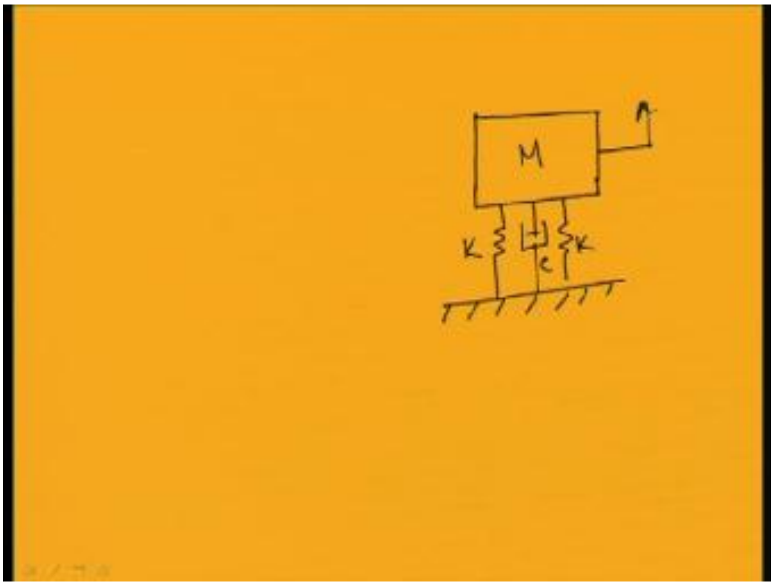

So, in this single degree of freedom systems let us take a very simple example and I will show you with the help of MATLAB that how you are getting the response of this system. So, we have taken this spring mass damper system. So, this is k, this is c, and this is m. Let us take some physical parameter m c k and let us see what is this vibration of this system? So, we will use MATLAB to find this response of the different cases.

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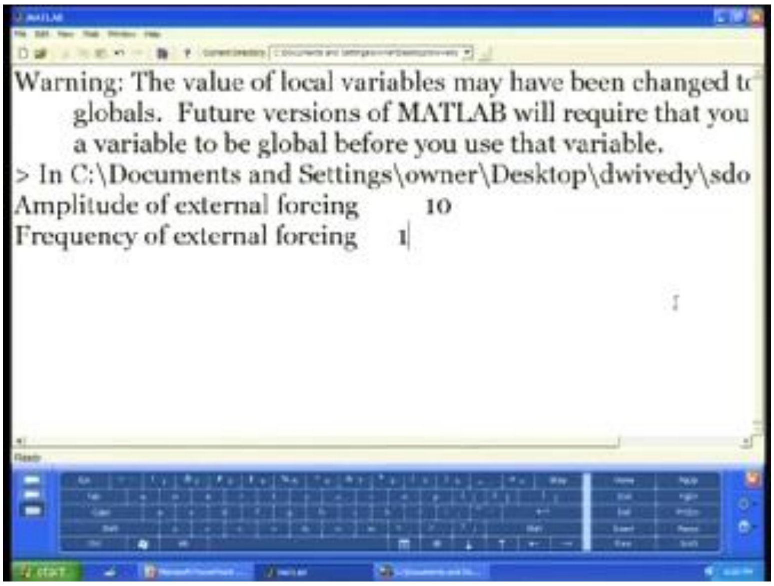

So, here let us give the amplitude of external forcing; let the amplitude of this external forcing let me give equal to 10, and the frequency of external forcing let it is equal to 1. So, I have taken a system with mass equal to I can tell. So, the mass in this case I have taken equal to 2, and I have taken the spring stiffness equal to 100 and damping equal to 20. So, I want to find the response of the system.

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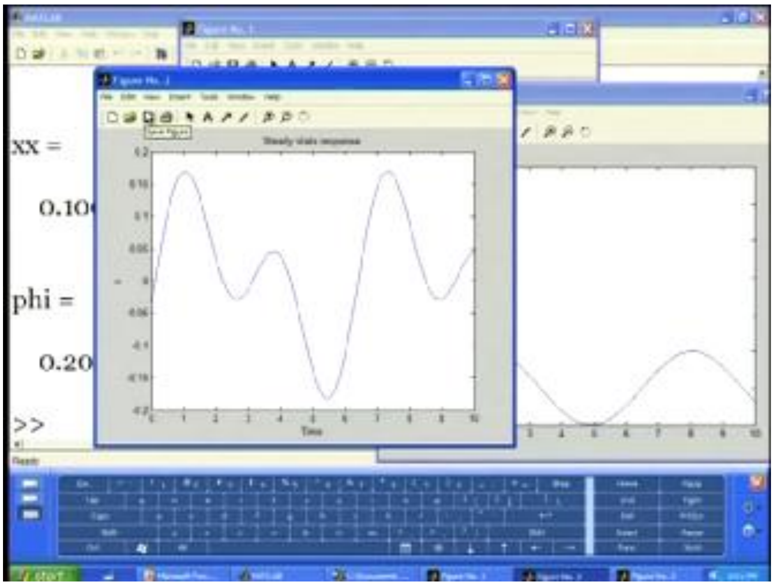

So, in that case the response of the system you can see that I obtain it in three different ways. So, I will show you three different curves. In this case I have given a forcing equal to F sin omega t plus F sin omega 2 t. So, when I have given these two forces to the system. So, this is the response.

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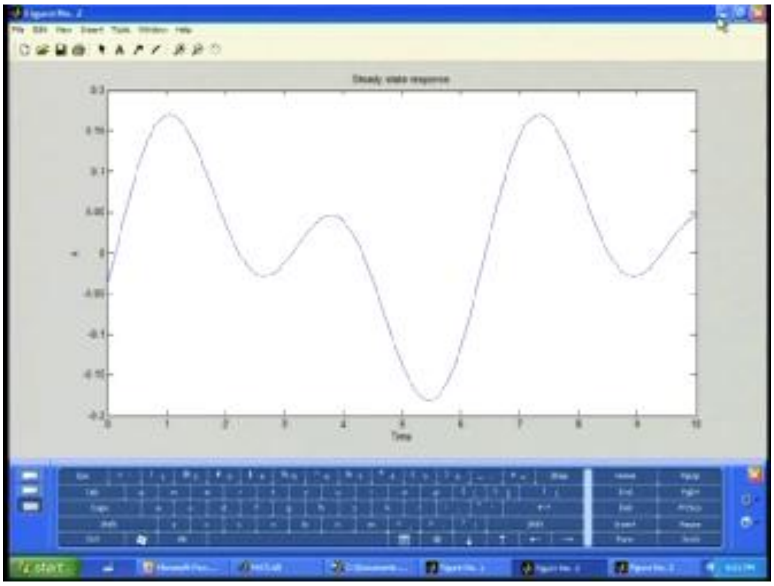

So, the response plot you can see. It contains two components, and if you plot this thing using this phase portrait. So, this is x versus time displacement versus time.

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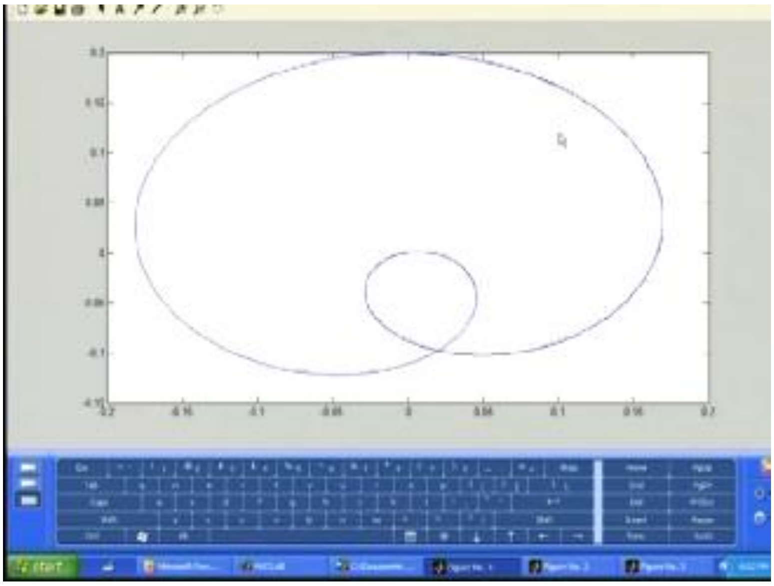

And if you plot that thing using this phase portrait. So, this is known as phase portrait. In case of phase portrait you can plot the velocity versus displacement. So, this shows the velocity versus displacement. So, you can see there is a loop here. So, in this case this contains two frequency of the system. So, as we have taken two frequency external frequency, the response is showing to be two frequency.

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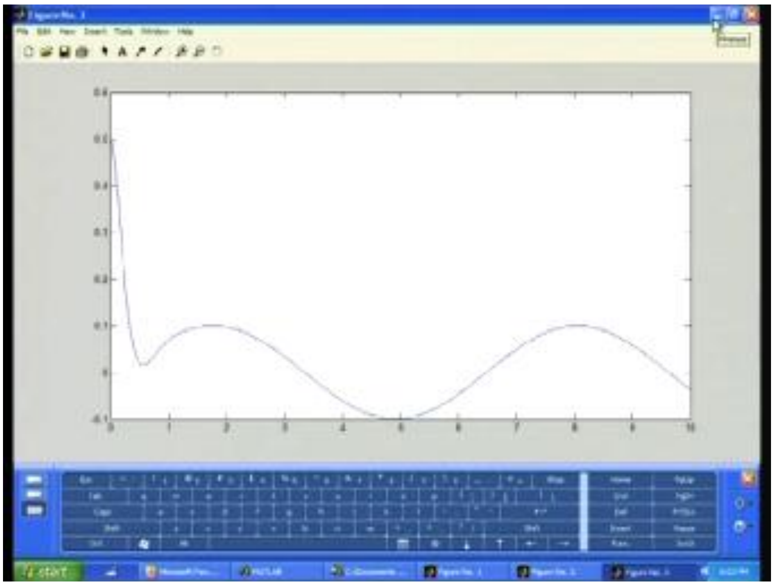

Also I obtain the same thing by solving it by Runge-Kutta method. So, in this case I have taken only a single frequency F sin omega t. So, in this case you can see that this p initial part is the transient response, and after that you have this steady state response. So, you can change this value and you can obtain different type of response of the system.

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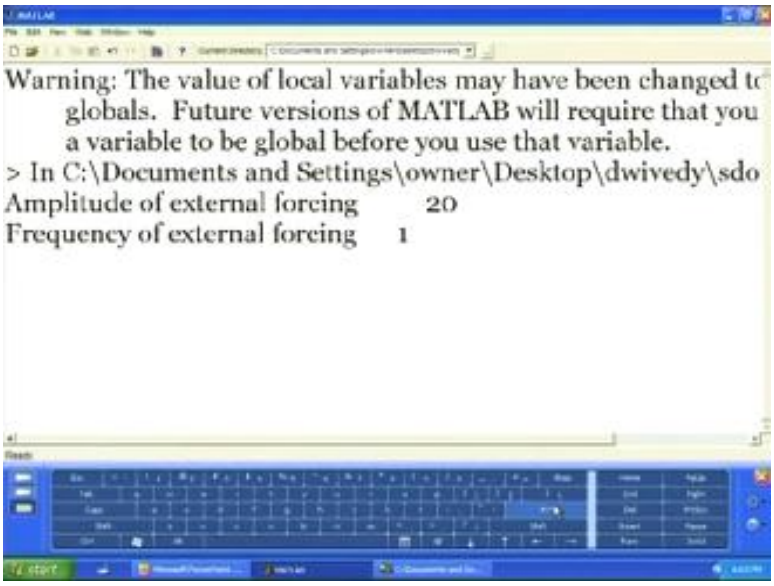

So, I can give you this external forcing. Let me change this forcing amplitude to 20 and forcing frequency let me change it to 1.

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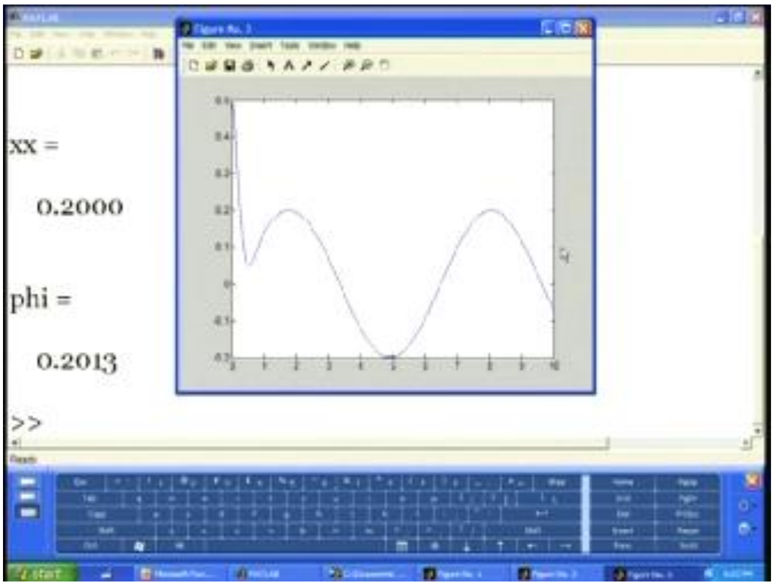

So, in this case also you can see the amplitude is increasing. Similarly, you can study for different cases.

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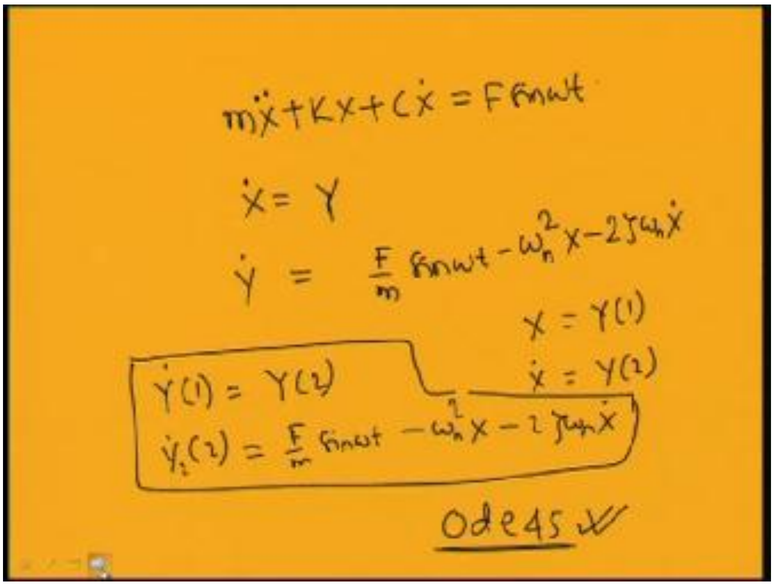

So, we have taken a system in which we have applied this force like this that is m X double dot plus k X plus c X dot equal to F sin omega t. So, in this case to use this Range-Kutta method to find the response numerically, you can write this equation in terms of two first-order equation. So, you can write this as let first equation you can write X dot equal to Y, and second equation you can write that is equal to Y dot equal to. So, Y dot will be equal to X double dot. So, that will be equal to minus F by m sin omega t minus this k by m will be equal to omega n square.

So, this is plus minus omega n square X, so minus 2 zeta omega n X dot. So, this is Y dot, and this is X dot, or you can write if you take this X equal to Y 1 and X dot equal to Y 2. So, this equation you can write in this form that is Y 1 dot equal to that is Y 1 dot let me write X 1 dot equal to Y 1 dot. So, Y 1 dot equal to Y or Y 1 dot equal to that is X dot that is Y X dot equal to Y 2, and the second equation you can write this is Y dot that is Y 2 dot equal to.

So, this will be F by m sin omega t minus omega n square X minus 2 zeta omega n X dot. So, in this case I have used these two functions to find the response numerically by using this Range-Kutta method. Here in MATLAB you have the option of ode45 to find the response of the system. So, either you can use this ode45 to find the response what I have shown or you can use the response what you already know.

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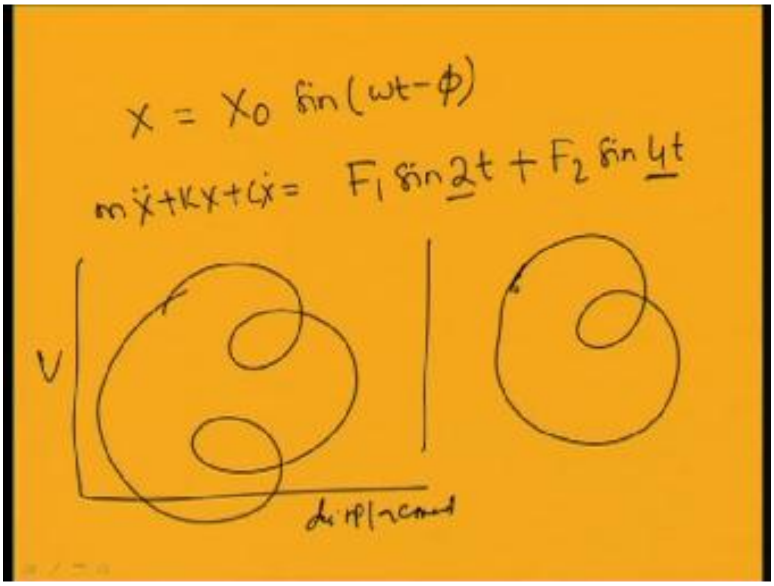

That is your X equal to X 0 sin omega t minus phi. So, let two forces are acting on this system; so one force equal to this is m X double dot plus k X plus c X dot. So, in this case this is equal to F sin one force is let this force I am giving equal to 2 t plus another force is acting that is equal to F 2 sin 4 t. So, in that case you can see that when this is 2 t and this is 4 t, already I have shown you the phase portrait and the response of the curve. So, in case of phase portrait you have clearly seen. So, there are two loops present in this case.

So, it clearly shows the response to be too periodic. Similarly, if you take three forces with frequency of integer multiple that is in this case two, four and five. So, in that case you can obtain three loops in this phase portrait. So, in case of phase portrait this will be your velocity versus displacement curve.

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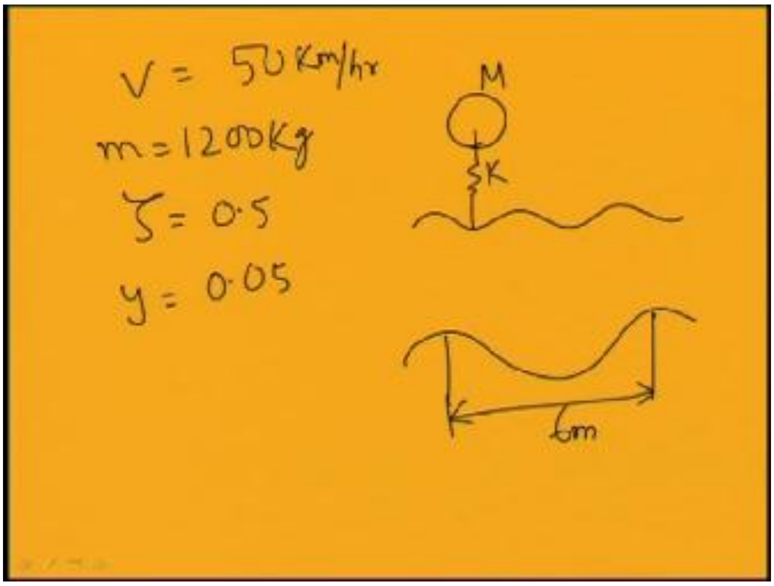

Similarly, you can use or let us take one more example that is a vehicle is moving on this wavy road. So, we can model this vehicle as a single spring and mass system. So, this is the mass of the vehicle, and let k is the stiffness of this vehicle, and it is moving on this wavy road. So, in this wavy road, I can have. So, this is the wavelength of this wavy road. Let the wavelength is 6 meter, and mass of the vehicle let it is equal to 1200 kg. And let me take the velocity of the vehicle equal to 50 kilometer per hour, and the damping factor let me take zeta equal to 0.5. So, if I will take this equal to 0.5. So, let me see. So, if you have an undulation of this y equal to 0.05. So, let us find what will be the displacement of this mass.

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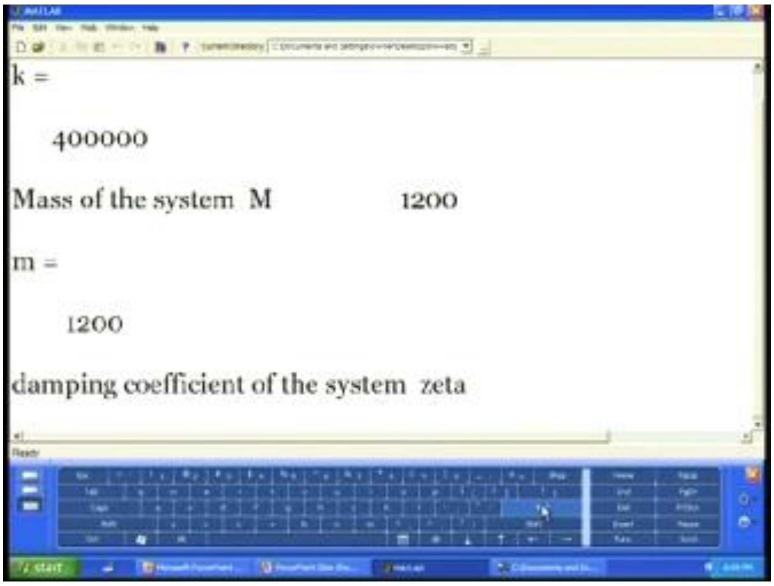

So, let us give the stiffness of this. So, let me give this stiffness to be 400 kilo Newton. So, let me give it 400 kilo Newton. This is the stiffness of the vehicle, then let us give the mass of the vehicle; the mass is 1200. So, when the mass is 1200, then the damping coefficient let us see the case for different damping cases zeta equal to 0.5. Now let us give this velocity of the vehicle equal to 50 kilometer per hour. So, you can see that when this velocity is 50 kilometer per hour for this root over k by m you can see this omega n equal to 18.2574.

Now let me give the amplitude of support motion. Let the amplitude of support motion is 0.05 meter. Then you can see let I am giving the wavelength of the road equal to 6 meter. So, you can find the transmissibility that is transmissibility equal to 1.4588, and then you can find the displacement of the vehicle equal to 0.0729. So, when you are taking a road with undulation 0.5 cm, then you can find that this displacement equal to 7 cm.

So, today class we have studied about this sharpness of resonance, and we have seen the complex stiffness of the system. And also we have studied about the vibration measuring instrument, and we have solved some problems which we have studied in this case of single degree of freedom system. The analysis of single degree of freedom system will be very much useful in the study of multi degrees of freedom system and continuous systems also. As you can reduce the multi degree of freedom and continuous system to that of a single degree of freedom system; so always you should start or you should understand the single degree of freedom system thoroughly, and you can then proceed for these two or multi degree of freedom systems.