Mechanical Vibrations
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Module - 4
Single DOF Forced Vibrations
Lecture - 4
Support Motion, Vibration Isolation

In the last three classes, we were studying about the harmonically excited systems. So, we are studying the single degree of freedom system.

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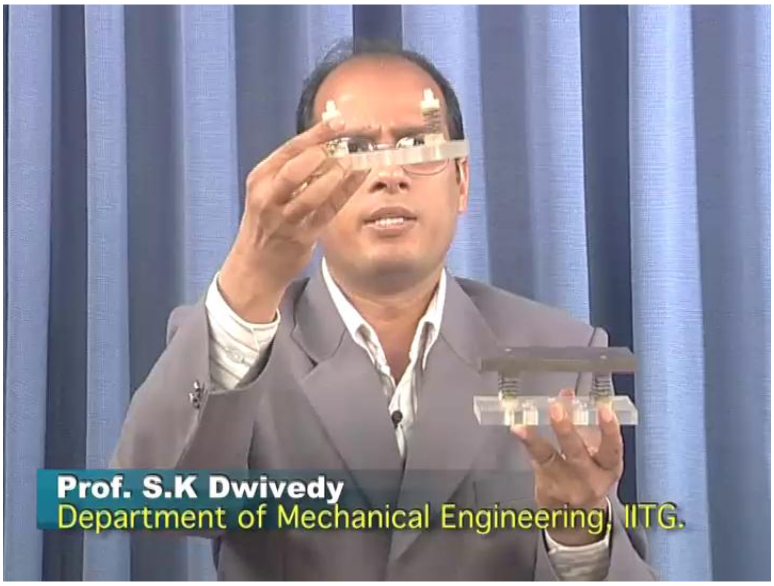

In the single degree of freedom system I have shown the example of a spring mass damper system. So, in this system you can see these are the springs, and this mass is supported on the spring. So, in case of free vibration, I am giving a force and leaving it. So, due to that this mass will vibrate, and in case of forced vibration you are continuously giving a force to the system.

So, when you are continuously giving a force with an amplitude f and frequency omega, then this mass will vibrate with the same frequency as that of the external excitation. And its amplitude will depends on the damping, also it will depends on stiffness, and it will depend on the mass of the system and the frequency and force in amplitude.

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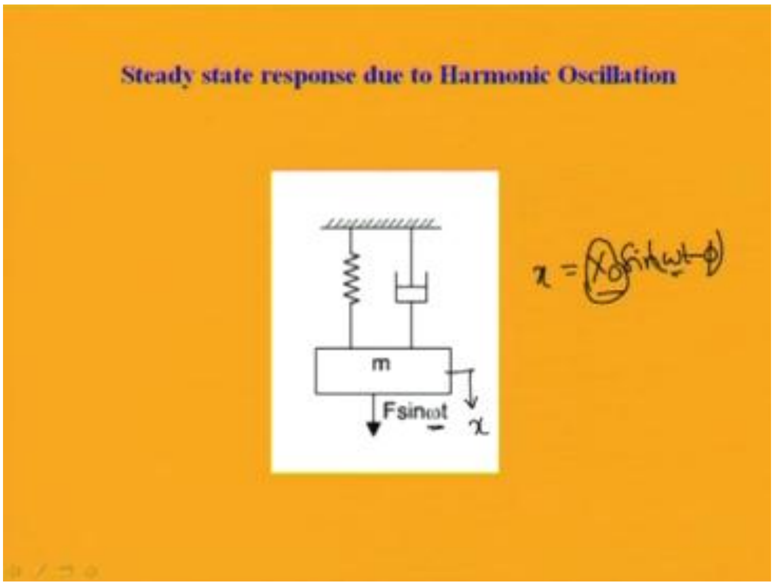

So, in this case already you have seen the response can be written like this X will be equal to X 0 sin omega t minus phi sin omega t minus phi. So, where this X that is the response of the system X equal to X 0 and this is the amplitude of the response, and this omega is the frequency of the response. So, this frequency equal to the frequency of the external forcing.

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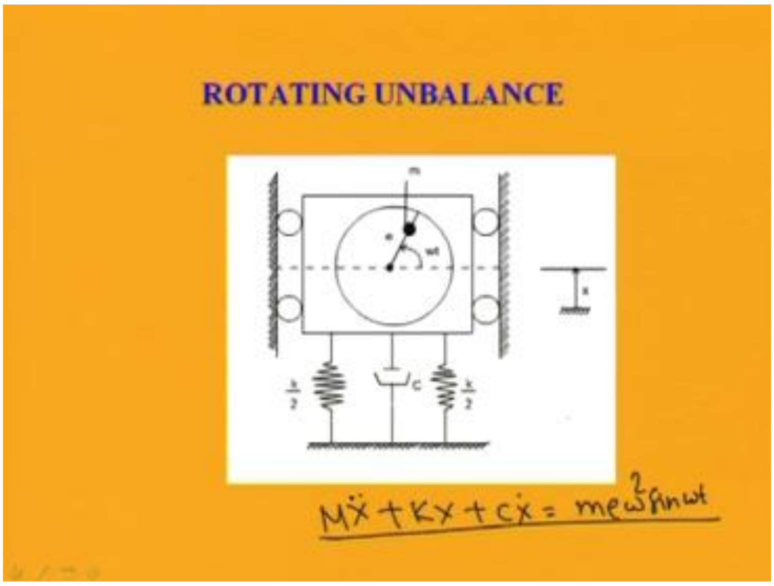

Also we have studied about this rotating unbalance. In case of rotating unbalance in the machine, there is an unbalanced mass placed at a distance e from the center. So, we have found. So, this system is equivalent to M X double dot plus or capital M this is the mass of the system. So, M X double dot plus K X plus C X dot equal to m e omega square sin omega t. So, this is the equation of the system. And last class we have studied about the response of the system, and also in the last class we have studied about the whirling of the shaft.

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So, this is the shaft. So, in this shaft when the line joining the bearing and the shaft axis are not coinciding; so this thing will happen when there is some unbalance mass present in the system. So, let you are attaching a mass at this position. So, due to this mass and in this mass let there is some unbalance or this mass center is not coinciding with the center of the shaft.

So, in that case it will be subjected to a centrifugal force, and that centrifugal force will try to bend the shaft. So, when the shaft will bend. So, the rotations of the plane containing this bend shaft axis or the line of the center that is known as whirling. So, the rotation of the plane containing the bearing center and the shaft center or the shaft axis is known as the whirling. So, in this case we have seen the equation for this whirling of shaft and we have derived the equation for this whirling of shaft.

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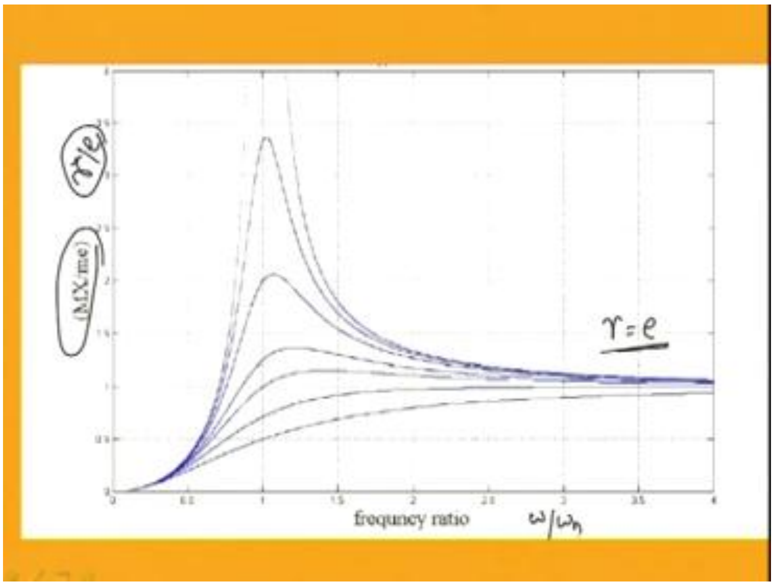

And we have found the response in this case. And in this case you can see that in case of the rotating unbalance, this M X by m e versus omega by omega n is same as r by e that is the r by e in case of whirling of shaft. So, in case of whirling of shaft, you have observed that when the machine starts. So, that is omega equal to zero. So, when the machine starts, then gradually it will goes on increasing this r that is the bend radius of the bend from the bearing line; the lines are in the bearing center and the shaft axis that is r and e is the eccentricity.

So, this r goes on increasing with increase in this frequency, and it will reach the maximum just after omega slightly greater than omega n. And for very high frequency you can observe that this amplitude r equal to e. So, r equal to e for omega very, very greater than omega n.

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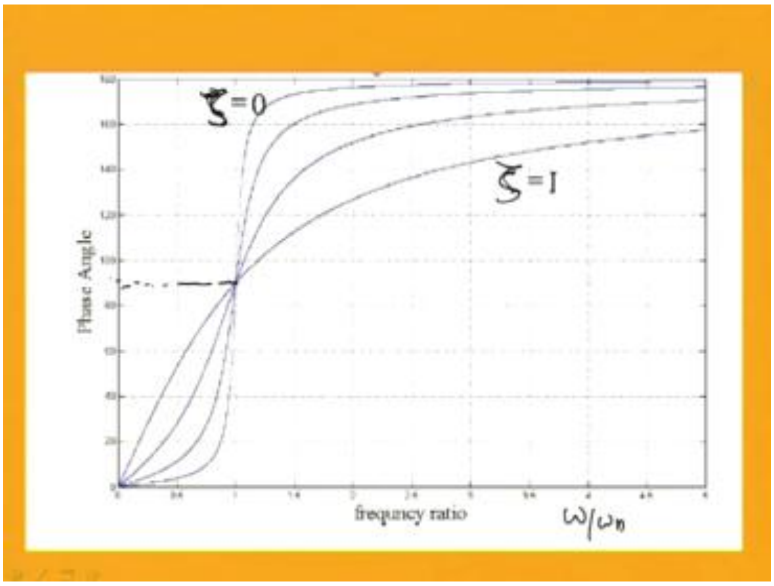

So, you have also seen about this phase plot. So, in this phase plot you can observe that when omega greater than omega n. So, this phase angle becomes 180 degree. So, the mass center of this rotor and the bearing center coincide in that case, and you can observe that omega equal to omega n. So, this phase angle equal to 90 degree.

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So, today class we are going to study about the support motion. Previously, you have studied about the system that is when the system is subjected to a force or the mass is subjected to a force that is F sin omega t. So, today class we are going to study about this motion of the support. So, when the support is getting some excitation, then what will happen to this mass? So, it is supported by some springs and dampers. So, in this case we have some springs attached here. And when this support is moving, then this force will be transmitted through the spring and damper through this mass, and ultimately, this mass will vibrate.

So, today class we are going to study on the support motion, vibration isolation and vibration measuring instruments also. So, this support motion will take place for example, in case of a vehicle moving on a road. So, that time the undulation of a road will be transmitted through the tyre through the body of the vehicle. So, that will cause the vibration of this vehicle. So, we are going to study about the support motion.

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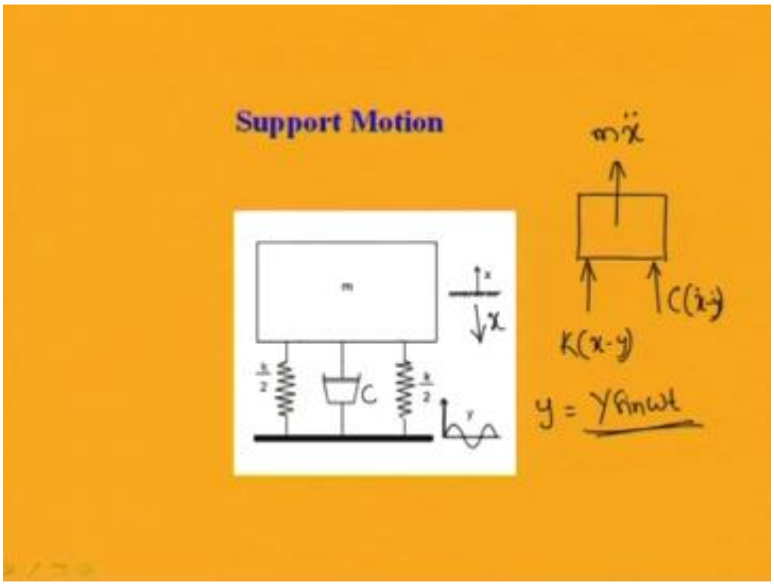

So, let this is the system. So, the system has a mass m. It is supported by the spring with spring constant K and a damper with damping C. So, let the support vibrate with y. So, assume this y equal to Y sin omega t. So, we just assume that the support vibrate in a harmonic way. So, when the support is vibrating in a harmonic manner, then we have to find what is this expression for this X.

So, when the support is moving. So, this force is transmitted or this displacement is transmitted to this mass. So, let the displacement of the mass is X and the support displacement is y. So, the spring and damper, the spring will have a displacement that is the relative displacement. So, this relative displacement will be X minus y, and the relating velocity associated with this damping will be X dot minus y dot.

So, if I will draw the free body diagram for this system. So, this is the mass. So, spring force will be K into x minus y, and similarly the damping force will be equal to C x dot minus y dot and this inertia force is upwards. So, this inertia force will be. So, inertia force when it is moving upward. So, I am considering when it is moving downward. So, I am considering when the mass is moving downwards or when the mass is moving downward, then it will be x minus y. So, this will be m x double dot.

So, mass is moving with x. So, acceleration will be x double dot. So, as acceleration is x double dot, then this inertia forces will be opposite to the direction of acceleration. So, this free body diagram is drawn when the mass is moving downward. So, this is m x double dot; this is the inertia force. The spring will have a motion K X minus y, because the spring will be compressed from the top side when it is moving downward. So, it will be compressed by an amount x.

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So, this is the spring and if I will compress the spring; so it will exert a force e in opposite direction. So, as the support is moving with y and we are giving a force from the top side that we are giving a displacement from the top side that is x. So, the relative displacement of the springs will be x minus y, and it will give this force in opposite direction to this. So, in this case it will become k x minus y, and similarly, for the damper it will be C X dot minus y dot.

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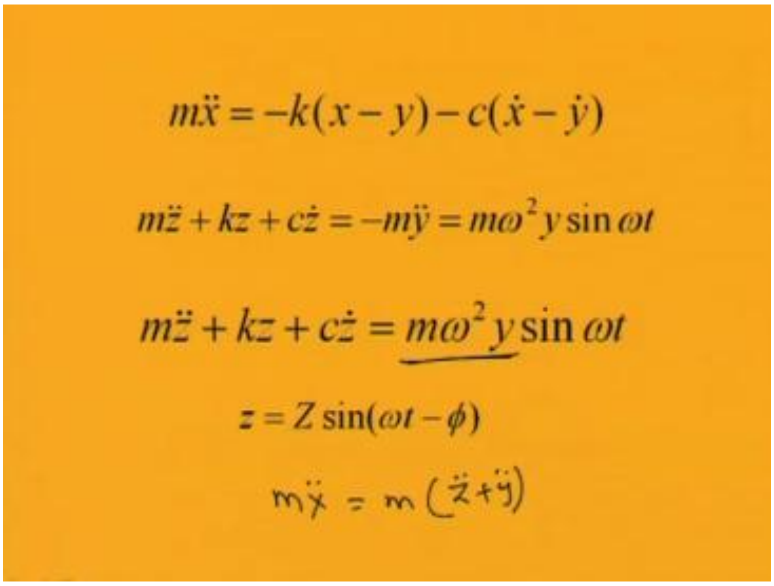

So, the equation motion you can write in this form. M x double dot will be equal to k x minus y minus c x dot minus y dot, or you can write this m x double dot plus k x minus y plus c x dot minus y dot equal to zero. So, now taking this x minus y equal to z, I can write this equation as m z double dot. So, when I am writing this x. So, x minus y equal to z. So, this x will be equal to z plus y.

So, this inertia force m X double dot I can write it equal to m into z double dot plus y double dot. Here I am taking this y equal to y sin omega t. So, y double dot will be equal to minus m y omega square sin omega t. So, I can take this term to the right hand side. So, this equation will become m z double dot plus k z plus c z dot equal to minus m y double dot; that is equal to m omega square y sin omega t.

So, this equation is similar to the equation we have previously studied that is m x double dot plus k x plus c x dot equal to F sin omega t. Here F is replaced by this m omega square y. So, the solution for this case will be equal to z equal to z sin omega t minus phi. So, here you can note that this z is the relative displacement between the response of the mass and the support. So, z is the relative displacement.

So, z will be the relative displacement of the mass of this mass and the support. So, you can write this relative displacement z equal to Z sin omega t minus phi, and we have to find this expression for z and for phi. So, you can draw the force polygon.

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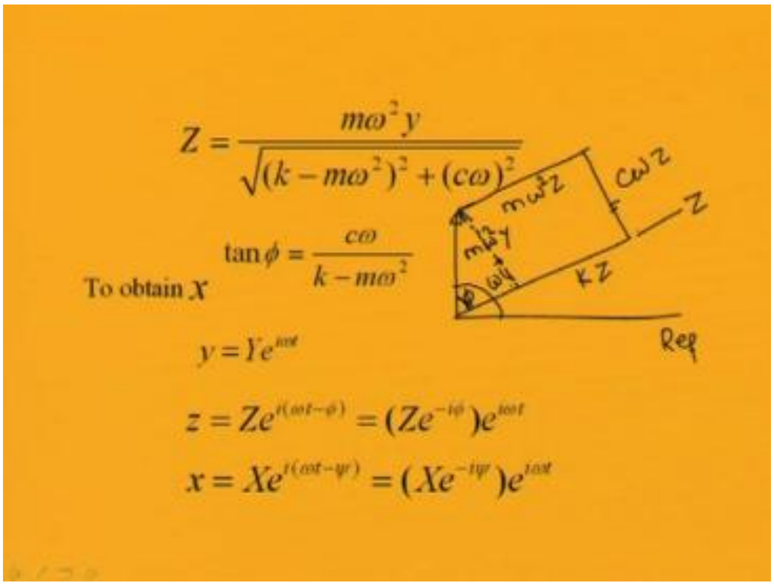

So, you know to draw the force polygon. So, I can draw the reference line. So, this is the k z. So, then this will be C omega z, and this will be the inertia force that is equal to minus m omega square z. And this is the closing side of this; that is the external force in this case that is equal to. So, in this case this is equal to m omega square y, and this angle is omega t, and this angle is phi. So, this inside angle is phi and this with reference line. So, this is the reference line, and this is the x direction. So, this is KX.

So, in this case this is the Z I am taking. So, this is Z. So, this is K Z, and this is C omega Z, and this is m omega square Z, and this side is m omega square y. So, now I can find from this right angled triangle. So, I can draw a line parallel to this. So, this line; so in this triangle I can find the expression for this Z. So, Z already you can see you have found similar expression for x. So, in this case you can find this Z will be equal to this force that is m omega square y root over k minus m omega square whole square plus c omega whole square, and this tan phi will be equal to c omega Z by K Z minus m omega square Z.

So, it will be c omega by K minus m omega square. So, to obtain x, we can proceed in different way. So, in this case let me proceed by taking the function as the exponential function. Let me assume that y in this form y e to the power I omega t. I can take this response as exponential function, and I will take the real part and imaginary part as you know that in case.

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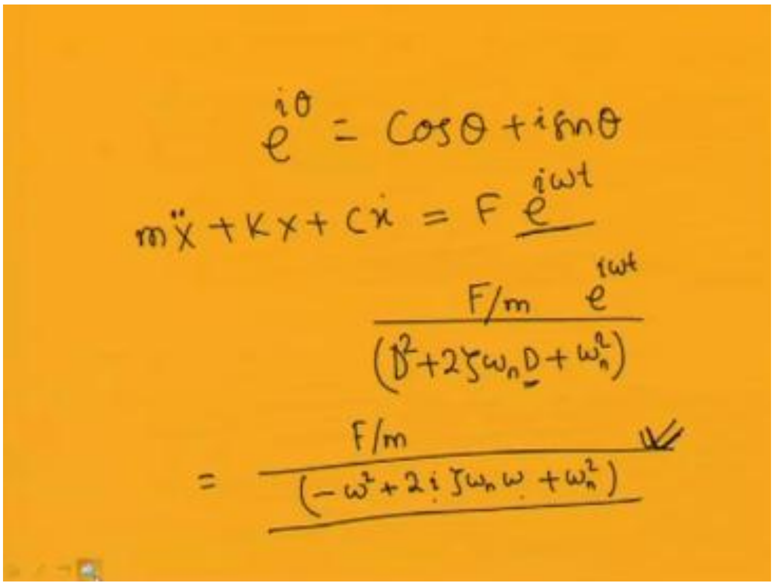

As you know this Y equal to or you know that exponential e to the power i theta I can write it as cos theta plus i sin theta. So, the response due to this, let me write we have this system m X double dot plus K X plus c x dot. So, it is subjected to a force F e to the power I omega t. So, already you know for the case when the force is equal to F sin omega t. So, in case of F sin omega t you have found the response and if you find the response for e e to the power I omega t. So, this will be. So, you just find the particular integral in this case. So, the particular integral in this case will be equal to…

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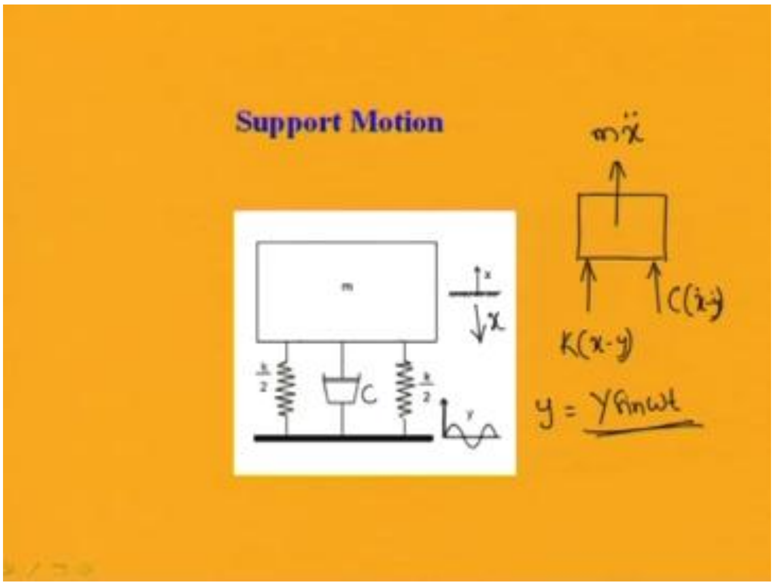

So, I can write this expression in this way. So, F by m I can write; I will divide this m. So, this becomes X double dot plus this is omega n square X plus 2 zeta omega n X dot. So, I can write this as D square; so for this d square plus 2 zeta omega n D plus omega n square. So, this is the particular integral part. So, it is e to the power i omega t. So, now I can substitute this i omega in place of D to find the particular integral. So, the particular integral in this case will be equal to F by m. So, for D if I will substitute this, then it will be minus omega square and plus this it will be 2 i zeta omega n omega.

So, for D I have to substitute i omega. So, I have substituted i omega for this and plus this is omega n square. So, this expression becomes F by m by omega n square minus omega square plus 2 i zeta omega n. So, it is a complex number, and this complex quantity you can separate it. So, you will have one real part and imaginary part. So, you can check that this real part is same as the force when you are applying F sin omega t. That will be equal to F by K root over 1 minus r square whole square plus 2 zeta r whole square.

So, where r equal to omega by omega n and the imaginary part also will be same. So, in this way you can find when you are applying a force e to the power i omega t. So, you can take the real part of that; that will give you the response if a sinusoidal force is acting, and the imaginary part will give the response when a cosine function or function of e is acting when the forcing is in terms of cosine function. So, you can take this imaginary part, and when it is in sin omega t terms, then you can take the real part of this. So, in this way, when you have one general forcing e to the power i omega t; so you can find the response to be in this form.

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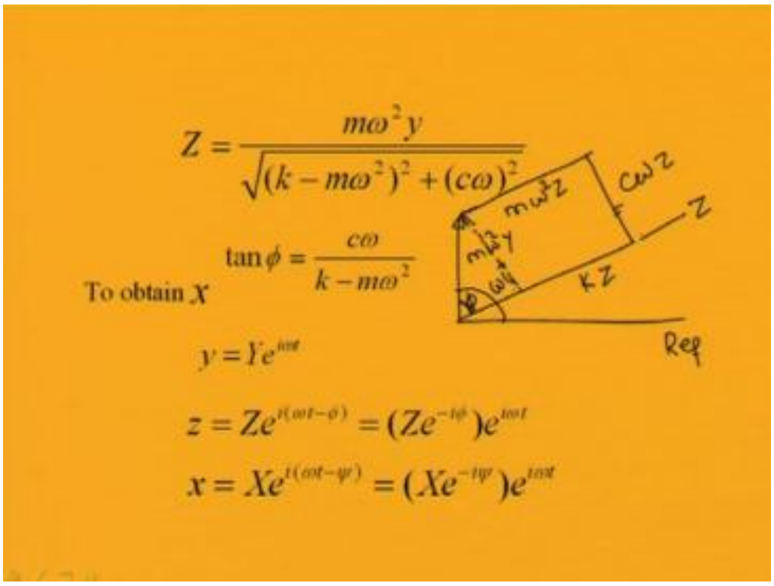

So, in this particular case let me take this y equal to y e to the power i omega t and Z equal to. So, in actual case the displacement I assumed. So, it is sin omega t.

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So, I have given a displacement of y equal to y sin omega t to the support, but for the time being I am considering a exponential displacement in this form y equal to e to the power i omega t. And I will find the response for sin omega t by taking the real part of that. So, if I am assuming a displacement y equal to y e to the power i omega t, then Z I can write equal to Z e to the power I omega t minus phi. So, it will take place after some time. So, I will take a phase of phi. So, this will be equal to z e to the power minus i phi into e to the power i omega t. Similarly, I can find x; so x equal to Z plus y. So, that will be equal to X e to the power i omega t minus psi. So, it will have another phase. So, this can be written as X e to the power minus i psi e to the power i omega t.

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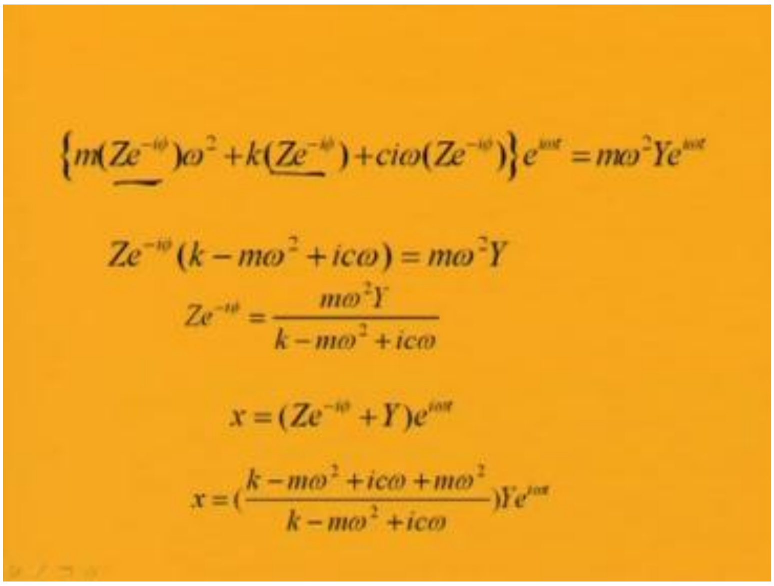

So, substituting this in this equation motion I can write the expression in this way. So, m Z e to the power minus i phi omega square plus k Z e to the power minus i phi plus c i omega Z e to the power minus i phi into e to the power i omega t will be equal to m omega square Y e to the power i omega t. So, I am writing this capital Y for this Y in this case. So, it will be Z e to the power. So, from this I can write Z e to the power minus i phi into Z e to the power minus i phi into k minus.

So, it contains e to the power minus i phi, and here also you have the same thing. So, you can write this K minus m omega square plus I c omega will be equal to m omega square Y or Z e to the power minus i phi will be equal to m omega square Y. So, I can divide this. So, I can write Z e to the power minus i phi equal to m omega square Y by k minus m omega square plus I c omega. So, x will be equal to Z e to the power. So, from this expression you have seen x equal to Y plus Z. So, I can add this two terms that is Z e to the power minus i phi e to the power i omega t plus Y e to the power i omega t.

So, by adding these two terms I can have this x. So, x is equal to Z e to the power minus i phi plus Y e to the power minus i omega t. So, x will be equal to k minus m omega square. So, this is the expression already you have written. So, it will be minus k omega square. So, from this I can write plus I c omega plus m omega square by k minus m omega square plus I c omega into Y e to the power i omega t. So, this x I can write it equal to X e to the power minus i omega t minus psi.

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$$= X(\cos\psi - i\sin\psi)e^{i\omega t}$$

$$\left|\frac{X}{Y}\right| = \sqrt{\frac{k^2+(c\omega)^2}{(k-m\omega^2)^2+(c\omega)^2}}$$

$$\tan\psi = \frac{mc\omega^3}{k(k-m\omega^2)+(c\omega)^2}$$

So, from that you can write. So, this is equal to x e to the power i psi; you can write cos psi minus i sin psi into e to the power i omega t. So, you can compare the real part and imaginary part in these two expressions. So, in the right hand side and the left hand side you can compare these parts and you can write this X by Y or you can find this X by Y equal to k square plus c omega whole square by k minus m omega whole square whole square plus c omega square and in this case this tan psi will be equal to. So, first you equate the imaginary part.

So, I can show you this thing. So, this is equal to. So, you can write this equal to k minus m omega square. So, this is k minus m omega square. So, this will cancel this m omega square m omega square. So, this is k minus I c omega by k minus m omega square plus I c omega into Y e to the power i omega t. So, this e to the power i omega t and this e to the power i omega t part will be cancelled, and from that you can get this X by Y term. So, this will be equal to this and this tan psi. So, as you are getting this cos psi and sin psi terms separately when you are equating the real part and the imaginary part. So, from that you can get the tan psi. So, tan will be equal to sin psi by cos psi. So, it will be m c q by k into k minus m omega square plus c omega whole square. So, in this way you can find this X by Y.

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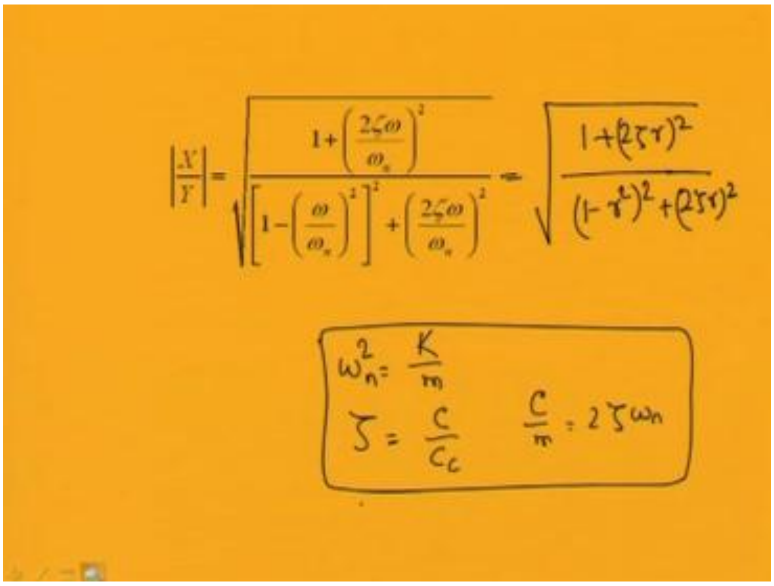

Now you can simplify this x by Y and you can write this. So, already you know this K by m can be written equal to omega n square, and this zeta it will be equal to C by C c. And you know this C by m you can write equal to 2 zeta omega n or you know that you can write this. So, using these expressions you can convert this X by this to this form. So, it will be equal to X by Y will be equal to 1 plus 2 zeta omega by omega n whole square root over 1 minus omega by omega n whole square.

So, this square plus 2 zeta omega by omega n whole square; so by taking this omega by omega n equal to r. So, you can write this expression equal to. So, this will be equal to 1 plus 2 zeta r whole square by 1 minus r square whole square plus. So, this is 2 zeta r whole square; so in this way you can find x by root over x by y.

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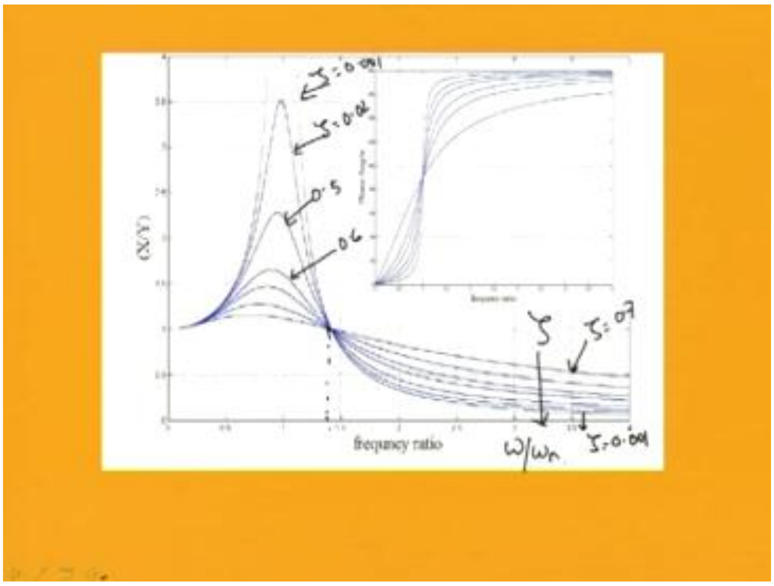

And if you plot this root over X by Y versus omega by omega n. So, let us see. So, in this case you can find that when omega by omega n equal to very small or omega is very very less than omega n, that is when the machine is starting. So, you can find this part equal to 0 and this is also equal to 0 and this is 0. So, this is 1 by 1. So, X by Y equal to 1.

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So, at that time when omega equal to 0, then you can see the support motion that is Y the amplitude of support motion will be equal to the amplitude of vibration of this mass, and when you are increasing this frequency you goes on increasing this frequency. So, the vibration of this mass will goes on increasing. So, the system will vibrate; the system vibration will goes on increasing with increase in this frequency and when it is near omega by omega n or when omega nearly equal to omega n. So, you can observe that this vibration is very huge.

So, in the absence of damping you can see this vibration tends to infinity. So, this is for a very low damping that is let damping equal to 0.001. So, in case of damping equal to 0, you can see that at omega by omega n. So, X will tend to infinity. So, by increasing this damping you can observe that. So, this is for zeta equal to 0.02, this is zeta equal to 0.5, this is 0.6. So, in this way you can find for different value of zeta of this curve. So, you can observe that at this point. So, irrespective of damping, this omega by omega n at this point, this X by Y equal to 1. So, irrespective of you can see that omega by omega n for this value of omega by omega n. So, this is equal to 1.

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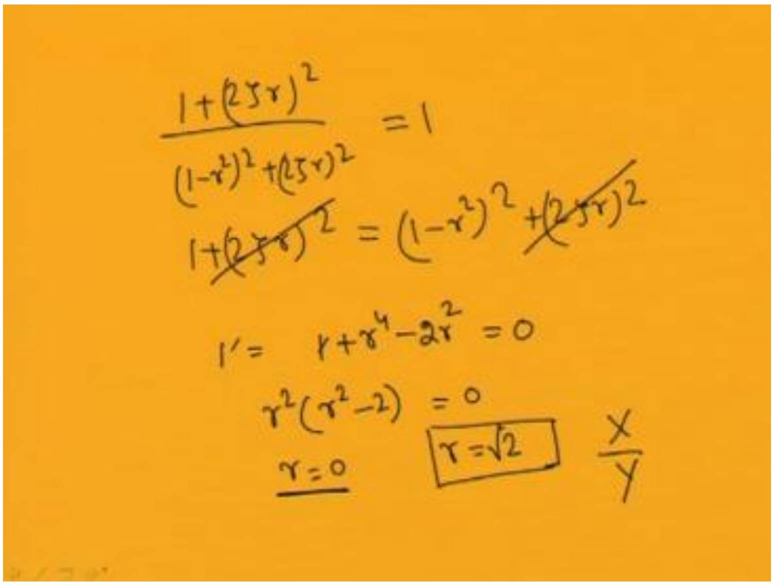

So, you can equate this expression to 1. So, you can equate this expression to 1; that is 1 by 1 plus 2 zeta r whole square. So, that is 1 plus 2 zeta r whole square by 1 minus r square whole square plus 2 zeta r whole square. So, this is root over. So, you can square it. So, this is equal to r. So, you equate this. So, you will find 1 plus 2 zeta r whole square will be equal to 1 minus r square whole square plus 2 zeta r whole square. So, in this way you can find this expression for the r.

So, you can see that in this case this side and this side get cancelled. So, this 1 will be equal to. So, this is the term 1 plus r fourth. So, you can find this r fourth minus 2 r square equal to 0. So, you can find this r square into. So, this one one will cancel, so r square. So, you can take r square minus 2 equal to 0 or either r equal to 0. So, you have X equal to Y or r square equal to 2 or r equal to root 2. So, when r equal to root 2. So, you can see that X equal to Y. So, irrespective of damping at r equal to root 2, this X equal to Y.

So, when the frequency of excitation of the base equal to root 2 times the natural frequency of the system, then the amplitude of displacement of the support will be the amplitude of displacement of this mass. So, you know this natural frequency that is omega n equal to root over k by m; k is the spring constant, and n is the mass of this system. So, when omega equal to root 2 times omega n. So, you can observe that this X equal to Y and also at starting that is omega equal to 0, and so this X equal to Y.

So, now, you can see in this case when you are increasing this frequency beyond this root 2 times omega n. So, you just observe that this with increase in damping. So, this is increase in damping. So, you can increase damping here. So, with increase in damping, this X by Y ratio decreases. So, this is for a higher value of damping; let it is for zeta equal to 0.7 and this is for zeta equal to. So, this upper line is for zeta equal to 0.7, and this is for this lower line zeta equal to 0.001.

So, you just observe that with increase in damping here, X is increasing. So, in this case X is increasing beyond this root 2. So, when omega is greater than root 2, omega is greater than root 2 times the natural frequency of the system. So, with increase in damping you just observe that a x by is increasing. So, a x by y is increasing with increase in zeta that mean this damping. So, that means this X is greater than Y. So, in this case after root 2; so you can observe that this damping it deteriorates the response of the system.

So, this damping is not beneficial to the system when it is operating at a frequency root 2 times greater than the natural frequency of the system. So, that thing also we will see in case of vibration isolation of the system. So, in this case you can observe that when the vehicle is moving. So, the amplitude of response increases with increase in frequency. Also you can observe from this phase diagram when omega equal to omega n, this phase equal to 90 degree. And when it is very, very higher than this omega n, this phase angle will become 180 degree.

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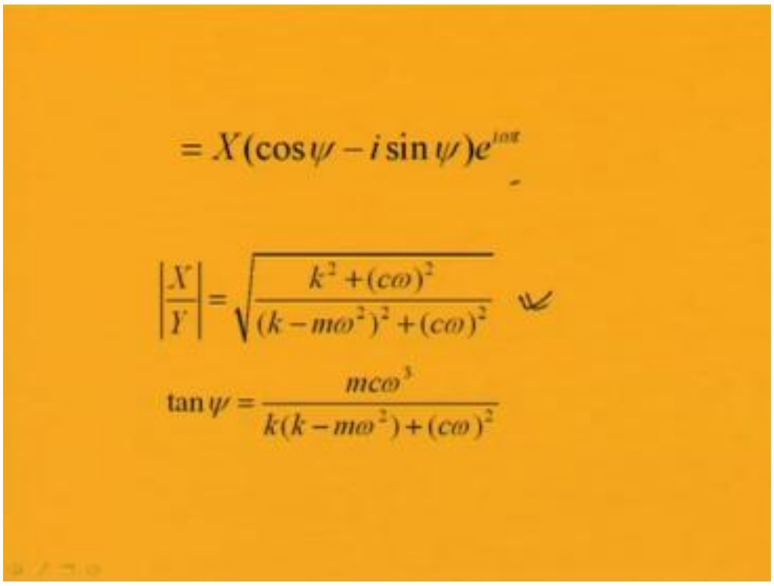

So, you can find from this expression X by Y, and from this expression you just note that with increase in frequency this amplitude increases, and after omega greater than omega n it decreases and at root 2 times omega n. So, X equal to Y and after that this decreases, but you can observe that with increase in this damping this X by Y or X is increasing. So, the damping is not beneficial in this; damping is not beneficial when the system is operating at a frequency greater than root 2 times the natural frequency of the system.

So, let us see the vibration isolation. Now we have studied that when the support is moving. So, the mass or the system will be getting vibration. So, when the support is moving, this mass is vibrating; also you have seen another type of thing when the mass is vibrating. So, this mass is vibrating, it will transmit some force to this ground or to the support. So, in both the cases either when the support is transmitting some force or motion to this mass or the mass is transmitting some motion to the support. So, we have to isolate the vibration.

So, for example, in case of a machine shaft; so due to the motions of this machine, the motion will be transmitted to the ground in turn. So, it will transmit that motion to the other machine. So, we should isolate this vibration of the machine to the ground. Similarly, in case of ground motion or support motion, we should avoid the motion of the system. So, in both the cases we have to isolate the vibration and for that we should study the vibration isolation.

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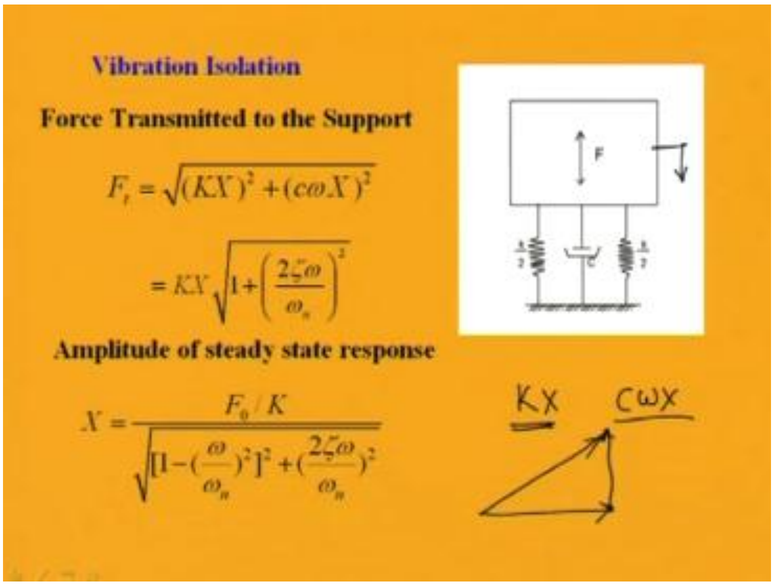

So, before studying the vibration isolation, we should know how much force is transmitted to the ground when the mass is subjected to a force F sin omega t.

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So, this is the mass; let it is subjected to F sin omega t. So, this force will be transmitted to the ground through the spring and damper. So, the force will be equal to. So, when it is vibrating with X. So, if the mass is vibrating with X, then the force transmitted to the ground will be equal to due to the spring it will be equal to K X, and due to the damper it will be equal to C omega X. And the force due to the spring will be equal to K X sin omega t, and due to this damper it will be at a quadrature that is C X dot. So, it will be C omega cos omega t.

So, this is the force due to damper C omega X cos omega T, and force due to the spring will be equal to K X sin omega t. So, the resultant will be C K X square. So, this is the spring force. So, this is the damping force. So, the spring force and damping force. So, the resultant will be this. So, this is the resultant, and it will be K X whole square plus C omega X whole square root over. So, this will be equal to K X root over 1 plus 2 zeta omega by omega n whole square.

So, we know this amplitude of steady state response when it is vibrating with or when this subjected to a force equal to F sin omega t. So, this expression for X equal to that is equal to F 0 by K root over 1 minus omega by omega n whole square whole square plus 2 zeta omega by omega n whole square. So, putting this expression in this, I can write this force transmitted to the support will be equal to K into F 0 by K. So, this K K will cancel. So, it will become F 0 into root over 1 minus 2 zeta omega n by omega n whole square by 1 minus 1 omega by omega n whole square whole square plus 2 zeta omega by omega n square.

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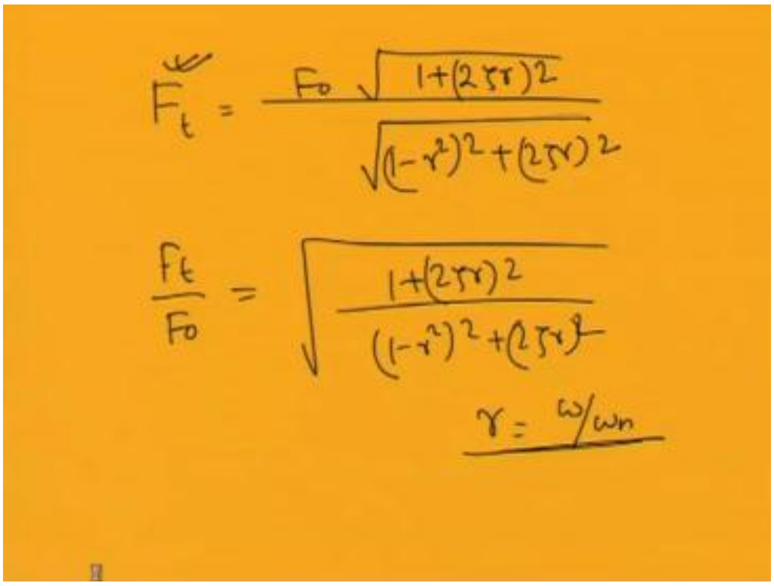

So, this way I can write F t will be equal to F zero. So, this is F 0 root over 1 plus 2 zeta omega by omega n or 2 zeta r I can write. So, 2 zeta r whole square by root over 1 minus r square whole square plus 2 zeta r whole square, or I can write this F t by F 0 that is force transmitted to the support by the amplitude of the force. So, F t is the magnitude of the force transmitted to the support and F 0 is the force applied here, amplitude of the force applied to the system. So, this F t by F 0 will be equal to I can write this equal to root over 1 plus 2 zeta r whole square plus 1 minus r square whole square plus 2 zeta r whole square. So, here r equal to omega by omega n.

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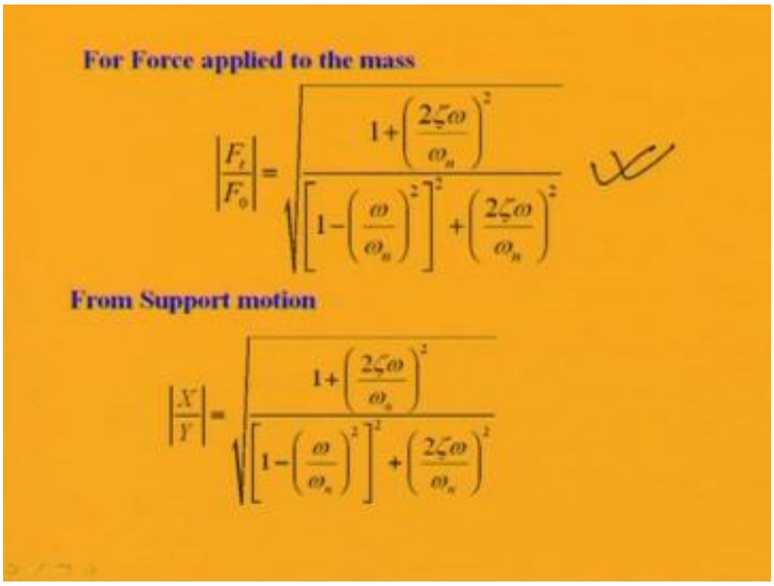

So, you can see this expression is written here F t by F 0 equal to 1 plus 2 zeta omega by omega n whole square minus 1 by omega by omega n whole square whole square plus 2 zeta omega by omega n whole square. So, this expression already you have seen in case of the support motion. So, this F t by F 0 will be equal to X by Y that is the. So, when support is moving with Y sin omega t. So, you have seen this amplitude of this motion by amplitude of this motion X by Y equal to the same expression as that we obtained in this case. So, this is equal to the force transmitted to the support by the force applied here as this is equal. So, the same curve you can plot for this case, and this F t by F 0 ratio is known as the transmissibility of the system.

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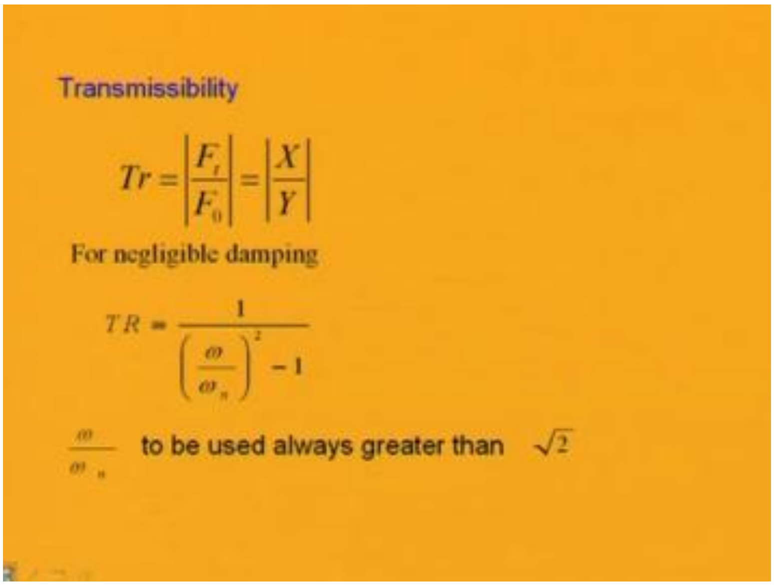

So, this Ft by F 0 equal to X by Y they are same. So, if you neglect this damping, then this transmissibility you can write it equal to 1 by. So, if you neglect damping then it will be equal to 1 by 1 minus omega by omega n whole square. So, this root over will go. So, it will be 1 by omega by omega n whole square minus 1. So, this omega by omega n used. So, you have seen in this case.

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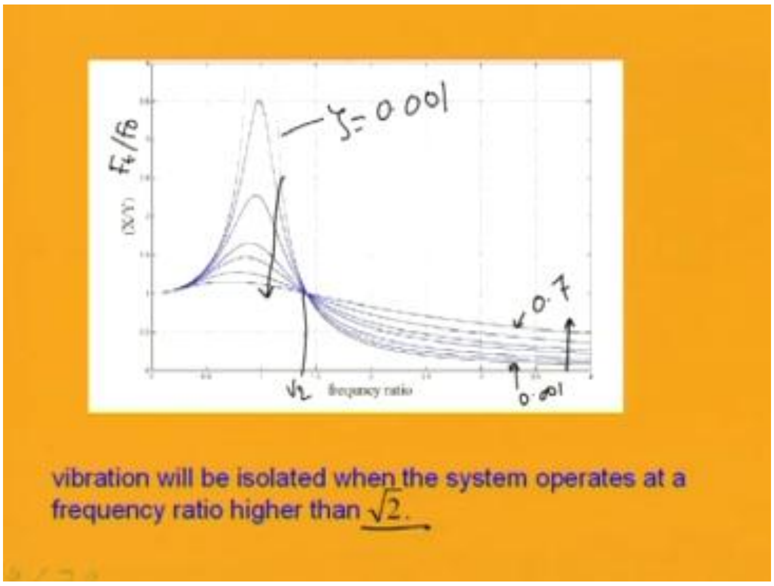

So, as both the system has both X by Y and this F t by F 0 are same. So, you can plot the same curve. So, you can observe that this is root 2. So, to isolate vibration you should operate the system at a frequency greater than root 2 times the frequency of the system. So, you have seen when it is greater than root 2, then this F t force transmitted to the ground is less than the applied force of the system. So, to isolate the vibration you should operate the system at a frequency root 2 times the natural frequency of the system

Also you observed that with increase in damping. So, this is with increase in damping you can see. So, this is for higher damping that is zeta equal to 0.7, and this is for lower damping zeta equal to 0.001 let me tell. So, this is zeta equal to 0.001. So, you can see that at lower damping. So, you can see that at. So, you can observe that this curve is for lower damping, and the other curve is for higher damping. So, if you put damping here. So, this will not be beneficial to the system.

So, with increase in damping here the response is increasing; the response is increasing with increase in damping. So, this side is for higher damping. So, that is for 0.7, and this is for 0.001. So, with increase in damping. So, you just see with increase in damping this amplitude is increasing. So, here in this side you just observe that with increase in damping, the response amplitude is decreasing. So, for omega less than root 2 you may use a damper, but when it is greater than root 2, you need not have to use a damper; that time you may use a stop to limit the vibration of the system.

So, in this way, you can isolate the vibration of the system. So, the principle of vibration isolation is to operate the system at a frequency more than root 2 times the natural frequency of the system. So, here we have seen this transmissibility equal to 1 by omega by omega n whole square minus 1. So, when we are taking very low damping. So, this is the case.

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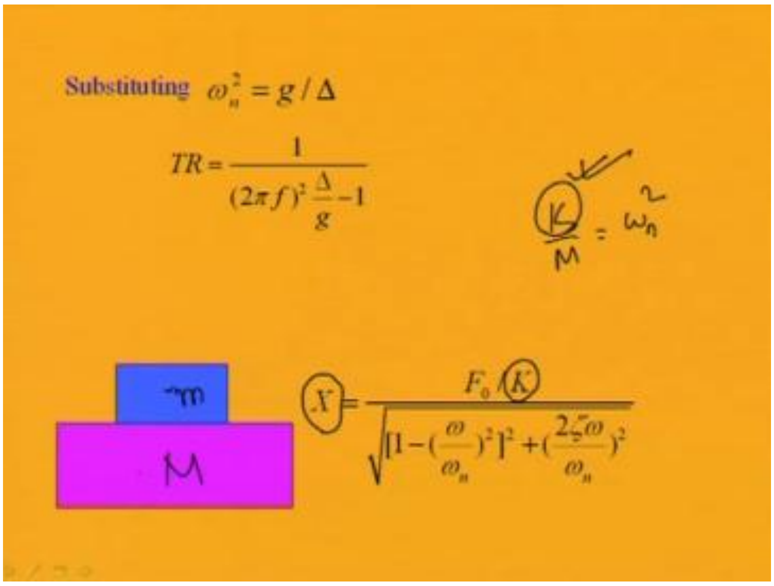

So, in most of the case you are finding this omega n square equal to g by delta; g is gravity, and delta is the deflection of this. So, this transmissibility you can write it equal to 1 by 2 by F square delta by g minus 1; this is omega by omega. So, this is equal to omega by omega n square minus 1. So, that way you can write this is equal to this is omega square; this is delta by g, and this is minus one. In most of the times to isolate the vibration, these small masses are generally kept on the higher mass. So, if you want to isolate the vibration for this.

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So, you can put the system or you can put this mass on a higher mass. So, when you are putting this mass, you want to isolate the vibration of this mass; you can put it on a larger mass. So, when you are putting this larger mass in that case. So, that omega n will change. So, to keep the omega n constant you have to increase k. So, as K by M equal to omega n square. So, as you are increasing this mass. So, when you are putting this mass on this small mass you want to isolate. So, you can put it on a bigger mass.

So, when you are keeping this on a bigger mass to keep this frequency same you should increase K. So, when you are increasing K you can note that this X amplitude of excitation equal to F 0 by K root over this. So, as you are dividing this K, this is in the denominator, then this X will reduce. So, in this way you can reduce the amplitude of vibration by putting the isolated mass on a bigger mass.

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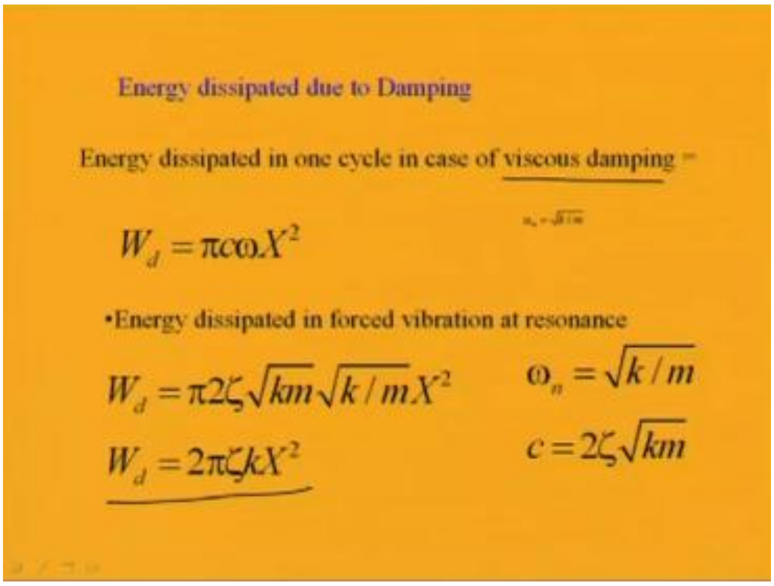

So, now we will study about the energy dissipation due to damping. So, already you know. So, when damping is present in the system. So, we have studied the case of the viscous damping. So, in case of viscous damping, the force we have taken equal to C X dot. In case of viscous damping, the force is proportional to the velocity, and I can show you that this force or the energy dissipated in one cycle it is equal to pi c omega into X square.

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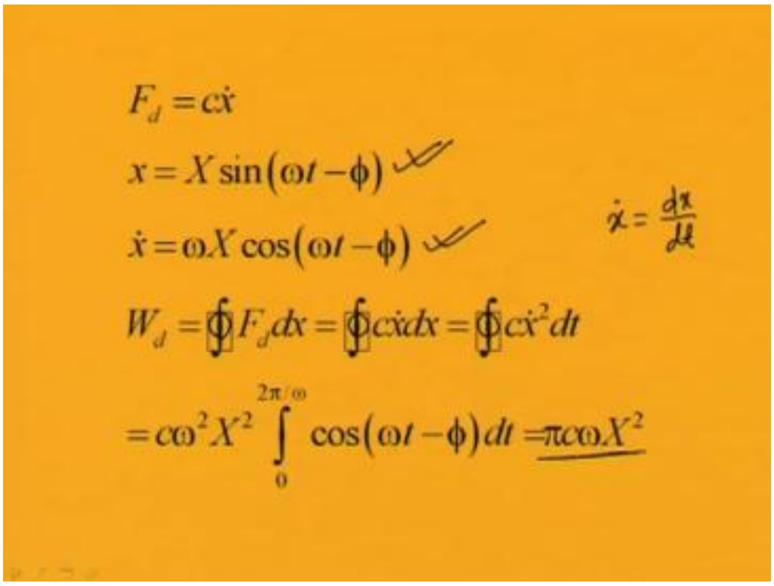

So, you can see in this way. So, you can write this F d equal to c x dot as we are taking this x equal to X sin omega t minus phi. So, this is the response of the system. So, then X dot will be equal to omega X cos omega t minus phi. So, the omega d that is in a cycle the energy loss in a cycle will be equal to F d d x. So, this will be equal to c x dot d x and this d x I can write as you know this x dot equal to d x by d t. So, I can write this d x equal to x dot d t. So, x dot multiplied by d dot will become c x dot square d t, and this time; time I can integrate it from 0 to 2 pi by omega.

So, this omega d that is energy lost per cycle will be equal to c omega square X square integration 0 to 2 pi by omega cos omega t minus pi d t. So, if you integrate this thing, this is coming to I c omega X square. So, energy loss per cycle equal to pi c omega X square and as there is maximum vibration at the resonance. So, we can study what is the energy dissipated at the resonance. So, energy dissipated at the resonance you can find by substituting this omega equal to omega n that is root over k by m and this damping equal to 2 zeta root over k by m.

So, omega equal to root over k by m and this c equal to 2 zeta root over k by m. If I am substituting this equation this becomes. So, pi into for c 2 zeta root over k by m and this omega equal to omega n equal to root over k by m and this is x square; so this omega d equal to 2 pi zeta k X square. So, this is the energy dissipated to a cycle.

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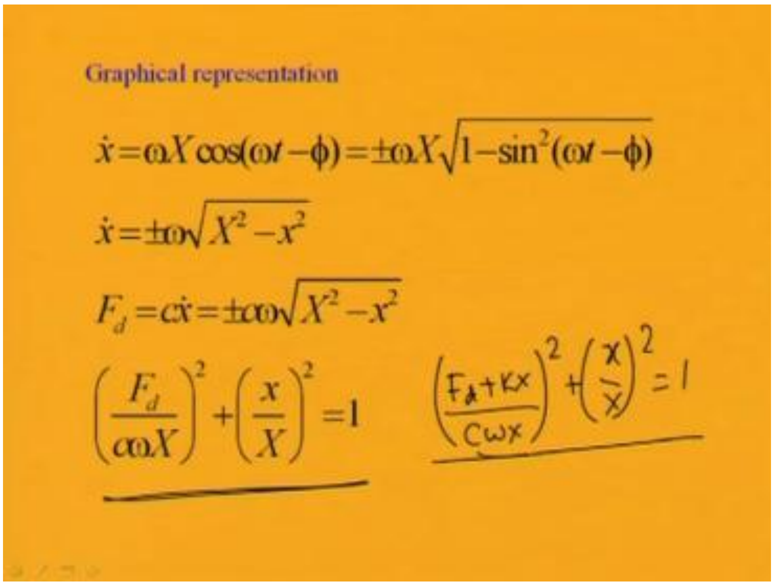

So, this energy dissipated in a cycle if I want to represent this graphically. So, I can write this X dot equal to omega X cos omega t minus phi as I have taken X equal to X sin omega t minus phi. So, differentiating that thing, I can write this X dot equal to omega X cos omega t minus phi. So, this will be equal to plus minus omega X root over 1 minus sin square omega t minus phi. So, I can write this equal to. So, this x dot can be written as plus minus this omega. So, I will put it inside. So, it will be capital X square minus capital X sin omega t minus phi equal to small x that is the displacement.

So, it can be written as x dot will be equal to plus minus omega root over X square minus small x square. So, if you arrange this thing. So, this thing can be written in this form. So, it will be equal to. So, F d if you write, that is the force in damping. So, that is c x dot. So, this thing will be written as c omega root over X square minus x square. So, rearranging that thing you square both the side. So, it will be F d square by c omega X square plus x by X whole square equal to 1. So, this is the equation for an ellipse.

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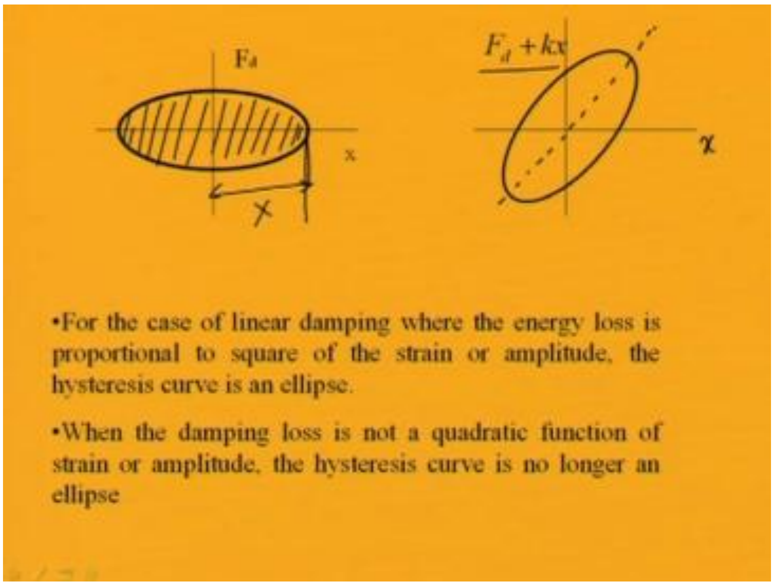

So, in case of this you can find or you can draw this thing. So, if you plot this x versus F d. So, then this will be the x that is capital X, and this part will be equal to c omega X, and area enclosed by this curve is the energy dissipated due to damping. So, this is the energy; this represents this loop or hysteresis loop represent the area; this represents the energy loss due to damping. So, this is due to damping. So, if you add the energy loss due to spring also, the spring force I can add it. So, if you will add it the spring force.

So, the total force can be written as F d plus K x by C omega X whole square plus x by X whole square will be equal to 1. So, in this case this ellipse will be rotated by this. So, the ellipse is rotated here, and this is similar to the Voigt model you have already studied. So, this is X; this is F d versus F d plus K X. So, the ellipse is rotated by an angle. So, in this case you now know that for the case of a linear damping where the energy loss is proportional to the square of the strain or amplitude, the hysteresis curve is an ellipse.

So, you have seen here the energy loss is proportional to square of the amplitude of response. So, in this case you can see that the energy loss per cycle or the hysteresis curve is an ellipse, but when the damping loss is not quadratic function of strain or amplitude, the hysteresis curve will no longer be ellipse. So, let us see some different types of damping.

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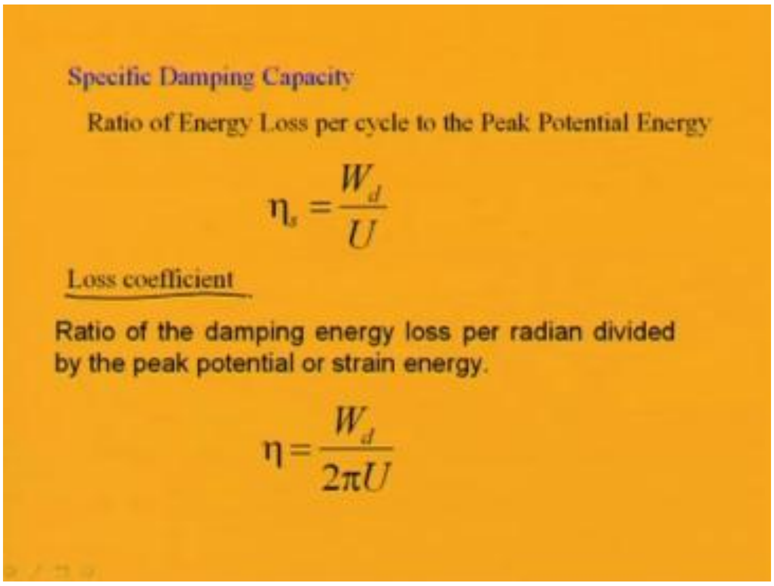

Also before that we should define two more terms that is specific damping capacity. So, it is defined as the ratio of energy loss per cycle to the peak potential energy that is eta s will be equal to omega d by U. Similarly, I can define another term that is loss coefficient. So, this is the ratio of the damping energy loss per radian divided by the peak potential or strain energy. So, eta will be equal to omega d by 2 pi U.

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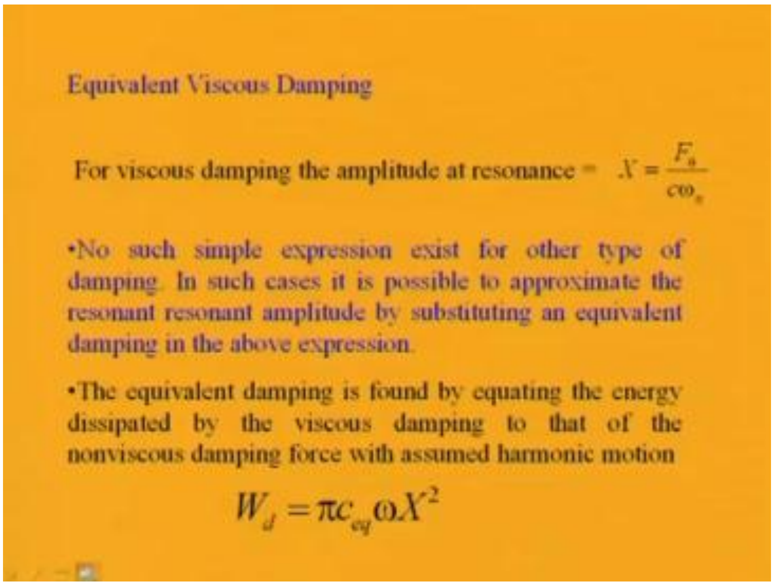

So, now let us find the equivalent viscous damping. In many cases, the system will not be subjected to this viscous damping. So, in the joints of the structure you will have structural damping and sometimes you may be having two surfaces in contact.

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Let this is one surface and this is the other surface. If the two surface are in contact and then due to this intermolecular interaction between these two surface, there will be dry frication. So, that dry friction is known as coulomb friction. So, we may model that coulomb friction also. So, we know that in case of the viscous damping, the amplitude at resonance equal to x equal to F 0 by c omega n. So, we got a very simple expression in this case, but other cases like this coulomb damping or structural damping this expression will not be that simple.

So, in those cases we can find an equivalent viscous damping. So, that equivalent viscous damping can be found by equating the energy loss in the system without viscous damping and the system with viscous damping, and we can find this omega W d equal to pi c equivalent omega X square; so in this way we can determine the c equivalent of the system.

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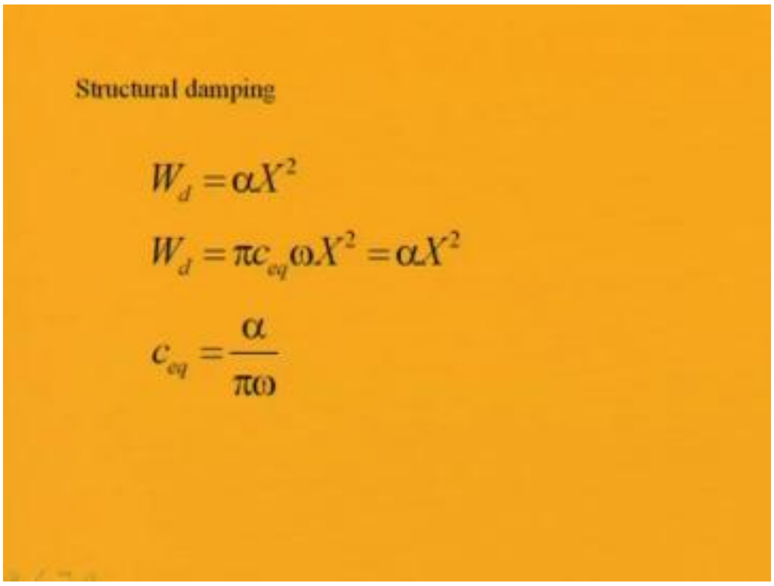

So, in case of structural damping, many materials like aluminum steel it has been observed that this energy loss per cycle is not a function of frequency. So, in this case it is proportional to the square of the amplitude of the response. So, in that case it can be written as W d equal to alpha X square or if I want to find the equivalent damping, I can write this W d equal to pi c equivalent omega X square. So, that will be equal to alpha X square, then the c equivalent will be equal to alpha by pi omega.

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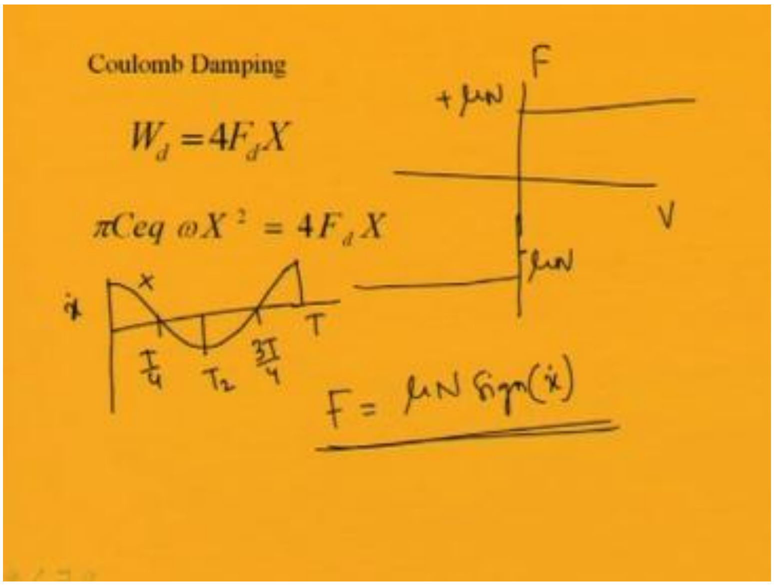

Similarly, in case of Coulomb damping, you know Coulomb damping can be written in this way. So, if I plot velocity versus Coulomb force. So, you know this is equal to this way I can represent. So, this is mu N; this is mu N minus mu N and this is plus mu N. So, for velocity less than 0, this friction force F will be equal to minus mu N, and for velocity greater than 0 this friction force can be written F equal to mu N. So, in this case you know this friction force F will be will be equal to mu N sign signum x dot, the Coulomb force is written in this way.

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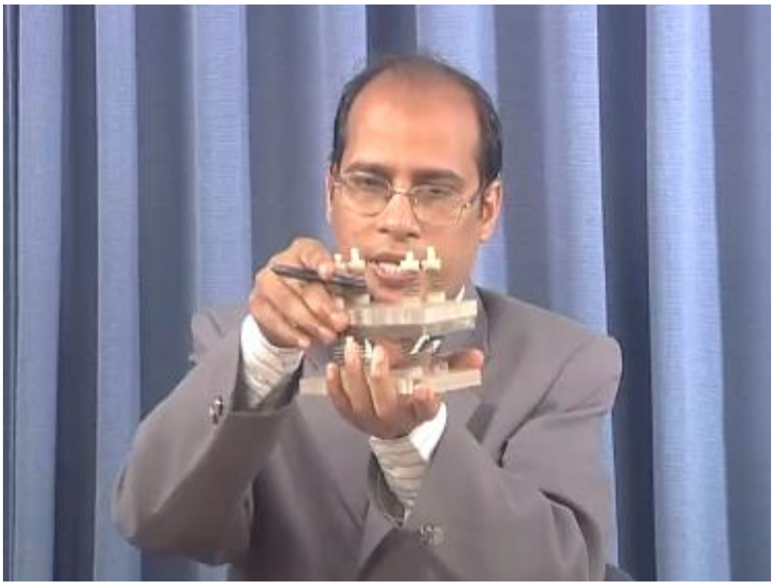

So, when there is surface to surface contact that time you can write this force. So, it will depend on the velocity and this velocity sign of the velocity curve, magnitude will be same. Magnitude will be the normal force normal force multiplied by this coefficient of friction that is mu, and normal force you can determine from the force acting on the system. So, this F will be equal to mu N sin x dot. So, in a cycle you can find the energy loss. So, if X equal to x sin omega t, then x dot will be equal to cos omega t. So, if you plot this cos curve then in a cycle you can plot it like this.

So, in the cycle you can divide it into four parts. So, this is your time and this is this x dot. So, you just see up to this rotation up to this pi by 4 rotation. So, this is pi by 2 cos pi by 2 equal to 0. So, if the rotation is up to this or then the sign if you are taking is positive, then this to this it is negative, this to this also it is negative. So, if you are writing in terms of time. So, this is one cycle. So, this is cycle time T. So, this is 4 cycle time T. So, this will become T by 2 and this is T by 4. So, this is T by 4; this is T by 2. So, this is 3 t by 4, and this is T. So, up to T by 4 you just observe that. So, up to T by 4 the sign is positive. So, if I will draw the force versus this time.

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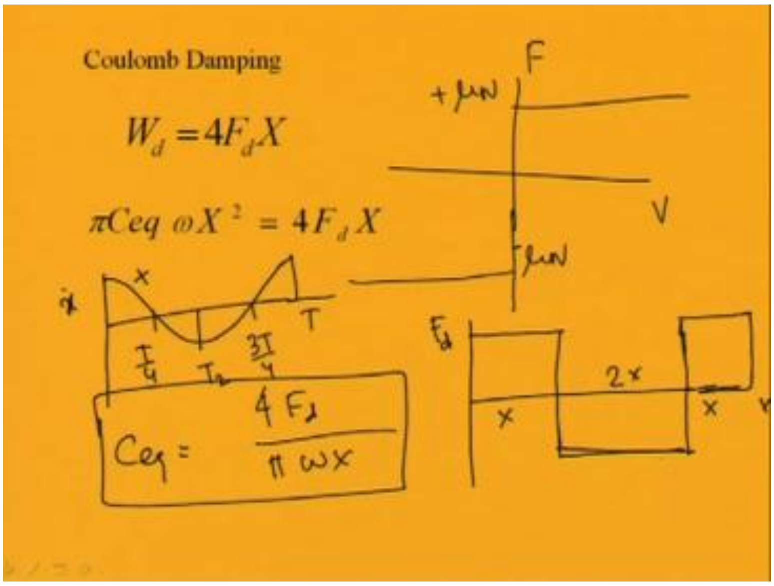

So, you can have it like this. So, this force versus time if you plot you can plot the curve like this. So, up to this it is positive. So, I can have the force equal to mu N. So, this is the force mu N. Then from here to here, up to this the sign is changing. So, this is negative sign. So, up to T by 2, this is the force mu N, and after this also you have a force that is mu N. So, this is the total force acting on the system. So, you can plot this F versus X. So, the area is the energy loss due to damping.

So, this area will be equal to. So, this is mu N. So, this is X; this is 2 X, and this is X. So, it will be 4 F d into X. So, if I am writing this as F d. So, this is this area will be 4 F d into X. So, C equivalent will be equal to 4 F d X by pi omega x square. So, in this case, you can find it equal to 4 F d by pi omega X. So, this is the equivalent damping in case of a coulomb damping.

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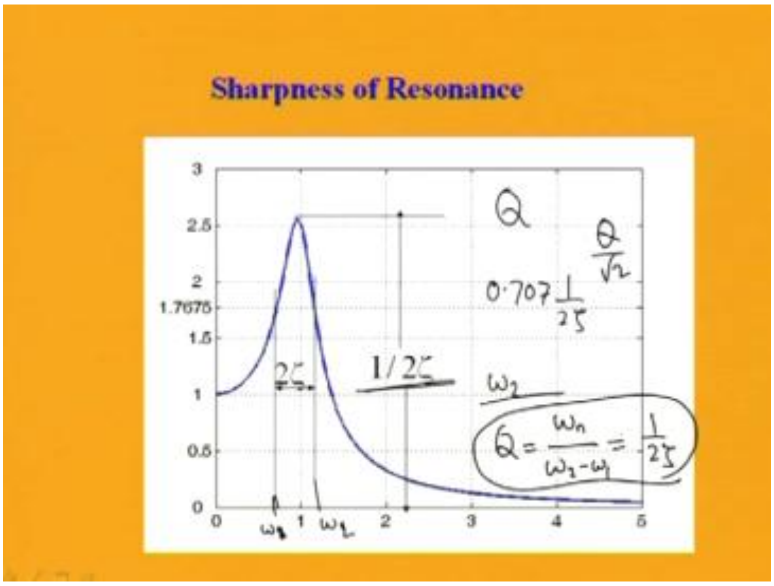

So, now let us study another thing that is sharpness of resonance. So, in this case, already you have seen when this is subjected to a force F sin omega t, when the system is subjected to the force F sin omega t. So, the maximum response will occur or the maximum response has the expression 1 by 2 zeta. So, the maximum amplitude equal to 1 by 2 zeta and if you take two points just near to this one; so left side and right side at an amplitude of 0.707 times this maximum amplitude that is 1 by 2 zeta.

So, you can find this value of omega 1; let this is omega 1, this is omega 2, or this is omega 1; this is omega 2. So, you can show that this omega 2 by omega 2 minus omega 1. So, this omega 2 by omega 1 will be equal to 2 zeta. So, you can derive this thing this is equal to 2 zeta. So, this factor that is 1 by 2 zeta is known as the quality factor of the system Q equal to 1 by 2 zeta is the quality factor of the system. So, you should not operate the system above the 0.17 that is 707 of this Q or you should not operate the system above this Q by root 2 or you should not operate the system in this range that is omega 1 to omega 2 in which the response amplitude is very high.

So, in that case when the response amplitude is very high; so you should not operate that system and in that case you can find that the expression for this that is omega n. So, this Q you can write it equal to omega 2 or omega n by omega 2 minus omega 1. So, it will be equal to 1 by 2 zeta. So, this is known as the sharpness of resonance. Today class we have studied about the support motion vibration isolation.