Mechanical Vibrations
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Module - 4
Single DOF Forced Vibrations
Lecture - 3
Rotor Unbalance and Whirling of Shaft, Transmissibility

In the last two classes, we have studied about the steady state response of harmonically excited single degree of freedom system. So, in a single degree of freedom system, it can be shown as a spring mass damper system.

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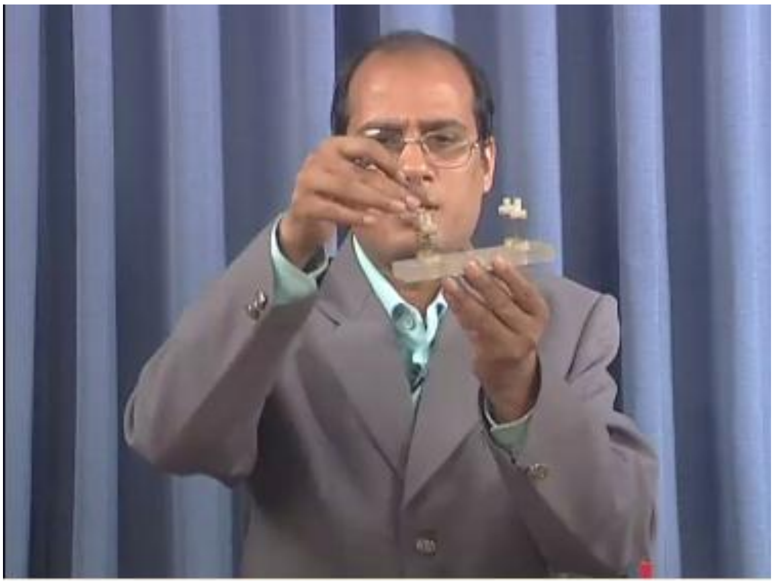

So, you can consider this. These are the springs. So, these springs and you can consider this as a mass. So, if the spring mass system is excited. So, let the system. So, let me consider this is the system.

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And the system is excited by a force. So, if we are applying a force F sin omega t on this system, then there will be lot of vibration or there will be vibration in this system. So, this system is a single degree of freedom system, and it is governing equation motion we have written in terms of m X double dot plus K X plus C X dot equal to F sin omega t.

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So, the system I have shown there is no damping present; only I have shown some springs, but you may consider the damping in the joint.

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So, when these springs are connected to this mass. So, you may consider some damping in the systems also. So, by considering this damping and spring, you can write the equation motion for the system equal to m X double dot plus K X plus C X dot equal to F sin omega t. Here m X double dot is the inertia force acting on the system; K X is the spring force, and C X dot is the damping force, and F sin omega t is the harmonically excited external force.

So, this external force you can give by using an exciter or it may be due to this undulation on this road when a vehicle is moving on a wavy road or on a road due to undulation of that road. So, it may transmit some force to the vehicle also. So, that type you can take that force as a harmonically excited force of the system. So, in this case you can write the equation motion and to solve this equation motion. So, you have to divide this equation into two parts. One is the homogeneous equation that is m X double dot plus K X plus C X dot equal to 0, and other part is the particular integral part that is equal to m X double dot plus K X plus C X dot equal to F sin omega t.

So, by putting right hand side of that equation of motion equal to zero, you are getting the transient part of the vibration. So, that is the free vibration response of the system, and by adding this particular integral part, you can have the steady state response of the system. So, last class we have seen how to determine the steady state response of the system; by using the force polygon you can find the response of the system.

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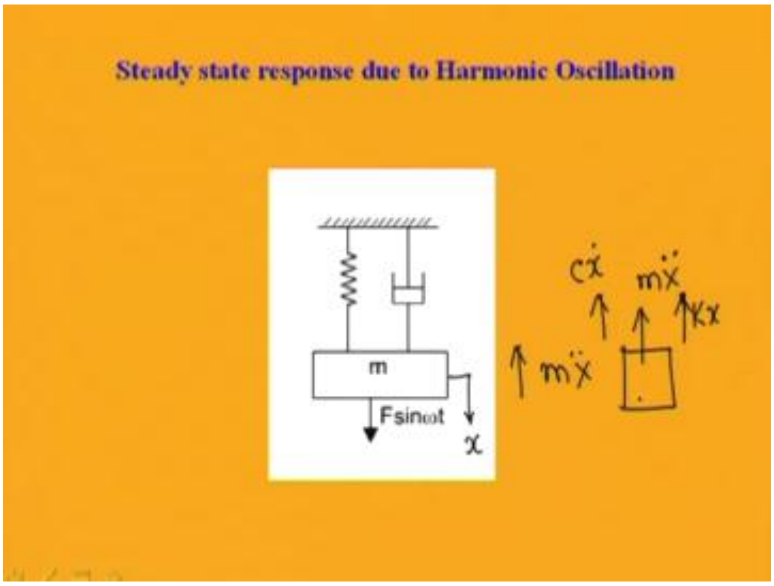

So, in this case of spring mass damper system, you have the inertia force acting on this. So, let the body is moving in this direction due to this force F sin omega t. So, when it is moving x, then it will be subjected to an inertia force that is equal to m X double dot. This will act in the upward direction. So, this is the mass. So, this is the inertia force m X double dot upward, and the spring is pulled by a distance x. So, it will push. So, as it is pulled down. So, it will pull up the mass by a distance x, and the spring force will be equal to K X. Similarly, the damper will pull this mass off by a force that is equal to CX dot.

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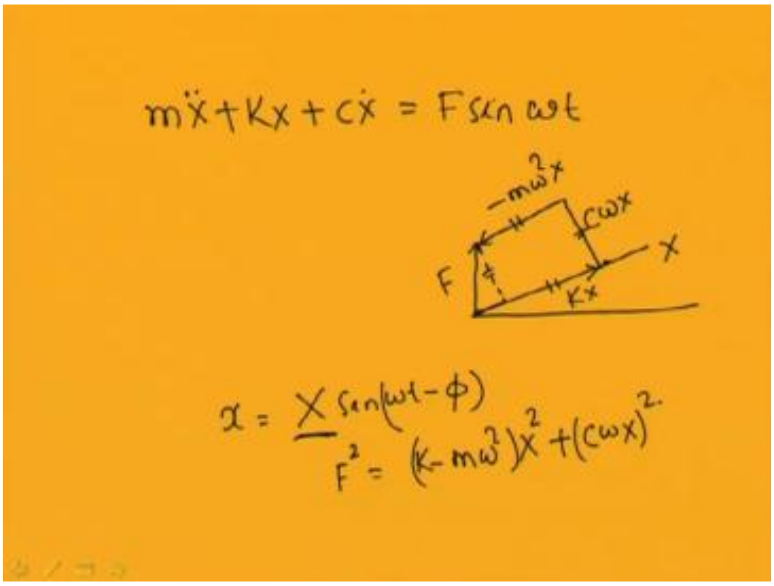

So, the resulting equation we have written this is equal to m X double dot plus K X plus C X dot equal to F sin omega t. And in this case we have drawn the force polygon by taking a reference line; let this is the reference line, and let our resulting displacement make an angle with this reference line, and this is X. So, we are yet to find what is this angle, and I will represent this spring force; as it is proportional to displacement, I will represent by this spring force K X.

So, this is K X, then damping force c omega x, and this is the inertia force which is parallel to the spring force that is equal to minus m omega square x. So, here we have taken a solution in the form F sin omega. So, F sin omega t minus phi. So, our solution is x equal to X sin omega t minus phi. So, we can represent or we can find these by this. So, this represent the force F; this is the external force; this is the spring force KX; this is damping force C omega X, and this is minus m omega square X.

And so you can obtain by drawing a line parallel to this here. So, you can obtain from this force polygon, the value for the response X and phi. So, this X can be written. So, you can find this X, it will be equal to this plus this. So, this is equal to K X minus m omega square X. So, this will be K minus m omega square, and this is C omega proportional to C omega. So, this is C omega X; this is K minus m omega square X. So, I can write this equation. So, this is a right angled triangle.

So, this F square will be equal to K minus m omega square X square plus C omega X whole square. So, from this already we have derived this expression for X. So, X can be written as. So, this is equal to F 0 by K root over 1 minus r whole square whole square plus 2 zeta r whole square, where r equal to omega by omega n. So, omega is the frequency of external excitation, and this omega n is the natural frequency of the system that is equal to root over K by m. And zeta equal to C by C c; C is the damping factor, and C c is the critical damping.

So, C by C c equal to zeta and omega n equal to root over K by m and this r equal to frequency receive. So, by using this thing, already we have plotted the magnification factor, and we have obtained the response of the system. So, this angle is omega t, and this angle is phi. So, as this angle is phi. So, this angle is omega t minus phi. So, this x that is the displacement makes an angle omega t minus phi with this reference line.

So, this x can be written as this x equal to small x that is the response of the system equal to capital X sin omega t minus phi. So, this phi equal to the C omega X phi K minus m omega square X; so you can cancel this X. So, you will get C omega.

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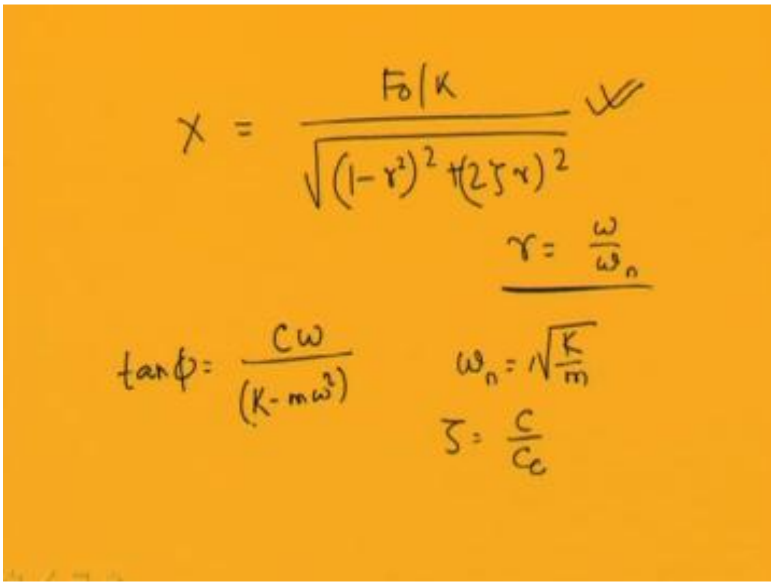

So, tan phi will be equal to. So, expression for tan phi also we can find. So, this is equal to C omega by K minus m omega square. So, tan phi equal to C omega X by K X minus m omega square X. So, this becomes C omega by this. So, in this way you can find the magnification factor and the response. So, this is the steady state response of the system. So, let us consider some different types of systems. So, today class I am going to tell you about the rotating unbalance in the machines and also about the pulling of shaft.

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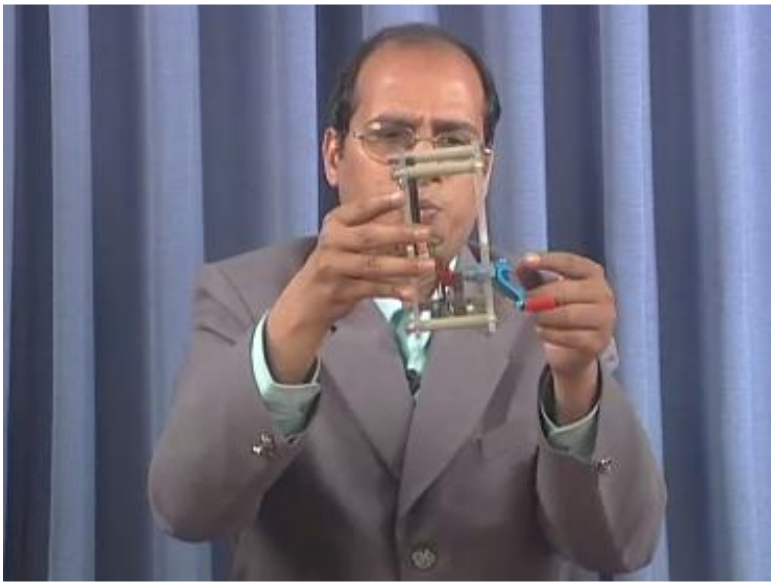

So, in many machines, for example, you just take this is a slider crank mechanism which is used in many IC engines, internal combustion engines or many other engines also it is used. So, there are many rotating parts associated with this. So, this is the crank which is rotating. So, this crank is rotating, and in turn it is rotating this or rotating this connecting rod. Then there is a piston; the piston is sliding. So, you can see this is the crank; I am rotating the crank. So, the crank is connected to this gear, and it is connected to the connecting rod which in turn it is rotating this or translating this piston.

So, in this machine there is unbalanced force. So, due to this reciprocating part there is unbalanced force. This reciprocating part or this inertia force of the piston will cause unbalanced force, and it will give raise to second moment in this main bearing. So, this will give raise to reciprocating unbalance in the machine but if you consider this crank itself. So, in the crank itself if there is some unbalance mass present in this. So, as when the crank is rotating it will create some unbalanced force, or it will create some unbalanced force on the crankshaft. And we are going to study about initially the rotating unbalance of the machine.

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So, here also in this rotating shaft, you can see this is an engine. So, in this engine, there are different clots. So, you can have these clots and other types of mechanisms also. So, you can see the engage and disengage of these clots. So, there are certain motor or this gears are mounted on this. So, when the shaft is rotating if there are some unbalance mass present on this rotors then when it is rotating this unbalance due to this unbalance mass, this rotor will be subjected to some force. So, in this way in the rotating machinery you may have different types of unbalanced forces associated in the rotating mass.

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So, this is a shaft. So, in this shaft you can add a mass. So, this mass when it is added to the shaft if it is put in an eccentric way or if this mass has some unbalanced mass or it has some this mass center is not coinciding with this geometric center, then when this shaft is rotating, the shaft will be subjected to a force in upward direction. Hence in the rotating machinery, if you consider the unbalanced mass in the rotating machinery. So, then it will create some unbalanced force in the machine.

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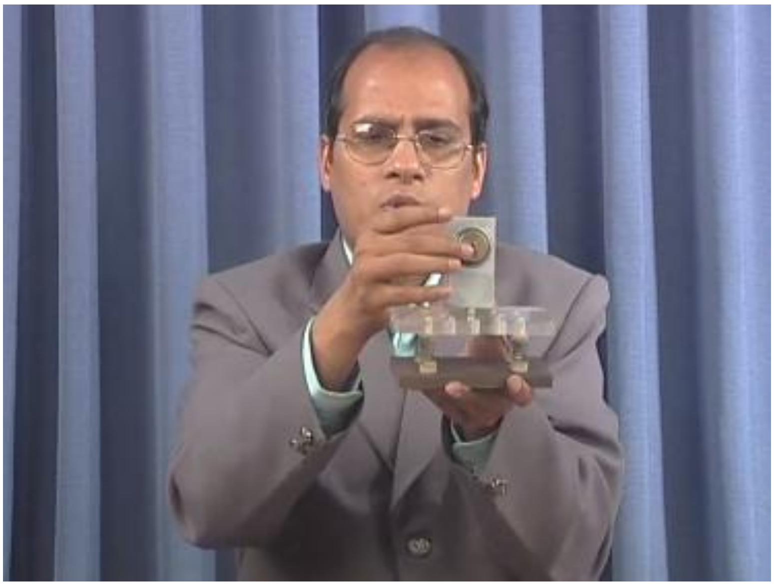

So, let this is the machine on which you have an unbalanced mass or unbalanced rotating part present in this. So, let this is the machine. So, on this machine you have a rotating part. So, this is the bearing I am showing. So, in this bearing, let there is a mass present, and it contains some unbalanced mass. So, due to that unbalanced mass let this unbalanced mass be small l. So, due to that when this is rotating on this, when this will rotate. So, it will be subjected to a force m r omega square.

If the radius I am considering equal to r, so there it will be subjected to a force of m r omega square. So, due to this centrifugal force m r omega square if it is constrained to move only in this vertical direction then the spring and if some dampers are used. So, there will be getting a force. So, due to that the whole system will vibrate.

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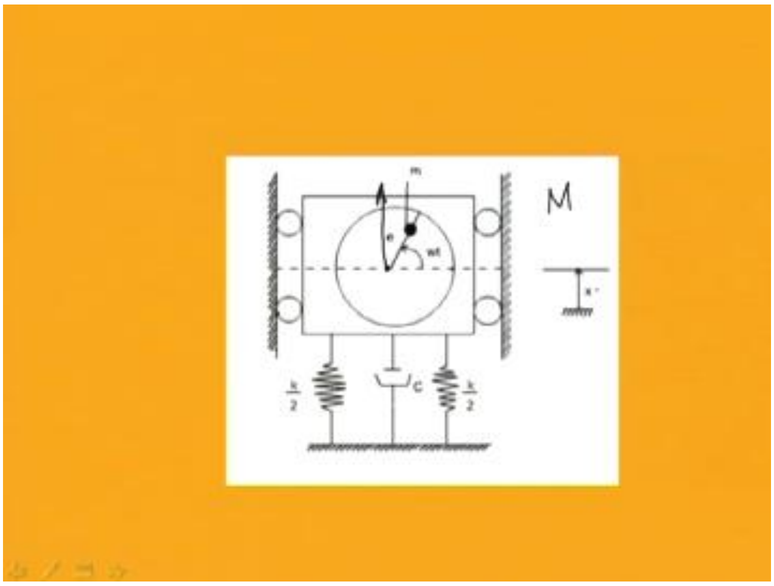

So, let us consider a system; let us consider this system. So, this is a machine. So, this machine has a small unbalanced force or small balance mass m. So, this is rotating machinery. So, in this rotating machinery you have a small unbalance mass m. Let this mass is rotating with an angular velocity omega. So, let it start from this position this reference line and at time equal to t it has come to this position. So, this angle moved by this equal to omega t. So, if you are restricting the motion in the horizontal direction by putting some guides. So, here some guides are put.

So, by putting some guides you are restricting the motion in the horizontal direction. The system will vibrate only in the vertical direction. So, you will have two component of this force. So, it will be subjected to a centrifugal force of m r omega square. So, this m r omega square force you can divide into two parts. One is m r omega square in the vertical upward direction that is m r omega square sin theta; other component is m r omega square cos theta.

So, this m r omega square sin theta component is responsible for this vibration of these machines. So, it is this machine let us consider it is supported on two springs of stiffness k by 2 and k by 2. So, the total equivalent stiffness will be k and the damping is C. So, in this case let capital M is the total mass of the machine out of which small mass is the unbalanced mass. And this unbalanced mass let it is present at a distance E from the center of this system mass.

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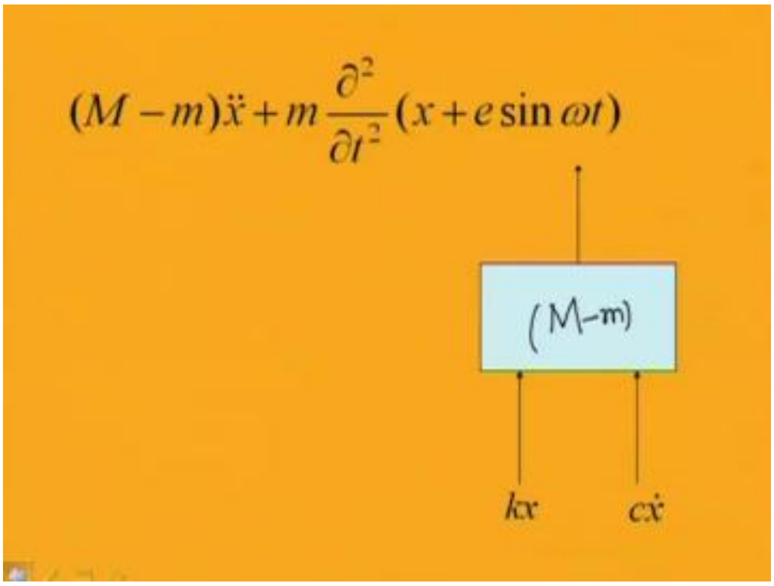

So, in this case I can write the equation also in this way. So, the total mass is capital M. So, this capital M minus small m mass we will have a displacement; let this mass has a displacement of x. This small mass will have a displacement of x plus in addition to this. So, this is the additional displacement it will have. So, this additional displacement will be let this omega t equal to theta. So, as it is at a distance e. So, this additional displacement will be equal to sin theta e sin theta.

So, the displacement of this small mass will be equal to x plus e sin theta. So, the small mass is moving with a displacement or it has a displacement of a x plus e sin theta and the other mass this capital m is moving with a displacement x. So, I can write the inertia force; inertia force will be equal to capital M into d square x by d square. For the small mass m I can write this equal to mass into d square x plus e sin omega t by d t square.

So, in that way I can find the acceleration or this inertia force of the system; total inertia force of the system becomes m minus m x double dot plus m into del square by del t square x plus e sin omega t. So, the other forces will be the spring force and the damping force. Spring force as this is moving with x, the spring force will be equal to k x, and this damping force will be equal to c x dot.

So, you just see the direction of the forces. So, when the body is moving in downward direction you can take the inertia force to be in the opposite direction of acceleration that is in upward direction and as the spring is compressed. So, it will exert a force in the upward direction. Similarly, the damper will also exert a force in the upward direction. So, taking these three forces; so these 3 forces will be equal to…

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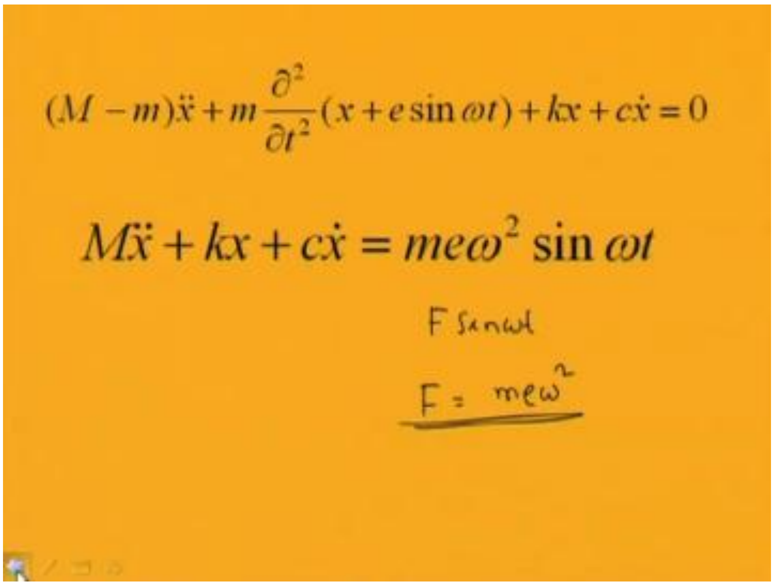

As no other forces acting on the system, so this will be equal to 0. So, I have written this thing. So, M minus m x double dot plus m del square by del t square x plus e sin omega t plus k x plus c x dot equal to 0 or if you write this thing. So, this will be m x double dot minus small m x double dot plus m into del square x by del t square that is equal to small x double dot. So, this is small m x double dot; this is minus small x double dot. So, they cancel out. So, the remaining thing becomes M x double dot plus k x plus c x dot equal to…

So, if you differentiate this e sin omega t twice. So, it becomes minus e omega square sin omega t. So, this term will become minus m e omega square sin omega t. So, I can take that term to right hand side. So, this equation becomes m x double dot plus k x plus c x dot equal to m e omega square sin omega t. So, this equation is same as the equation we have already studied. So, there it was m x double dot plus k x plus c x dot equal to F sin omega t, and here your expression becomes m x double dot plus k x plus c x dot equal to m e omega square sin omega t. So, here F equal to m e omega square.

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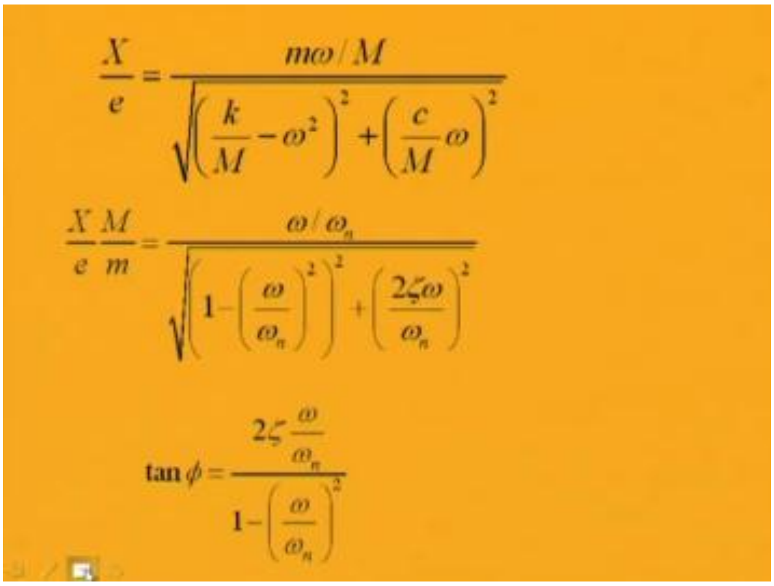

So, as F equal to m e omega square similarly proceeding in the previous way, you can draw the force polygon.

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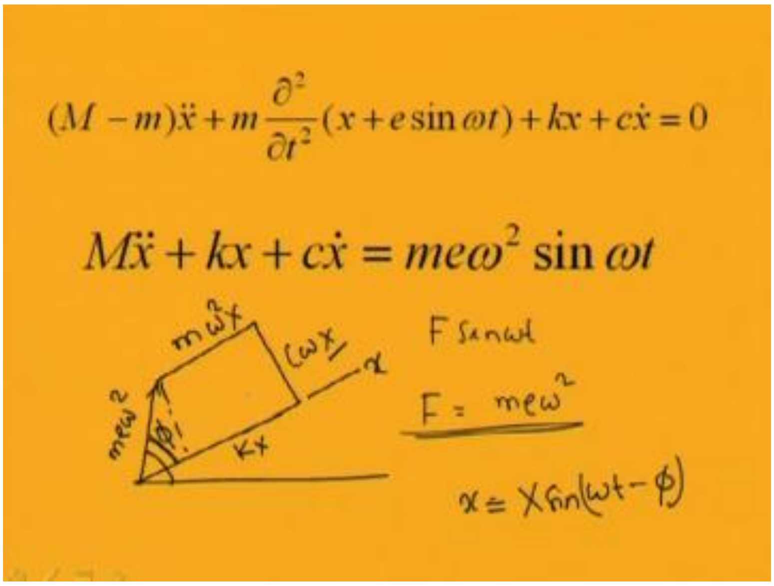

So, you can draw the force polygon; that is this is the reference line, and this is the k x. Then c omega x, and this is minus m e omega square. And this is minus M x double dot; that is m omega square x. And this is the resulting force that is equal to m e omega square. So, this is m e omega square, and this is k x. This is c omega x, and this is x m omega square. X is the inertia force, c omega x is the damping force, and k x is the spring force on the system.

So, these three forces will be equal to the m e omega square force. Now this angle is omega t, and this angle is phi. So, you can write the resulting expression that is k x minus m omega square x whole square plus c omega x whole square will be equal to m e omega square whole square. As this is a right angle triangle, from this force polygon you can find the response x. So, here x is the amplitude of the response and the response will be. So, this is the x; so your x equal to x sin. So, this is the response x. So, this is the reference line we have taken. So, x equal to X sin omega t; so this is phi. So, this is omega t minus phi. Here tan phi equal to c omega x by k minus m omega square x.

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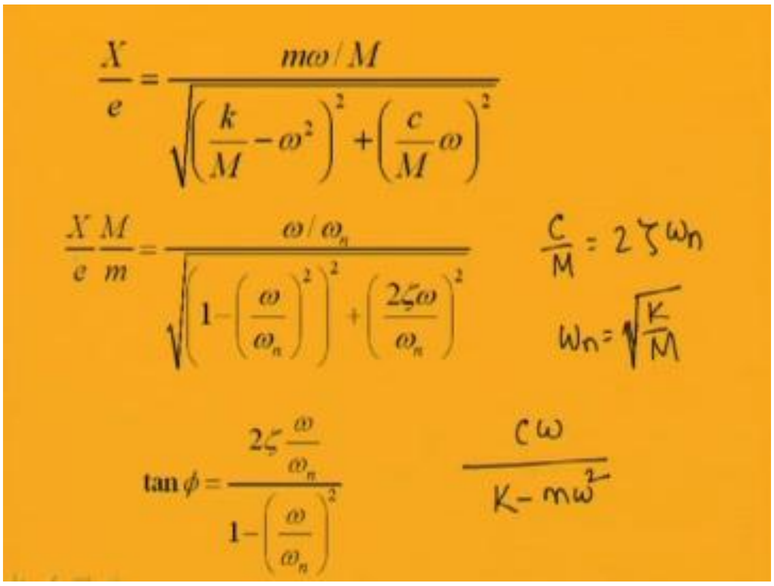

So, that expression if you write, then you can write this X by e will be equal to m omega by M and root over k by M omega square whole square plus c by M omega whole square, or you can write this as X by e into capital M by small m. So, this will be equal to omega by omega n root over 1 minus omega by omega n square whole square plus 2 zeta omega by omega n whole square. So, you may note that this C by M equal to 2 zeta omega n and omega n equal to root over k by m.

So, root over k by capital M is the omega n. So, substituting that thing, you are getting this expression where X e M by m equal to omega by omega n root over 1 minus omega by omega n whole square plus 2 zeta omega by omega n. And tan phi, tan phi which we obtained equal to C omega by K minus m omega square. So, from this you can find simplifying this thing by dividing K. So, you can write it equal to 2 zeta omega by omega n by 1 minus omega by omega n whole square. So, from this you may observe that X by e, e is the eccentricity of this mass of the rotor.

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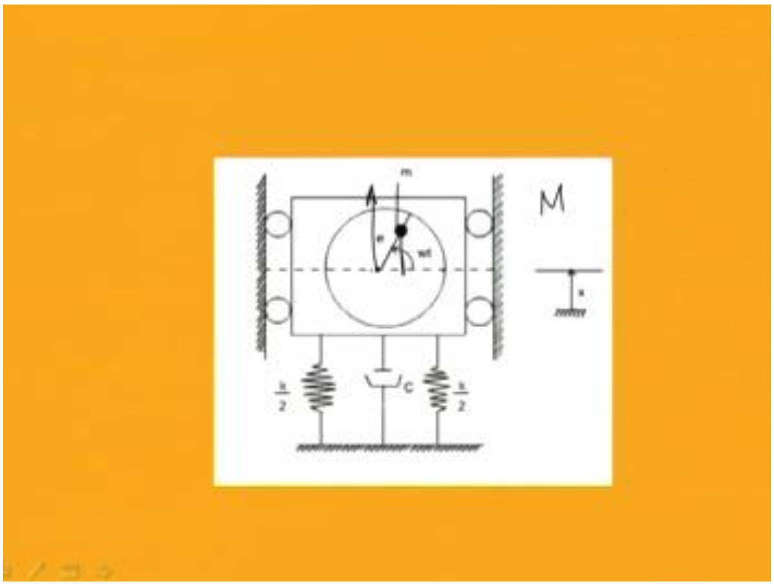

So, e is the eccentricity of the unbalanced mass. So, x by e x is the displacement or capital X is the amplitude of vibration.

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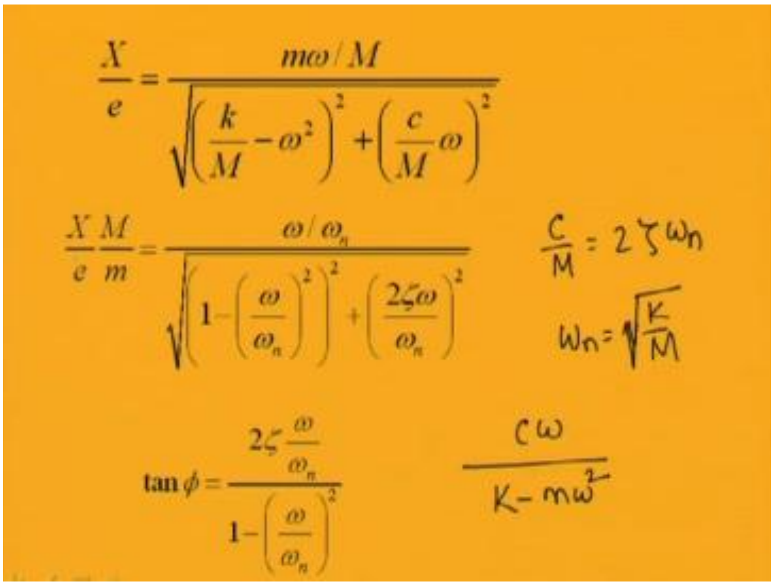

So, amplitude of vibration by this eccentricity into total mass of the system by the eccentric mass equal to omega by omega n root over 1 minus omega by omega n whole square whole square plus 2 zeta omega by omega n whole square. So, when you are starting the machine that is omega equal to 0.

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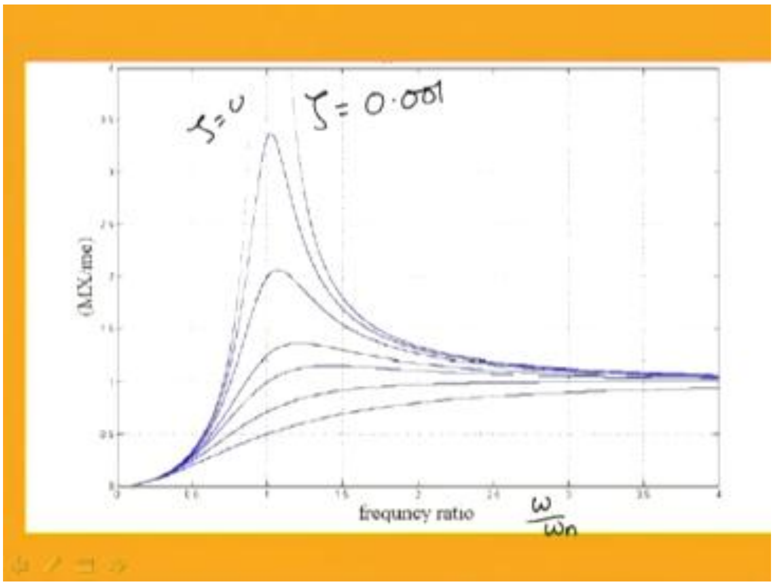

So, you can observe that this equal to zero. So, when you are starting the machine. So, if I am plotting this M x by m e versus frequency received that is omega by omega n for different value of zeta. So, this is zeta equal to 0.001 for very less volume of damping or you can take zeta equal to 0.

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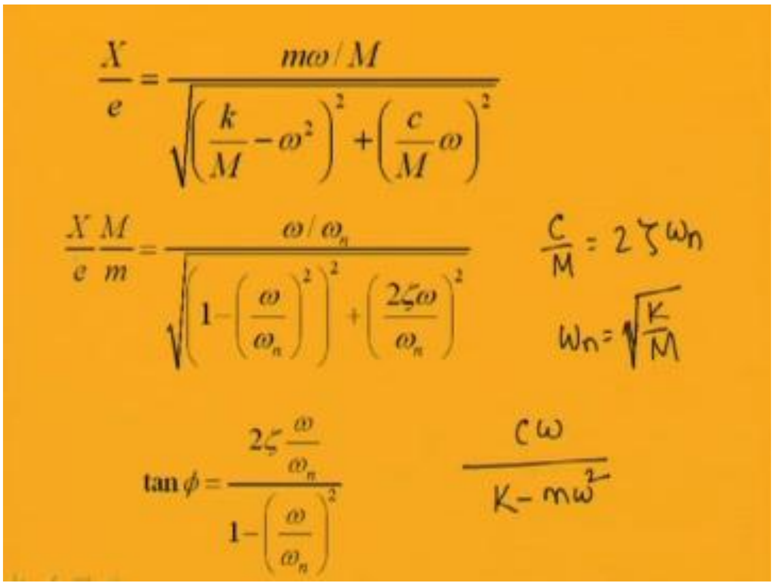

For zeta equal to 0 you can see that, this term becomes. So, this part equal to 0 and this will become omega by omega n by 1 minus omega by omega n whole square.

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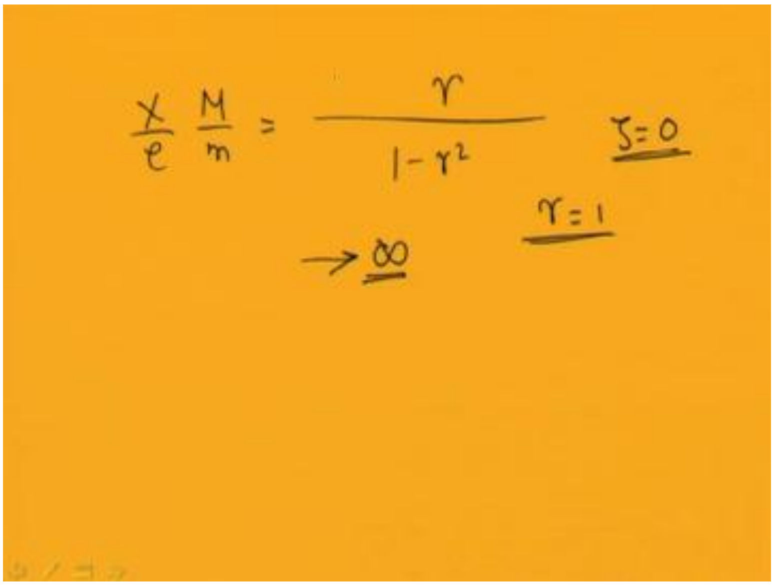

So, for zeta equal to 0 I can write this expression as X by e capital M by small m will be equal to omega by omega n; let me write omega by omega n equal to r. So, this becomes 1 minus r square whole square root over. So, this becomes 1 minus r square. So, this becomes r by 1 minus r square. So, you may observe that when this r equal to 1 that is omega equal to omega 1 in the absence of damping. So, this expression is for zeta equal to 0.

So, when damping is not present in the system when r equal to 1. So, you just observe that this term tends to infinity. So, the system will be having infinity vibration or the system response will be infinity in the absence of damping. But in case of real system always damping is present in the system. So, you may take a small volume of damping.

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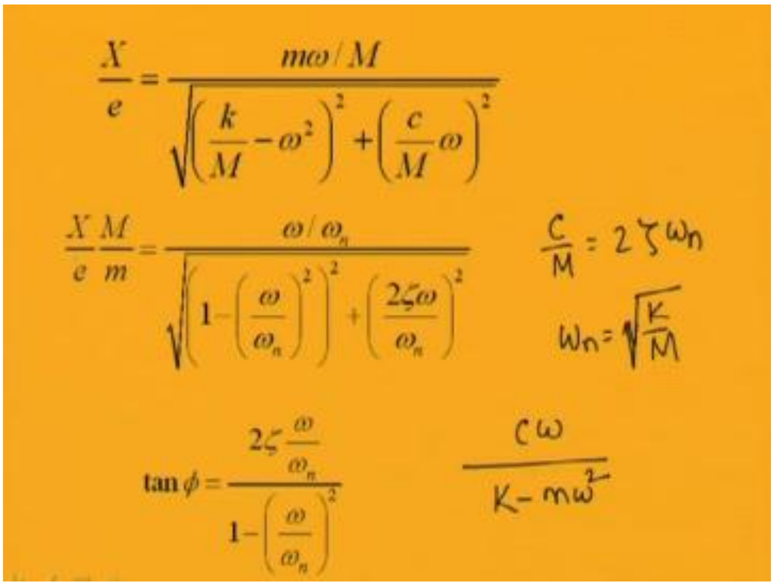

So, taking a small value of damping by inserting this, so you will have a finite value of amplitude of response. So, without damping present in the system, the system will have infinity amplitude of response.

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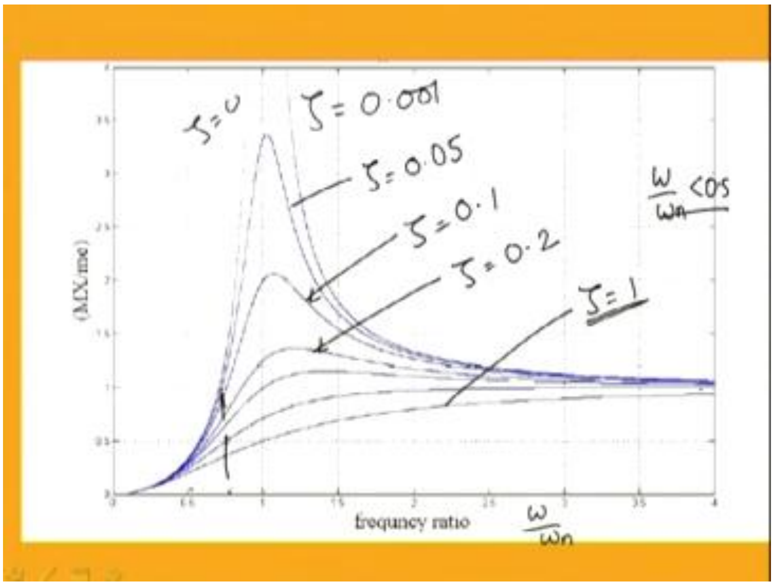

And when damping is present, then the response will have a finite value. So, with different value of damping let it is damping equal to 0.05. SO, this is for damping zeta equal to 0.1. Similarly, you can take this as damping equal to 0.2, and you can go on increasing that thing. And this value this value is for zeta equal to 1 that is for over damped system. So, in this case you just observe that with increase in damping, the amplitude of response gets decreased.

And also in the previous case you have seen when you have taken m x double dot plus k x plus c x dot equal to F sin omega t. The response was started or the magnification factor versus frequency ratio when you have plotted, then it has started from 1. So, now at omega equal to omega n that is when you are starting the machine you can observe that when you are starting the machine the response start from 0 and slowly it goes on increasing.

So, it goes up to 0.5 you just see omega by omega n when up to 0.5. So, it is very very less. So, omega by omega n when it is less than equal to 0.5. So, you can observe that the amplitude of response is very less. So, you can take up to this value, up to this value you can see that this M x by m e term is less than equal to 1. And after this you can see that up to 1 or slightly more than 1 the response goes on increasing, and after this value you can observe that the amplitude of the response get decreased.

So, it get decreased, and finally, you can observe that late after 2, you can observe that all values are converging to 1; that means for omega by omega n greater than 2, you can observe that this M X by m e tends to 1.

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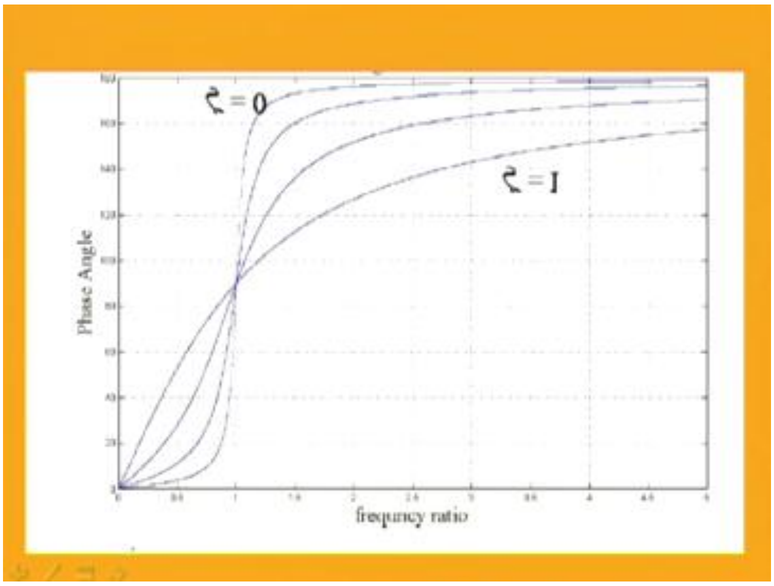

This MX by me tends to 1, and in this case you can find for all the cases this value tends to 1. So, using this equation, you can find the response of the system when a rotating unbalance of mass m with an eccentricity e present in the system. Also you can observe from the phase plot let us see the phase plot. So, from this expression you can see the phase plot that 2 zeta by omega by omega n 1 minus omega by omega n. So, when zeta equal to 0. So, this tan phi equal to 0, and when you have in presence of zeta, you can observe that when omega equal to omega n.

So, this lower part becomes 0, and it tends to infinity. So, tan phi becomes infinity. So, phi becomes 90 degree. So, in this case when omega equal to omega n, so you will have the phase angle is always phi by 2 or 90 degree. So, you can plot this phase from this.

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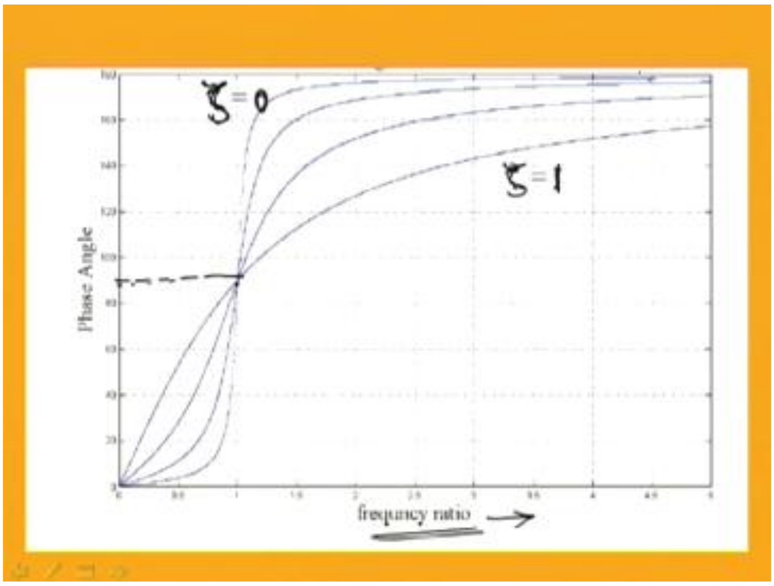

So, the phase angle when we are plotting. So, this is for zeta value equal to 1 and this is for zeta value equal to 0. So, for zeta equal to 0 you can observe that when frequency ratio is higher value, it tends to 180 degree, and for omega equal to omega n, it is equal to 90 degree. So, the phase angle is 90 degree when omega equal to omega n and it tends to 180 degree when for higher value of frequency. So, when the machine starts. So, you will have a very less phase angle which is less than 90 degree, and after that the phase angle becomes more than 90 degree. So, finally, with higher frequency it becomes 180 degree. So, if you take into account the transient response of the system.

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The complete solution becomes

$$x(t) = x_1 e^{-\zeta\omega_n t}\sin(\sqrt{1-\zeta^2}\,\omega_n t + \psi) + \frac{me\omega^2}{\sqrt{(k-M\omega^2)^2 + (c\omega^{2)}}}\sin(\omega t - \phi)$$

So, the total response of the system can be written by writing the transient part of the solution or transient response of the solution, and this is the particular integral or steady state response of the solution.

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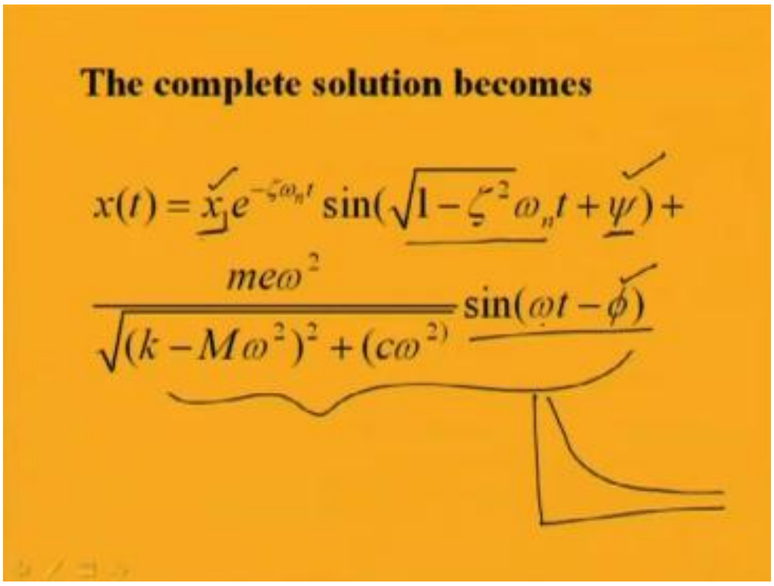

So, the transient part equal to x 1 e to the power minus zeta omega n t sin 1 minus zeta square root over 1 minus zeta square omega n t plus psi. So, this is the damped natural frequency that is equal to omega n into root over 1 minus zeta square. And this psi and x 1 depends on the initial condition and initial condition of displacement and velocity. So, taking the initial condition of displacement and velocity, one can obtain this x 1 and psi, and this part is the steady state response of the system.

So, due to presence of damping, you can observe that this e to the power minus zeta omega n t terms will decay the response. And finally, the system will vibrate with a frequency of omega, and it will vibrate as a sinusoidal function sin omega t. And the displacement will start at a time difference of this phi by omega. So, this is the phase angel. So, from this you can find the time. So, it will take that time after which the response will start. So, in this way you can find the response of a system when rotating unbalance present in the system.

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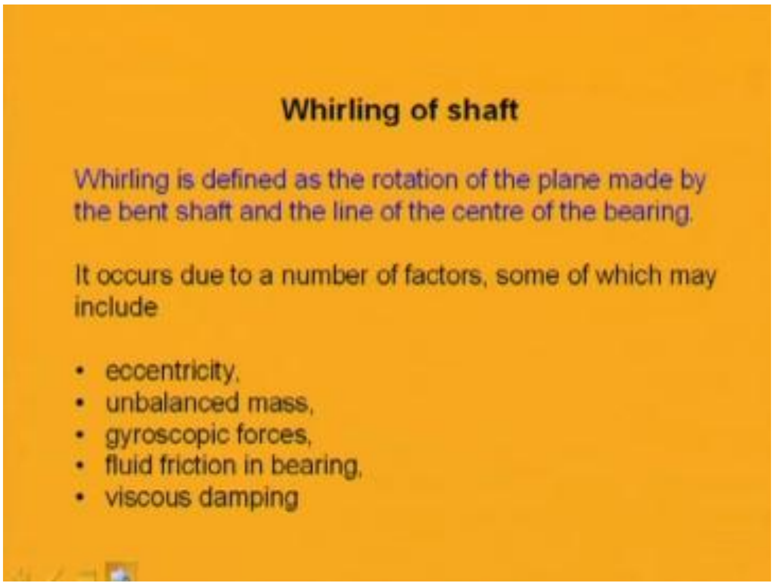

So, let us now study about the whirling of shaft and in case of whirling of shaft. So, let me show you.

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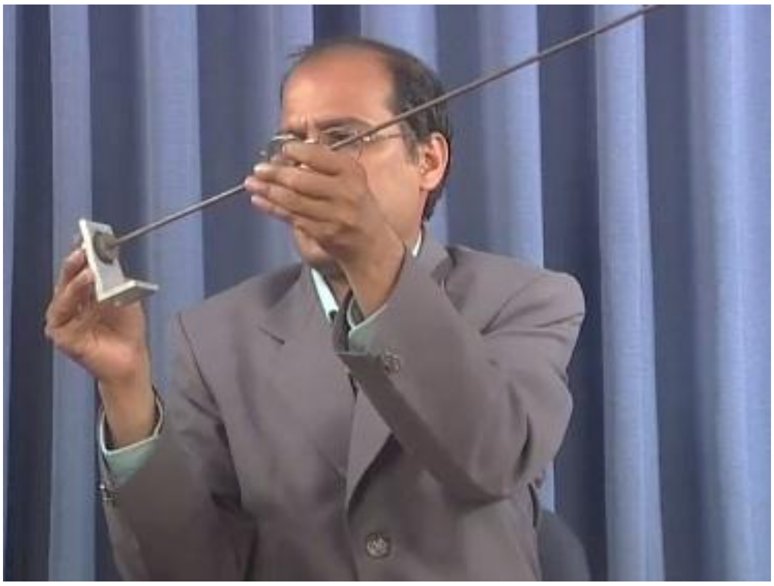

So, this is a shaft. So, the shaft may be in the bearing. So, I can show you two bearings. So, this is one side of the bearing, and this is the other bearing. So, in this bearing you can mount the shaft. So, these are the bearings also. So, in this bearing you can mount this for holding the shaft. So, for holding the shaft you can mount this thing in this bearing and after putting this shaft in the bearing. So, this side is connected to the. So, you have to put this side of the bearing.

So, this side is coupled to the motor. So, the motor will rotate the shaft. So, the shaft will be rotated by using this motor. So, in the bearing when the shaft is rotating let me take. So, this is the shaft. So, when the shaft is rotating in this bearing. So, if some unbalanced mass present on this.

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So, let I am putting this mass on the shaft. So, if this mass is put on the shaft and this mass has some unbalance, then when it is rotating due to this unbalanced mass, the shaft will experience a centrifugal force that is equal to A m e omega square. So, due to this centrifugal force, the shaft will bend. So, the shaft will bend and the rotation of the bend is r. So, this is the straight shaft; when it is straight it is rotating like this.

Now when it is bend it will make. So, you can observe that, the bearing axis the lines here in the bearing centers and this shaft axis will not coincide. So, when the line joining this half bearing center and the shaft will not coincide. So, when this is rotating and it will subject to additional centrifugal force. So, due to that centrifugal force you can see a plane containing this bend shaft, and the center line of the bearing is rotating. So, that rotation of the plane is known as the whirling of the shaft.

So, whirling of shaft is defined as the rotation of the plane made by the bend shaft and the line of center of the bearing. So, the line of the center of the bearing and the shaft line will differ due to the eccentricity; it may differ due to the unbalanced mass or it may differ due to this gyroscopic process, and in case of fluid friction in bearings also it may differ; also sometimes due to this viscous damping present it may differ.

So, also you can you can note that when the shaft is rotating. So, in case of continuous systems you can see the shaft has infinity degrees of freedom, and the shaft will have infinity number of natural frequency. So, if you have infinity number of natural frequency. So, when the shaft is rotating beyond its critical value or past critical value, then it will start bending.

So, the rotation of the shaft, so whirling you can define as it is the rotation of the plane made by the bend shaft and the line of the center of the bearing. So, when it is rotating below its first critical limit. So, you can tell it as a rigid rotor. So, if there is no unbalance mass present or there is no other unbalance forces present on the system. So, below this first critical limit, you can tell this as a rigid rotor and after that thing you may tell it as a flexible rotor as it will try to bend. So, if the shaft is very slender like this.

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So, in this case the shaft is very slender; in comparison to this shaft with this you can tell this shaft is very slender, because this l by d ratio you can see in this case it is l by d ratio very large. So, in that case this shaft will be. So, you can consider the shaft as an Euler-

Bernoulli shaft or Euler Bernoulli beam and you can consider the transverse vibration of the shaft. So, this thing we will study in case of the continuous system, and that time you will find, there is infinite number of frequencies or critical frequencies associated with this shaft.

So, when you are running the shaft above the critical speed or when you are running this shaft where it crosses the critical speed. So, you will find that there is transverse vibration of this beam. So, due to that you can see the whirling of the shaft in that case. So, the whirling you can define as the rotation of the plane continuing. So, it is the rotation of the plane continuing this bend shaft. This is the bend shaft. So, those shaft axis and this line. So, the center line of the bearing this line and this bend shaft; so they make a plane

So, this plane is rotating. So, when the shaft will rotate. So, this plane will rotate and the rotation of this plane you can take it as the whirling of the shaft. So, now we will find the expression for this whirling of shaft for a shaft weight on eccentric mass. So, let us consider a shaft.

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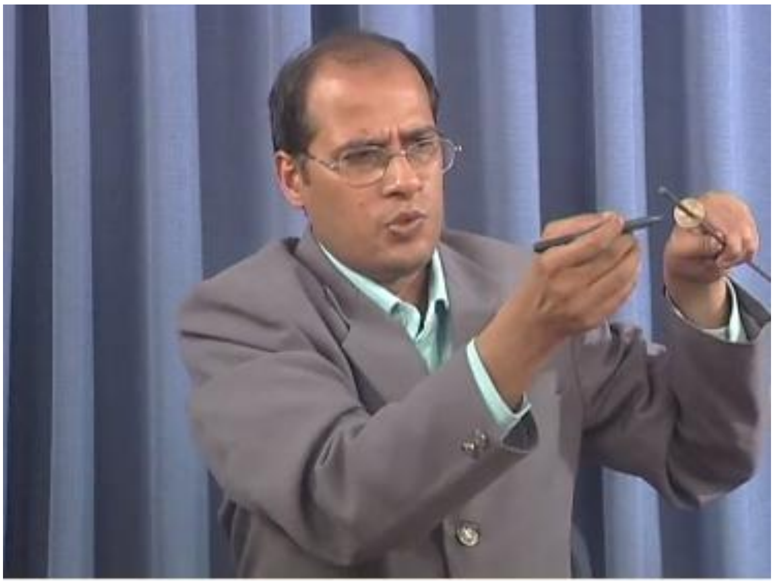

So, this is a shaft on which you have an attached mass; let this mass is attached to the shaft. So, you have attached this mass to the shaft and in this mass. So, this way you can put this mass, and we are assuming that there is an unbalance or the mass center of this rotor is not coinciding with the geometric center of the shaft. So, as the mass center of this rotor is not coinciding with this geometric center of the shaft, then let me assume that it is at a distance e from the mass center or this shaft center. So, I can take it in this way.

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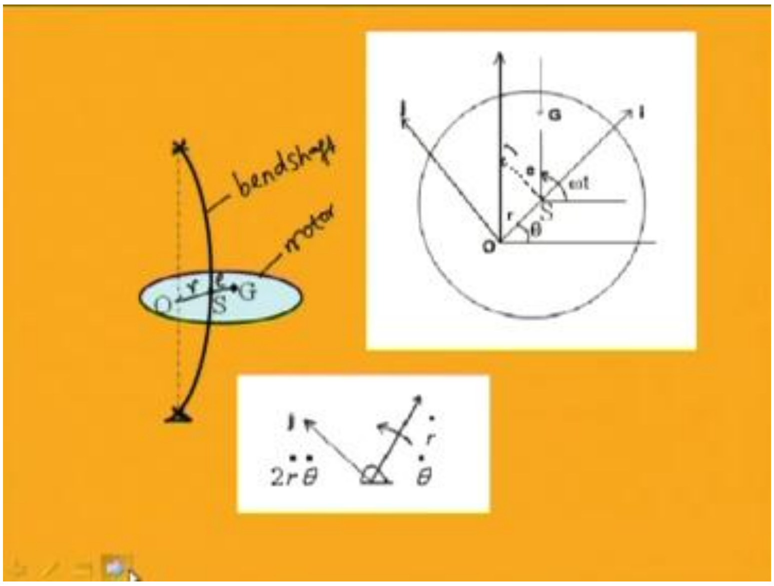

So, this is the bend shaft, and this is the rotor mounted on this, and this is the bearing. So, this is the bearing line, line joining the bearing when the shaft is got bend. So, the shaft axis and this bearing axis are same. So, the lining joining the bearing are same. So, now as the shaft is rotating and I am assuming a disc mounted on the shaft, which has a mass center here. So, the mass center instead of on the shaft, it is at a distance e from the shaft center.

So, when it is rotating. So, due to this unbalanced force as the mass is present here. So, it will be subjected to a force m force. So, let this distance is capital R. So, it will be subjected to a force m into capital R into omega square. So, due to that centrifugal force, the shaft will goes on bending. So, let this at particular instant this point on the shaft where the disc is connected be at a distance R from this bearing center. So, let o is the bearing center. So, S is the point at which we are attaching the shaft, and G is the point where we are putting or where the mass center is located.

So, now we can find the acceleration of this point and to find the acceleration of this point. So, let we can divide this acceleration into two components; one component along this direction, along this direction. So, this is R and this is along the line R, we can put one force. So, this is the i th direction I can take, and I can take a direction perpendicular to that also as the j direction. So, I can put I and j. So, the acceleration will be. So, the acceleration of e point will be acceleration of S point with respect to o loss acceleration of G with respect to S.

So, acceleration of point S with respect to o will have these following components. So, this distance is r. So, as this r is changing with time. So, I can find this velocity; I can tell this velocity as r dot, and it is simultaneously rotating. As it is simultaneously rotating and there is change in velocity of this. So, this point will be subjected to a Coriolis force; this Coriolis force will be equal to 2 r dot theta.

Let me explain it again. So, this point r is as the shaft is bending this length r is changing. So, let the change in r will be equal to r dot and simultaneously as this is rotating with velocity of theta dot. So, it will be subjected to a Coriolis acceleration that is equal to 2 r dot theta dot in addition to this. So, you will have this centripetal acceleration and tangential acceleration. So, the tangential acceleration, centripetal acceleration and Coriolis acceleration by taking all these acceleration into account.

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$$a_G = a_S + a_{G/S}$$

$$= \left[\ddot{r} - r\dot{\theta}^2 - e\omega^2\cos(\omega t - \theta)\right]\boldsymbol{i} + \left[r\ddot{\theta} - e\omega^2\sin(\omega t - \theta) + 2\dot{r}\dot{\theta}\right]\boldsymbol{j}$$

$$m\left[\ddot{r} - r\dot{\theta}^2 - e\omega^2\cos(\omega t - \theta)\right] + kr + C\dot{r} = 0$$

So, we can write a G that is the acceleration of point G equal to acceleration of point S plus acceleration of point G with respect to S. So, that thing can be written as r double dot. So, acceleration of point it will be r double dot minus r theta dot square minus. So, let me show you. So, this is r. So, r is changing in this direction. So, r double dot will be in this direction. So, this is r double dot minus r theta.

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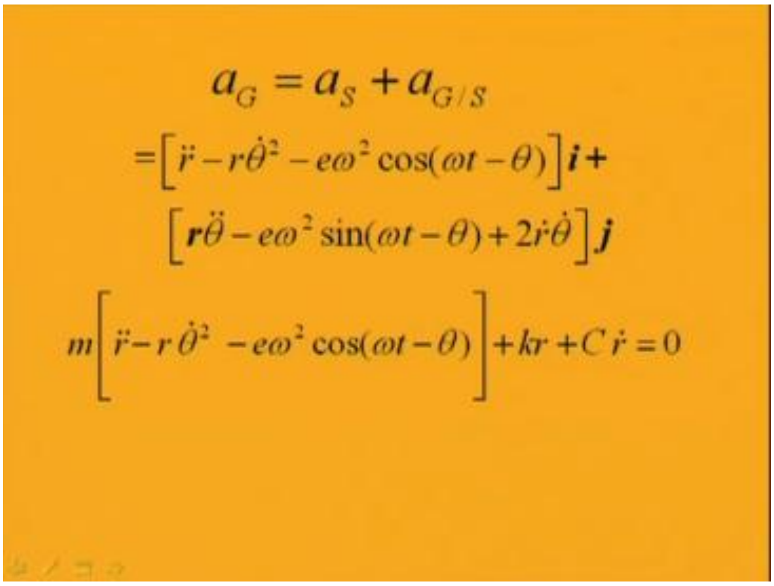

So, you can see another term is r theta dot square. So, r theta dot square will be. So, as theta is rotating in this direction. So, theta dot it is taking place in this direction, and it will be subjected to a centripetal acceleration that is from S to o, and that acceleration is r theta dot square. So, r theta dot square acceleration is from S to o, and this r double dot is in the direction of o to S and again another component. So, this is the mass. So, this mass will be.

So, this will have an acceleration that will be from G to S. So, this acceleration will be. So, let this angle is omega t. So, as this angle is omega t. So, it will have this acceleration, you can divide this acceleration into two components. So, that acceleration can be retained as E omega square cos omega t minus theta. So, this angle is omega t, and this angle is theta. So, this angle I can write it as omega t minus theta.

So, you will have two components, one component along this direction. So, this acceleration equal to m e omega square. So, this acceleration you have two components, one along this direction and other along this direction.

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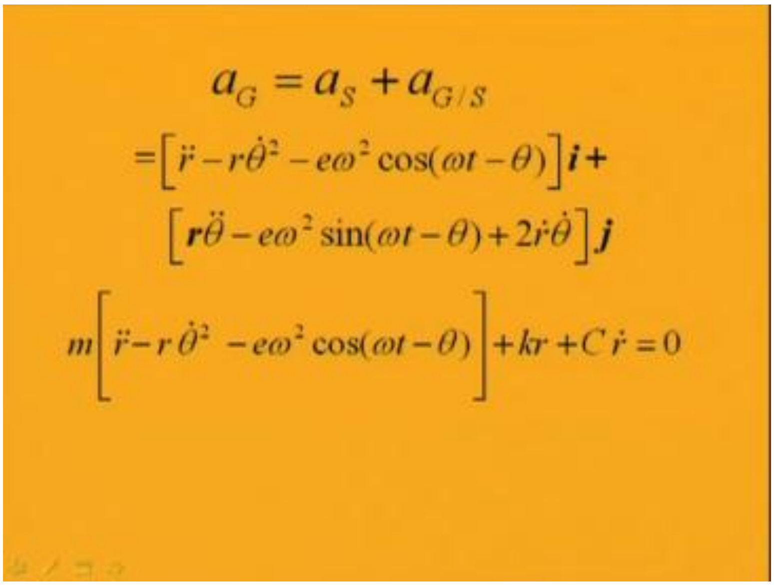

So, these forces you can write that is equal to e omega square. So, that component is e omega square into cos omega t minus theta in i direction. So, this acceleration that is e omega square here. So, you can put this component cos omega t minus theta in i direction and sin omega t minus theta in j direction. So, putting that way, you can have i direction acceleration equal to r double dot minus r theta dot square minus e omega square cos omega t minus theta. So, this is in i direction.

And j direction you will have the force acceleration. So, in j direction as we can consider this theta is changing. So, if theta is not uniformly changing, then you will have this theta dot and theta double dot, this angular acceleration also you will have. So, this angular acceleration will take place in the direction perpendicular to this OS. So, this component will be r theta double dot. So, that is along z direction.

Then the other components are already we found this Coriolis component that is equal to 2 r dot theta dot; that is also along j direction and the component of the acceleration of G with respect to S in j direction. So, these three components you can write. So, this becomes r theta double dot minus e omega square sin omega t minus theta plus 2 r dot theta dot. So, this is the Coriolis component.

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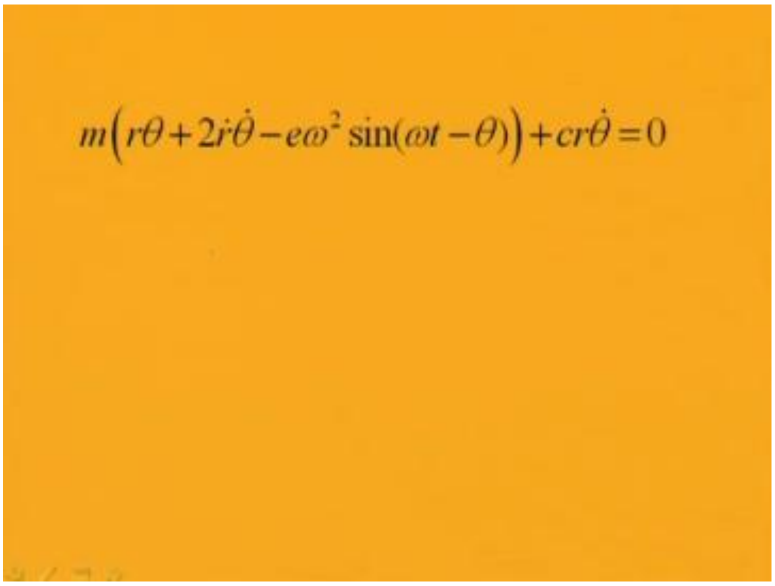

And this is the tangential component due to the rotation or considering the angular acceleration on the shaft, and this is the component due to the eccentricity in the j direction. So, we have two component of acceleration; one in i component, other is the j component. So, I can write the equation motion by writing. So, in this case the forces acting are. So, I will have three different forces. So, one force will be the inertia force; the inertia force will be equal to mass into acceleration, second will be the…

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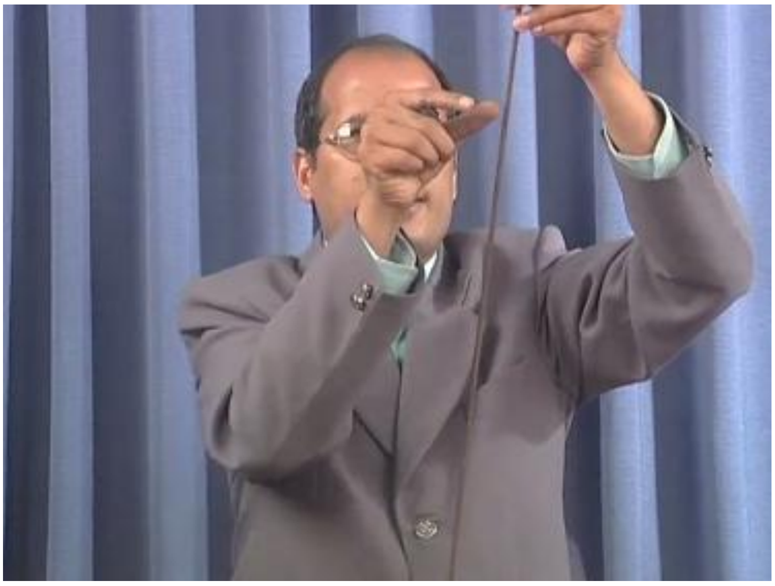

So, I can consider the stiffness of the shaft. So, due to this rotation when it is subjected to this transverse vibration; so it is moving in this direction outward from the center. So, the elasticity of the shaft will resist its motion, and it will try to pull the shaft towards the center. So, I can write that resistance force by stiffness k. So, this force will be equal to. So, as it is it will try to pull this towards the center that is at distance r. So, this force will be equal to k r.

And also I can assume a damping force there. So, that damping force I can take equal to C r dot. So, the total force acting on the system will be equal to mass into acceleration plus k r plus C r dot equal to 0. So, this is the equation in the r direction. So, similarly we may have the equation in the theta direction. So, in the theta direction the shaft will have no resistance. So, only damping is present in that direction.

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$$a_G = a_S + a_{G/S}$$

$$= \left[\ddot{r} - r\dot{\theta}^2 - e\omega^2 \cos(\omega t - \theta)\right]\boldsymbol{i} + \left[r\ddot{\theta} - e\omega^2 \sin(\omega t - \theta) + 2\dot{r}\dot{\theta}\right]\boldsymbol{j}$$

$$m\left[\ddot{r} - r\dot{\theta}^2 - e\omega^2 \cos(\omega t - \theta)\right] + kr + C\dot{r} = 0$$

So, in the j direction you can write it will be equal to mass into this acceleration in j direction plus C r theta dot. So, we can assume damping in both the direction. So, this is equal to zero. So, this equation you can have a two degrees of freedom system equation.

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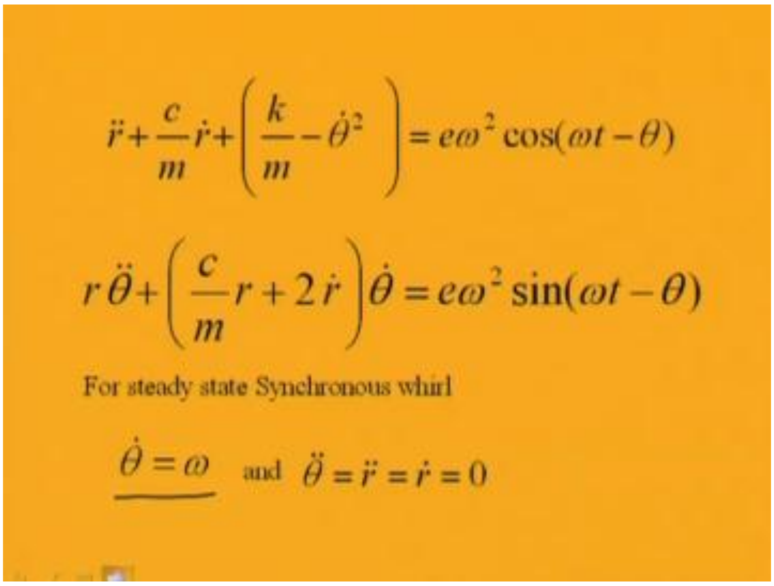

But by simplifying that thing we can write this equation as r double dot plus c by m r dot plus k by m minus theta dot square equal to e omega square cos omega t minus theta. And second equation you can write it in this form that is r theta double dot plus c by m r plus 2 r dot into theta dot equal to e omega square sin omega t minus theta. So, as we are considering only the single degree of freedom system, we can limit our study to the synchronous whirl.

So, we have to find the response for the synchronous whirl that is when the whirling and the rotation of the shaft equal to same or the whirling speed equal to the speed of the shaft that is theta dot equal to omega. So, considering the case when theta dot equal to omega and theta double dot equal to r double dot equal to r dot equal to 0.

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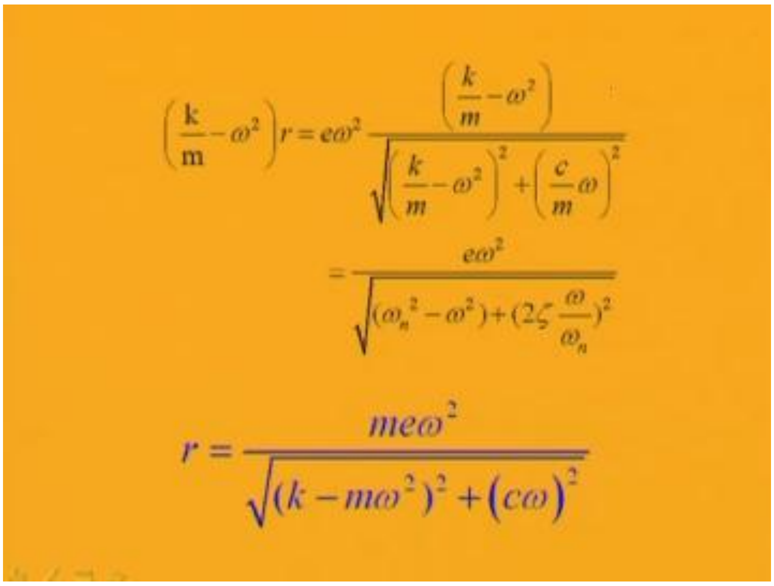

So, I can reduce these two equations to this form, and I can write the expression in this way so that is k by m. So, by substituting this thing in this equation, this equation will reduce to. So, as theta double dot is equal to 0. So, this terms equal to 0 and c by m r plus 2 r dot, r dot I am putting equal to 0, theta dot equal to omega. So, this gives me c by m by r omega. So, let me write for the fast equation fast equation r double dot equal to 0, then r dot also equal to 0. So, these two terms are not there; so k by m minus theta dot square.

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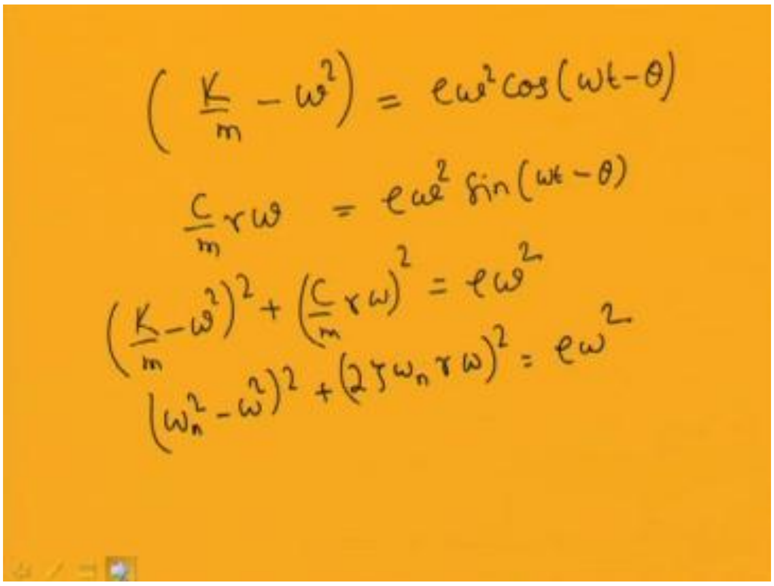

So, I can write this as K by m minus theta dot. So, for theta dot I can write omega. So, minus theta dot square it will be minus omega square.

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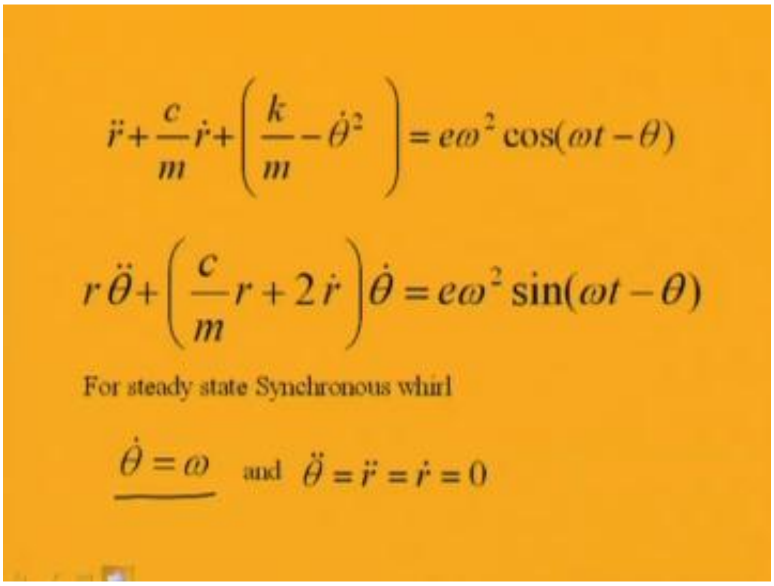

So, K by m minus omega square equal to e omega square. So, this is equal to e omega square cos. This is equal to omega t minus theta. And from this equation I can write this as r theta double dot is 0; this theta c by m r theta dot. So, this will be equal to c by m r theta dot I can write it equal to e omega square. So, this is equal to e omega square sin omega t minus theta.

So, this way I can write. So, c by m r omega equal to e omega square sin omega t minus theta, and the other expression equal to k by m minus omega square equal to e omega square cos omega t minus theta. So, I can square and add these two. So, by squaring and adding I can have this k by m minus omega square whole square plus c by m r omega whole square. So, this will be equal to. So, the sin square and cos square equal to 1.

So, if I add this sin square omega t minus theta plus cos square omega t minus theta. So, this becomes 1. So, these become e omega square. So, I can have this expression k by m minus omega square whole square plus c by m r omega whole square equal to e omega square. Similarly, I can find the expression for phi by dividing this.

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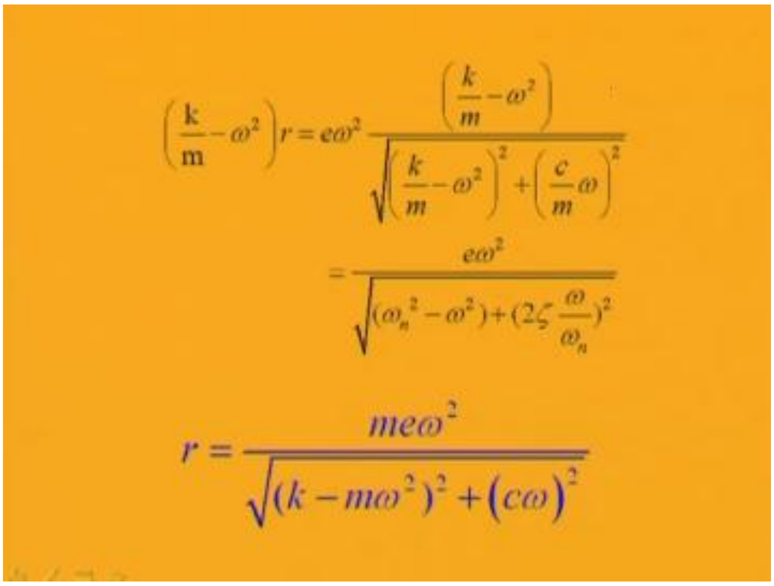

Or I can write this equation K by m minus omega square r equal to e omega square. So, from this I can write this. So, from this I can write this r will be equal to m e omega square by root over k minus m omega square plus c omega square I can find it from this expression. So, I want to find the expression for r. So, I can find the expression for r. So, this k by m will be equal to omega n whole square. So, this is omega n square minus omega square whole square plus this c by m equal to 2 zeta omega n r; this is omega. So, this whole square is equal to e omega square.

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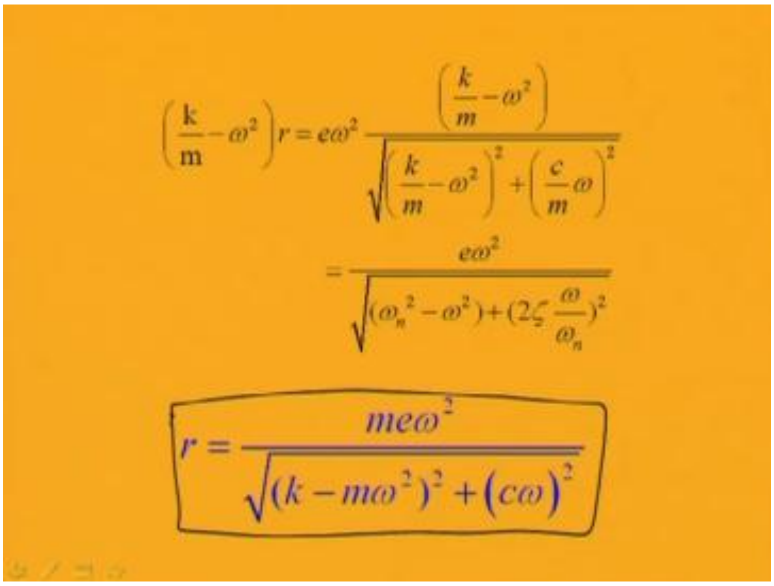

And from this I can have this expression for r. So, this r will be equal to m e omega square by root over k minus m omega square whole square plus c omega whole square.

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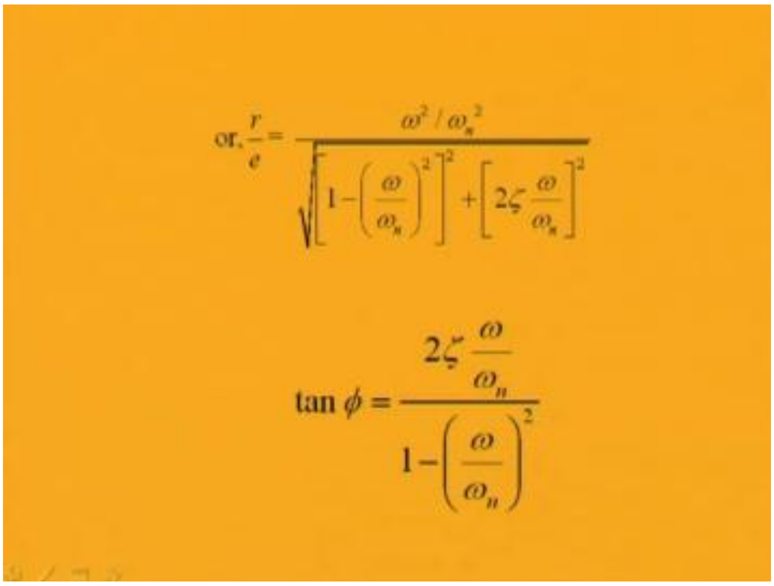

So, in this way I can find this r by e ratio equal to omega square by omega n whole square by root over 1 minus omega by omega n whole square whole square plus 2 zeta omega by omega n whole square, and tan phi I can obtain from that. So, tan phi will be equal to 2 zeta omega by omega n 1 minus omega by omega n whole square. So, in this way you can study the whirling of shaft, and also form the continuous system we will see that by expression for this transverse vibration of beam can be given by this way.

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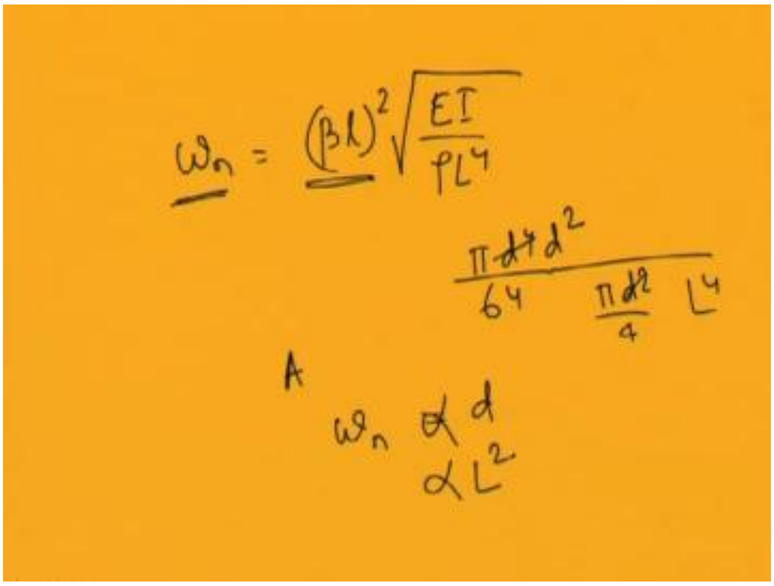

So, you can write the expression for transverse vibration of beam which can be retained, or this omega n can be written equal to beta l square root over E I by L fourth. So, this beta l will depend on different boundary conditions and as this I content. So, by taking a circular shaft it will be pi d fourth by 64. And this row content row will be equal to area into. So, this is row is mass per unit length. So, this is equal to pi d square. So, this is proportional to pi d square by 4.

So, this is pi d square by 4 and you have an l fourth term in the denominator. So, you will have d square term here. So, this omega n is proportional to. So, root over d square that is omega n is proportional to d and also it will be proportional to L square also.

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So, in this case of whirling of shaft if you increase the diameter of the shaft; so this is a shaft I have shown. So, this shaft as this is a thin shaft with less thickness or less diameter; this is a shaft with more diameter. So, in this shaft if you compare the natural frequency of this shaft and this shaft; so you can see that a shaft with higher diameter will have a higher critical speed and similarly, if you change the length of the shaft. So, you have a shaft with the length is less than the other shaft.

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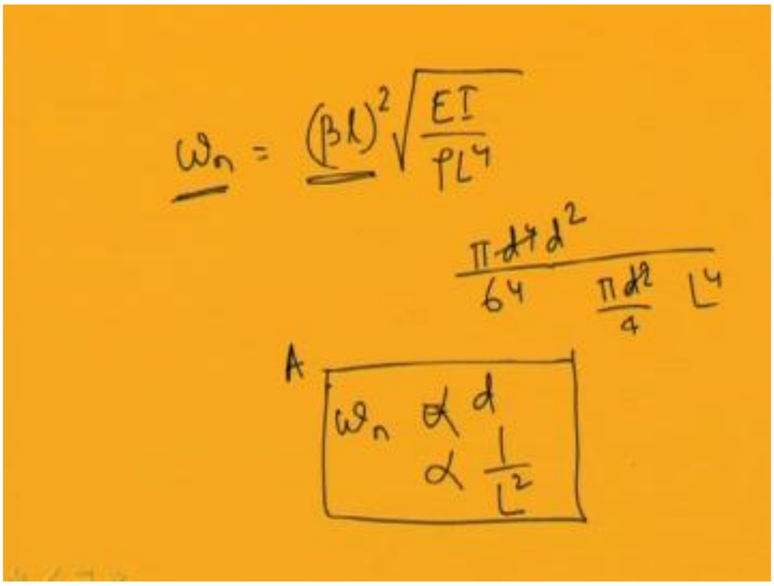

So, in that case this omega n is inversely proportional to L square or omega. So, if you increase the length of the shaft the natural frequency decreases. So, for a long shaft you will have a critical speed at a lower frequency, and for a shaft with less length you will have a critical speed at a higher frequency. So, in this way you can study the natural frequency of the shaft, also you may mount several shaft on this shaft. So, when mounting several shaft, you can find the natural frequency of the shaft by using the Macaulay formula which we will study in the approximate method.