Engineering Mechanics

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Friction-1

Module 4 Lecture 9
Friction

Today the lecture is on friction. For your reference, this is module 4, lecture 9 of the web-based engineering mechanics course. In all our preceding discussions, we have assumed that the contacts between two rigid body surfaces are either smooth, that is, they are frictionless, or they are fixed - that means there is no relative motion. But in practice, between any two contacting surfaces of rigid body tangential forces develop that we call frictional forces.

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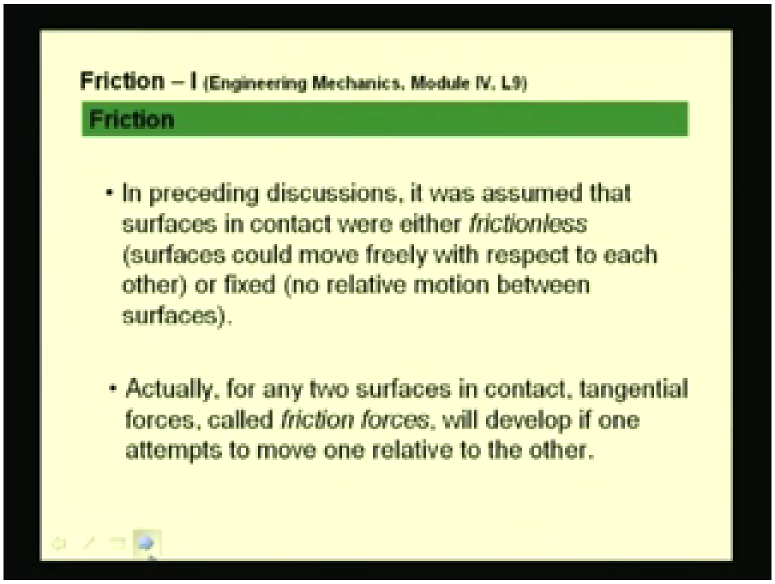

They prevent the relative motion between the two and offer resistance to some degree before a motion impends and starts. This is for all surfaces, irrespective of how smooth they are though the degree of resistance offered for rough surfaces is more and for smooth surfaces, it is less. The origins of these frictional forces are primarily due to the non-uniform nature of the contacting surfaces in the microscopic levels.

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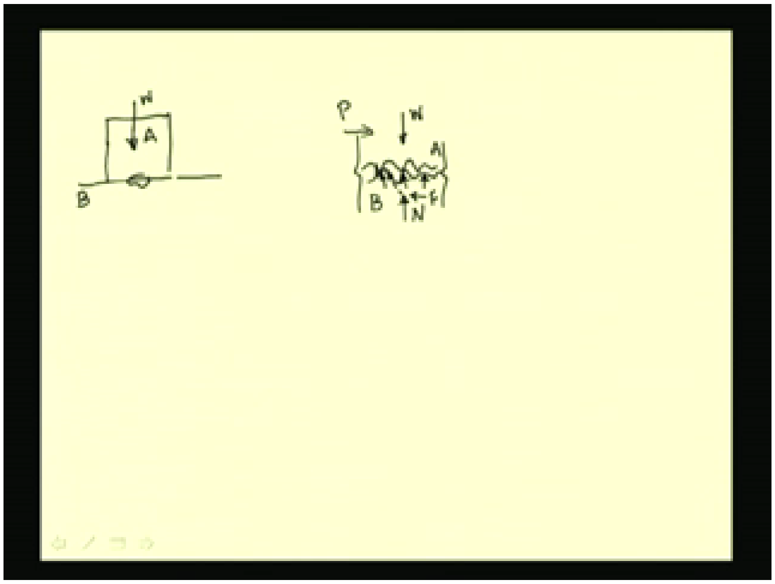

Let us take an object A, which is in contact with another object B. Let us say this is the weight of this object. If we consider microscopically a portion of this contact, there are undulations on the surface of both A and B. The contact actually happens at few of these places and not as seen macroscopically.

At these contact points, the intensity of the contact forces is large and they lead to deformation and thus interlocking of these material features. For the object under stationary condition, the sum of these, let us say the reactive forces which are happening wherever the two surfaces are in contact, the resultant of all these forces has to be equal and opposite to the weight of this object A, so that the equilibrium is maintained.

Let us assume a small force now acts on this object A. Since these material features resist the motion, these reactive forces change, the point of contacts may also slightly shift, and their orientation will also shift; so that the resultant of all these forces is having some horizontal component also.

If P is the force then if the horizontal component of these reactive forces is F, it balances this force P and this happens only for small amounts of horizontal force P. As this force increases, ultimately, these material features deform and the interlocking is no longer maintained and the object A starts moving or sliding on the object B.

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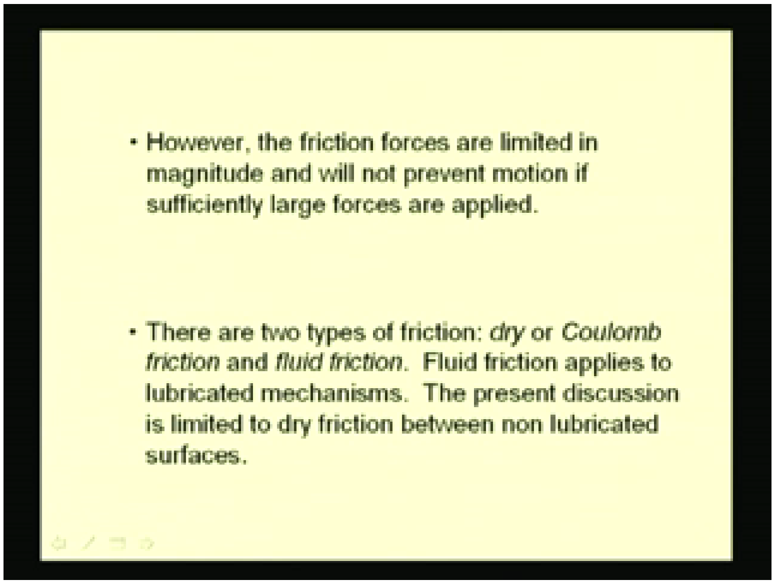

These frictional forces are limited and they prevent only motion to certain degrees and if the forces are large then the motion actually takes place. The kind of friction that we just saw is called as the drive friction or the friction existing between two surfaces directly in contact without any fluid interface. But in mechanisms or machines, we have lubricated surfaces where a film of a lubricant is present between the meeting pads to reduce the friction. To analyze these problems, we need to also take care of the fluid friction or the friction between successive layers of the fluid; the resistance offered by these layers of fluid for the relative motion.

In the current lecture, we will limit our scope to dry friction that means the friction existing between non-lubricated parts.

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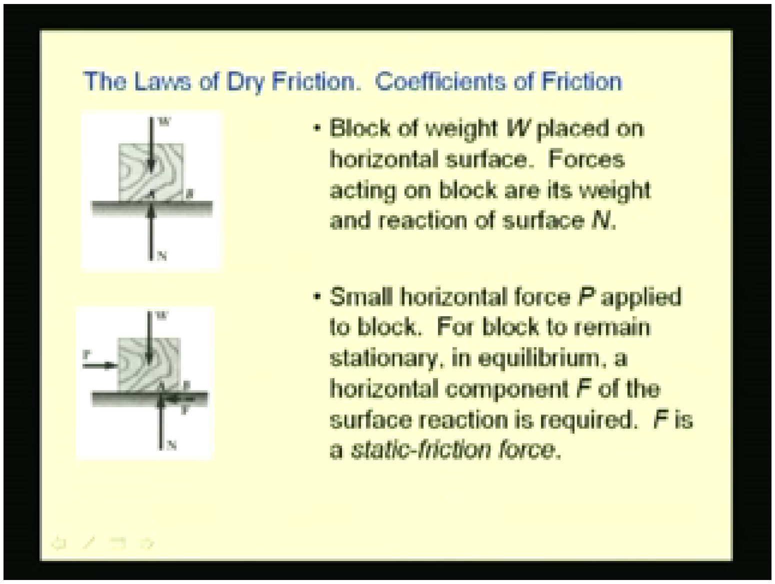

The frictional forces that are developed are governed by certain laws. We will see that by considering this example. We take a block of weight W which is being supported by the surface B. When there are no horizontal forces, the weight of this object is balanced by the normal reaction then a small amount of force P acts on this body, then a corresponding amount of frictional force develops between the contact between A and B. This is what we call as the static-friction because the object is still in rest and so this frictional force that is developed is the static-frictional force. As this force, P increases this frictional force F also increases to a maximum limiting value.

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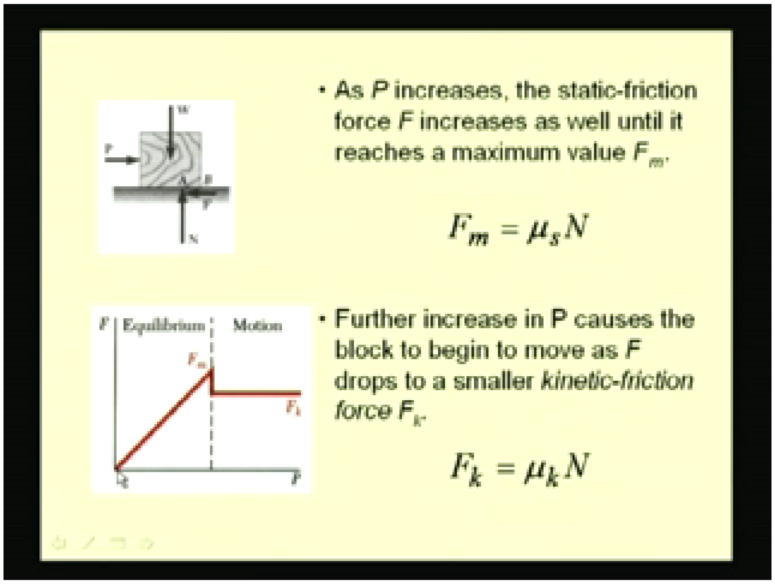

If we call that limiting value as F_m for a pair of surfaces then the same is related to the normal reaction by what we call as the coefficient of static-friction mu_s. This has been found by series of experiments. Further, when this force P is increased the block starts to slide and the frictional force drops to a lower value and remains constant. That frictional force which exists in the bodies in motion is known as the kinetic-friction. Let us designate it as F_k. It also bears a constant relation with the normal reaction N and the constant of proportionalities $mu_{k,}$ which is the coefficient of kinetic-friction. These two coefficients that is the coefficient of static-friction and the coefficient of kinetic-friction are dependent on the types of surfaces; the roughness of the surface that are in contact.

This behavior of the frictional force is depicted by this graph. When there are no horizontal forces trying to move the object there is no frictional force and thus the frictional force is 0. But as this horizontal force P increases, correspondingly the frictional force also increases to a maximum limiting value which is given mu_s times of N, the normal reaction. When the body starts to move, it drops to a lower value and almost remains constant for the entire motion. It has been found that even for small changes in velocity the force remains constant, though at very large velocities this force can drop to a slightly lower value.

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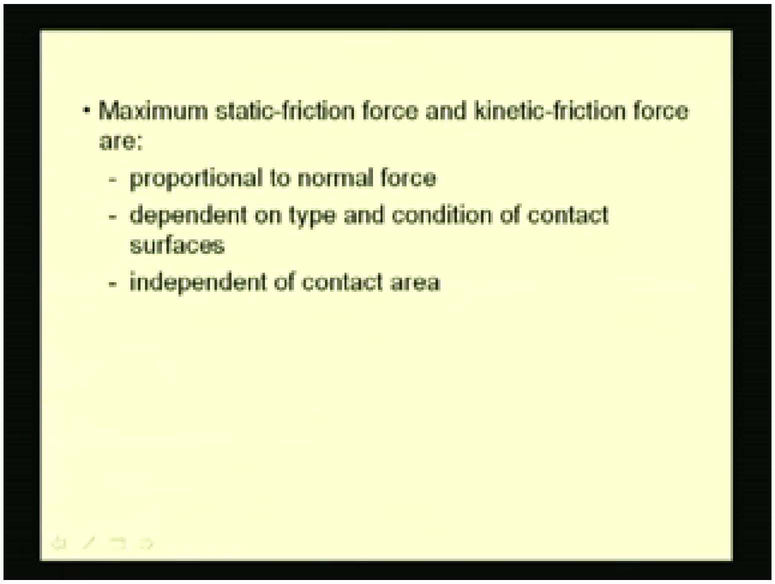

This maximum static-frictional force and the kinetic-frictional force that we have seen are proportional to the normal force, and depend only on the condition of the contacting surfaces, and they are independent of the contact area. This has been found by again series of experiments.

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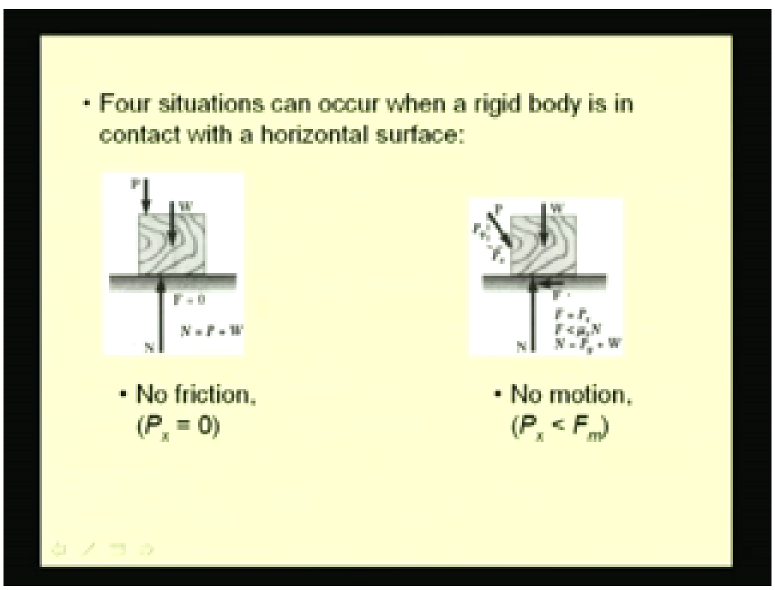

In the entire region where the friction is developed, right from the body which is in equilibrium and which is static to the point when it impends to move, the behavior can be divided into four different regions. We have initially no horizontal component of the force leading to any tendency of this body to move and, so there are no frictions.

We have a horizontal component of the force which is $P_{x,}$ which is smaller than the limiting value, which is mu$_s$ times of N. So the frictional force F developed is equal to and opposite to this force P_x. Here one should know that this frictional force F is not equal to mu times of N. mu$_s$ times of N is the maximum limiting value and when the object is not in the impending motion state this frictional force is less than that. It is only equal and opposite to the forces that tend to cause the motion, so here this force P_x is balanced by this force F.

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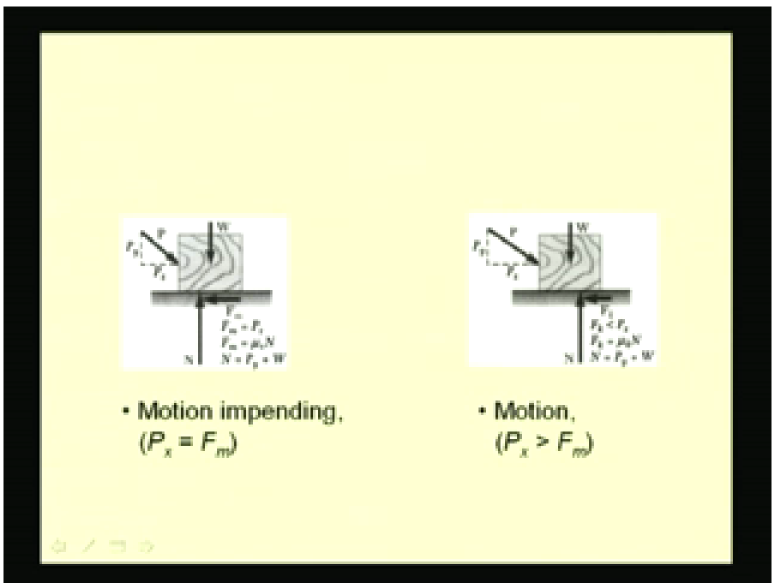

When the motion is impending, at that case we have this P_x equal to the maximum limiting force F_m. Only for this impending motion case we have the relation between this frictional force, which is equal to F_m and N as F_m equal to mu$_s$ times of N.

One has to be careful in solving the equilibrium problems with friction. If only the motion is impending, we can use this relation between the horizontal force F and N using the coefficient of friction. If we are not sure, whether the motion is impending then this relation cannot be used. When the body is in motion, this force F_k is less than the force that causes motion that is P_x and this force is equal to mu$_k$ times of N.

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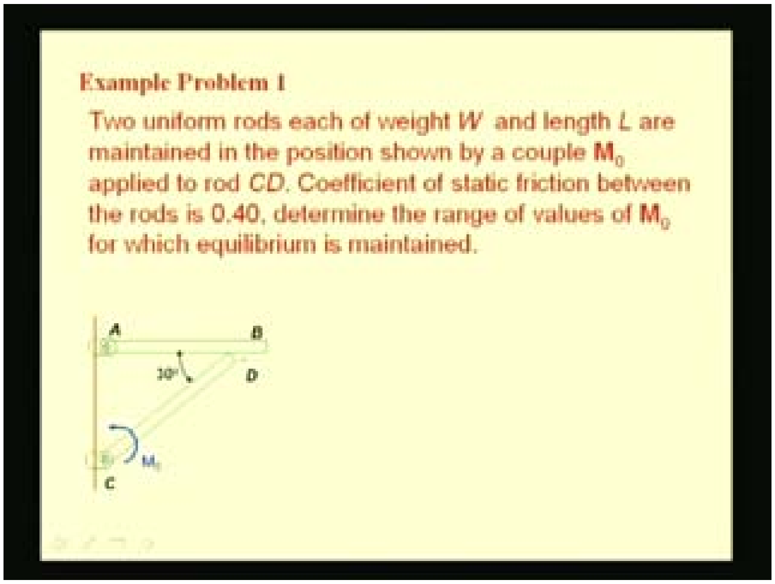

Let us see an example. Here you see two rods AB and CD pinned at A and C. They have a uniform weight of W and they are maintained in a position by the couple M_0 at C. The coefficient of friction occurring between the contacting surfaces is 0.40. We are interested to determine the range of this moment which can maintain this equilibrium.

We see that if this moment is large then this rod CD starts rotating in the counter clockwise direction, thereby this point D starts moving against this rod AB towards A. If this moment M is small and is not able to support these loads, then this rod CD starts to rotate in the clockwise direction, thereby this point of contact D moves from A to B. We can consider the impending motion in the clockwise and counter clockwise directions in order to find the moments that are required.

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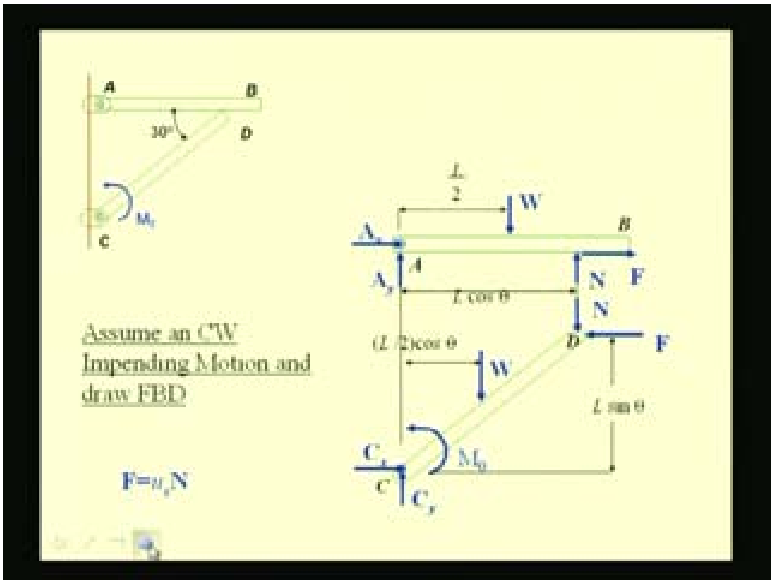

Let us consider the impending motion in the clockwise direction. That means this rod CD tends to rotate in the clockwise direction. So to construct the free body diagram, we should know the direction of the frictional force that exists at this contact point. If we see this rod CD, when it tends to move in the clockwise direction the point D tends to move towards B.

The frictional force will resist always the motion so it has to be in the opposite direction of the impending motion, so that means the frictional force will be from B to A. From this, we can draw the free body diagram. For the rod CD, we have the pin reactions C_x and C_y, the applied moment M_0 at C, and the weight of this rod CD acting through its CG. At D, since the impending motion is that the point D tends to move towards B this force is against the same; so the force is marked from B to A.

On rod AB this force will be equal and opposite; so it is marked from A to B. We have the pin reaction at A, the weight of the rod, and the normal reactions as the other forces occurring in the free body diagram.

From this, let us write the required equations to find the unknowns. We are interested to find these frictional forces that exist when the rod impends to move and the required moment. We can take the force summation and moment summation equations and for the impending case relate this F with this N using this equation.

We see that we have a total of two unknowns here, and two unknowns here; so four plus two that is six unknowns in this equation. Additionally, we have one moment which is also unknown so totaling to seven unknowns. But we have only two free body diagrams, which results in six equations only. For the impending case we have an additional relation that is the relation between this F and N, which are related by this mu_s; so leading to one more equation. The seven unknowns can be found using these seven equations.

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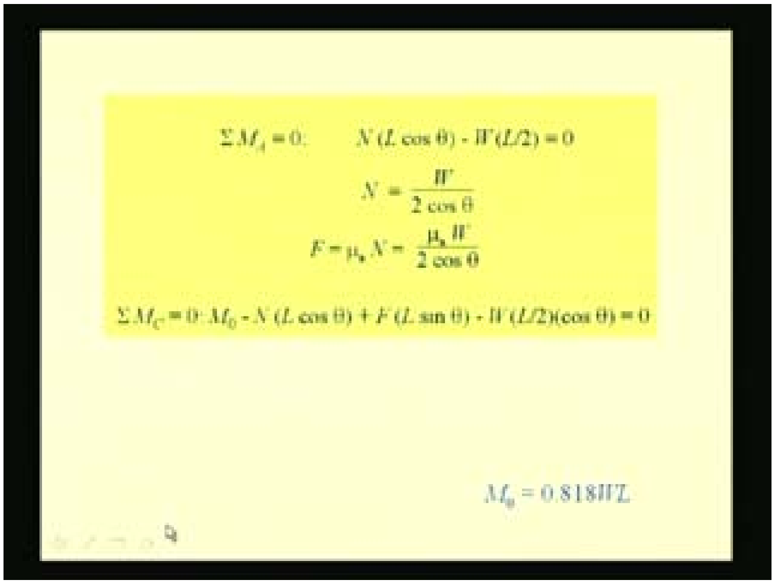

Let us write only those equations which are of interest. We sum the moment about A and equate it to 0. From this we obtain this normal reaction N from the free body diagram 1, which is equal to W by 2 cos theta. We know the relation between the frictional force and the normal force; they are related by this coefficient of static-friction. So the frictional force becomes mu_s W by 2 cos theta.

Let us sum the moments about C. If we sum the moments about C, we have M_0 the moment of this weight which is W times L by 2 cos theta, and the moment of this force N whose momentum is L cos theta, and the momentum for this frictional force F is L sin theta. These are taken care in this moment equation. By solving this equation, we know these values, i.e., N, the frictional force F and W. We get this required moment and it is 0.818 times of WL. In the similar way, now we can consider the other impending motion.

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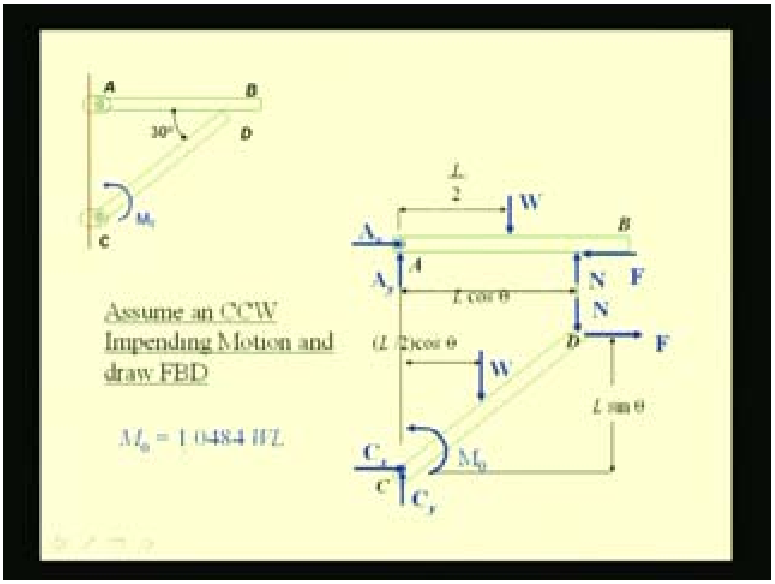

The other impending motion is that this rod CD has a counter clockwise motion. That means this point B tends to move from B to A, thereby the frictional force developed at this contact point is from A to B which is opposite to the impending motion.

For this impending motion case, the free body diagram is shown in this picture. This frictional force F is marked in the direction A to B because at this point D for the impending counter clockwise motion this D moves from B to A. So this completes the free body diagram and again we have the set of equations which can be solved to find this moment. In this case, it has been found as 1.0484 WL.

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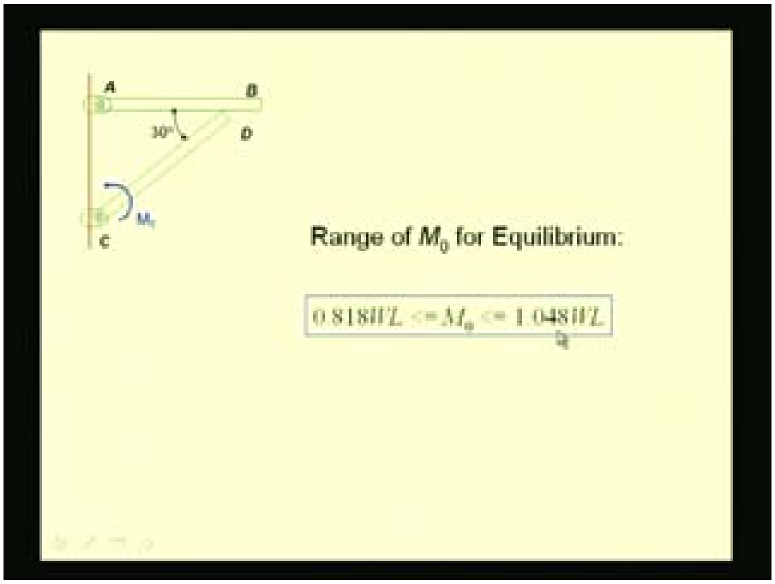

The range of moments for which these members are maintained in equilibrium is between 0.818 WL and 1.048 WL. We see that when friction exists for equilibrium, there are a range of moment values for which equilibrium is maintained.

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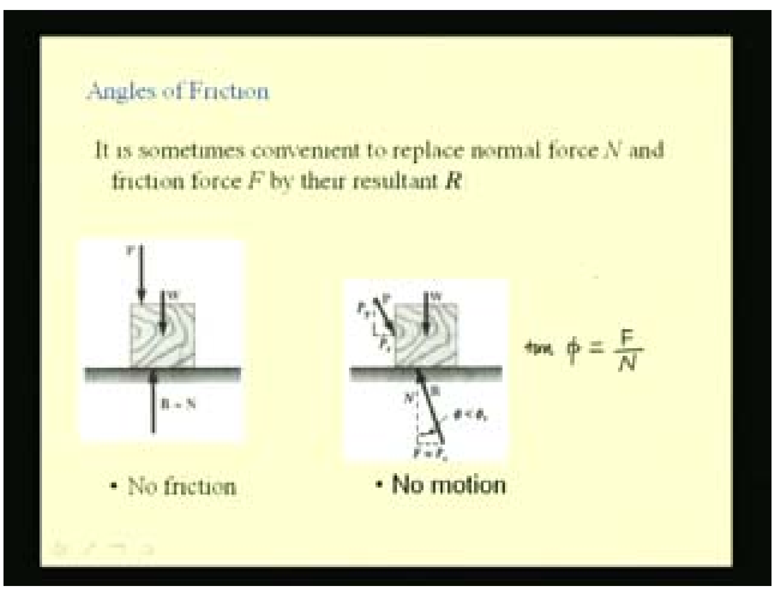

In certain problems, it is convenient to represent this frictional force F and the normal force by their resultant and relate the angle made by this resultant force with respect to the normal. This helps in certain analysis where this angle, what we call as the angle of friction, helps in the analysis procedure.

Let us consider the same example of the block. Now the applied force is vertical, thereby there is no horizontal component of the reaction; that is, there is no frictional force. So the resultant of the frictional force and the normal force is R, which is same as N. Let us consider a horizontal force P_x acting on the body and still, this force P_x is less than the limiting value of the frictional force F_m; so the body is still stationary. For this condition a frictional force F develops, which is equal to this P_x in magnitude and opposite in direction of this load P_x. The resultant of this frictional force and the normal reaction is R and now this R is inclined to this N. The angle of the same phi is given by this relation where tan phi is equal to this F by N.

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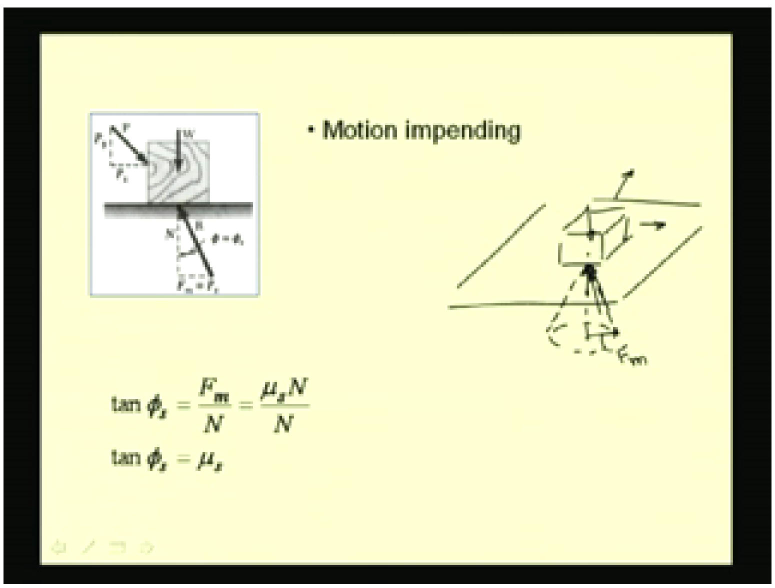

If the motion is impending then this angle is equal to phi_s, what we call as the angle of static-friction, in which case the developed frictional force is equal to the maximum limiting force that is F_m. So we have tan phi_s as F by N.

In this case, it is the limiting force F and which is equal to mu_s times N by N and which simplifies to mu_s. So for the limiting case, the angle made by the resultant is related to the coefficient of static-friction. So if we consider the motion in all directions of this plane then we have this force R making a cone, having a half cone angle of phi_s. If the resultant is within that cone then the motion is not impending.

Let us assume that there is a plane and there is a block and this is the contact point. If the forces are applied such that this block tends to move in any of these directions and if the resultant of the normal reaction and the developed frictional force lies within this cone, say like this, then the motion is not impending. But when the frictional force reaches this limiting value that is F_m then the resultant lies on this cone which we call as the cone of friction; then we say that the motion is impending. This kind of an analysis is sometimes useful in certain classes of problems.

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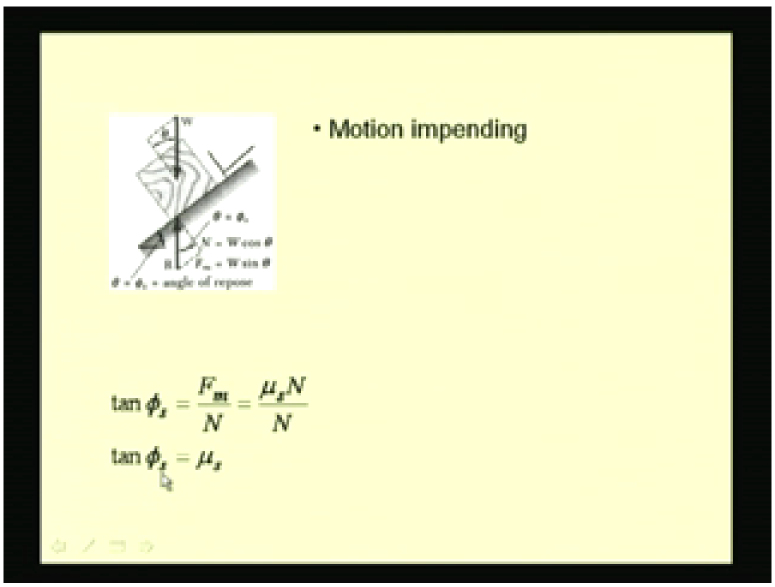

Here we see a block in an inclined plane. So the horizontal component or the component of the weight along the plane, if we take these as our coordinates, then the component of this weight along this plane tends to move this object in the downward direction. So a frictional force develops which acts against this force which is this F_m.

We have for the limiting case this F_m equal to W sin theta and the normal reaction equal to W cos theta. Also for this limiting case, we know that this angle between the normal and the resultant force is phi_s. From geometry, we see that this angle phi_s has to be same as this angle of inclination theta. For the impending motion case, we have theta equal to phi_s which is equal to the angle of the force. For this case, again, tan of this angle phi_s is equal to the coefficient of friction mu_s.

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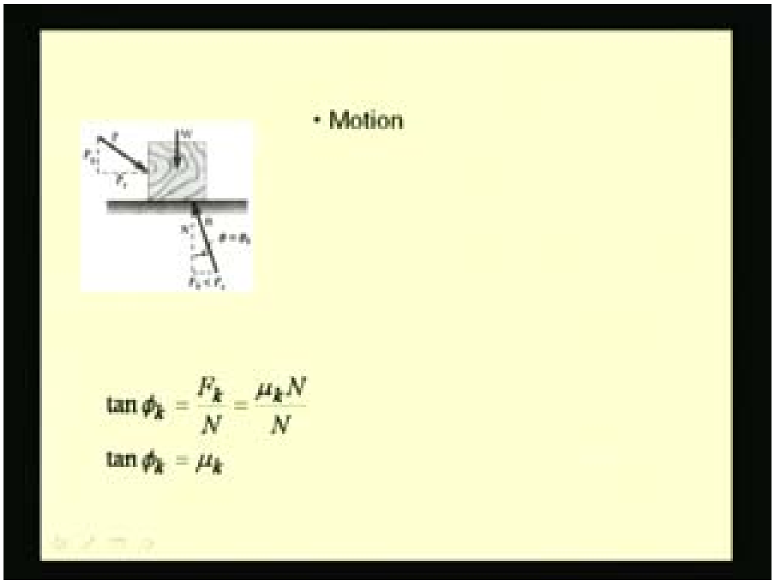

When the motion starts, we know that this reaction drops that means its inclination comes down because the horizontal component of the force reduces and becomes the kinetic-frictional force F_k which is less than the applied horizontal force. The angle of this resultant with the normal is equal to the angle of kinetic-friction phi_k. This angle phi_k is related to the coefficient of friction mu_k by considering these force relations.

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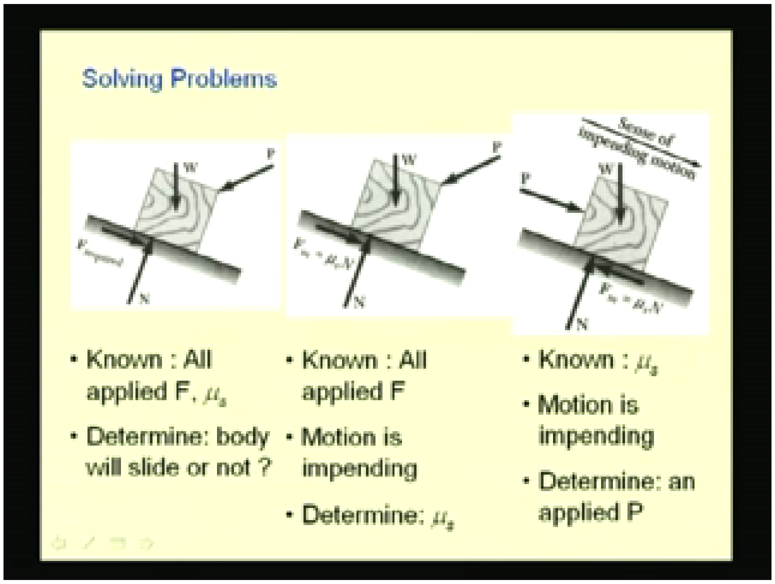

When we solve problems involving friction, we may have different situations. That means we may be interested in certain cases to determine the coefficient of friction that is needed, or in some other cases we may be interested in finding the required force in order to cross a required motion, etc.

Let us see the class of problems. There could be class of problems where we know all the forces as well as the coefficient of friction that exists between the various surfaces and we are interested to find whether the body will move or not. The other class of problems can be that we know these forces and we also know that the motion is going to be impending and we are interested to find the required coefficient of friction. There are other classes of problems where we know this coefficient of friction and the condition that the motion is impending and we are interested in determining this applied force which is required to cross the motion.

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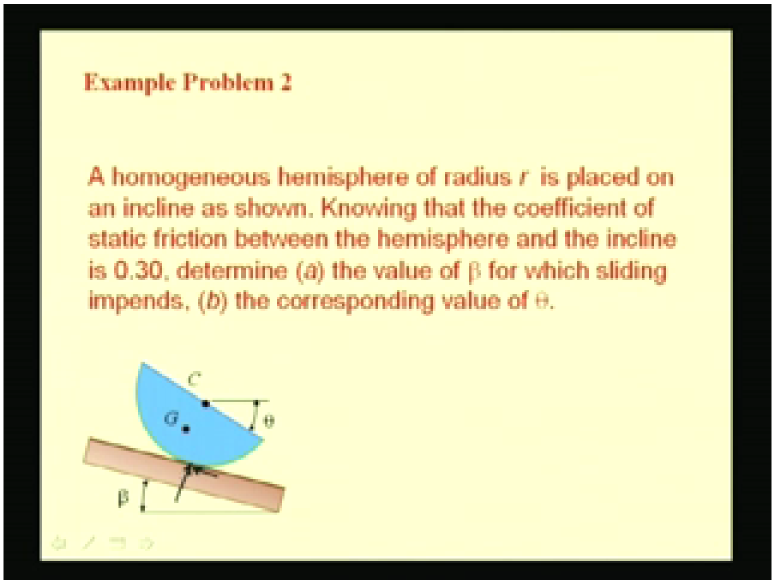

Let us take one example. Here you see a homogeneous sphere of radius r placed on this inclined plane. This is making an angle of beta with respect to the horizontal. For any given position this hemisphere also makes an angle theta that means the diametrical plane makes an angle theta with respect to the horizontal plane. The coefficient of friction is given. The radius of the sphere is given.

We are interested for the condition when this hemisphere starts to slide or impends to slide then what are the corresponding angles, that is, beta and theta? For this body to be in equilibrium, the weight of the sphere which acts through its CG has to be supported by the resultant of the reactive forces that occur at the point of contact. That means the normal reaction plus the frictional force that is along the plane and the normal force; the resultant of these two forces should be able to support the weight of the sphere. Obviously, the point of contact should lie just below the CG of the hemisphere.

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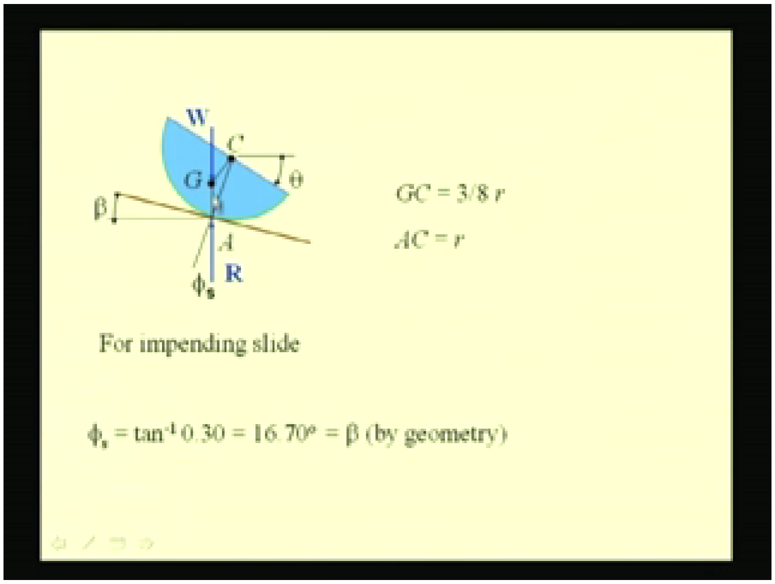

This picture shows the free body diagram of the hemisphere; the weight of the hemisphere passing through it is CG and the resultant of the normal reaction and the frictional force passing through the point of contact A which is lying just beneath G. From this geometry, we know this GC which is nothing but the location of the center of mass which is equal to 3 by 8 times of r from the diametrical plane. This distance AC is nothing but the radius of the hemisphere. From the geometry, we also see that the angle between the resultant r and the line AC is equal to this angle beta, which is nothing but the inclination of this surface.

For the impending motion case where the hemisphere starts to slide along the inclined, this angle, that is, the angle between the resultant and the normal to the inclined plane is equal to $phi_{s,}$ the angle of static-friction. Since we know the coefficient of the friction as 0.30, we can determine the value of phi_s and that is equal to this angle beta from geometry.

We have to find what this angle theta for this geometry is. To find that let us consider this triangle AGC.

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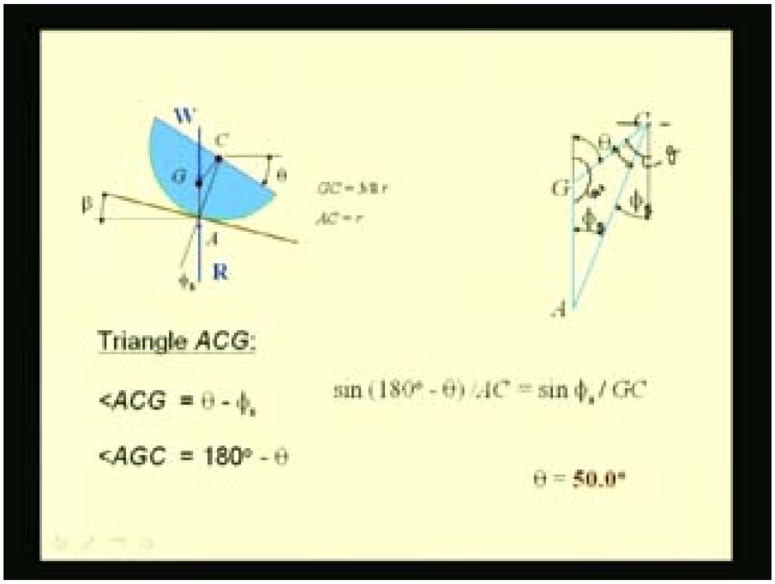

This picture shows the point of contact, the CG and the center of the hemisphere and the various angles that are involved. From geometry, we can see that the angle between the horizontal and this diametrical plane is equal to the angle between this line GC and the vertical line that is GA. This picture marks the various angles theta and phi_s. From this, we get this angle ACG, the angle ACG, as theta minus phi_s because we know that if we consider this triangle, if this is the angle theta then this angle is also theta. So this angle is theta minus phi_s.

This angle AGC is nothing but, this total angle is 180 degrees, so 180 degrees minus theta. We know the relation that sin of the angle divided by the opposite edge of the triangle is equal to the sin of the other angle and the other edge, so the relation between the sin of the other angle and the opposite edge.

We take this angle opposite to this edge AC that is sin of 180 degrees minus theta divided by AC equal to sin of this angle phi_s divided by this length GC. Since we know this dimension GC and also we know this dimension AC and we know this angle phi_s, from this we can determine this theta. The same has been determined as 50 degrees.

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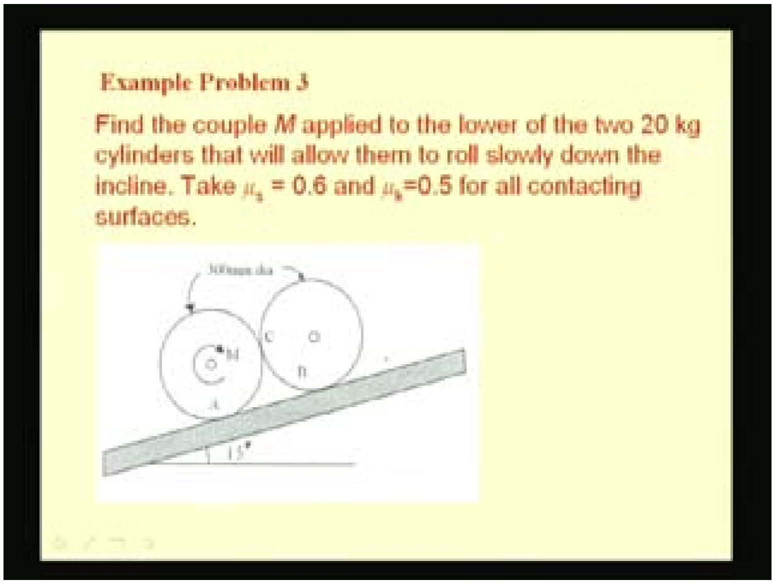

Let us consider another example. Here you see two cylindrical rollers each weighing 20kg kept on an incline whose inclination is 15 degrees. Their diameters are given as 300mm. The values of coefficient for friction, both static and kinetic are given for all the contacting surfaces.

Moment of M is applied to this cylinder in order to allow for these two cylinders to roll slowly down this incline. So these are rolling such that they are rolling slowly down the incline. We are interested to determine the value of this moment M that is required to allow for this slow motion to be possible.

In order to solve the problem, let us consider the free body diagram for each of these cylinders. Since these cylinders are rolling without slipping at these points A and B, the frictional forces that are developed at these contact points are less than the limiting value. But in order for the slipping to occur, these two cylinders roll down, this point at C has to have a relative motion. At this point, slippage is occurring and since these rollers are moving slowly, we have to consider the coefficient of friction mu_k because it is not the impending motion case, but actually the motion is occurring.

The frictional forces that are developed at this point of contact are the kinetic-frictional forces. With this, let us draw the free body diagram.

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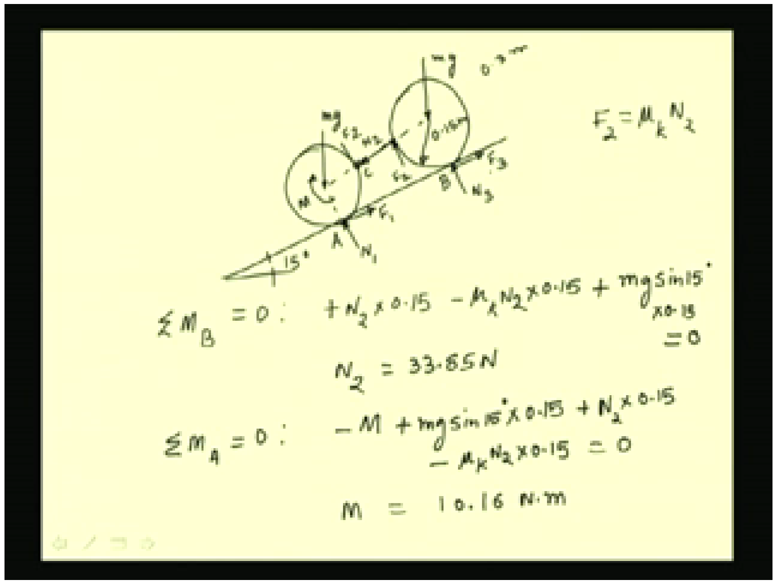

We have the incline, inclined at 15 degrees and the two rollers. At this point of contact for the roller, let us say the normal reaction is N_1 and we have a frictional force F_1 along this incline; because, this cylinder if allowed will try to slip in this direction and the frictional force has to prevent this slippage. At this point of contact, the frictional force is upward along the incline.

We have the applied moment M and the weight of the cylinder that is mg, which is acting. Let us say this is the line connecting the centers of the rollers. At this contact point C we have the normal reaction, let us call it as N_2. Since this roller is slipping at this point C in the counter clockwise direction, the direction of the frictional force is opposing the same and the same is marked here.

On the other cylinder, the direction of this frictional force will be opposite and this will be the direction of this force N_2. At this contact, we have the normal force, the frictional force and the weight of the cylinder, say mg. We should keep in mind that at A and at this contact point B there is no slippage, so we do not have the relation between the frictional force and the normal reaction. These frictional forces F_1 and F_3 are less than the maximum frictional force that is possible.

From this free body diagram, let us write the equations. We can sum the moments about this point B in order to determine this normal reaction $N_{2.}$ This frictional force F_2 is related to this normal force by the kinetic coefficient of friction because here slippage is happening and they are in constant motion. We relate this force to the normal force by the coefficient of kinetic-friction.

Let us write the equation, moment summation about B and equate it to 0. We have minus N_2 times the momentum. We know that these rollers have 0.3 meters diameters, so this radius is 0.15 meters. The momentum for this force N_2 is 0.15. This force has a moment about this point B, which is equal to N_2 times of 0.15. Then we have minus mu_k times N_2, so this force is positive. This is the negative force mu_k times of N_2, which is this force F_2 which is also having a momentum of 0.15. Then we have the moment due to this force which is plus mg sin 15 degrees into 0.15. From this, we get N2 as 33.85 Newtons.

Let us consider the other free body diagram and write the moment summation about point A. We have sigma M_A equal to 0. We have this moment which is clockwise, so we have minus M. Then the moment of this weight mg about this point A, so we have plus mg sin 15 degrees. The momentum for the same is 0.15 meters plus N_2 times 0.15 minus mu_k times of $N_{2,}$ which is the frictional force F_2, times 0.15 equal to 0. From this, we determine M as 10.16 Newton meter. This is the required value of the moment in order to allow these cylinders to roll slowly down the incline. These problems of simple contact nature can be solved in similar sense.

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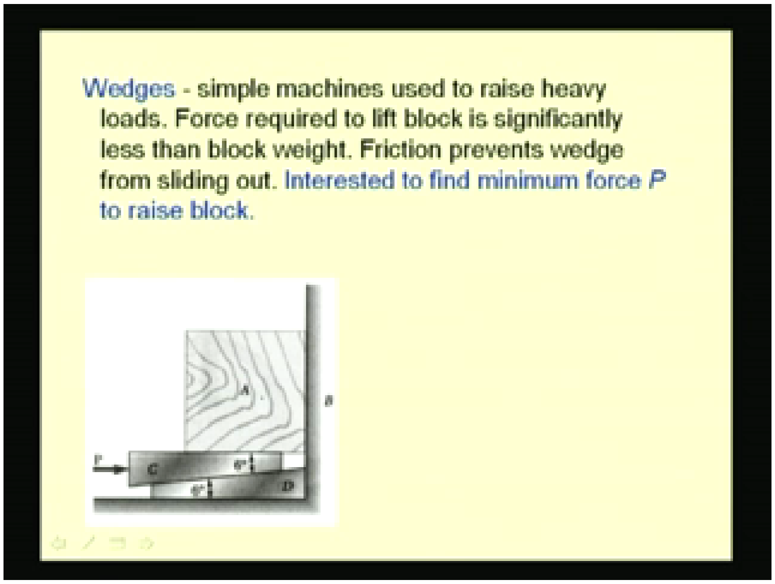

The simplest machines that use friction in order to do some work are wedges. These are devices that are used to raise loads or to keep loads in certain position. They advantageously use friction to accomplish this job. Here in this picture you see this block A which has to be raised or supported in this position. We use these wedges C and D in order to accomplish this task.

A typical problem could be that we are interested to find the force P that a human has to apply, in order to keep this block in this position or to raise this block. In order to analyze this problem, we have to consider the free body diagrams of the individual blocks and wedges and represent the frictional forces in proper sense that means we need to know the impending motion direction, in order to draw these frictional forces in the free body diagram.

Let us consider this wedge C, which is being pushed by the human by this force P in order to raise this load or to keep this load in this position. Since the wedge tends to move in the direction from left to right, the frictional forces on these surfaces will oppose this motion, thus they will be acting from right to left.

Let us mark this on the free body diagram.

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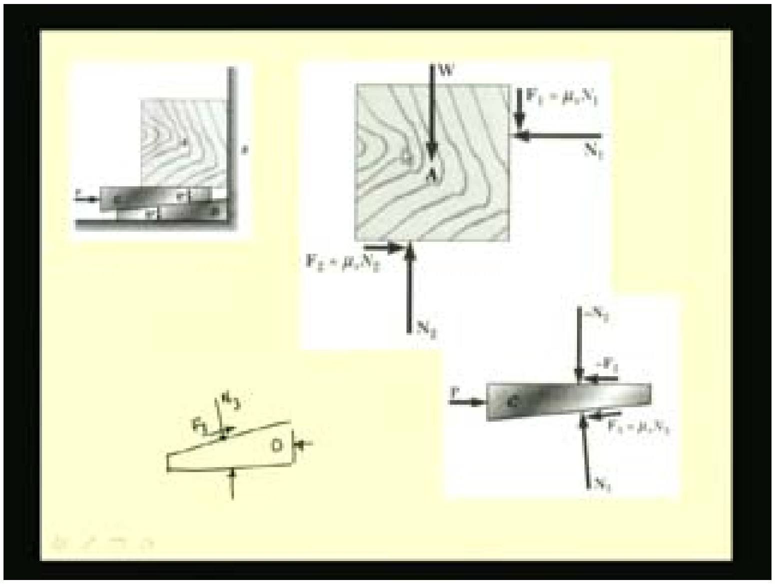

We have this block and the free body diagram of the wedge. Since at both these contact points this wedge C is trying to move from left to right, these frictional forces oppose the same and thus they are marked from right to left. These are the normal reactions. For the impending case this can be related to the normal reaction using this coefficient of friction, mu$_s$.

Once we know this, we can construct the free body diagram of these other two components that means the wedge D. The frictional force F_3 is acting in the opposite direction and we have this normal reaction N_3. We have the normal reaction on this face and the reaction from the wall.

We can consider the free body diagram of the block. On the block face, this force F_2 is equal and opposite to the same, so it is marked in this direction. We have the normal reaction and the weight of the block and since this block is tending to rise against this vertical wall, we have this frictional force opposing the motion. We have this force F_1 and for the impending case, this can be related to the normal reaction by the coefficient of static-friction mu$_s$. Once we construct these free body diagrams, it is possible to solve for the same.