Engineering Mechanics

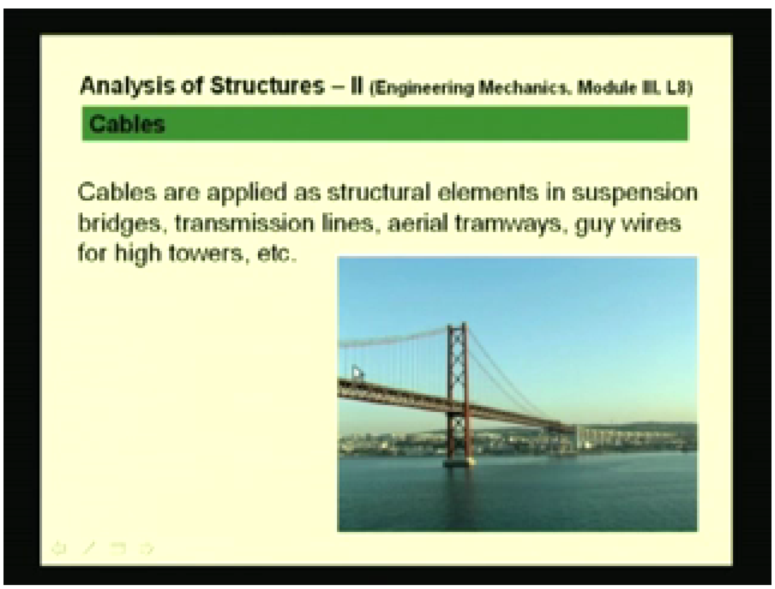
Dr. G. Saravana Kumar
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module 3 Lecture 8
Analysis of Structures - II

Cables

Today, we will continue our discussions on internal forces. In the previous two lectures, we saw how to determine internal forces in the structural members and particularly we discussed about finding these internal forces in beams. The other class of structures that are predominantly used are cables.

(Refer Slide Time: 01:35)



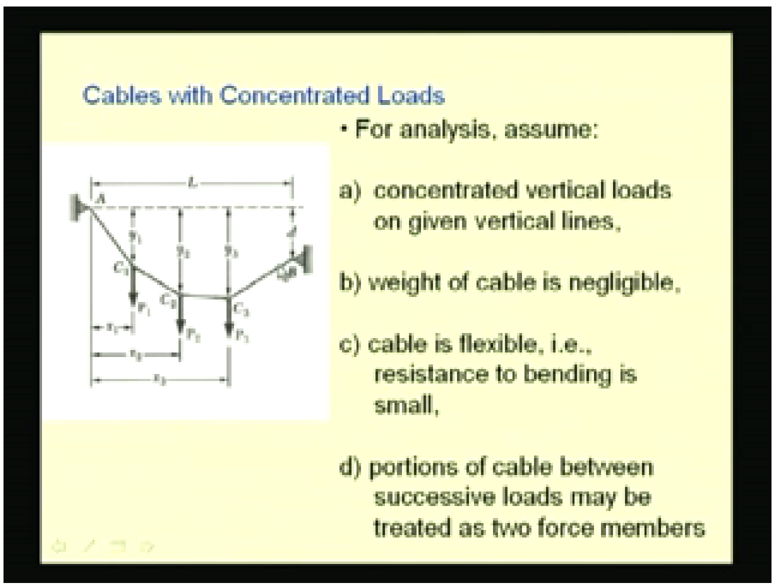
We will see how to analyze the internal forces in cables in this lecture. The cables, being flexible elements, are subjected to tensile forces only because they cannot resist any compressive force and they cannot also take any bending moment.

Some examples of cables are: suspension bridges, transmission lines, aerial tramways, guy wires for high towers, etcetera. In this picture, you see a bridge that is being supported by these cables;

so this is the suspension bridge. You have these vertical cables tied to the platform of the bridge and supported by these cables.

The primary concern in the design of such structures is to know the shape of the cable when the cable is loaded with various kinds of loads that could be concentrated or could be a distributed load. Accordingly, we would like to know the lengths of the cable to be used so as to maintain, let us say, the platform in the horizontal position or in the required position. So, we will start the discussion with concentrated loads and then move on to distributed loading on cables.

(Refer Slide Time: 03:41)

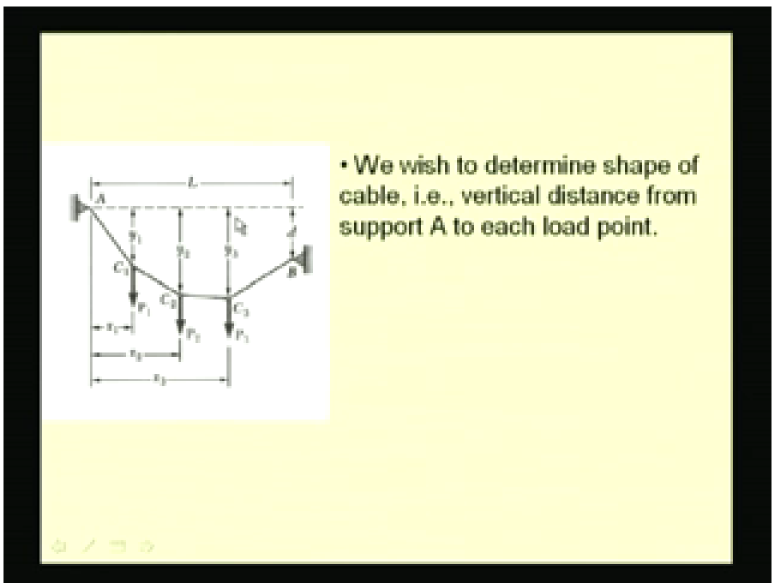


Here you see an example of a cable that supports concentrated loads. The cable is supported at A and at B and carries some concentrated loads say P_1, P_2, P_3, etcetera at various locations C_1, C_2, and C_3. These loads as well as their horizontal spans are known. The interest is to know these y coordinates, that is, the shape the cable will take once these loads are being applied to this cable.

Certain things we can note for analysis, we assume that these locations are given. That means the horizontal spans x_1, x_2 and x_3 where these loads are applied to the cable are known. We also assume for the current analysis that the weight of the cable to be negligible when compared to the concentrated load; else, the shape of the cable will be different if we also consider the weight of the cable because it becomes a uniformly loaded cable in that case. But for the current analysis, we assume the weight is negligible compared to the concentrated loads. This resistance to bending is also very low for the cables, so these end reactions will be only force reactions and not any bending moment.

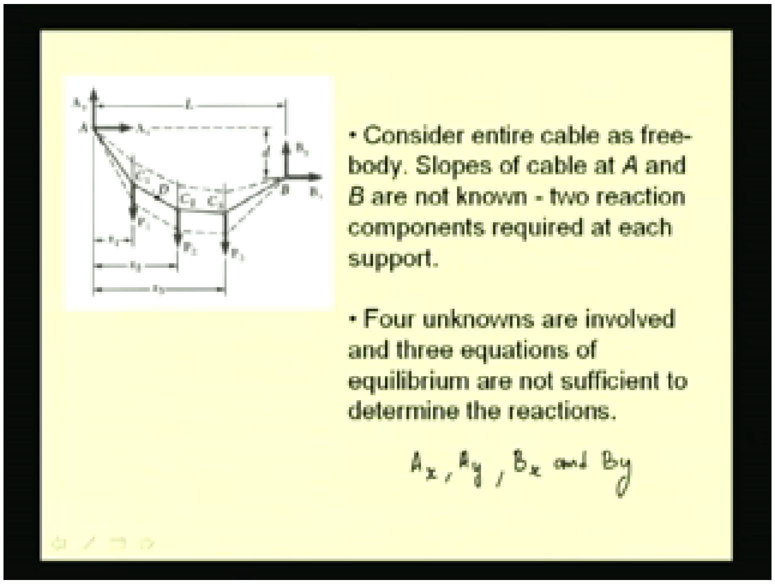
Between two portions, say C_3C_2 or C_3B the cable can be assumed to be a two force member. This is because the loads are applied at C_3 and at B and we are neglecting the weight of this portion so it becomes a two force member. Since we have seen that the cables can only be loaded in tension, the forces developed in this portion are tensile and their direction is that of the pulling nature.

(Refer Slide Time: 06:41)



Our interest is to determine these vertical coordinates of the point of loading or in other sense the shape of the cable.

(Refer Slide Time: 06:55)



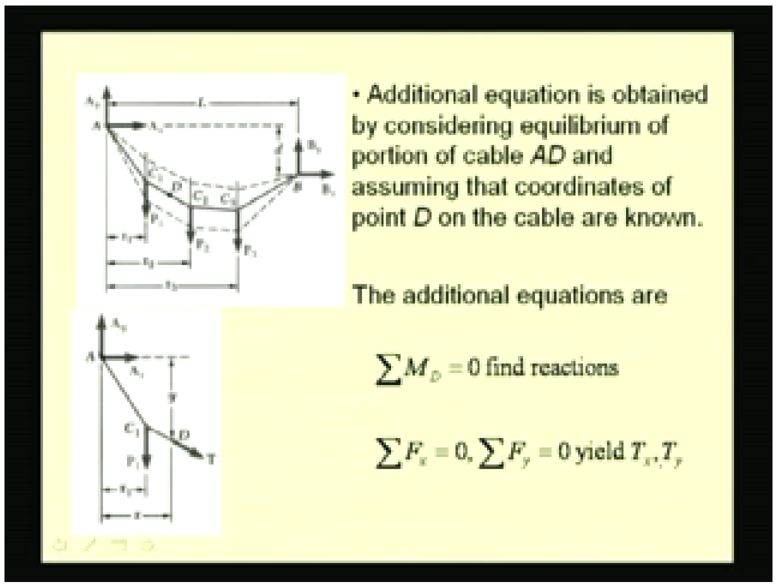
Let us consider the free body diagram of the complete cable in order determine the end reactions. In this picture, you see the free diagram of the cable. You have at A, two components of reactions A_x and A_y because we saw that the cable cannot take any bending moment; since the cable is fixed to this point A, the end reactions have to be only two force components and no bending moment. In same way at B, the cable is fixed; so, there will be two components of reaction B_y and B_x.

The shape of the cable is unknown; that means, it could be something like this: the dotted line shown or it could be something like this or any other position. So we do not know the end tangent of this cable, that is, the direction of this line AC_1. We also do not know the direction of this line that is C_3B. So we have two unknown components at B and two unknown components at A.

We see that we have a total of four unknowns involved in this: that is, A_x, A_y, B_x and B_y. If we consider the equilibrium of this portion for the two-dimensional cable layout, we can write three equations. From the force summation equation, we get two scalar components: that is sigma F_x equal to 0 and sigma F_y equal to 0. For the moments to be 0, we have one scalar component that is the moment about the z-axis that is perpendicular to the plane of the problem.

With these three equations, we can only find three unknowns. Since this problem has four unknowns we need one more equation to solve this; so it could be as a known point on the cable. So, if we know the y co-ordinate of let us say a point D on this cable, then it is possible to write additional equations to solve for the unknown.

(Refer Slide Time: 10:05)



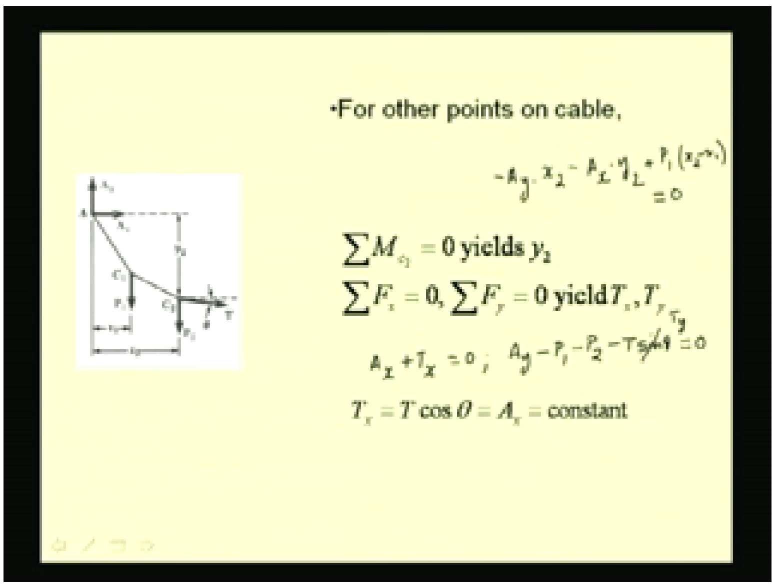
Let us assume that we know the coordinate of this point D on the cable. It could be any point either in this segment or in this segment. If we know this, let us draw the free body diagram of one portion of the cable: that is, either AC_1D or DC_2C_3B. Let us consider this portion AC_1D and draw the corresponding free body diagram.

At D, we have the tension of the cable which is both unknown in magnitude as well as direction. That is depicted by this arrow. We have these two unknown forces A_x and A_y; the reactions at A and these loads whose horizontal location is known and not the vertical location. We know this coordinate D - both x and y. So now, it is possible to write an additional equation, say the moment summation equation about point D. Now we will have three equations from the earlier free body diagram and one more equation from this free body diagram totaling to four, to determine the four unknowns.

This is the additional equation that we have and now we have four equations to determine the four unknowns. The two more equations that is the force summation will give the horizontal and the vertical component of this tensile force T, that is T_x and T_y.

Now that we have determined these end reactions it is possible to determine the tension in each of the segments and also the location, the vertical coordinate of the location of the concentrated loads by considering the free body diagrams of sections of the cable.

(Refer Slide Time: 12:33)



Let us consider this section AC_1C_2 and draw the free body diagram. Since we have determined both A_x and A_y, we know the tension that will be occurring in this segment, that is, AC_1; so we do not need to consider the free body diagram of the portion AC_1 again. But now we do not know the tension in this segment, that is C_1 C_2; so we take this segment and consider the free body diagram.

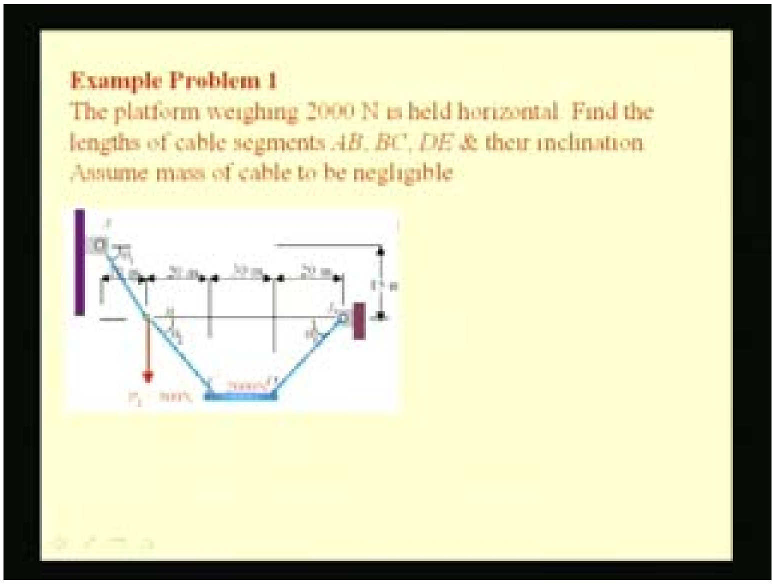
The tensile force T is both unknown in magnitude and direction; so we mark the same as both T_x and T_y as unknown. We also do not know this y co-ordinate that is the location of the point of concentrated load in the vertical direction. So we can now write the equations for equilibrium. We take the moment summation about this point C_2; since this unknown force T passes through this point, we only have moments because of the concentrated load P_1 and the moments due to this A_x and A_y, the end reactions.

This equation sigma M_{C2} equal to 0 is A_y times the momentum, which is x_2 and this leads to a clockwise moment about this point, so it is negative. A_x has a momentum of y_2 again which is clockwise, so it is negative. This force P_1 has a counter clockwise moment so it is positive. P_1 times the momentum is x_2 minus x_1 equal to 0, so in this we know A_y, A_x, this location x_2, as well as P_1. We only have this y_2 as unknown and that can be determined.

The other two equations, that is, sigma F_x equal to 0 and sigma F_y equal to 0 yields the other two unknowns that is T_x and T_y. So if we write this equation let us say sigma F_x equal to 0, it is A_x plus T_x. We have only these two forces equal to 0. For this equation that is sigma F_y equal to 0 we have A_y minus P_1 minus P_2 minus T sin theta equal to 0, this is nothing but T_y the vertical component. So from these two equations, since we have only this Tx and Ty as unknown, the same can be determined.

It is interesting to note that the horizontal component of the tension that is T_x, which is T cos theta, is equal to this reaction A_x for all the segments. So even if we consider the free body diagram of the cable for the portion AC_1 or for a portion in between C_1 and C_2; at any point the horizontal component of the tension is equal to A_x. That is because we do not have any horizontal loading in the cable, so the horizontal component of the tension is always equal to the horizontal component of the end reaction. This is useful in certain analysis.

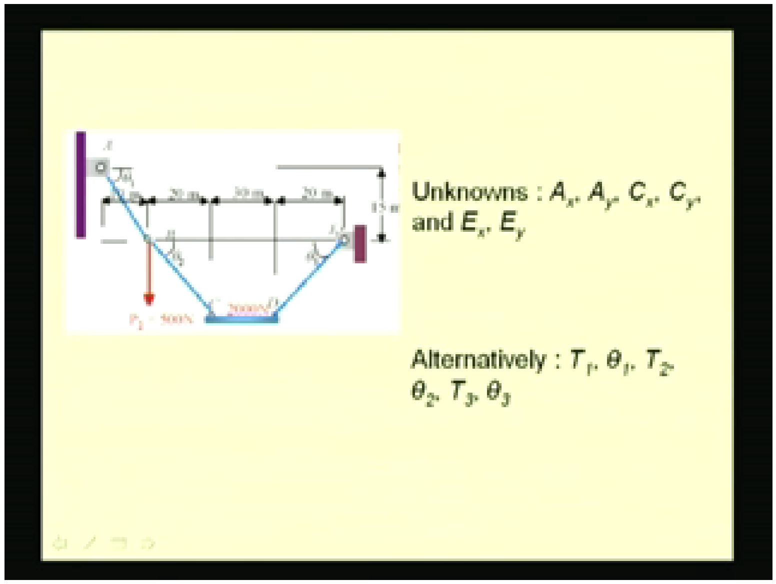
(Refer Slide Time: 18:14)



Let us see an example to illustrate the complete procedure. Here you see a platform which weighs 2000 Newtons and is held in a horizontal position using cables. The cables are tied at this point E and at point A. In order to maintain this platform in the horizontal position, we have the concentrated load P_2 which is 500 Newtons acting at a point B. This is to make sure that the cable takes a shape such that the platform is maintained in the horizontal position.

The lengths of these cables AB, BC and DE and their inclinations are to be found. We know these horizontal spans: that is the span between AB, between BC, CD and DE. For this analysis, we assume the mass of the cable to be negligible. So in order to proceed we consider the free body diagram of the complete cable.

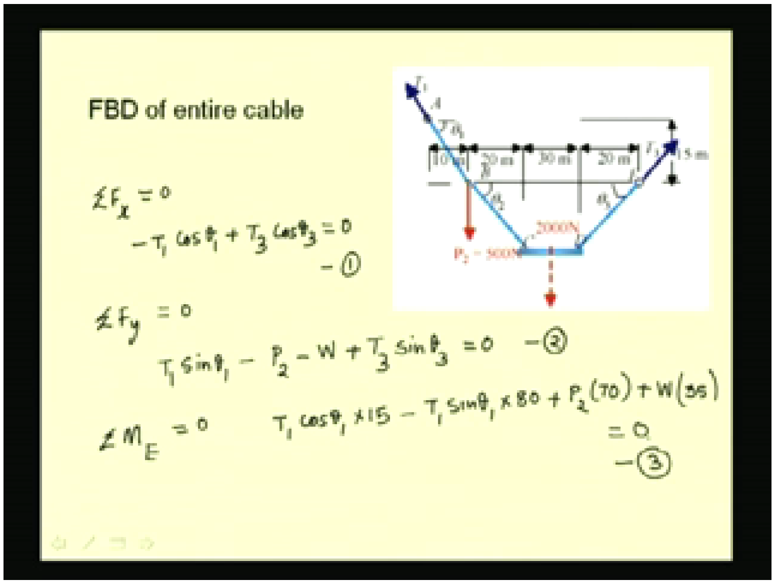
(Refer Slide Time: 19:47)



The unknowns for this problem are the end reactions, that is A_x, A_y, E_x and E_y as well as the forces at this point C, that is C_x and C_y, because if we know these forces E_x and E_y, then D_y and D_x are also known. So the unknowns for this problem are six that is the horizontal and vertical components of forces at A, C and at E. It could be otherwise stated in terms of the magnitude and the direction of the tension that occurs between various segments of the cable.

Here we have three segments AB, BC and DE. So the unknowns can be restated as the magnitude and angle of these tensions in these segments that is, say T_1 theta$_1$ for the segment AB, T_2 theta$_2$ for the segment BC and T_3 theta$_3$ for the segment DE.

(Refer Slide Time: 21:12)



Let us draw the free body diagram of the complete cable. This figure shows the free body diagram of the complete cable. At E we have this end reaction which is equal and opposite to the tension that occurs in this portion DE that is T_3, so that is what the end reaction has been marked as T_3 in the opposite direction.

Same way, at A the reaction is equal and opposite to the tension that occurs in the segment AB, so that is why we have marked T_1 in this direction. Let us replace this platform CD by a concentrated load of 2000 Newton acting through its CG.

From the free body diagram, for the equilibrium of the cable to exist, the sum of the forces has to be 0 and sum of the moments has to be 0. Let us write the corresponding equations. Let us write the force summation along the horizontal direction and equate it to 0. We have the horizontal component of this force T_1 acting in the negative direction, so we have minus T_1 cos $theta_1$. Then we have the horizontal component of this force T_3 in the positive direction, so plus T_3 cos $theta_3$. We do not have any other horizontal forces so this has to be 0. Let us say this is our equation 1.

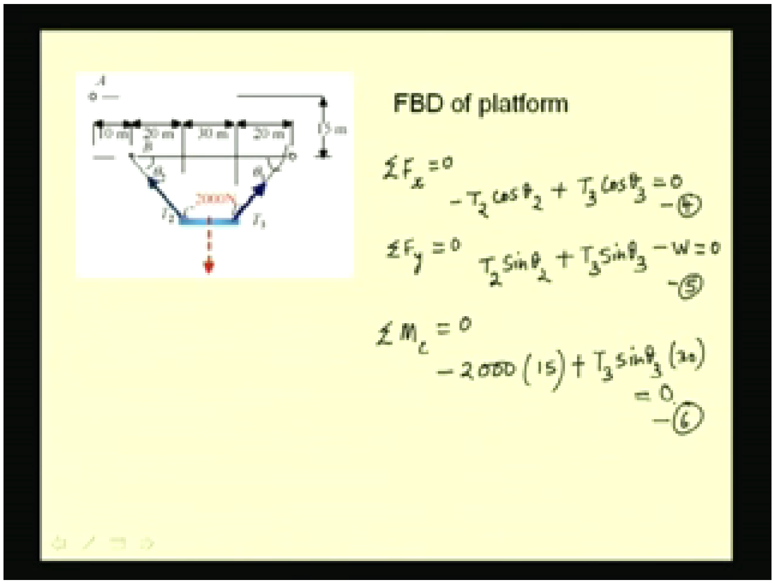
For the forces to be 0, the vertical component has to be also 0. We have the vertical component of this force at A which is T_1 sin $theta_1$ in the positive direction. We have this concentrated load P_2, so minus P_2 which is known. The equivalent concentrated load of the platform CD so minus, let us say, W which is the weight of the platform which in this case is 2000 Newtons. Then we have the vertical component of this tension plus T_3 sin $theta_3$. Let us say this is equation 2.

Then the moments have to be 0. So let us take the moment summation either about E or A where we have the unknown forces. Let us take the point E and equate it to 0. We have the moment of this force T_1. The horizontal and vertical components of this force have moments about E. Let us consider the horizontal component first, which is T_1 cos $theta_1$. The momentum for the same is 15 meters and it causes a counter clockwise moment about this point E; so it is a positive moment.

Let us consider the vertical component that is T_1 sin $theta_1$. It causes a clockwise moment about point E. So it is a negative quantity, minus T_1 sin $theta_1$. The momentum for the same is this complete span, which in this case is equal to 80 meters. Then we have the moment due to this force P_2; so we have plus P_2 times 70, which is the momentum. This also causes a counter clockwise moment, so it is positive. We have the moment due to this weight which is also a counter clockwise moment so it is positive. So we have plus W times the momentum is 35. This has to be 0.

We have got the three equations of equilibrium. From this, we can determine three unknowns but we see that we have six unknowns in this problem, so we have to consider one more free body diagram in order to get three additional equations. Let us consider the free body diagram of the platform that is CD.

(Refer Slide Time: 27:06)



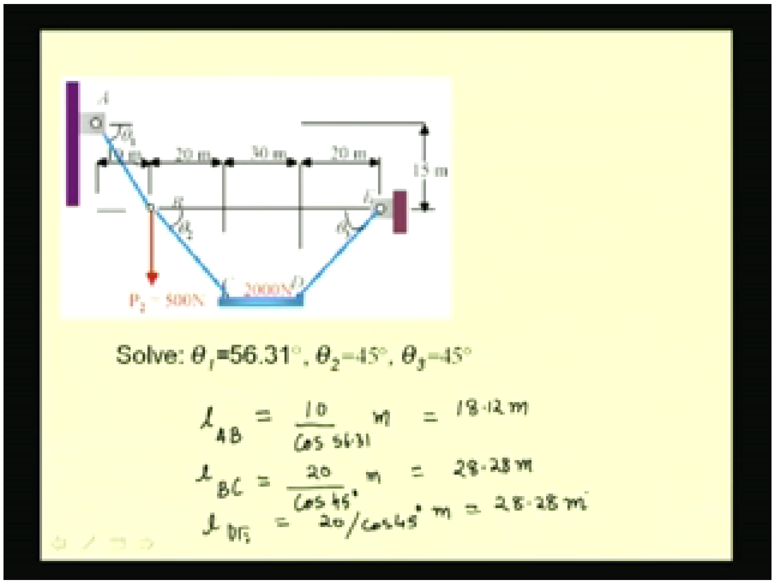
Here you see the free body diagram of the platform CD. The unknowns are the tension occurring in the portion DE and the portion BC which is T_2. From this free body diagram for the equilibrium, we can write three equations, two force summations and one moment equation.

Let us first write the force summation that is sigma F_x has to be 0. We have the horizontal component of the force T_2 cos $theta_2$, which is negative, plus T_3 cos $theta_3$, which is in the positive direction has to be 0. For the forces to be 0, the vertical component has to sum to 0. We have the vertical components; T_2 sin $theta_2$ plus T_3 sin $theta_3$, the two vertical forces at C and D minus the weight W of the platform to be 0.

For equilibrium to exist the moments have to be also 0, so either we can take moment about C or D. Here let us take the moments about C to be 0. At C this force T_2 passes through this point C. So it does not cause any moments but we have moments due to this concentrated load, the equivalent concentrated load of the platform, and this force T_3. The moment of this force is clockwise and so it is negative, so minus 2000. The momentum is 15 meters.

The moment due to the vertical component of the force T_3 is plus T_3 sin $theta_3$ times the momentum, which is 30. The horizontal force that is T_3 cos $theta_3$ passes through the point C and so it does not has any moment; so this has to be 0. So if we say that this is equation 4, 5 and 6, we have now the six equations to solve for the six unknowns.

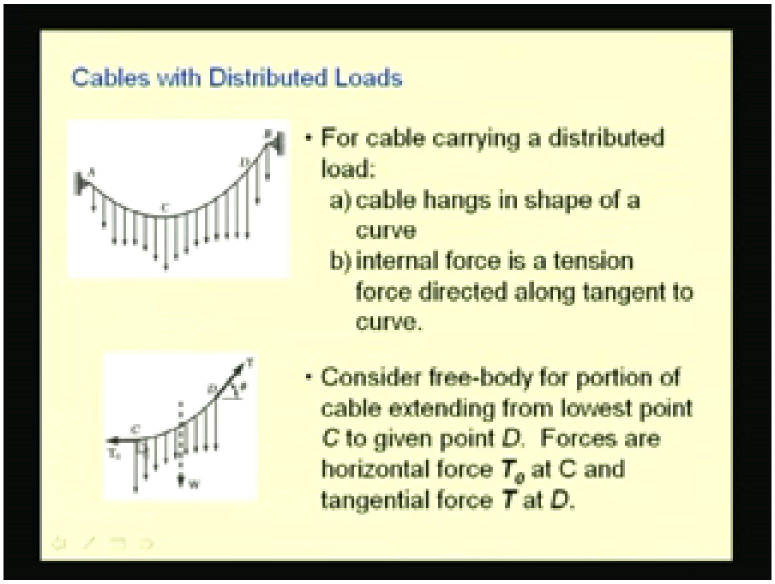
(Refer Slide Time: 30:41)



If we solve these equations, we can determine the unknowns. Since our interest is only in the shape of the cable, so here after solving we have found these angles: $theta_1$ as 56.31, $theta_2$ as 45 degrees and $theta_3$ as 45 degrees.

Once we know these angles, we can determine the lengths. The length of the portion AB is equal to 10 divided by cos $theta_1$, 10 meters which is the horizontal span divided by cos 56.31, so many meters. It is equal to roughly 18.12 meters. Similarly, the length of the portion BC is the horizontal span divided by cos $theta_2$ which is 45 degrees. It turns out to be 28.28 meters. The length of the portion DE is equal to 20 divided by cos 45 degrees, which is again 28.28 meters. This example completely illustrates the way we solve the cables when subjected to concentrated loads.

(Refer Slide Time: 32:33)

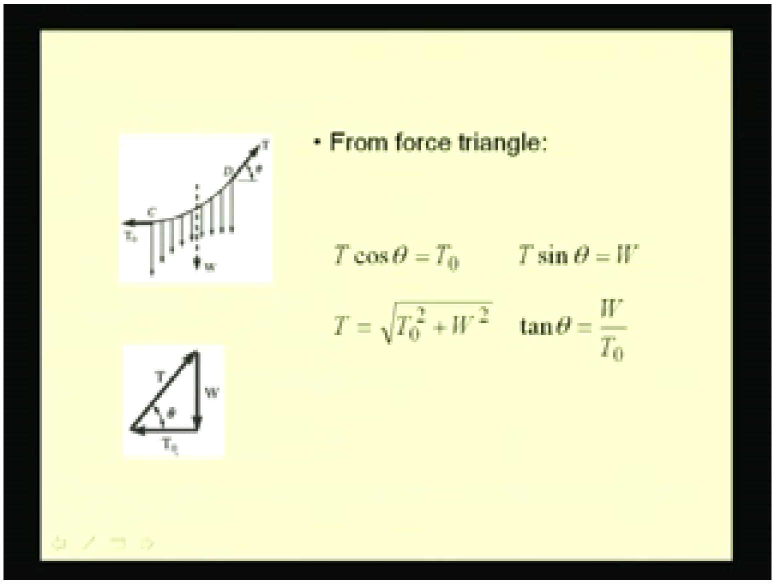


Let us move on to discuss cables under uniform loading. This kind of situation occurs in the cables which support the bridges. In the earlier example, we have seen a cable supporting a bridge. We see that there are at equal distances the cable is tied to the platform of the bridge, so it is equivalent to a uniformly distributed load in the horizontal direction for the cable. Let us consider a general cable ACDB which is tied at this point A and B, subjected to some kind of a uniform loading. It could be varying in magnitude along the cable either along the cable or along the horizontal direction.

We are interested to find the shape of the cable under the load. The internal forces that develop in the cable at any location are tangent to the shape of this curve. Let us say at C, where the tangent of this curve is horizontal, the tension in the cable is directed in the horizontal direction. At B, where it is fixed, the direction of the tensile force is tangent at this point and which is also equal to the reaction at this point B.

In order to solve let us consider a portion of this cable, let us say this portion CD and draw the corresponding free body diagram. At C since the tangent to this curve is horizontal, the tension is T_0 which is directed in the horizontal direction. At D, this tension force T is along the tangent and we do not know this angle, so this force T is both unknown in the magnitude and direction. This W is the equivalent load of all these distributed loads.

(Refer Slide Time: 35:41)

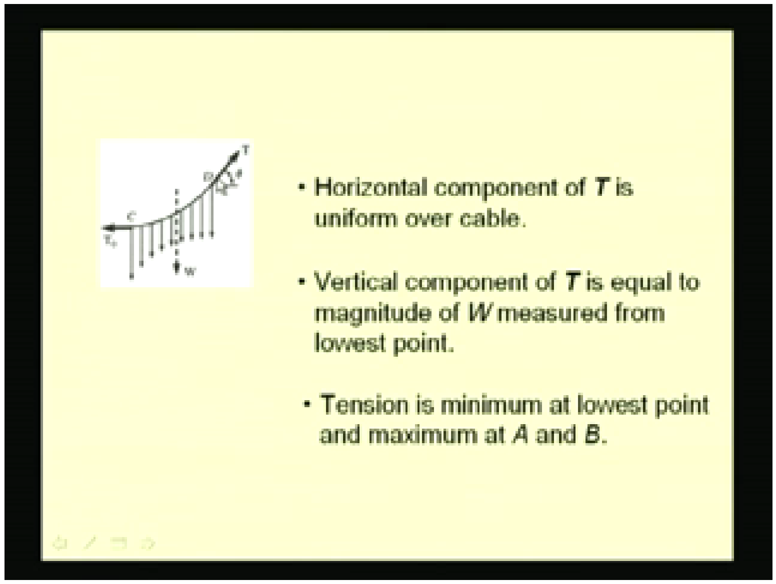


Let us construct the free body diagram and from this free body diagram write down the equations relating these forces. One graphical method of solving of the equilibrium particularly, in case of two-D is using the force polygons or force triangles. Here you see that method applied. You mark these forces T_0, T and W. For equilibrium to exist the sum of all these forces has to be 0. So that means they have to form a closed polygon where the ends of the vectors have to meet.

From this force triangle, we have the horizontal component of this tension T that is T cos theta should be balanced by this force T_0. So we have this T cos theta equal to T_0. The vertical component of this force T which is T sin theta has to be balanced by the vertical force W that is T sin theta is equal to W. Also, since this is a right angle triangle we know that T_0 is in the horizontal direction and weight acts in the vertical direction and so this forms a right angle triangle.

We have the magnitude of T as square root of sum of the sides T_0 square plus W square; also this angle theta is nothing but tan inverse of opposite side that is W by T_0. So these equations come from this force triangle.

(Refer Slide Time: 37:49)

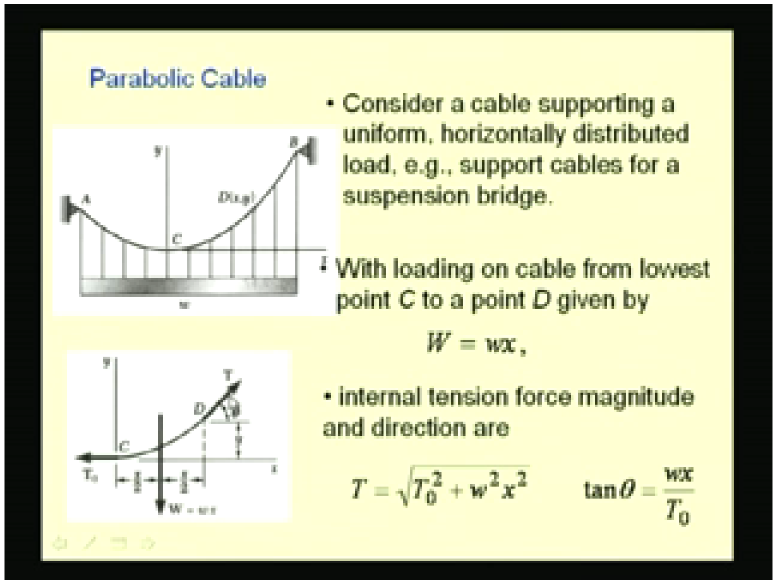


If we move this point along the cable to any other location, the tension in the cable changes, but we see that the horizontal component of this tension T which is equal to T cos theta has to be equal to the tension at this point C throughout the length of the cable. So the horizontal component of this tension force T remains constant and equal to T_0.

The vertical component of this tension equals to the total load carried between the portion C and D, where C is the minimum point on the cable. The total load carried in this portion is equal to the vertical component.

From this we see that the tension in the lowest portion on the cable that is at C is the lowest and as we move along the cable towards the end support, the tension increases and becomes maximum at the end supports that is at A and B.

(Refer Slide Time: 39:26)

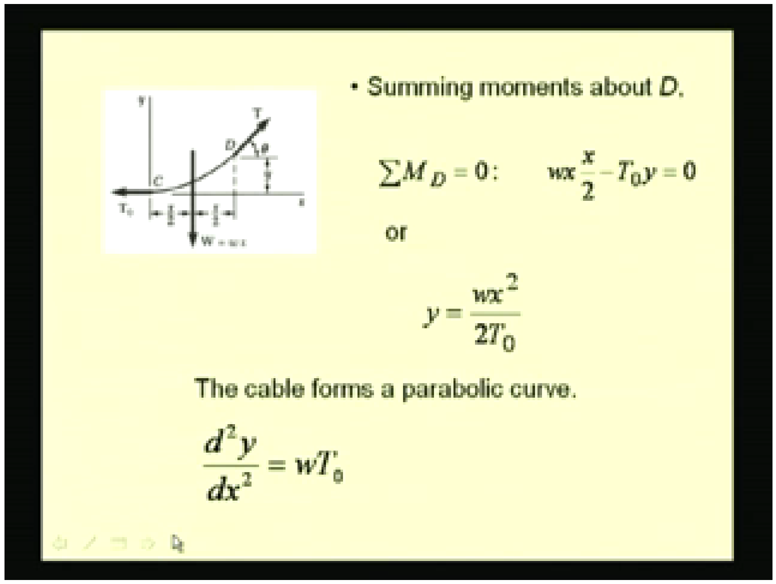


Let us consider few examples of uniformly distributed loads on cables. One, we call it as a parabolic cable because the shape of the curve taken by the cable is a parabola. This occurs when the cable is loaded uniformly along the x direction. This kind of a situation can be approximated for cables which support the bridges.

Here in this picture, you see the cable ACB supporting a bridge platform which is having a weight w Newtons per meter length of the bridge. This is connected to the cable through these vertical strings or cables, so the loading is like a horizontally uniformly loaded cable. If we consider this portion CD and draw the free body diagram, C being the lowest point on the cable we have the horizontal tension T_0. At D, the tension is tangent to this path, which is both unknown in magnitude and direction. The total load carried in this portion which is equal to w times the x span of this cable CD that is wx, which acts through the center span between C and D.

Let us find what these values are through the force triangle. The magnitude of this tensile force T has to be equal to square root of T_0 square plus capital W square, which is w square x square. This angle is given by tan theta which is equal to wx divided by T_0.

(Refer Slide Time: 42:27)



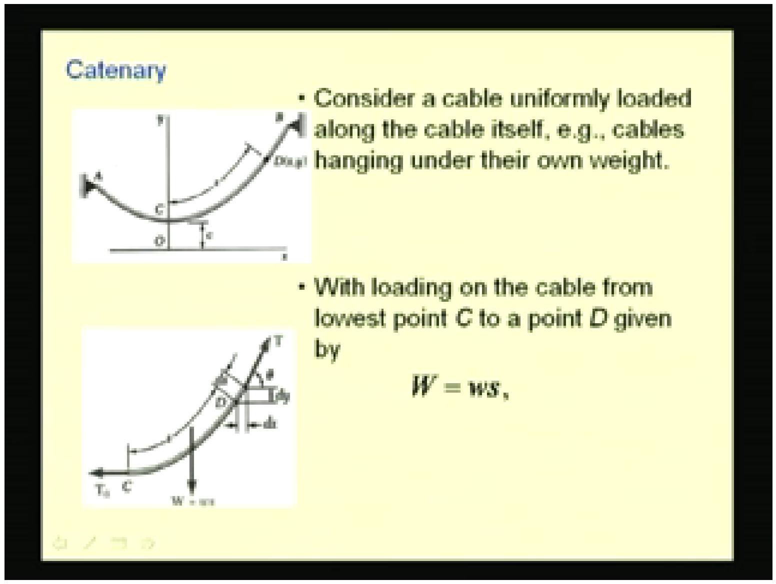
Let us sum the moments of these forces about this point D. We have this vertical force wx having a momentum of x by 2 which causes a counter clockwise moment; so it is positive. So we have wx into x by 2. The moment of this force T_0 is a clockwise moment; so we have a negative moment minus T_0 times y which is the momentum of this force T_0 about this point D.

This equation relates the y and x coordinates of the various locations of the cable. Rewriting this equation we have y is equal to wx square by 2 T_0. So this is an equation of a parabola. We have it in the form y equal to A_x square, which is the equation of a parabola, where A is in this case w by 2 T_0 square.

That is why we called it as a parabolic cable since the shape of the curve taken by the cable is a parabola. In some problems, we may be given the end conditions in terms of the tangent or in terms of the curvature, so in that case it is convenient to use the differential forms of this equation, i.e., is either dy by dx or d square y by dx square. For this cable, we have d square y by dx square as w T_0.

Sometimes there maybe some constants in the equation. Since here we have considered our origin at C, these constants becomes 0, else we generally have this equation with some additional constants. We will see some example later.

(Refer Slide Time: 45:13)

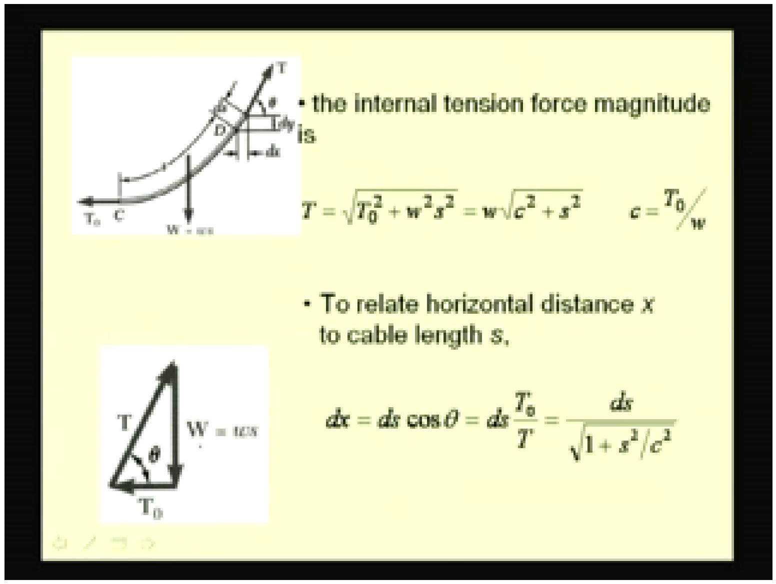


Let us consider now the cable which is loaded uniformly along its length because in all the previous cases we have not concentrated on the effect of the weight of the cable itself. The weight of the cable acts along the length of the cable and in the previous discussions we have not considered the same, so let us see how the shape of the cable changes if we consider the self-weight of the cable.

Here you see a cable ACB where the self-weight is considered and it acts along the length of the cable. At any point D which is having a coordinate x, y, has a coordinate along the length of the cable as s. Let us also assume that the lowest point along the cable that is C has a vertical coordinate of, let us say, c.

Let us draw the free body diagram of a portion that is CD. We have the horizontal component of the force T_0 at C, the tension at D which is both unknown in direction and magnitude and the weight of this cable which is ws; s is the coordinate along the length of the cable. This is total vertical weight for this portion CD.

(Refer Slide Time: 47:06)



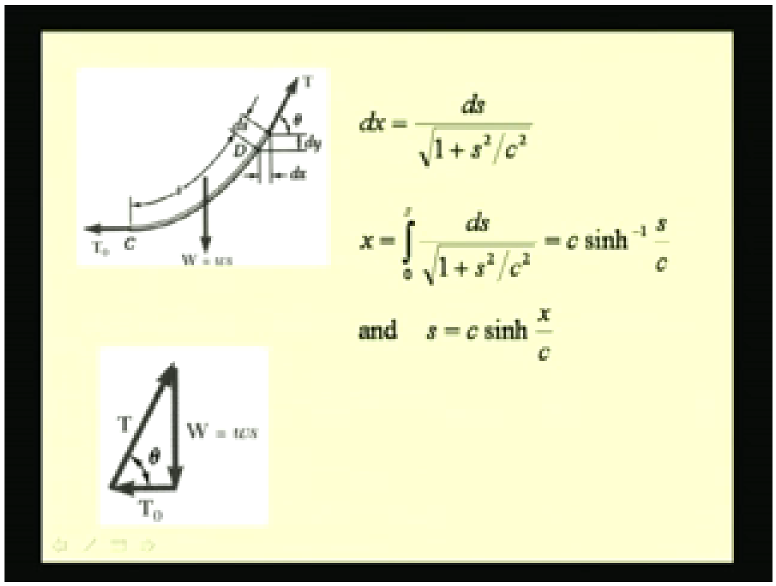
The internal tension force T's magnitude can be related to the vertical force w and the horizontal force T_0 or the tension at C as T equal root of T_0 square plus w square s square from the force triangle. If we write T_0 by w as c then this can be rewritten as w times root of c square plus s square, where c is the ratio of this horizontal tension to the uniform weight w. So this is obviously from this force triangle.

Let us relate the horizontal and vertical coordinates with the coordinate along the curve because the weight is distributed along the length of the curve, so we should be able to correlate the x and y coordinate with the path coordinate that is the length of the curve.

First, let us relate this x with the cable length s. For that, we consider this differential element d_s, which spans d_x and d_y along the horizontal and vertical directions and write the equation. For a very small element, d_x is equal to ds times cos theta and cos theta can be obtained from this force triangle which is equal to T_0 by T. If we find what is the value of T_0 by T from this equation, we have it as 1 over root of 1 plus s square by c square. This can be quickly found by substituting the value of T and T_0 from these two relations and solving.

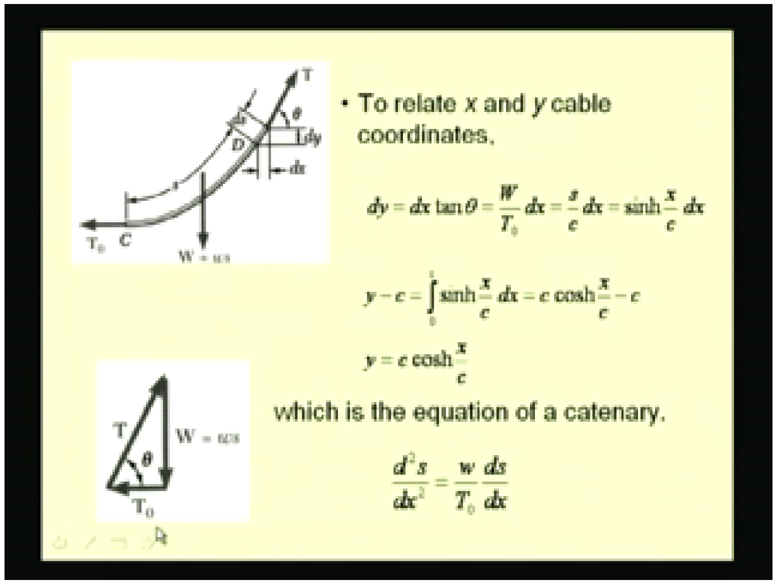
We have this d_x as ds divided by root of one plus s square by c square. If we integrate this from this point C to D we get the relation between the total length s spanned by the segment CD and the horizontal distance spanned by this curve segment CD.

(Refer Slide Time: 50:14)



We integrate it between this point and the point of our interest. So the integral is 0 to s d_s by root of 1 plus s square by c square which is equal to c times of sin hyperbolic inverse s by c. Rewriting this, we have s equal to c sin hyperbolic of x by c. This equation relates the horizontal coordinate x to the path coordinate or the length of the cable s.

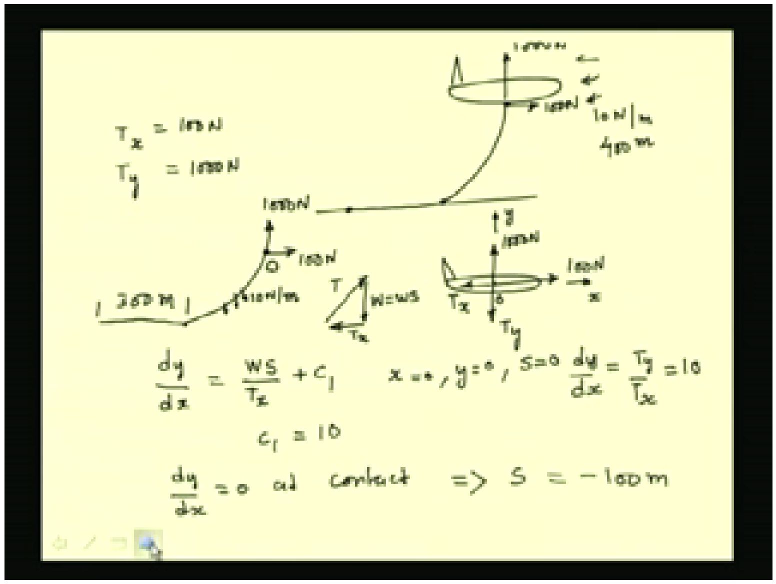
(Refer Slide Time: 51:04)



Let us relate the x and y coordinates of the cable. We may be interested in some cases to relate the y coordinate with the length of the cable also, so in that case we can again write the equations in the similar way. We write again from this differential element d_y is equal to d_x tan theta and tan theta we get from this force triangle as W by T_0, which is equal to s by c. We know that this is equal to sin hyperbolic of x by c from the previous equation that we derived for the relation between x and c.

Let us integrate this between the limits so we have y minus c equal to the integral between the limits 0 to x sin hyperbolic x by c dx, which is equal to c cos hyperbolic x by c minus c. If we rewrite this, it is y equal to c cos hyperbolic x by c which is nothing but the equation of a catenary. In some problems, we may be interested by knowing the end conditions as the tangent and the end location, so it is possible to write the differential form of this equation and solve for the same. We have the differential form as d square s by dx square which is equal to w by T_0 times ds by dx.

(Refer Slide Time: 53:33)



Let us solve one example problem. Consider a blimp which is tied to a cable such that certain portion of the cable is lying on the ground and certain portion of the cable is hanging. The weight of the cable as 10 Newtons per meter and the length as 400 meters is given. As this blimp is flying, it is developing a forward thrust of 100 Newtons against the air resistance and it is developing a lift of say 1000 Newtons.

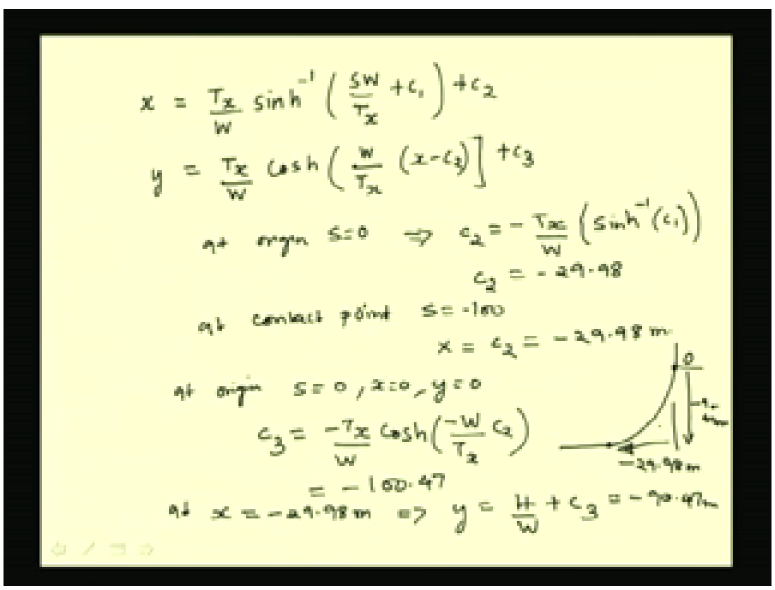
We are interested to find at what height the blimp is flying and what length of the cable is lying on the ground. In order to solve this problem, let us consider the point where the cable is attached to the blimp as our coordinate reference. If we draw the free body diagram of the blimp, we have the 1000 Newtons lift generated by the blimp, 100 Newtons forward force developed by the blimp and the tension in the cable say T_y and T_x. This is our origin and this is the positive x and positive y direction. From the equilibrium equation, we have T_x as 100 Newtons and T_y as 1000 Newtons from this free body diagram.

Let us draw the free body diagram for the cable where certain portions of the cable are lying on the ground and the remaining cable is hanging. We have these forces developed by the blimp and we are considering this as our origin. This cable is uniformly loaded by its self-weight which is 10 Newtons per meter.

In order to solve this problem, let us take the differential relations between y and x which is equal to ws by T_x plus C_1. This comes from the force triangle. If we consider any portion of the cable, we have the horizontal component of the tension force T_x balanced by the tension force T, the weight of the cable which is ws and the horizontal component T_x balancing the forward thrust of 100 Newtons. So from this force triangle, we have this equation in the differential form.

We know that at the origin, where x equal to 0 and y is equal to 0, we have the length of the cable to be 0 and dy by dx which is a tangent is equal to the ratio of T_y and $T_{x,}$ which is 10. If we substitute this, we find C_1 as 10. At the contact point where this cable contacts the ground, we have the tangent to be 0; so, we have dy by dx equal to 0 at contact. From this, we get s equal to -100 meters. That means the length of the cable between this portion, where the coordinate is 0 to the point where it is in contact with the ground, is 100 meters. So of the total length of the cable which is 400 meters, 100 meters of the cable is hanging and so the remaining 300 meters is lying on the ground.

(Refer Slide Time: 59:51)



Let us consider the relations between the length and the x coordinate from the equations that we have derived for the catenary. This is the general equation and the equation that relates the y co-ordinate with the length. H is the horizontal component which is nothing but T_x. From this equation at origin s equal to 0, so we get C_2 as minus T_x by W times sin hyperbolic C_1 We know this C_1 and we also know this horizontal component and the weight carried, so from this we find C_2 as - 29.98. At the contact point we know that s is equal 100 and from this we get x equal to C_2 which is equal to —29.98 meters. If you remember that this is the cable and this is our coordinate 0 and we have found that at the point where the cable is in contact the x distance is —29.98 meters.

If we solve this other equation at origin, that is at this point, we have s equal to 0, x equal to 0 and y is equal to 0.From this we find this constant C_3 as cos hyperbolic minus W by T_x C_2. We have already found this constant C_2 and from this, we find C_3 as —100.47.

At the point of contact with ground, x equal to —29.98 meters and we substitute this in this equation to find that y is equal to H by W plus C_3 which is —90.47 meters. So at the place where the cable is in contact with the ground the y coordinate is —90.47 meters. So the height at which the blimp is flying is 90.47 meters. So this example shows how to compute the shape of the cables; loaded uniformly along the cable or uniformly along the x direction. We have to use sometimes the differential forms because sometimes the end conditions are known in terms of the tangents or their locations.