Engineering Mechanics

Dr. G. Saravana Kumar

Department of Mechanical Engineering

Indian Institute of Technology, Guwahati

Analysis of Structures-2

Module 3 Lecture 7
Internal Forces in Beams

Today we will continue with our lecture on internal forces in beams. For your reference, this is lecture number 7, module 3 of the web-based engineering mechanics course. In the last lecture, we saw how to determine the internal forces in a member, particularly in beams.

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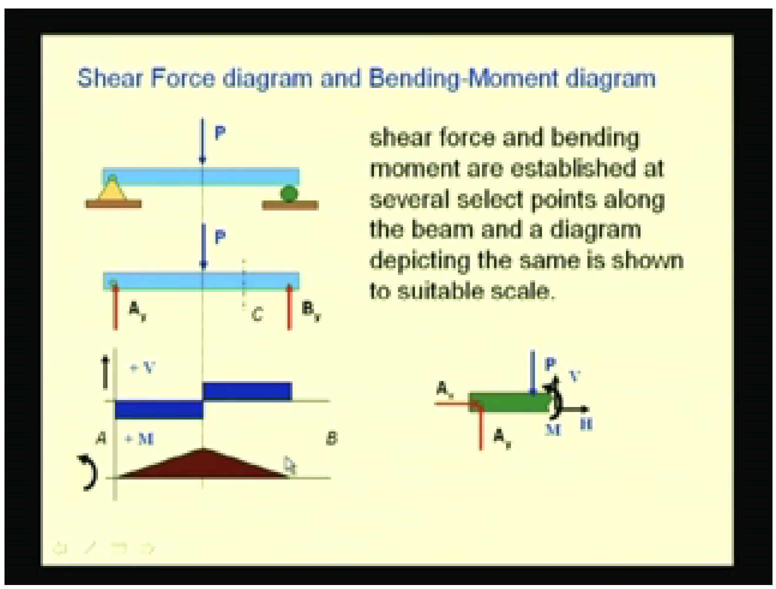

Today, we will continue with the discussion on the internal forces and see how to depict these forces as a diagram. We have the shear force diagram and the bending-moment diagram depicting the shear forces and the bending moment, which are the predominant internal forces in a beam, along the entire length of the beam for a particular loading and support condition.

These diagrams help a civil engineer in designing the beam because in one glance the diagram shows the shear forces and bending moment that occur in the entire length of the beam. Today we will see how to create these diagrams for beams under various loading and support conditions.

Let us take for discussion an example. Here you see a beam which is pinned at one end and supported by a roller at another end and we have say one concentrated load P. First, in order to construct the diagram we determine the support reactions. We construct the free body diagram to do that. At A, we have a pin connection, so we have both a vertical component and a horizontal component of a force, but since we do not have a horizontal component of an applied load, this component vanishes. We quickly see that for the equilibrium of this beam, the reaction at A and the reaction at B both should be equal to half of this load that is P by 2.

Once we have determined the reactions, let us consider a section at C to construct the free body diagram of a portion of a beam in order to determine the shear force and bending moment at this section. As we move this section from A to B, we get the shear force and bending moments along the length of the beam. The same is plotted in order to obtain the shear force diagram and bending-moment diagram.

This is a free body diagram of the portion of the beam AC. We have this shear force, the bending moment and axial force, if any, in the positive convention. For this case, since A_x is 0, the axial force H is also 0 and we have only the shear force and bending moment as the internal forces. As we move this section, we get the shear force and bending moment along the beam and the same is depicted as a diagram. So this diagram shows the shear force variation from A to B; this diagram shows the bending moment variation from A to B.

Let us see this little more clearly.

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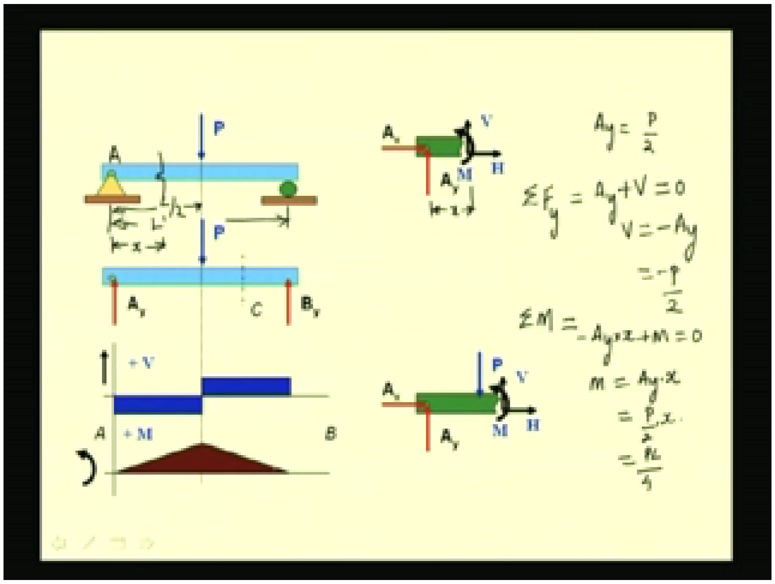

If we take a section between A and the midsection of the beam, then this is the free body diagram of any such section because this load P is not available in the free body diagram of that portion of the beam. From this, we determine the shear force and bending moment. We have already determined A_y to be P by 2. If we consider L to be the length of the beam and this concentrated load is acting at a distance of L by 2. Let us say we have considered a section at a distance of say x; so this is a section at the distance x from A.

For equilibrium of this section, we can equate the forces to be 0 and moment to be 0 to find the shear force and bending moment. Equating these vertical components, we have A_y plus V is equal to 0; thus, V is equal to —A_y.

We see that as the section moves from A to any point up to L/ 2 of the beam, the shear force does not vary and it is equal to —A_y and remains constant. So we plot the same. Here to some scale we have plotted this A_y which is equal to —P by 2. We have plotted the same and it remains constant up to the mid-section.

From the moment equation, summing the moments about this point we have the moment of this force A_y which is clockwise and positive. We have A_y whose momentum is x and the clockwise moment, so it is negative and we have this moment M which is positive, equal to 0.

We get M is equal to A_y times x which is P by 2 times of x. As x increases as we move from A the bending moment increases. At A, x equal to 0, so bending moment is 0. As we increase x the bending moment increases linearly. It reaches a maximum value at the midsection where x is equal to L by 2 and the bending moment is PL by 4, the maximum value. The same is plotted here.

Now for plotting the shear force and bending moment in this section that is from midsection to B, we consider this free body diagram. We set up similar equations, that is, the force summation and moment summation to determine the shear force and bending moment and draw the remaining portion of the diagram. So, in this way we can construct the shear force and bending-moment diagram for beams under various loading and reaction conditions.

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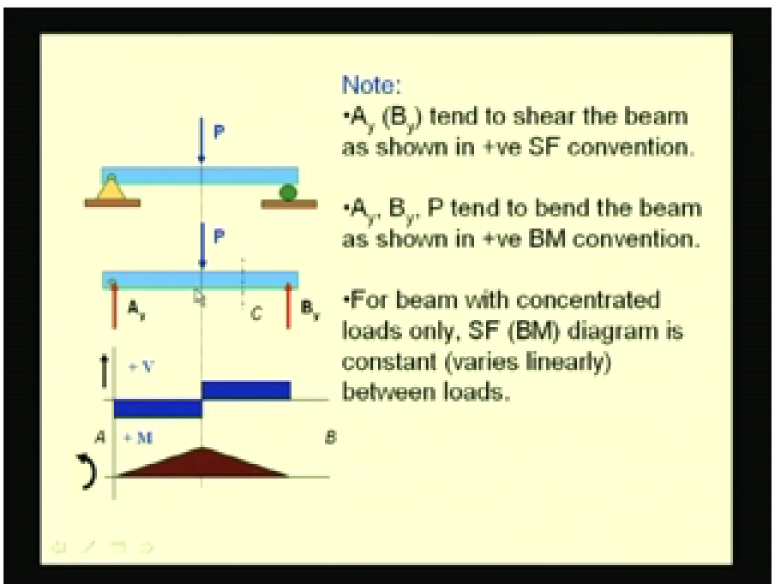

Let us note certain behaviors of these diagrams. The shear force that has been depicted is the shear force in the positive convention. We have seen in the last lecture the positive conventions of shear force and bending moment, the way these forces tends to shear or bend the beam. For the beams which have only concentrated loads, as we have seen in this example, we see that the shear force is constant between the loading points. We have the reaction at A and a concentrated load at the midsection. We see that the shear force remains constant between these two points of loading. The moment varies linearly - either it increases linearly or decreases linearly between the points of loading.

These behaviors can be used to plot the shear force and bending-moment diagrams conveniently by determining the values of shear force and bending moments at locations where we have concentrated loads. In between, we can plot using these behaviors that is the shear force remains constant and bending moment varies linearly for these concentrated loads.

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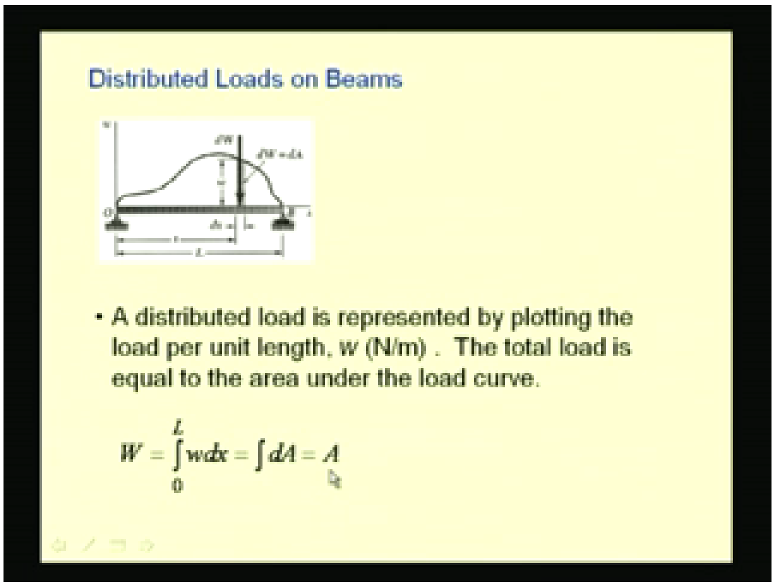

If the beam is loaded with uniform loading like the loading pattern shown in this picture, we have this beam OB which has a loading pattern defined by this load curve.

If we know this equation of the load curve, then we can determine the total load on the beam by computing the area under this loading curve. So if we take a small element of the beam B_x, the corresponding load is dW which is equal to the area of this incremental rectangle. In order to compute the total load, we have to integrate to find this area. The total load is computed. So we have this W, the total load, as the sum of these areas of incremental rectangles which is integral dA. If it is equal to A then that is the total load.

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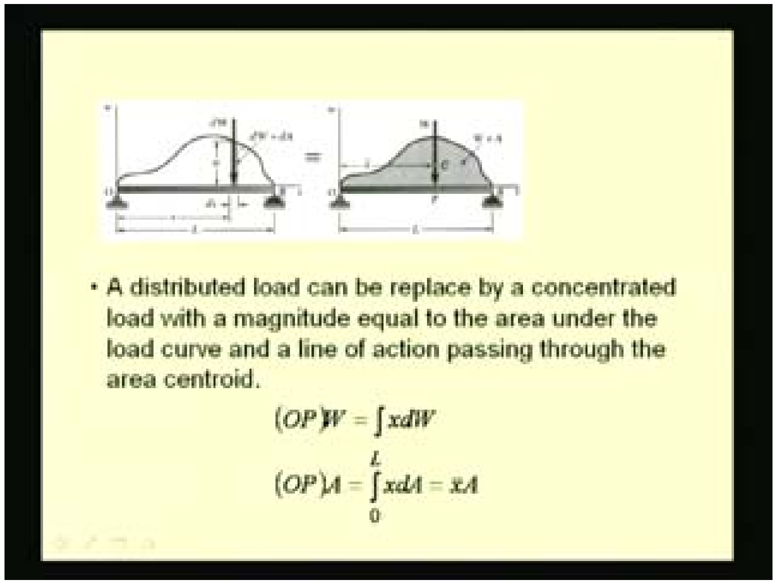

If you want to represent this distributed load by a concentrated load, so that we can use the shear force and bending moment drawing procedures of concentrated loads for the distributed loads also. So in order to do that we have to find an equivalent concentrated load for any given distributed load.

We have already seen that the area under this curve gives the value of the total load that is W, but now we have to determine the point of action for this concentrated load. In order to determine that we can sum the moments of these individual incremental forces about say a point O and equate it to the moment of this concentrated load about the same point O. By doing this we can determine the momentum for this concentrated load W.

We can find that for the first moment of this weight to be the same, this load W has to act through the centroid of this area. So the moment of this force W is OP times W, should be equal to the sum of the moments of these individual elemental weights. From this, we know the distance at which this concentrated load has to be placed.

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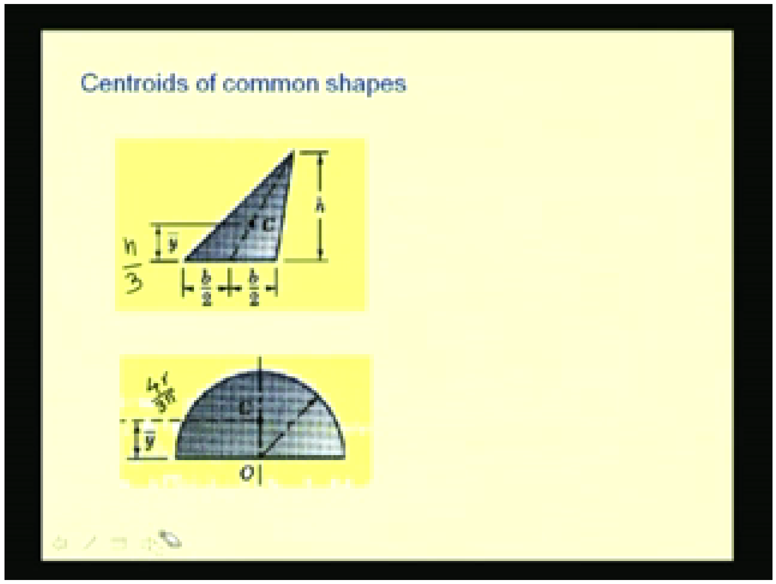

For common shapes we know the location of centroids. Like for this triangular lamina, the centroid lies along the angle bisector and at a distance of h by 3 from the base. Another lamina is shown; here, from this geometry we say that the centroid lies along one of the diametrical lines and at a distance of 4r by 3 pi, if r is the radius.

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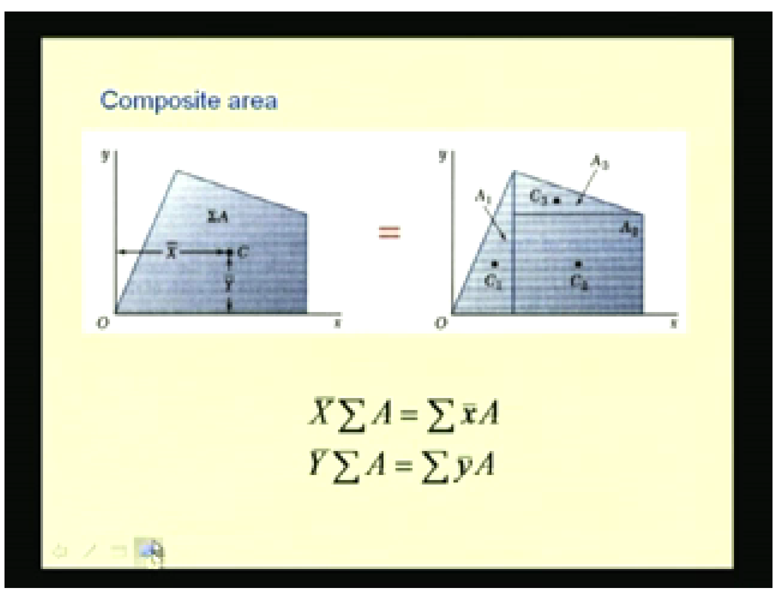

For loading curves which have complicated shapes, we can determine the equivalent load by considering the area under the curve as constituting of areas of simpler regions for which the centroids can be determined. Like in this case, we have this area A_1, A_2 and A_3 for which the centroids C_1, C_2 and C_3 are known. Thus, by equating the first moment of these areas about O to the first moment of this composite area, we can know these momentums say x bar and y bar that are necessary for computing the equivalent load and its location.

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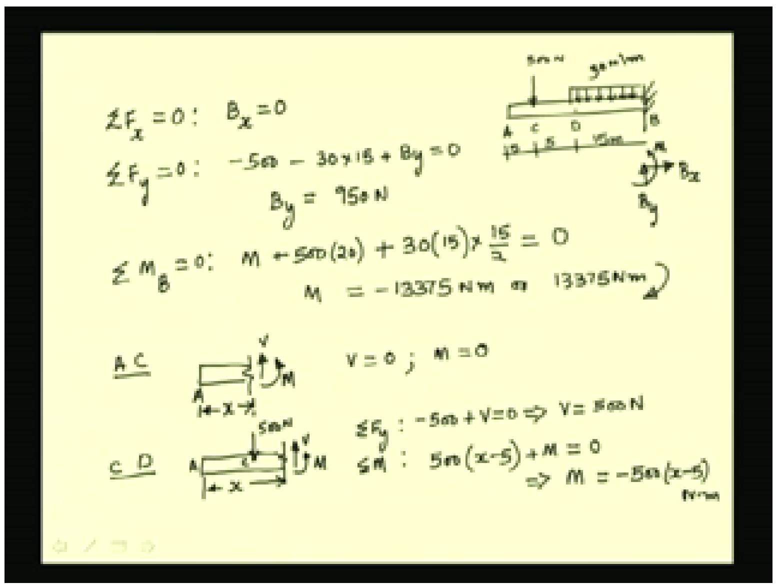

Let us see one example. Let us consider a beam which is cantilevered at B and supporting a point load of say 500 Newtons at C and supporting a uniformly distributed load up to point D from B. The dimensions are known and all the dimensions are in meters.

In order to determine the shear force and bending moment we first determine the end reaction. So in this case, we have at B, which is the fixed end, the three components of reaction: B_x, B_y and M the bending moment. A is a free end so we do not have any reaction. From the equilibrium of this body, we write these equations. The first is summing the force component along the x direction, from which we find that B_x has to be 0 because we do not have any horizontal component of the loading, so B_x has to be 0.

Next for the sum of the forces to be 0, the vertical components have to be 0. From the diagram, we see we have 500 Newton force at C and a distributed load, say in this case let us take this to be 30 Newtons per meter acting from D to B.

We can find the equivalent concentrated load, which is nothing but the area under the curve, which is 30 Newtons per meter for 15 meters. We have the vertical component of reaction B_y equal to 0. From this, we get B_y as 950 Newtons.

For equilibrium, the sum of the moments of all these forces has to be 0 about any point. Let us consider the point B and sum the moments of various forces and equate it to 0. We have the moment at B which is a counterclockwise moment, so which is positive. We have the moment of this 500 Newton force whose momentum is 20 meters, so we have 500 into 20. Then we have the moment of this distributed force.

The equivalent force is the total load acting through which centroid. So here, in this case, this centroid lies midway from D to B that is at 7.5 meters from B. So we have the moment as 30 into 15, which is the total load, and whose momentum is 15 by 2. So the sum of all these moments has to be 0. From this, we get the moment as —13375 Newton meter or it is 13375 Newton meter in the clockwise direction. So now, we have found the reactions, that is, the moment as well as force reactions at B. Now we can proceed to find the shear force and bending moments.

In order to construct the shear force and bending moment, we can take sections for which the loading patterns are similar; because, the behavior of the shear force and bending moment curves will be constant or same for a portion where there is no change in the loading behavior.

Here we see for this case that AC, CD and DB are portions where the loading behavior is consistent for drawing the shear force and bending-moment diagrams. So let us consider first the section AC. The free body diagram of the same is that we have no end reactions at A and also we do not see any forces in the section.

We take this shear force and bending moment in the positive sense. For equilibrium of this section, let us say at any distance x between A and C, the sum of the forces and sum of the moments has to be 0. From that, we find that V has to be 0 and M has to be 0 as well because we do not see any other forces; so these components have to be 0 for this section CD.

Let us consider now the portion CD and draw the free body diagram. Here we have a 500 Newton force acting at C and no other force is there. The shear force and bending moment are in the positive sense.

Let us consider this section at a distance of x from A between C and D. For the equilibrium of this section, again, we write the force summation and moment summation and equate it to 0. So we have from the force summation —500 plus V has to be 0, from which we find that shear force has to be 500 Newtons. Summing the moments about this point we have the moment of this 500 Newton force, which is counterclockwise and which is equal to 500 times this distance minus the distance AC which is 5 meters. So we have x minus 5 and the moment M which is also positive because it is counter clockwise to be 0. From this, we get M as —500 times x minus 5 Newton meter.

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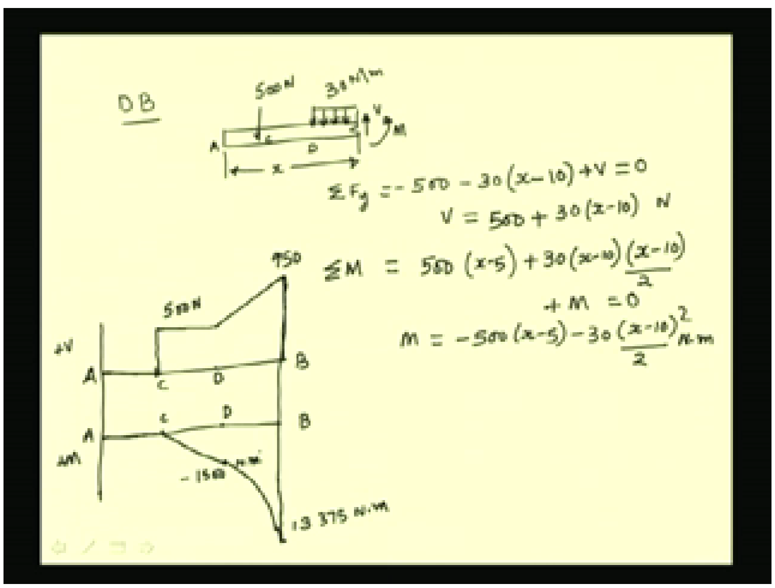

Let us consider the portion DB and draw the free body diagram of the same. We have a concentrated load of 500 Newtons at C and from the point D, we have this uniformly distributed force of 30 Newtons per meter.

We are considering this section at a distance of x between D and B. We have the shear force V and the bending moment M. So this completes our free body diagram. For equilibrium we can write the force summation and moment summation to 0 to find the shear force and bending moment.

Let us sum the forces. From this, we see that we have a 500 Newton force which is negative and we have this 30 Newton meter force acting for this distance; so which is —30 times x minus the distance AD, which is 10, plus the shear force V equal to 0. From this, we get V as 500 plus 30 into x minus 10 Newtons. Summing the moments, we have the moment of this 500 Newton force which is counter clockwise, so we have 500 into its momentum, which is x minus 5 meters and the moment of these distributed forces. The equivalent force of this distributed force lies in its centroid. We have the same as 30 Newton into x minus 10 is the total load, which is acting at a distance of x minus 10 by 2 from this section and we have the moment M. The sum of these has to be 0. From this, we have M as —500 into x minus 5 minus 30 into x minus 10 squared by 2, so many Newton meter.

Now we have for the complete length of the beam the shear force and bending moment equation from which the shear force as well as bending moments can be determined to draw the shear force and bending-moment diagram. So, let us draw the shear force and bending-moment diagram.

Let these be the points, so we have the beam section AC, CD and DB. In the section AC, we have seen that the shear force is 0; so, let us say to some scale we are drawing the shear force. For the section CD, we have found the shear force to be 500 Newton; so it jumps to 500 Newtons and remains constant in this portion CD; the value of the same is 500 Newton. For the portion DB, we have seen that the shear force starts from 500 Newtons and linearly varies to a shear force value of 950 Newtons at B. This completes our shear force diagram.

Let us draw the bending-moment diagram to some scale. We have already seen for the section AC we have bending moment to be 0; for the section CD, the bending moment is linearly varying; for this section DB, it varies as a second-degree curve. The value of the bending moment at this point and this point can be found from these equations and can be marked for the given scale. We know that this value is 13375 Newton meter and the same way this will be 1500 Newton meter. So, in this way we can construct the shear force and bending-moment diagram for beams with concentrated or distributed loads.

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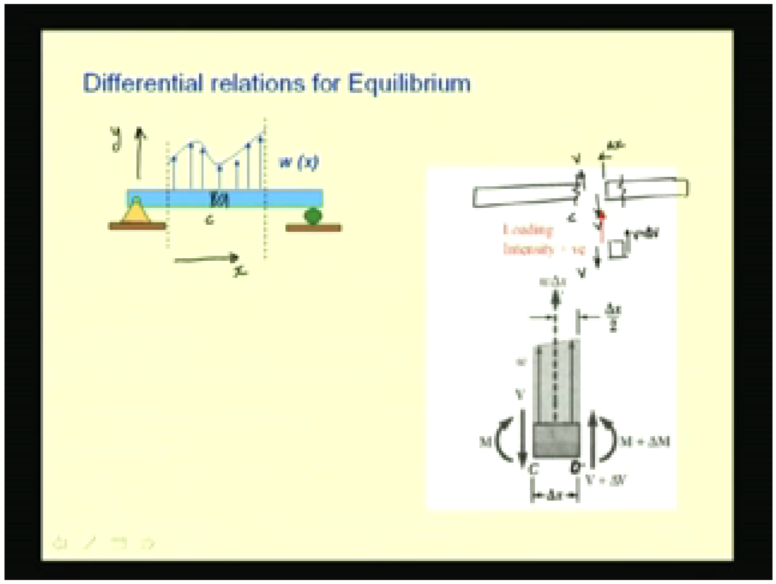

For the distributed force, if we consider the differential relations of equilibrium rather than the algebraic relations of equilibrium, the construction of shear force and bending-moment diagrams becomes easier. So, let us formulate the equilibrium of a portion of a beam subjected to a distributed load.

Here in the picture you see a continuous loading function say w(x) which varies with respect to the length of the beam. For the purpose of deriving the relations, we have considered the loading to be in the positive y direction.

Let us consider a free body diagram of a small portion of this beam. If we take the free body diagram of a portion up to C and let us say for the beam, we have the shear force in the two sides of the beam as depicted in this picture.

Now, let us take a section at a very small distance, say delta x, on the right hand portion of the beam. If we consider the free body diagram, we will have a shear force which has changed also incrementally. In the same way, we can consider the moments also.

This picture shows the free body diagram of such an incremental beam section. At C, we have this shear force and bending moment, all in the positive sense. For a section D, which is at a distance of delta x we have the shear force and bending moment incremented by a small value delta V and delta M. We have a loading curve and the total load in this section is w delta x. We can write the equilibrium equations for this elemental beam section to get the differential relation.

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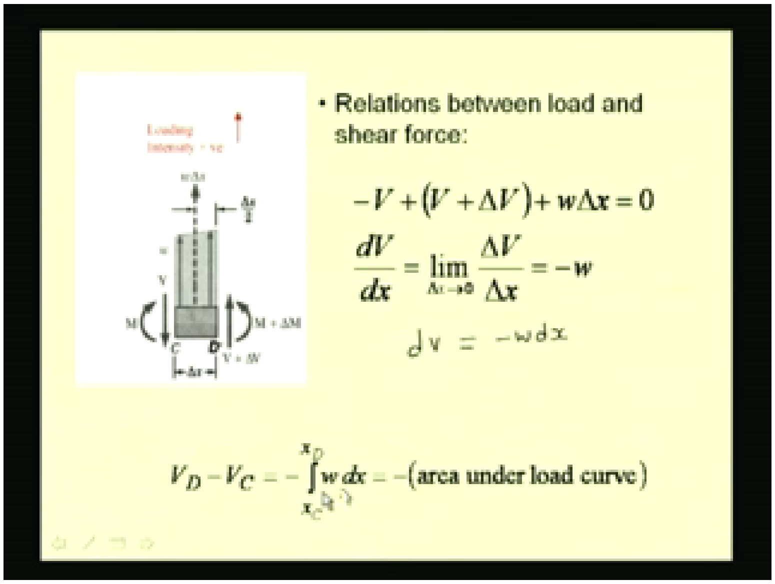

Let us first derive the relation between the load and the shear force. For equilibrium of this element CD, the total forces have to be 0 from which we get minus V, the force at C, plus the upward force V plus del V at D plus the upward forces which are applied, that is w delta x, on this incremental beam portion to be equal to 0.

In the limit, we get dV by dx as —w; so, if we integrate this quantity we get the shear force for any particular section between CD. If we integrate this value from C to D, we get the difference in the shear force between the section D and C as the negative of the area under the load curve because this value, that is integral wdx between C and D, is nothing but the area under the load curve. So we have this relation that the change in the shear force is equal to the negative of the area under the load curve. This property can be used to construct the shear force diagram.

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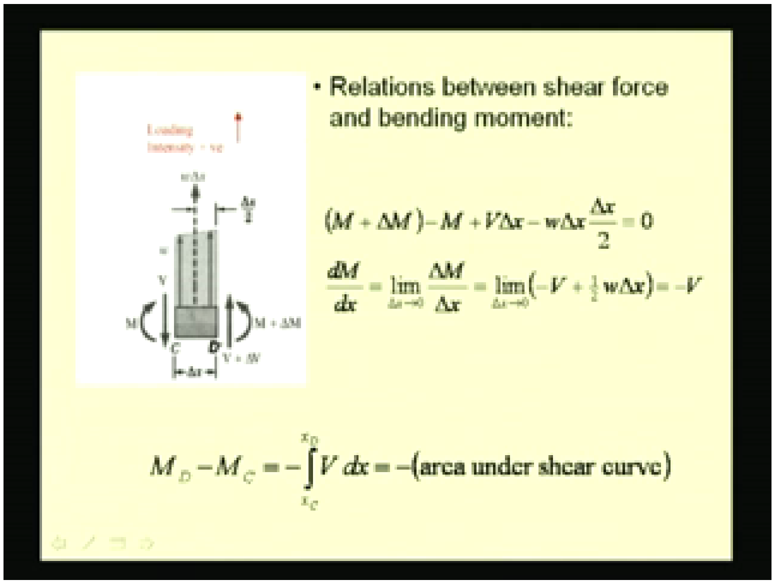

Similarly, let us consider the bending moment and shear force relation. We have the moments M plus delta M, which is counterclockwise, and so a positive value and a clockwise moment of M, so which is negative plus the moment due to this force V, which is V times delta x. We are taking the moments about this point D minus the moment of the load about this point; so we are considering an equivalent load passing through its centroid. So we have —w delta x into delta x by 2. In the limit we get the relation between dM by dx is equal to —V.

If we integrate this relation between C and D, we find that the difference of the moment between D and C is equal to the negative of the area under the shear force curve. So these two relations can be used to draw the shear force as well as bending-moment diagrams.

We will consider an example.

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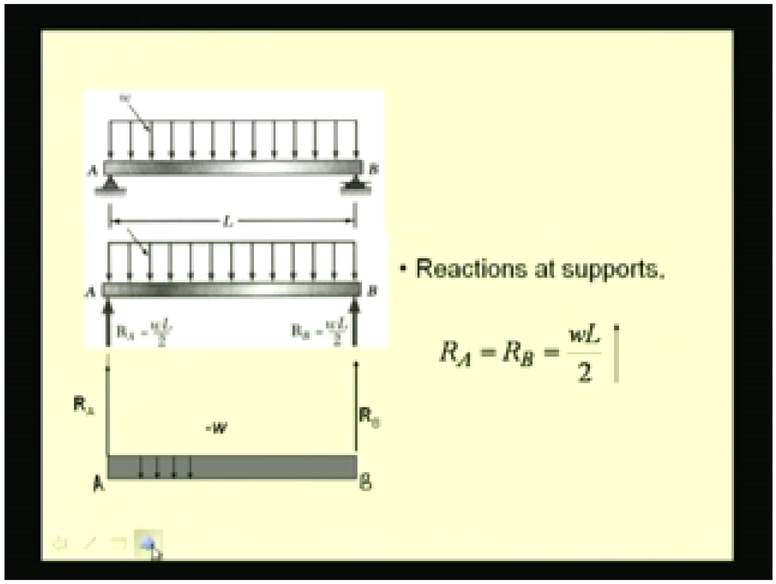

Let us take a beam AB with an uniform loading of w Newtons per meter for the entire length of the beam L. First, we determine the end reactions and it can be quickly found that these reactions have to be equal to half of the total load. The total load is area under the load curve which is w times of L, so we have the reactions at A as well as B as wL by 2.

Once we have determined this reaction, let us construct what we call as the load curve where we depict all the loads, both the applied load as well as the reactions in the diagram. We have the reaction R_A at A which is a positive value of magnitude wL by 2; so, we mark it as a point load or a concentrated load at this point A. At B, we have the reaction R_B whose magnitude is again wL by 2, so we mark it as a concentrated load. In between A and B we have this uniform load which is negative because it is a downward load and so we mark —w Newton per meter. This shows the negative loading between A and B.

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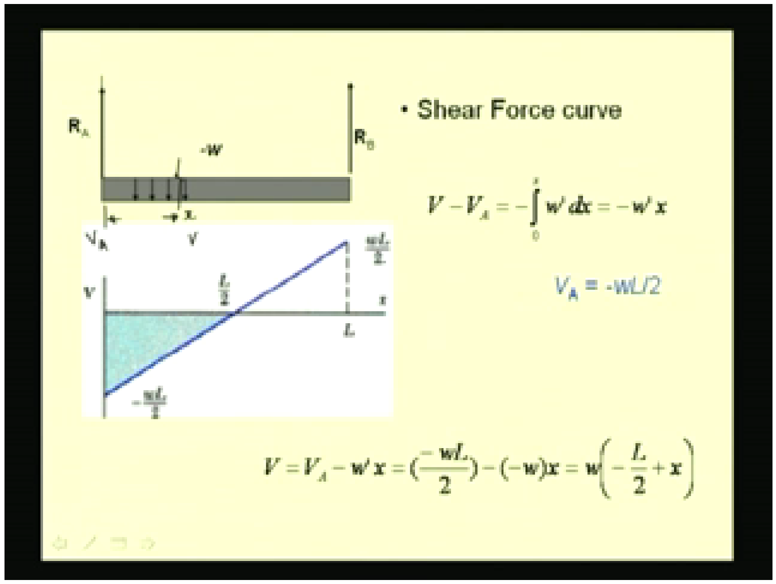

Now let us see how to determine the shear force curve from this loading curve. For the points where we have concentrated loads, we have to use the algebraic relations to find the shear force. So we can find the shear force at A.

Now let us see how to determine the shear force between A and B. If we consider a section at any distance x from this end, the shear force at this section is let us say designated as V, then we have the relation V minus VA, which is the shear force at A, equal to the negative of the area under the load curve which is nothing but the area under this curve.

We can find that the shear force at A is —wL by 2 and so the shear force at any section is equal to w times of minus L by 2 plus x. We have at A the shear force minus wL by 2 and it linearly changes to wL by 2 at B. At any point, the value of the shear force is equal to the negative of the area under the load curve.

Once we have constructed the shear force curve, we can determine the area under the shear force curve and thus, we can move on to draw the bending moment curve.

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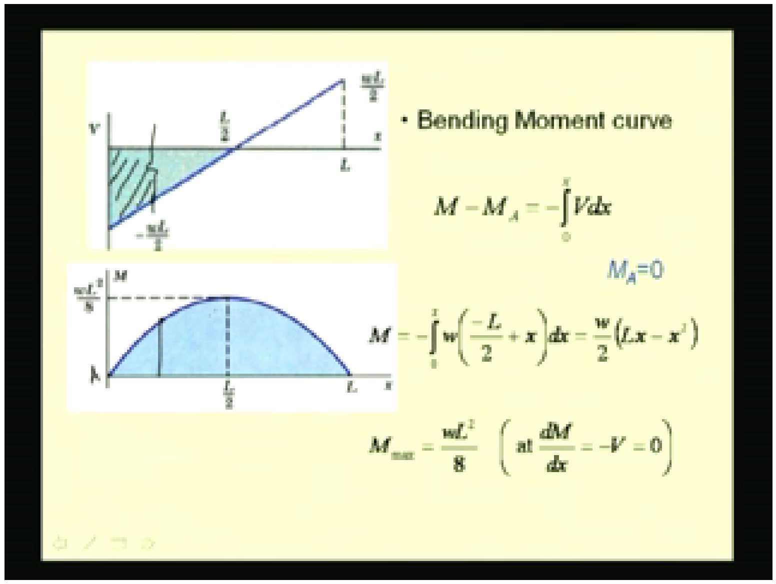

Here we see that the shear force is linearly varying. The area under this curve can be found out and the bending moment at any section, say a section at this point, is equal to the negative of the area under the shear force curve, that is, the area of this portion and that will be the bending moment at this section.

We can find that the bending moment at A is 0; so the bending moment at this point will be the bending moment at A minus the area under this shear curve. Already the area under the shear curve is negative, so we have the positive value of M. In this way, we complete the bending-moment diagram for the entire portion of the beam.

From these diagrams, we can note certain properties. We see that the maximum value of the bending moment occurs at a point where the shear force curve crosses the abscissa. The maximum value of the same is equal to wL square by 8 in this case.

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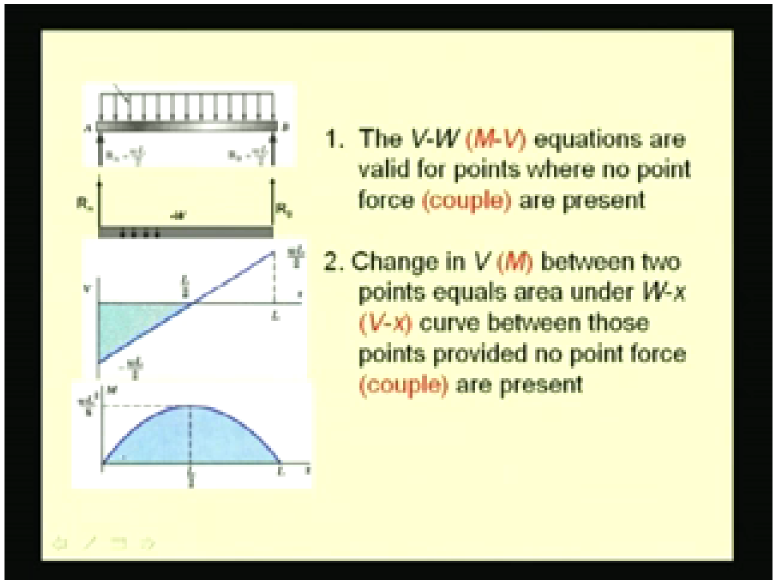

Certain other things that we can note from these shear force and bending-moment diagrams are that these equations are obviously valid for points where we do not have any concentrated loads or concentrated couples; because, there we will have a discontinuity and we cannot use this differential relation. The change between say the shear force between two points is equal to the area under the load curve and the same is valid for the bending moment curve also provided there are no point loads or point couples.

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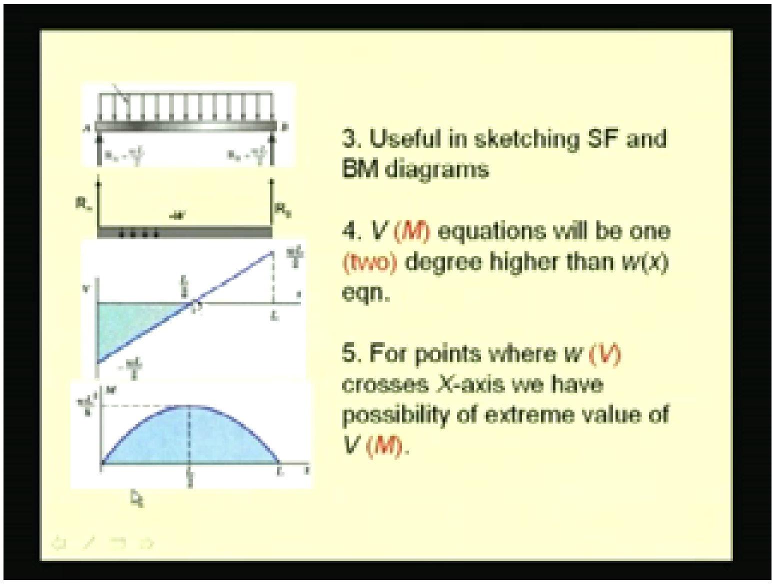

We have found from this example that this can be conveniently used to draw the shear force and bending-moment diagrams. One more interesting thing that we note is the degree of the curve is one degree higher between the load curve and the shear curve and for the load curve and the bending moment curve the degree is two degrees higher.

Here we see that the load curve is a constant, so we have a constantly varying shear curve. The bending moment curve varies as a two-degree curve. In this case, it is a parabola. This way it is possible to construct the shear force and bending-moment diagram. We have also seen that at places where the shear force curve crosses the abscissa, we have the possibility of the extreme M value for the moment.

These characteristics can be used to draw the shear force and bending-moment diagrams for the beams. Let us consider one example.

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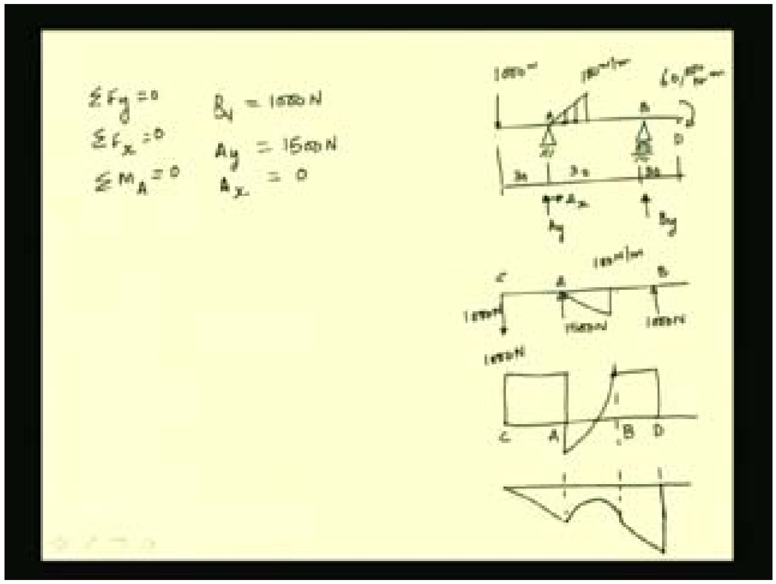

Let us say we have a beam with a pin reaction at A, the roller reaction at B, the uniformly varying load of say 100 Newtons per meter and a point couple at say the point D which is equal to 60000 Newton meter. Various dimensions are 30 meters, 30 meters and 30 meters

First, we determine the end reactions. From the equilibrium of the complete beam, we know that we have two components of reaction: A_y and A_x at A. At B, we have a single component of reaction that is B_y. By considering the free body diagram, we can determine these values. Summing the moments about A and equating it to 0, we find that the reaction B_y has to be 1000 Newtons and the reaction A_y has to be 1500 Newtons. We do not have a horizontal component of the reaction that is A_x is 0.

Now that we have determined these reactions, we can construct the load curve. So we have a 1000 Newton force at A or let us say this point is C. At A, we have the reaction which is 1500 Newton. Then we have the negatively applied load which is uniformly varying. At B, we have the upward reaction of 1000 Newton force. So from this load curve let us try to determine the shear force curve.

At C, we get the shear force as 1000 Newtons from the algebraic relations by considering the free body diagram of a portion between, let us say, this C and A. Now, we see that between this point C and A there is no change in the load and we do not have any loads; so the shear force remains constant. At A, we have a concentrated load of 1500 Newtons which is an upward force in this diagram; so the shear force curve goes down by 1500 Newtons and the shear force at this point is —500 Newtons. This can again be found by considering the equilibrium of the portion AC.

For this portion, where we have this uniformly varying load, since the load curve is linearly varying, the shear curve will be varying with one degree higher. That means it will be quadratic in nature. At this point, it will rise by a value equal to the total area under this curve. Let us draw it. (Refer Slide Time: 1:00:14 min)

We know that it has to be a quadratic curve. The total area under this curve is 1500 Newton and thus it rises to the point with positive 1000 Newton shear force value. Between this portion and B, we do not have any change; at B, we have this 1000 Newton positive force, so the shear force curve comes down. These are the various points on the beam.

From this shear force curve, we can determine the area under this curve and we can use the same to construct the bending-moment diagram. So at C we have a 0 bending moment. The area under this curve in this portion CA constantly increases and the bending moment is negative of the area under the shear force curve, so it has to be linearly increasing. Then for this portion, it is a little more difficult to directly determine it. So let us consider the portion BD. At D we have a negative moment and the area under the curve constantly increases and between this portion AB it has to be one order higher, so it becomes a cubic curve. In this fashion, we can construct the shear force and bending-moment diagrams using the differential relations.

In the next class, we will continue our discussion and we will see determining the internal forces in the other class of commonly used element, that is, the cable.