Engineering Mechanics
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Module 3 Lecture 6
Internal Forces

Today, we will see analysis of structures part II which is lecture number 6 in module 3 of engineering mechanics course. Today, our topic of discussion will be to determine the internal forces that are developed in a member when it is subjected to some external force.

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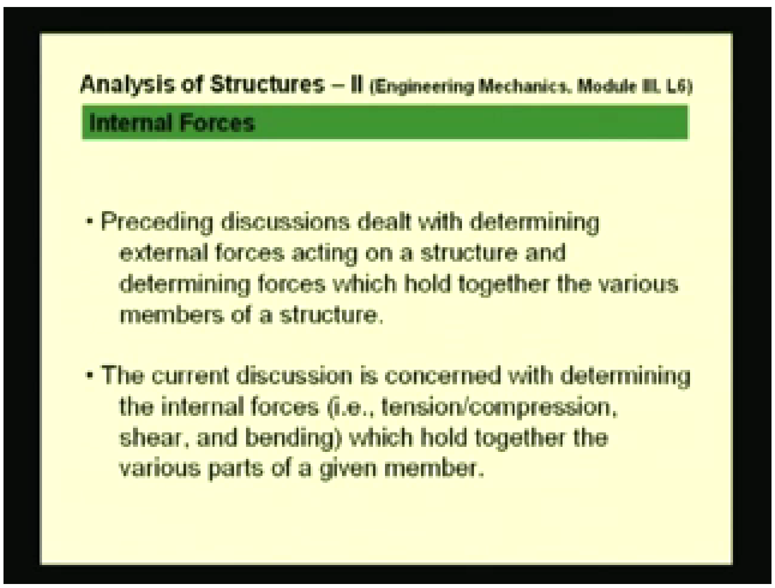

In the preceding discussions, we have dealt with determining the external forces that occur in a member, or the forces that occur in the joints, in case of connected to rigid bodies. In the preceding lectures, we saw some examples of connected rigid bodies like trusses, frames and machines. We saw procedures to determine the forces that hold them together or the forces that occur in the joints.

Today's interest will be to determine the internal forces that occur in a rigid body when external forces are applied. These forces are nothing, but inter atomic or inter molecular forces that hold together, the various parts of a member from deformation or failure when subjected to external loads. We have seen that in structures, predominantly, we have two-force members and multi force members. We will see how to determine the internal forces for the two-force members and then the multi force members.

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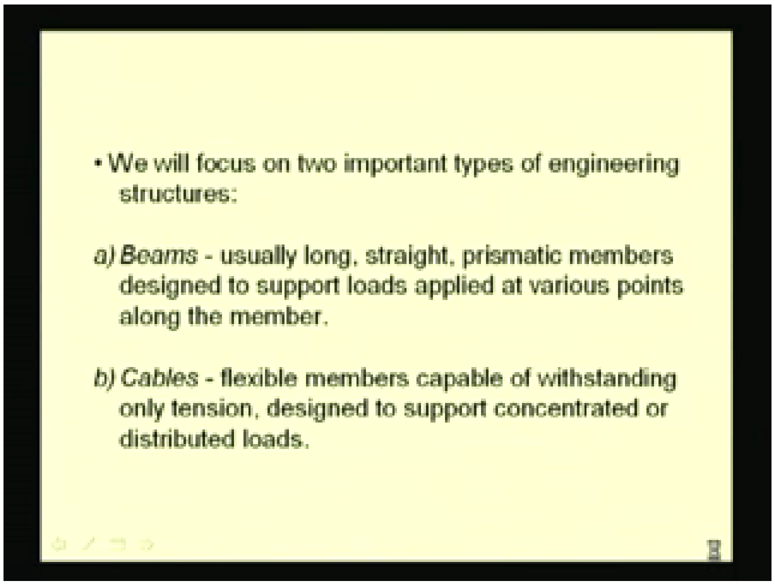

The interest in determining these forces lies in analyzing the internal forces for predominantly two types of structures that we use in our civil construction or other mechanical constructions. They are the beams which are usually long, straight and prismatic members with supports, and which carry loads either distributed or concentrated loads.

The other kinds of structures are cables which are used to tie the electric poles or the cables that are used to carry winches. So, these members have predominantly the tensile forces.

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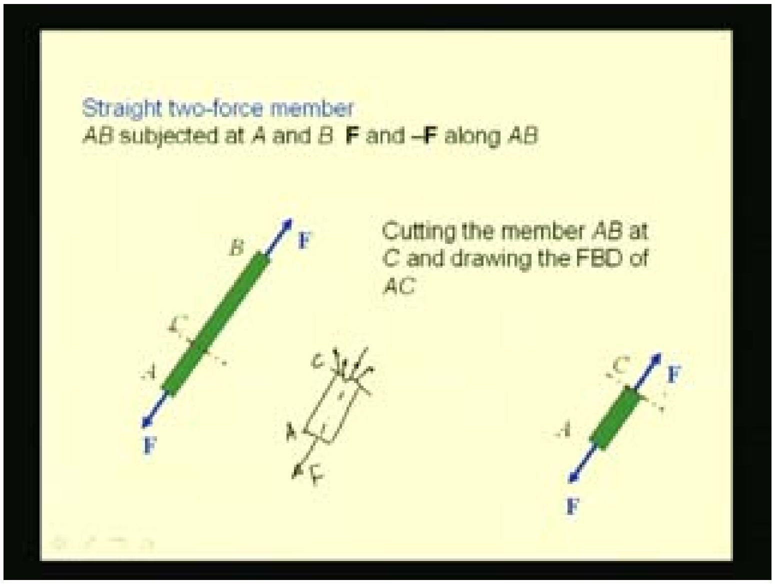

We will see first, how to determine the internal forces in a two-force member?

Let us consider first, a straight two-force member AB which is subjected to the force F at A and an equal and opposite force at D. If we are interested to know what kind of forces are set up at a section C then let us create a virtual section at C and create the free body diagram of one of the portions AC.

Let us draw the free body diagram of the portion AC. We have the external force acting at A and at C. The various parts of these members develop internal forces that hold them together with the parts or portion of CD. So, these forces can be represented by these quantities. These are nothing but the various internal forces that hold the portion AC, with the portion CB at this section C. These forces are inter-atomic forces or inter molecular forces.

If we consider the equilibrium of this portion that is the portion AC, of this two-force member then for equilibrium to exist, the sum of all the forces should be 0, and sum of the moments has to be 0. It becomes apparent that if we consider the axis of this member and if we sum all these forces at this section C, they have to be equal and opposite to this force, because this portion is again a two-force member. We have the resultant of all these forces as F which is equal and opposite to the applied force.

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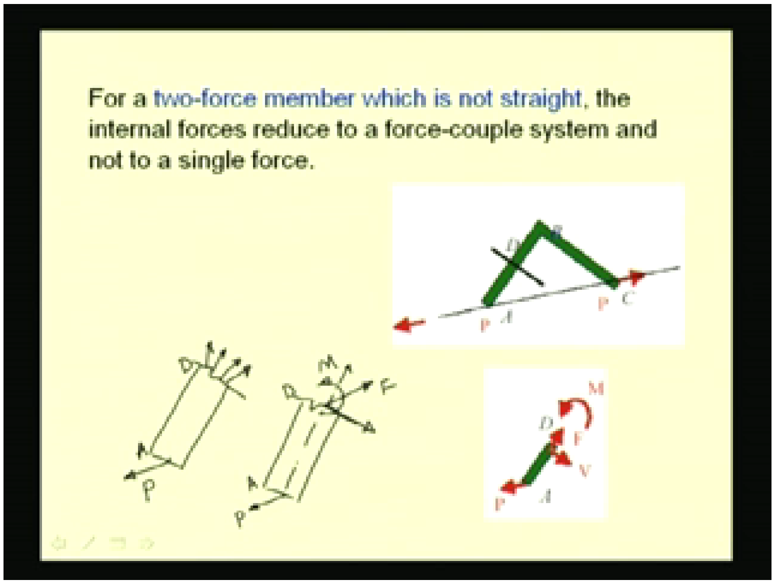

Let us consider a two-force member which is not straight and try to determine the internal force that is developed in a non-straight two-force member. Here, you see a member A B C which is bent, but it is a two-force member, because loads are applied at two points at A and at C. We have these loads P that are being applied to this member. For equilibrium to exist, we know that these two forces have to be collinear. Let us take a section D and draw the free body diagram of this portion AD.

We consider this portion AD; we have the external force P that acts at A and at D. We have these internal forces that hold together this portion AD, with the remaining portion DBC. For this portion AD to be in equilibrium, the sum of the forces and sum of the moments has to be zero; that means, the resultant of these forces should balance, the force as well as the moment due to this force P. Let us draw the resultant of these internal forces at D. At A, we have the external force P and at D. The resultant of all these forces is equal to a force F and a moment M.

How do we say that this is the resultant of these forces? If we consider this free body diagram, for the portion to be in equilibrium, the sum of these two forces has to be 0. The moment of this force P about this point should be equal to this anticlockwise moment M that we have marked at D. For a two-force member which is not straight, the internal forces reduce to a force and a couple. This force F can be represented by its two components; one along the axis and one transverse to the axis. This represents the resultant of the internal forces at D. For these two-force members, we have designated the axial force and the transverse force as F and V, and the moment as M.

Let us see how to determine these internal forces for a multi-force member.

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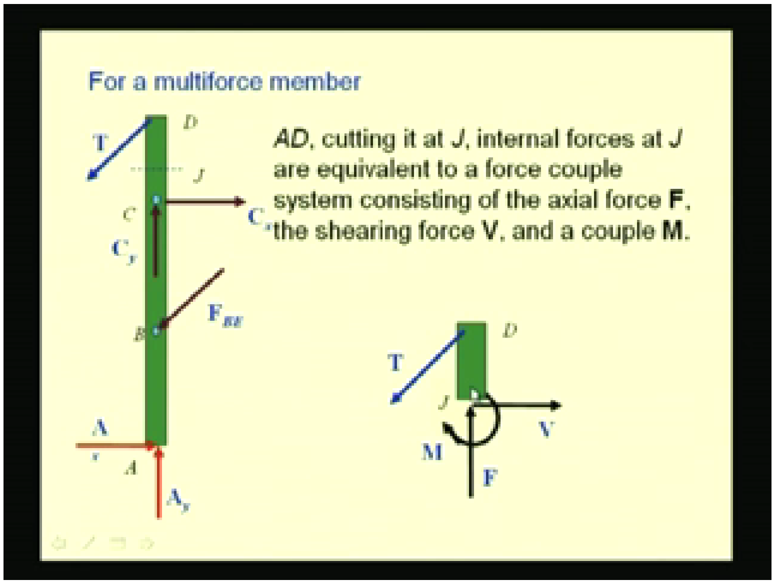

We see a multi force member A, B, C, D. We are interested to find the internal forces at this section J. Let us consider the free body diagram of the portion DJ. We know that the resultant of all these internal forces will be a force and a couple. The force is represented by its two components, V which is transverse to the section and F which is perpendicular to the section, and the moment is M.

This force V tends to shear this member at this section J and it is known as the shearing force. This moment M tries to bend the member at this section. So, it is known as the bending moment. This axial force F tends to compress or extend the member. It is an axial force, either tensile or compressive in nature.

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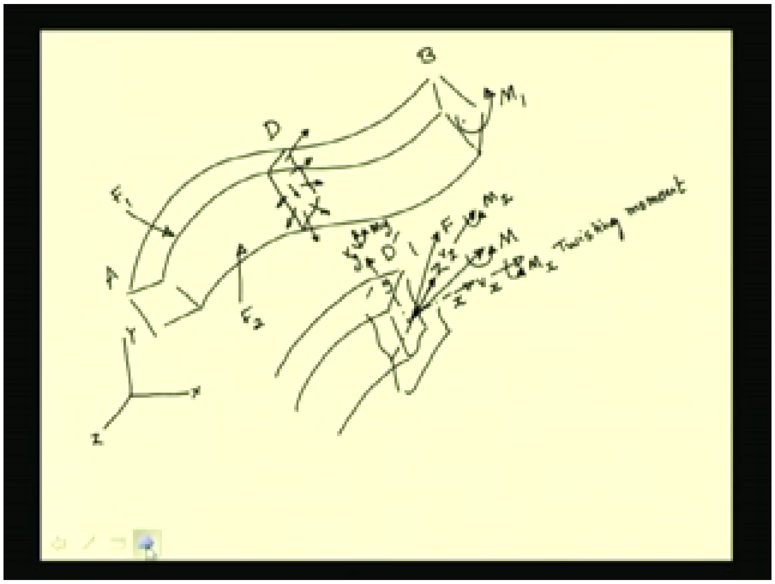

Let us see how to determine this shear force and bending moment for a three dimensional structure or a three dimensional beam. Let us consider an arbitrary structure in space. Consider a section D in this member AB. Let this member be subjected to some forces F_1, F_2 and maybe some moments M_1 and so forth. For the equilibrium of this member, the internal forces that will be developed at this section DE can be represented by all these forces.

All these forces are three dimensional. They can have any orientation. Let us consider this section. The resultant of all these forces at this section D, if these are the local coordinates of this section. Let me call this as small x, this as small y, and this as the z axis. So, the resultant of all these forces will be a force and a moment. These are the internal forces at this section D. If we take the components of this force, we have a component along x, we have a component along y, and we have a component along z.

Let me designate these as V_y V_x and V_z. The force V_x is along the instantaneous axis of the section, or which is perpendicular to this section and thus, causes a tensile or a compressive force on the member. These two forces that is V_z and V_y which lie in the plane of this section tends to shear this structure in the plane $z_{y.}$ The force V_z tends to shear the structure along the z direction. The force V_y tends to shear the structure along the y direction. So, these forces are the shearing forces.

Similarly, consider the components of this moment. We will have F moment component $M_{x,}$ a moment component M_z and a moment component M_y along the x, y and z direction. This moment that is M_x tends to twist the member at this section D. So, this is known as the twisting moment. This moment M_z and the moment $M_{y,}$ along y and z directions tend to bend the member. So, the moments M_z and $M_{y,}$ bend. The two moments are the bending moments. We see that the resultant of the internal forces tend to twist, bend are axially compress or extend or shear the member at any given section.

Knowing the behavior of these internal forces, it is possible to design the member to resist these forces. The aim of design of various members of the trusses or any machine or beams, is to select suitable material and the cross sections that can resist these internal forces.

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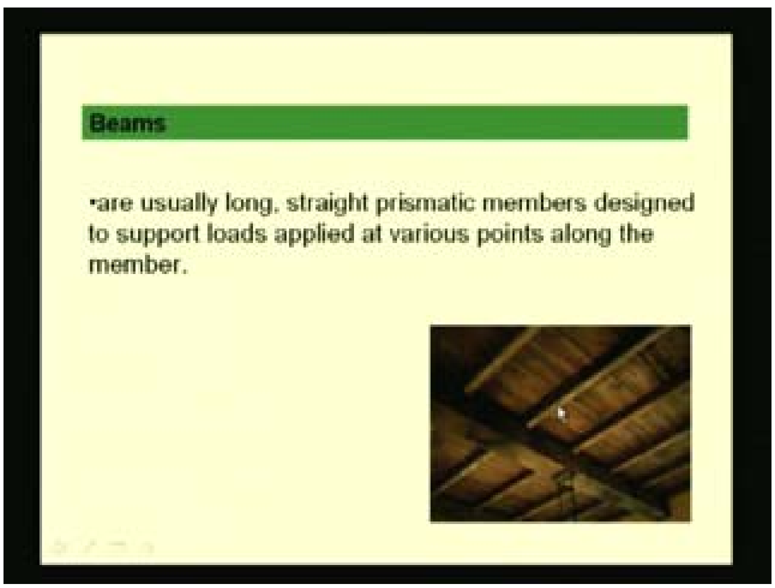

With this understanding, let us move to analyze, beams for the internal forces that are developed in them. Then we can move onto design a beam which obviously is not covered in this engineering mechanics. But later on, in your second course, you will be studying the design of beams or design of various other structures.

Here, you see in the accompanying picture, what we call as beams that are used to support the roof of this structure. So, beams are generally long straight members which are used to support loads in the transverse direction. So, here you see this beam supporting the slender beams. These slender beams in turn, are supporting the roof of the structure.

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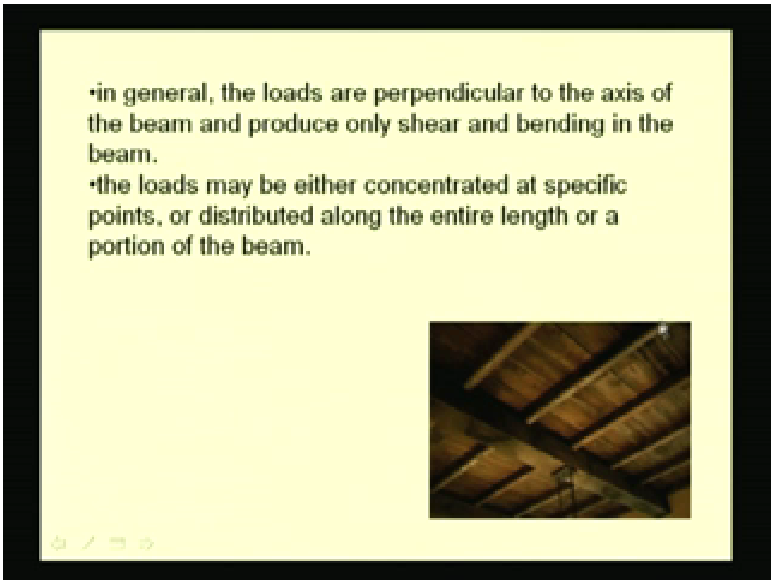

The loads are perpendicular to the axis of the beam, like for this big beam, you see that the forces from the slender beam are perpendicular to the axis. Also, for the slender beams, you see the loads from the roof come in the transverse direction. We have already seen that if we have only loads in the transverse direction, they will cause only shear and bending, and they will not cause any axial force. If it is a straight member, the loads could be either concentrated as you see for this big beam, where the slender beams apply the load at the these points, or it could be a distributed load, like you see for this slender beams, where the roof planks apply continuous loading all along the member.

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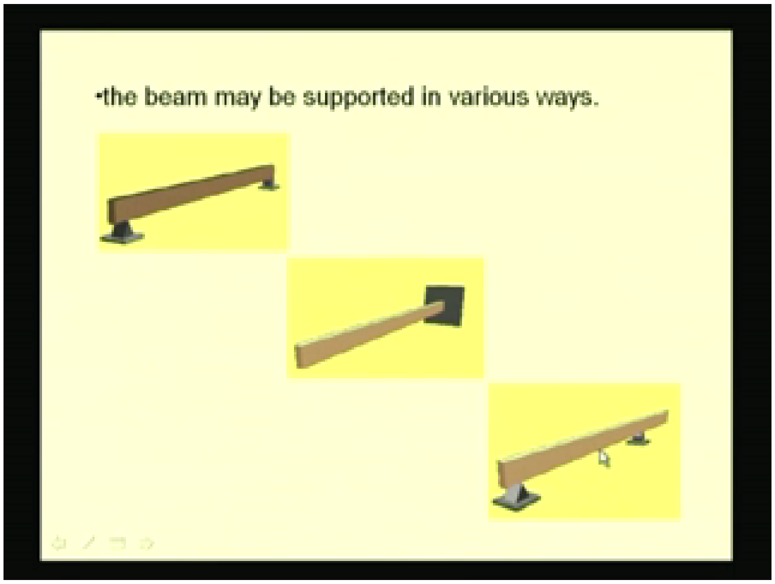

The beams can be supported by various ways.

Here, you see a beam that is being supported by a roller and a pin, or a beam could be supported by embedding it into a wall like this picture, or the beams could be overhanging; that means, they extend beyond the support in order that we are able to determine the internal forces. The supports should be statically determined.

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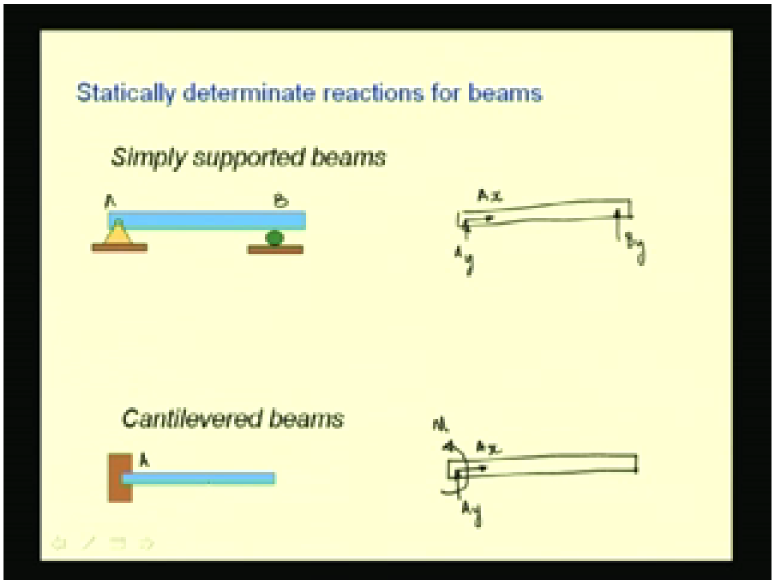

Let us see the kinds of supports that result in statically determinate reactions for beams.

First, let us consider this simply supported beam. We have a pin connection at this end and the roller connection at the other end. If we draw the free body diagram of this beam, as a complete structure at A, since it is pinned, we have two components of reaction. If this point is A, let us call it as B. We have A_y and $A_{x.}$ At B, we have a single component of reaction $B_{y.}$ We know that for the equilibrium of this member, the sum of the forces and the moments have to be 0 in plane. This results in three equations from which we can determine the three unknowns, A_x, A_y and B_y.

This combination of support is a statically determinate support reaction. Here, we see cantilevered beams, or beams which are embedded into walls at one of their ends. Let us draw the free body diagram for this beam. Let us designate this point as A. At A, since all degrees of freedom are arrested, we have the two components of force A_y and $A_{x,}$ as well as there is a moment reaction. From the equilibrium of this member we have three equations which can be used to determine these three unknowns namely A_y, A_x and M. So, cantilevered beams are also statically determined.

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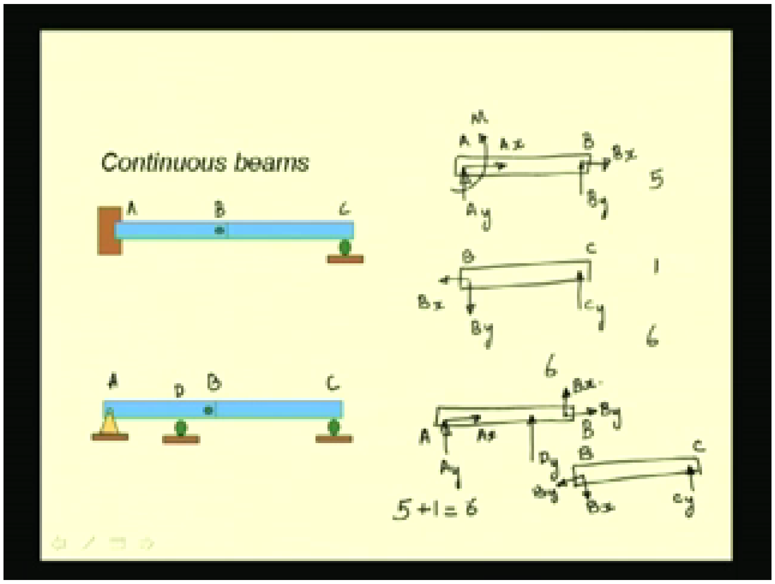

This kind of a support configuration leads to continuous beams.

Let us designate this point as A B and C. Here, we see two beams are pinned at B and this beam is supported. BC, the beam BC supported by a roller at C, and the beam AB is cantilevered at A, that means fixed at A.

Let us draw the free body diagram. In order to develop the equilibrium equations, we have to dismember it as two members AB and BC, because we have already discussed that if we remove the structure from its supports, if it collapses then we cannot consider the free body diagram of the complete structure. In this case, if we remove this structure ABC from its support, obviously it collapses; that means, the beam can rotate about this point B.

We consider the free body diagram of the dismembered beam. First, let us consider the beam AB. At A, since it is fixed, we have the two components of reaction and the moment reaction. At B, since it is pin connected, we have two joint forces say B_y and B_x. Since the beam BC can rotate freely about this point B, there is no moment force at this joint.

Let us consider the free body diagram of the beam BC. At B, we have an equal and opposite joint force as shown in the free body diagram of AB. We have these forces B_x and $B_{y.}$ At C, since the beam is supported by a roller, we have a vertical component of reaction say C_1. Let us see how many unknowns we have.

We have three unknowns at A and two unknowns at B, totaling to five. From this free body diagram, this B_x and B_y have already been accounted for in this free body diagram. They are only opposite in signs. So, they do not constitute a new unknown. We have one more unknown that is C_y. We have a total of six unknowns to be determined. For the equilibrium of this rigid body AB and rigid body BC, from the equations, we get to know three unknowns for each free body diagram. Since we have two free body diagrams, we get six equations. Since the number of equations and number of unknowns are equal, the problem is statically determined.

Let us consider another example of continuous beam. Here, we have a beam ABC which is pinned at A, supported by a roller at some point between AB and at C. Again, we see that if we dismember or if we remove this beam from its support it collapses. So, we dismember and construct the free body diagram. Let us draw the free body diagram for the beam AB. At A we have two unknown reactions A_y and $A_{x.}$ At the roller, we have this point D, we have a vertical component of reaction, and at B we have two joint forces say B_y and $B_{x.}$ For the next member that is BC, we have these joint forces B_y and $B_{x,}$ and at C we have the vertical component of the roller reaction.

Let us count the unknowns that we need to determine and see whether we have sufficient equations to determine. From this free body diagram, we see that we have to determine 2 plus 1 plus 2, a total of 5 reactions. From this free body diagram, only C_y is the additional unknown, since B_x and B_y have already been taken care of. So, we have a total of six unknowns and from the two free body diagrams, we can write six equilibrium equations and all the six unknowns can be determined. So, this is also a statically determinate support reaction.

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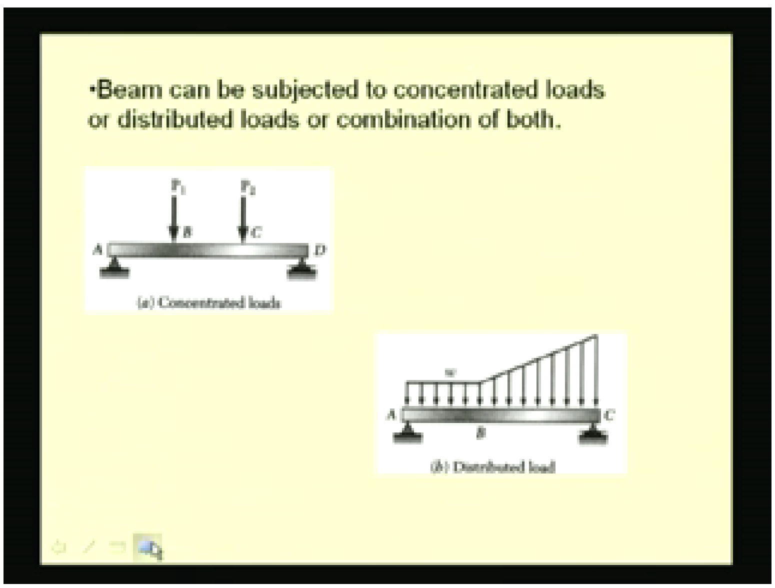

Beams can be subjected to point or distributed loads, as shown in these figures. Here, you see a point loading configuration like a force of P_1 acting at B and the force of P_2 acting at C. Here, you can see a distributed load either uniform distributed load or a varying distributed load acting on this beam A, B, C. For all these configurations, if we have statically determinate support reactions, we can determine them and thus, it is possible to proceed to determine the internal forces.

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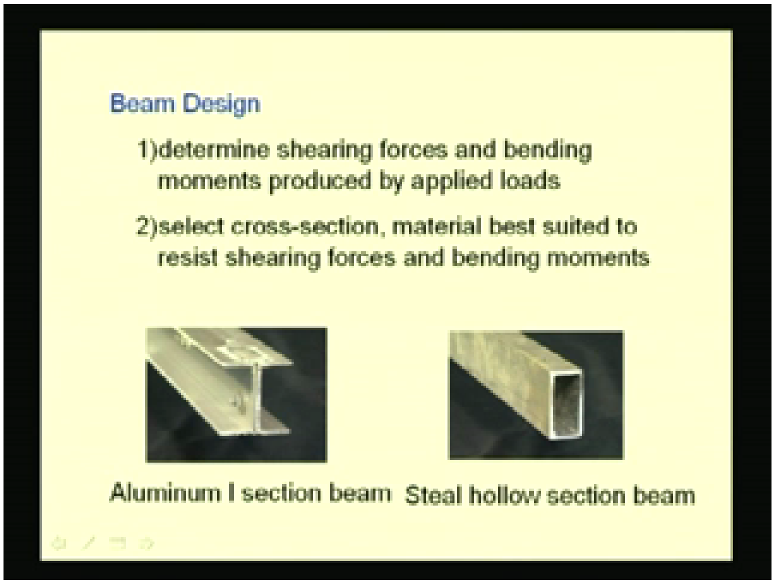

The problem of beam design can be stated as that first we are interested in finding the internal forces that are caused, because of the externally applied force; that is, we are interested to know the shearing forces and the bending moments. Once we have determined these internal forces, we now design the beam cross section and we select the beam material which can resist these forces under the applied load.

Here, you see some examples like, here you see, an I section beam which is made of aluminum, and here, you see a hollow section beam. The material could be steel or wooden beams which are solid.

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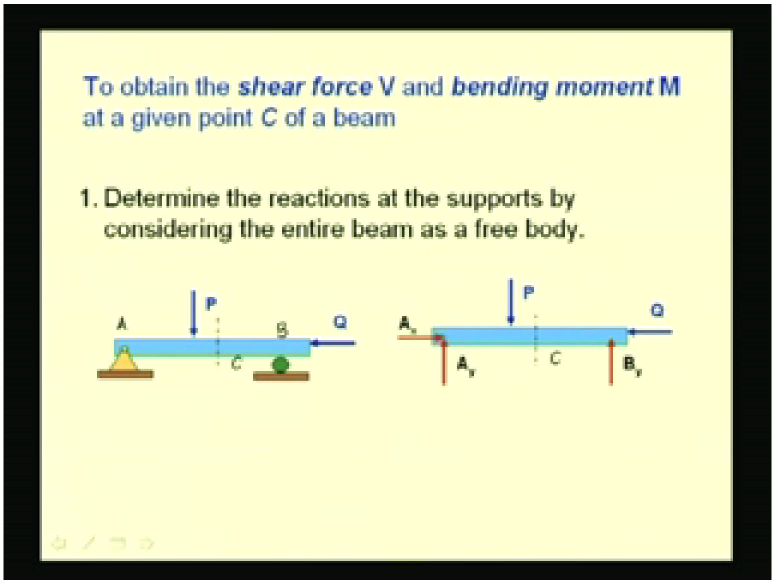

In order to determine the shear force and bending moment, we consider the free body diagram of a portion of a beam and determine the unknown internal forces. Let us see an example.

Here, you see a beam which is pinned at A and has a roller support at B. We are interested to find the shear force and bending moment at a section C. We have some applied loads P and Q. First, we determine the external reactions and at B we have the vertical reaction B_y. At A, since it is pinned, we have A_x and A_y. We know that from the free body diagram, from an equilibrium equation these three unknowns can be determined.

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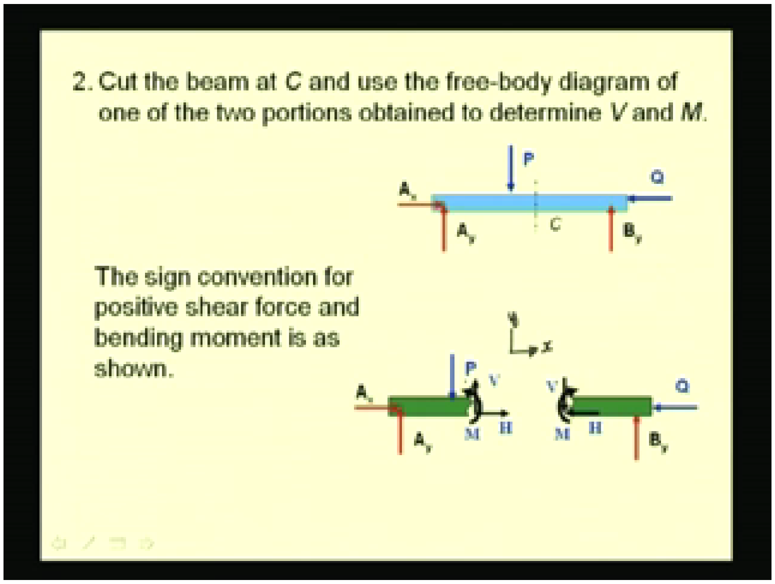

Now, we consider a portion of the beam that is sectioned at C, either it could be the portion AC or the portion CB. Construct the free body diagram and determine these internal forces. Here, you will see the free body diagram of the portion AC at section C. We have the resultant of the internal forces having components V which is the shear force, transverse to the axis of the beam, and H, the axial force along the axis of the beam, and the moment M which is the bending moment. In the other free body diagram that is the free body diagram of the portion CB, these forces are equal, but they are of opposite signs.

Like here, V is vertically upward then in the other free body diagram, it is vertically downward. These directions that have been shown are taken as the positive convention; that is, the shear force mark vertically upward on the left hand portion that is AC is taken as the positive shear force, and the axial force is taken positive along the positive direction of x. If we designate this local coordinates x, y and z perpendicular to the plane of paper and the moment M which is the counter clockwise moment is taken as the positive bending moment. The positive convention has been chosen from their effects.

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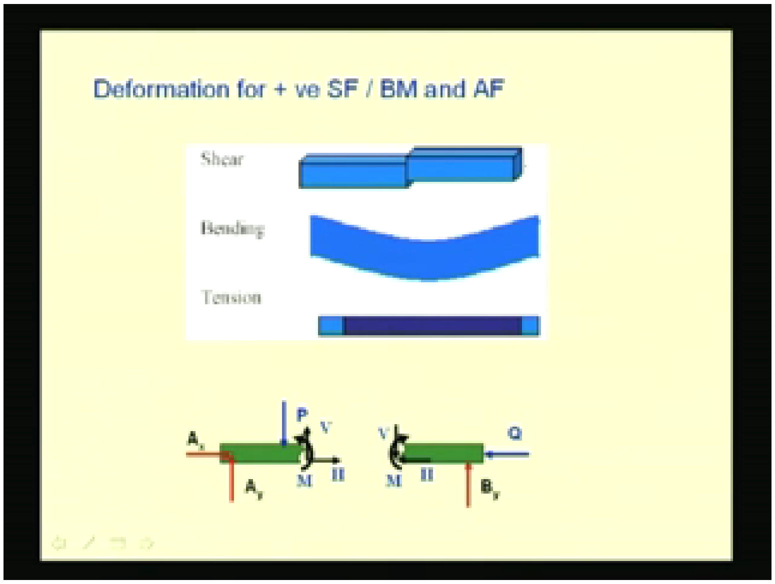

We have seen already that the the shear force V tends to shear a beam and the direction of the force V tends to shear the beam in this fashion. Let us consider this bending moment M. This tends to bend the beam in this fashion. The axial force H tends to pull this member, thereby the member is subjected to tension. So, these effects are taken for the positive conventions.

Let us consider a simple example problem.

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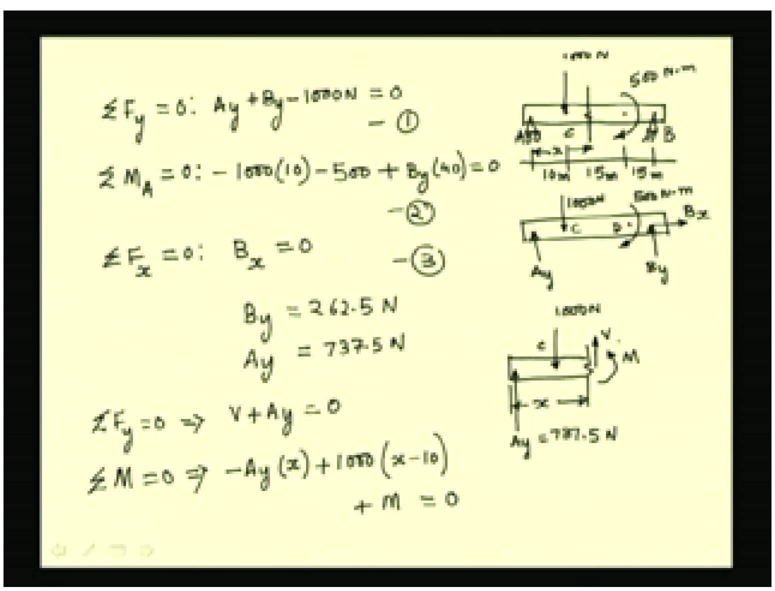

Let us consider a beam AB with a roller support at A and a pin support at B, and a concentrated load of 1000 Newtons at C and a point moment of 500 Newton meter. The various dimensions, all dimensions are in meters. We are interested in finding the shear force at a distance x from the end A. How do we proceed?

We first construct the free body diagram of the complete beam to determine the external reactions. So, let us draw the free body diagram. At A, we have a roller support. So, we have one component of unknown reaction. At B, we have two components of unknown reaction. We have the applied loads 1000 Newton and a point moment of 500 Newton meter, at C and D. Now, from this free body diagram, we can write three equations which can be used to determine the three unknowns. Let us write those equations.

We can write the force summation equation for the vertical component. We have the force $A_{y,}$ the force $B_{y,}$ the two unknown vertical reactions. The vertically applied load which is downwards equal to 0. Let us write a moment summation equation about A. We have the 1000 Newton force having a clockwise moment about A which is a negative moment. So, we have minus 1000 into 10 which is the momentum. At D, we have a clockwise moment of 500 Newton meter which is also a negative moment. At B, we have two unknown reactions that is B_x and $B_{y.}$ The moment of B_x is 0, because it passes through A, but the moment of B_y is counter clockwise, so it is positive. The momentum is 40 meters. So, we have plus b_y times 40 equal to 0.

Let us write the force summation for the x-axis. We have only one horizontal component of the force that is B_x and so, it has to be 0. From these three equations, we can determine the three unknowns that is A_y, B_x and B_y. We already found that B_x is 0. By solving 1 and 2, we find B_y to be 262.5 Newtons and A_y as 737.5 Newtons.

Now we can proceed to determine the internal forces that are the shear force and bending moment. To determine the shear force and bending moment at this section C, or a section at a distance x form A, we consider the free body diagram of the section which is at x meters from A. We have the applied load of 1000 Newton at C and the reaction A_y which has been found to be 77.5 Newton. The resultant of the internal forces at this section will be the shear force.

Since there is no horizontal force, the axial force is 0. Even if you will consider the axial force, from the free body diagram, it will turn to be 0. We consider the moment M. Both have been taken in the positive sense, as discussed.

From this free body diagram, let us write the equations. First, let us write the force summation equation which gives that V plus A_y should be 0, because we have thea vertical component of the force V and the vertical reaction A_y.

Let us take the moment summation about this point that is the point on this section. Why we would like to take it? We can take it about any point, but since V is an unknown, we can avoid it in the moment equation. If we take the moment summation about a point on that section of our interest, let us take the moment and equate it to 0. So, that states we have this force A_y which causes a clockwise moment. At this section, the momentum is x meters.

We have A_y times x which is a negative moment, because this is a clockwise moment about this section. We have the moment of this 1000 Newton force which will be counter clockwise about this point. So, we have plus 1000 times the momentum will be x minus 10 meters, because this force of 1000 Newton is being applied at 10 meters, from this end A. We have the momentum as x minus 10 and we have this moment M, which is a positive moment. Since we have already determined $A_{y,}$ now we can find V as well as this M. These two are the internal forces at this fraction. Similarly, we can consider any section along this beam and determine the shear force and bending moment at that section.

Let us consider another example.

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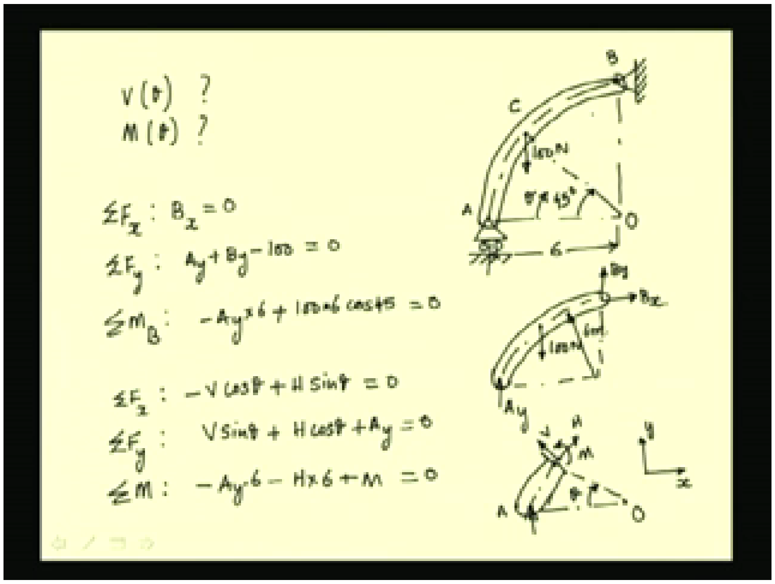

Here, we have a curved beam which is supported by a pin at B and at A by a roller. This is a circular beam with radius equal to 6 meters. There is a concentrated load of 100 Newton applied at the midsection of this circular arc beam. We are interested to find the shear force and bending moment that occur in this beam as a function of theta, where the theta is taken from the point A towards the point B. So, we are interested to find V, the shear force and bending moment as a function of theta.

Now, how do we proceed?

We first determine the external reactions. Then, we construct free body diagrams for portions of the beam and develop the equation. First, let us consider the free body diagram of this complete beam. At A, we have a roller support, and at B we have a pin support. We have an externally applied load of 100 Newtons. The radius is 6 meters. Let us write the equations resulting from this free body diagram.

If we sum the horizontal component of the forces, we find that we have only one horizontal force and that is B_x, and it becomes 0. Let us take the force summation along y axis. We have this reaction B_y and the reaction A_y plus the externally applied force 100 Newtons along the y direction. We have, A_y plus B_y minus 100 the downward force, equal to 0.

Let us take a moment equation and it is convenient to take the equation about B. Then, we will able to find this reaction $A_{y,}$ and equate it to 0. The momentum of this reaction A_y is 6 meters and this causes a clockwise moment about B. So, we have minus A_y times 6 which is the momentum. The 100 Newton force has a counter clockwise moment about D which is positive. So, we have plus 100 times the momentum of this is this. This load is applied at a radial angle of 45 degrees. So, we have the momentum as 6 cos 45. From these two equations, we can find A_y and B_y.

After we have found these external reactions, we can move on to determine the internal forces. Let us consider a portion of the beam before the mid section. Let me call this as C and O. So, we are considering a section of this beam before the mid section C. At A, we have the external reaction and we have the shear force V transverse to the member, the axial force H and the moment M. So, V, H and M are the resultant of the internal forces at this section. V is radial and H is along the tangent of the axis of the beam, because this is a circular arc and M is the moment of all these internal forces.

From this free body diagram, let us write the equilibrium equations. If we sum the forces along the x direction, we have, these are the positive x and y directions; we have the component of this shear force in the negative x direction which is minus V cos of theta. We have the horizontal component of this axial force H which is H sin theta, and this has to be 0. Now, summing the vertical components, we have V sin theta which is positive plus H cos theta which is the component of this force in the vertical direction plus A_y equal to 0.

Let us take the moment summation about this point O, because this V, the shear force is radial and passes through this point O. It can be avoided in the equation and is minus A_y times 6, the clockwise moment of this reaction A minus H times 6, because this force, H is tangential at this point to this radial line. So, the momentum of this force H is 6 meters again; this plus the moment M has to be 0. From these equations, we can find the three unknowns that are V, H and M.

We can consider the free body diagram of a portion between C and D, and we can write the equations for shear force and axial force and bending moment. Now, we have the shear force, axial force and bending moment as a function of theta as interested by us. We will continue with our discussions, on determining the shear force and bending moment in beams in the next lecture.

Thank You