Engineering Mechanics

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Analysis of Structures-1

Module 2 Lecture 5
Analysis of frames & machines

Today, we will continue our lecture on analysis of structures. For your reference, this is lecture number 5 of module 2. In the last two lectures, we have seen how to analyze interconnected rigid bodies particularly the trusses. We have seen that a truss constitutes of interconnected two force members. Today, we will see other class of structures where the individual members which are interconnected; they are both two force members as well as multi force members.

We will see what we call as a multi force member before going to analyze the structures containing multi force members and two force members.

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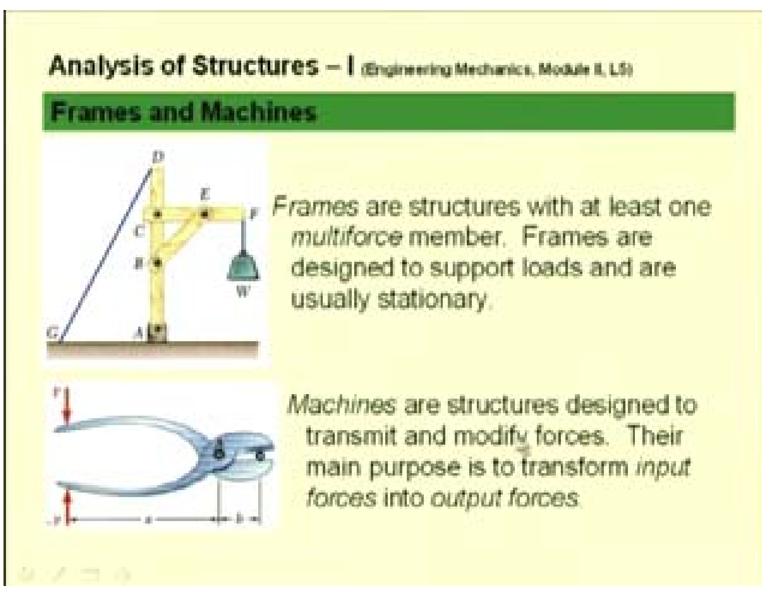

We have particularly frames and machines which come under this class of interconnected rigid bodies. Let us see first what we mean by a frame: Frame is a structure similar to that of a truss which is used to support static loads. The difference between a frame and a truss is that the frame has apart from two force members what we call as multi force members. So here, you see in the picture a frame which is used to support this load W which is connected at this point F. The frame is supported by a pin at A and it is tied to the point G on the ground by a taut string at D.

Here we see that this structure has this member B which is connected by a pin at B and a pin at E. Since forces are only applied at B and E, this member BE is a two force member. But let us see this member ABCD; here we have the pin reaction at A, and the tension in the string GD at D and the pin reactions at this point C as well as B; since we have lateral loads applied at these two points not in the extremities but in the middle also, the loading in this member is not axial but it has other components of internal forces. So multi force members are subjected to not only tensile and compressive internal forces but two other kinds of internal forces also. We will see them a little later.

Right now, let us analyze for the forces that act on such members; that is two force as well as multi force members in the frame. The other class of structures which also come under this category are machines which also have two force as well as multi force members, but the primary purpose of machines is to modify and transmit forces, as you see here, an example of a plier which is used to hold this wire. This plier helps in magnifying this force P, which is applied by the human, so that enough force is generated to hold the wire or it could be to cut the wire, etc. The primary purpose of such interconnected bodies, which we call as machines, is to modify the input forces to desirable output forces, both in terms of magnitude as well as direction in some cases.

Before going to analyze these kinds of structures let us see in little more detail the multi force members.

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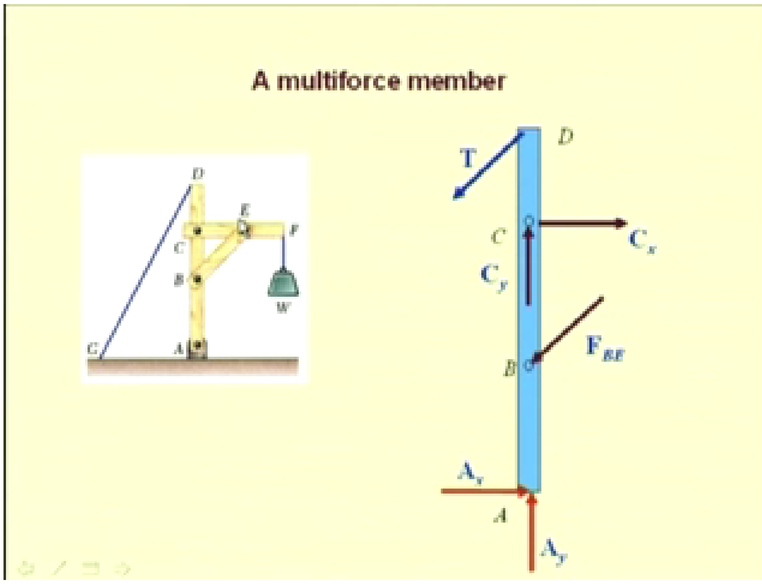

In the same frame example that we have shown let us consider the free body diagram of the member ABCD. This is the free body diagram of the member ABCD. We have the pin connection at A, so the two components of reaction A_x and A_y are shown at A. At D, we have the strength tension. Here the direction of this force is known, only the magnitude of this force is unknown; because, we know that strings can only be in tension and their direction of the tension is along the direction of the string. At C, we have a pin connection and we do not know both the magnitudes as well as the direction of the forces that act at C; so we have both C_x and C_y. In other words, we have a force at C which is unknown in magnitude as well as in direction.

Let us see this point at B. Here at B, this member ABCD is pinned to another member BE which is a two force member. Since we see that this member is being subjected to forces B and E only and for the equilibrium of this member the force has to be collinear and it has to be along this direction BE. At B, we have a force which is unknown in magnitude but known in direction. All such combination of forces can act in a multi force member: forces with known direction and unknown magnitude forces, forces with unknown magnitude as well as unknown direction. With this understanding of multi force members, let us see how to determine the forces in the various interconnected members of frames.

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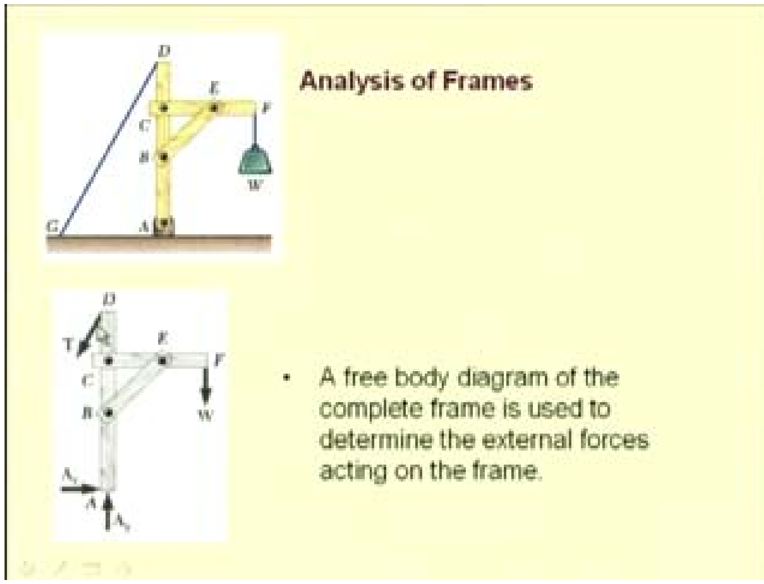

We will take the same example to illustrate the analysis procedure. In order to proceed, first we have to determine the external reactions for the frame under the given loading. We have this frame ABCDEF, with applied load at F, and reactions at A as well as at D. Let us construct the free body diagram.

We have shown at A, the two components of the pin reaction A_x, A_y and at D we have the force in the string GD that is T. We know that this force T is of unknown magnitude but known direction. For the equilibrium of this rigid structure, the sum of the forces has to be 0 and sum of the moments has to be 0. This condition results in three equations which can be solved to determine three unknowns. We have three unknowns, that is: A_x and A_y - the two components of reaction at A and the magnitude of the tension T. So the problem is statically determinate and one can determine these forces.

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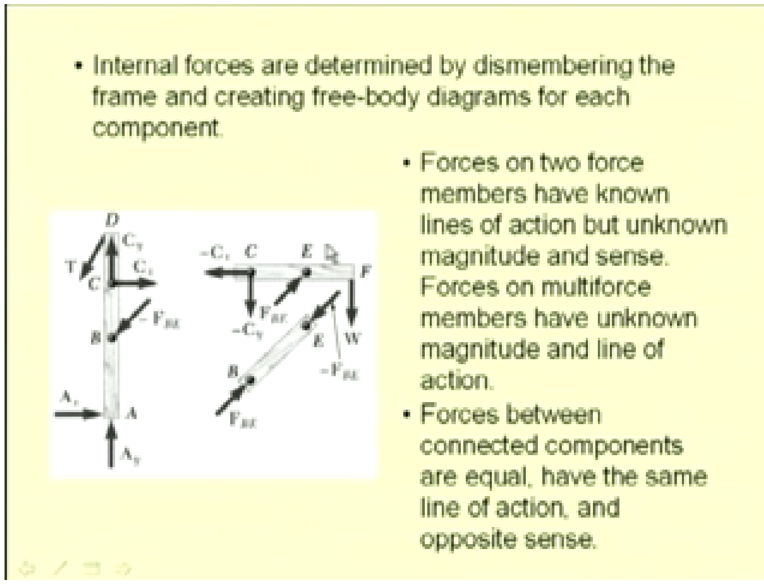

Now, in order to analyze the forces that act at various points of these interconnected bodies, we have to dismember and construct the free body diagram of each of the constituent members; both multi force as well as two force members and solve the resulting equilibrium equations.

For this example, these are the free body diagrams of the constituent members. We have the free body diagram of a multi force member ABCD where we have this pin reaction at A, the force from the two force member F_{BE} which is known direction but unknown magnitude. At C, we have the pin reaction coming from another multi force member. So, this pin reaction is both unknown in magnitude as well as direction; at D we have the tensile force of the string of unknown magnitude and known direction.

This is the free body diagram of the other multi force member in the frame CEF. The forces at C are equal and opposite to the forces that we have shown in this free body diagram. The force at E is because of the force in the two force member F_{BE}, which is equal and opposite to the force shown in this free body diagram and the applied load W. This is the free body diagram of the two force member BE where we have the axial force of F_{BE}.

By solving the equations resulting from these free body diagrams, we have two free body diagrams, so it will result in six equations from which we can determine six unknowns. This free body diagram just shows that this force F_{BE} is equal and opposite to this force F_{BE} at this point B. From these two free body diagrams, we have six equations to determine the six unknowns, that is: C_y, C_x, F_{BE}, A_y, A_x and T. You should remember that these equations are not independent of the equations that we used to determine the external reactions by considering the complete free body diagram of the frame.

We see that these pins are assumed to be associated with one of the members while constructing this free body diagram. So when dismembering, we associate a pin with one of the members; because at E, the C_{EF} and the member B_E are connected at E by a pin at E. The pin at E is assumed to be either associated with this multi force member C_{EF} or with the two force member BE. While solving, we always start from a multi force member and proceed.

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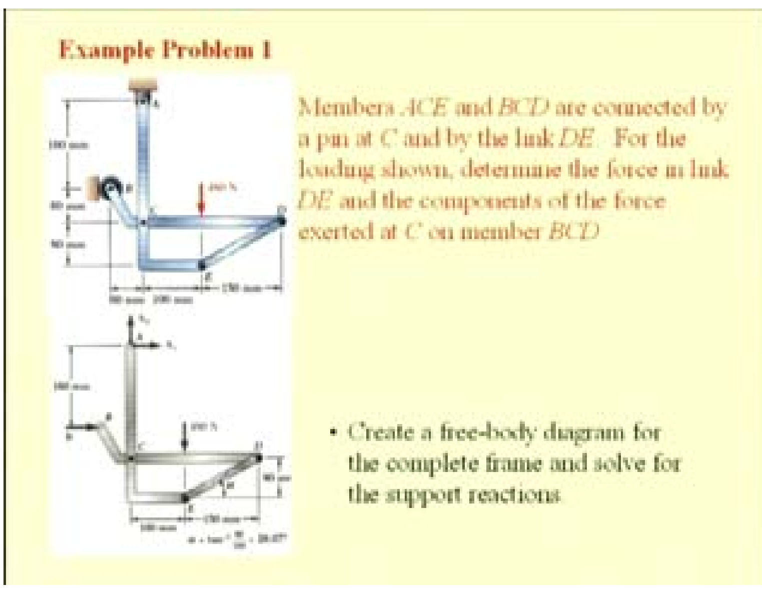

Let us see an example on how to analyze frames. Here you see a frame constituting of members ACE, ED and BCD connected by pins at A, C, E and D. This frame is supported by a pin at A and a roller at B; various dimensions are also given. An externally applied load of 480 Newtons is laterally applied to this member BCD. We are interested that under this loading condition, what are the various forces developed in the joints and in the members.

Let us see first whether this member is statically determinate, the frame is statically determinate, by considering the free body diagram of the complete frame. So we draw the free body diagram of the complete frame. At A, since we have a pin, so we have the two components of reaction A_y and A_x. At B, since we have a roller we have a single component of reaction B. 480 Newton of applied force is also shown in the figure. Various dimensions are shown to compute the moments and other required quantities. We have three unknowns: A_y, A_x and B; from the rigid body equilibrium equations, we can determine the three unknowns. It can be two force summation equated to 0 and one moment summation equated to 0.

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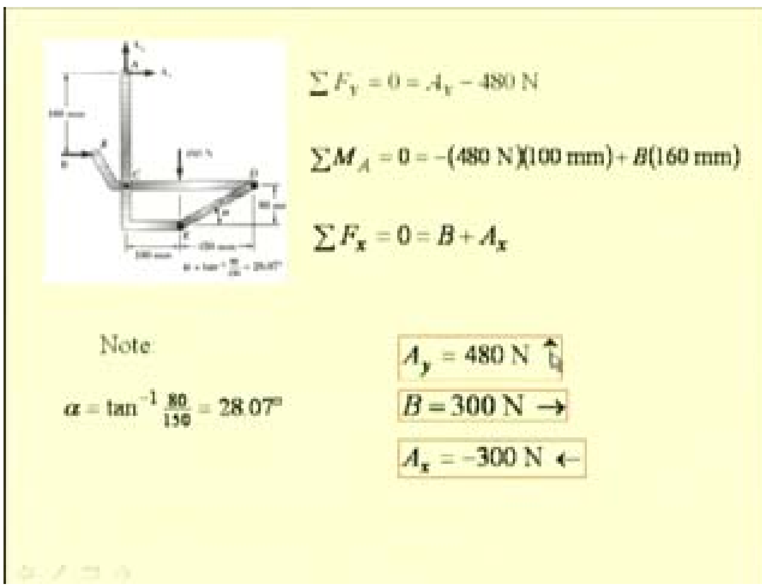

Once we determine the supports using these equations, we can proceed further to determine the internal forces. These are the three equations that result from the free body diagram of this complete frame. The force summation about the y axis when equated to 0 predicts the force A_y; because we have A_y, the vertical component of reaction at A and the 480 Newton which is downward so —480 Newton equated to 0 can be used to find the force A_y.

We can conveniently use the moment summation about this point A because these two unknown forces A_y and A_x are concurrent at A and the equation can be used to find this force at B. So the momentum of this force 480 Newton is 100 mm about this point A and it is a clockwise moment. So we have the negative sign; so the moment of this force is —480 into 100 mm. The moment of this force B, the reaction because of the roller at B, is B times 160 mm which is the momentum. This is a counter clockwise moment about A; so thereby it is a positive moment. From this equation, we can determine B.

The third equation which is the force summation about x-axis can be used to relate this force at B and the horizontal component of reaction at A. Since we have determined B, now it is possible to determine the horizontal component A_x. For this problem, these three have been determined as: A_y as 480 Newton, B as 300 Newton and A_x as —300 Newton. Now that we have determined these external reactions, let us dismember the frame and construct the free body diagram for the constituent members.

First, let us consider one of the multi force members say BCD.

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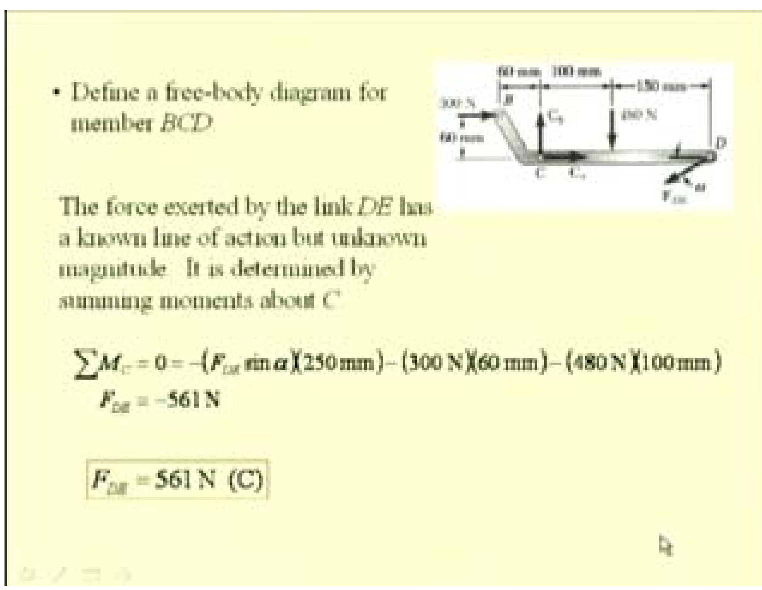

This diagram shows the free body diagram of the multi force member BCD. At B, we have the externally applied reaction coming from the roller which has been determined as 300 Newton; at C, we have an unknown pin reaction coming from the other multi force member; so both magnitude as well as direction is unknown; so we represent it by two components of reaction C_y and C_x. At D, we have the force coming from a two force member which is unknown magnitude and known direction; so the direction of this force is along the member DE and from geometry we know the direction of this member DE. We have this externally applied force 480 Newtons. So from this free body diagram, we can write three equations and determine the three unknowns. Here the three unknowns are C_y, C_x as well as the magnitude of this force F_{DE}.

From the equilibrium equations resulting from this diagram we can determine these three forces. Let us proceed to determine the same. We see that the point C can be conveniently chosen to take the moment summation because these two forces are concurrent at C and this equation can be used to determine the force F_{DE}.

Let us take the moments about C. The force F_{DE} is taken in its two components - that is F_{DE} cos theta along this member CD and F_{DE} sin alpha perpendicular to this member CD. So the moment of the horizontal component about C is 0 because there is no momentum, but the vertical component that is F_{DE} sin alpha has a momentum of 100 plus 150, that is 250 mm and that causes a counter clock clockwise moment which is negative.

The 480 Newton force also has a clockwise moment; so again it is a negative quantity; its momentum is 100 mm. The 300 Newton force from the roller is also a clockwise moment about C; so again it is a negative quantity. The momentum for the same is 60 mm. So we write these three quantities and sum them and equate it to 0. In this equation we have only one unknown F_{DE} and the same has been determined as —561 Newton. Since we have originally assumed the force to be of tensile in nature, but since the answer has turned out to be negative, the forces are compressive in nature. Let us move to another multi force member that we have in this frame.

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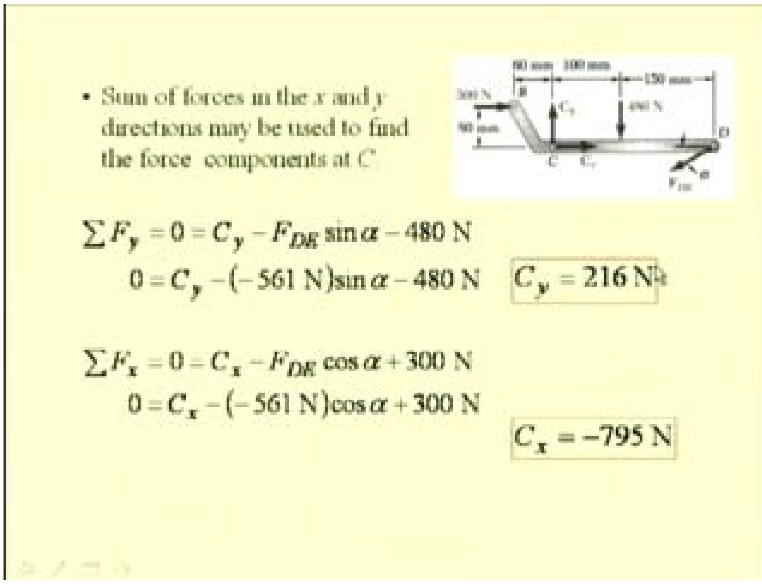

Before moving on, we can determine the other two reactions that are C_x and C_y from the two force summation equation. Summing the vertical components of the forces, we can determine this force C_y as 216 Newton. Summing the horizontal component of forces we know the horizontal component of the reactions C_x; the same has been determined as 216 Newton and minus 795 Newton.

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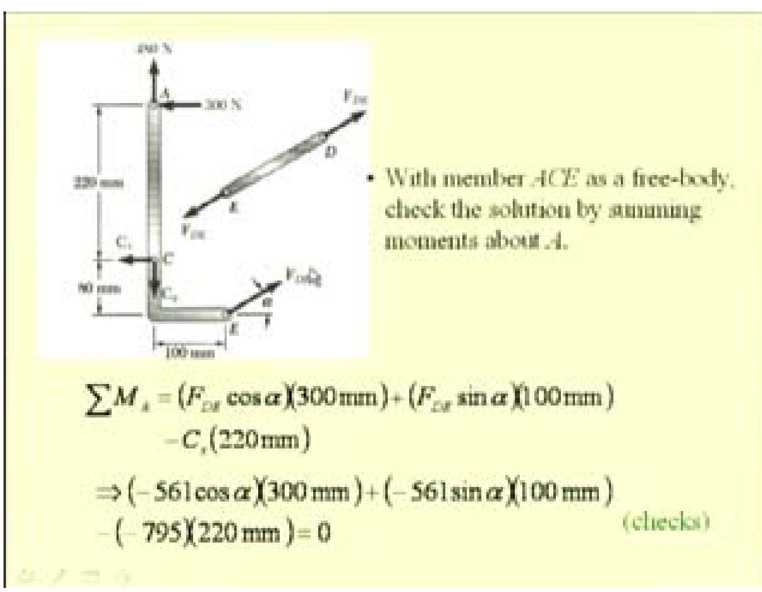

Now we move to the next multi force member that is ACE and we draw the free body diagram of this member. We have this pin reaction at A; so the two unknowns are which have been determined earlier from the free body diagram of the complete frame are shown here. The two reactions C_x and C_y have been determined from the free body diagram of the previous multi force member. The force from this two force member ED, F_{DE} again has been found from the previous free body diagram.

We do not have anything new to be found from this free body diagram, but the equations that result from this free body diagram can be used to check whether our results are correct or not. Let us do that by summing the moments about A. We know this force C_x, C_y as well as F_{DE} determined from the previous free body diagram of the multi force member; so now we use those values and sum the moment of these forces about this point A. We verify whether the sum becomes 0 because this member ACE is in equilibrium, the sum of the moments has to be 0 about this point or any other point.

In this equation, we have the moment from this force F_{DE}; so we have both the vertical as well as horizontal component of the force having the moment about A. The momentum for the vertical component that is F_{DE} sin alpha is 100 mm and the momentum for the horizontal component that is F_{DE} cos alpha is 220 plus 80; that is 300 mm. These two components of moments cause counter clockwise moment about A; so both are positive quantity. At C, we have C_x and $C_{y.}$ C_y does not cause any moment because it is collinear with A and the moment of this force C_x whose momentum is 220; is a clockwise moment; so it is a negative quantity. So we have this minus C_x into 220 mm. We substitute the values as found from the previous free body diagram. We find that the sum turns to be 0; so the forces that we have determined are correct.

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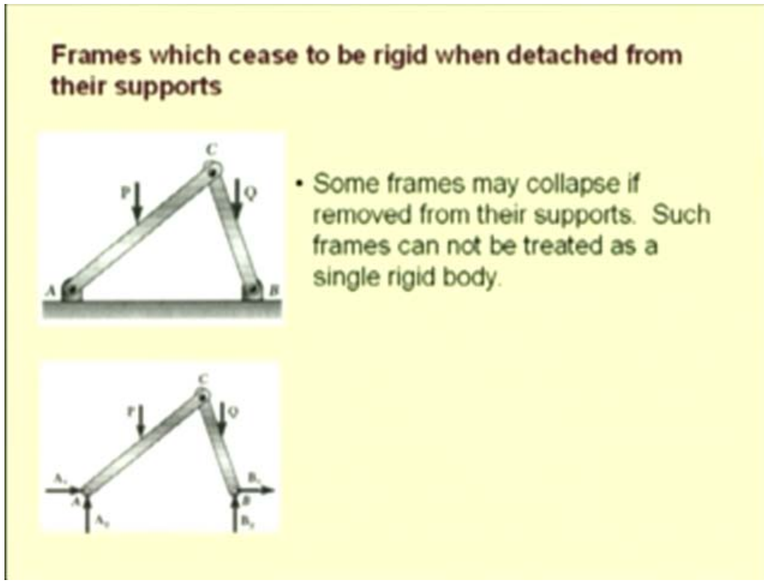

Sometimes, frames collapse when they are removed from their supports; such frames are not rigid; we cannot consider the free body diagram of the complete frame to determine the external reactions. In that case, the analysis procedure has to be little different. Here we see an example of a frame which is not rigid and collapses when removed from the support. So, here we have a pin reaction at A, as well as a pin reaction at B and two members AC and CB. If these two externally applied forces P and Q were applied at let us say joints, then this becomes a truss, but since these forces have been applied lateral to the member AC and CB these members become multi force members and thus this is a frame.

Let us consider the free body diagram of this complete structure. At A we have two components of reaction A_x and A_y and at B we have two components of reaction B_x and B_y. We know that from the equilibrium equations, it is possible to write three equations and determine three unknowns, but here we see that we have four unknowns.

The problem is that this member ACB once removed from the support is not rigid. Thus we cannot write the rigid body equilibrium equations for a structure which has the tendency to collapse under the action of the load.

How do we proceed? We dismember this frame and create components of the frame which are rigid and then construct the free body diagram of the same and from that we proceed further.

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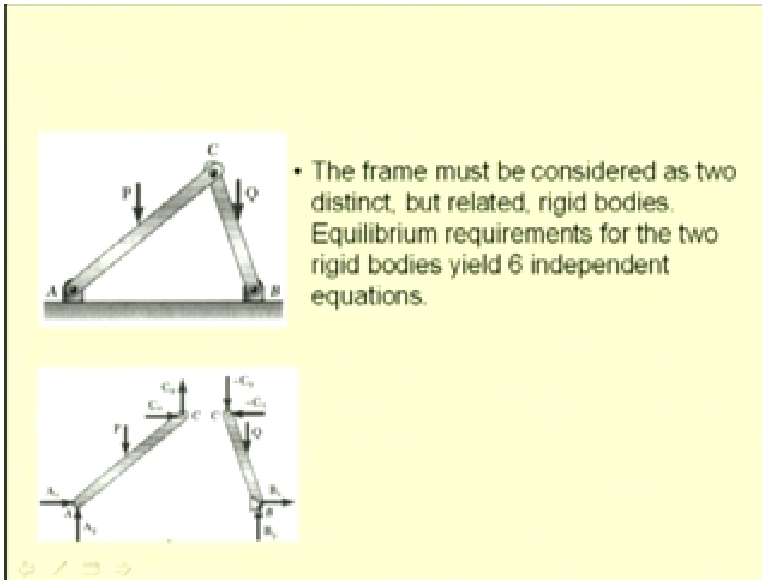

Let us dismember this frame and construct the free body diagrams. We have dismembered the frame here. The frame has only two members AC and CB and we construct the free body diagrams of these individual members. At A we have the pin reaction A_x and A_y; at C we have the joint forces C_x and C_y ; at B again we have the pin reaction that is B_x and By.

How many unknowns do we have? A_x, A_y, C_y, C_x, B_x and B_y. These two forces are equal and opposite to the forces in the other free body diagram; so, they do not constitute a additional set of unknowns. Totally, we have six unknowns to be determined from two free body diagrams. Since both these members AC and CB are rigid we can use the equilibrium equations. We have three equilibrium equations from each free body diagram. So we have six equations to determine six unknowns; thus, the problem becomes statically determinate.

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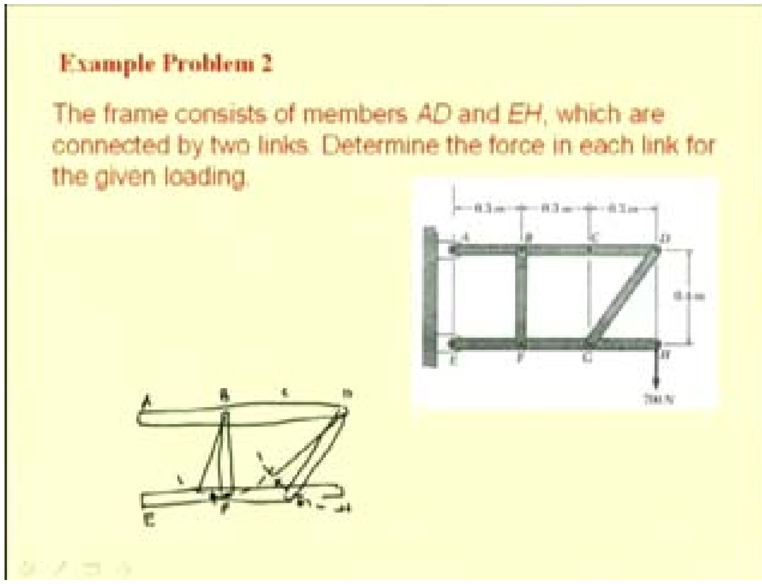

Let us see an example. Here you see a frame ABCDEFGH. The member ABCD is connected to the member EFGH by two, two force members: one is this BF and other is this GD. This frame is connected by pins at A as well as at E and we have an external load of 700 Newton applied at H. First, let us see whether the frame under consideration is a rigid body or not.

Let us take this frame ABCD having a two force member BF and another two force member BG and then multi force member EFGH. In order to analyze whether this frame will collapse or not after it has been removed from the support, let us fix one of these members, say ABCD and try to see if the other members are having some degrees of freedom for moment. If this structure has to move, since we have this two force member BF connected at this point F, this part of the frame EFGH can move such that the point F traces a circular part because this body BF is rigid, so its dimension cannot change. So if any moment that is possible is such that the point F moves along a circular path. In the same way we also see that we have another two force member that is DG which is also rigid; so any moment is possible such that the point G traces a circular path with radius as DG. If at all any moment is possible, it is only possible if F traces this path and G traces this path, but we also know that on this member EFGH the distance between these two points F and G has to be same. So let us take any point on this locus and try to see if we can find a point on this locus such that the distance is FG. We take the distance and mark the distance on this. We see that it is possible to find and the new position of this member could be something like this -

the position of this EFGH. We see that once this frame has been removed from the support, it is possible to collapse this frame if you apply some forces. So this frame is not a rigid frame and we cannot consider the free body diagram of the complete frame.

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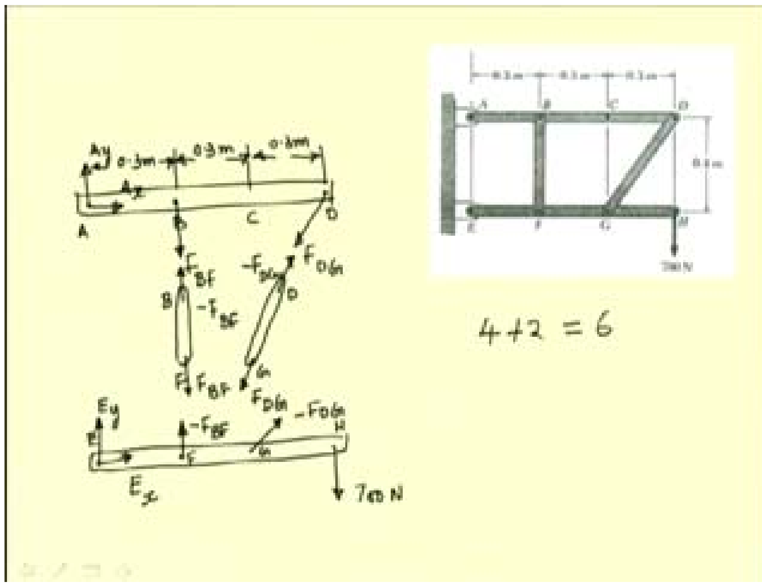

Let us construct the free body diagram of individual components after dismembering. First, let us consider this multi force member ABCD. At A, we have a pin connection, so two components of forces are unknown, that is A_y and A_x. At B, we have a two force member, so we have this force FBF of known direction and unknown magnitude. At D, again we have a two force member, so the direction of this force is known, but the magnitude is unknown. This completes the free body diagram of this member ABCD. Obviously, one has to mark the various dimensions in order to solve any moment equation.

Now, let us move on to the two force members. We have this member BF. This force is $-F_{BF}$ and the force at F is F_{BF}. Same way we have another two force member DG; here we have the force $-F_{DG}$ and here we have the force F_{DG}. So all these forces have known direction but unknown magnitude; because, they are the forces acting on a two force member and these forces are along the member BF and DG.

Let us move to the last member that we have, that is EFGH. At H, we have an applied load of 700 Newton. At E, we see that we have a pin connection and so we have two components of unknown reaction: E_y and E_x. At F, we have the force from the two force member that is —F_{BF} and at G we have the force from the two force members GD which is —F_{DG}.

Now we have completed all the required free body diagrams. Let us see how many unknowns we have to determine: A_x, A_y, the force in the member F_{BF} and force in the member DG. We have four unknowns from this free body diagram ABCD. From this free body diagram of EFGH, we have unknowns E_y E_x, but these forces, that is F_{BF} and F_{DG} are equal and opposite to these two forces shown in the earlier free body diagram. This comes from the fact that these forces are the forces of the two force member and so they are equal and opposite. So we have only additionally two more unknowns from this free body diagram.

We have a total of 6six unknowns to be determined. From the free body diagram ABCD and EFGH it is possible to obtain three plus three equations, so the problems is statically determined and solvable. So this is the way one has to proceed to solve frames, either frames which are rigid when dismembered from its support or frames which are removed from the support and become collapsible.

In case the frame collapses when removed from the support, we have to dismember and consider the free body diagram of individual components rather than considering the free body diagram of the complete frame.

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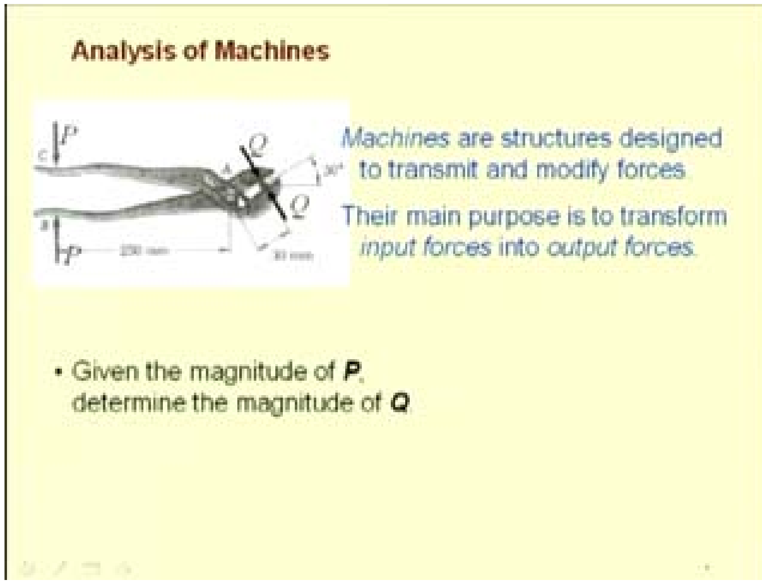

Let us move on to analyze machines. We have seen that machines are structures used to modify forces and provide mechanical advantage. Like here you see a plier used to hold a rod or a wire. The force required to hold or let us say to cut this wire may be a large quantity Q, which is not convenient for a human to supply. So we have this plier which provides a lever arm, so that a force which is of considerably less magnitude can be applied and the job of either holding or cutting the wire can be accomplished.

Machines are also inter-connected rigid bodies with two force as well as multi force members. The difference between a frame and a machine is that machine has movable parts; so we cannot consider the complete free body diagram of a machine, but rather we should always dismember a machine and consider individual constituent to rigid bodies, draw their free body diagrams and write their equations to solve.

A typical problem could be that in this plier. given the magnitude P, can we determine the force Q that is applied on this rod or for a given magnitude P what are the kinds of forces that are being supported by the pin at A; because, the design of this machine may need that the pin should be sufficiently strong and should not shear under the action of the forces in the constituent members.

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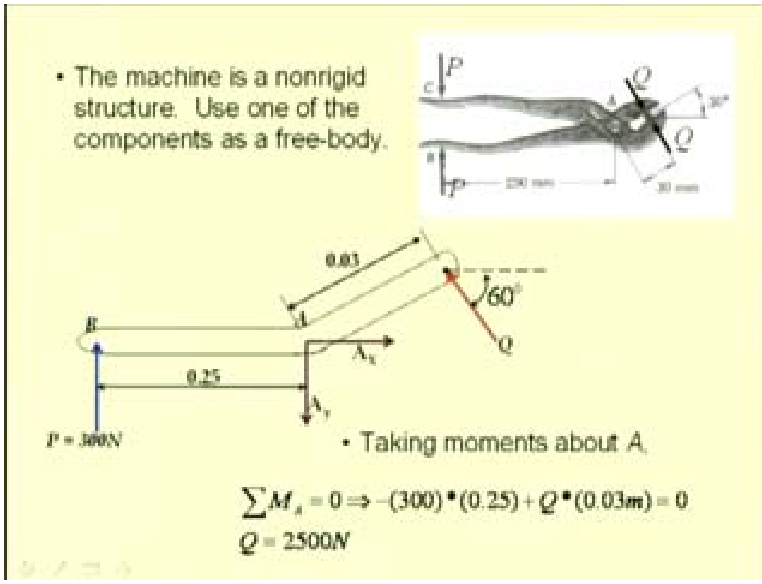

Let us consider this problem. We use one of the free body diagrams since I have mentioned that machines are movable, the individual components move and so we cannot consider the complete machine as a rigid body. We have to only consider constituent members which are rigid.

In this case, the member CA is rigid member and this member BA is also a rigid member. We can consider the free body diagrams of these to determine the required forces. Here we have considered this member BA. At B, we have the applied force P and at A we have a joint reaction coming from another multi force member. So the reaction is both unknown in magnitude as well as direction. We represent it by two components A_y and A_x. At Q we have the force that is being applied on this rod or wire that is being held by the plier.

This force is of known direction because the force is applied normal to the jaw of this plier; so the direction of this force is known but the magnitude of the force is unknown. We see that in this free body diagram, we have three unknowns that is A_x, A_y, as well as magnitude of this force Q and it is possible that if you know the geometry of this plier arm we can determine those forces.

Let us say, we can consider the moment summation about this point A in order to determine this force Q. If we are interested to find these reactions A_x and A_y, we can consider the force summation equations of equilibrium to determine the two components of reaction A_x and A_y.

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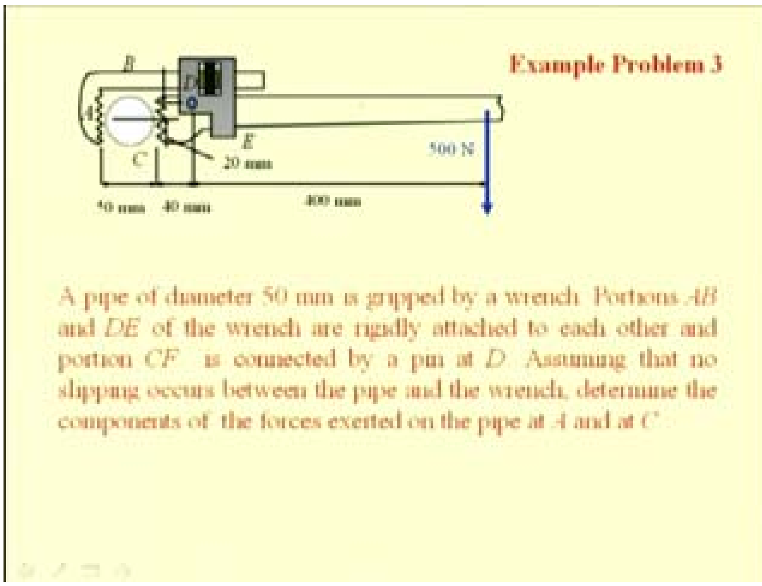

Let us see an illustrated example. Here we see a pipe wrench. We would have seen that plumbers use these kinds of wrenches to hold pipes or pipe fittings like taps or elbow joints to fit these fittings with the pipe. At pipe ends, we have threads in which the taps or the elbow joints can be screwed on. In order to fit these pipe fittings to a pipe, one has to hold the pipe as well as apply a twisting moment in order to drive the pipe into the fitting. These wrenches are used for holding as well as supplying these twisting moments to the pipe segments.

In this example, you see a pipe wrench. It consists of an adjustable nut, so that the position of this member DE can be adjusted on this arm, so that variable diameter pipes can be held. We have a saw tooth jaw at A as well as at C, so that no slip occurs between the pipe and the wrench face. We are interested that if a plumber applies a force of 500 Newton how much twisting moment is generated on the pipe or what are the reactions at A and C. Let us consider the structure of the constituent members.

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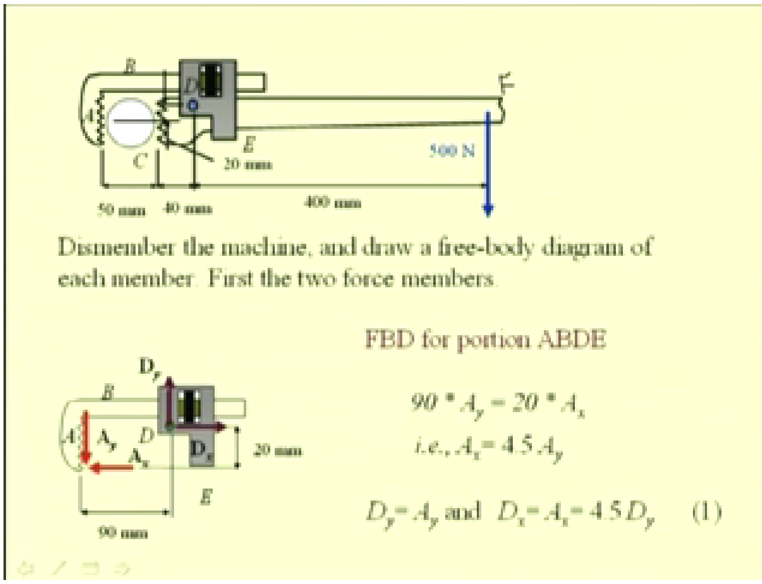

Here we see that one of the members of this machine tool is ABD and the other member is CDE. Let us say F is the point where the plumber applies the load. So these two are the constituent members of this wrench connected by a pin at D. If you see this member ABD, we see that forces are applied at A and at D only. Since forces are applied at only two points, this member ABD is a two force member. Though it does not come from the first hand visualization of the problem, but when we see the nature of the forces, we are able to predict that ABD is a two force member. Let us consider the free body diagram of these constituent members.

This is the free body diagram of the member ABD. At A, we have an unknown reaction, both in magnitude and direction, which has been represented by the horizontal as well as vertical components of reaction. At D, we have the pin joint; so we have two unknowns that are D_y and D_x.We see that this member CDF is a multi force member because forces are applied at this point C, at this point D, as well as this point F.

Since this member is a multi force member the force at D is both of unknown direction as well as unknown magnitude. That is why we represent both the components D_y and D_x at D. So we write the equilibrium equations. If we take the moment about this point D, then the moment of this horizontal component A_x has to be balanced by the moment of this vertical component A_y and that gives the relation between A_x and A_y.

Here we find from the geometry that A_x is 4.5 times of A_y. Next, from the force summation equation, we have the relation that D_y should be equal to A_y and D_x, has to be equal to A_x. Let us consider these set of equations as equation 1, and these may be used to solve the equations that we determine from the multi force member. Let us consider the free body diagram of the multi force member CDEF.

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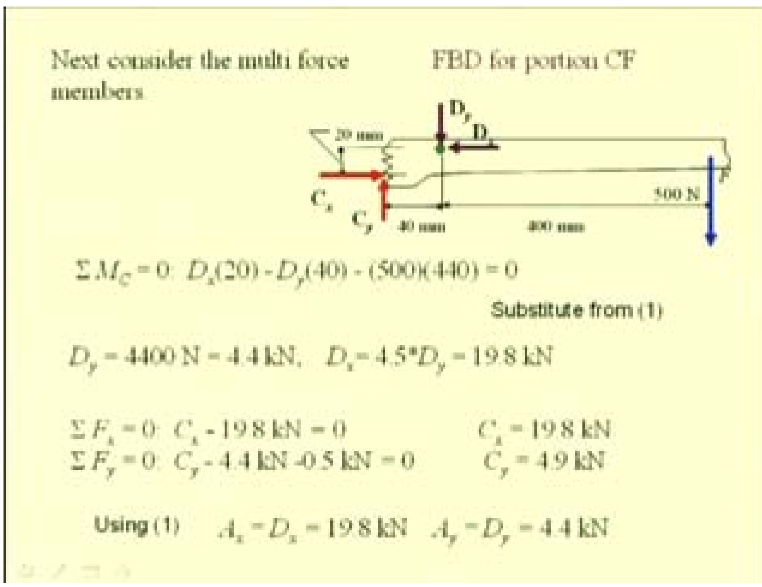

This is the free body diagram. At C, we have a reaction which is both unknown in magnitude as well as in direction; so we represent that by C_y and C_x - two components. At D, we have the joint forces that is D_y and D_x, which has been accounted for in the earlier free body diagram and at F, the plumber applies a force of say 500 Newtons.

This is clearly a multi force member. From this member for equilibrium, we can write three equations: one moment equation and let us say two force summation equations. Let us take the moments of these forces about this point C. We take this point since we do not know these two forces C_x and C_y but we have some relation between D_x and D_y from the earlier free body diagram. So the momentum of this force D_x about this points C is 20 mm; it causes a counter clockwise moment and thereby it is a positive quantity.

This force D_y causes a clockwise moment, the momentum for this is 40 mm; so we have $—D_y$ into 40; the moment of this applied force 400 Newton is again a clockwise moment about this point C, thereby it is a negative quantity.

The momentum of this force is 400 mm plus 40 mm; that is 440. We have another relation between D_x and D_y; from the free body diagram that we have considered earlier, we have one more relation. So let us substitute the values from that equation to determine the two unknowns D_x and D_y. So D_y has been found to be 4.4 kilo Newton and D_x can then be found to be 19.8 kilo Newtons.

Now that we know this D_x and D_y, we can determine these two forces C_x and C_y by considering the force summation equation. If you sum the forces along the horizontal that is along the x-direction, we determine the force C_x to be 19.8 kilo Newton. We use the force summation along the y direction and we determine this unknown C_y to be 4.9 kilo Newton. Again, from equation 1 since we now know D_x and $D_{y,}$ we can know the two reactions A_x and A_y and they are found to be 19.8 kilo Newton and A_y as 4.4 kilo Newton.

Now we have found all the reactions that are being applied to the wrench by the pipe.

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In order to know the forces on the pipe, they will be the equal and opposite forces that are being applied at A and C. We have these forces 19.8 kilo Newton, 4.4 kilo Newton, 4.9 kilo Newton and 19.8 kilo Newton force being applied on to the pipe.

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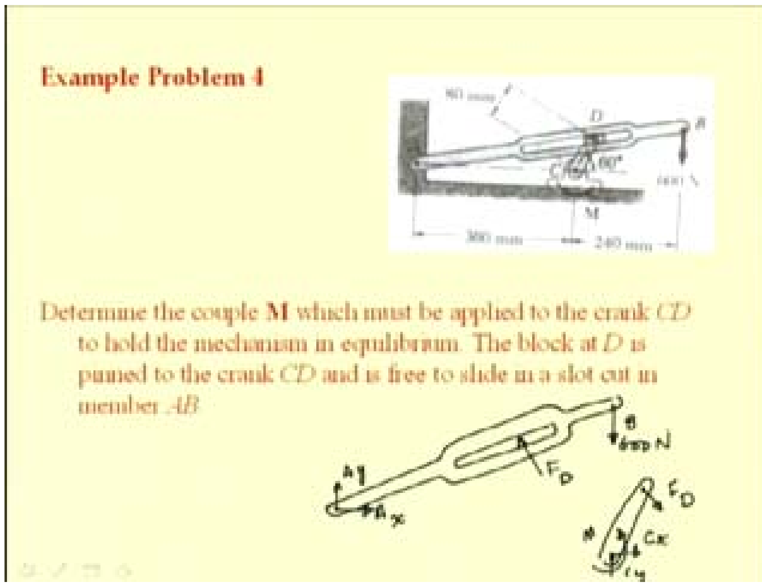

Let us consider another example. We will just consider how to draw the free body diagrams. Later on, one can proceed the way we have illustrated in the earlier example. Here, you see a mechanism which has a slotted arm ADB and a crank CD. We are interested to know the moment that has to be applied on to this crank at C, in order to support this load of 600 Newtons.

This block is free to slide in this slot; thereby, it offers a reaction only along a direction perpendicular to the slot. Let us consider the free body diagram of this member ADB We have the applied force at B as 600 Newton, and at A we have the two pin reactions A_y and A_x and in the slotted arm at D we have the reaction; let us say we call this as force at D. From the geometry of this slotted arm, it is possible to find these forces FD as well as A_y and A_x because we only have three unknowns. From the equilibrium equations of the slotted arm, it is possible to find this force FD as well as A_x and A_y.

Once we have found this force on this slotted arm, that is FD, now we can consider the free body diagram of the crank at C, where we have two reactions C_x and C_y, a moment is being applied that is M. At this point D we have an equal and opposite force FD. These two forces are equal and opposite. From the earlier free body diagram, we have already determined this force FD. From this free body diagram, it is possible to determine the three unknown quantities C_y, C_x as well as M.

In this lecture, we have seen the analysis of frames and machines which are interconnected rigid bodies. In the earlier lectures, we have seen the analysis of trusses which are also interconnected rigid bodies but with only two force members. So, we will proceed with the analysis of the internal forces that occur in constituent members in the next lecture.