Engineering Mechanics

Dr. G. Saravana Kumar

Department of Mechanical Engineering

Indian Institute of Technology, Guwahati

Module 2 Lecture 4
Truss analysis Part- 2

Today we will continue our lecture on analyzing trusses. In the previous lecture, we saw how to analyze the trusses and determine the forces that occur in the members, by the method of joints. The method of joints is conveniently used to determine all the forces in all the constituent members of a truss.

In certain cases, we maybe interested in finding only the forces in certain members or a single member. In such a case, the method of joints is not an efficient method and we use another method called as the method of sections. Today, we will see how to analyze trusses by method of sections.

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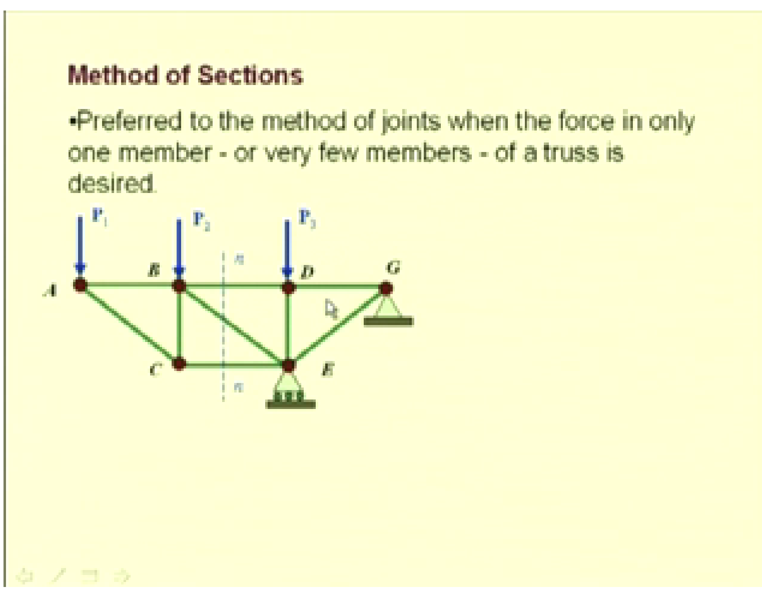

As the name suggests, this method involves creating sections and the free body diagrams of the sections to determine the various forces in the truss.

Let us consider an example: here you see a truss AB, BG, and CE connected at G by a pin joint and at E by a roller joint and on some external forces P_{1}, P_{2}, and P_3 acting on the truss. If say, we are interested to find the forces in the member BD, BE and CE alone, then it is not very efficient to find these forces by method of joints because, in order to determine this force in BD, first we have to move from either joint G or E where only two forces are involved and we have to move one joint to another joint.

The method of section uses a portion of the truss and the free body diagram of the portion of the truss, to determine the internal forces. Let us consider this section nn, which passes through the members BD, BE and CE for which we are interested to find the forces. Then we can consider the equilibrium of either the section ABC or the section DEG to determine these forces. But if we use the free body diagram of this portion, that is DEG, then one has to determine the reactive forces at G and the reactive forces at E. But if we consider the free body diagram of this portion ABC, then we do not need to determine either the reactions at E or G.

Let us use the portion ABC to analyze this truss.

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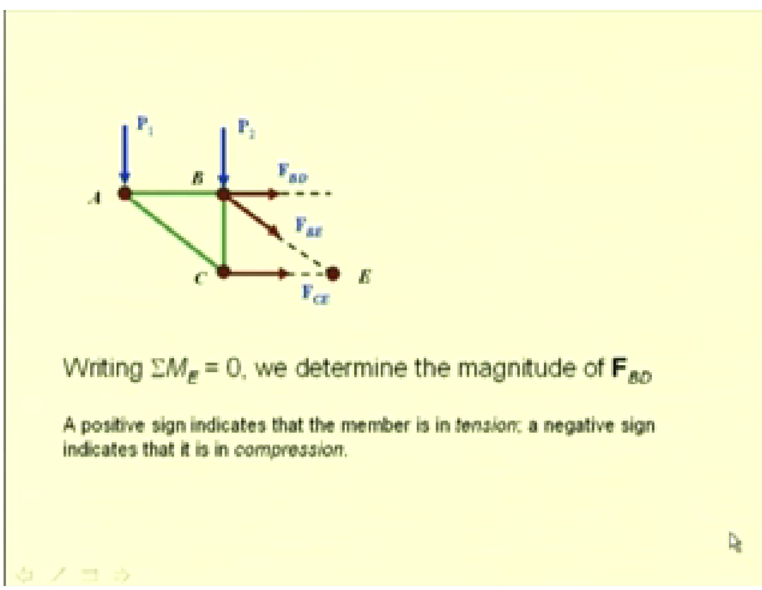

This is a free body diagram of the portion ABC. Since the section passes through the member BD, BE and FC, we have to include the forces setup in these members in the free body diagram.

We assume that all forces are tensile in nature; so the forces are acting away from the joint. That means the force FBD and the force FBE are acting away from B and the force FCE is acting away from the joint C.

The equilibrium of this portion is maintained if the sum of all the forces is 0 and the sum of the moments of this force about the point is also 0. So, we have three equations from this free body diagram and the three equations can be used to determine the three unknowns. Since we have only three unknown forces in this free body diagram, this problem is a statically determinate problem and we can solve it.

Here we can note that the forces FBE and the force FCE are concurrent at E because the member BE and CE are connected at this joint at E. It is convenient or judicious to use the moment summation about this point E, in order to find this force FBD, because the moments of the force FBE and FCE will be 0 about this point E.

In the same way, the members BD and BE are concurrent at B. So the moments of this force FBD and FBE are 0 about this point B. If we take a moment summation about this point, it is possible to find this force FCE. The third equation could be a force summation; either the summation of the vertical components or the summation of horizontal components can be equated to 0 in order to determine the force FBE.

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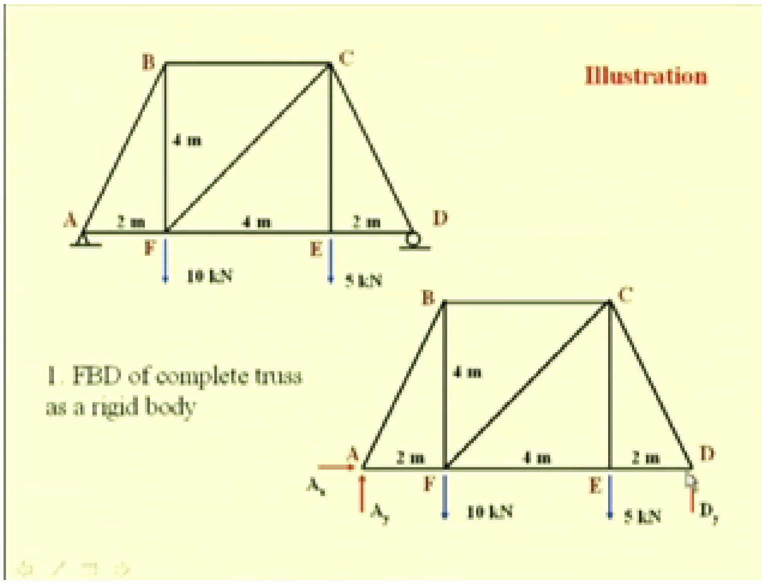

Let us see an illustrated example, to see how we use this method of section for determining the internal forces in a truss. Here we have considered the same example that we used for illustrating the method of joints. You see a truss ABCD supported at A by a pin joint and at D by a roller joint and some external forces, say 10 kilo Newton force at F_1, 5 kilo Newton force at E.

Let us say, we are interested in determining the force in this member BC and the force in this member FC and the force in this member FE. We see that it is possible to create a section passing through the member BC, FC and FE such that the section passes through only three members. If we consider, the portion of the truss ABF then we will have only three unknown member forces if we have determined the pin reactions from the free body diagram of the entire truss.

In this problem, we can also consider this section because it is also possible to determine this reaction D_y. We can consider either the portion containing ABF or the portion containing CDE. In order to proceed, first let us find the reactions that is at A and D from the free body diagram of the complete truss. Here you see the free body diagram of the complete truss; we have replaced the pin connection at A by the two reaction components A_x and A_y and the roller has been replaced by a vertical component of the reaction force and the problem is statically determinate. We can determine the three unknowns that is A_x, A_y and D_y from the three equations.

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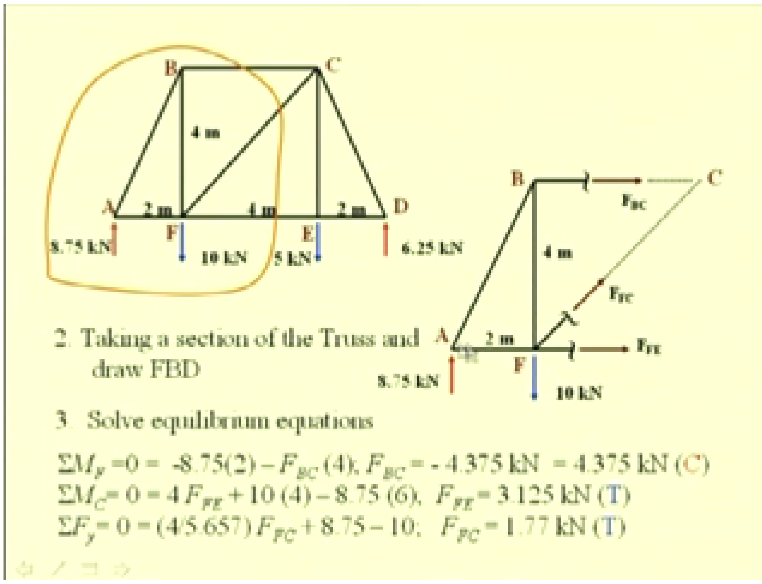

Let us consider this section ABF to determine the forces in the members BC, FC and FE. So this is the section that we are going to consider. Let us create the free body diagram of this section. Since this section passes through BC, FC and FE, the forces in these members have to be included in the free body diagram of the portion of the truss.

Let us assume that all the members are subjected to a tensile force because of the loading. Thus, these forces act away from the corresponding joints. That is the force F_{BC} acts away from the joint B and the force FC and the force FE acts away from the joint F. Here we see that the members BC and FC are concurrent at C. So it is judicious to use a moment summation equation about point C to determine the force FE.

Same way, we see that the forces in the member FC and the force in the member FE are concurrent at F; it is judicious to use a moment summation about this point F to determine the force F_{BC}. The third equation could be a force summation equation to determine the force in FC. We have first the moment summation about the point F. So, when we compute the moment of the various forces about this point F, we first have the reactive force at A which is 8.75 and the moment of this force about this point is a clockwise moment which is a negative sense and the momentum is 2 meters. Hence, we have 8.75 times 2 which is a negative moment and the moment of this force F_{BC} is also a clockwise moment and negative. We have $-F_{BC}$ and the momentum is 4 meters, so we have - F_{BC} times 4. From this equation we have the force in BC determined as -4.375 Kilo Newton.

Since the answer has turned out to be negative, our original assumption that the member is in tension is wrong; actually, the member is in compression. So the force in the member BC is 4.375 Kilo Newton's of compressive nature.

In the same way, we take the moment summation about this point C. We have 4 times of force in FE because the momentum of this force is 4 meters. This is a counter-clockwise moment and thereby a positive moment plus 10 times of 4. The moment of this force 10 Kilo Newton, again a positive moment, because it is counterclockwise about C. The moment of this force 8.75 Kilo Newton reactive force is a clockwise moment about C. Thereby it is a negative moment and this equation gives the value for the force in the member FE as 3.125 Kilo Newton.

The third equation we can take either a force summation about x-axis or a force summation about y-axis. Here we have taken the force summation about y-axis and we have equated to 0. From geometry, we get the direction cosine of this force F_{FC}. Since we are summing up the vertical components, we need to know the sine of this angle which is 4/5.657 times of force in FC plus the vertical reactive force which is positive in direction and the applied force is negative because it is a downward force, so -10.

From this equation, we have this force in FC as 1.77Kilo Newtons. This problem illustrates how we can use the method of section to determine the forces in the members. Few things you would have noted that we create a section; such that, the section only passes through three members. Since we know from the equilibrium equations of a rigid body in plane, we can only determine three unknowns. So it is judicious to pass a section, passing through only three members, because the internal forces in the members are unknown. If we pass the section through more number of members then it is not possible to determine those forces by only considering the free body diagram of the section.

Let us see few problems: a simpler problem first and then a little more complex problem to see how to use this method of section to solve the problem on truss analysis.

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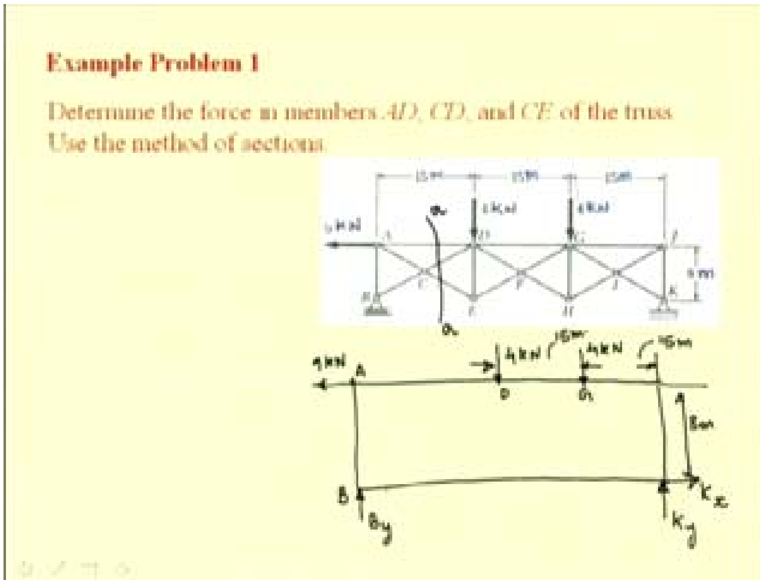

Let us consider this truss ABJK, which is a planar simple truss connected by a roller at this joint B, by a pin connection at joint K, and certain applied loads say 4 Kilo Newton at D and 4 Kilo Newton at G. The various dimensions and geometry of the truss is given.

Let us say we are interested in determining the forces in the members AD, CD and CE only. From the geometry of the truss, we can see that it is possible to create a section that passes through. Let us consider this section aa that passes through the member AD, CD and CE. Since the section passes through only three members, if we consider the free body diagram of this portion containing ABC, then it is possible to determine the forces in the member AD, CD and CE. Before we can proceed to do that, we have to determine the reactive force at B.

First, consider the free body diagram of the entire truss, to determine the reactive force at B. Let us consider the free body diagram of the complete truss. We replace the roller at B by a vertical reaction B_y. We have an applied force of 9 Kilo Newton's at A and we have the 4 Kilo Newton forces applied at D and G. The pin connection at K is replaced by two components of reaction K_x and K_y. The various dimensions are needed to compute the moments, like this is 15 meters and this is again 15 meters and so forth.

If we consider this free body diagram and since we are going to use this section ACB to determine the forces, we are interested in determining this reaction B_y. From this free body diagram, we can see that if we take a moment about this point K then only B_y is the unknown force and it can be determined.

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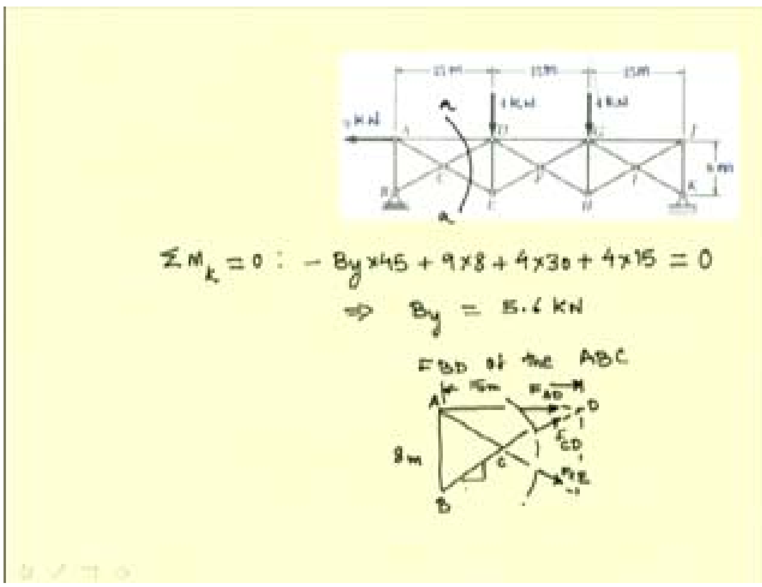

Let us write the equation, taking the moments about K and equating it to 0. Since the force at B is an upward force and the moment of this reaction is clockwise about this point K, it is a negative moment. B_y times the momentum is 15+15+15 that is 45 meters. The moment of this 9 Kilo Newton force is a counterclockwise moment about this point K with a momentum of 8 meters. Since this moment is a counterclockwise moment, it is a positive moment; we have plus 9 into 8.

The moments of this 4 Kilo Newton forces at D and G are also counterclockwise about K and thus they are positive moments. So we add them, 4 into the momentum of this is 30 meters plus the momentum for this 4 Kilo Newton force is 15 meters. We equate this to be 0 because for equilibrium to exist, the moment of all the forces about a point has to be 0. From this equation, it is possible to determine B_y and it has been found to be 5.6 Kilo Newton.

Now we have determined this reaction force at B. We can consider the free body diagram of the section aa; the portion of the truss ABC to one side of this section aa. Let us draw the free body diagram of the portion of the truss containing ABC. Since the section passes through the member AD, we have to include the force in the member AD and it also passes through the member CD so the force in that member has to be also included.

We assume all these forces to be tensile forces; so the forces will be acting away from the corresponding joints. That means, the force AD will be acting away from A, force CD will be acting away from C, and force CE will be acting away from C. We also see from geometry that these two forces are concurrent at D, and these two forces that are F_{CE} and F_{CD} are concurrent at C, and the force F_{CE} and F_{AD} are concurrent at A.

From this free body diagram, we can now write the equilibrium equation to determine these three unknowns. We need also certain dimensions in order to find the momentums. This is 8 meters, and this distance is 15 meters, and this is half of that which is 7.5. So, the slope of this line is 7.5 and 4.

Let us write the equilibrium equations from this free body diagram. Since we have seen that the forces are concurrent like the forces F_{AD} and F_{CD} are concurrent at D and these two forces are concurrent at C and these two forces are concurrent at A. It is convenient to consider the moment equations.

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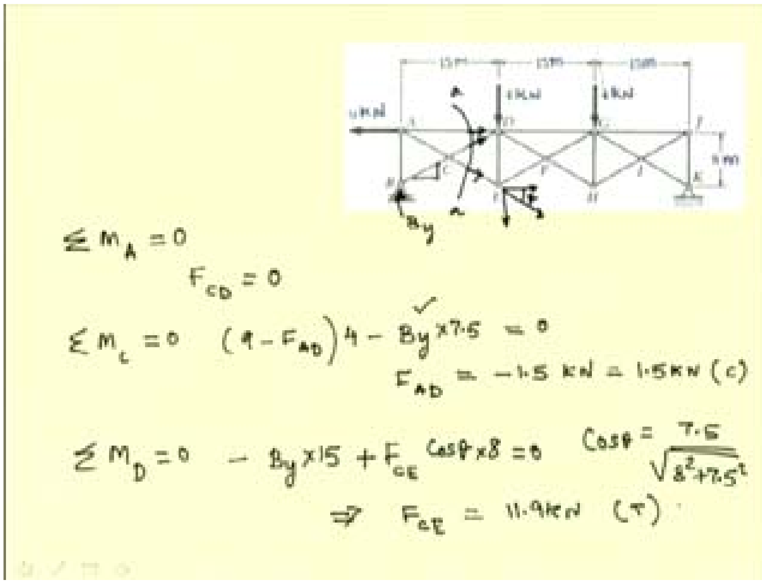

The first equation that we write is the summation of the moments about A. For your convenience we have considered this section aa. If we take the moment summation about A, since these two forces are concurrent at A we have only this force in the member CD, which is unknown. All other forces: that is the reactive force at B, the applied force of 9 kilo Newton at A are all concurrent at A. Thereby, the force in the member CD has to be 0 because the moment of this force cannot be balanced by any of these forces. From this equation, we find that the force in the member CD is to be 0.

Let us consider the summation of the moment about C, where the force in the member CD and the force in the member CG are concurrent. This can be used conveniently to determine the force in this member AD. We have the 9 kilo Newton force and the force in AD; so 9 minus the force in AD is the resultant force at A and the momentum of these forces about C is half of this distance that is 8 meter which is 4.

Then we have this force that is a reactive force B_y and the momentum of this force with respect to the point C is half of this distance that is 7.5 meters it is minus B_y times 7.5. We see that this force B_y has a clockwise moment and this force has a counterclockwise moment. So this is a positive quantity and this is a negative quantity and we equate this to 0.

Since we have already determined this force B_y, now from this equation we can determine the force in the member AD and it has been determined to be -1.5 kilo Newtons. Since the answer has turned out to be negative, our original assumption that the force in the member AD is tensile in nature is not valid and the force is compressive in nature. That means the force is 1.5 kilo Newton compressive force.

Let us consider the moment about D because the forces F_{AD} and F_{CD} are concurrent about D it is possible to determine the force in the member CE from this equation. The equation is the moment of this reactive force B_y is minus B_y times 15 and since this is a clockwise moment about D it is a negative quantity. The moment of this force 9 kilo Newton force about D is 0, moment of these two forces is also 0. The moment of this force F_{CE} is not 0.

The force F_{CE} can be resolved into two components and the horizontal component of this force is going to have a moment about D and the vertical component of this force is not going to have a moment. In order to determine this, we need to know this angle theta and the cos of this angle. We already know the slope of these lines and from that we know that cos of theta is equal to 7.5 times by root of 8 square plus 7.5 square. The horizontal component of this force is going to have a counterclockwise moment; so it is a positive quantity. F_{CE} cos theta and the momentum is 8 meters into 8 is 0, and from this equation we determine F_{CE} as 11.9 kilo Newton. Since this is a positive force, it is a tensile force. This problem illustrates how we can use this method of section to determine the forces.

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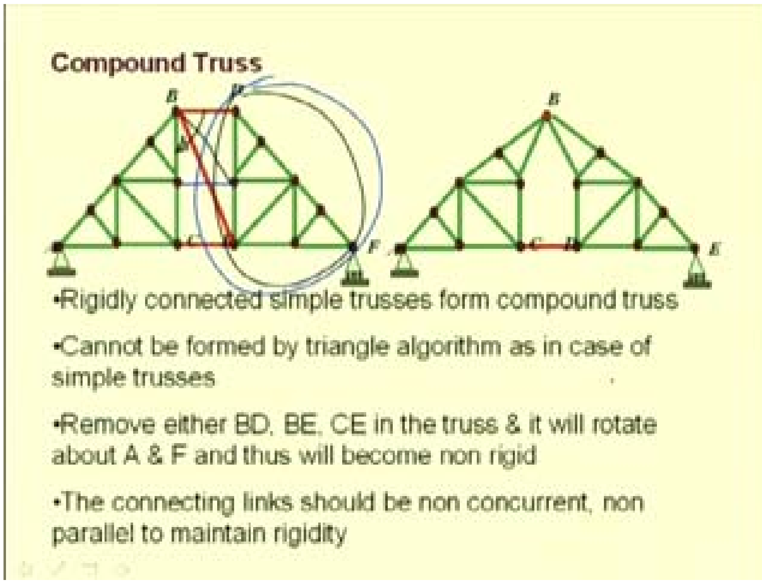

Let us consider another class of trusses that we call the compound trusses. A compound truss is a rigid truss constructed by connecting two or more simple trusses. We have already seen how to construct simple trusses.

A simple truss, let us say ABC is shown here, which can be constructed by taking a triangular truss and connecting two members and a joint and in the same way this complete truss ABC has been constructed. This green truss what you see is a simple truss. The same way we see another truss DE and F which is again a simple truss. These two simple trusses have been connected by the links BD, BE and CE to form a rigid truss and this complete truss formed by connecting the two simple trusses is known as the compound truss and it has been supported by a pin at A and a roller at F .

There is another configuration by which also it is possible to connect the two simple trusses. Here you see another compound truss. The two simple trusses are ABC and BD and E and here they have been connected by a pin at B and a member CD and this combination also results in a rigid connection of the two simple trusses.

Let us see some conditions that have to be satisfied for the compound truss to be also statically determinate as well as rigid. We have already seen that simple trusses are rigid as well as statically determinate.

Now, let us derive some conditions that are necessary for the compound truss to be statically determinate as well as rigid. We see that this kind of trusses cannot be created by the triangle algorithm that can be used for creating simple trusses. The triangular algorithm is that for a simple truss we connect two members and a joint in order to build this simple truss. The same algorithm cannot be used to create this compound truss. We see that even if we remove one of these members, let us say in this compound truss the member BD or the member BE or the member CE the rigidity of this compound truss is lost and either this portion DEF starts to rotate with respect to the simple truss ABC.

This three connection that is BD and BE and CE are optimal connections to ensure rigidity. Here we see that these three links BD, BE and CE are non-concurrent; that means, the three links do not pass through a single point. Instead of having this connection let us say CE, if we had a link connecting B to this joint, then this complete truss can rotate about this point B, or if we had instead of these three connections, if we had a link connecting these two members then also this portion of the truss can rotate because all these three links – BD, this link as well as CE - are of equal lengths and this complete truss can rotate with respect to the simple truss ABC.

This condition that the three links should be non-concurrent as well as they should not be non-parallel should be satisfied for the connection between two simple trusses to be rigid. If two simple trusses are connected in a rigid way, then they have similar properties of static determinacy and rigidity as for the simple trusses.

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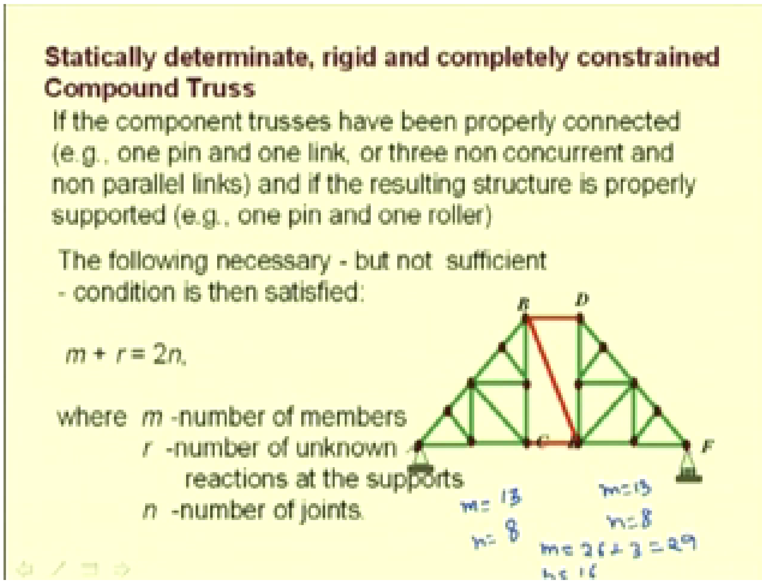

That is the equation, that m plus three, if m is a number of members and r the number of external reactions to be determined and n the number of joints; then the equation m plus r is equal to 2n, which was satisfied for the simple trusses, is also satisfied for a compound truss.

Let us see this example: here we have two simple trusses ABC and DEF connected by three links BD, BE and CE. Let us count the number of members; we see that the truss ABC has 13 members and 8 joints. So this is the number of members and this is the number of joints for this simple truss. The same way we have 13 members for this simple truss and 8 joints for this simple truss.

Additionally, we have three members connecting the two simple trusses and the joints are not increased. So the total number of members in this case is 26 plus the 3 connecting links, that is 29 and the total number of joints is 16. If we see in this equation, the number of members is 29 plus the external reaction; here we have a pin connection, so two unknowns, and here we have a roller connection, one unknown, so r is 3.

We have m plus r is 29 plus 3 which is 32, which is equal to 2 times of n, where n is the number of joints which is 16 and it is equal to 32. So this equation we see that is satisfied for this compound truss.

Let us see how to analyze such compound trusses by say, method of sections. If you are interested to determine all the forces in the members, then we may use the method of joints; but if we are interested to determine only certain forces in the compound truss then we can use the method of section.

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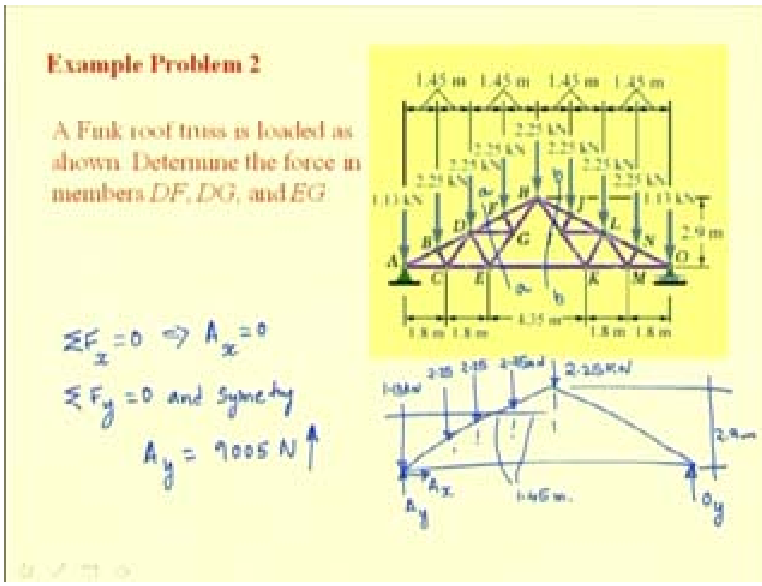

Here you see a roof truss, which is known as Fink roof truss. You see that it is a compound truss constituting of two simple trusses, one AHE and the other HKO connected by a pin at H and the member EK. We are interested in finding the forces in the members DF, DG and EG. In order to determine these forces, we can conveniently use the method of sections.

In order to use the method of section, we have to see if we can pass a section which passes through the members of interest, that is DF, DG and EG. Let us consider this section aa. If we consider this portion of the truss, we see that we have four unknowns, that is: the force in the member DF, the force in the member DG, the force in the member EG and the force in the member EK. We know that in two dimensions from the equilibrium equation of rigid body it is not possible to determine more than three unknowns.

Now we have to see how we can determine one of the unknowns, say the force in this member EK has to be determined. In order to determine this, we can consider another section say bb, which passes through HJ and this member, and this member EK. We see that since this section passes through only three members, it is possible to determine all the three forces. Once we have determined this force EK, we can go back to the previous section. Now only we will have three unknowns; that is DF, DG and EG and the problem is solvable.

Let us try. Before solving this we have to determine the reactions at A which can be done by considering the free body diagram of the complete truss. Let us draw the free body diagram of the complete truss. Here I am only showing the salient features. We have two components of reaction A_x and A_y at A, since at A the truss is connected by a pin joint. At O, since we have a roller, we have a single component of reaction O_y. Other forces are acting at various points. Since the entire loading is symmetrical, I am just drawing one-half of the truss. All the forces are in Kilo Newton.

Some dimensions are required to compute the momentum. All these individual distances are 1.45 meters. Let us use this free body diagram to compute the reaction at A. If we use this equation sigma F_x equal to 0, this results in A_x to be 0 because all the applied loads are vertical in nature and we do not have a horizontal load. The horizontal reaction also vanishes; so A_x is 0. By using this equation sigma F_y equal to 0 and the symmetry that exists, we see that this reaction A_y has to be half the total load that is acting on the truss.

We determine A_y to be 9005 Newtons upward in this case. Now that we have determined the reaction, we can use the free body diagram of the portion of the truss constructed by the section bb, to determine the force in this member EK.

Let us draw the free body diagram of the portion of the truss constructed by using the section bb.

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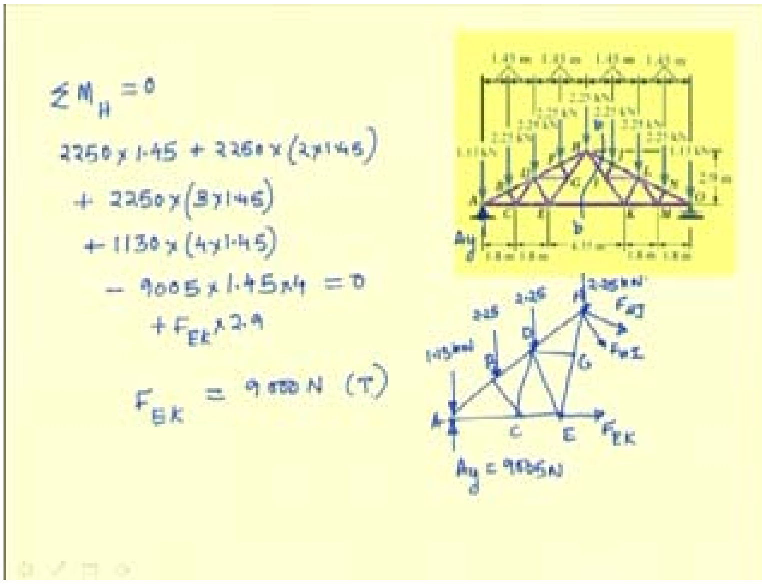

We are interested in constructing the free body diagram of this section bb. Here we have this force A_y which has been determined. Let me draw this diagram for clarity. Since this section passes through the member HJ, we have the force in the member HJ, and we have the force in the member HI, as well as the force in the member EK.

All the forces have been assumed to be of tensile in nature. We know this reaction A_y which has been determined as 9005 Newton and various applied loads are also known: 1.13. All forces are in kilo Newtons: 2.25, 2.25 and 2.25 Kilo Newton. The horizontal suppression between all these forces is 1.45 meters from the geometry of the truss.

Let us write the equilibrium equations from this free body diagram. Since we are interested in determining the force in the member EK, it is convenient to take the moments about this point H where the force HJ and HI are concurrent. The only unknown will be the force in the member EK and it can be determined.

Let us write the moment summation about H and equate it to 0. This results in the moment of the 2250 Newton force. Forces are: the first moment of the force is the counterclockwise moment of the force at this joint F, which is 2250 times this distance that is 1.5 meters and this is a counterclockwise moment, so it is positive, plus the moment of the applied force at D which is again 2250 into 2 into 1.45 which is the momentum. Again, this moment is a counterclockwise moment plus the moment of the force at joint B, which is again 2250 and the momentum is 3 into 1.45 meters.

The momentum of this force is 1130 times 4 into 1.45, again counterclockwise. The moment of the reactive force at A is a clockwise moment, so it is negative. The force is 9005 Newton as determined times 1.45 into 4. This has to be equated to 0.

We also have the moment of this force in EK, which is a positive moment FEK and the momentum is 2.9 meters. Sum of all these moments is equal to 0 and from this equation, the force in the member EK can be determined and it is determined as 9000 Newton. Since it is positive, it is tensile in nature.

From this free body diagram, we have determined the force in the member EK. It is possible to move to the free body diagram of the portion which is sectioned by the section surface aa, that is it passes through the member DF, DG, EG and EK.

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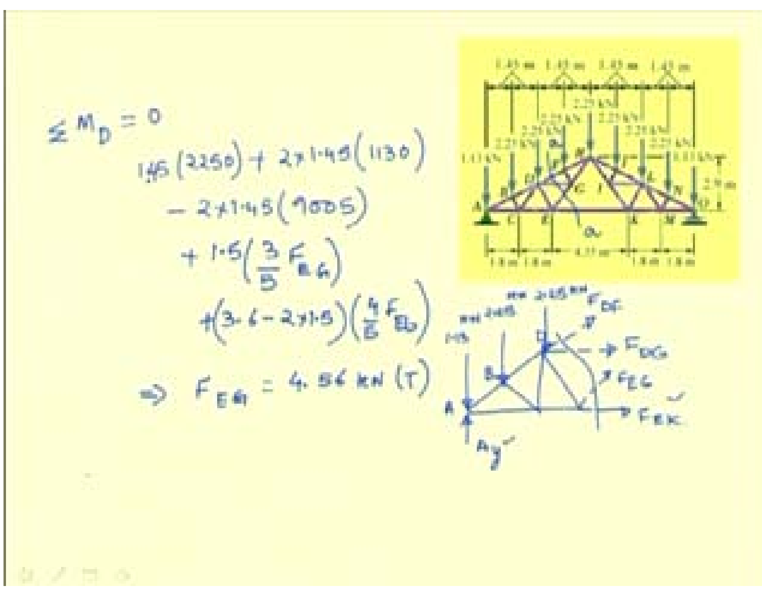

Let us draw the free body diagram of this section. We are assuming all forces to be tensile in nature, so they act away from the corresponding joints. We have the force in DG, and we have the force in EG and the force in EK. The reactive force A_y has been already determined, this is a known quantity and so we only have three unknowns to be determined. We see that these two forces that is the force in the member DG and the force in the member EG are concurrent at G, so it is convenient to take moment about G to determine the force in DF.

Same way we see that these two forces, that is the force in DF and the force in DG are concurrent at D; so a moment about this point can be used to determine this force in EG. Let us start by considering first the moment about D to be 0. Since these two forces F_{DF} and F_{DG} are concurrent at D, their moments are 0 and only we have the moments of this force $F_{EG,}$ the moment of this force $F_{EK,}$ and the moment of this force A_y in the moment summation equation. That is equal to 1.5 times 2250 because we have some applied loads at these points: the 1.13 kilo Newton force, the 2.25 kilo Newton force at B, A and D.

When we take the moment of all these forces about this point D the moment of this force is 0. The moment of this force which is 2.25 kilo Newton force about D is 2250 times 1.5 meters, which is the momentum 1.45 meters about this point D and which is a counterclockwise moment and a positive quantity; plus 2 into 1.45 times of 1130 because that is a momentum of this force minus, because the moment of this force is A_y is clockwise and so it is a negative quantity, 2 into 1.45 which is a momentum times the reactive force which is 9005 Newtons. The moment of this force FEK is a counterclockwise moment so it is a positive quantity plus 1.5 times the direction cosine of this force EG plus 3.6 minus 2 into 1.5 times the force in the member EG. This results in finding the force in EG as 4.56 Kilo Newton and since this is a positive quantity, it is a tensile force. Now we can use the force summation equation to determine the other two forces.

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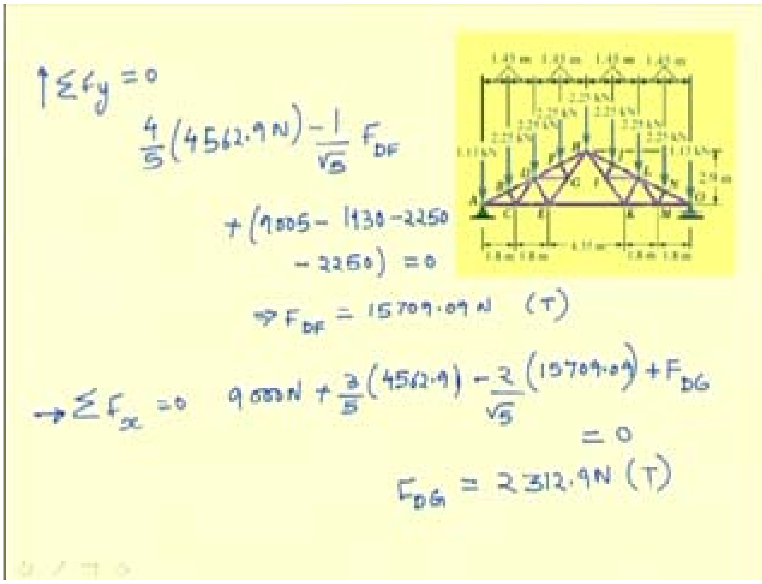

First, we use the force summation about y-axis. The various direction cosines are available. This is for the member EG. The direction cosine is 4 by 5 times the force in this member minus 1 by root 5 for the force in DF; for this member this is the direction cosine, plus all these vertical forces that is the reaction force at A_y which is 9005 Kilo Newtons. Then all the applied forces which are downward we have 1130 Newton at A, we have 2250 Newton at B and another 2250 at D. The sum of all the vertical components has to be 0 and this results in the force in DF as 15709 Newton. Since this is a positive quantity, again it is a tensile force.

We can use the force summation about x to be 0. The positive axis is the positive x-axis. We have this 9000 Newton force in a member EK, plus the component of the force in the member EG, plus the component of the force in the member DF, plus the force in the member DG, which is a horizontal member and so the force in the DG has to be added.

From this equation, we have this force in DG is determined as 2312.9 Newton of tensile nature. We have seen how to analyze the method, how to use the method of sections to solve compound trusses. Any problem on trusses, either a simple truss or a compound truss can be solved for determining the internal forces, if the problem is statically determinate by the method of joints or the method of sections. The method of section is conveniently used if we are interested in determining all the forces and the method of section is used if we are interested in only in certain of the members.

In the method of sections, the key to the solving is to find suitable sections; such that, at a time only three forces are unknown. If more numbers of forces are unknown then we may use additional sections to determine the unknowns, and then solve the problem. With this, we complete our discussion on analysis of trusses and we will move to analyze other kinds of civil structures like beams in the next lecture.