Engineering Mechanics

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Introduction to vibration

Module 15 Lecture 36
Free Vibration

Today, I am going to speak on free vibration. The study of vibration is concerned with the oscillatory motion of bodies and the forces associated with them.

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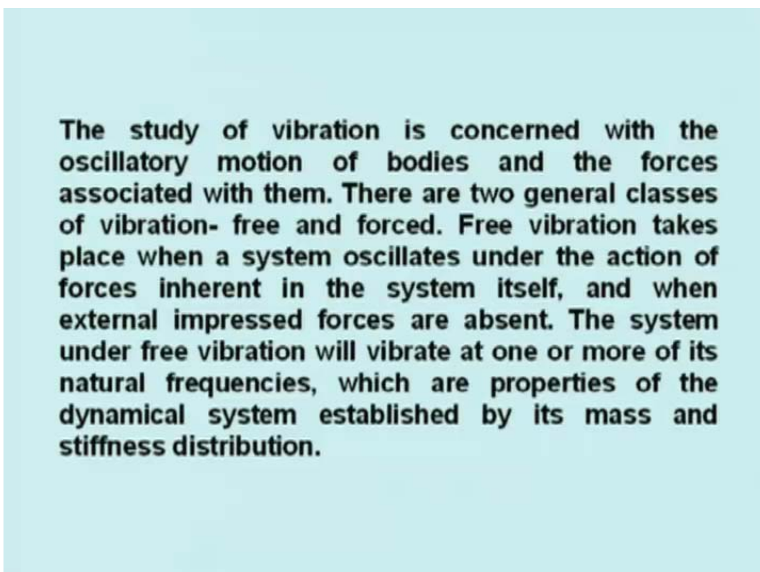

You might have seen number of bodies which will keep on oscillating. In some bodies, even if there is absence of force, they keep vibrating for sometime. For example, if you have a tight string and you displace it slightly, then after you remove the force also, your hand will keep vibrating for sometime. Similarly, if you have a spring mass system and you displace the mass slightly, it will start oscillating. These are examples that without of presence of any force, the body is vibrating, but at the same time, you may have another example that there is disturbing force.

The force is like, if you would have travelled in a bus, the bus is travelling, the road is rough. So, because of the rough road, the forces are transmitted through your tyres and they come up to the seat. So, you feel those types of vibrations. There are vibrations; they may be because of some external force, or without external forces also, that body can oscillate between two positions. The study of vibration is divided into two general classes; free vibration and forced vibration. Free vibrations take place when a system oscillates under action of forces inherent in the system itself and when external impressed forces are absent.

If there are no forces, then body, obviously will be in equilibrium. Therefore, there must be some forces which will cause the acceleration of the body and body's velocity, if we change like that it will keep vibrating. However, there are no external forces. The system under free vibration will vibrate at one or more of its natural frequencies.

If we take a spring mass system and disturb it free, for some distance, that means we displace the mass by some amount, then in that case, there is where motion of the mass starts and this is called free vibration. There, the spring force is always present and at the same time, you have inertia force, bodies under equilibrium, under the spring force and inertia force. So, it can be seen that it starts with two, with one particular natural frequency. That natural frequency, the property of the system is independent of the how much you have displaced. We are interested to find out the natural frequency.

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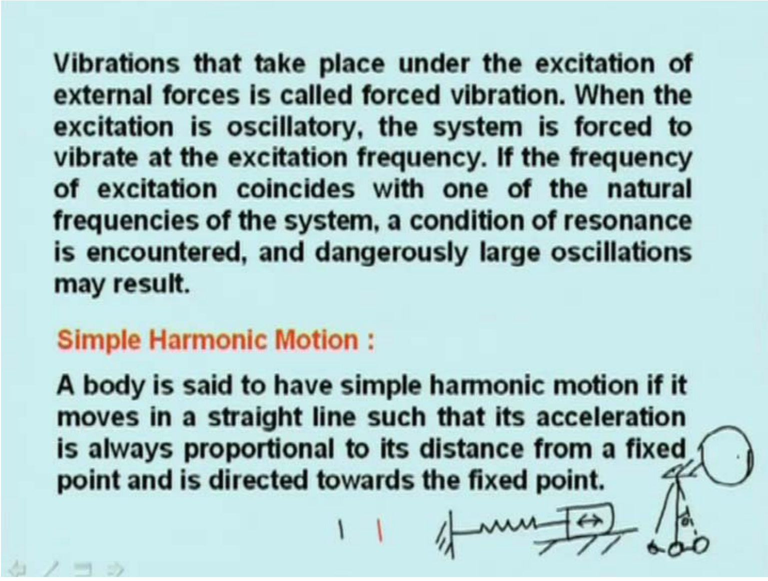

Let us go to the next slide. Vibrations that take place under the excitation of external forces are called forced vibrations. When the excitation is oscillatory, the system is forced to vibrate at the excitation frequency. If the frequency of excitation coincides with one of the natural frequencies of the system, a condition of resonance is encountered and dangerously large oscillations may result. You may see that, suppose there is a spring mass system and you apply some oscillating force, in the beginning the motion can be of some different kind, but after some time, the system will vibrate only with the natural frequency; only with the frequency of the forces itself, whatever forces you have kept. So, it will vibrate with that; that is, in steady state, the frequency of excitation coincides with one of the natural frequencies of the system.

A vibrating body can vibrate in a number of ways. However, first we will study that very simple case, that is called simple harmonic motion, in which that displacement can be represented by a sin function or a cosine function, because sin theta is equal to cos 90 plus theta. So, simple harmonic motion, a body is said to have simple harmonic motion, if it moves in a straight line such that its acceleration is always proportional to its distance from a fixed point and is directed towards the fixed point. This is one definition of simple harmonic motion.

In this, let us pay attention to each and every part. The body has to move in straight line; that means, if you have a spring mass system, mass may vibrate and it may move. It moves in a straight line. Therefore, this is satisfied. So, this type of motion can qualify for simple harmonic motion. Acceleration is always proportional to its distance from the fixed point. That is another condition and is directed towards the fixed point. If the acceleration is not directed towards the fixed point or at least one particular point, then body will not come back to that point. The vibrations will not take place. So, that condition is also required.

Suppose you might have done one experiment, seen, this pendulum. A pendulum is attached, if you slightly disturb it, it goes to new position and then if you release, it comes back but it over shoots the material position and reaches here. It keeps oscillating. Naturally, this ball is moving on the arc and it is not moving in a straight line. So, it should not be a simple harmonic motion. However, you can see that if angle theta is very small then it can be considered a straight line only. This segment, if you take a bigger circle and cut it here then the small portion that will appear is more or just like very close to a straight line. Therefore, approximately, simple harmonic motion can be obtained by this.

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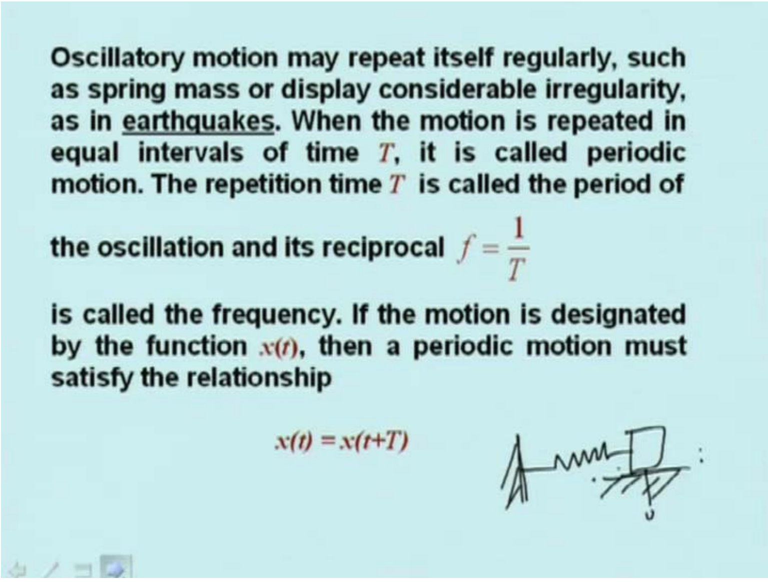

Oscillatory motions may repeat itself regularly. Suppose you have force vibration such as spring mass. This is or even if that in free vibration, you have a spring mass system and it may repeat regularly. We know that it goes, and after from 0 second it was here, at 5 second it went there, after 5 second 5 more second it came here then it went here. So, this type of behavior keeps continuing or it may display considerable irregularity like in earthquake, earthquake vibrations are there, but they are irregular.

When the motion is repeated in equal intervals of time, it is called periodic motion. That equal interval of time, when you have periodic motion, the repetition of time t is called the period of the oscillation. Its reciprocal f is equal to 1 by T is called the frequency. So, t is the time period, f is the frequency. If the motion is designated by the function x(t) then a periodic motion must satisfy the relationship, x(t) is equal to x(t plus T), T is the time period. At time period T, it should come back to the same position. So, x(t) is equal to x(t plus T), and that is the condition for a periodic motion.

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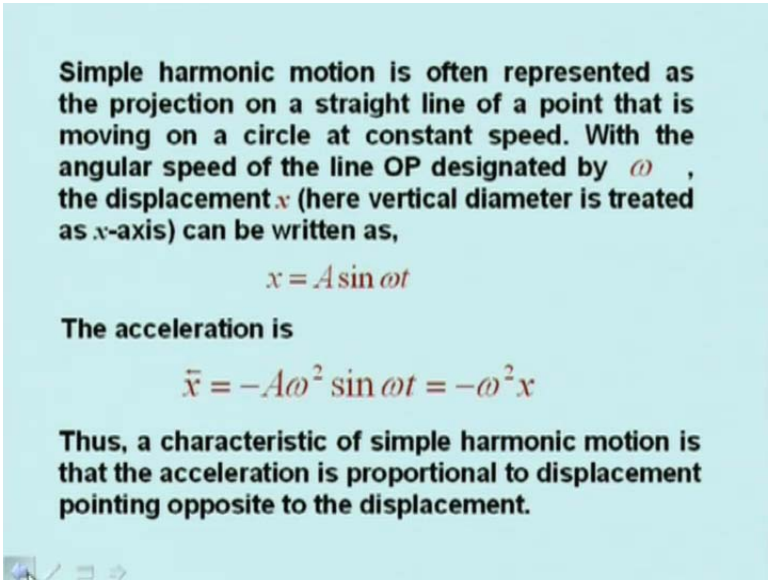

Simple harmonic motion which is a periodic motion is represented as the projection on a straight line of a point that is moving on a circle at constant speed with the angular speed of the line OP designated by omega that means the simple harmonic motion.

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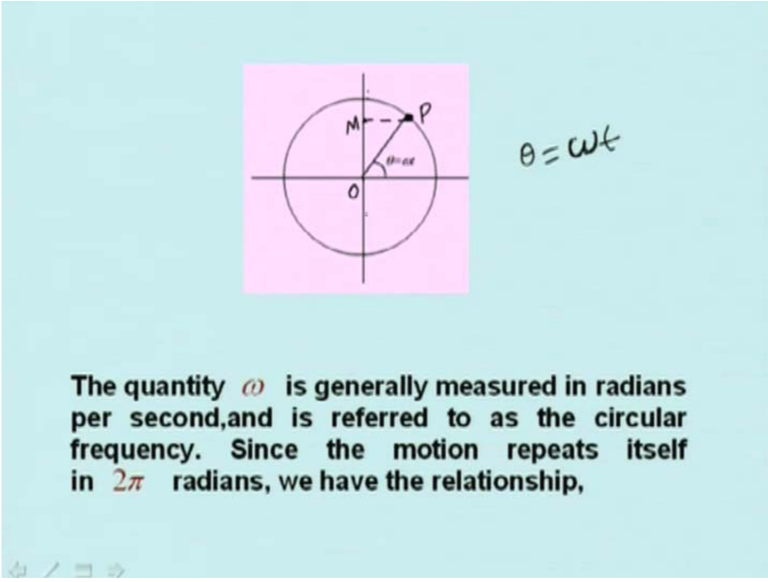

There is a circle on which a particle is moving. It is connected by a string. You can assume that in a horizontal plane, we attach this string with the mass and it is moving in the circle. So, if it starts at time t is equal to 0, then theta at time t is equal to omega t, because we have said that it is moving. So, its angular position at any instant is indicated by theta is equal to 2 pi theta is equal to omega t. Why theta is equal to omega t? Because, at any point it is moving with a uniform velocity, so, at any point between two time intervals, the displacement should be same. So, omega must be some constant. So, theta is equal to some constant times time.

Then, what you do that draw a projection. So, this point may be called O, this may be called P and this is may be M. Now, study the motion of M, how the point M moves. Do not study the motion of P, study the motion of M and see what type of motion is this. P is obviously moving in the same way it is going, but M is like this. In the beginning, M was here because P was here; so P. Then, as it moves here (Refer Slide Time: 13:52 min), the M keeps rising and finally when P is here, then P and M coincide and M goes here. After that, P goes from here to here, then M will move from top position to here. Then P goes from here to here; M goes from here to here; then, P goes from here to here; then M goes from here to here (Refer Slide Time: 14:16 min).

Naturally, when it is completing a circle, by that time M has gone from this point to this point, came back from this point to this point, came back from this point up to this point and then went to this point (Refer Slide Time: 15:00 min). So this motion is oscillatory motion type in one evaluation itself it went from here.

Naturally, when it is completing a circle by that time M has gone from this point to this point came back from this point to this point came back from this point up to this point and then went to this point. So, this motion is oscillatory motion type. In one revaluation itself it went from here, came here then this this like that so this type of motion will repeat. Therefore, with the angular speed of the line OP designated by omega the displacement x can be written as x is equal to A sin omega. That we can write, displacement of this point can be written x, because A sin omega t. If A is the radius then this is naturally, this will be A sin omega t. Theta is equal to omega t. Therefore, acceleration is x double dot is equal to minus A omega square sin omega t is equal to minus omega square x.

Thus, characteristic of simple harmonic motion is that the acceleration is always proportional to displacement pointing opposite to this displacement. If x is equal to A sin omega t then x double dot acceleration will be minus A omega square sin omega t that is minus omega square times x. Therefore, in a simple harmonic motion, acceleration is proportional to displacement pointing opposite to the displacement and it is that is the this thing that means pointing towards the origin whatever you have taken.

The quantities omega, is generally measured in radians per second and is referred to as the circular frequency. Sometimes, term circular is dropped and it is just called frequency.

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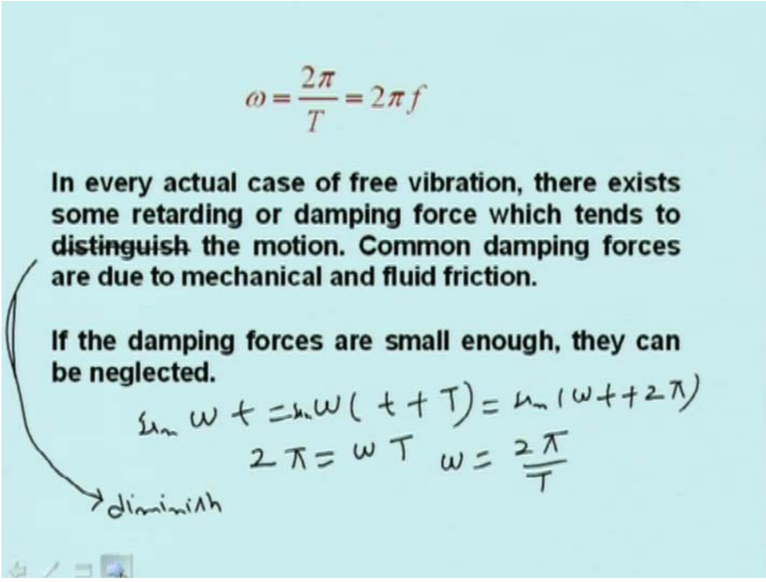

Since the motion repeats itself in 2pi radians, we have the following relation; omega is equal to 2pi by T is equal to 2pi f. Omega t must be same as omega t plus T, where T is the time period and not omega t. It is sin omega t, because we know that, it is same as sin omega t plus 2 pi. So, what happens that you have 2pi is equal to omega T, where T is the time period. Therefore, omega is equal to 2pi by T, 1 by T is called the frequency. Therefore, this can be written as 2pi f.

In every actual case of free vibrations, there exists some retarding or damping force which tends to diminish the motion. Otherwise, the body will always keep oscillating. You have seen that if you take a tight string and displace it slightly, it keeps vibrating for some time, but after that it stops. Why? Because of the presence of damping. Similarly, if you take that simple pendulum and displace it slightly, it keeps oscillating for some time and after that it stops. Why? Because of the damping. Therefore, if the damping forces are small enough, they can be neglected, because they are very small. So, sometime, we can study for considerable period of time, the motion keeps taking place and it can be neglected.

Moreover, from designer’s point of view, it is conservative first to study the motion without damping. After that we see, what is the effect of damping. Damping will have effect on reducing the vibration.

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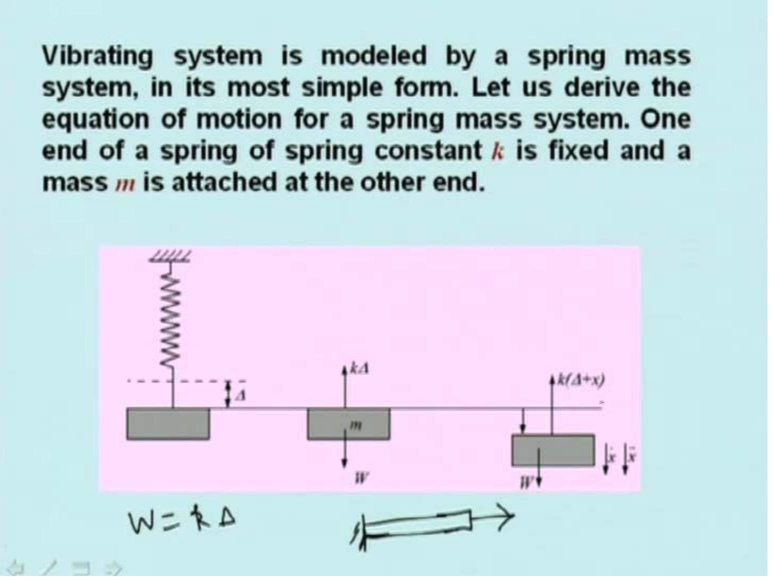

A vibrating system is modeled by a spring mass system, in its most simple form. You can take a spring and hang it from a ceiling. At the other end, if you put a mass, it becomes a spring mass system and you can study this one. But it is not that the study of this spring mass system is limited only to this type of thing. Many other systems can be approximated by a spring mass system. You can always say that this is something like a spring. Suppose tight string is there and you displace. So, middle point behaves like, as if there is some spring. Similarly, if you have a rod and you have continuous rod, and if you are opposing by a force P, it displaces because of Hooke's law, elasticity is removed, the force goes back. So, it is basically a type of spring, the rod also can be modeled by a spring.

Similarly, the mass may be distributed, but mass, you can concentrate at one point. So, you can make a simple model of that real system. Let us derive the equation of motion for a spring of spring constant, k. Here, one end of a spring, of a spring constant k is fixed and a mass is attached at the other end. When you put the mass, there is a displacement. This is the static displacement, delta d. This is the equilibrium position. At this point, W will be equal to k delta. Make the free body diagram of this one.

This is mass, this is W and this k delta. So, it is in equilibrium. Now, disturb it slightly. You give the motion x. Then what happens, that the total restoring force is what spring has stretched by an amount k delta plus x. Therefore, this is k delta plus x is equal to W and this is x dot, this is x double dot, x dot denotes velocity of that mass. x double dot denotes the acceleration of that. Those forces have been shown here. W and k delta plus W_x.

We see that W is equal to k delta, but here, there is a force W and other side, the force is k delta plus x. Naturally, it is unbalanced. So, unbalanced amount is kx. So, that is a restoring force. That will try to bring it back to the original position.

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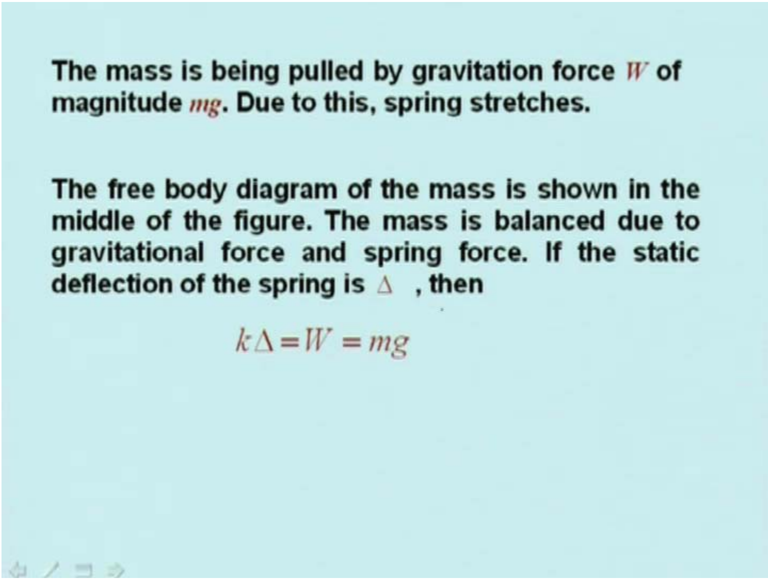

In the free body, let us summarize, the mass is being pulled by gravitational force W of magnitude mg, due to this spring stretches. The free body diagram of the mass is shown in the middle of this figure. The mass is balanced due to gravitational force and spring force. If the static deflection of the spring is delta, then k delta is equal to W is equal to mg.

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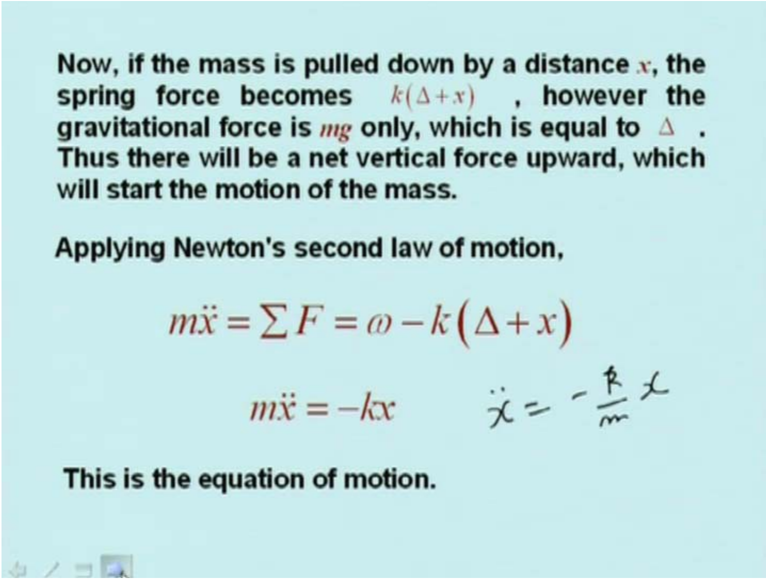

If the mass is pulled down by a distance x, the spring force becomes k delta plus x. However, the gravitational force is mg only, which is equal to delta. Thus, there will be a net vertical force upward, which will start the motion of the mass. Applying Newton's second law of motion, mx double dot mass times acceleration is equal to net force acting on the particle, that is W minus k delta plus x or mx double dot is equal to minus kx. This is the equation of the motion. So, we observe here that x double dot is equal to minus k divided m into x, k by m is a constant. It is a system property, k is the spring stiffness of the system and m is the mass of the system. So, this is constant. Therefore, acceleration is, where minus constant times x, acceleration is proportional to x and it is directed towards origin, because it is negative sign and the motion is obviously in a straight line, because we are talking about x-coordinate. If the motion was in two coordinates, then we would have used x y. So, this is the simple harmonic motion.

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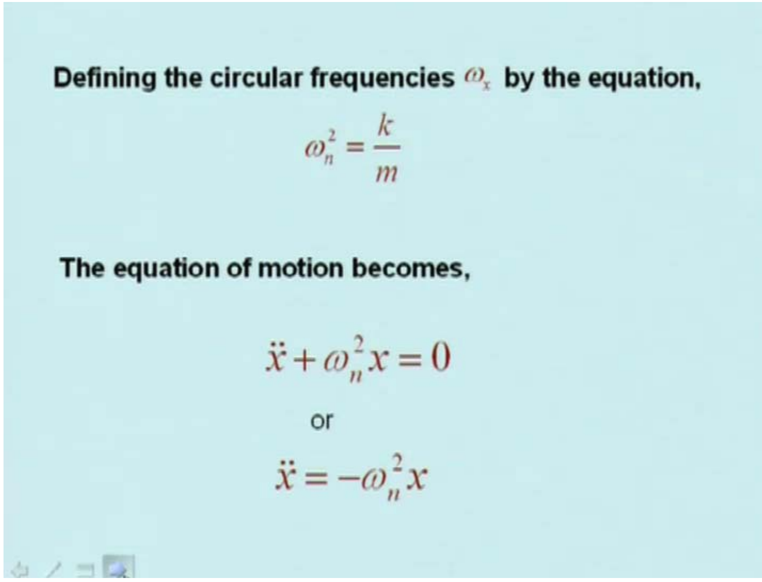

If we just define the circular frequency omega$_n$ by the equation n, omega$_n$ square is equal to k by m, or consider that since k times m is a constant, therefore, let me represent it by omega$_n$ square. Time being let us consider it a constant. The equation of motion becomes, x double dot plus omega$_n$ square x equal to 0 or x double dot is equal to minus omega$_n$ square into x.

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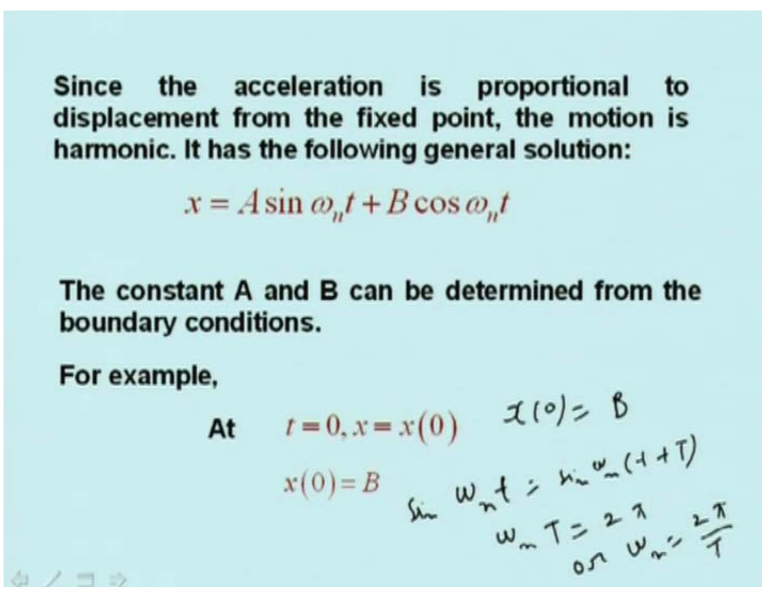

It has the following general solution. If you solve this equation, this will be x is equal to A sin omega$_n$ t plus B cos omega$_n$ t; that is the general solution of this type of problem. You can get this from differential equation or substitute this equation into that previous equation. You see that the equation is satisfied. So, the general solution of this equation is basically A sin omega$_n$ t plus B cos omega$_n$ t. The constants A and B can be determined from the boundary condition.

For example, at t is equal to x is equal to x(0), if you put that boundary condition then x(0) will be equal to B, because what will happen, put t is equal to 0; so x(0), x at 0 is equal to 0. This becomes B cos omega. So, x(0) is equal to B. So, that is one thing. It is basically sin omega$_n$ t. This naturally has got a period of 2pi omega, because omega$_n$ must be… So, this must be the frequency, because omega$_n$ repeats.

So, sin omega$_n$ t is equal to sin omega$_n$ t plus T. Therefore, we can say omega$_n$ into t is equal to 2 pi or omega$_n$ is equal to 2pi by t, where this is time period. So, time period is equal to 2 by omega$_n$.

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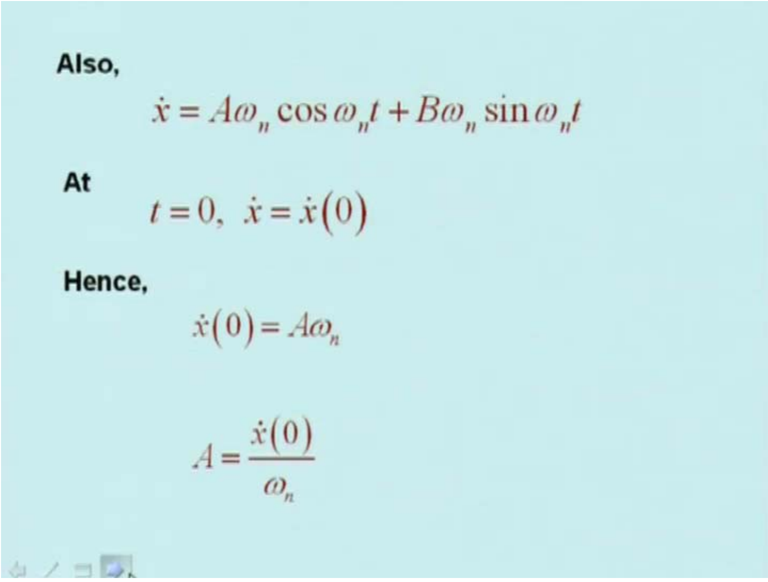

Let us say, at t is equal to 0, x is equal to 0 and x(0) is equal to B. Similarly, x dot is equal to A omega$_n$ cos omega$_n$ t plus B omega$_n$ sin omega$_n$ t at t is equal to 0 x dot is equal to x dot(0).

Therefore, x dot(0) is equal to A omega$_n$. If you put this thing, because this term, at t is equal 0 is 0 therefore, A is equal to x dot (0) and divided by omega$_n$.

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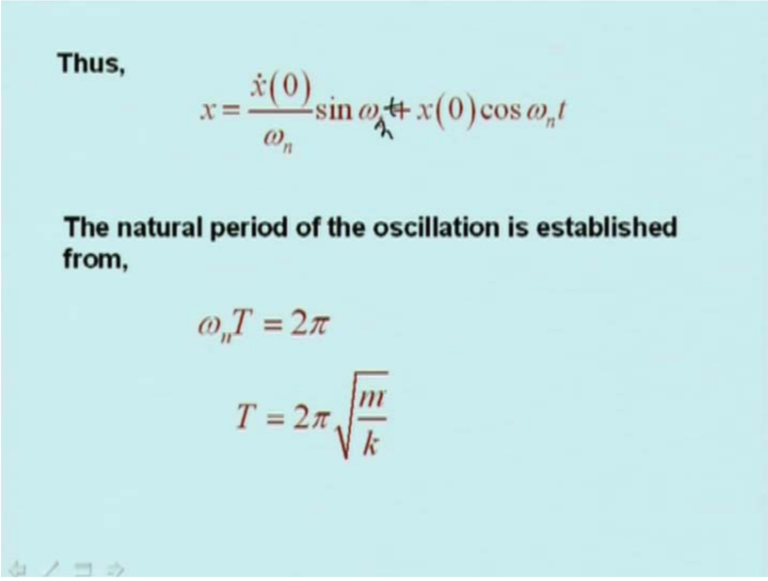

Thus, the general solution of this problem can be written as x is equal to x dot (0) divided by omega$_n$ multiplied by sin omega$_n$ t plus x(0) cos omega$_n$ t. Here, we see that this is the general solution of the problem. If initial velocity is 0 then this will be the solution. If initial displacement is 0 only you provide the velocity and this will be the solution. The natural period of the oscillation is omega$_n$ into T is equal to 2 pi or T is equal to 2pi divided by omega$_n$ that means under root m by k. Therefore, what happens, the time period is proportional to 1 by under root k. It is proportional to under root m.

If you take a spring mass system and suppose the spring is very stiff, and k is very high then time period will be almost 0, that means motion it will immediately come back to the same position. So, time period is 0. It vibrates and if k is equal to very small then time period becomes very high similarly about the mass. So, it is like this.

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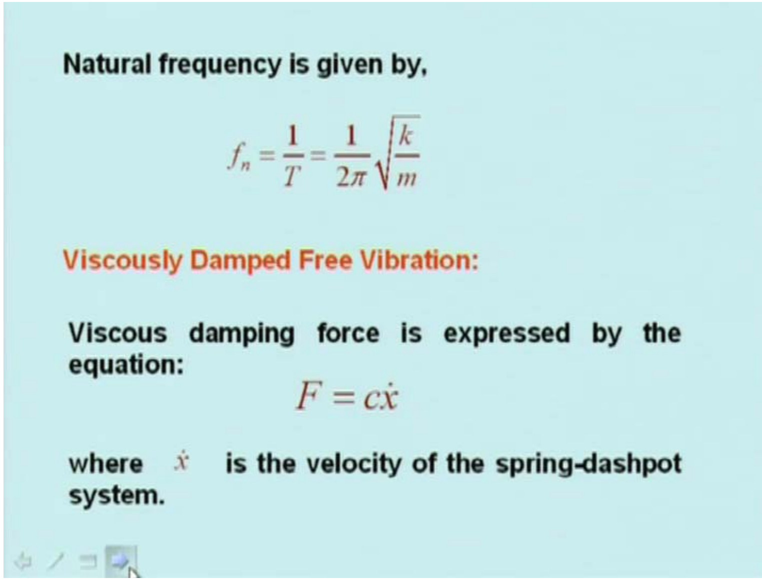

Natural frequency is given by f_n is equal to 1 by T that means it is 1 by 2pi under root k by m. So, this is the type of motion and it keeps occurring. These are the simple equations of that. Now, we have to talk about viscously damped free vibration. In previous example, we considered only the restoring force of the spring that is kx. Now, we are going to consider a damping also. Viscous damping force is expressed by the equation, f is equal to cx dot, where x dot is the velocity of the system. We should also put minus cx dot, because it is opposite direction. So, this is that way.

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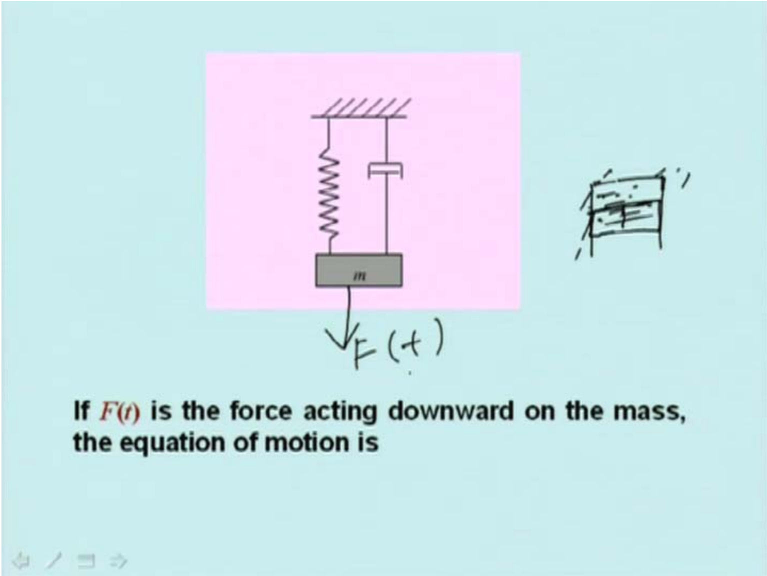

Then we represent this system in which viscous damping, viscous damping means there is damping; that means, resistance is proportional to the velocity, but in the negative direction minus cx. So, that means acceleration is proportional to this is force, damping force is proportional to F(t). Take the mass then represent by a spring. Then again, we have a dashpot.

Dashpot is one thing that suppose you take a system part which is filled with oil, and in this a piston is moving. So, this type of system is called dashpot. Here, the force is proportional to the velocity more, because you know that in a viscous material, if this object is moving, therefore, the force is proportional to velocity. Here, we can put F(t) as the force acting downward on the mass. Actually, you will not observe that in a system we always have dashpot but we are only modeling. That effect, the same type of effect which this dashpot provides, may be provided by some other mechanism. If a cantilever beam is vibrating, so many particles are having that influence. So, this is very complicated, but we can actually put like this.

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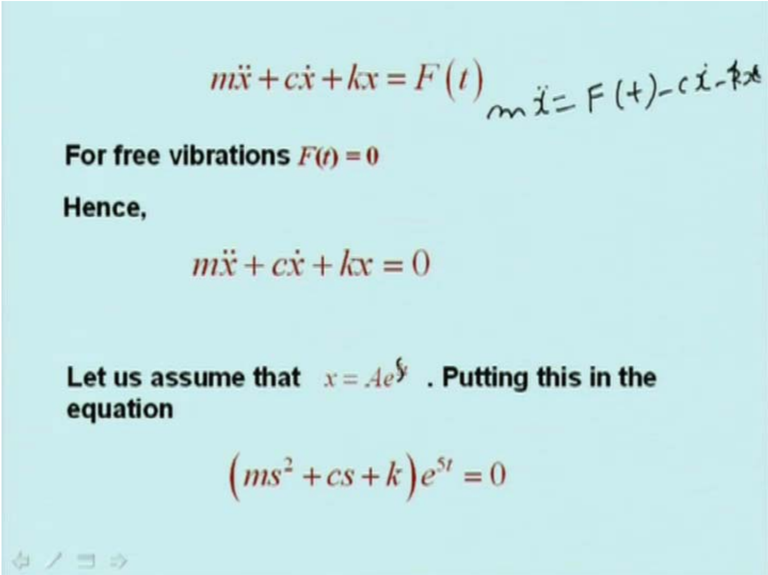

Therefore, equation of motion is m x double dot plus c x dot plus kx is equal to F(t). That means, because if we can apply the D'Alembert's principle, we can say, the F applied force, or we can say mx dot. Newton’s law mass times acceleration is equal to F(t) minus cx dot minus kx. Now for free vibrations F(t) is equal to 0. Hence, m x double dot plus c x dot plus kx is equal to 0 for free vibration. This is the equation for free vibrations in the absence of damping.

Let us assume that x is equal to A into e to the power st, where s is any number. So, it is any variable you define. Let us assume that, one solution is x is equal to A to the power st of this solution. So, see what happens, we have to put this value in this expression then ms square plus cs plus k e to the power st, is equal to 0.

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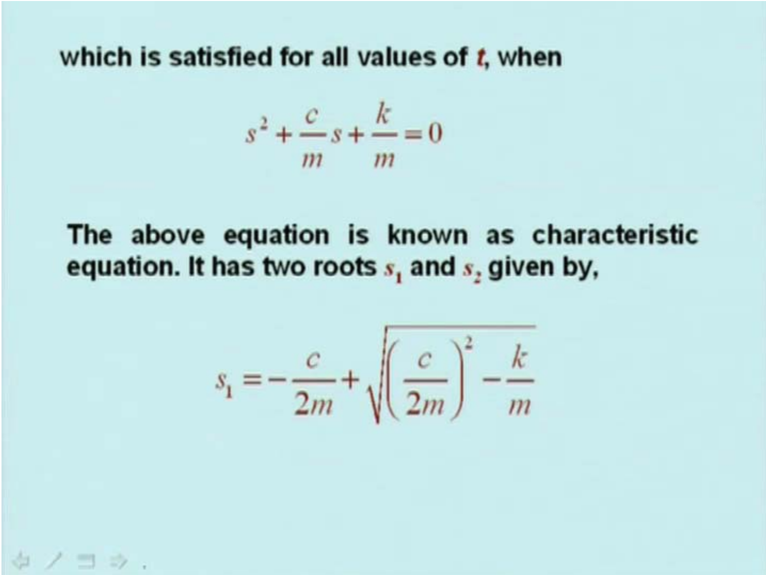

This equation has been put which is satisfied for all values of t, when s square plus c by m s plus k by m is equal to 0. Then it is satisfied for all values of t. So, this is that type of a thing. Otherwise, because e st cannot be 0, e to the power st cannot be 0, except when t becomes minus infinity. This cannot be 0. Therefore, this must be equal to 0. If this equation is known as characteristic equation, it has two roots s_1 and s_2 given by s_1 is equal to minus c by 2m plus under root c by 2m whole square minus k by m. This is one root s_1.

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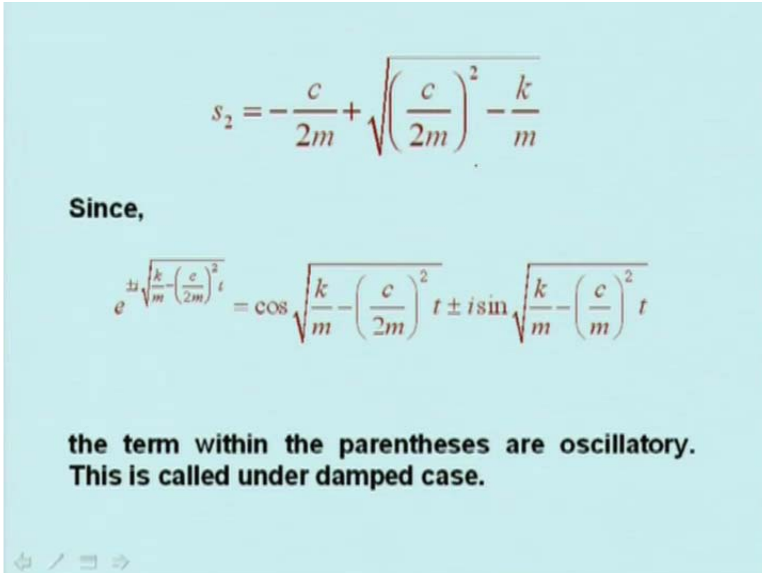

Similarly, s_2 will be equal to minus c by 2m plus under root c by 2m square minus k by m. Let me show you some more steps. Now, you have got two roots of this problem; that means both are the solutions.

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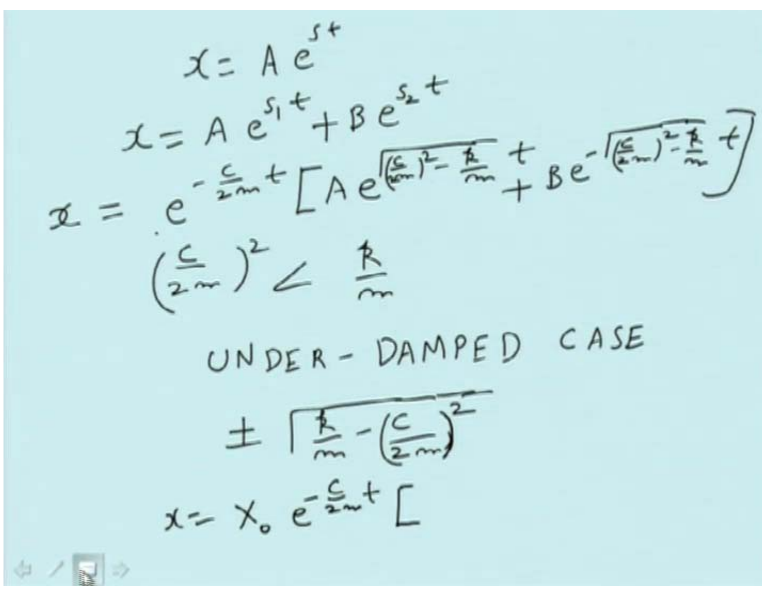

We assumed that the solution was x is equal to A e to the power st, but it has got two roots. So, the solution will be what? That solution can be written as A e to the power s_1 t; this is also satisfied and B e to the power $s_{2\ t;}$ that will also satisfy. Therefore, we can say A plus B e s 2 t also satisfies this one and if you want only this, put A is equal to 0. If you want only this, put B is equal to 0. A and B are just constants. Therefore, putting the value of s_1 and s_2, x can be written as e minus c by 2 m into t. This is A e to the power under root c by 2 m whole square minus k by m multiplied by t plus B e to the power minus under root c by 2 m square minus k by m, and t outside this square root term.

We get this type of term; let us see what happens. When the damping term c by 2m square is larger than k by m, it is larger than k by m then the exponent in the above equations are real numbers. This is A to the power some real number t and this is equal to this one and in this case there are no oscillations are possible. It cannot provide the oscillatory motion. This case is called over damped case; this is over damped. So, no oscillations are possible in over damped case.

When the damping term c by 2 m square is smaller than k by m that means this is smaller. If it is smaller then this will be called under damped case. This is under damped case. In this case, this is square root of a negative number. Therefore, it becomes imaginary number this is this one. Therefore the exponent becomes basically you can say in this case I can write exponent becomes plus minus under root k by m minus c by 2 m whole square. So, this becomes like this; k by m minus c by 2 m whole square and this is imaginary number.

Therefore, in this case what happens? See you have got s_1 then s_2 is equal to this one. Now you consider e plus minus under root k this one this can be written as cos under root k by m minus c by 2 m square t plus minus i sin under root k m minus c m square by t, because e i theta is equal to cos theta plus i sin theta. You have to use that relation.

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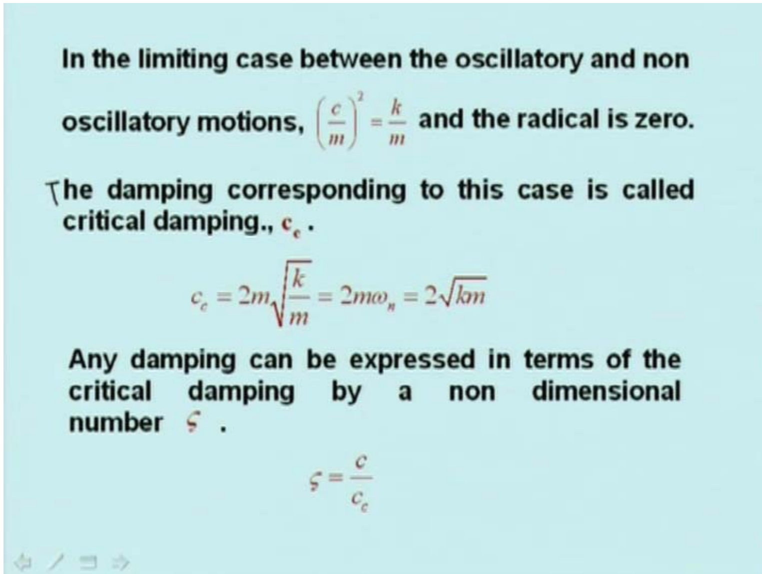

In this case, in the limiting case, between the oscillatory and non-oscillatory motions, we have c by m square is equal to k by m and the radical is 0. The damping corresponding to this case is called critical damping; that is $c_{c.}$ Therefore, c_c is equal to 2 by m under root k by m or it is 2 m times $omega_n$ or this is 2 under root km. So, critical damping is a property of the system. If you know that it depends that this is the thing So, k by m is critical damping. Any damping which is more than the critical damping is called over damping. Any damping which is less than the critical damping is called under damping. Therefore, any damping can be expressed in terms of the critical damping by a non-dimensional number, zeta.

We say the zeta is equal to c by c_c or we can use any other symbol c by $c_{c,}$ where c_c is the critical damping. If c by c_c is more than 1, this is the case of over damping. If c by c_c is less than one, this is the case of under damping.

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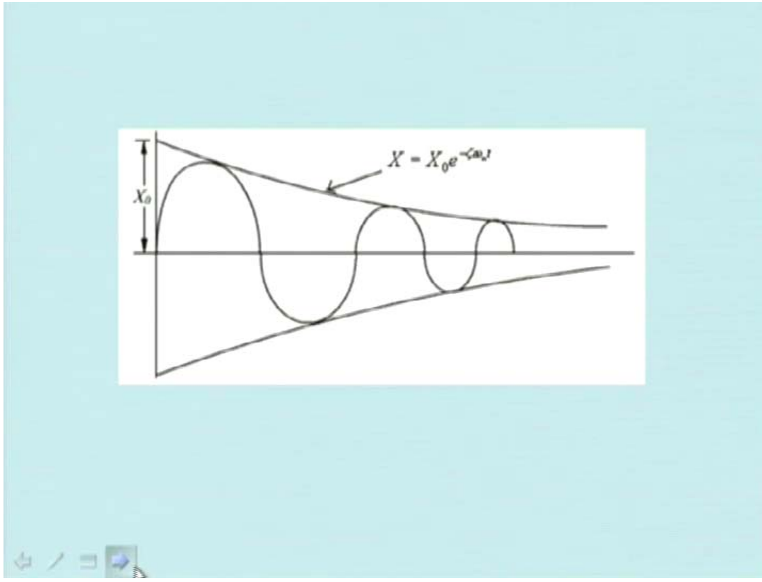

We see, what about the amplitude? In the case of that we say we started with this equation, we discussed about one point, as I already showed here, that you have e minus c by 2 m. So, that means, A can be taken. This term can be written in terms of sin omega t cos omega t. You have the amplitude term that means you can write x is equal to something like $x_{0.}$ You can always have some term. So, this is x_0 e minus c by 2mt and then may be inside, you may have p sin omega t and this thing.

I am not writing that term. So, x is equal to x_0 e minus tau $omega_n$ into t. That term is e minus this is the thing tau $omega_n$. In this case, if there is 0 damping, tau 0 then x is equal to $x_{0;}$ that means amplitude remains constant. But if tau is some number then it will be exponentially decreasing. Therefore, the amplitude keeps on decreasing in exponential fashion. It never becomes 0, but it keeps on decreasing. This is a profile and this is what has happened.

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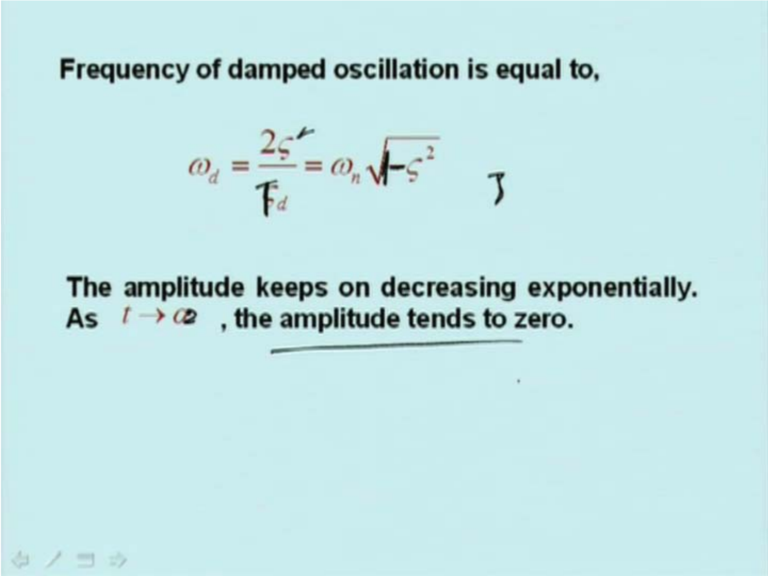

Frequency of damped oscillation is equal to, $omega_d$ is equal to 2 tau divided by $T_{d.}$ This is T_d that is $omega_n$ 1 minus tau square. That is not tau. This is basically zeta. The amplitude keeps on decreasing exponentially as t tends to infinity, the amplitude tends to 0. So, that way, you know damp motion will take place and this is how it will be covered.

Having discussed about the free and forced vibration, let us discuss simple cases. Let me just see, what you have learnt.

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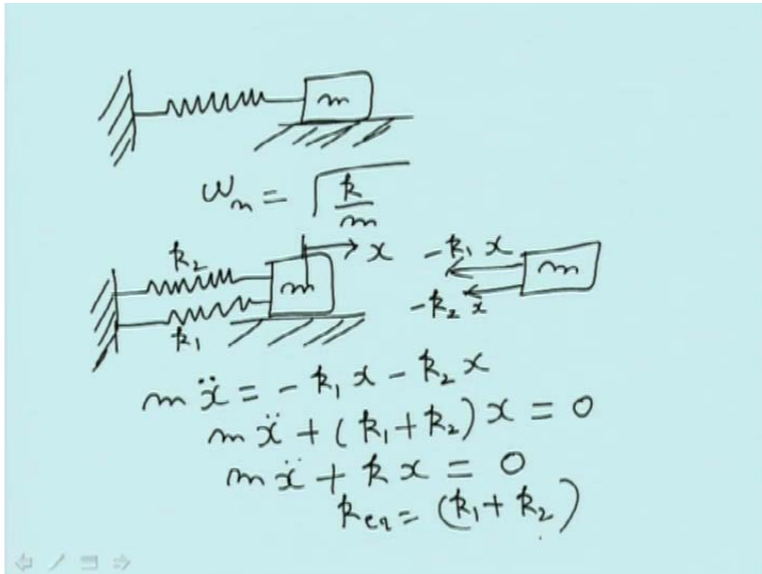

This is the spring mass system, here omega$_n$ is equal to under root k by m. What happens if we put two springs here? What type of equations we will get?

This is m. In this case, suppose you displace it by distance x from here, mx double dot is equal to minus k_1 x minus k_2 x. If you make a free body diagram of this mass, you indicate you can say minus k_1 x and minus k_2 x will be the forces. So, you have mx double dot minus k_1x minus k_2x that means mx double dot plus k_1 plus k_2 x. so it is the same type of equation.

Here, compare this equation with previous equation mx dot plus kx equal to 0. So, we see that we need not solve this again. We can see, equivalent k. In this case, we can say k equivalent is equal to k_1 plus k_2. So, these two springs are in parallel. Therefore, their stiffness gets added up.

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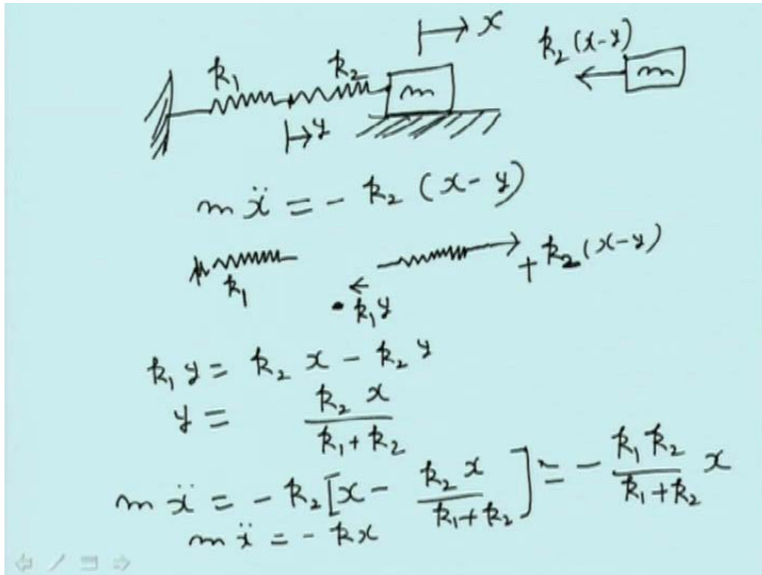

Let us see another case, in which there is a mass. This is the mass m, this is k_1, this is k_2 and it is x. If you give that this x. So, let us make the free body diagram of this. Let us say that this point displaces by y. So, we have mx double dot. Newton’s law applied. mx double dot is equal to minus k_2 x minus y. We can say, because mass is this one, separately we can draw and this is accelerating. So, its acceleration is mx is x double dot and this is mass.

Here, this becomes k_2 x minus y, because this end of the spring has moved at a distance x and the other end has moved at a distance y. So, mx double dot is equal to minus k_2x minus y. If you consider the spring k_1 so k_1 spring is there. Now, this gets stretched by an amount y. So, this force is minus k_1, so minus k_1 y.

Consider that spring k_2; spring k_2 is subjected to a force minus k_2 (x minus y) and it must be the same, because the force is getting transmitted. From transmissibility principle, this is same as basically, this is k_2 plus. This is k_1 and this is y; that means, k_1 y is equal to k_2 x minus k_2y. Therefore, y is equal to k_2x divided by k_1 plus k_2. Therefore, your equation becomes mx dot is equal to minus k_2x. k_2x minus k_2y. So, this becomes k_1 plus k_2 and this becomes k_2x. So, minus mx dot is equal to minus k_2x and this is k_2x minus k_2y. k_1 plus k_2 is equal to k_2x divided by k_1 plus k_2. This is y and this is x minus k_2 x divided by k_1 plus k_2. So, simplify it. This will come out to be minus k_1 k_2 divided by k_1 plus k_2x. Compare it with mx is equal to mx dot is equal to minus kx.

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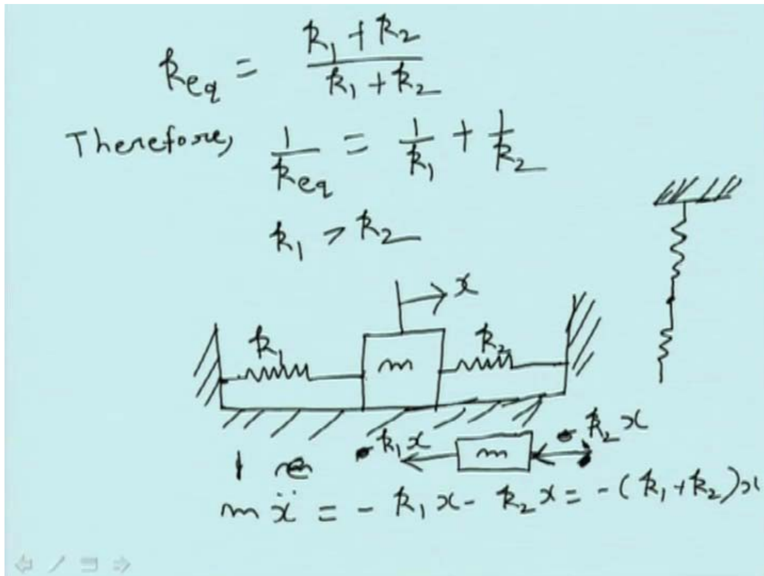

Therefore, we see that in this case, the k equivalent comes out to be k_1 plus k_2 divided by k_1 plus k_2. Therefore, 1 by k equivalent is equal to 1 by k_1 plus 1 by k_2. When these things are in series then they are added in this fashion. You can easily see that k equivalent cannot be; suppose you have k_1 k_2 and in this case, k_1 is greater than k_2 then naturally the k equivalent will be cannot be more than k_2 actually. Therefore, when they are in the series then equivalent stiffness reduces. So, when the springs are in series, then this is 1 by k equivalent 1 by k_1 plus 1 by k_2.

Let us see, say for example, this case, here this is k_1, this is k_2. How should I solve this problem? This is the x position from here. If this mass is displaced by some distance, therefore, k_1 mx dot so, free body diagram. Let us see making the free body diagram. This is m. This is minus k_1x. This spring gets compressed, because, suppose this gets stressed by x, another spring gets compressed by x. Therefore, this becomes k_2x. So, minus k_2 this is k_2x.

Let me make k_1x like that, but this is k_2x.This gets compressed. So, it puts opposite force. So, when we consider the force coming on the mass, this has to be put k_2x. Therefore, mass mx dot x dot is equal to minus k_1x minus k_2x. That means minus k_1 plus k_2x. So, equivalent stiffness is basically, minus k_1 plus k_2. Therefore, considering that springs are in parallel does not mean that physically they will look in the same line and this thing that they are basically like this because they undergo same type of displacement. If they are going with the same amount of displacement then they are in parallel. If they are undergoing the different displacement then they are not in parallel. They are in series. Here, they are in series, because here this is the thing and then after that another spring is there. It gets stretched. So, they are in the series.

This goes by some x amount. This point, with respect to this point, it may move some another distance. So, it is like that.

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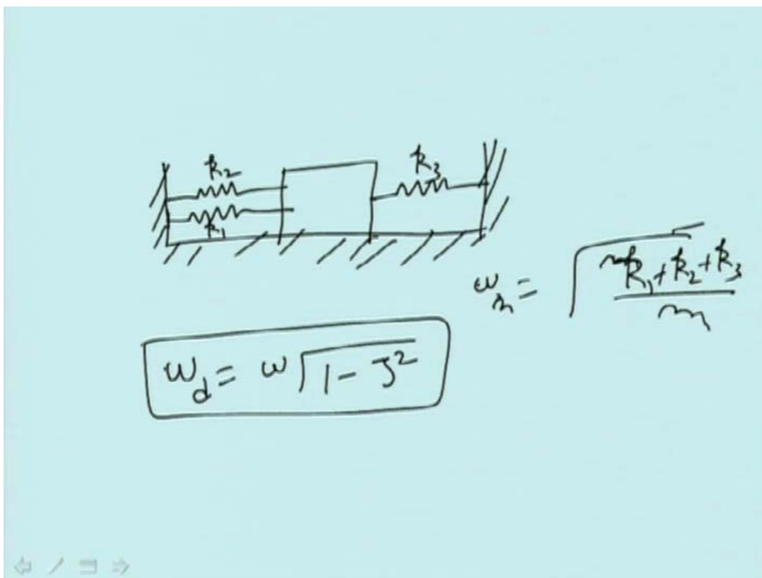

Therefore, if you have finally this type of problem; k, k_1, k_2, k_3 here all the springs are in parallel. Therefore, equivalent stiffness is k_1 plus k_2 plus k_3 and omega_n will be equal to under root m divided by k divided by k_1 plus k_2 plus k_3 divided by m. By this, you can find out the vibrations of this one. So, we have discussed about the free vibrations in the absence of damping. We also have discussed the vibrations in the presence of damping. In presence of damping, you can easily derive that in presence of damping, the natural frequency will be omega times under root 1 minus zeta square; that is, damping. Therefore, the natural frequency and damped natural frequency reduces by that thing.