Engineering Mechanics

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Module 14 Lecture 35
Kinetics in 3D

In this lecture, we will discuss the kinetics of rigid body. You know that it has been already stated that the rate of change of linear momentum of a body is equal to the net applied force. This is valid in inertial frame of reference.

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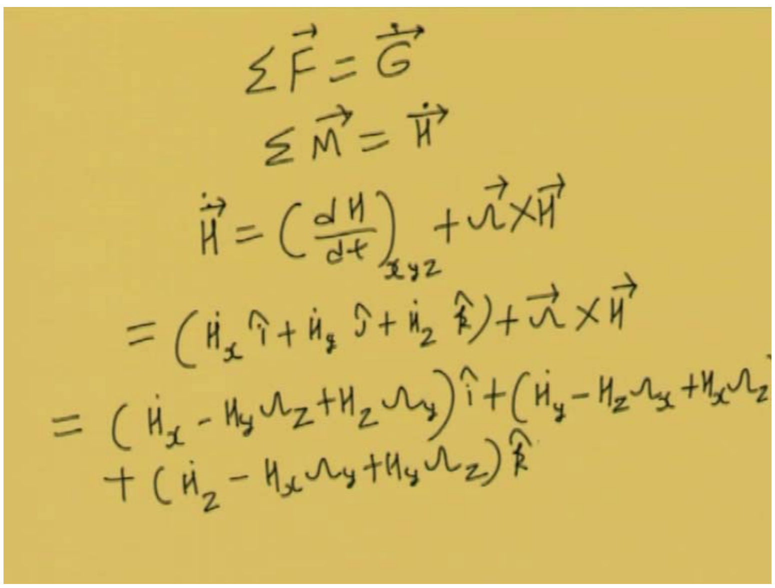

That means sigma F is equal to G dot. This is vector this also vector. Dot represents time derivative. Similarly, the rate of change of angular momentum is equal to net applied moment, when the terms are taken either about a fixed point O, or about the mass center that is sigma M is equal to H dot. This equation is also valid, if the angular momentum H and the moment M are taken about fixed point O, or mass center, or a point which is moving towards or away from the center of mass. When H can be expressed in terms of components measured relative to a moving coordinate system then H dot can be written as H dot is equal to dH by dt x y z plus omega cross H. Axis system itself is rotating with angular velocity omega.

Then you have to write like this; H dot is equal to dH by dt x y z axis itself is rotating then this is the thing. That means I have to write like this. This becomes H dot $_x$ i plus H dot $_y$ j plus H dot $_z$ k plus omega cross H.

You know the cross product, and suppose omega is having three components; omega$_x$ i then omega along y direction, omega y component and then omega z component, then you can do the cross product and finally you will be getting these equations, H dot $_x$ minus H_y omega $_z$ plus H_z omega $_y$ i plus H dot $_y$ minus H_z omega $_x$ plus H_x omega $_z$ j plus H dot $_z$ minus H_x omega $_y$ plus H_y omega $_z$ k. Here, omega $_x$, omega $_y$, and omega $_z$ are the components of the angular velocity omega of the axis.

If the axis system is attached to the rigid body itself, in that case, omega is equal to omega i, small omega; that means, the angular velocity of the body itself. In that case, we can write these equations and you know that this is equal to moment.

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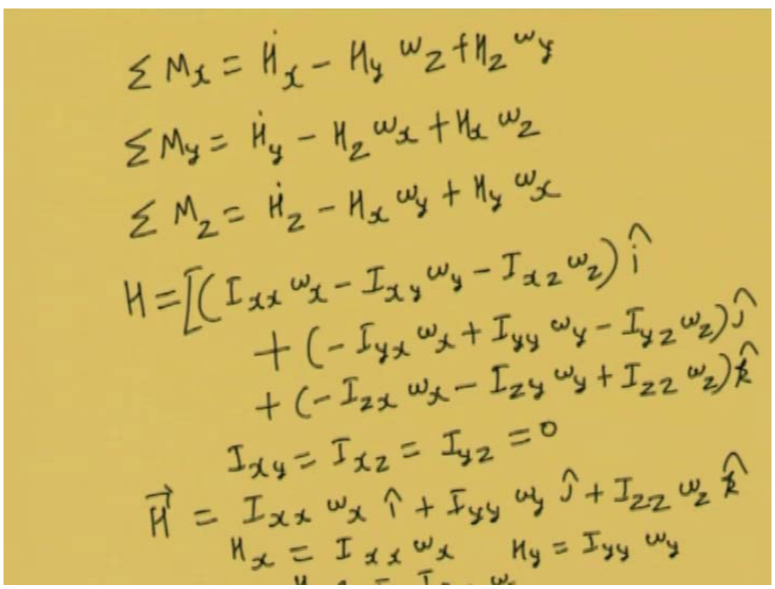

So, sigma M_x comes out to be H dot $_x$ minus H_y omega $_z$ plus H_z omega $_y$. Similarly, sigma M_y comes out to be H dot $_y$ minus H_z omega $_x$ plus H_x omega $_z$, sigma M_z is equal to H dot $_z$ minus H_x omega $_y$ plus H_y omega $_x$.

We have already developed these expressions. If we write that H is basically, I_{xx} omega$_x$ minus I_{xy} omega$_y$ minus I_{xz} omega$_z$ I, these expressions we have developed plus minus I_{yx} omega$_x$ plus I_{yy} omega$_y$ minus I_{yz} omega$_z$ j plus, minus I_{zx} omega$_x$ minus I_{zy} omega$_y$ plus I_{zz} omega$_z$ k.

If a xyz axis are principal axis, if xyz axis chosen are the principal axis then I_{xy} is equal to I_{xz} is equal to I_{yz} is equal to 0. Thus, H is equal to I_{xx} omega $_x$ i plus I_{yy} omega $_y$ j plus I_{zz} is equal to I_{zz} omega $_z$ k.

You see that how simplified it becomes. Then, in that case, if you take that just like that, and that is the advantage of finding out the principal axes. Now, expression has become very compact. Hence then in this case if xyz are the principal axis then H_x component is I_{xx} omega$_x$ H_y component is I_{yy} omega$_y$ and Hzz is basically I_{zz} omega$_{z.}$

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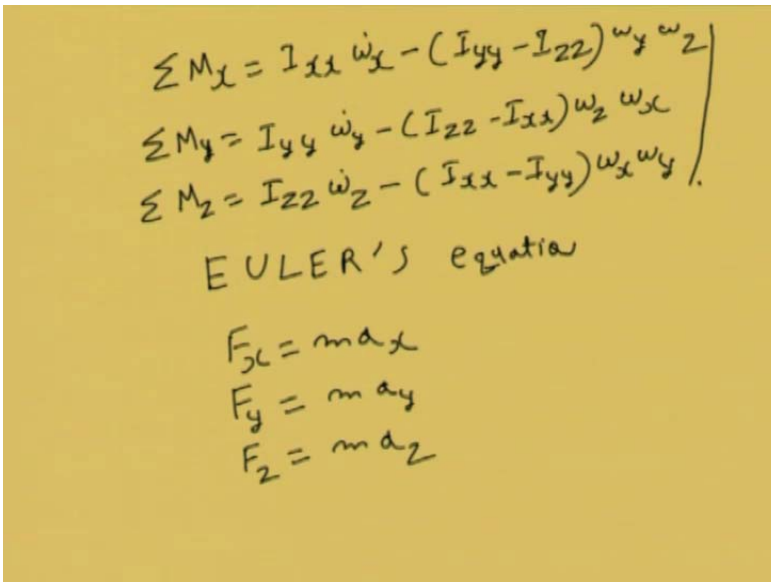

If we put these in F, then we get sigma M_x is equal to I_{xx} omega dot $_x$ minus I_{yy} minus I_{zz} omega $_y$ omega$_{z.}$ sigma M_y is equal to I_{yy} omega dot$_y$ minus I_{zz} minus I_{xx} omega$_z$ omega$_{x.}$ sigma M_z is equal to I_{zz} omega dot$_z$ minus I_{xx} minus I_{yy} omega$_x$ omega $_{y.}$ So, we get these set of equations somewhat simplified. Had we not used the principal axis, the expressions would have got very complicated.

In this case, we got simplified expressions that is sigma M_x is equal to I_{xx} omega dot_x minus I_{yy} minus I_{zz} $omega_y$ $omega_{z.}$ sigma M_y is equal to I_{yy} omega dot_y minus I_{zz} minus I_{xx} $omega_z$ $omega_{x.}$ sigma M_z I_{zz} omega $_z$ minus I_{xx} minus I_{yy} $omega_x$ $omega_{y.}$

These equations are known as Euler's equation. They are known as Euler’s pronounced as oiler, Euler’s equation. These are called Euler’s equation and they have been named after a Swiss Mathematician. The conditions under which these equations are applicable are these as follows: Number one: The reference point where the body axes are fixed is a point fixed in space. These are called body axes, why? Because they are moving with the body only. So, they have angular speed as the body itself, speed of that body.

The reference point where the body axes are fixed is a point fixed in space, or is the center of mass itself. That can be either you take a point in the body, either that point is the fixed point or that point is the center of mass; however, apart from that, it can also be a point which is accelerating, but the acceleration is either towards the center of mass or away from the center of mass. Two things are there. There are three types of things. Number 1: point should be either fixed. Then you know these equations, Euler’s equations are valid. Number 2: point can be center of mass, whether it is fixed or not fixed does not matter; then also the equation is valid. The third thing is that the point may not be fixed also. It may not be center of mass, but it is moving. It is accelerating towards the center of mass or away from the center of mass. So, this is the first condition under which these Euler’s equations are applicable.

Number 2, condition is that the xyz reference frame is fixed on the body and is directed along the principal axis at that reference point. Otherwise, we would have got complicated expression. So, these xyz axes are the principal axes.

Third point is that the moments of the forces are taken about the reference point. About the reference point, we take the moment and also the components of the inertia matrix are determined with respect to the body axis at that point. Moments of the forces have to be taken about that reference point. Similarly, components of the inertia matrix are also determined by that. So, we have got these three sets of Euler’s equation. We supplement these three sets of equation by three equations of motion. Three equations of motion, for the motion of center of mass that is Newton's law that is F_x is equal to m times $a_{x,}$ F_y is equal to m times $a_{y,}$ F_z is equal to m times $a_{z.}$ So, total six. So, these six equations are enough to solve different types of problems.

For example, we take various examples. We take one example.

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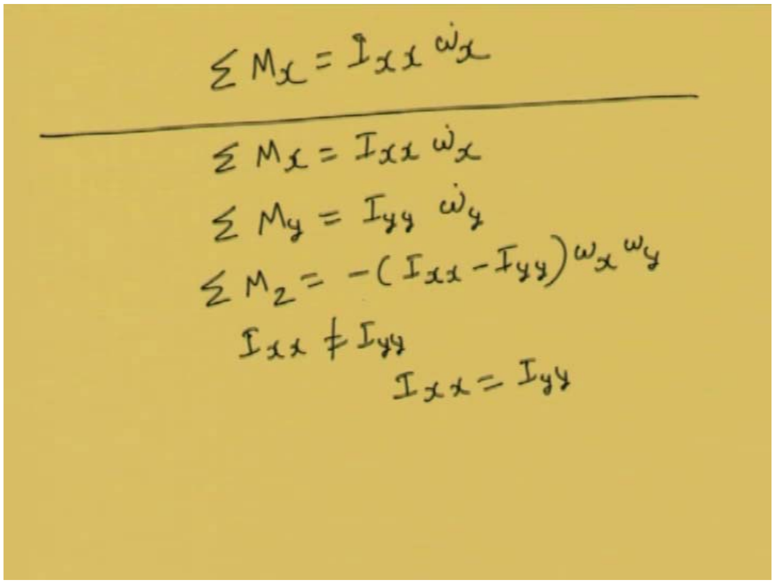

Suppose body is just rotating about x axis only. Then, we get sigma M_x is equal to I_{xx} omega dot $_{x,}$ Euler's equation. Body is rotating with a constant angular velocity then no net moment acts on it. If body has the component of angular velocity along x and y axis, both, then what happens? If the body has the component of angular axis velocity along x and y, then 1 equation; sigma M_x is equal to I_{xx} w comma $_x$, sigma M_y is equal to I_{yy} omega comma $_y$, sigma M_z is equal to minus I_{xx} minus I_{yy} omega_x $\text{omega}_{y.}$ In this case, we get these types of things. Now, if I_{xx} is not equal to $I_{yy,}$ then in that case, body experiences net moment in z direction also. Even if suppose omega_x is constant, therefore sigma M_x is I_{xx} w omega dot $_x$; that means, this is 0 sigma M_y is 0. However, sigma M_z will be present. If I_{xx} is not equal to I_{yy} then sigma M_z will be 0. If I_{xx} is equal to $I_{yy,}$ that means I_{xx} is equal to $I_{yy,}$ then sigma M_z will be 0 in the case of constant rotation about two axes like x and y. Otherwise, this will be this one. It is like this that you know this phenomena comes.

Then we discuss torque free motion. In the absence of torque, what all motion we get that is called torque free motion.

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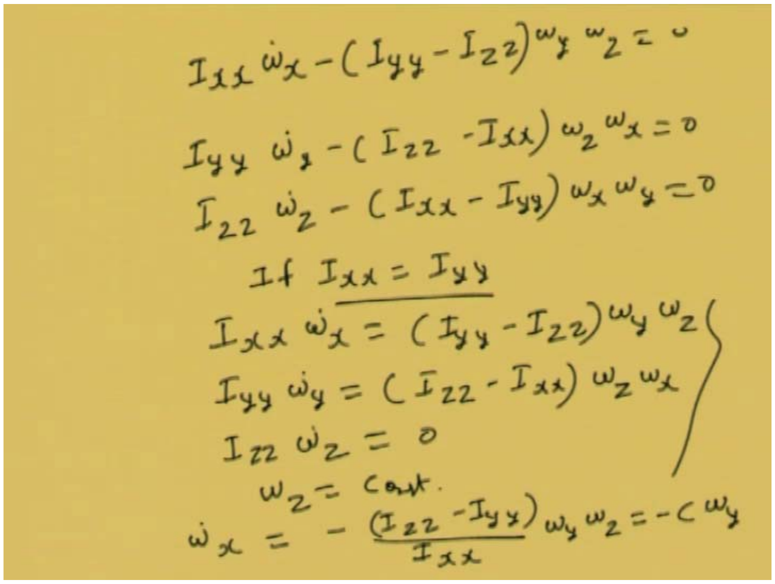

If we write the Euler's equation then this is I_{xx} w dot $_x$ minus I_{yy} minus I_{zz} omega $_y$ omega $_z$ equal to 0 is the first equation you will get. Second will be I_{yy} omega dot $_y$ minus I_{zz} minus I_{xx} omega $_z$ omega $_x$ is equal to 0. Then I_{zz} omega dot $_z$ minus I_{xx} minus I_{yy} omega $_x$ omega $_y$ equal to 0.

These three equations can be solved. If in this case, if Ixx is equal to I_{yy} then you get I_{xx} omega dot $_x$ is equal to I_{yy} minus I_{zz} omega $_y$ omega $_z$ and you get I_{yy} omega dot $_y$ is equal to I_{zz} minus I_{xx} omega $_z$ omega $_x$. Now, I_{zz} omega dot $_z$ is equal to 0. These are the three equations you get. If you know I_{xx} is equal to I_{yy} in torque free motion then omega $_z$ is equal to constant. Third equation gives omega $_z$ is equal to constant and omega dot $_x$ is equal to basically minus I_{zz} minus I_{yy} I_{xx} omega $_y$ omega $_z$ is equal to minus C omega $_y$.

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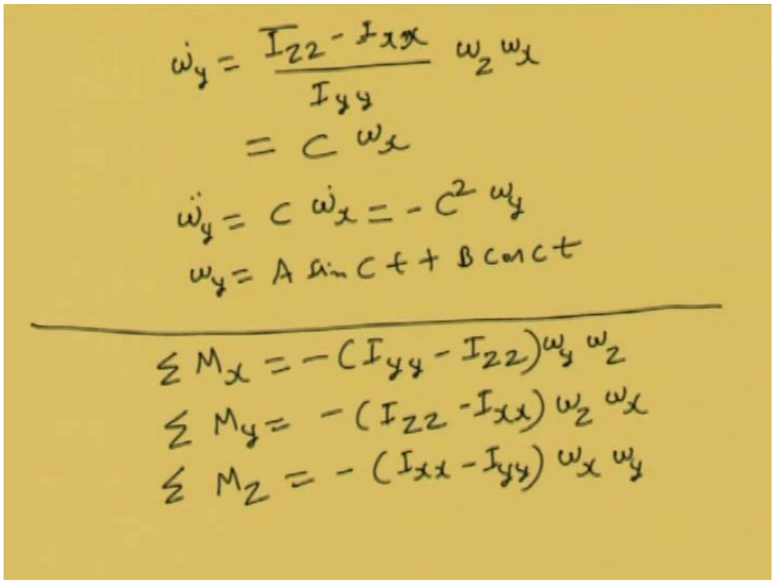

Similarly, omega dot $_y$ will be equal to I_{zz} minus I_{xx} I_{yy} omega$_z$ omega$_x$ and this is equal to C times omega$_{x.}$ If we differentiate the above equation, this equation then you will be getting omega dot $_y$ is equal to C times omega dot $_x$; that is equal to minus omega dot $_x$ is equal to minus C omega $_y$. Therefore, this becomes minus C square omega$_y$ which gives you omega$_y$ is equal to A sin Ct plus B cos Ct, where constants A and B can be found from the initial conditions.

Similarly, we can find out the expressions for omega$_x$ also. If angular velocity is constant then what happens, there will not be any dot terms. Sigma M_x will be minus I_{yy} minus I_{zz} omega$_y$ omega$_{z,}$ sigma M_y is equal to minus I_{zz} minus I_{xx} omega$_z$ omega$_{x,}$ sigma M_z is equal to minus I_{xx} minus I_{yy} omega$_x$ omega$_y$.

In case I_{xx} is equal to I_{yy} is equal to $I_{zz,}$ in that case, of course the torques will be 0. Suppose I_{yy} and I_{zz} are different, in that case, even if the angular velocity is constant then also there is torque acting. Therefore, this is what the point I want to emphasize. This point, you know that even if you have a constant angular velocity, it is possible to have moments; that means, there can be moments and still there can be constants. That is what we get from the Euler equations.

Suppose there is a sphere about which if you take a point at the center mass centre, I_{xx} is equal to I_{yy} is equal to I_{zz}. In that case, of course that sigma M_x will be 0, sigma M_y will be 0 and sigma M_z will be 0. So, sphere can keep rotating without any moment, but in general it is not true.

Suppose we discuss about steady rotation of an asymmetric body.

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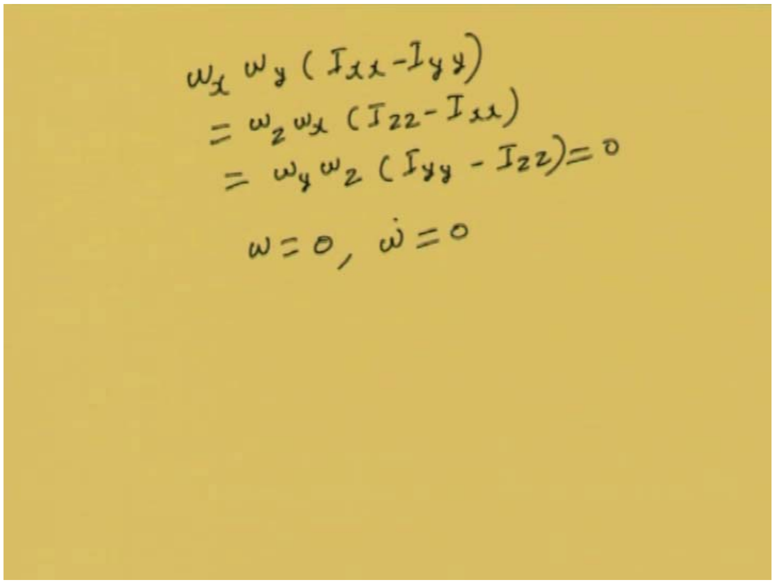

From the Euler's equation, we see that steady rotation of an a symmetric body is possible if omega$_x$ omega$_y$ I_{xx} minus I_{yy} is equal to omega$_z$ omega$_x$ I_{zz} minus I_{xx} is equal to omega$_y$ omega$_z$ I_{yy} minus I_{zz} is equal to 0. This requires at least two of the components of angular velocity is 0 that is the angular velocity is along only one of the principal axis. Then it can be without any moment.

If we discus about the translation of a rigid body then you get omega is equal to 0, omega dot is equal to 0. In this case, Euler's equations are automatically satisfied and the motion can be studied by Newton's law only. So, this is what you think if you have got that. So, we have understood the Euler's equation and we will discuss about the very interesting phenomena that is called gyroscopic motion.

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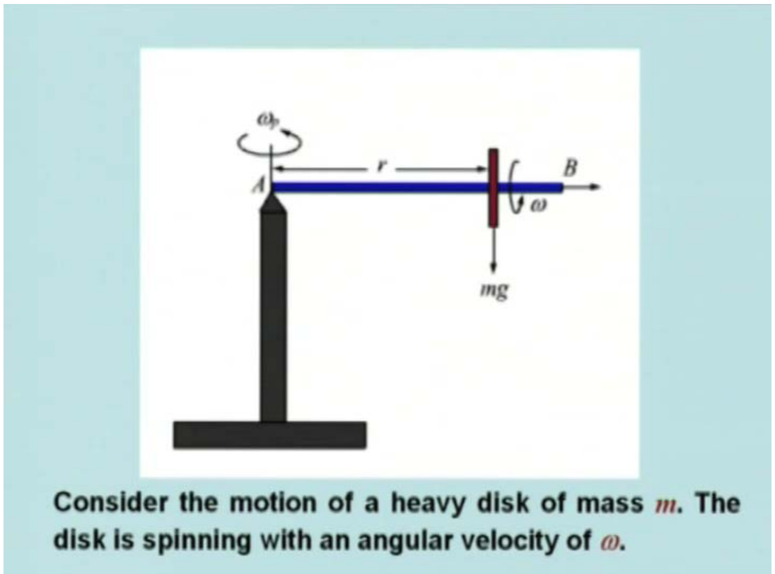

We discuss the gyroscopic motion. Without giving any formal definition to what is gyroscopic motion, I would draw your attention to the following figure.

Consider the motion of a heavy disc of mass m. This disk is spinning with an angular velocity of omega. So, the mass of the shaft, for the time being you assume that it is negligible, but you have got a heavy disk mg. Its velocity is omega and the whole shaft is précising with an angular velocity omega$_{P.}$ Then what happens? If you just put a pin joint here; that means, this link is by a pin joint then you may think, because this is a heavy disk, naturally it should fall; but you will see that surprising thing that this will not fall actually and the motion will continue. How it happens? This is called gyroscopic effect.

What is the reason for this?

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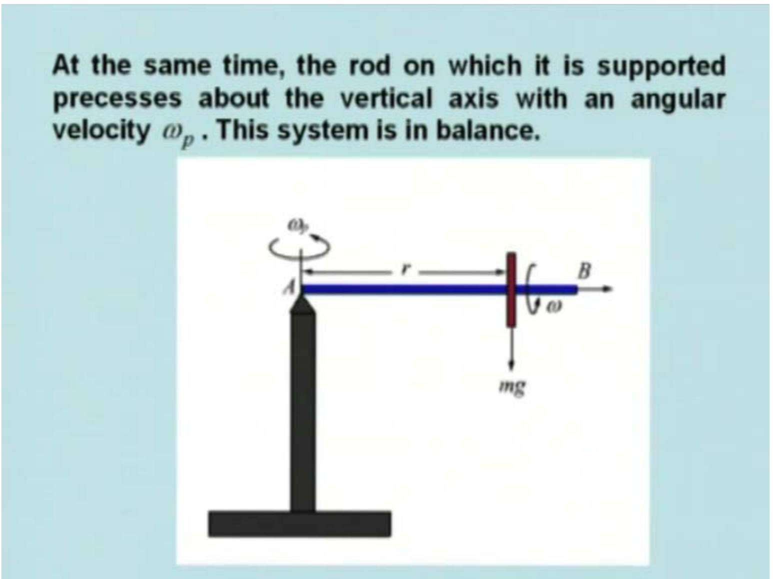

Motion of the disk is spinning with an angular velocity of omega which is high, but at the same time, the rod on which it is supported precesses about the vertical axis with an angular velocity omega$_P$. The precession velocity, time being you can assume is smaller; that spin velocity omega. This system is in balance. How this system is in balance? Why it is not falling downwards?

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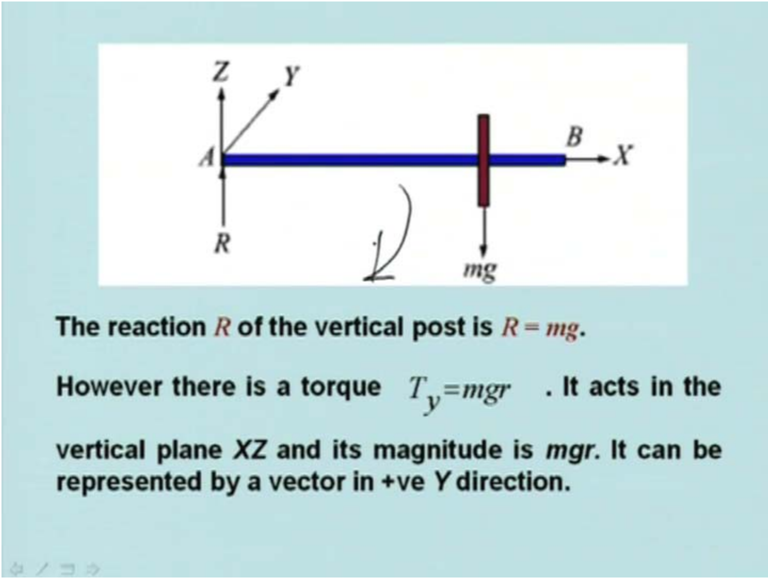

We have to see this thing; if we make the free body diagram of the system and we have to show the reaction R of the vertical post and this reaction is balancing, the force due to gravity mg is equal to R.

However, there is a torque that is T_y is equal to mgr. It acts in the vertical plane xz and its magnitude is mg times r. It can be represented by a vector Y direction, because like this mgr. So, it is no torque. It is trying to, it acts clockwise. Therefore, it is going away from you. So, that means it is along the plus Y direction.

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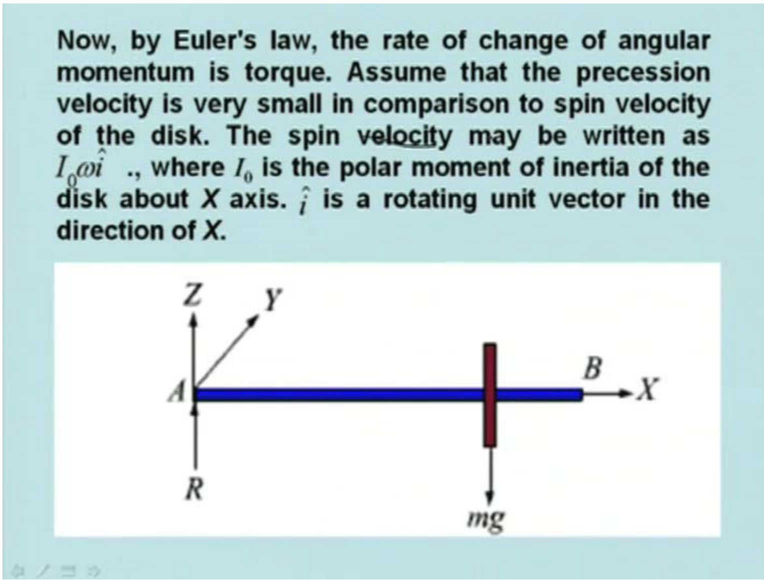

By Euler's law, the rate of change of angular momentum is torque. Assume that the precession velocity is very small in comparison to spin velocity. Then the spin velocity may be written as I_0 omega$_{I,}$ I_0 omega$_i$ is not the spin velocity, spin momentum along the spin velocity direction. Where I_0 is the polar moment of inertia of the disc about x-axis and i is a rotating unit vector in the direction of X.

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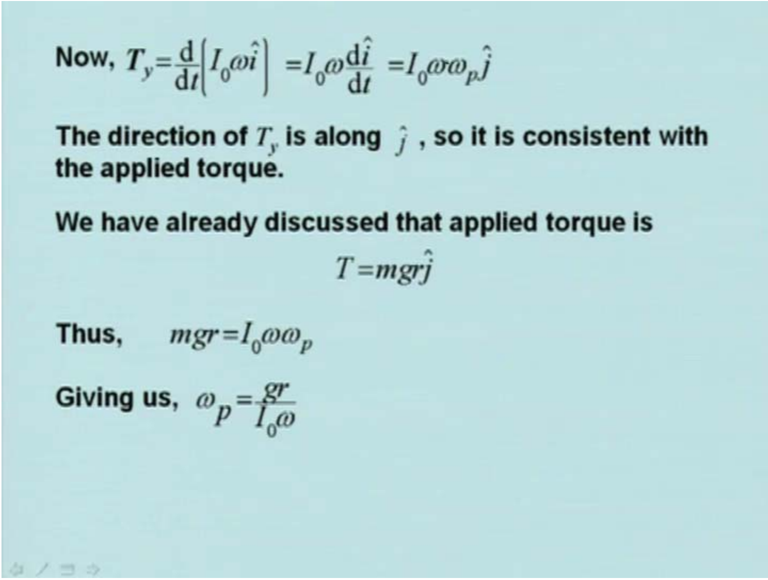

Therefore, T_y is equal to d by dt I_0 omega_i is equal to I_0 omega di by dt; that means, I_0 omega omega_p into j. The direction of T_y is along j, so it is consistent with the applied torque; that means, I_0 omega omega_P can balance T_y. We have already discussed that applied torque T is equal to mg times rj. Thus, in this case, mg into r is equal to I_0 omega into omega_P which gives us omega_P is equal to gr divided by I_0 omega. Therefore, due to its weight, spinning, that rotor will also start precession. So, instead of falling, you know this type of thing.

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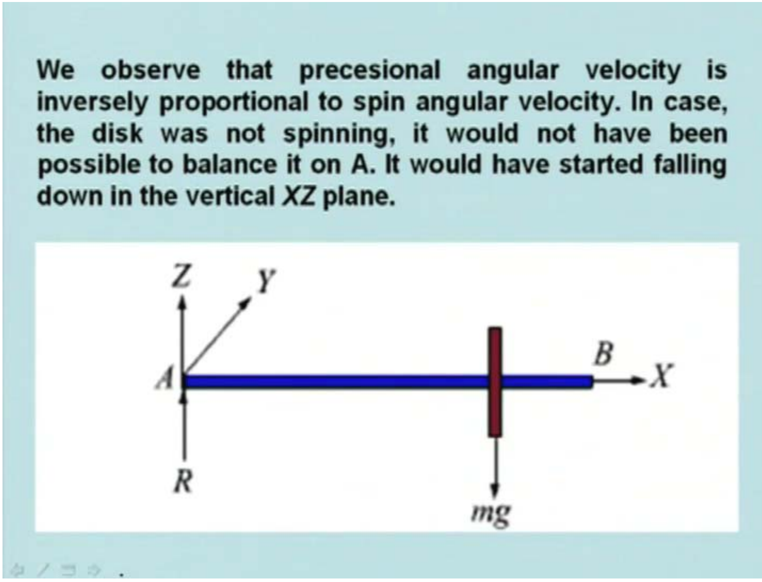

We observe that precessional angular velocity is inversely proportional to spin angular velocity. It is inversely proportional to that thing. That means if the rotor is spinning at a very high speed then it will not precess that fast. So, that means spin provides some sort of stability. You require more torque to precess it. We observe that precessional angular velocity is inversely proportional to a spin angular velocity. In case the disc was not spinning, it would not have been possible to balance it on A. It would have started falling down in the vertical XZ plane, because if you see that equation, theoretically, if omega is equal to 0 then omega_P will become infinite; it cannot be infinite. Actually what happens in that case, it will basically fall. You know that this one would have started falling down in the vertical XZ plane. That is the phenomena will happen.

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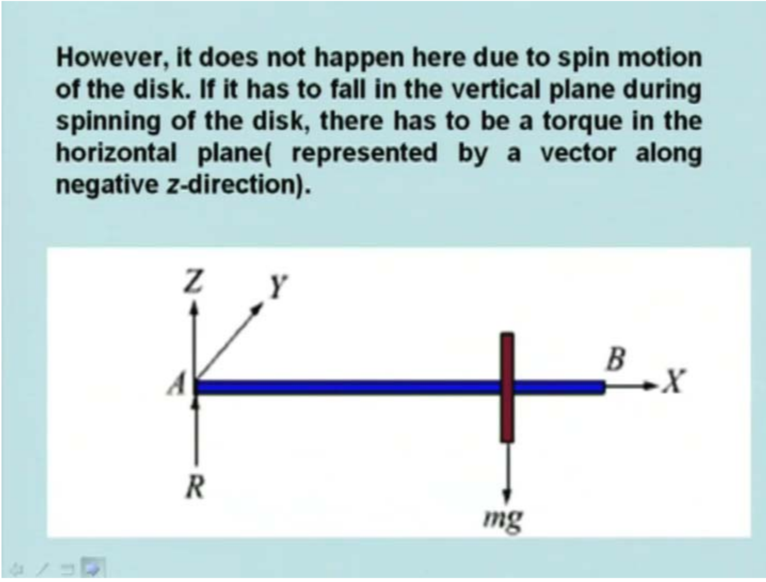

However, it does not happen here due to spin motion of the disk. If it has to fall in the vertical plane during spinning of the disk, there has to be a torque in the horizontal plane represented by a vector along negative Z direction. Suppose it is falling, you know that it will start creating. So, there has to be a torque in the horizontal plane for the spinning one. Why?

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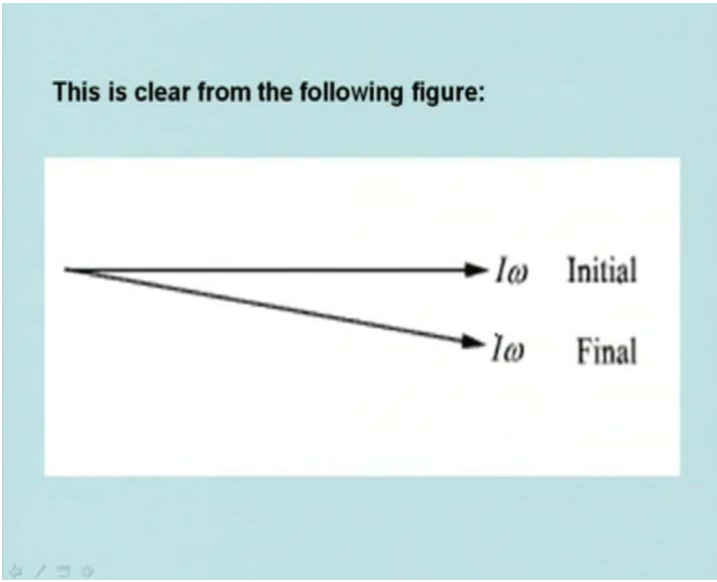

This is clear from this figure, because spinning rotor has got angular momentum. Initially its angular momentum is like I_w and after it has slightly tilted, means it started falling and the rotor has moved down. Therefore, finally it goes like this.

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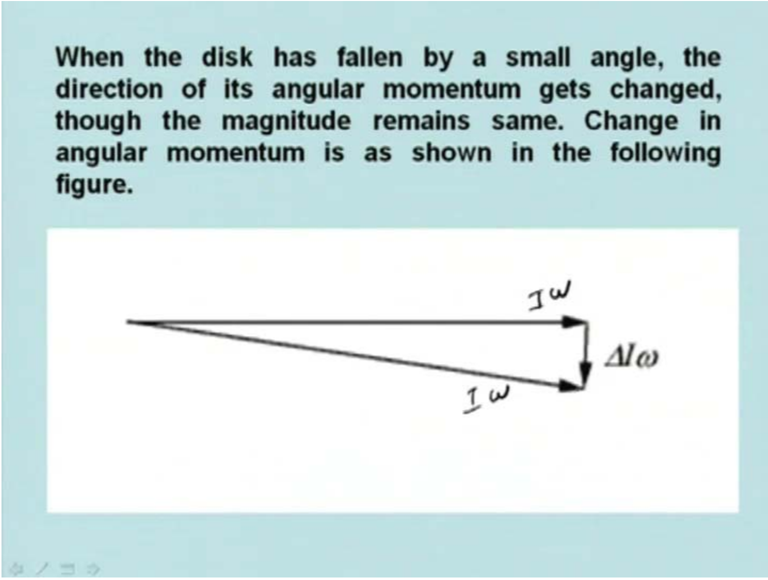

When the disk has fallen by a small angle, the direction of its angular momentum gets changed, though the magnitude remains same. Change in angular momentum is as shown in the following figure. So, this is delta Iw. So this is the thing.

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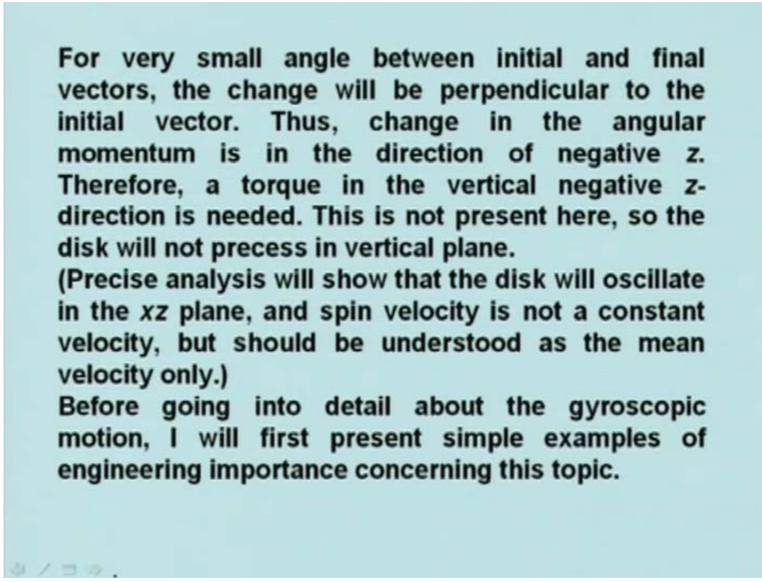

For very small angle between initial and final vectors, the change will be perpendicular to the initial vector. You have got Iw and you know it started falling and at this point this is also $I_{w.}$ This is the change and the change is almost perpendicular. Thus, change in the angular momentum is in the direction of negative Z. Therefore, a torque in the vertical negative Z direction is needed. Otherwise, this cannot take place. This is not present here. So, the disk will not precess in vertical plane. That is the way it has been shown.

This is actually based on the assumption. Of course, that spinning velocity is much more than the precession velocity. If we do the precise analysis then we will see, the disk will start oscillating in the x-z plane. Spin velocity is not a constant velocity but should be understood as the mean velocity only. So, precession velocity is not a constant velocity but should be understood as this one.

Before going into detail about the gyroscopic motion, we will first present simple examples of engineering importance concerning this topic.

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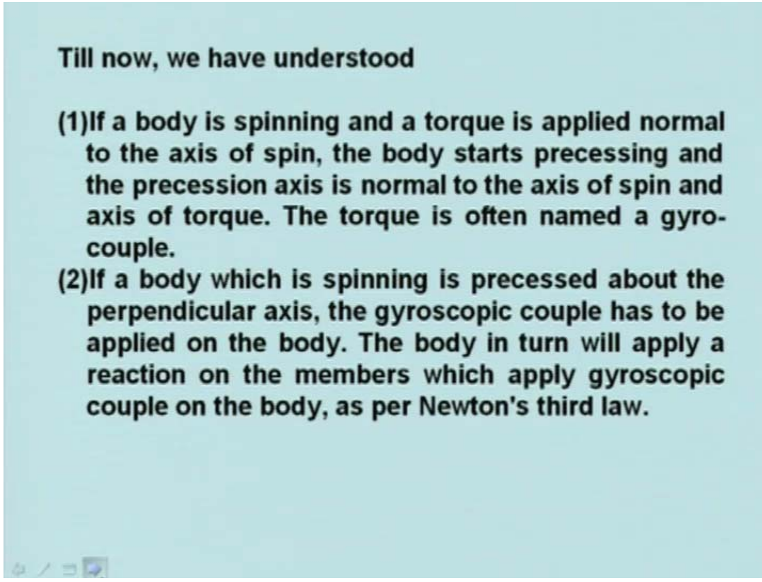

We have understood that if a body is spinning and a torque is applied normal to the axis of spin, the body starts precessing and the precession axis is normal to the axis of spin and axis of torque. This torque is often named as that gyro couple. If a body which is spinning is precessed about the perpendicular axis, the gyroscopic couple has to be applied on the body. If the body in turn will apply a reaction on the members who apply gyroscopic couple on the body, as per Newton's third law. So, these things have to be kept in mind.

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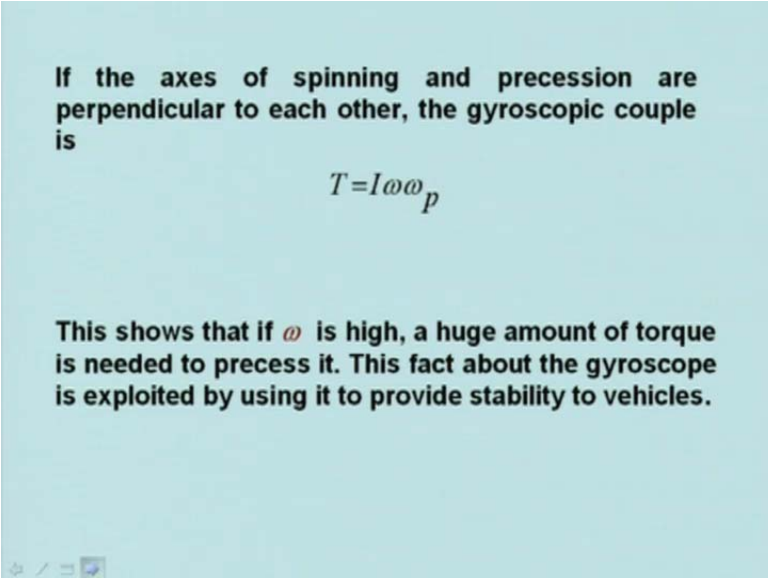

If the axis of spinning and precession are perpendicular to each other then gyroscopic couple is T is equal to I omega into omega$_{P}$, where omega is spin velocity and omega$_{P}$ is precession velocity. This shows that if omega is high, a huge amount of torque is needed to precess it. This fact about the gyroscope is exploited by using it to provide stability of vehicles.

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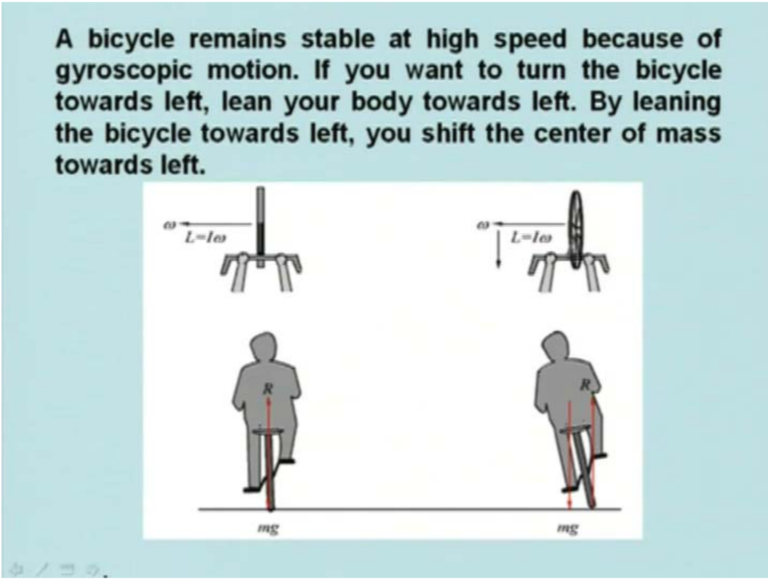

A bicycle remains stable at high speed because of gyroscopic motion. You cannot balance yourself on a stationary bicycle; that everybody you have experience, but when you start moving then you do not fall. This is gyroscopic effect and it has a role to play here. See this person sitting on the bicycle; this is the reaction of the ground and this is the weight of this one and it is spinning. So, w L is equal to I times w.

If you want to turn the bicycle towards left then you lean your body towards left. By leaning the bicycle towards left, you shift the center of mass towards left. Therefore, you generate a couple mg. So, couple is generated in anticlockwise direction. That vector, couple vector is now directed towards you. Therefore, you get L is equal to I omega. So, angular momentum also changes and therefore, you start precession that means you shift.

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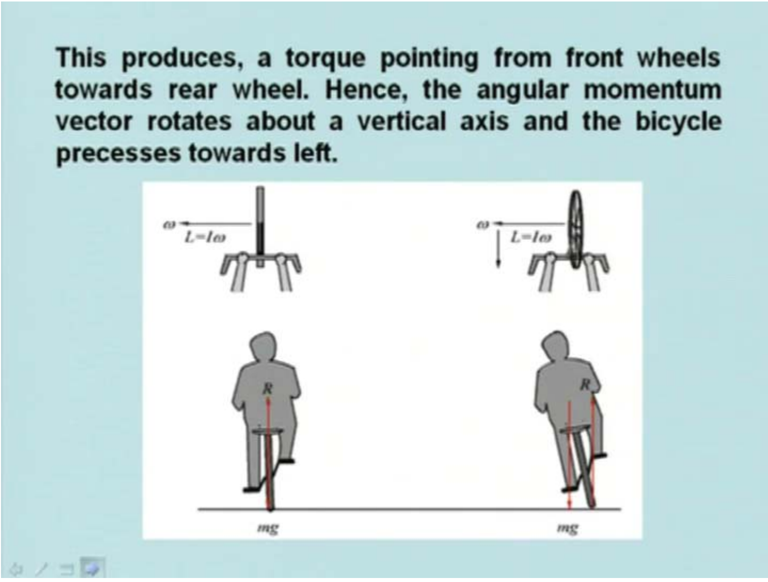

This produces, a torque pointing from front wheel towards rear wheel. Hence, the angular moment vector rotates about a vertical axis and the bicycle precesses towards left. So, this is due to gyroscopic motion.

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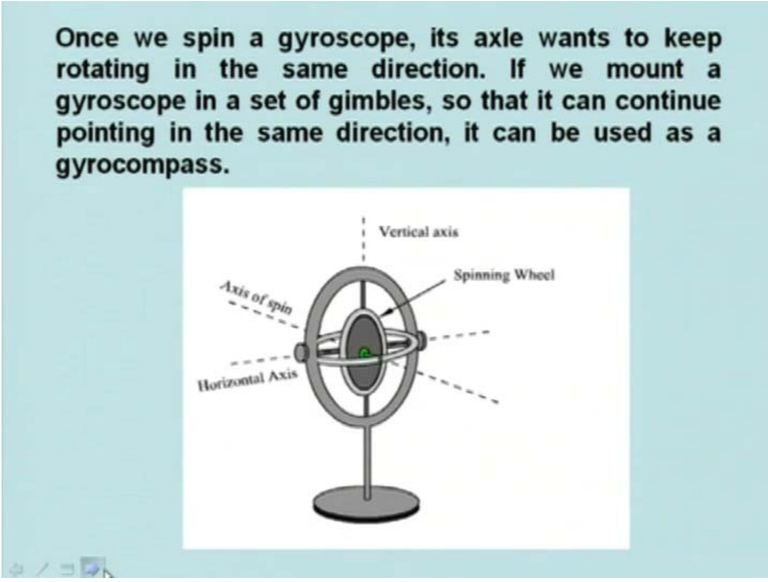

Once we spin a gyroscope, its axle wants to keep rotating in the same direction. If we mount a gyroscope in a set of gimbles so that it can continue pointing in the same direction, it can be used as a gyrocompass. You have this type of this thing and it is the vertical axis.

Suppose it can rotate about the vertical axis. On that, you have mounted another gimble which can rotate about the horizontal axis. Then you have mounted a shaft. This is the axis of spin. It can spin about that. Mass center always remains fixed, but these things can rotate about vertical axis. Then this can rotate about horizontal axis. However, once you have imparted some motion and since there is no torque to precess, it will keep rotating in the same direction. Therefore, it will act as a compass.

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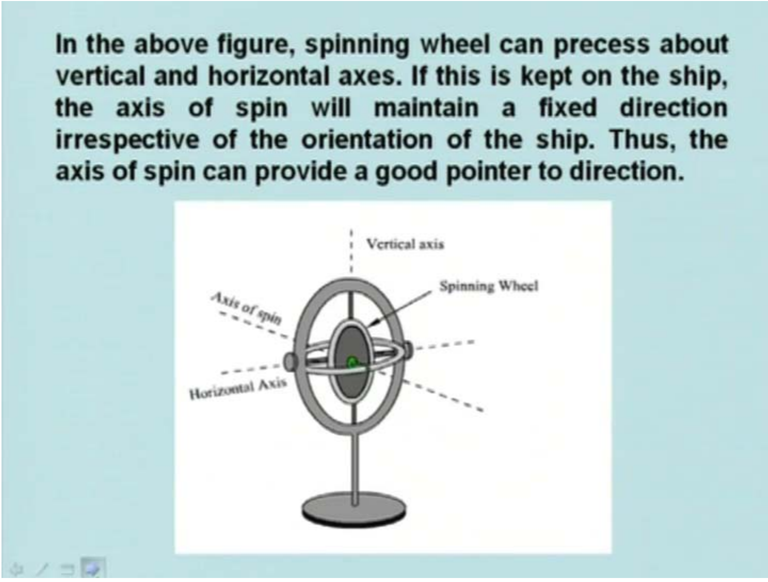

In this figure, a spinning wheel can precess about vertical and horizontal axes. If this is kept on the ship, the axis of spin will maintain a fixed direction irrespective of the orientation of the ship. Thus, the axis of spin can provide a good pointer to direction.

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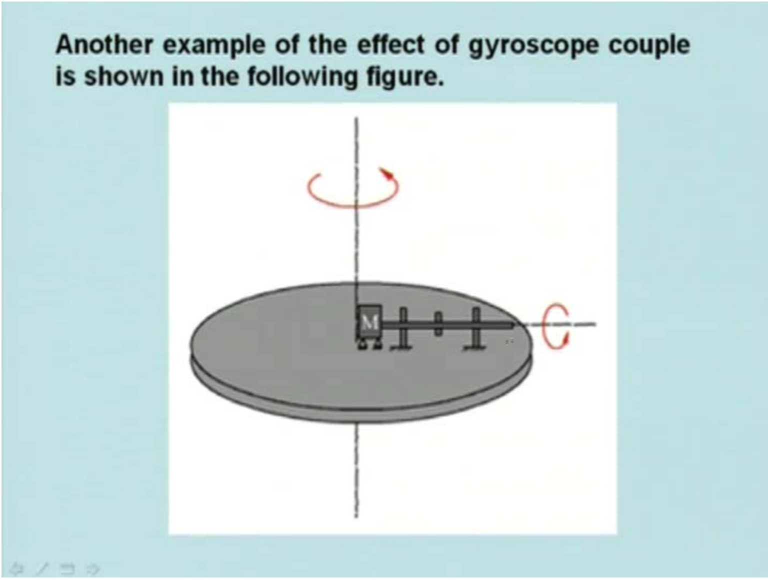

Another example of the effect of gyroscope couple is shown in the following figure. This is the figure and disc is rotating. This is the axis, this is a motor and this is rotating about that. There is a spin motion of the rotor but at the same time whole disc is rotating.

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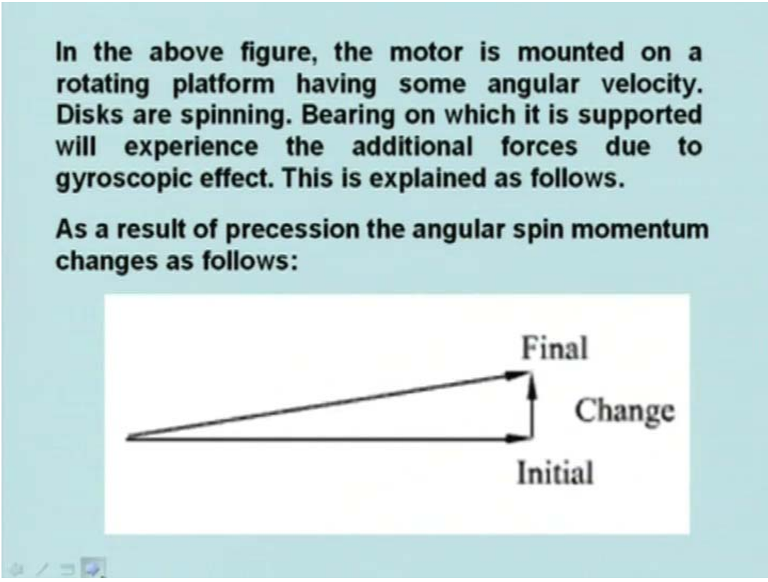

Motor is mounted on a rotating platform having some angular velocity. Disks are spinning. Bearing on which it is supported will experience the additional forces due to gyroscopic effect. This is explained as follows.

As a result of precession, the angular spin momentum changes as follows: initially, the situation is like this and you give a small change. So, final velocity is this.

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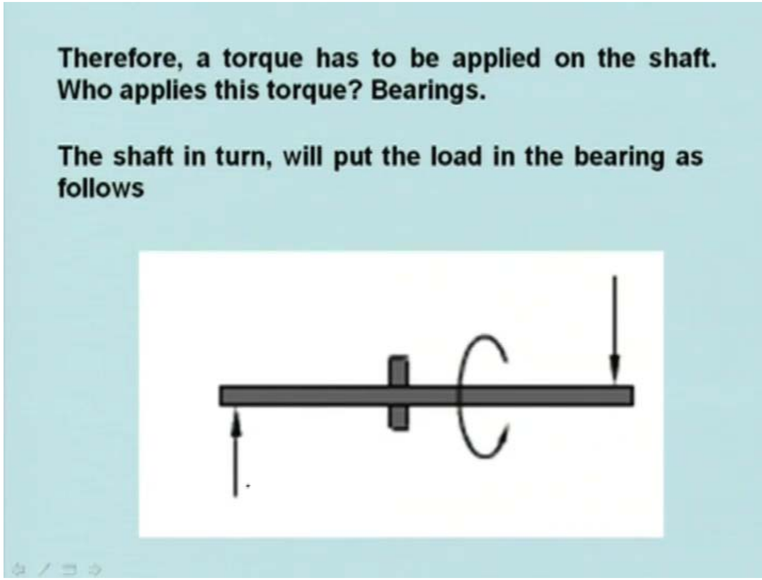

Therefore, a torque has to be applied on the shaft, because its angular momentum is changing. Initially, the angular momentum is like this. Finally, after sometime, it has become like this. So, this much is the change. Since there is a change, the torque has to be applied. Who applies this torque? Of course, the bearings. This shaft, in turn will put the load on the bearings as follows.

Therefore, the bearings will apply the torque. So, the shaft in turn will put the load. So, if we make the free body diagram of the shaft, this is the torque coming on this because this is clockwise because it is going away from the paper.

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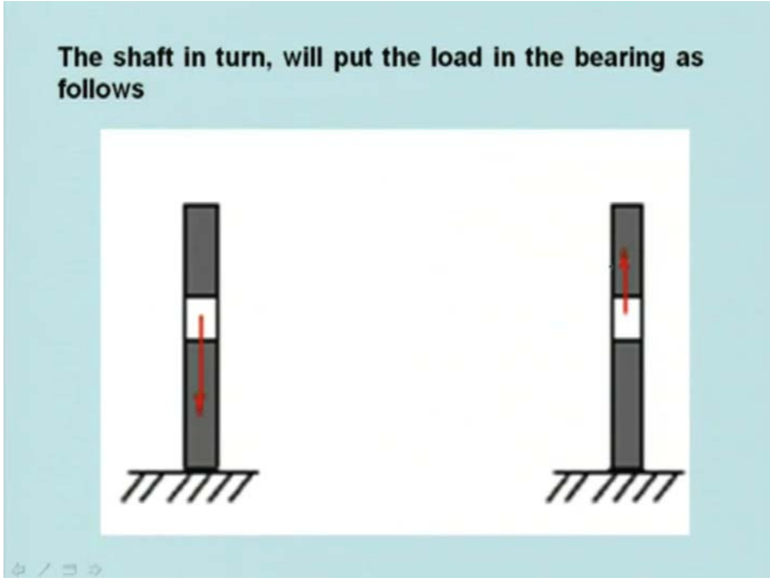

The shaft, in turn will put the load on the bearing as follows: Newton's third law, opposite way. So, this is a thing you will be getting this one. Then you are applying torque and that is what you are getting. Thus, the left bearing will be loaded downward and right bearing upward.

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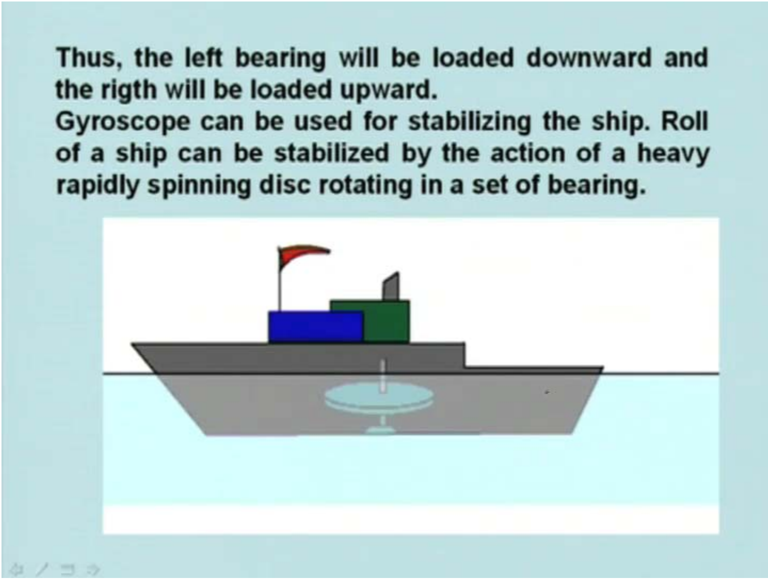

Gyroscope can be used for stabilizing the ship. Roll of a ship can be stabilized by the action of a heavy rapidly spinning disk rotating in a set of bearings, like, this is the ship and here you have put this in the bottom, that horizontal rotor which is rotating about a vertical axis. So, this is what is being done. Suppose the ship now tilts, so angular momentum will change. For that, there has to be presence of a heavy torque. You know that that torque cannot be provided. Therefore, the ship remains stable.

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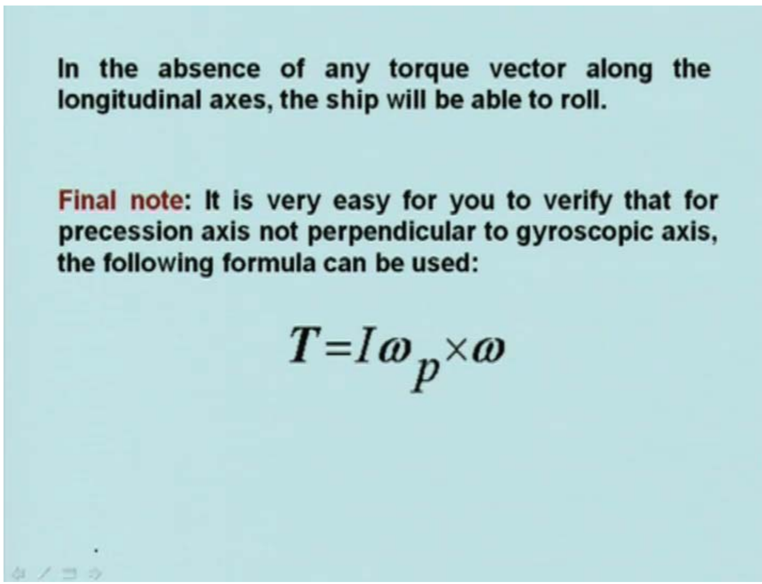

In the absence of any torque vector along the longitudinal axes, the ship will not be able to roll. Finally, about this thing, now you can easily verify. We have said T is equal to I omega$_p$ into omega. If omega, if directions of precession angular velocity and spin angular velocity are perpendicular to each other, in that case. Otherwise, the formula is T is equal to I omega$_p$ cross omega, cross product of the precession angular velocity and this one.

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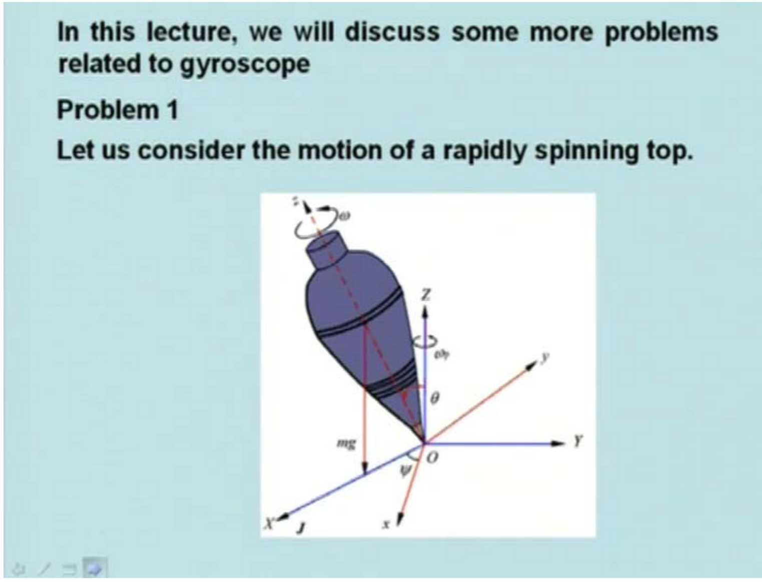

We present some more examples of gyroscopic motion. We discuss, some more interesting problems related to gyroscope. Let us consider the motion of a rapidly spinning top. Every one of you might have played in your childhood with the top. In this case, it is symmetric that atleast I_{zz} is different, but I_{xx} is equal to I_{xyy}. Its mass is, mass times gravity, mg. So, it is directed downward and this is omega$_{P.}$ Then you have got, these are the fixed axes system, X Y and Z and this is omega$_{P.}$ This is X Y and Z. At any point of time, this is small z- axis which is attached spin axis of the top, which is a making an angle theta with the Z axis, capital Z axis. At the same time, if I make here, in this plane, perpendicular to that if I have this XY, these are the body axis attached. This is making angle psi.

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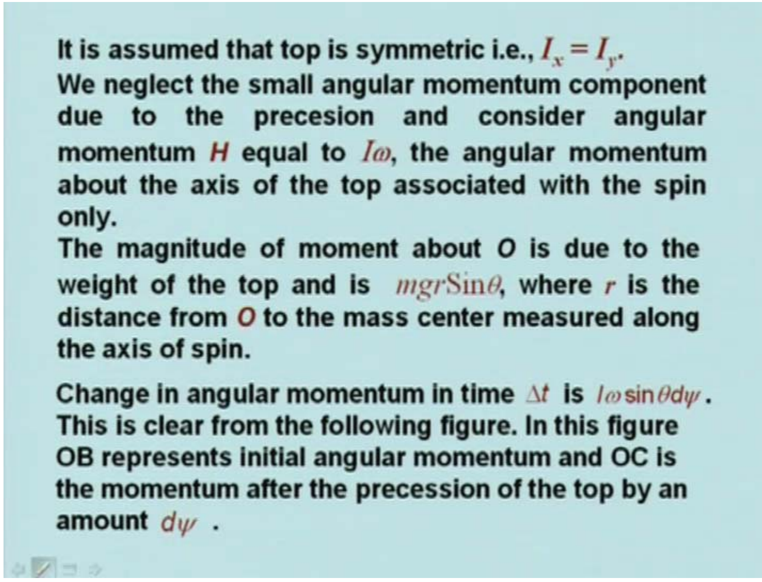

So, then in that case, if it is assumed that top is symmetric, I_{xx} is equal to I_{yy} then we neglect the small angular momentum component due to the precession and consider angular momentum H equal to I omega, the angular momentum about the axis of the top associated with the spin only. The magnitude of moment about O is due to the weight of the top and is mgr sin theta, where r is the distance from O to the mass center measured along the axis of spin. So, mgr sin theta. Change in angular momentum in time t is actually, I omega sin theta d phi. This is clear form the following figure like this.

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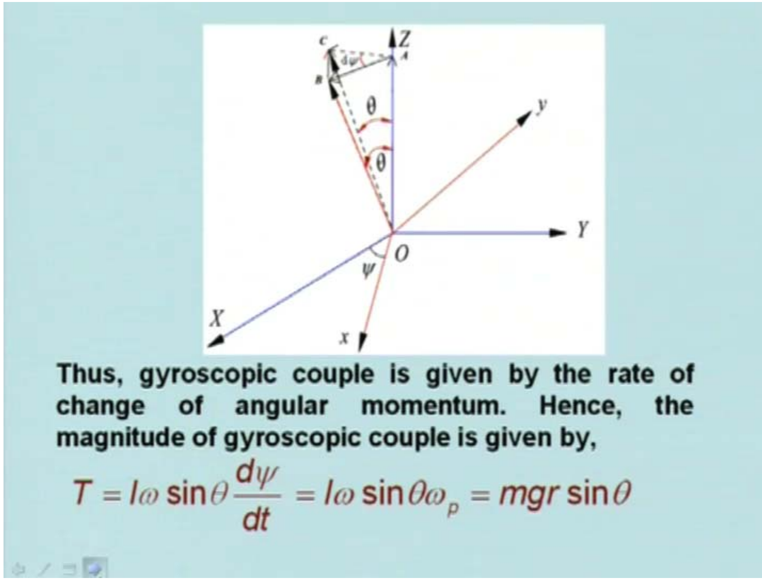

Suppose this is I omega and it has changed like this, I omega dot. Here, this is, I omega sin phi component that only changing. That vertical component along the z direction is not changing; that means, you have I omega, but it can be this. One component is along this, another component is along this. So, this is what? In the figure, OB represents the initial angular momentum and OC is the momentum after the precession of the top by an amount d psi.

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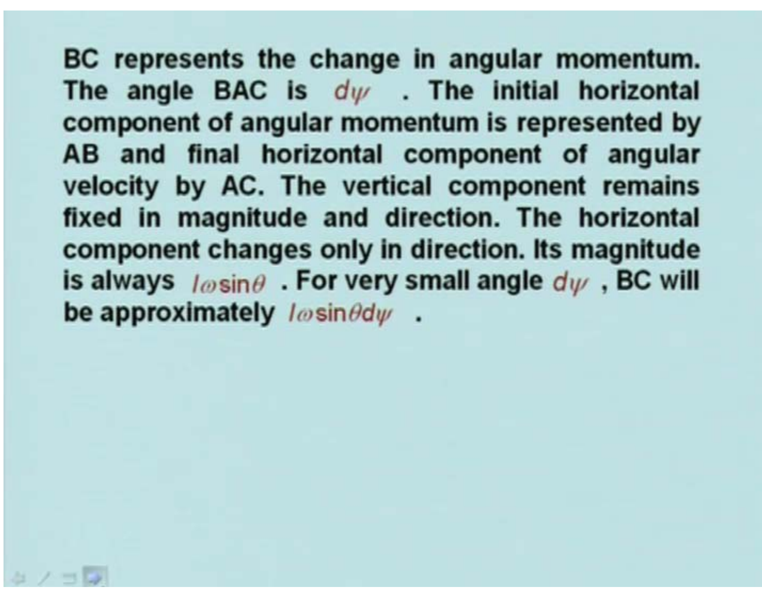

BC represents the change in angular moment. The angle BAC is d psi. The initial horizontal component of angular momentum is represented by AB and final horizontal component of angular velocity by AC. The vertical component remains fixed in magnitude and direction. The horizontal component changes only in direction. Its magnitude is always I omega sin theta.

We assume theta remains same. Axis of spin is inclined for very small angle d psi BC will be approximately, I omega sin theta d psi. Thus, the gyroscopic couple is given by the rate of change of angular momentum. Hence, the magnitude of gyroscopic couple is given by T is equal to I omega sin theta d psi by dt is equal to I omega sin theta omega$_P$ and that is equal to mgr sin theta.

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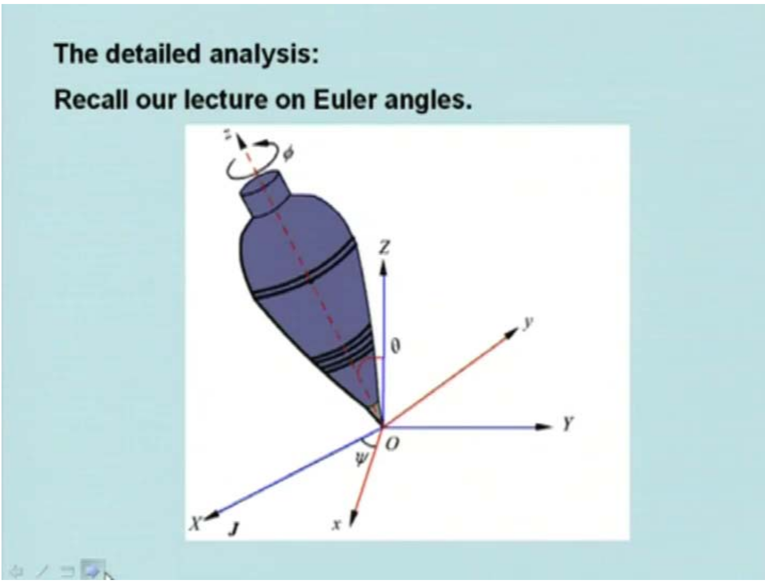

In this case, mgr becomes I omega omega$_P$. It is not dependent on theta, or you get omega$_P$ is equal to mgr divided by I omega. This expression shows that as omega reduces, the spin velocity reduces and omega$_P$ increases. That is why it is observed that at the end of its motion the top runs much faster.

The general motion also involves. If you do precise analysis, you will find out that it also involves wobbling about the mean precession. This wobbling is called nutation. In this, the angle theta does not remain fixed, but keeps on oscillating. Now, we can do somewhat detailed analysis. Also, we remember about Euler's equation and we have given that lecture on Euler angle.

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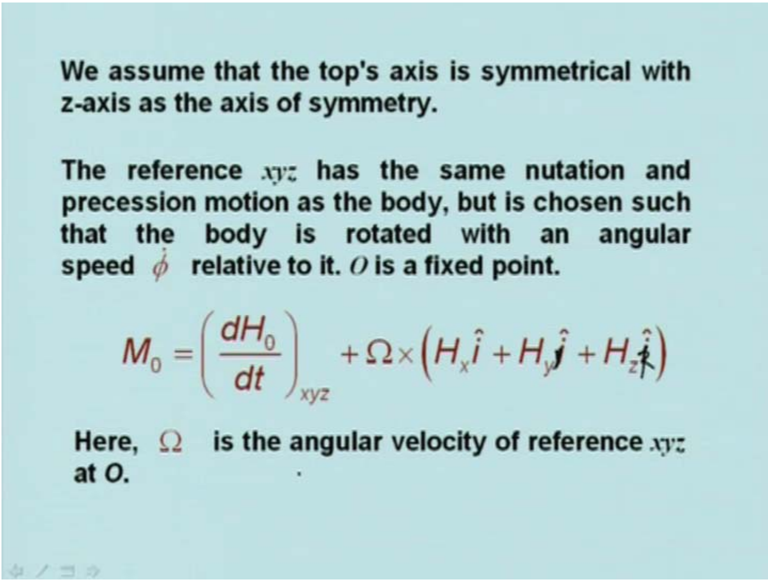

In that we assume that top's axis is symmetric with z-axis as the axis of symmetry. The reference xyz has the same nutation and the precession motion as the body, but is chosen such that the body is rotated with an angular speed phi dot relative to it. So, that means xyz are not exactly fixed on the body. There is some difference O. That is, phi dot O is a fixed point then M_0 is equal to dH_0 by dt xyz plus omega cross H_xi plus H_yj plus H_zk. Here, omega is the angular velocity of reference xyz at O.

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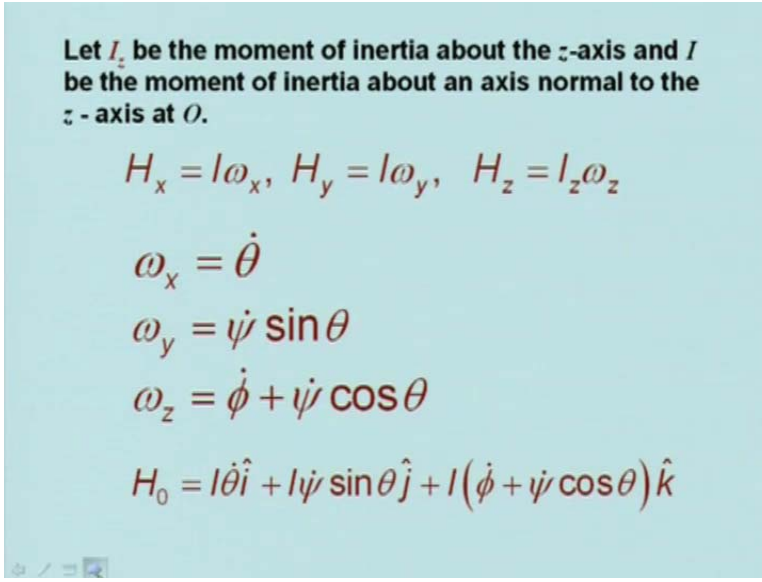

Let I_z be the moment of inertia about the z axis and I be the moment of inertia about an axis normal to z-axis at O. Then you will be getting H_x is equal to I omega $_x$ H_y is equal to I $omega_y$ H_z. You will be getting I_z $omega_z$ and $omega_x$ will come out to be theta dot, $omega_y$ will come out to be psi dot sin theta, and $omega_z$ will be phi dot plus psi dot cos theta. Therefore, H_0 will be I theta dot i plus I psi dot sin theta j plus I phi dot plus psi dot cos theta k.

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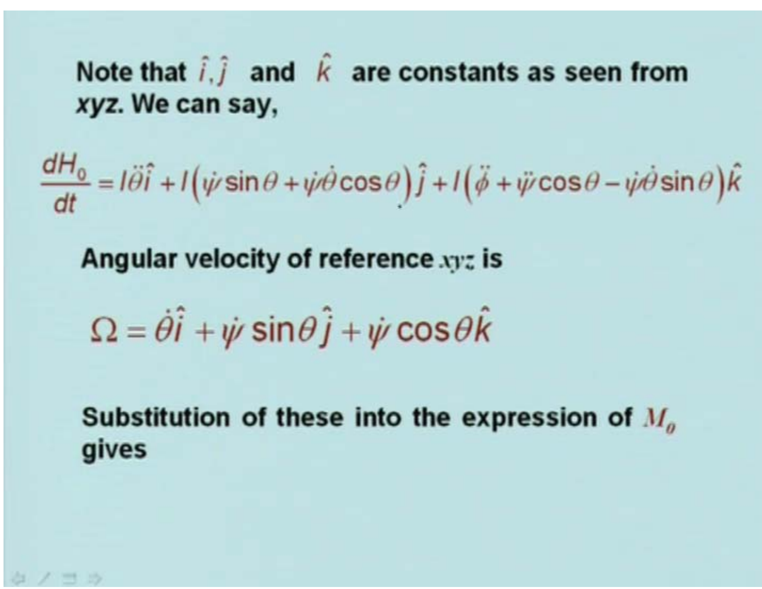

We note that I, j and K are constants as seen from xyz. Then we can say that dH_0 by dt is equal to I theta dot dot i plus I psi dot sin theta plus psi dot theta dot cos theta j plus I phi dot dot plus psi dot dot cos theta minus psi dot theta dot sin theta k. Angular velocity of reference xyz is omega is equal to theta dot I plus psi dot sin theta j plus psi dot cos theta k.

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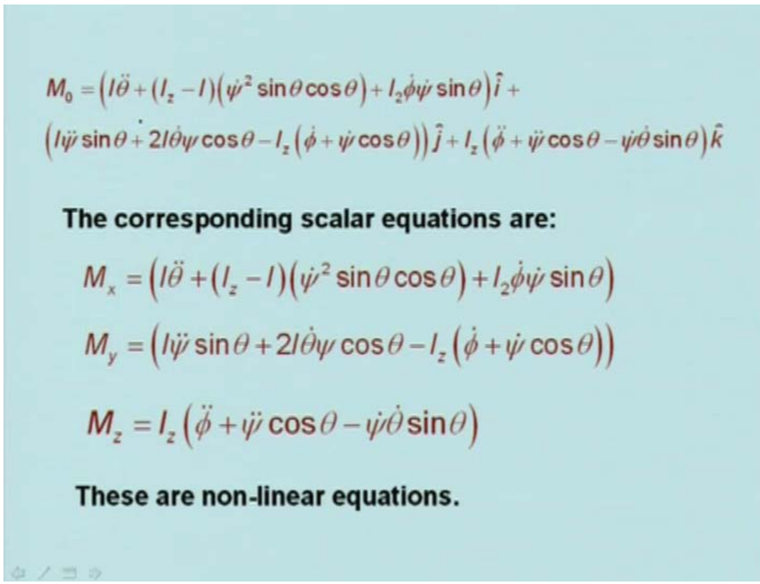

Now, substitution of these into the expression for moment M_0 gives, M_0 is equal to I theta dot dot plus I_z minus I psi dot square sin theta cos theta plus I_z phi dot psi dot sin theta I plus I psi dot dot sin theta plus 2 I theta dot psi cos theta minus I_z phi dot plus psi dot cos theta j plus I_z phi dot dot psi dot dot cos theta minus psi dot theta dot sin theta k.

The corresponding scalar equations are: M_x is equal I theta dot dot plus I_z minus I psi dot square sin theta cos theta plus I_z phi dot psi dot sin theta and M_y will be I psi dot dot sin theta plus 2I theta dot psi cos theta minus I_z phi dot plus psi dot cos theta and M_z will be I_z phi dot dot plus psi dot dot cos theta minus psi dot theta dot sin theta. These are the there equations and these are non- linear equations. So, you see that solving them is a bit complicated term.

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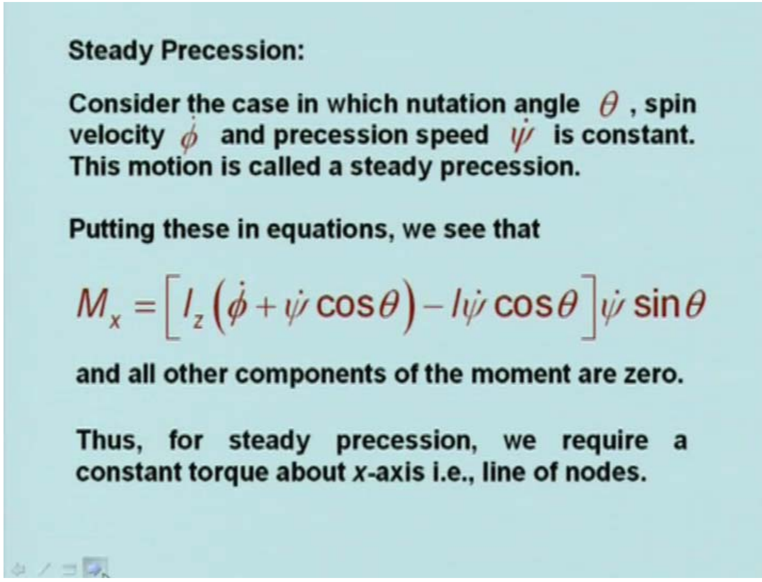

Let us discus about the steady precession. Now, consider the case in which nutation angle theta, spin velocity phi dot and precession speed psi dot is constant. Precise analysis will show something different and all other components of the moment are 0. Thus, for steady precession we require a constant torque about x axis that it will lie in.

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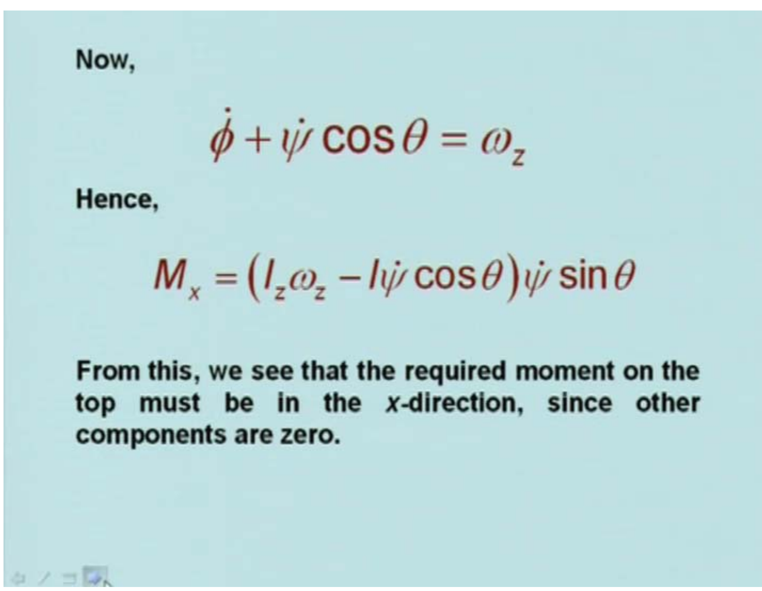

Now, psi dot phi dot plus psi dot cos theta is equal to omega$_z$. Hence, M_x is equal to bracket I_z omega$_z$ minus I psi dot cos theta psi dot sin theta. From this, we see that the required moment on the top must be in the x-direction, since other components are 0.

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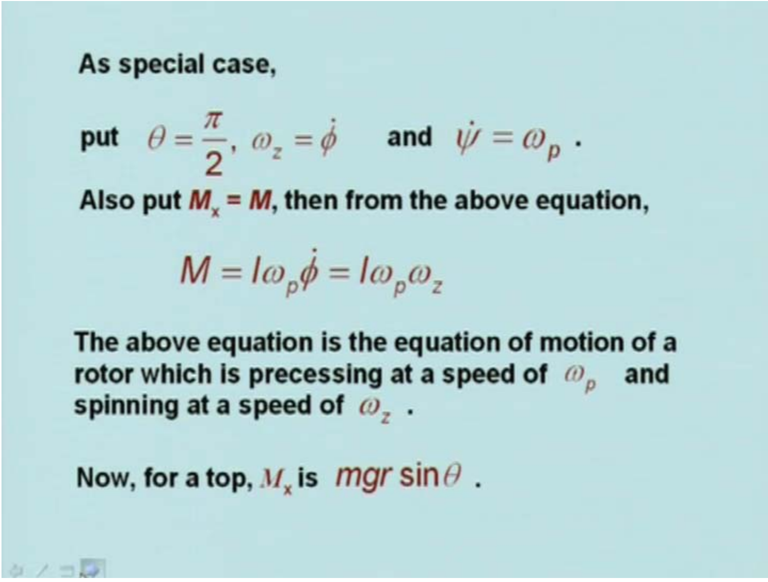

As a special case, if we put theta is equal to pi by 2 and omega$_z$ is equal to phi dot and psi dot is equal to omega$_P$ and we put M_x is equal to M. Then from the above equation, we get M is equal to I omega$_P$ phi dot that is I omega $_P$ into omega$_z$. The above equation is the equation of motion of a rotor which is precessing at a speed of omega$_P$ and spinning at a speed of omega$_z$.

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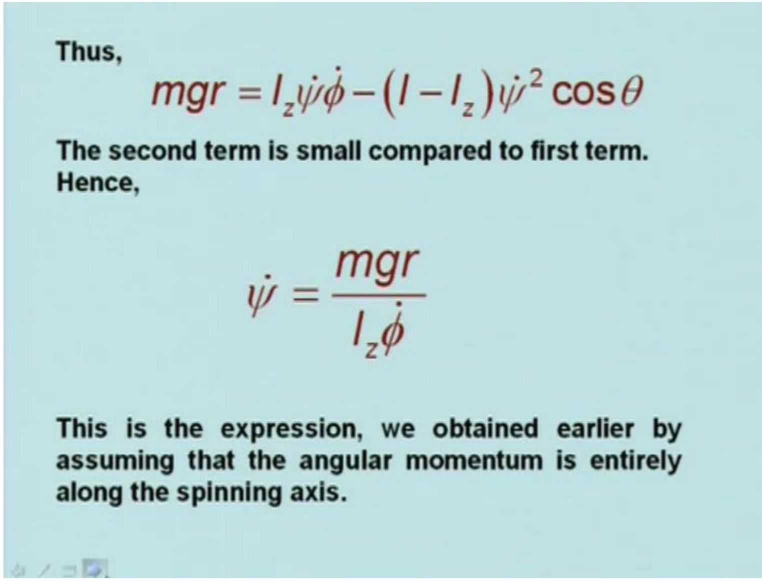

For a top, M_x is mgr sin theta. Therefore, we get mgr I_z psi dot phi dot minus I minus I_z psi dot square cos theta. Second term is small compared to first term. Hence, we get psi dot is equal to mgr divided by I_z phi dot. This is the expression we obtained earlier, by assuming that the angular momentum is entirely along the spinning action. So, we had got that expression based on the assumption about this thing.

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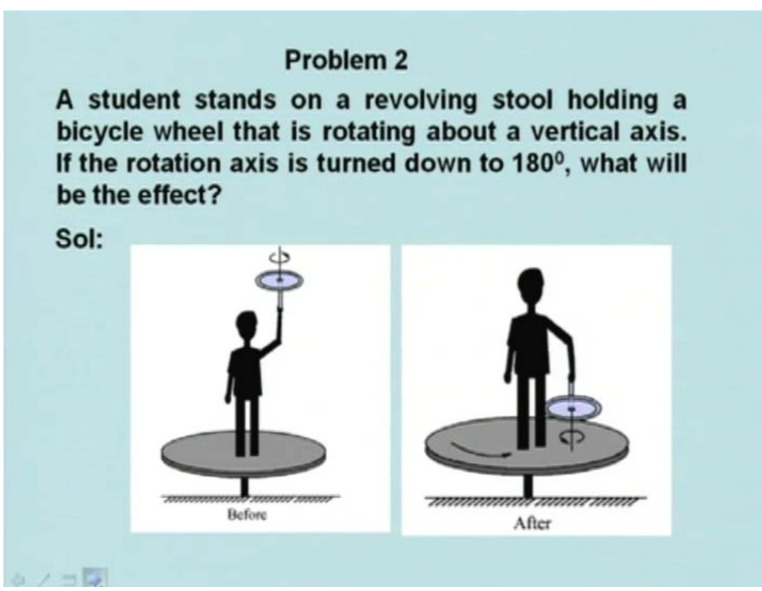

Therefore, we are getting almost the same equation. I will end this lecture by giving a simple, another example. A student stands on a revolving stool holding a bicycle wheel that is rotating about a vertical axis. If the rotation axis is turned out to 180 degree, what will be the effect?

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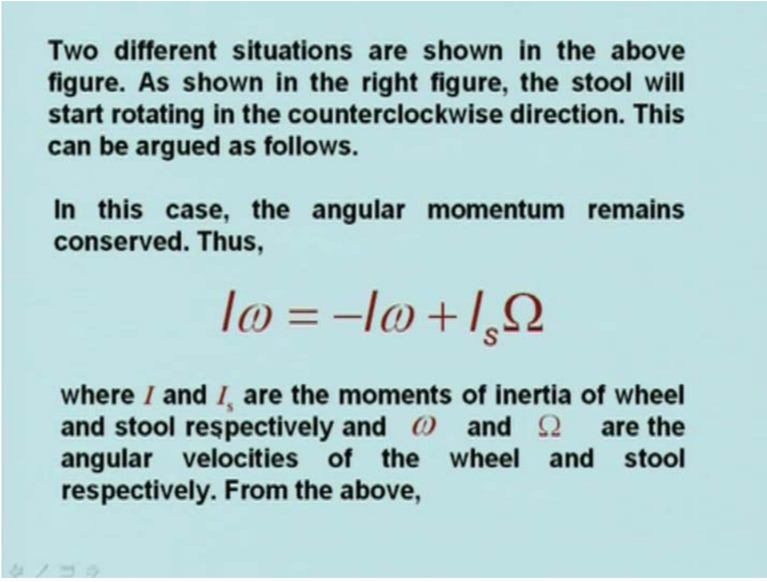

We know that it is before, and after that it is like this. So, in this, two different situations are shown. In this case, the angular momentum remains conserved. Thus, I omega is equal to minus I omega plus I_s big omega where, I and I_s are the moments of inertia of wheel and stool respectively. Omega and big omega are the angular velocity of the wheel and stool respectively. From this, you can find out that you know that since it has changed, you can find out omega. This is how it can be done.