Engineering Mechanics

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Module – 13
Kinematics in 3D
Lecture - 34
Kinematics in 3D

We will be discussing three dimensional motions. We will discuss three dimensional kinematics, and then three dimensional kinetics. We have studied quite a lot of dynamics; we have studied Newton's law, Euler's law. There is no reason to be afraid of the three dimensional things, but generally, students do not find comfortable in doing the problems related to three dimensional kinematics or kinetics. The reason is that in three dimensions, the first difficulty is that if you want to solve the problem with a piece of pen and paper, then three dimensional objects have to be suitably represented there. So that is the difficulty of visualizing. Sometimes, you will see some figures in three dimensions and you will not be able to visualize properly what these things are? The second is, as the number of dimensions increase then the required number of equations to be solved also increase and also the corresponding terms in the equation. Sometimes they increase, not in the proportional way, but by the order of n square or more. So, there will be computational difficulties. Keeping those aside, the basic principle remains same. There is no reason, why we should not be able to understand the three dimensional dynamics. Only thing is that we may not be able to solve very lengthy problems here itself, but we will get this thing. Nowadays, computational tools have become quite advanced. Those problems can be solved by computers also.

The objective of these lectures is to understand the basic concepts, how to develop the equations and understand some very interesting phenomena. In two dimensional, suppose a disk is rotating with a constant angular velocity then there is no torque acting on this, because there is no angular acceleration; but in three dimensional, even if there is constant angular velocity, there can be some torque. These types of things we will study here.

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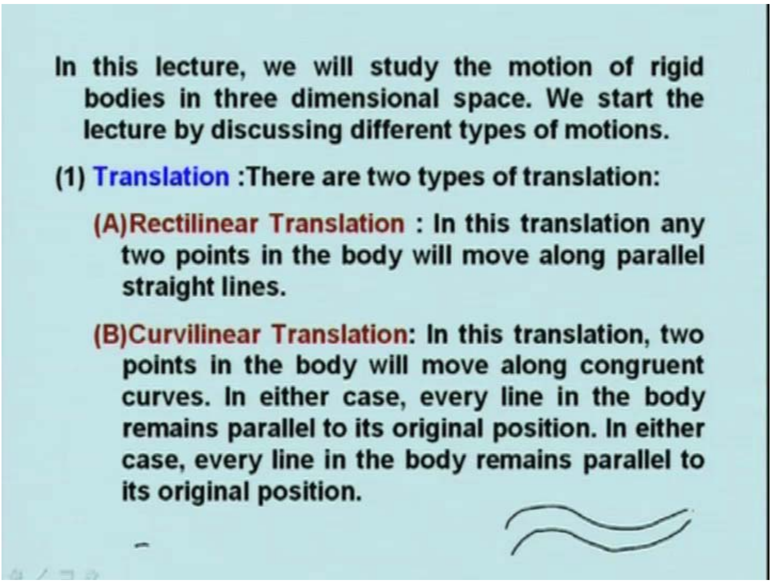

If I start from 3D kinematics, in this we first study the motion of rigid bodies in three dimensional space. We start the lecture, by discussing different types of motions, which we have already discussed in the previous one. Mostly, we gave examples from 2D space. Now we will give examples from 3D.

There are basically, different types of motions; one is translation. In translation, there are two types of translations. One is the rectilinear translation in which the translation between any two points in the body will move along parallel straight lines. Suppose there is a line in the body, that line remains parallel to itself. Naturally, in this translation angular velocity is zero. But in a rectilinear translation, the two points move along parallel straight lines; that is the point. They have to move in a straight line. Then it is rectilinear translation.

Then the other type mode of motion is curvilinear translation. In curvilinear translation, two points in the body will move along congruent curves. There will be two types of curves; this is one type of curve and then another type is this one. Suppose this is a point, it is moving and similarly a parallel type of curve is there which also is moving. In either case, every line in the body remains parallel to its original position. In either case, that means, in both in rectilinear and curvilinear translation every line in the body remains parallel to its original position.

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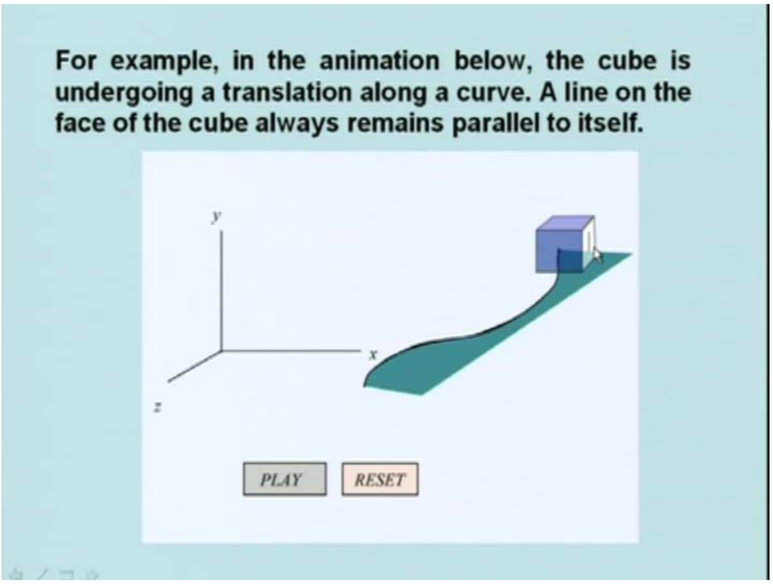

In this animation, I am going to show you one animation. The cube will undergo translation along a curve. This is not a planner curve, this maybe space curve; that means one plane is not there which can encompass this curve. A line on the face of the cube will always remain parallel to itself. I am playing this animation. Just see this line. Although this cube is moving in a curved path, this line always remains parallel. So, this is an example of a translation. Sometimes if you sit on a merry go round, in that you actually move in a circular path, but your body remains basically in the same position. That means a line in the body will remain parallel to itself. Therefore, what happens in translation is that a line on the face of the cube always remains parallel to itself . That is what this animation shows.

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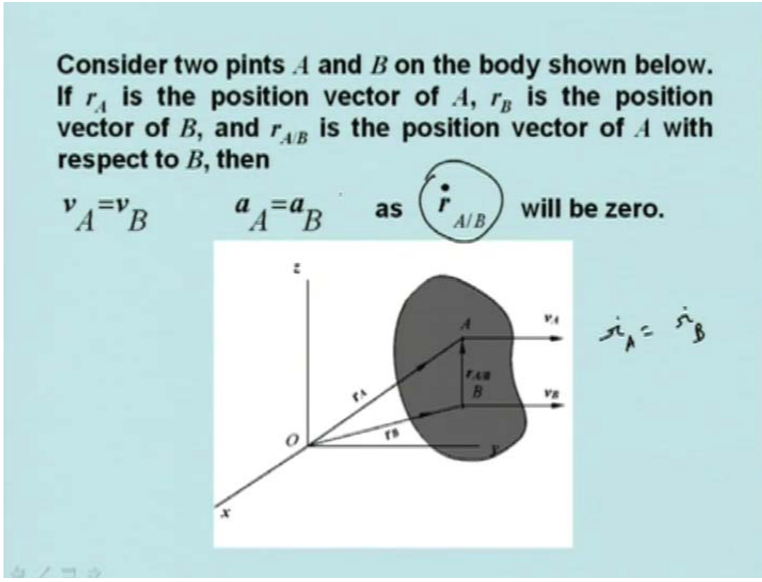

If you see this figure, consider two points A and B on the body; that body has been shown. If r_A is the position vector of A, this point A has been shown here; r_B is the position vector of B and $r_{A/B}$ is the position vector of A with respect to B, then v_A is equal to v_B. Because r_A is equal to r_B plus $r_{A \text{ by } B}$, sum of these vectors and r_{AB} is always constant, because the r_{AB} is not rotating. If you take the derivative, you can always say that r dot A will be equal to r dot B; that means, v_A is equal to v_B. Any two points on the body will have same velocity, because r dot AB will be 0. Similarly, if you take the derivative of this, we can say a_A is equal to a_B. Therefore, acceleration of two particles will be same that is the speciality of translation.

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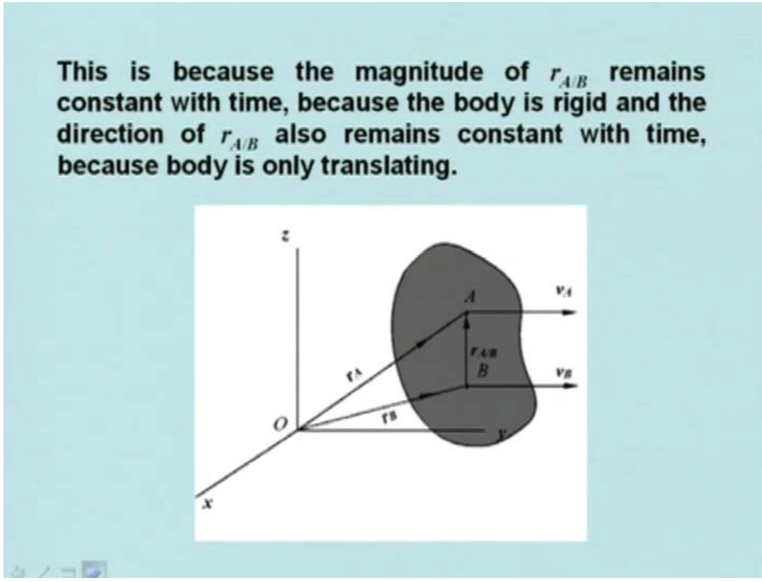

This is because the magnitude of $r_{A/B}$ remains constant with time, because the body is rigid. In case if a body would have been flexible then this relation was not correct because there may be r dot AB. In the flexible body even in translation, different points may have different velocity, but we are studying the rigid body mechanics. In rigid body, two points are having same velocity, same acceleration and therefore that is the speciality.

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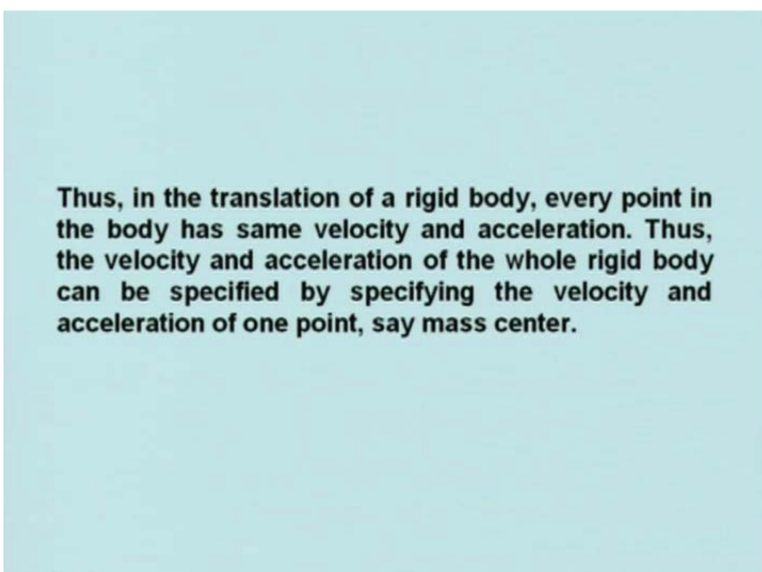

Thus, in the translation of a rigid body every point in the body has same velocity and acceleration. Thus, the velocity and acceleration of the whole rigid body can be specified by specifying the velocity and acceleration of one point. So, usually we specify the velocity of the mass center. That means, if we say that mass center is moving with velocity then the whole body is moving. All points in the body are moving with the same velocity and acceleration also, that is the mass center.

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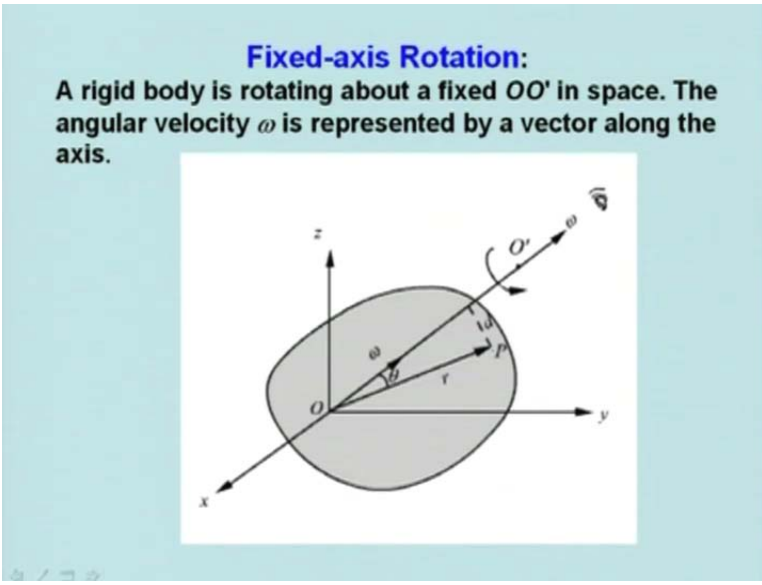

Let us discuss another type of motion. This was the translation and the other type is fixed axis rotation. A rigid body is rotating about a fixed axis OO dash in space. Suppose this OO dash is a fixed axis that is rotating. The angular velocity omega is represented by a vector along the axis. It is rotating about that axis. Our convention is that about the axis about which the body is rotating, that axis will represent the vector of the angular velocity. Angular velocity vector is having the direction of that vector and magnitude is the magnitude of omega. From here, you are observing that this is a body and x y z system has been shown. This is theta and this is any arbitrary point P which is at distance r from this one, and this distance is d and after that we go to the next.

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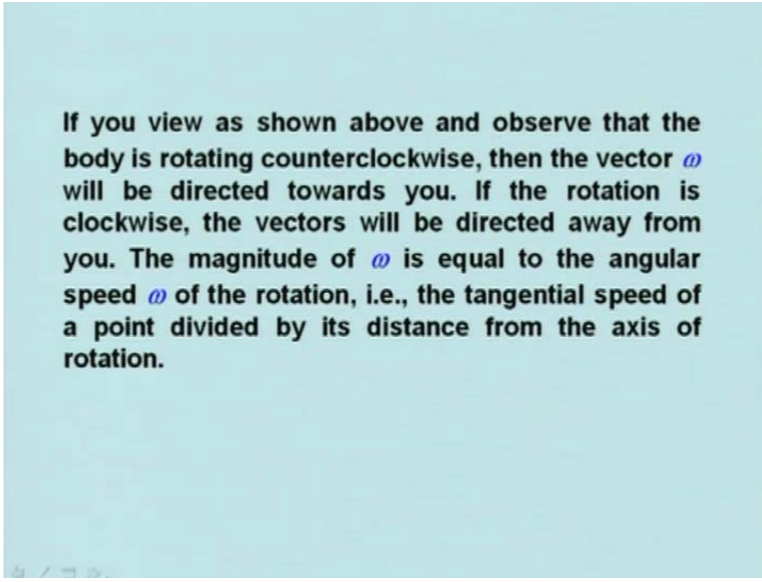

If you view this one like this from this one and you see by viewing from this side from top, you see that it is rotating counterclockwise. If it is rotating counterclockwise then your angular velocity vector will be directed towards you. If you observe that some body is rotating and from the top you see, its direction of rotation is counterclockwise then the angular velocity vector will be towards you. If the body was rotating clockwise then the angular velocity vector will be away from you that is in the opposite direction.

If you view as shown above and observe that the body is rotating counterclockwise then the vector omega will be directed towards you. If the rotation is clockwise, the vectors will be directed away from you. The magnitude of omega is equal to the angular speed omega of the rotation; that is, the tangential speed of a point divided by its distance from the axis of rotation. Magnitude will be like this. If you find out that point P is there, point P has got some tangential linear velocity. You divide it by this small d and that becomes the angular velocity that is, the magnitude.

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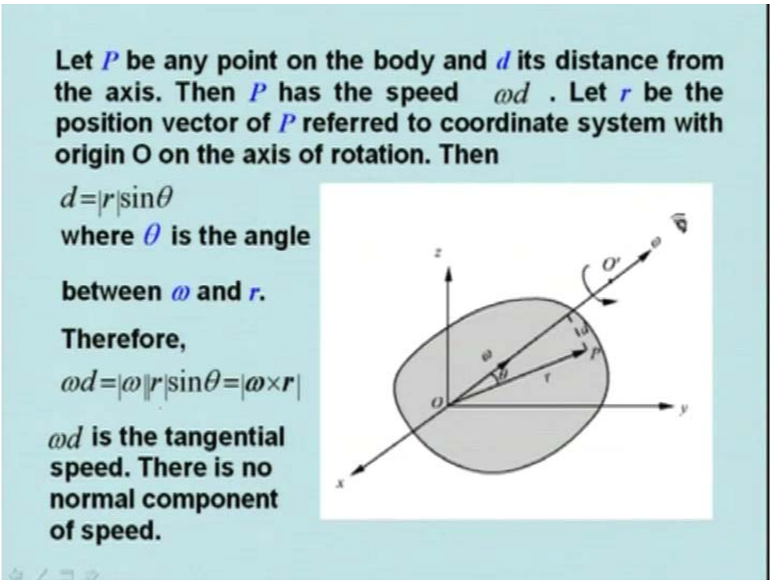

Let us go to this point. Let P be any point on the body. Now, concentrate on this figure and d is the distance from the axis. Then P has the speed omega times d. Let r be the position vector of P referred to coordinate system which you have chosen with origin O on the axis of rotation. Then, if this is theta then d is equal to magnitude of r times sin theta, where theta is the angle between omega and r; omega is the angular velocity vector and r is the position vector. In this case, omega dot d will be omega times r sin theta, which is equal to omega cross r. This will be omega cross r, because omega d can be written as magnitude of omega r sin theta. It is same as the magnitude of omega cross r, because cross product of two vectors, omega and r is omega r sin theta.

Omega d is the tangential speed. There is no normal component of speed, because it is rotating. This point is moving in a circular fashion.

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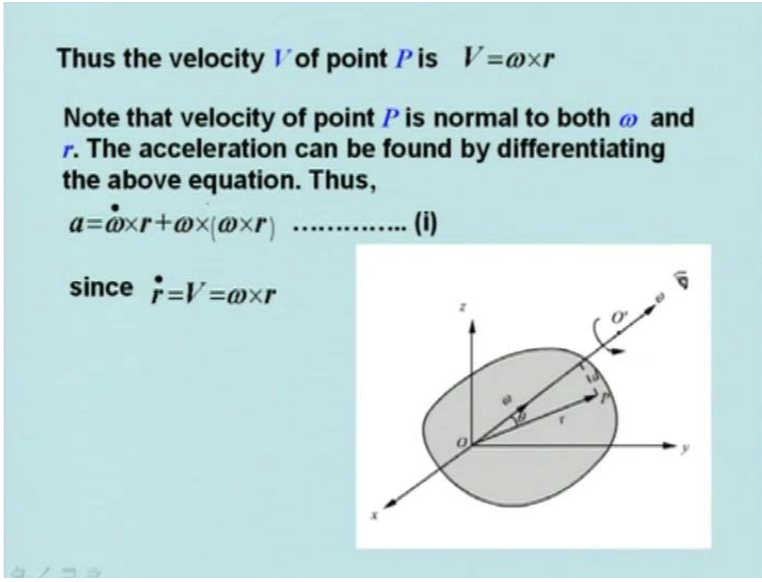

The velocity of point P is basically V is equal to omega cross r. So, you have got a formula that velocity is equal to omega cross r. Note that, the velocity of point P is normal to both omega and r; that means, the tangential velocity will be normal to omega as well as position vector. The acceleration can be found by differentiating this above equation; that means, differentiate V is equal to omega cross r and you can get the acceleration. a is equal to omega dot cross r plus omega if you differentiate r, r is rotating. So, this is r dot. r dot will be again omega cross r, since r dot is equal to V that is omega cross r. We have seen that r dot equal omega cross r that is V. Therefore, acceleration comes out to be omega dot r plus omega cross omega r.

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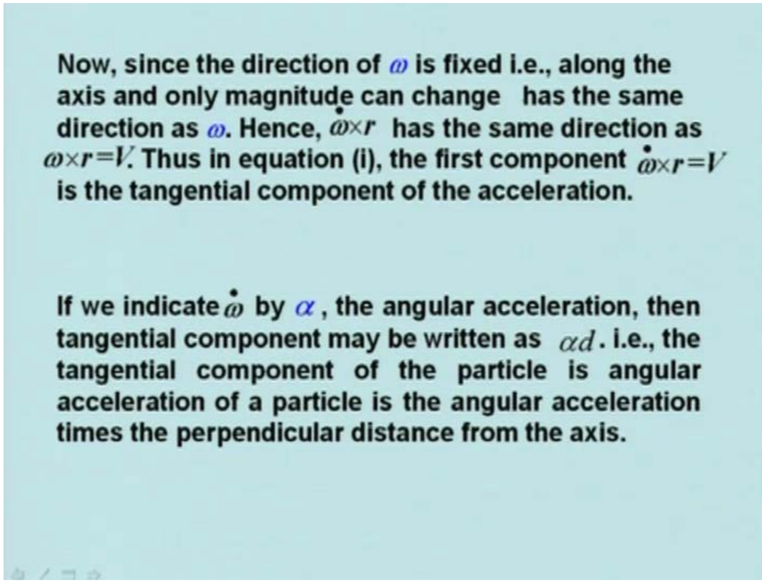

Since the direction of omega is fixed, as we are discussing the case in which the axis of rotation is fixed, that is along the axis and only magnitude can change, it has the same direction as omega. What happens is omega dot has same magnitude as omega. So, omega dot is also along that one and therefore omega dot r and has the same direction as omega cross r and is equal to V. Thus, in the equation, the first component that is omega dot r which can be written as alpha times r is equal to V is the tangential component of the acceleration. That is the tangential component of the acceleration and if we indicate omega dot by alpha, the angular the angular acceleration, then tangential component maybe written as alpha times d.

So, omega dot is equal to alpha and alpha cross r. Its magnitude will be alpha r sin theta that means alpha times d; that is, the tangential component of the particle is the angular acceleration of the particle times the perpendicular distance from the axis.

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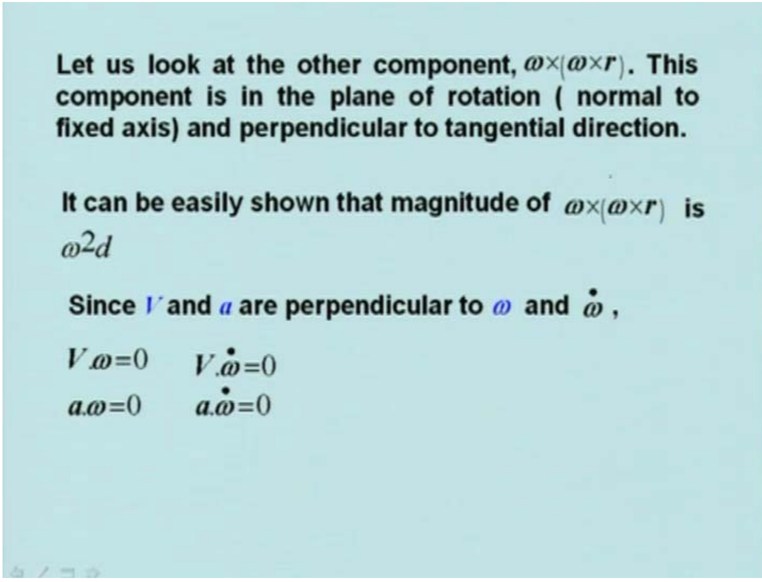

Let us look at the other component that is omega cross omega cross r. This component is in the plane of rotation that is normal to fixed axis and perpendicular to tangential direction. Therefore, it is basically in the plane but in the normal direction. It can be easily seen that the magnitude of omega cross omega cross r is omega square d, because magnitude of omega is omega times d and omega is there, so omega square d. This is that component, centripetal component of this one. Since V and a, are perpendicular to omega and omega dot, V dot omega is equal to zero. Why? Because V, tangential velocity is always perpendicular to the direction of omega; so V dot omega is equal to 0.

Similarly, V dot omega dot is also 0; dot product of velocity and angular acceleration, and tangential velocity and angular acceleration is 0. Similarly, the acceleration into omega is also equal to 0 and then, acceleration into omega dot is also 0. These are the equations. So, these are important things.

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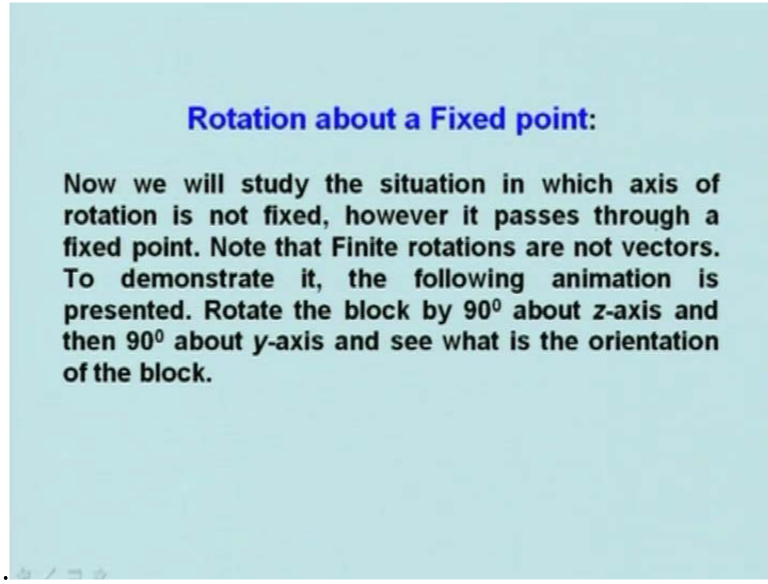

We have discussed the rotation about fixed axis; that means axis of rotation is fixed. Now, we can discuss the case in which the rotation is about a fixed point. Point may be fixed but axis may not be fixed; axis maybe changing, like in the rotation of a top, axis is there so that axis keeps changing. Now, we will study the situation in which axis of rotation is not fixed. However, it passes through a fixed point. The interesting point, I will bring here is that finite rotations are not vectors. Suppose you know that there are rotations, axis rotating. We are going to discuss the rotations. So, the point I would like to emphasize is that, like displacements can be added like a vector, can we add finite rotations in three dimensions like a vector? No. To demonstrate it, we have shown some animation.

Here, I will show animation in which you rotate the block by 90 degree about z-axis first and then 90 degree about y-axis and see what the orientation of the block is.

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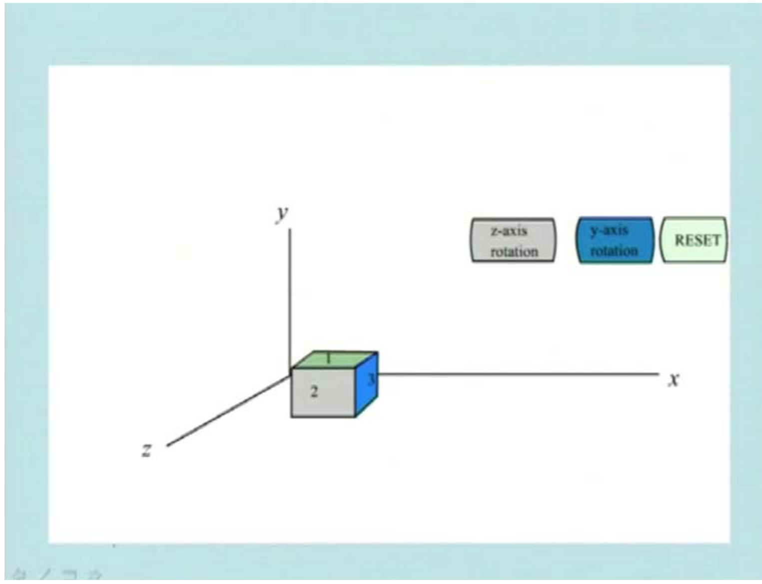

This is a block. In this cube, different faces have been shown by different colors. Top is green, 2 is slightly brown and 3 is blue. Here, I show z-axis rotation. Rotate this by 90 degree about z-axis. Now, this face has come like this and if I say y-axis then you got this type of orientation. You can see it again; z-axis rotation 90 degree and y-axis rotation like that.

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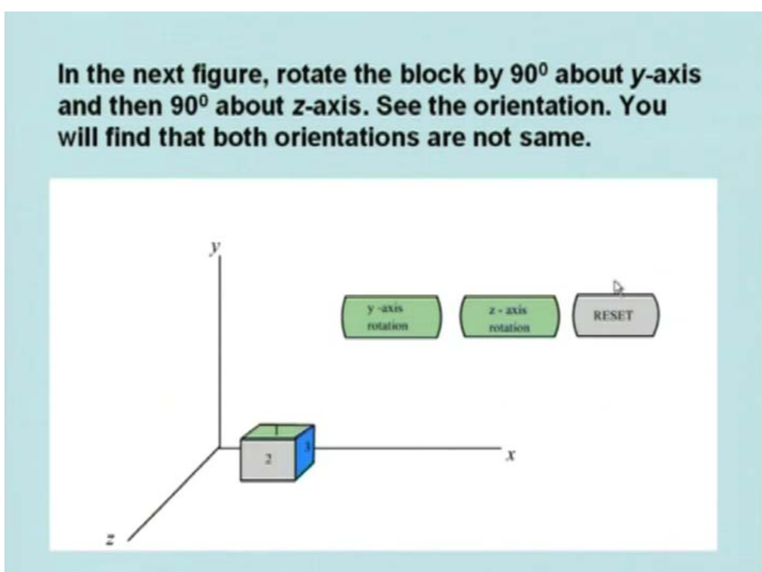

Now, in the next figure, rotate the block by 90 degree about y axis first and then 90 degree about z axis. See the orientation. You will find that both orientations are not same. Suppose I rotate it at y-axis, you got like this and z -xis I am rotating, I got like this. So, I get totally different type of things. See this animation again; about z-axis, rotation by 90 degree. You can do these types of experiments, with your learning and more clarity by taking some book, and rotating it 90 degree and then again 90 degree about some other axis like that. You see that finite rotations are not commutative. It depends in which order you rotate.

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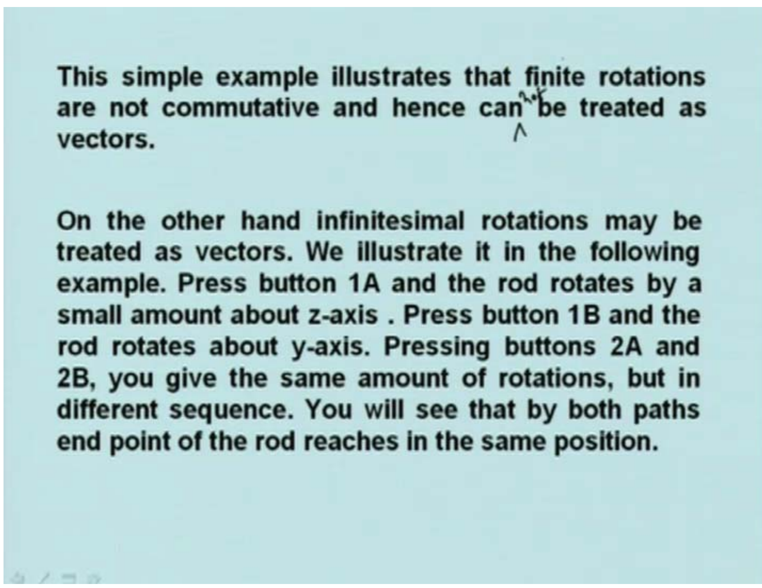

This simple example illustrates that finite rotations are not commutative and hence cannot be treated as vectors. They cannot be treated as vectors. On the other hand, infinitesimal rotations maybe treated as vectors. For that, we illustrate something.

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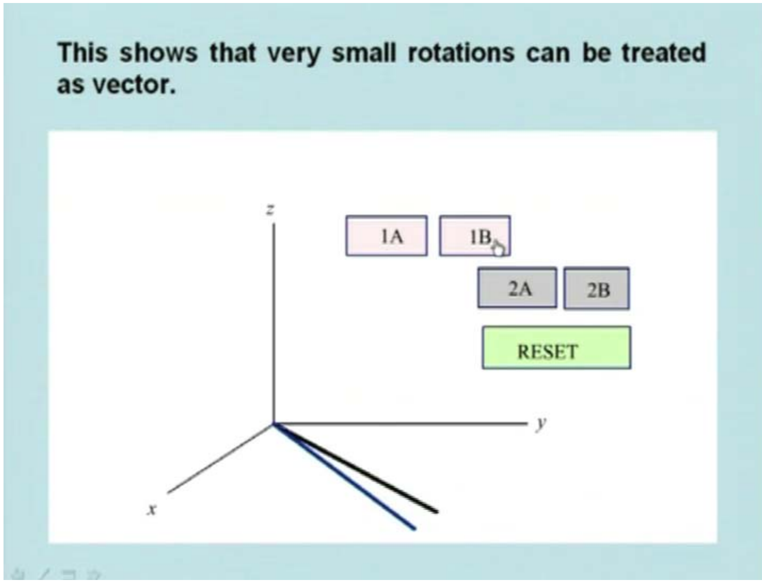

We will see that in the animation it is like this, suppose you press 1A first. I am rotating it first here, then about different axis like that and in this I am changing this axis. I am rotating it like this and then here I am rotating it about z-axis here. Here, basically the rotations have produced the same result. We can see it again. First rotate it about z-axis and then rotate about y-axis; Here, first rotate about y-axis and then about z-axis, we get the same thing. Again, these are small rotations. This is like this and then after that you get like this and after that reset.

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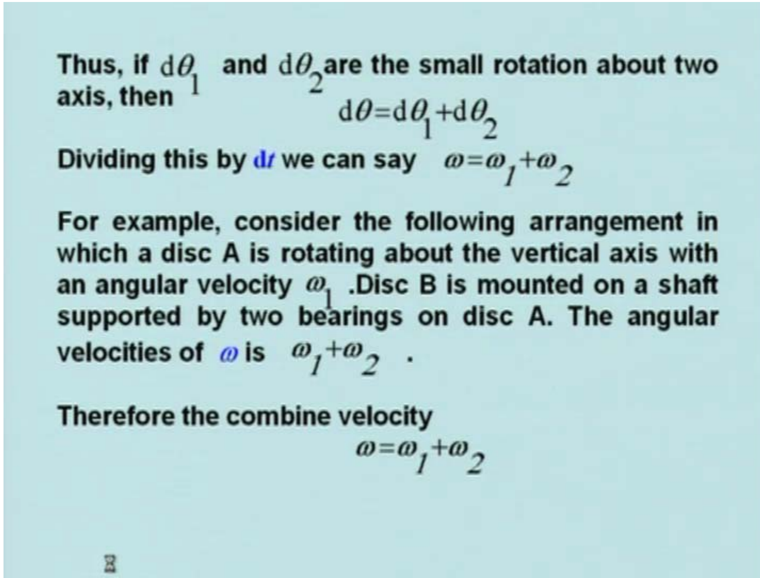

Thus, if d theta_1 and d theta_2 are small rotations about two axes, then d theta is equal to d theta_1 plus d theta_2. If you divide this by dt then omega is equal to omega_1 plus omega_2. We can divide it by dt, limit dt tends to 0. Therefore, what happens is that in this case, the velocities can be added like vector.

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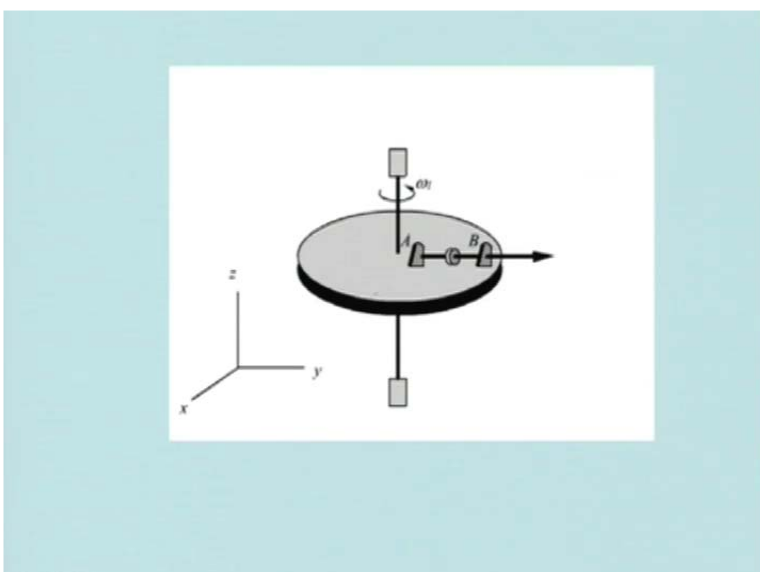

For example, consider the following arrangement in which there is a disc which is rotating at ω_1. There are two bearings on which this shaft is mounted. There is a disc that is also spinning about another axis. If I show x y z axis, one rotation of the whole disc is about z and this is spinning about y-axis. In this case, what happens is that disc A is rotating about the vertical axis with an angular velocity ω_1. Disc B is mounted on a shaft supported by two bearings on disc A. The angular velocity of omega is ω_1 plus ω_2. Therefore, the combined velocity omega is equal to ω_1 plus ω_2 vector. Therefore, combined velocity is like this.

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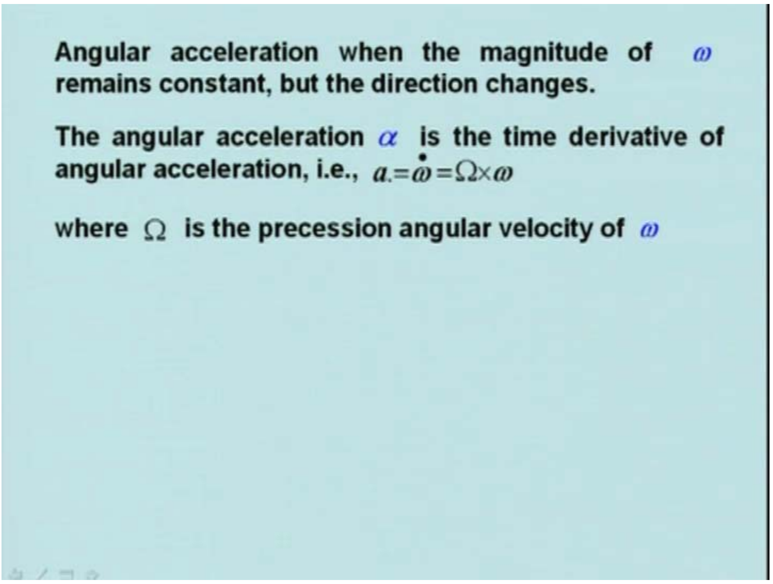

Angular acceleration: when the magnitude of omega remains constant but the direction changes; we can find out the angular acceleration in that case. The angular acceleration alpha is the time derivative of angular acceleration that is a is equal to omega dot. We can write this big omega cross this small omega, where this big omega is the precession angular velocity of omega. That is the expression for angular acceleration if the axis is also precessing. That is called precessing motion.

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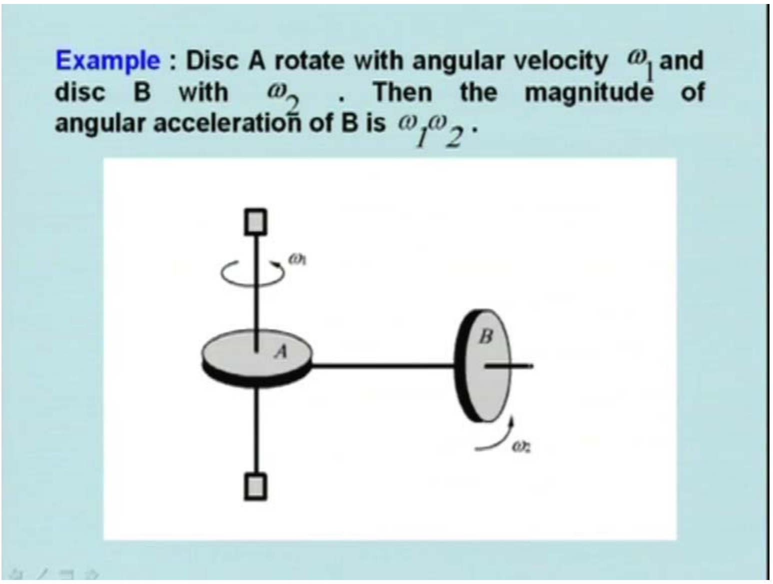

For example, in this you see that there is disk which is rotating. On this, is mounted some shaft and on which there is a rotor which itself is spinning with omega$_2$, but the axis of rotation of omega$_1$ is fixed. However, the axis of rotation of omega$_2$ is changing then the magnitude of angular acceleration of B is omega$_1$ into omega$_2$. We discussed about translation, then about rotation about a fixed axis and then we discussed rotation about a point. Now, we discuss the general 3D motion.

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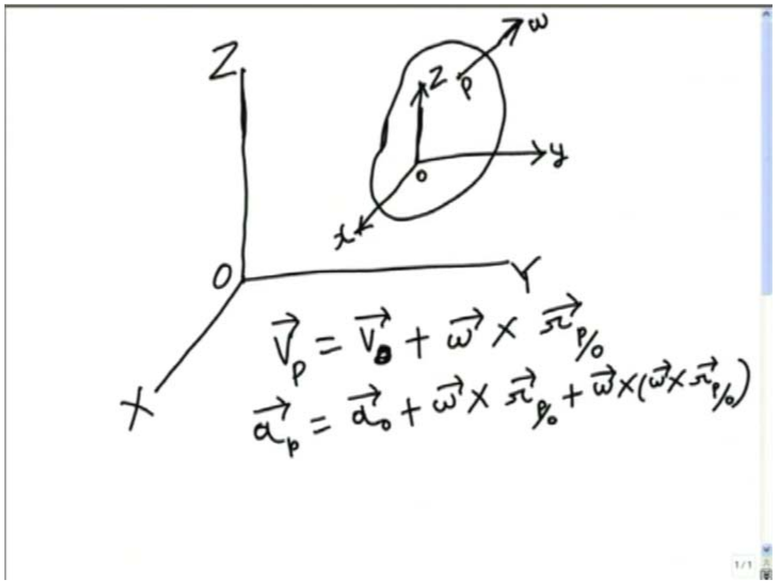

Consider a rigid body which has an angular velocity omega. First, I show the axis system in 3 dimensions; this is Y, Z, X and O. This is a body I am showing. This body is having angular velocity omega. I can show the angular velocity vector omega. It is in this direction; at the same time, the body maybe translating also. If I choose an axis system X Y Z on the body, this is y, then z and this is x where small o is the origin, a point on the body. We choose another axis system that is oxy which is considered to be fixed; that is the velocity and acceleration of point o is always zero and we assume that x y z axis remains parallel to X Y Z. That means we have attached an axis system which remains parallel to x y z. Then the velocity of a point P, this point support there is a point P, with respect to o maybe written as velocity of point P, V_P is equal to V_o plus omega cross r $_{P \text{ by } o,}$ small o.

You get V_P is equal to V_o plus omega cross this one and similarly, the acceleration, a_p can be written as $a_{o,}$ small o plus omega cross $r_{p \text{ by } o}$ plus omega cross omega cross $r_{p \text{ by } o}$. These are the expressions which we developed while studying kinematics also.

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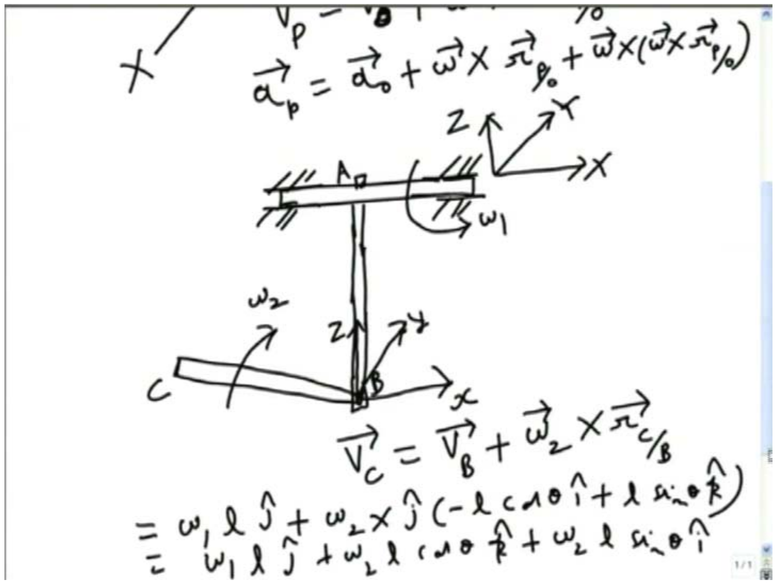

Now, I will take one example. Consider this link. Suppose there is shaft and these are the bearings. Here also these are bearings. Suppose this just supported that axis and then in this you have attached one link like this and here by pin joint another link is there and if it is rotating with omega$_1$ and this axis is x y z. This axis if I attach another axis system if this point I call A, and this link is B the third link is BC. Here I attach a small x then you have y and then this is z. This is omega$_2$. y-axis which I have shown is perpendicular to the plane of your screen. xyz and capital XYZ both are parallel. In that case, we can always write that V_C is equal to V_B plus omega$_2$ cross $r_{c \text{ by } B}$. This is the expression and V_B is equal to basically omega$_1$ times l because it is rotating by omega$_1$ times j plus omega$_2$ cross j minus l cos theta i plus l sin theta k. If you simplify that, you would get w_1 l j plus w_2 l cos theta times k plus w_2 l sin theta times i. So, that type of expression you get.

I could have taken another reference system that is rotating reference axis.

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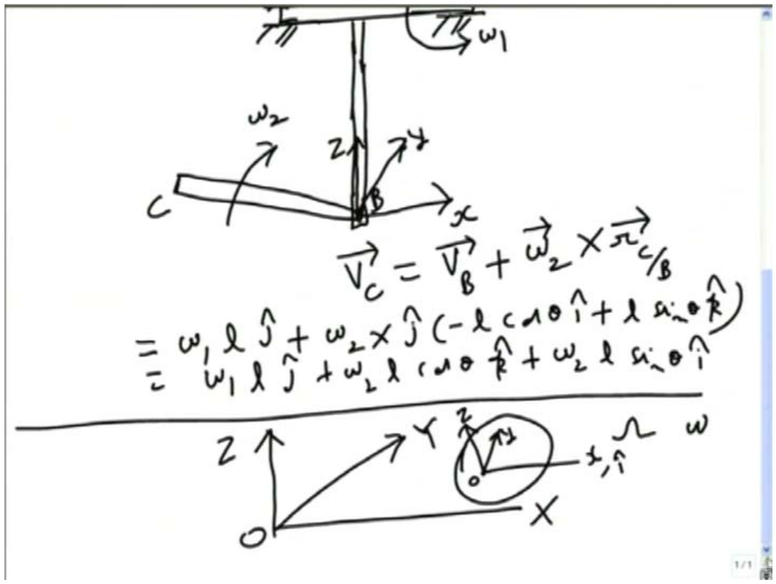

If we take XYZO, this is a fixed axis system and if we take a body like this in 3D, but if we take a small o origin and this is x y and z. This axis system is also attached, but this time it is not parallel; instead it is rotating also. Therefore, what happens in this, suppose xyz rotates with angular velocity omega that means rotation of axis is by big omega, which is different from the angular velocity small omega of the body, omega and omega in general can be different.

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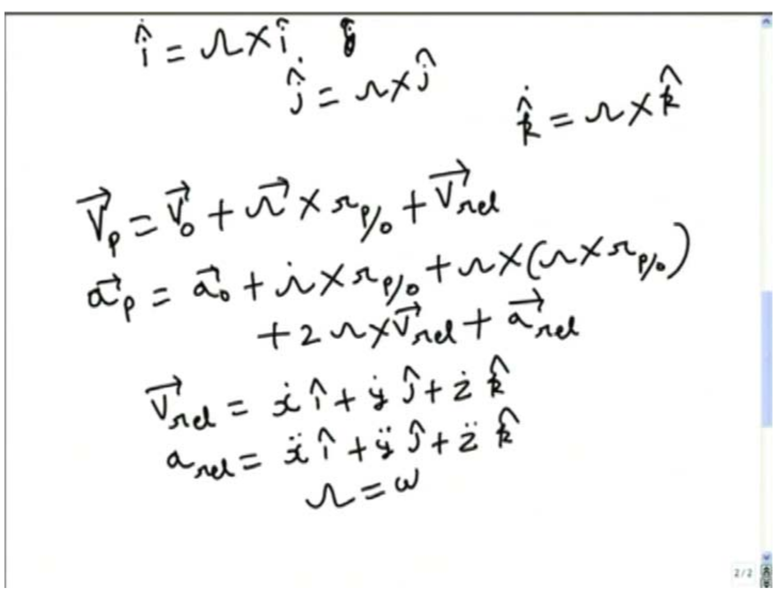

Then, you can easily prove that if I take i axis about this and j about this y, then you can easily prove one point that i dot unit vector dot is equal to basically big omega cross I; angular velocity of the axis system cross product with i. Similarly, you can say j dot is equal to omega cross j. Similarly, k dot is equal to omega cross k. Here, dot indicates the time derivative. The expression for the velocity and acceleration of point P then become, V_P is equal to, suppose we have point P on the body that become V_P, then V_o small plus omega cross $r_{p \text{ by } o}$ plus $V_{rel,}$ relative velocity of that point. a_p becomes equal to a_o plus omega dot cross $r_{p \text{ by } o}$ plus omega cross omega cross $r_{p \text{ by } o}$ plus 2 omega cross V_{rel} plus a_{rel}. These expressions we have already discussed, but we are just looking at them again from the point of view of 3D kinematics.

In this case, V_{rel} is basically in 3D. It is x dot i plus y dot j plus z dot k and a_{rel} is x double dot i plus y double dot j plus z double dot k. If the axes are rigidly attached to the body then omega will be equal to omega. In that case, since they are rigidly attached V_{rel} and $a_{rel,}$ both will be 0. In this case, the equation reduces to your previous equation. That is omega cross $r_{p \text{ by } o}$.

We now solve the same problem which we just solved. We will go back to the previous slide. We have solved this problem. Now, this time we attach the axis system to link 1 itself that means it rotates with the link 1.

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$$\vec{V}_{rel} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

$$a_{rel} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$$

$$\Omega = \omega$$

$$V_C = \vec{V}_B + \omega_1\hat{i} \times (-l\cos\theta\hat{i} + l\sin\theta\hat{k}) + (\omega_2\hat{j} - \omega_1\hat{i}) \times (-l\cos\theta\hat{i} + l\sin\theta\hat{k})$$

$$= \omega_1 l\hat{j} + \omega_2\hat{j} \times (-l\cos\theta\hat{i} + l\sin\theta\hat{k})$$

In that case, we will get V_C is equal to V_B plus $omega_1$ i cross minus l cos theta i plus l sin theta k plus omega 2j minus $omega_1$i l cross minus l cost theta i plus l sin theta k. If we simplify then this becomes basically $omega_1$ lj plus omega 2j cross minus l cos theta i plus l sin theta k. We get the same equations as we got before; same velocity. So, one can take different type of axis system and he can get the same result.

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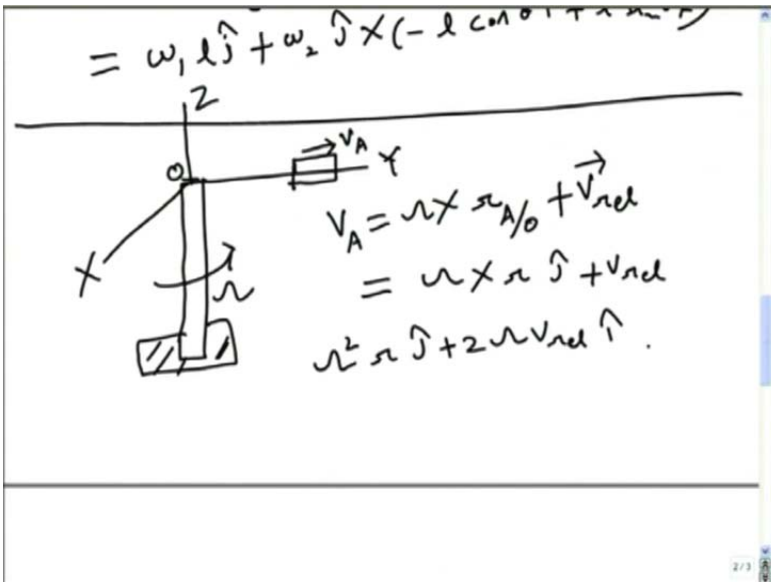

Now, we take another example. Suppose there is shaft here. Let us put in some bearing and it is rotating with omega. You have axis attached here z and this is x and this V_A. Then the relative velocity of this is V_A is equal to omega cross $r_{A \text{ by } o}$ plus V_{rel} that is omega cross r times j, this is in y direction plus V_{rel} and it's absolute acceleration will be omega square r_j plus twice omega V_{rel} i, where 2 omega V_{rel} is the Coriolis acceleration. These equations which we discussed can be used for 3 dimensional things also.

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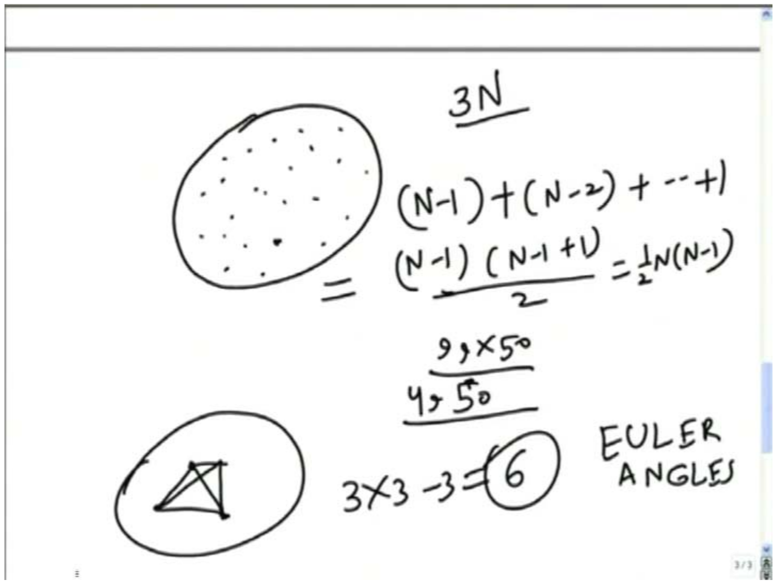

We discuss about that. Just pay attention that how many independent coordinates a body has; rigid body. A rigid body is composed of so many particles. Supposing a rigid body has got N particles. If there are N particles a rigid body is composed of, although we will say that in rigid body N tends to infinity, otherwise it will not be continuum, we cannot have discrete set of particles.

Suppose there are N particles then, each particle has 3 degrees of freedom, xyz. So, total degrees of freedom are how much? 3N; since, each particle has 3 degrees of freedom. Therefore, 3N coordinates are needed to specify the body. However, these coordinates are not independent; they are related, because in the rigid body distance between two particles is fixed. So, one particle is connected to other N minus 1 particles. Thus, we write N minus 1 equations relating the coordinates and distance.

Similarly, for the second particle, we can write N minus 2 equations, since the equation relating this and the first particle has already been written. For third particle, we can write N minus 3 equation. Thus, total available equations are N minus 1 plus N minus 2 plus, plus ….. plus 1. This is equal to N minus 1 into N minus 1 plus 1 by 2; that is half N and N minus 1. For large N, these equations, total available equations these are constraint equations. They are more than 3N; suppose you take N is equal to 100. So 3N is 300, but this will be 100 times 99 by 2 that means

50 into 99. So, 4950; these are much more than this thing. But all of these equations are not independent.

This way we cannot ascertain. Another way to understand that is if we know the coordinates of 3 points; let us say there is a rigid body and you know the coordinates of three points; three points coordinates are known. Then for any point, if you take any fourth point in the body, there are like, if I take this point then there are three equations relating coordinates with the distance of the points. This is like this, this is like this and this is like this. Once you have fixed up three points on the body then all points get constrained. They cannot have their independent degrees of freedom. We can only take three reference points but the three reference points themselves are also not independent. There are three equations of distance actually. Thus, the independent degrees of freedom are 3 into 39 minus 3, that means 6. So, there are only 6 independent degrees of freedom.

Since there are 6 degrees of freedom, if we fix a point o on the body and some coordinate system, then 3 coordinates are needed to specify the origin of the coordinate system. Since total 6 degrees of freedom are required to specify the body, 3 coordinates are needed to specify the orientation of the axis. There are many ways of specifying the orientation of a Cartesian set of axis, relative to another set with common origin, but most common is the Euler angles. So, most common is Euler angles. We will explain this by animations.

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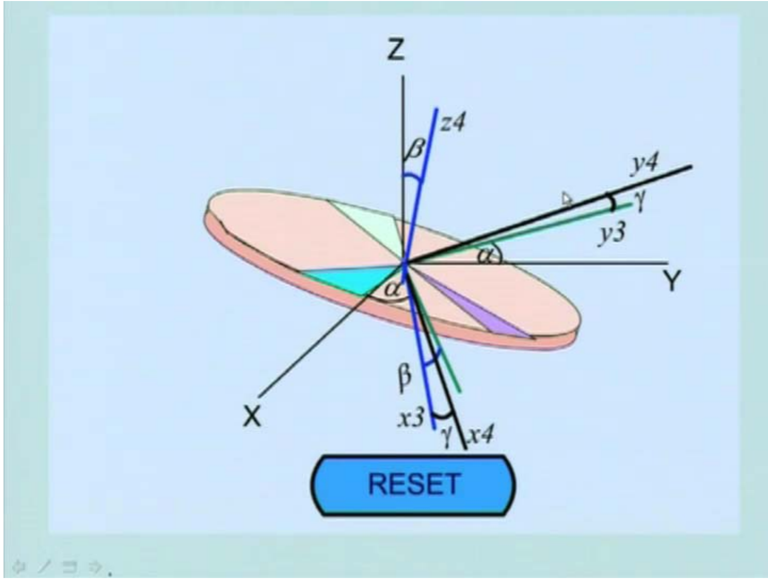

Euler angles are defined as 3 successive angles of rotation. Many conventions are followed but the sequence implied here, it started by rotating the initial system of axis XYZ. Suppose you have an angle alpha about X axis, the resulted coordinate system is labeled as x2 y2 and z2. The z2 axis is same as z1 axis. Suppose, you have a system that it is x1 y1 and z1; you rotate and this axis is same as that one.

You give the first rotation, you got another system x2 y2 z2. It has rotated about the z1 axis by an angle alpha but the z2 is same as z1 in that direction. See this animation again. Initially x1 y1 z1, now rotate it. First rotation has given x2 y2 z2. Then we go to the next we give the second rotation. In the second stage the intermediate axis x2 y2 z2 is rotated about y2, about this y2 to produce intermediate set x3 y3 z3. Now, it is rotated about y2. This is how it will rotate. Now, let us see the second rotation. So, by second rotation, y3 is same as y2 in the same direction but these has rotated, x3 and z3 and those angles are beta. So, that rotation is beta. The intersection of x2 y2 and x3 y3 planes, this is x2 y2 and x3 y3, is called as the line of nodes. These two planes intersect. So, that is called line of nodes. Will see it again; this was x2 y2 plane and then this gets rotated. This is x3 y3 plane. Their intersection will be called line of nodes.

Now, we see finally x3 y3 z3 is rotated above z3 by an angle gamma. Therefore, what happens we reset that and see third rotation about z3; so, gamma.

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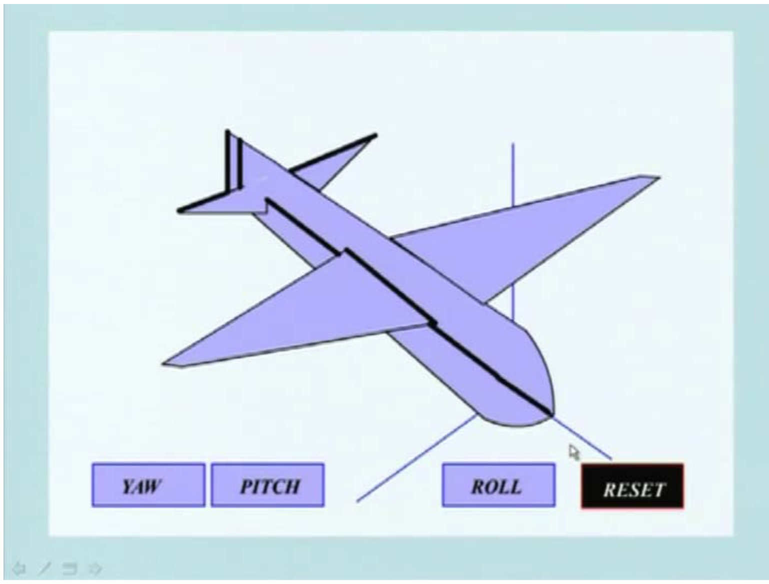

Euler angles are not the only way to specify the motion of this thing. There are others. Once we have the rotations, these rotations can be represented by the matrices and finally we can multiply the matrices to get the final outcome. Now, we can have another type of rotation. More common convention in engineering application is like this. We can have the axis system and we can have yaw. This is called yaw, yaw motion; that means, about this vertical axis the plane is moving that is yaw motion. Then we have pitch motion. This is pitch motion. Yaw motion is this one and pitch is about this upward. Pitching motion is about this transverse axis. This one is called pitch and roll is about the longitudinal axis; this is roll motion. These types of conventions can be done. These are the motions.

We have discussed kinematics and now, we have to discuss kinetics also. Before that, let me just show that in 3D. Earlier you got some appreciation of the visualizing of the 3D kinematics etc. and why you have to understand the type of angle system and all these things.

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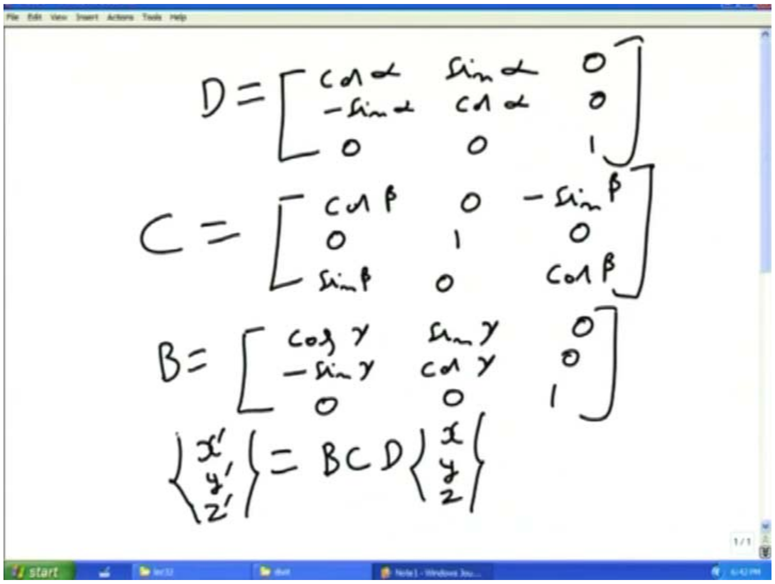

Now in the case of Euler angle, here suppose you have got D is equal to cos alpha sin alpha 0, minus sin alpha cos alpha 0, 0 0 1 and then the rotation about another axis can be specified by cos beta 0 minus sin beta, 0 1 0, sin beta 0 cos beta, and the rotation about the third one B is cos gamma sin gamma 0, minus sin gamma cos gamma 0, 0 0 1. If we denote the old axis system by x y z and new by x prime y prime z then you have these equations; x prime y prime z prime is equal to BCD xyz. Like that we can get the relations.

We will get the coordinate systems relating one axis system with the other axis system. This way it can be done. So, once we understand these things, we can have, for any particular body, we can have three coordinates xyz of the origin and we can also have alpha beta gamma. These are the angles. With this, we can describe this thing.

In the next lecture, we will be discussing the Euler’s equation; that means we will start dynamics. Then, we will also study one phenomena of that gyroscope. We will study gyroscopic motion and that will be the topic of the next lecture.