Engineering Mechanics

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Module 12 Lecture 32
Kinetic Energy

We are going to discuss about work and energy of rigid bodies.

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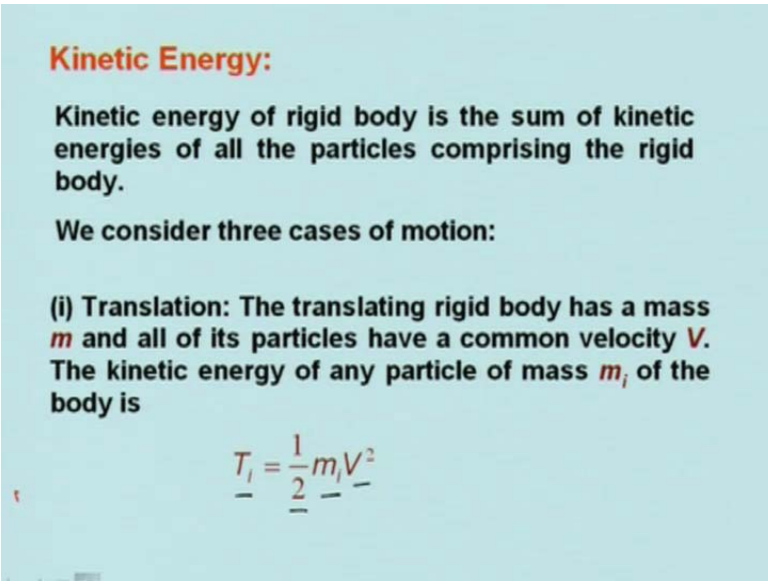

We have discussed about kinetic energy that is half mV square and we also discussed the work and energy principle. Work and energy, that means work done on the particle by the external forces, increase the kinetic energy equal to the amount of the work only. So, kinetic energy of a rigid body is the sum of kinetic energies of all the particles comprising the rigid body. A rigid body consists of so many particles.

We can consider that a rigid body consists of infinite number of particles, but that does not matter that each particle is of mass tending zero and therefore, we will have some kinetic energy. Instead of summation, we have to do integration, but actually the definition is that kinetic energy of rigid body is the sum of kinetic energies of all the particles comprising the rigid body. In the continuous system integration is basically summation.

We consider three cases of motion. One is the translation. You know that in translation, each line in the body moves parallel to itself. So, there is no rotation, omega is 0. The translating rigid body has a mass m, total mass maybe m and all of its particles have a common velocity V then the kinetic energy of any particle of mass m_i of the body is T_i is equal to half m_i V square.

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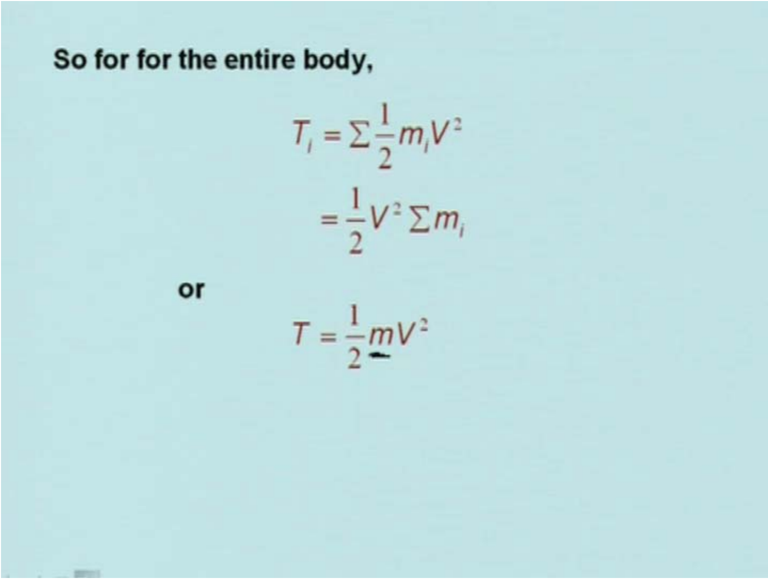

For the entire body, we have T_i is equal to sigma half m_i V square and you can take this common. So, it becomes half V square into this sigma m_i and sigma m_i is the summation of that. That becomes the total mass m. So, kinetic energy then becomes equal to half mV square; that means, kinetic in this expression is similar to that expression for a particle. So, instead of particle mass, now I am putting the mass of the whole body. Therefore, T is equal to half mV square. In translation, we do not have to worry, but still we talk about the velocity of the mass center. We say, body is moving in the translatory motion, what is the velocity of the mass center? Velocity of mass center should be same.

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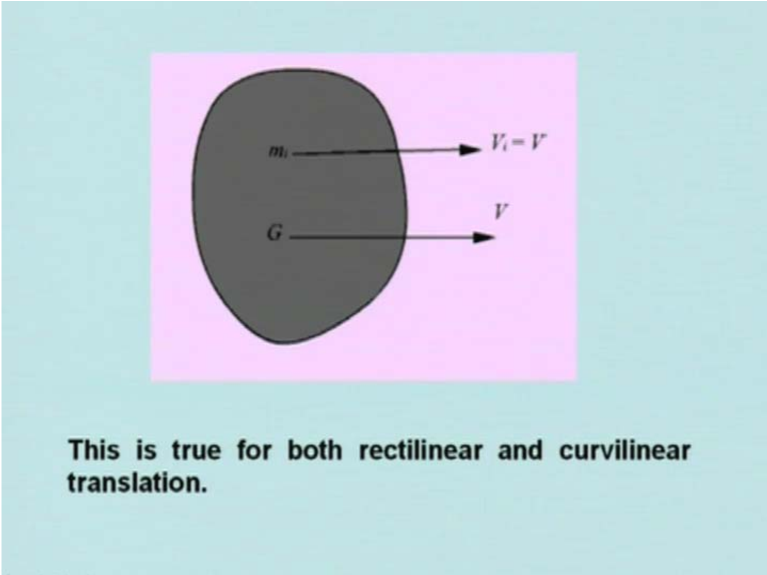

Here, the velocity of the mass center G is like this and velocity of any other particle is also same. So, V_i is equal to V. This is true for both rectilinear and curvilinear translation. No matter body is moving in a straight line or body is moving in a curved path, you have got the same type of relation. So, there is no problem in this.

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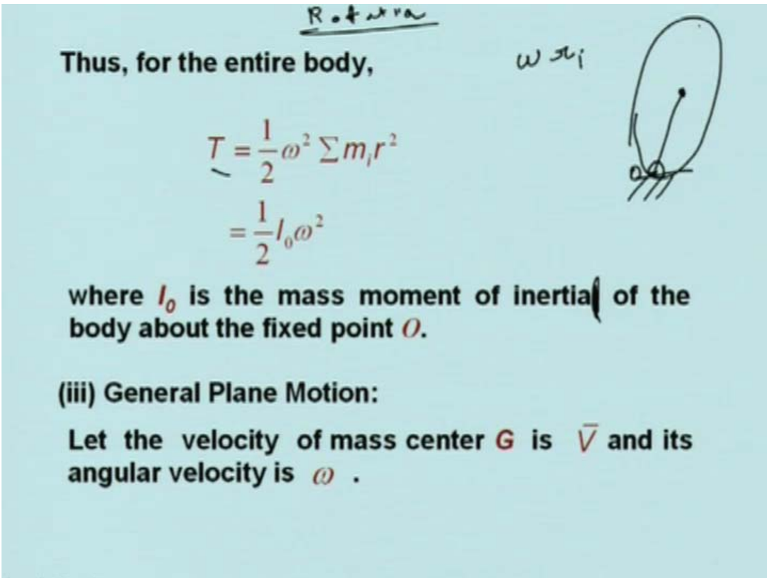

In this case, the body is only rotating. Now, let us discuss the second case that is rotation; body maybe rotating about an axis. In that case, for a particular particle you have got the velocity that is omega times r_i, where r_i may be the distance from the fixed point. Suppose this is the body and this body is rotating about this point. What happens is, its velocity is omega times r_i. So, omega r_i. Velocity square is omega square r_i; omega square is common for all the particles and so, it can be taken outside. Then for the entire body, we still have the expression, T is equal to half omega square sigma m_i r square and that is equal to half I_0 omega square, where I_0 is the mass moment of inertia of the body about the fixed point O. This is the fixed point. If the body is rotating about a fixed point, we know a fixed point about which the body is rotating; then T is equal to basically half I_0 omega square. It maybe rotating about that fixed point or it may be some instantaneous center of rotation. Then also, we can find out half I_0 omega square. Here you have to find out the mass moment of inertia about the fixed point.

Now, we discuss the third case that is general plane motion. We know that there is a theorem of [….], that motion of a body in a plane can be decomposed into the rotation part and translation part. So, let the velocity of mass center G be V and its angular velocity is omega.

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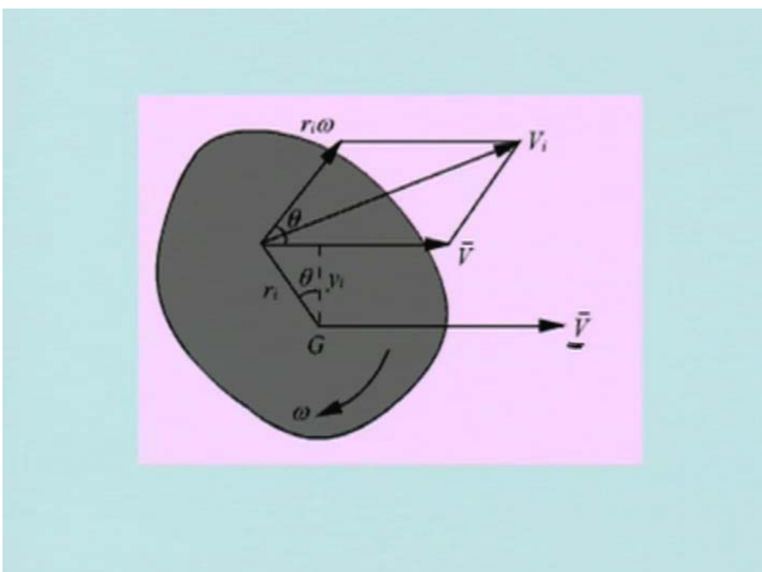

This type of situation is there. See this body. It is rotating and at the same time, the mass centre G is going with a velocity of V bar. It has been indicated. So, any particle on the body that is at a distance of r_i and this is inclined. Now this is at a distance of $r_{i.}$ This is having a relative velocity with respect to G in the perpendicular direction, 2 $r_{i.}$ Because in the rigid body, the particles do not move towards each other, they cannot have any velocity component towards any other particle. Two particles cannot have relative motion towards their line of joining. Therefore, the relative velocity has to be in this direction and that is r_i omega.

We have shown this velocity as horizontal for our convenience. This does not affect our general treatment. Therefore, what happens is that here you have V bar and r_i omega. If they are making an angle theta, then we have resultant velocity is V_i and that has been obtained by parallelogram law of addition of two vectors. So, vector has this. Velocity is represented by one side of the parallelogram and this is represented by the other side and the resultant is coming like this.

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$$V_i^2 = \bar{V}^2 + r_i^2\omega^2 + 2\bar{V}r_i\omega\cos\theta$$

$$T = \Sigma\frac{1}{2}m_iV_i^2$$

$$= \Sigma\frac{1}{2}m_i\left(\bar{V}^2 + r_i^2\omega^2 + 2\bar{V}r_i\omega\cos\theta\right)$$

This is the expression you have got and then, we get V_i square. By parallelogram law, we only get V_i square is equal to V square plus r_i square omega square and plus two Vr_i omega into cos theta, where cos theta is what? Cos theta is the angle between V bar and r_i omega. V bar is considered horizontal. If we draw and if we consider that as x-axis and this was y-axis, vertical, r_i makes theta with y_i. This has been indicated in the figure. This is for one particle. So for one particle, the energy will be we say that the mass of that particle is m_i and we can do summation instead of integration sign. Now, I am just using summation continuous system. It will convert to integration. That is half mi V_i square and like that you have to sum. This is sigma half m_i V bar square plus r_i square omega square plus 2V bar r_i omega cos theta. Let us discuss these terms here.

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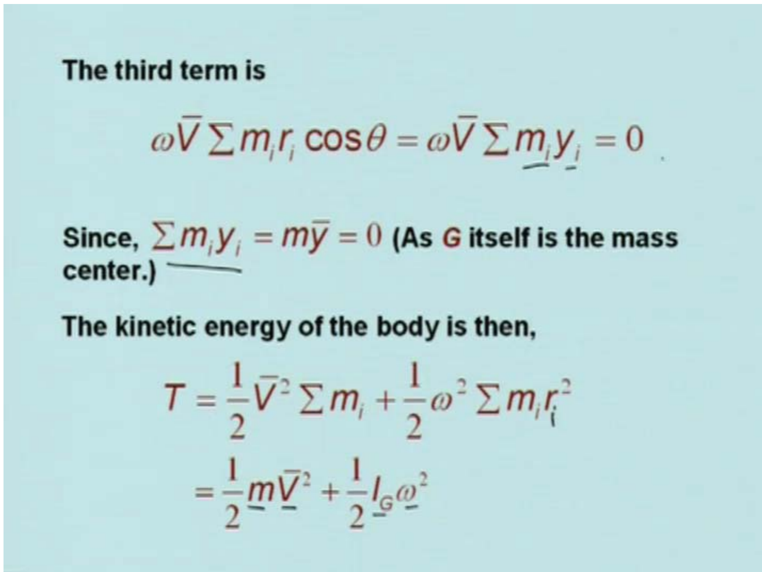

The third term in this one is r_i omega cos theta. If we see this figure, then r_i cos theta becomes equal to $y_{i.}$ This is r_i cos theta and that is becoming equal to y_i. In this case, the third term is omega into V bar sigma m_i r_i cos theta; that means, omega V bar sigma m_i $y_{i.}$ That is equal to 0, because basically sigma m_i y_i bar will be mass times y bar, where y bar is the y-coordinate of the center of mass. So, that is equal to 0, because G itself is the mass center.

In this figure, you have to see that. If we take the y_i times that mass and then sum, it will become 0. Here in the first term half m_i V bar square, V bar is common. Because it is the velocity of the mass center, it can be taken outside and omega is also common and so it can be taken outside. We get kinetic energy of the body as kinetic energy is equal to half V bar square. This is sigma m_i plus half omega square sigma m_i r square and this is half m V bar square plus this is sigma m_i r square half I_G omega square. So, this is mV square and this is the thing. Because I am taking r is the distance in this figure from the mass center, so therefore naturally this is r_i;, r_i is the distance from the mass center. So, this becomes $I_{G,}$ so half I_G omega I square.

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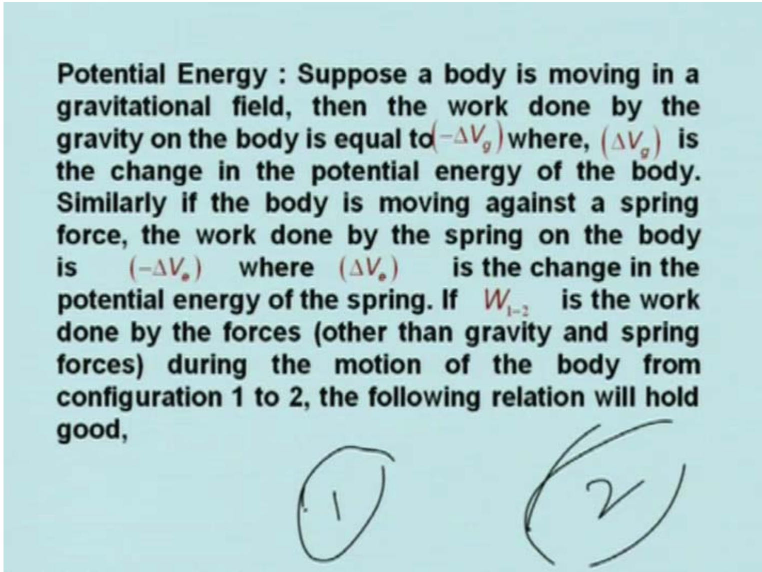

We have got a nice expression for the kinetic energy of the body that is half mV square plus half I_G omega square; translational kinetic energy and rotational kinetic energy. Rotational kinetic energy is half I_G omega square and translational kinetic energy is half mV square and kinetic energy is basically capacity to do work because of its motion. Both the things can be there.

We will discuss about the potential energy. Suppose a body is moving in a gravitational field, then the work done by the gravity on the body is equal to minus delta V_g we know. That means when the gravity does some work on the body then the body's potential energy decreases basically. But if the work is done against the gravity then the body's potential energy increases because that work is stored there as potential energy. If a body is moving in a gravitational field then the work done by the gravity on the body is equal to minus delta V_g, where delta V_g is the change in the potential energy of the body. Similarly, if the body is moving against a spring force, then work done by the spring on the body is minus delta V_e, where delta V_e is the change in the potential energy of the spring. These are the conservative forces. Generally, work done by these forces can be expressed by potential.

If W_{1-2} is the work done by the forces, other than gravity and spring forces, other than these conservative forces, during the motion of the body from configuration 1 to 2, any two configuration body may be here, that is configuration 1, it may move here, which I will call configuration 2; same body in a different orientation or different place, the following relation will hold good.

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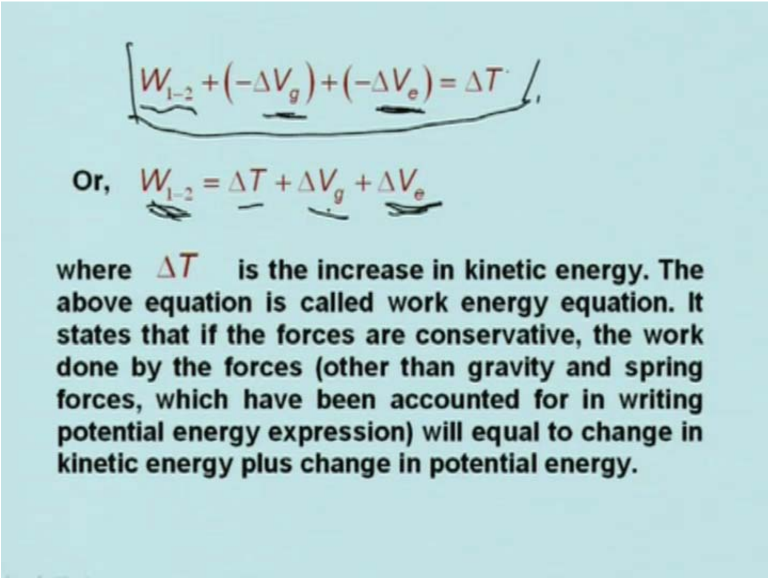

Total work done actually on the body is basically W_{1-2}, because that is the work done by some other forces and this is the work done by the gravity forces minus delta V_g, where delta V_g is the increase in the potential energy due to gravity, and here minus delta V_e, delta V_e is in the increase in the potential energy due to spring; that is equal to delta T. W_{1-2} is equal to delta T plus delta V_g plus delta V_e.

From this, delta T is basically increase in the kinetic energy. We had Newton's law; only from that, we derived this in the previous lecture that work done is equal to increase in this one and in this case W_{1-2} is equal to delta T plus delta V_g delta V_e. So, delta T is the increase in the kinetic energy. This equation is called work energy equation. It states that if the forces are conservative, the work done by the forces other than gravity and spring forces which have been counted for in writing potential energy will be equal to change in kinetic energy plus change in potential energy. Delta V_g and delta V_e are conservative and even if we consider the other forces, the non-conservative forces then, also these relations remains valid because this directly comes from Newton's Law. But you have to recognize all the forces properly.

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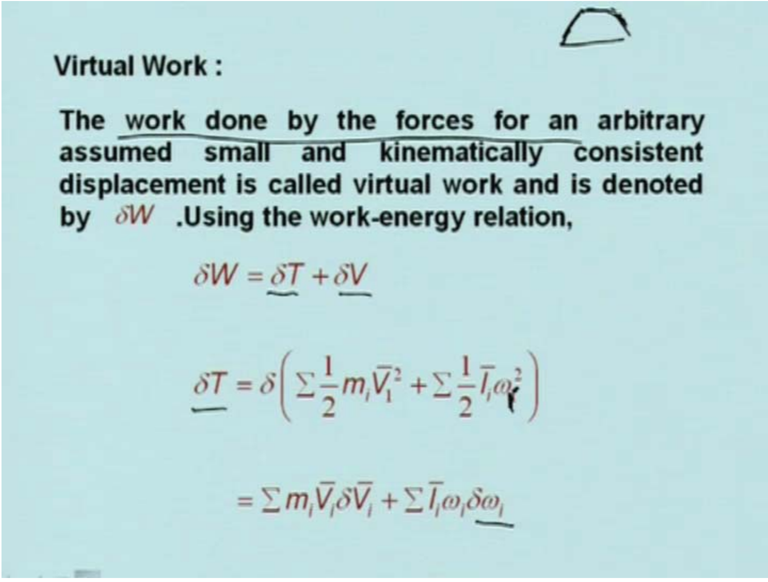

We come to the virtual work principle. We have studied that in statics. Here also, the same type virtual work principle can be used. The work done by the forces for an arbitrary assumed small and kinematically consistent displacement is called virtual work and is denoted by delta omega.

You pay attention to each and every line of this work done by the forces. Body may be subjected to the forces that are there. We assume that there are number of forces and we assume arbitrarily small displacement you give that. Actually, it is not the real displacement. Real displacement will change the forces but this displacement is so small that it does not change the forces. Force situation remains same and kinematically consistent. Kinematic conditions are not changed; actually, that means the boundary conditions and the velocity and other things. If there are two linkages, between two linkages, the relations of the velocities remain same. Then, that is called kinematically consistent and this work is called virtual work and is denoted by delta W.

If we use the work energy relation, delta W will be equal to delta T plus delta V. Here, delta T is basically delta sigma half m_i V_i bar square plus sigma half I bar omega square is equal to sigma m_i V_i delta V_i plus sigma I_i omega$_i$ delta omega$_i$, this is omega$_i$, like that and there may be number of linkages for that.

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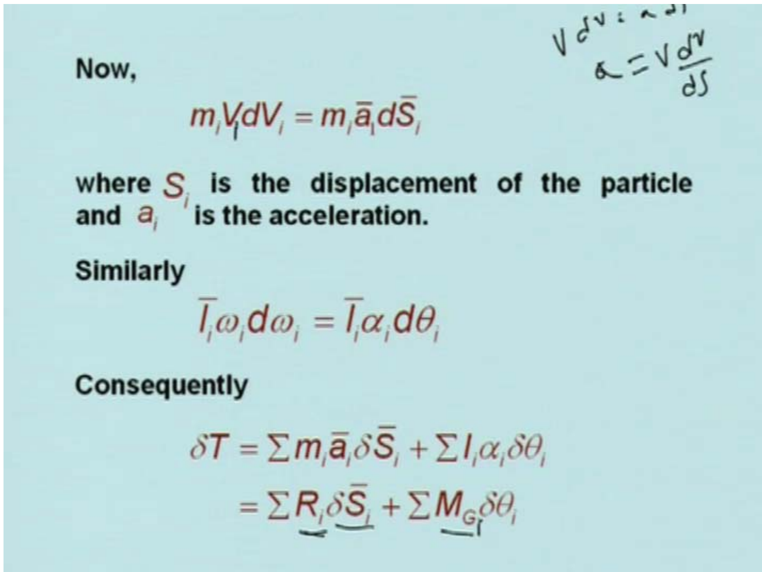

Now m_i V_i dV_i is basically m_i a_i dS_i, because we know that we have a relation V dV is equal to adS, because a is equal to V times dv by dS. That relation is there. We make use of that relation and you say that for this particle, because this is a simple kinematic relation, S_i is the displacement of the particle and a_i is the acceleration. Similarly, I_i omega$_i$ d omega$_i$ can be written as I_i alpha$_i$ d theta$_i$, where alpha$_i$ is the angular acceleration.

Consequently, we get delta T is equal to sigma m_i a_i delta S_i plus sigma I_i alpha i delta theta i, Now, m_i a_i is basically R_i on that and this is delta S_i and I_i alpha i is basically M_G and delta theta i for that $M_{G\ i}$ may be we can say.

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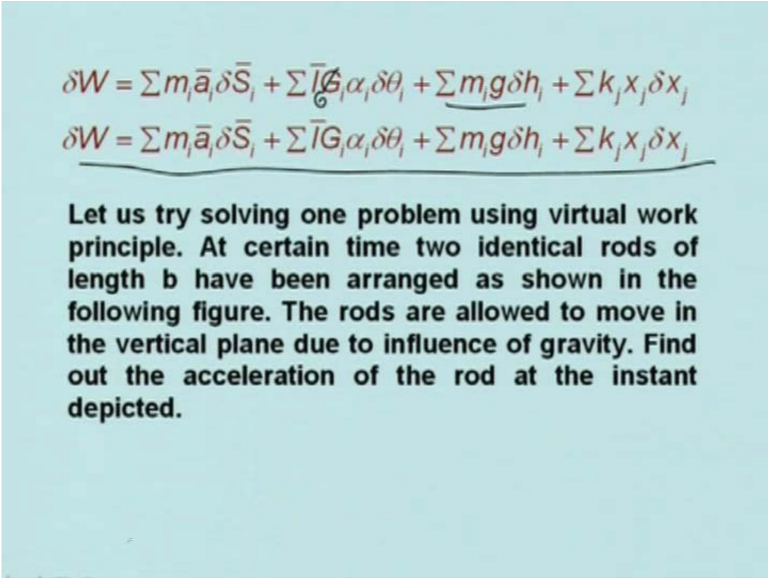

So, delta W comes out to be sigma m_i a_i delta S_i plus sigma I_{Gi} this is I with respect to G, the subscript G_i and alpha$_i$ delta theta$_i$ plus if we consider that m_i g delta h_i, virtual change in the height and sigma K_j x_j delta x_j, K is the spring constant and x_j is the displacement. Similarly, this can be written as so this expression has been written. So, this is the expression for the virtual work. By this, we can do. Now, we will try to solve one problem by virtual work principle.

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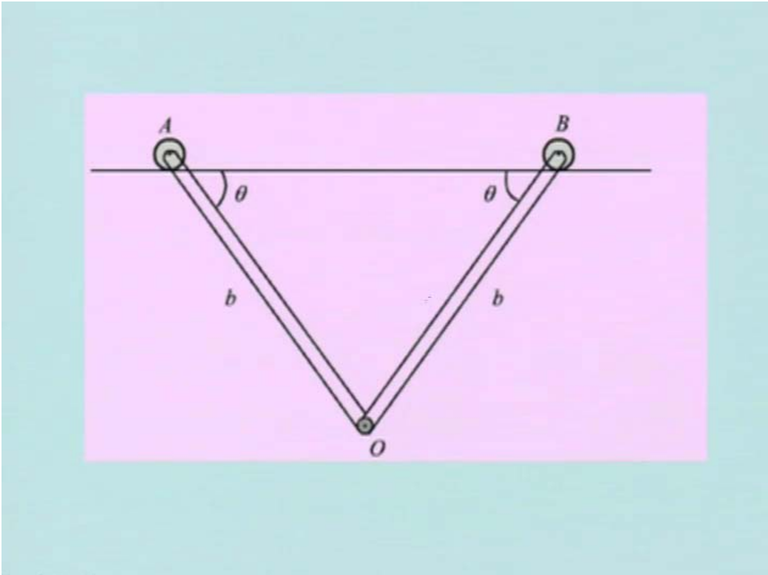

This is the problem. There are two rods here, this one of length B and these are rollers which are moving on a rail, and this is theta, this is theta and this is b and this is this one. At certain time two identical rods of length b have been arranged as shown in the figure. The rods are allowed to move in the vertical plane due to influence of gravity. Now, find out the acceleration of the rod at the instant depicted. It is this thing here that you have to find out the rod and this is.

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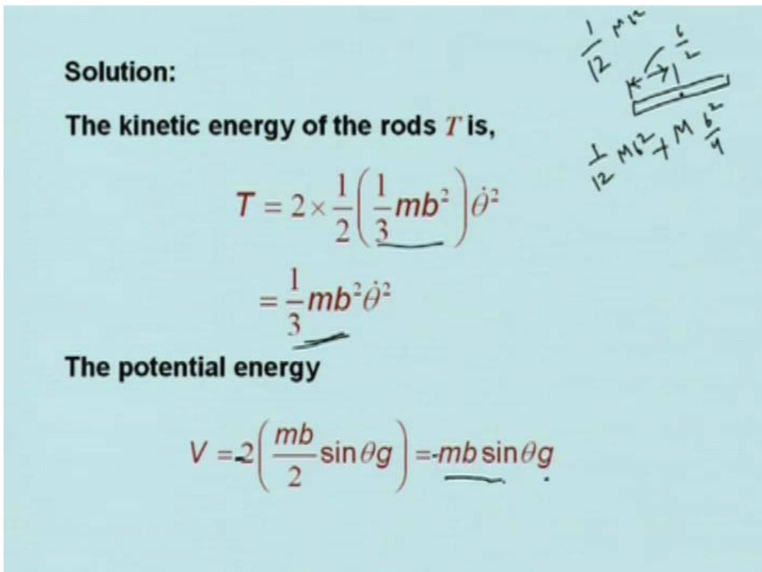

The kinetic energy T is equal to 2 into half and we take here this point, which is the point of zero velocity because this is instantaneous center. So, here instantaneously the velocity is zero. We can say that it is rotating about this fixed point. The kinetic energy is half I omega$_i$ square, but two rods are there, so, 2 times half and about that fixed point, it is 1 by 3 mb square. About its mass center, its moment of inertia is 1 by 12 mb square, but about the fixed point, it may be half mb square, because this is the rod and this is about the mass canter it is 1 by 12 mb square and this distance is, b by 2.

This will be mass times b by 2, b square by 4; so, you get 1 by 3 mb square theta dot square. That gives you 1 by 3 mb square theta dot square. The potential energy is given by V is equal to 2 times mb. If we take this as a datum and say that this is the zero potential energy, this line, then the potential energy of the rods will be negative. Because mass is we know that thing. So, we will put a minus term here; minus 2 mb by 2 sin theta g, because theta is this angle. It becomes mb times sin theta into g. So, you get minus mb sin theta.

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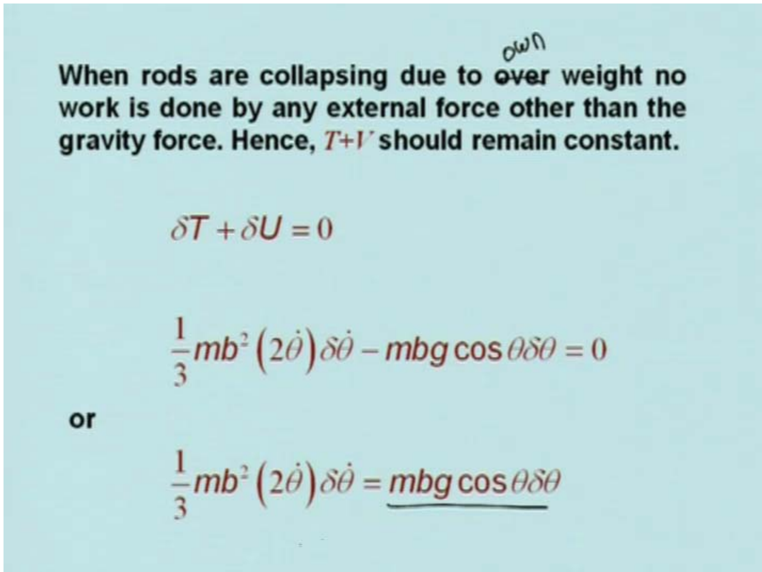

When the rods are collapsing due to own weight then no work is done by external forces other than the gravity forces. Hence, T plus V should remain constant. Therefore, delta T plus delta U is equal to 0; that means, 1 by 3 mb square 2 theta dot delta theta dot minus mb g cos theta delta theta equal to 0, or 1 by 3 mb square 2 theta dot delta theta dot that is equal to mbg cos theta delta theta.

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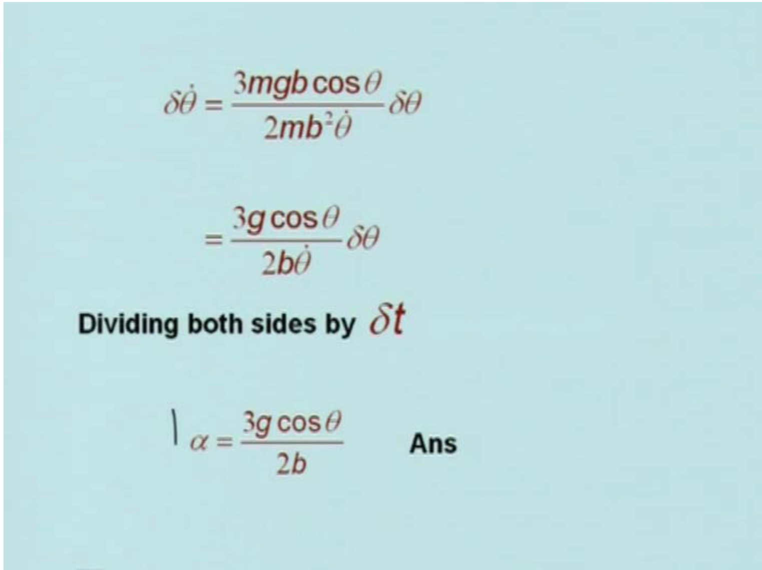

Then we get, delta theta dot is equal to 3 mgb cos theta divided by 2 mb square theta dot delta theta; that is 3 g cos theta divided by 2 b theta dot delta theta. If you divide both sides by delta t then alpha is equal to 3 g cos theta divided by 2 b. We get like this and this is the answer of this one.

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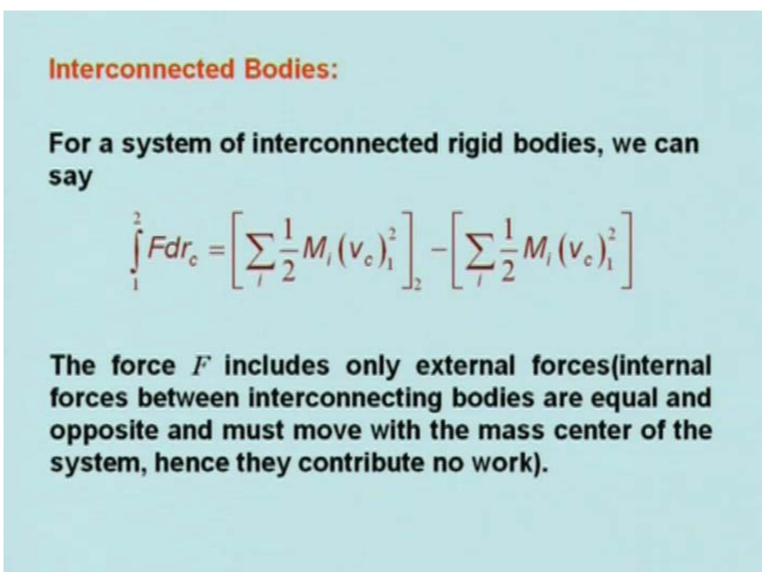

If we have interconnected body, for a system of interconnected rigid bodies, we can write 1 to 2 Fdr_c and this half M_i V_{c1} square and this is half M_i V_{c1} square. The force F includes only the external forces, because internal forces do equal and opposite work. They contribute to do work basically.

We give this example.

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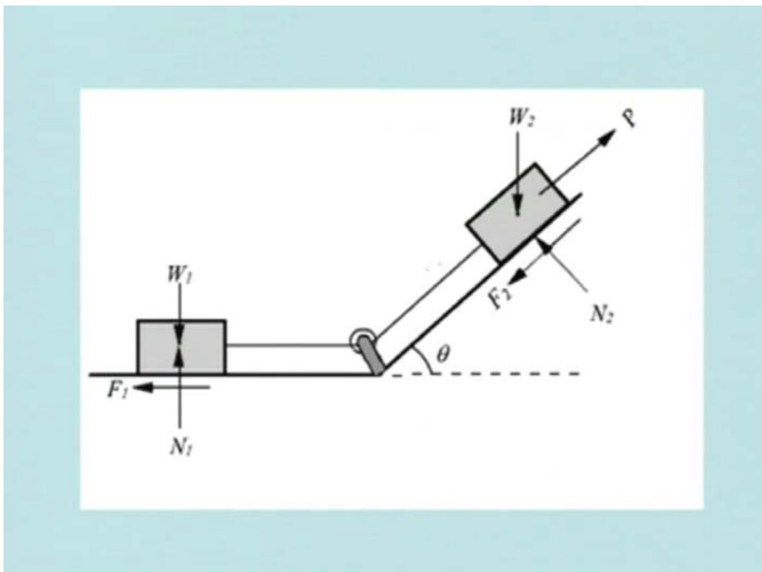

The two bodies are connected in this case. This is the problem.

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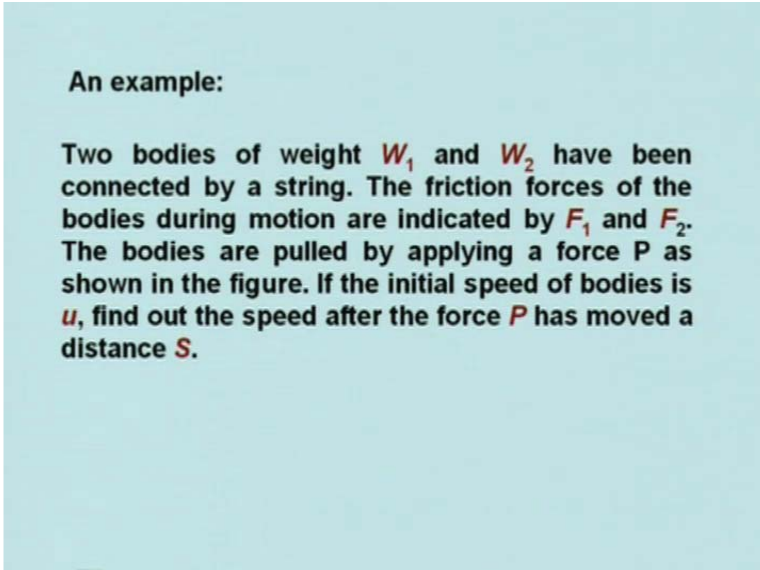

Two bodies of weight W_1 and W_2 have been connected by a string, this one and then this one, it is being pulled here and there is a friction force also acting.

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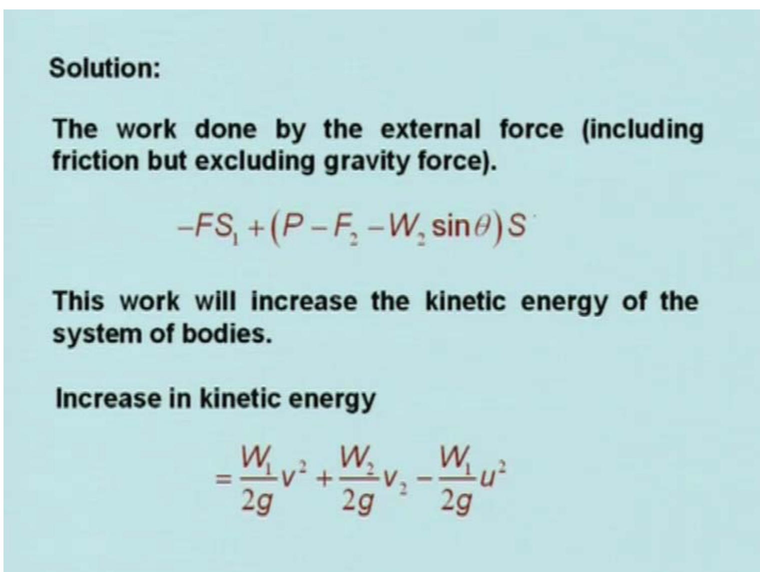

The work done by the external forces including friction but excluding gravity force is, minus FS_1 plus P minus F_2 minus W_2 sin theta into S. This work will increase the kinetic energy of the system. So, increase in the kinetic energy will be W_1 by 2g v square plus W_2 by 2g v square again and minus W_1 by 2g u square.

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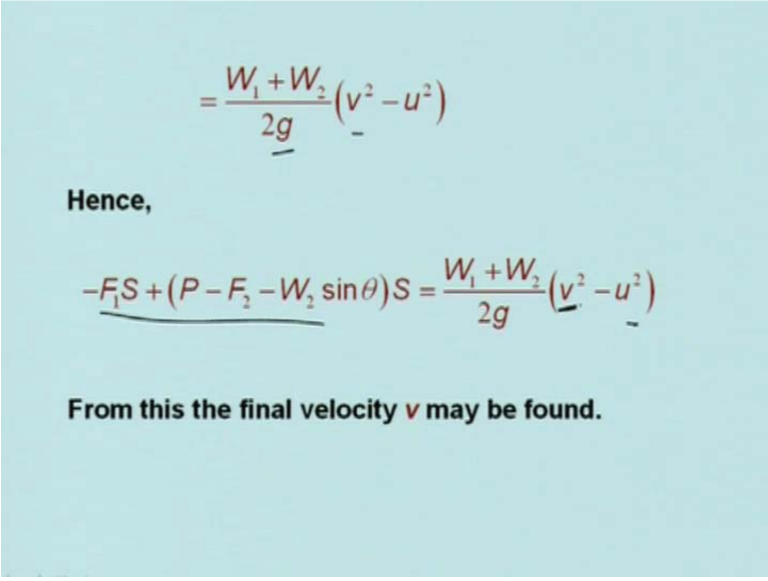

Therefore, this becomes W_1 plus W_2 by 2g v square minus u square. Hence, this work done is equal to this one. So, from this, we can find out the final velocity, given the initial velocity u.