Engineering Mechanics

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Kinetics-1

Module 11 Lecture 31
Plane Kinetics of Rigid Bodies

Having described the kinetics of particles, we will discuss plane kinetics of rigid bodies. We have not taken any lectures on the kinetics of system of particles, because I thought that the same thing gets covered if I describe the kinetics of rigid bodies. A rigid body is after all a system of particles in which continuous distribution of particles is there. So, instead of algebraic additions, here you will get integrations. Actually, same thing can describe the kinetics of system of particles, but instead of integration, you will have discrete additions. Otherwise, essential thing remains same, procedure is entirely same. So, therefore it gets covered here.

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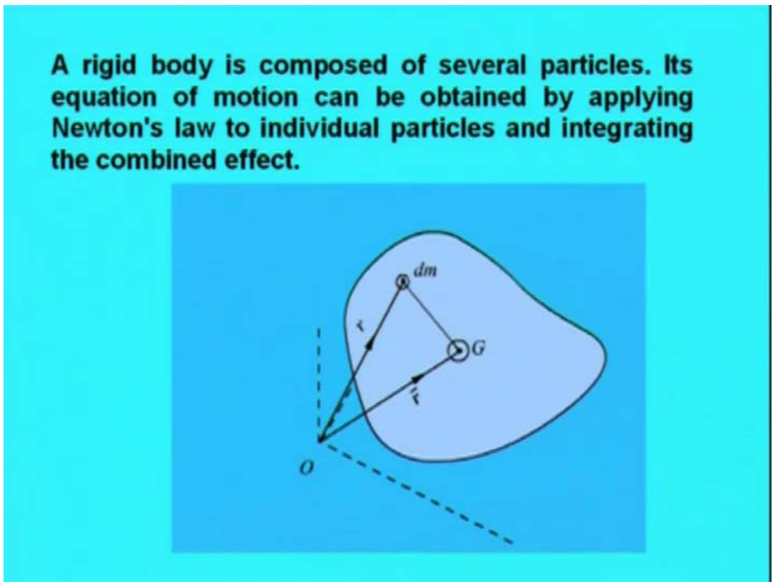

A rigid body is composed of several particles. Its equation of motion can be obtained by applying Newton’s law to individual particles and integrating the combined effect. So, you have got that. So, we have got a lot of particles here and they are of different this one.

We can take small particle of size dm, where dm is tending to 0; that means rigid body is composed of infinite number of particles. However, its mass is finite, its individual particle's mass is 0. So, 0 multiplied by infinite can still give you a finite number. That is how we do that. Therefore, we take a small particle of mass dm which is very small differential element.

The mass center is denoted by G and if we take any axis system, like O x y z, then o times, O to G this distance is r bar and this is O to dm that is r. So, this distance is r and then G to dm this distance is there. Now, here we apply Newton's law.

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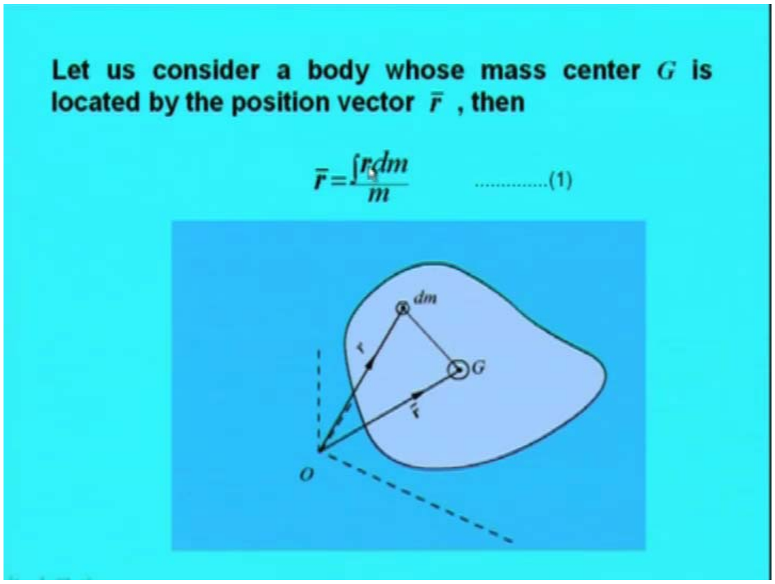

Let us consider a body, whose mass center G is located by the position vector r bar. Then, r bar is actually rdm divided by m where this is r. We have written by bold letters; that means it is a vector. So, rdm divided by m, m is the total mass of the body and rdm, this is the mass equation of the mass center we have integrated.

This is based on the continuum hypothesis. We assume that all the particles, this whole body is in a continuum and there are no voids in between the particles. That is our assumption and we have been able to integrate.

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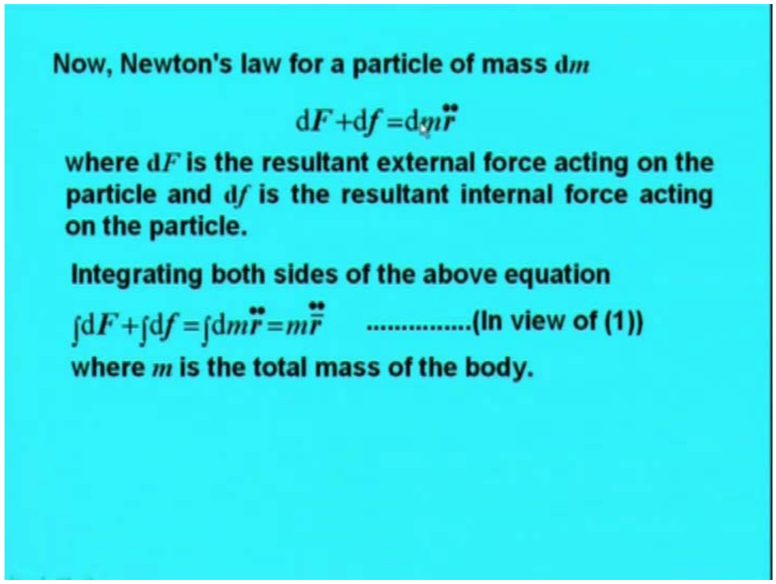

Now, we apply Newton's law for a particle of mass dm and then we have dF plus df is equal to dm times r double dot. For that particle, the force acting on that particle is dF. It is the resultant external force acting on the particle and df is the resultant internal force acting on the particle. That is equal to dm times r dot dot, because r is the position vector of that particle with respect to that outside inertial frame of reference. O is some stationary point, r dot dot will give acceleration and you will have dF plus df is equal to dm into r dot dot.

Integrating, if we integrate both sides of these equations, we integrate dF also. We integrate df and here we integrate dm times r dot dot. Basically if we integrate dm r dot dot that is equal to m into r bar double dot, where we have equation one. Equation one says that r bar is equal to integration of rdm divided by m. So, we get dF plus integral df is equal to dm r dot dot; that is mass times r bar dot, where r bar is the distance of the center of gravity from the fixed point and m is the total mass of this thing.

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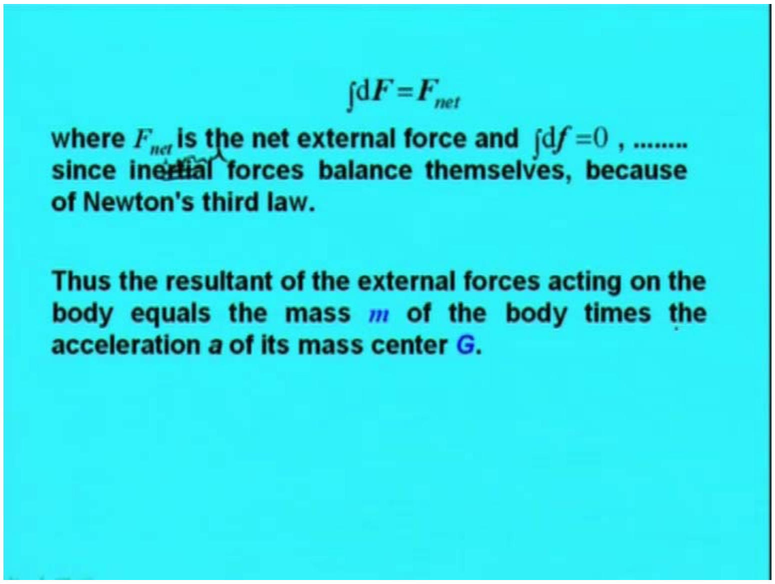

If we integrate dF we get F_{net}, where F_{net} is net external force and if we integrate df, small f then this will become equal to 0, since internal forces balance each other. So basically, the internal forces will balance each other and therefore this is the thing. Internal forces will balance each other. The resultant of the external forces acting on the body equals the mass m of the body times the acceleration a of its mass center G. In that case, you get only this equation that resultant of the external forces are mass times the acceleration of the mass center.

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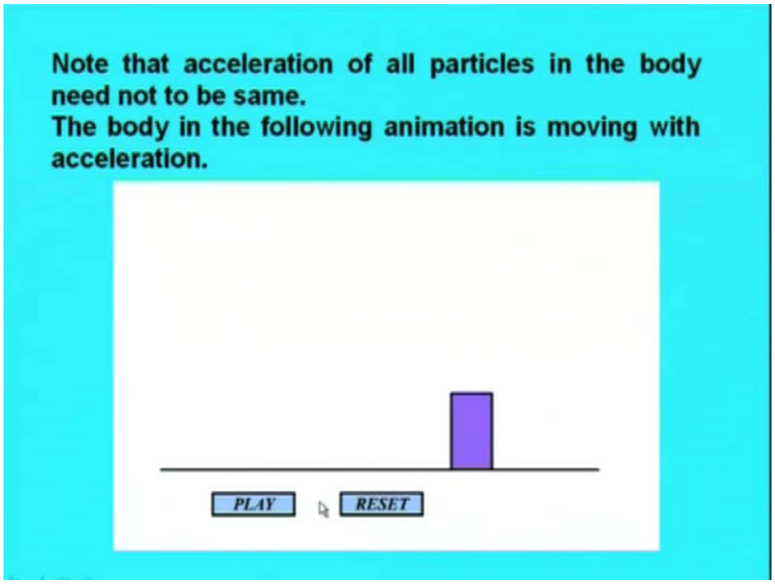

It has to be noted that the acceleration of all particles in the body need not be same. The body in this animation is moving with acceleration. So, you can see the animation. This is undergoing rectilinear motion; that means this body is moving in a straight line. In that case, all particles are moving with the same velocity and same acceleration. Therefore, if we talk about the acceleration of mass center or of any other particle, that does not matter.

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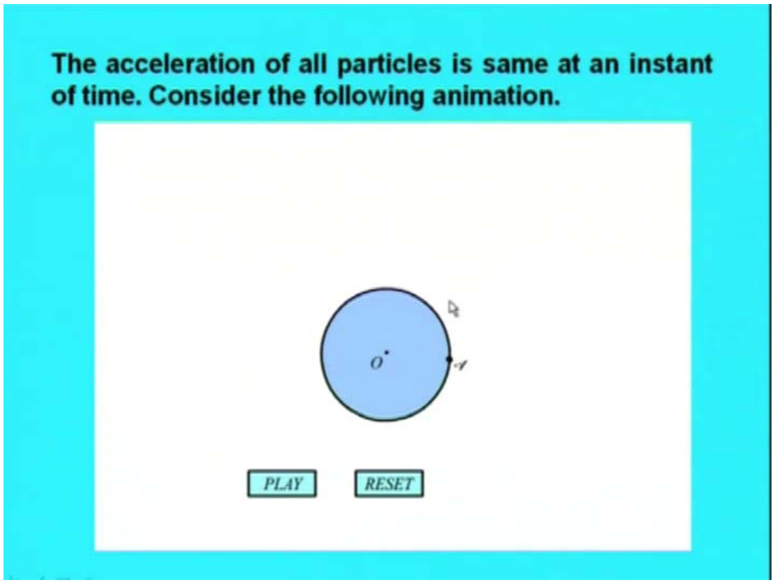

However, in the other case, the acceleration in this animation, this disk is rotating. The acceleration of all particles is different. Particle O is not moving. So, its acceleration is 0; particle A is the having the maximum acceleration. Mass center is O. Although there is acceleration of body; that means some particles are having acceleration, but overall, that mass center is not having acceleration. Therefore, net resultant force, external resultant force must be 0. That condition is there.

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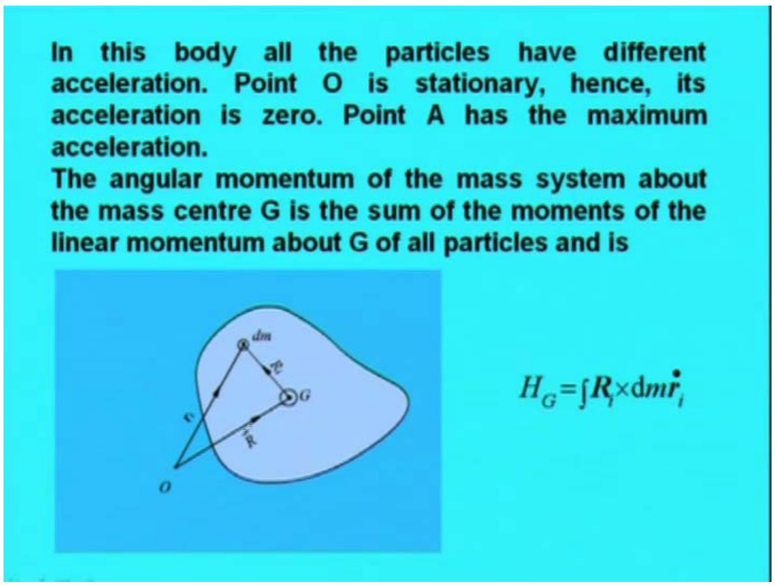

Let us talk about the angular momentum. The angular momentum of the mass system about the mass center G is the sum of the moments of the linear momentum about G of all particles and is given like this; suppose you have a fixed point O here and this is a mass center. This distance OG is r bar, and from G to dm, I have denoted by $R_{i,}$ this capital $R_{i.}$ Then small r_i is the absolute displacement vector of dm with respect to O. You get H_G is equal to R_i cross dm R_i dot. So, this is the angular momentum about mass center, about G. But here, I have taken the absolute velocity. So, dm; that means, you write the absolute linear momentum and cross product it with $R_{i.}$ that means distance from the mass center. Then, you get expression for angular momentum.

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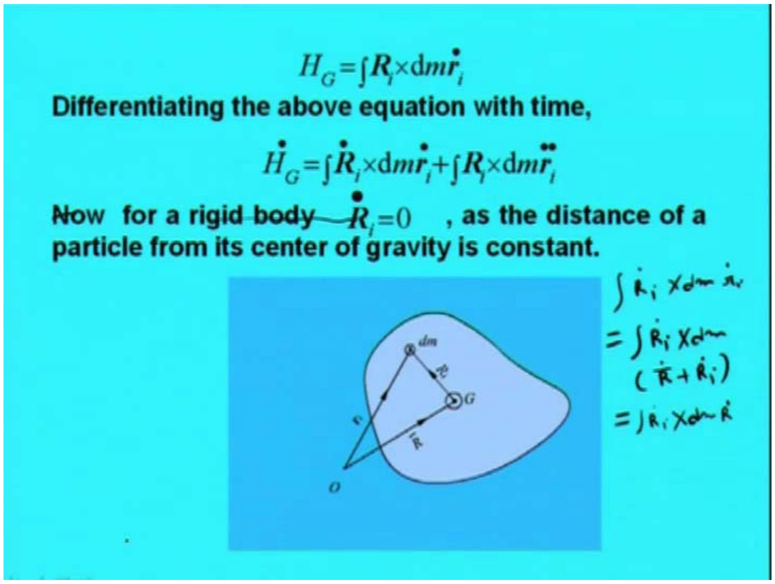

H_G is equal to R_i cross dm r dot i. If you differentiate this expression with time, then you will get H dot G is equal to R dot_i cross dm r dot i plus R_i cross dm r_i dot dot. We applied the product rule of differentiation. It can be easily shown. It does not mean although if the body is moving in a translator mode, then R_i remains in the same direction and R dot_i is 0, because the distance between the particles of the rigid body cannot change. However, if the body is doing rotation, in that case R dot_i need not be 0. This can be very easily shown. Actually this first term, that means R dot_i cross dm R_i will be equal to 0 and only the second term will remain. Now, this can be shown like this, that if we can write $R_{i\,,}$ this is like this.

If this term R_i dot cross dm R_i dot is equal to integral R_i dot cross dm, this will be R bar dot plus R_i dot. Like that we have decomposed. The second term will obviously become 0, because cross product of R dot_i with R do t_i is 0. The first term becomes 0 because by the definition of the mass center. You have R_i dot dm can be taken. Because it is a scalar quantity, you can take it like this. Then what happens, you are left with R dot $_i$ cross dm and R dot.

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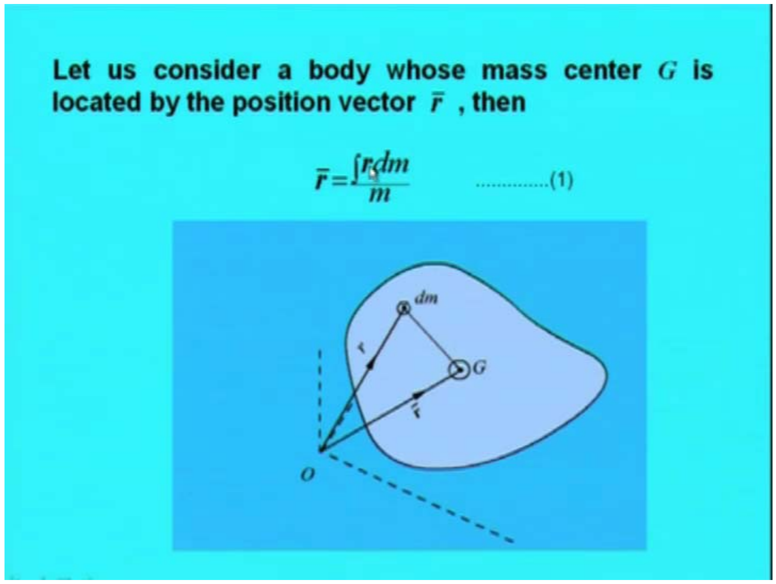

This is 0, because we have that equation number one whereas here, it was shown that rdm. So, if you can show that if you have taken this and differentiate this, obviously this will come out to be 0. So, that way it can be shown. So, first term becomes 0 and the second terms remains R_i cross dm into R_i dot.

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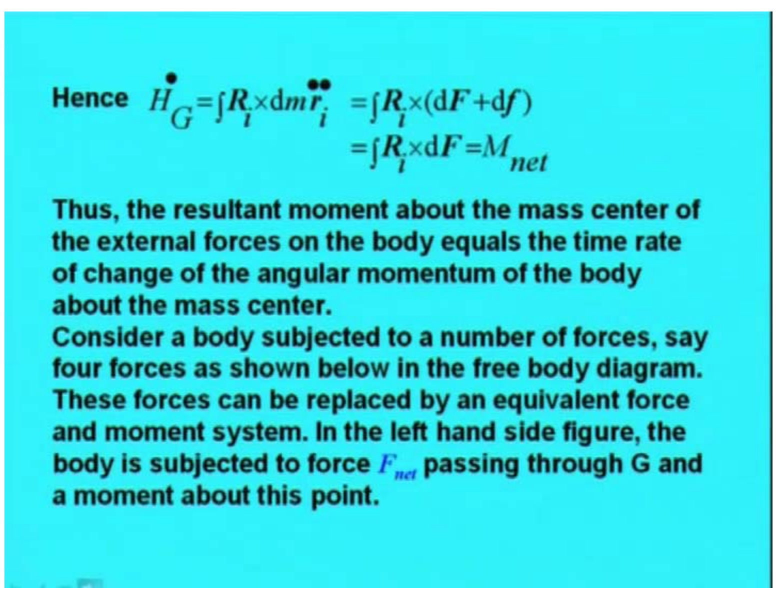

Hence, we have H dot G is equal to R_i cross dm into R_i double dot. That is equal to basically R_i cross dF plus df, because dF is the external force acting and this is internal force acting, so dm times R_i dot dot. Here you can apply the Newton's law and you get this thing. These are the internal forces. So their moment will obviously be 0 and you are left with R_i cross dF and that will equal to $M_{net.}$ Thus, the resultant moment about the mass center of the external forces of the body equals the time rate of change of the angular momentum of the body about the mass center. So, you have got H dot G is equal to $M_{net.}$

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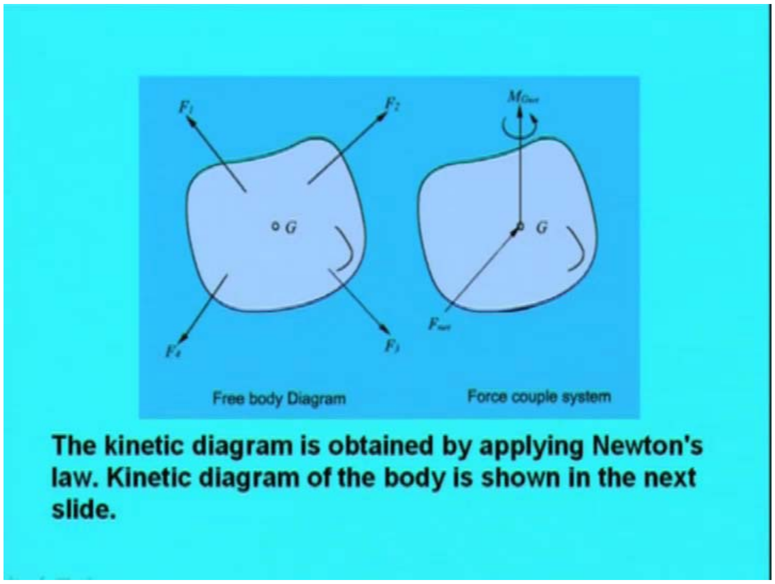

We consider a body subjected to a number of forces. We can consider, it is subjected to number of forces as shown in the free body diagram. Here, that we make the free body diagram, in the free body diagram, we will show all the forces acting on the body. That is called free body diagram. So, we see an arbitrary body with mass center G. We have shown just four forces and one point has to be noted that these forces can be replaced by an equivalent force and moment system.

We can take the effect of all the forces here and we can find out $F_{net.}$ That resultant is passing through G and then we have one couple MG_{net}, because we discussed in the first lecture itself that the force system can be replaced by one resultant force and then the net moment. So this is like that. Then we can obtain the kinetic diagram. Kinetic diagram shows basically the inertia forces you can say that mass into acceleration and mass into linear acceleration and mass into angular that moment of inertia into angular acceleration. So, kinetic diagram is obtained by applying Newton's law. Kinetic diagram of the body is shown here.

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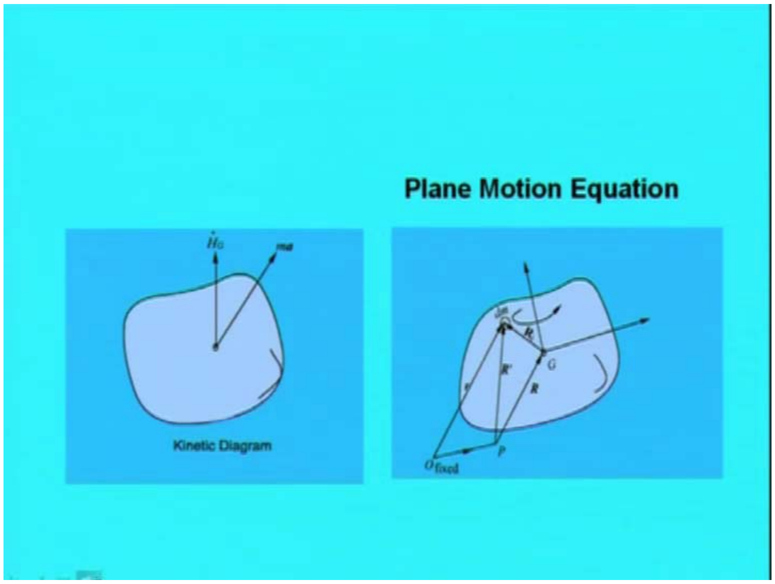

We can see that net force is in some direction. In the same direction mass into acceleration will be active then you have H dot G that is the angular momentum that angular momentum is also shown and it is in the direction of $MG_{net,}$ because we have established that equation that rate of change of angular momentum will be in the direction of $M_{net.}$ So, you have got here rate of change of angular momentum in this direction.

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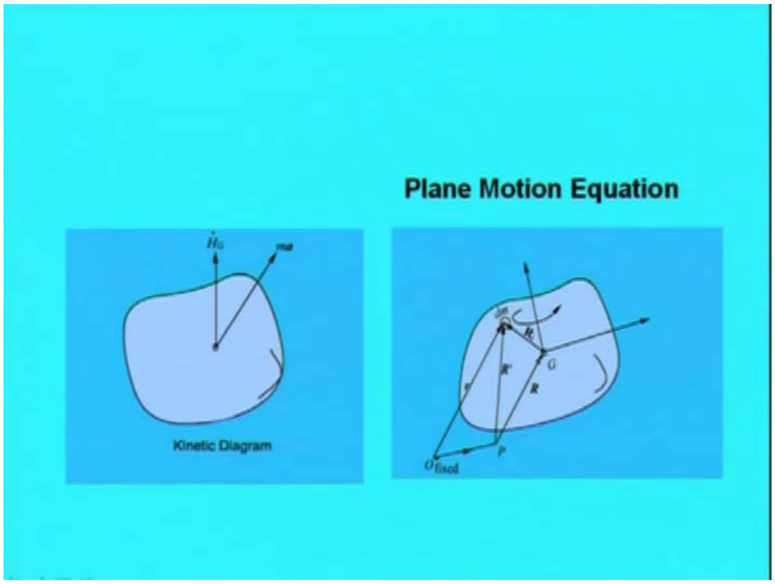

If we consider the plane motion in this slide, the mass center is shown as G and we choose any other point, this point is O which is a fixed point. We take any other point P which maybe on the body or it maybe on the hypothetical extension of the body. In that case, the distance of P to G is R; that is this distance, the fixed distance. Then you have this d small mass, dm that is at a distance of R dash from here. Then, we have G to dm is R_i. We can attach axis system at G also and body maybe undergoing translation as well as rotation.

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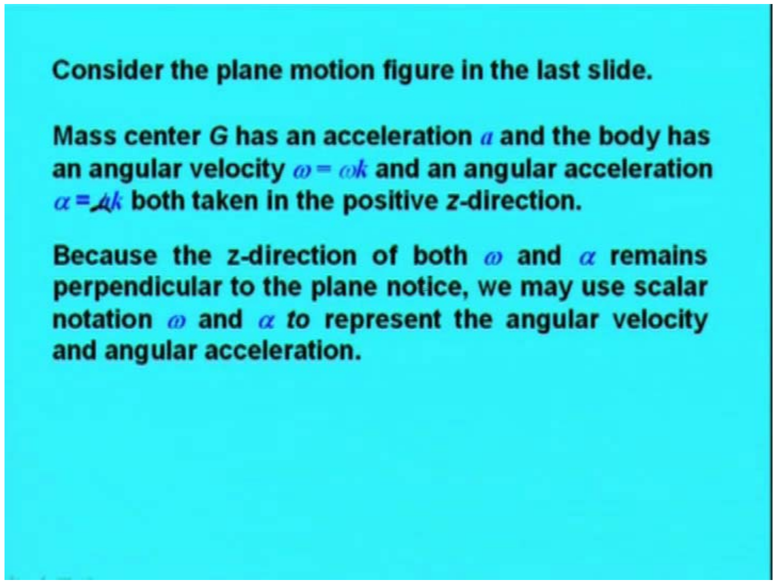

In that case what happens, the mass center G has an acceleration of a and the body has an angular velocity omega is equal to omega k and angular acceleration alpha is equal to alpha k, both taken in the positive z direction and so this will be alpha k. Because the z direction of both omega and alpha remains perpendicular to the plane of this one, we may use scalar notations omega and alpha to represent the angular velocity and angular acceleration. We just can write these scalar equations. That will be enough.

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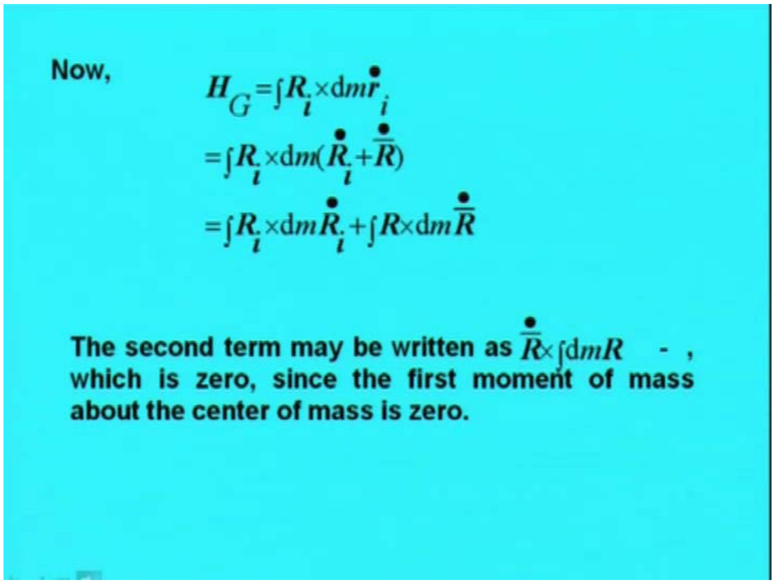

In this case, if H_G is equal to basically R_i, if we talk about H_G is equal to R_i cross dm, R_i cross dm R dot i plus r dot and this is R_i cross dm cross R dot i plus R cross dm R dot. This term becomes 0, because this second term maybe written as R cross into dmR with a minus sign which is 0, since the first moment of mass about the center of mass is 0.

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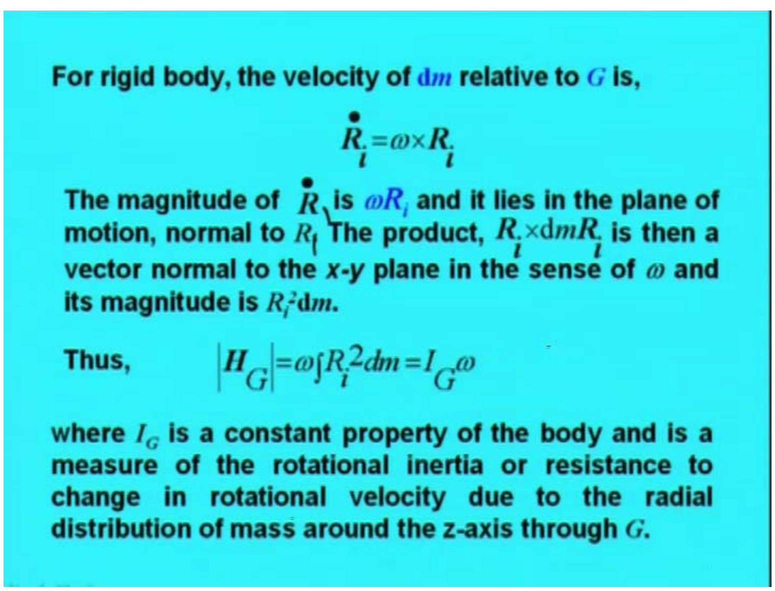

For a rigid body, the velocity of dm relative to G is R dot i is equal to omega cross $R_{i.}$ So, the magnitude of R dot i is omega R_i and it lies in the plane of motion normal to $R_{i.}$ The product R_i cross dmR_i is then a vector normal to the x-y plane in the sense of omega and its magnitude is R_i square into dm. Thus, magnitude of H_G is equal to omega into R_i square dm; that means, it is I_G into omega, where I_G is a constant property of the body. It is measure of the rotational inertia or resistance to change in the rotational velocity due to the radial distribution of mass about this thing.

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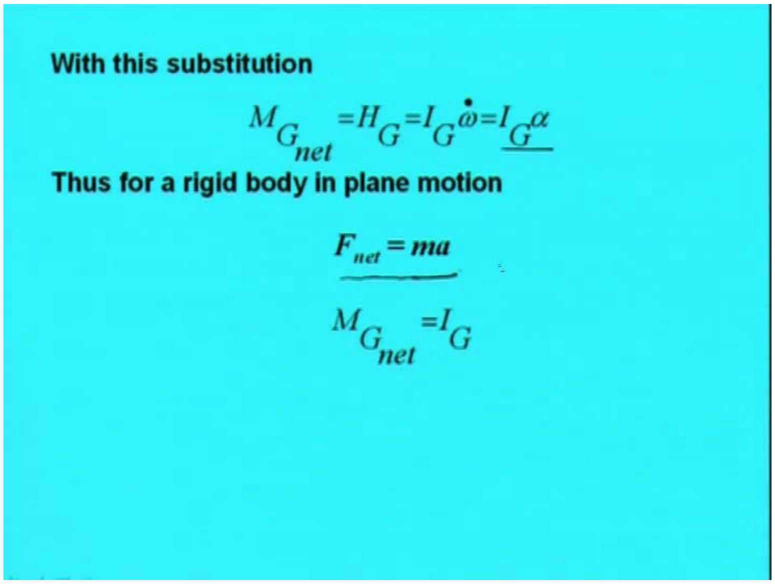

Now M_{Gnet} is equal to H_G that is I_G times omega dot or this become I_G into alpha. For a rigid body in plane motion, you have two equations of motion. F_{net} is equal to ma, that is a vector equation. You can have two scalar equations, F_{net} in x-direction is equal to mass times acceleration in x direction and F_{net} in y-direction is equal to mass times acceleration in y direction, and M_G net is equal to $I_{G.}$ So, these three equations are enough to determine the motion in a plane.

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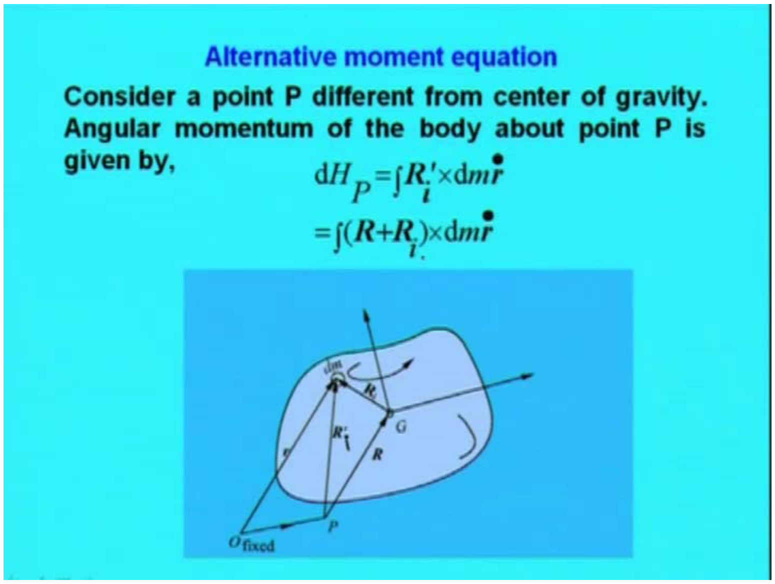

We develop some alternative moment equation. You consider a point P different from center of gravity. Angular momentum of the body about this point P is given by dH_P is equal to R_i prime into dm into r, because this distance is R_i prime from here. So, this is R_i prime and that can be written as R plus R_i cross dm into r dot. So, this can be written as this is equivalent to that.

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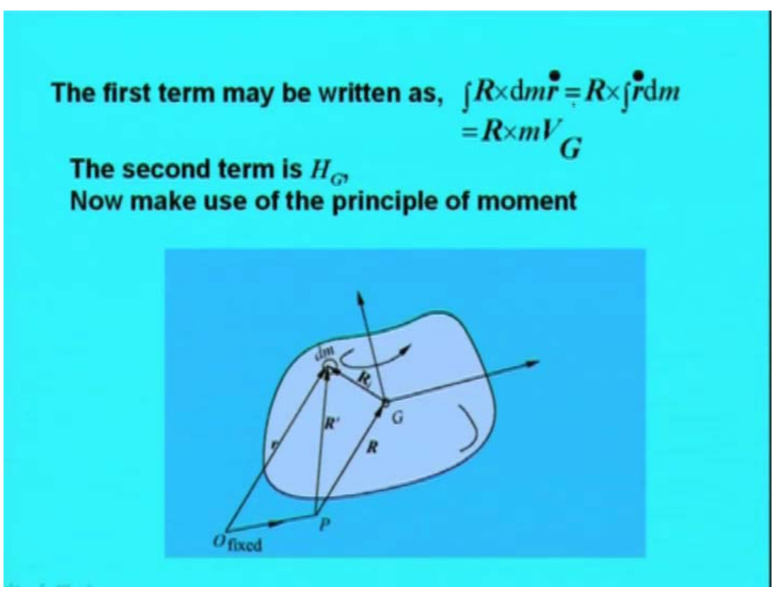

Like, the first term maybe written as R cross dm into r dot is equal to r cross r dot dm or R into $mV_{G,}$ where the V_G is the velocity of the mass center. Second term is $H_{G.}$

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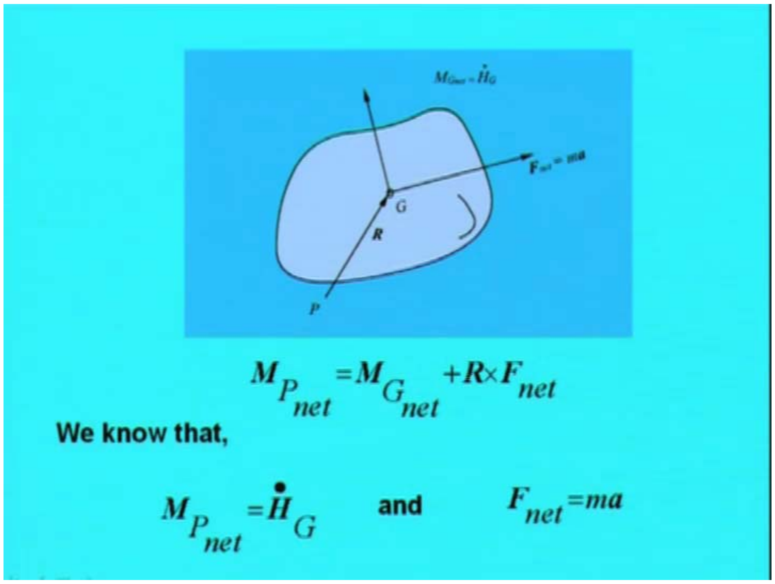

We make use of the principle of moment. So, we can also write $M_{P\ net}$ is equal to $M_{G\ net}$ plus R cross $F_{net.}$ We know, in this case, M_{Pnet} will be H dot G and F_{net} will be equal to mass times acceleration.

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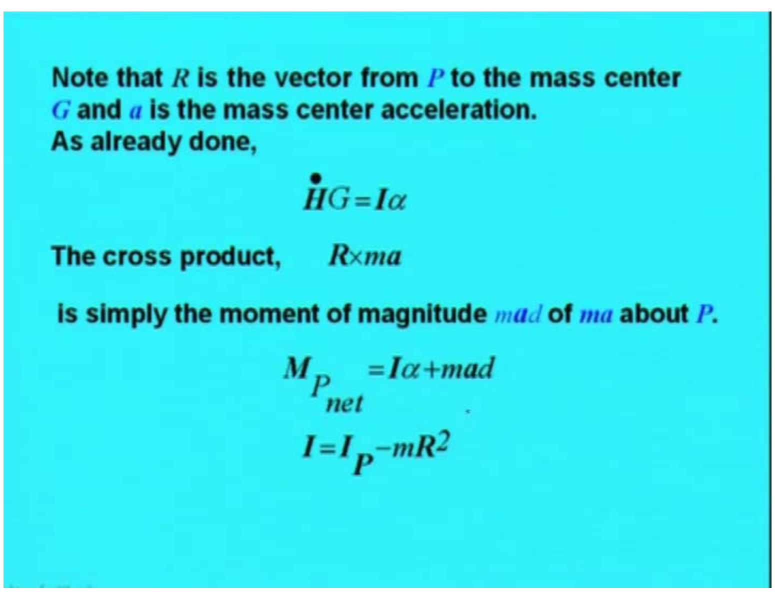

Therefore, R is the vector from P to the mass center G, and a is the mass center acceleration. We already know that H dot G is equal to I alpha and the cross product R cross ma is simply the moment of magnitude m times a times d of ma about aP where d is the perpendicular distance of acceleration vector from that point. Therefore, M_{Pnet} is equal to I alpha plus mad.

We can write, by parallel axis theorem I is equal to I_P minus mR square, because I is the moment, second moment of inertia about the center of mass. Therefore, we can write like this and then it becomes M_{Pnet} is equal to IP minus mR square into alpha plus ma times d and this becomes equal to IP times alpha plus R cross.

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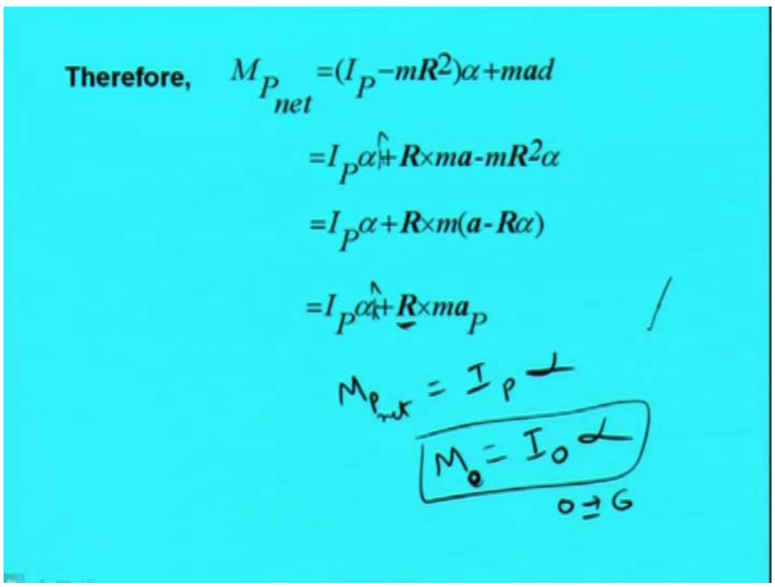

If write that I_P into alpha in the vector form, that means just put k, then after simplification, this will be I_P times alpha k plus R cross mass times acceleration. In this case, if about any point P, P maybe on the body or it may be on hypothetical extension of the body, you have the equation that M_{Pnet} is I_P times alpha. Then you have got R cross mass into acceleration product.

If P is a fixed point, it is not moving. So, this portion goes to 0. We have M_{Pnet} is equal to I_P times alpha; that means, as you find out the equation about the mass center, that moment about the mass center is $M_{G\ net}$ is equal to I_G times alpha. So, you get I_P times alpha. So, that becomes valid. So, first condition is that P should be stationary and then this is valid.

The second is that if aP and all are having the same direction, then also it is valid. Thus, we say that this equation, if M is equal to general form that O is any point, and we write M_0 or M_O equal to I_O times alpha. In that case, this mass where M_O is basically the moment about O and I_O is the mass moment of inertia about point O. So, this equation is valid, provided three conditions; this O is mass center, O is same as G then also it is valid. O is a fixed point or O is having one acceleration which is in the direction of mass center, because R is the point having towards acceleration towards the mass center, then also this equation is valid.

Having discussed that kinetics of rigid bodies in plane motion, now we describe the kinetics of body in different modes.

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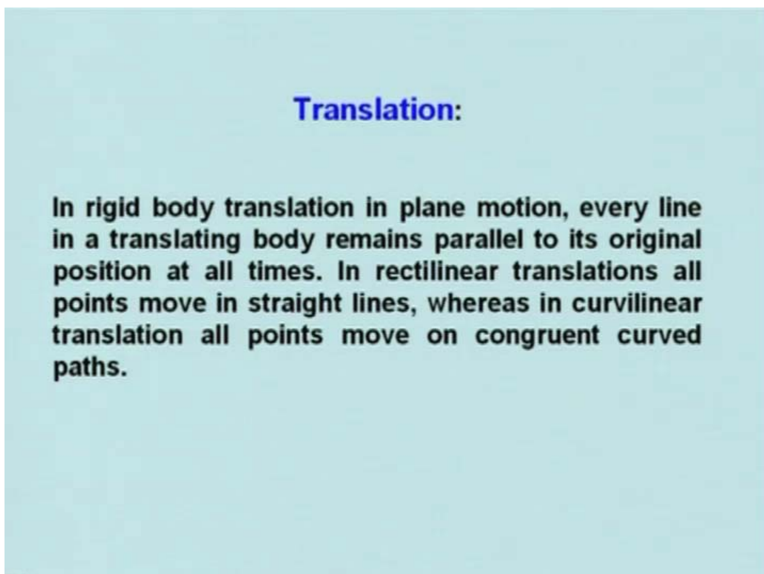

First, we decide that a rigid body in plane can move in two modes; one is the translation other is rotation. Any motion can be broken down into two parts. One is the translatory motion and other is rotary motion. In rigid body translation in plane motion, every line in a translating body remains parallel to its original position at all times. That is the definition. In rectilinear translation, all points move in straight lines, whereas in curvilinear translation all points move on congruent curved paths. So, that means you can have completely straight line type of motion or you can have a motion which will be curved, but still that every line remains parallel to its original position and therefore this can be called as this one.

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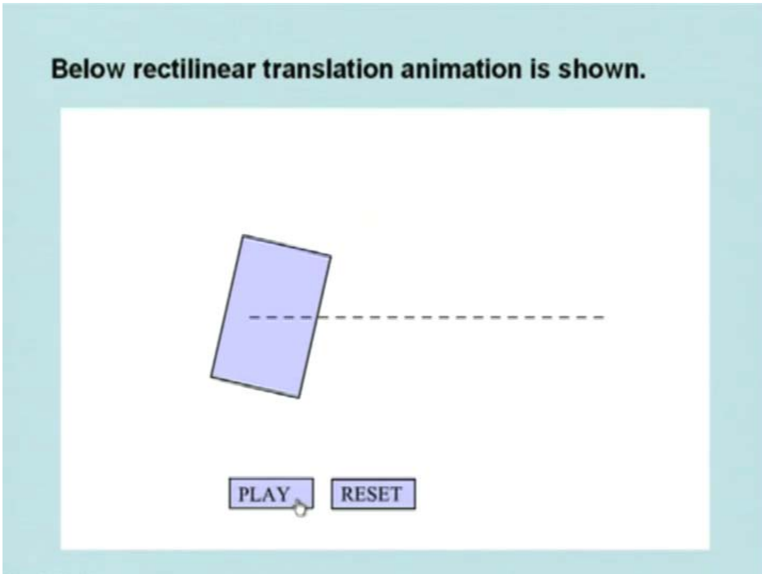

I am showing one animation for rectilinear translation motion. Here, any particle of the body is moving in a straight line. So, the particles here are moving in a straight line. Similarly, the particles here are moving in another parallel straight line. So this is the example of rectilinear translation.

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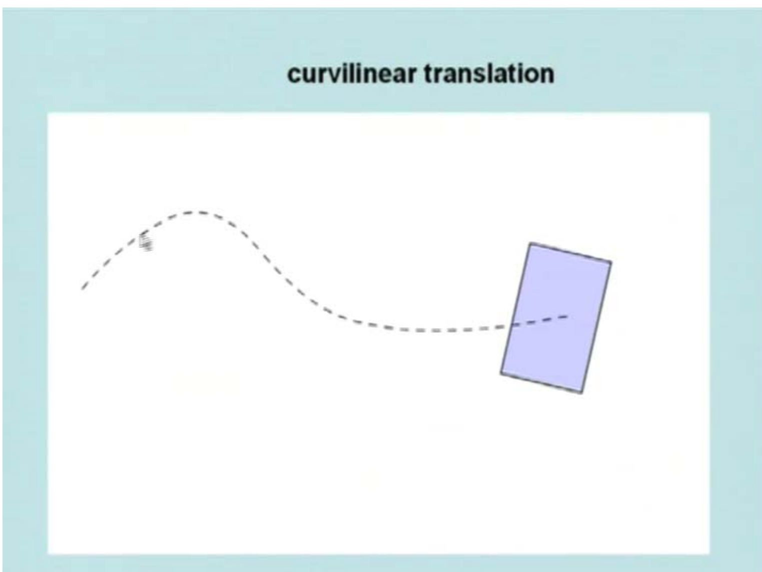

This is a curvilinear translation. Here, if we follow any typical particle say mass center, we will see that this mass center keeps on moving on the curved path. This is example of curvilinear translation. However, during the motion, anytime if you see any line, this particular edge of the body remains parallel to itself. So, it is not rotating. That angle theta remains same. Therefore, omega, in this case is 0. So, translating body has angular velocity 0 and angular acceleration is also equal to 0.

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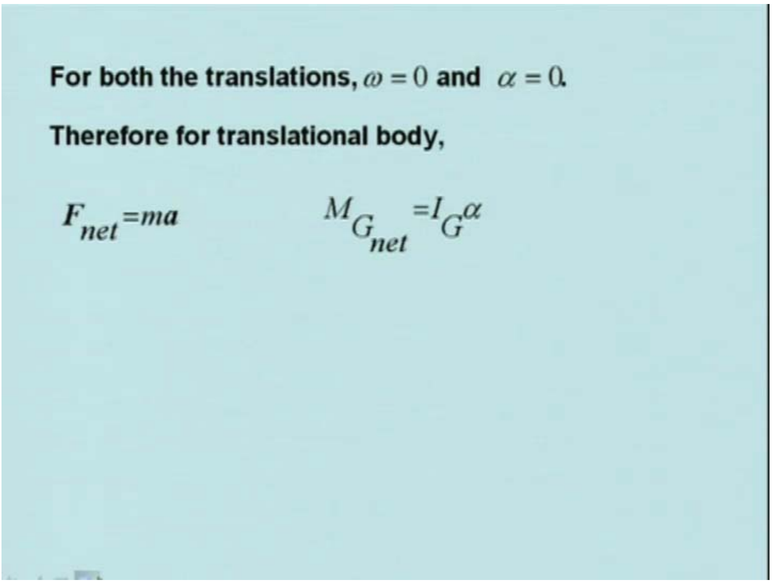

Therefore, for both translations omega is 0 and alpha is 0. So, for translation of a body, F_{net} then becomes equal to mass times acceleration, because that was one equation. This is a vector equation, F_{net} is a vector and that is equal to mass into a and M_{Gnet} is equal to I_G times alpha. So, M_{Gnet} is this one and so alpha is 0. Therefore M_{Gnet} is 0, means if you take about the mass center that M_{Gnet} will be equal to 0.

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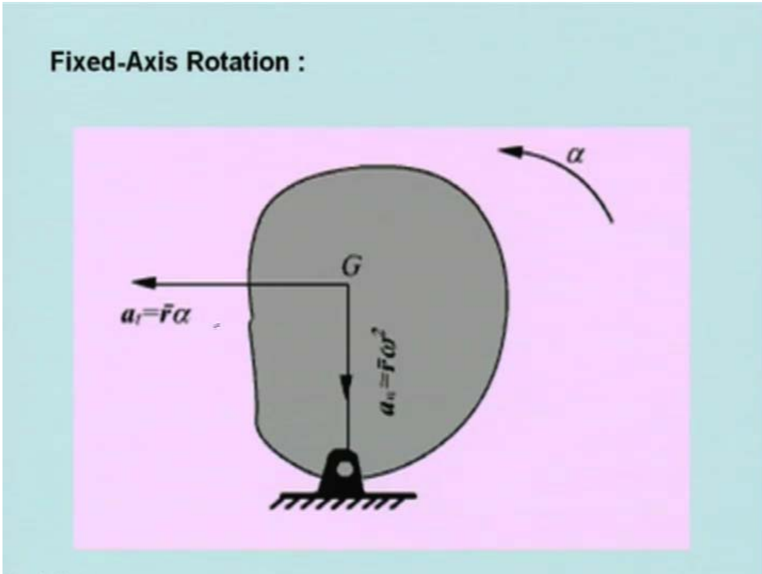

Let us discuss another type of motion; that means fixed axis rotation. Now, in the fixed axis rotation, this is the body which is rotating about the point O. I have shown the mass center G. Here, this is the mass center G and then you have got that a_t is equal to r times alpha. It is rotating about this point. So, angular acceleration is given by alpha and the linear tangential acceleration is equal to r times alpha, where r is the distance of O to G. Similarly, the normal acceleration will be directed towards center that is equal to r bar square, where omega is the angular velocity of rotation and alpha is the angular acceleration. Now, in this body let us see what these equations are.

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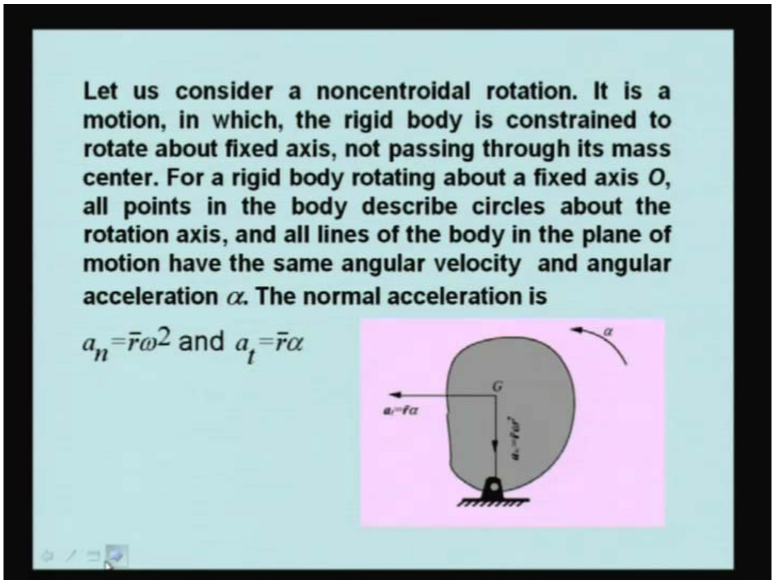

We consider noncentroidal rotation that means assume that O is not the center of mass. In noncentroidal rotation, rigid body is constrained to rotate about fixed axis, not passing through the mass center. For a rigid body rotating about a fixed axis O, all points in the body describe circles about the rotation axis, at all times of the body in the plane of motion and all lines of the body in the plane of motion have the same angular velocity and angular acceleration alpha. This type of things we have discussed actually in the last classes.

The normal acceleration a_n is equal to omega square r bar and a_t is equal to r bar into alpha. Therefore, these things will hold good. Then the equations of dynamics are basically this. We get always these two equations; F_{net} is equal to mass times acceleration and M_G net is equal to I_G times alpha.

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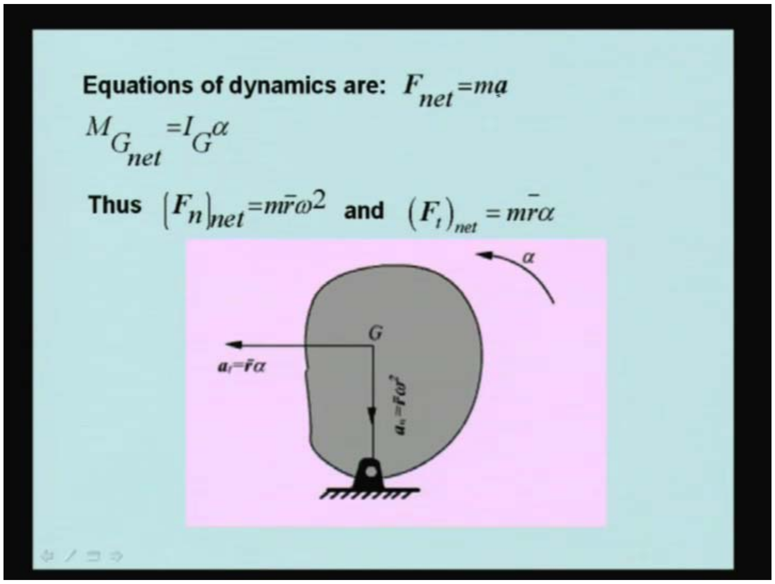

Thus, we have $F_{net.}$ Now, F_{net} is equal to m alpha. This is basically a vector equation. We can make out two scalar equations here one is the F normal net is equal to mr omega square and F tangential net is equal to mr times alpha. Then, we get M_{Gnet} is equal to I_G alpha, which is of course scalar equation. Otherwise, we can write it as a vector equation also. If we attach k bar, here k is the unit vector in the z direction. Since it is same, actually both side we eliminate k and we write in a scalar form, M_{Gnet} is equal to I_G times alpha. These are the three equations we have got and by this, we can get the required things.

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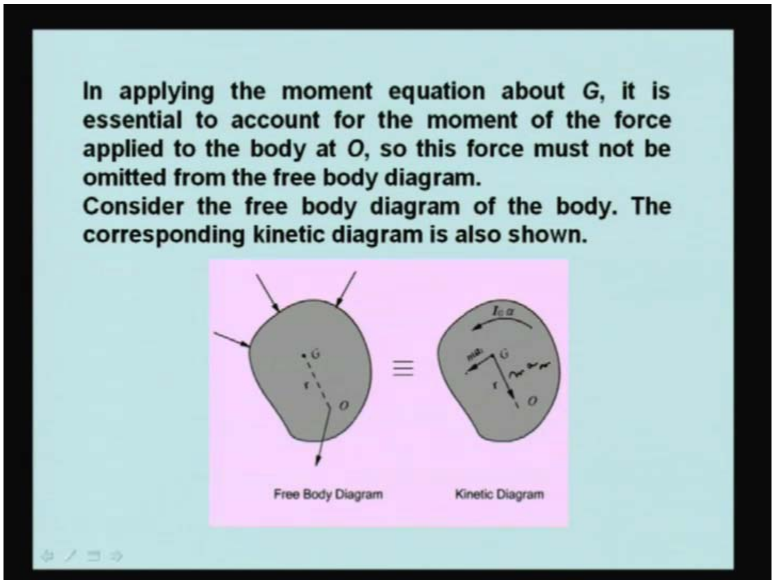

When you apply the moment equation about G, like you have to apply $M_{G\ net}$ is equal to I_G times alpha, then always remember that you have to account for the moments of the all the forces applied to the body. That means, even you have to consider the forces, internal forces which are applied at O. When you make a free body you have to understand that there are some internal forces at O. These also have to be reaction forces and they have to be taken into account.

This force must not be omitted from the free body diagram. Even if you omit some force which is passing thorough G, then maybe it is alright, because when you take the moment about G that force in fact vanishes, but in general you should show all the forces. Now, consider the free body diagram of the body. For example, this body is acted by these forces and it was fixed at O. There must be some reaction force acting. That force also has been taken into account here and this is r and this is G. We make the equivalent kinetic diagram here. So, this is I_G times alpha. We show that this is I_G alpha and then here mass times the tangential acceleration passing through G and mass times that normal acceleration is basically m a n.

In kinetic diagram, we will be showing these things m a t and m a n and I_G alpha. This is a kinetic diagram. From free body diagram, one can easily obtain the kinetic diagram by applying Newton's law. The resultant force passing through G, you have to transform these forces into a resultant force passing through G and then you can find out mass into acceleration. Here, mass into acceleration vector maybe in the same direction as the resultant force passing through G and then you can find out the I_G into alpha, because if you can find out the moment of all the forces about G, then you can get this thing.

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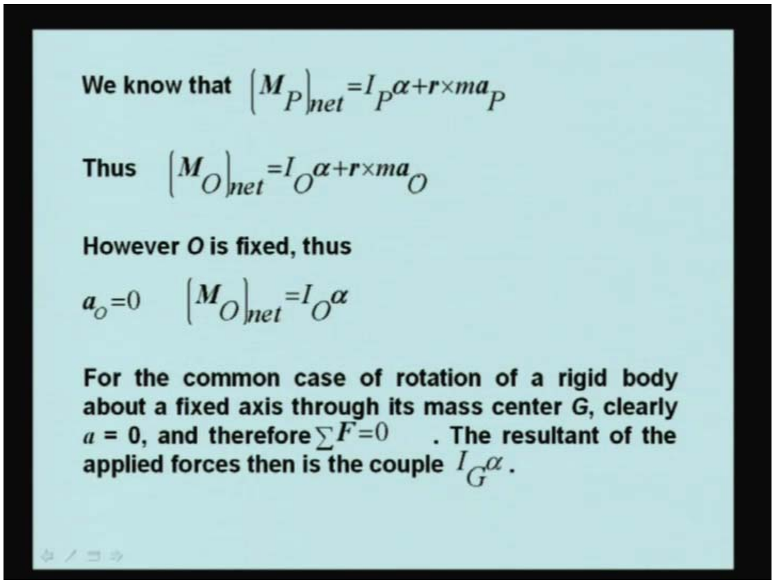

We also have discussed in this lecture, this equation that if you take any point P, say P is any point, then you get M_{Pnet} is equal to I_P times alpha plus r cross mass into a_P. Here, I_P is the moment of inertia of the body, second mass moment of inertia about the body. But it is about point P, and this is r cross m a_P here a_P is the acceleration of point P, not the point G, it is point P. So, we get this equation.

If we are talking about the fixed point O, then we can use this equation. Instead of P just write O. You get M_{Onet} is equal to I_O times alpha plus r cross ma_O. However, O is fixed. Thus, acceleration of a_O is 0; so, a capital O is equal to 0. Therefore, you get M_{Onet} is equal to I_O times alpha. So, this is the equation for the moment. For the common case of rotation of a rigid body about a fixed axis through its mass center G, if you have a rotation of a rigid body about a fixed axis which is passing through the mass center G, then in that case a will clearly be 0 and therefore sigma F is equal to 0. The resultant of the applied force is the couple I_G times alpha. In that case, it will be simply I_G times alpha, because that is the thing. This way, these equations can be used to study the rotation of a body.

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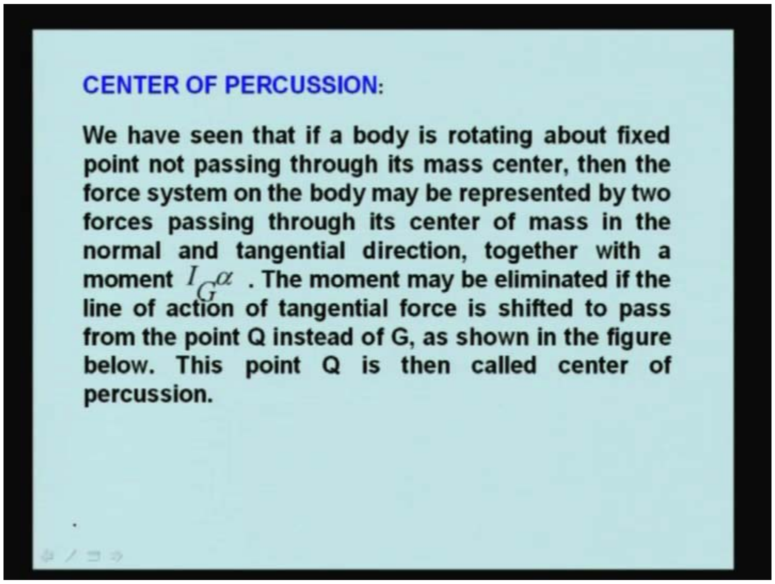

We are going to introduce one concept that is called center of percussion. We will tell about the center of percussion here. We have seen that if a body is rotating about fixed point, not passing through its mass center, then the force system on the body maybe represented by two forces passing through its center of mass in the normal and tangential direction, together with a moment I_G times alpha. You see always, in this case for example, you know kinetic diagram has been shown. From kinetic diagram, one can obtain a resultant free body diagram. Here, one can see the forces.

Since in the kinetic diagram, you are having that I_G into alpha; that means, the moment should be present here. That moment is equal to I_G into alpha and mass times a_t is equal to F_t and mass times a_n is equal to F_n. Therefore, what happens that if a body is rotating about fixed point not passing through the mass center, then the force system on the body maybe represented by two forces passing through its center of mass in the normal and tangential direction, together with a moment I_G times alpha.

The moment maybe eliminated if the line of action of a tangential force is shifted to pass from the point Q instead of G. In the same line you join O and G and then after that you shift. So, as shown in the figure, the point Q is called the center of percussion.

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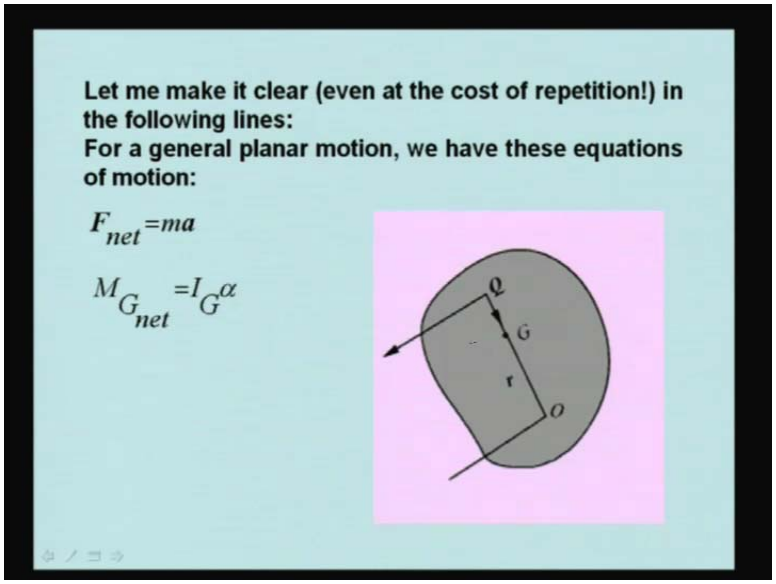

I have made another diagram. In this diagram, the moment has eliminated; why because, there was a force passing through G. I have shifted it in a parallel manner to another point Q on the same line. So, this is, in effect if you take the moment about G you will still get a moment, because the force is not passing through point G. So, there is no need to show the force. Therefore, this has been shifted and then this point Q will be called center of percussion.

Let me repeat these points, because this is very important. So, this point Q is a center of percussion, we have described. Now, from another angle, for a general planar motion, we have these equations of motion that is F_{net} is equal to mass times acceleration and then M_{Gnet} is equal to I_G times alpha. So, these are basically three scalar equations F_{net} is equal to mass times acceleration and M_{Gnet} is equal to I_G time is alpha.

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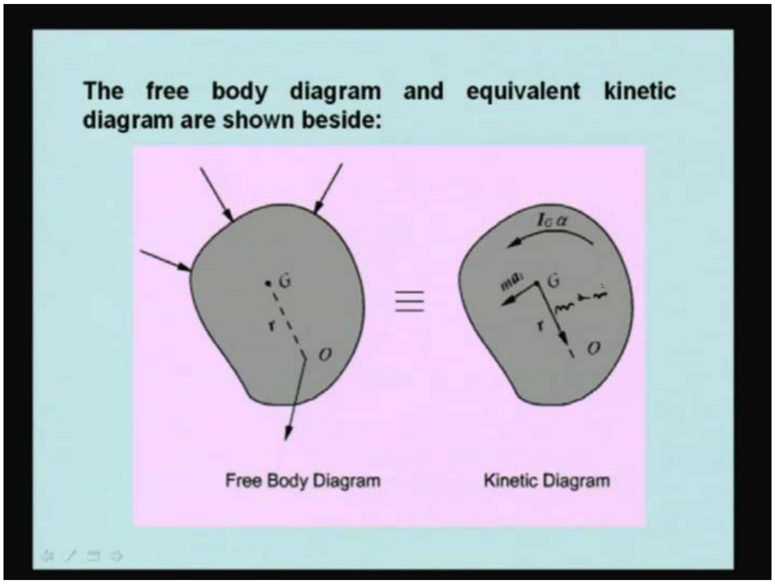

Now, free body diagram and equivalent kinematic diagram are shown here. Free body diagram shows the forces, externally applied forces and also the reaction forces passing through O. G is the mass center and then the kinetic diagram is this; mass times tangential acceleration, mass times normal acceleration is here and then you have got I_G times alpha.

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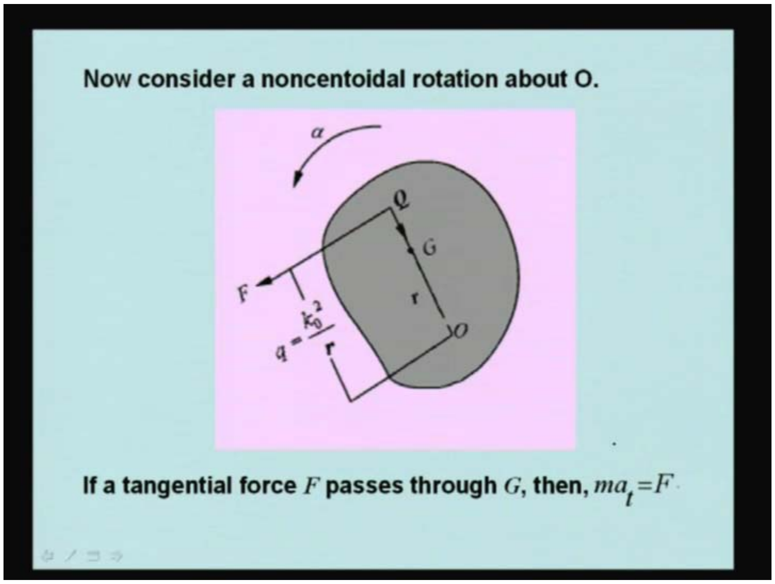

Consider a noncentroidal rotation about O. Like in this case, if we pass a tangential force through G, if the tangential force was passing through G then mass times a_t will be equal to F.

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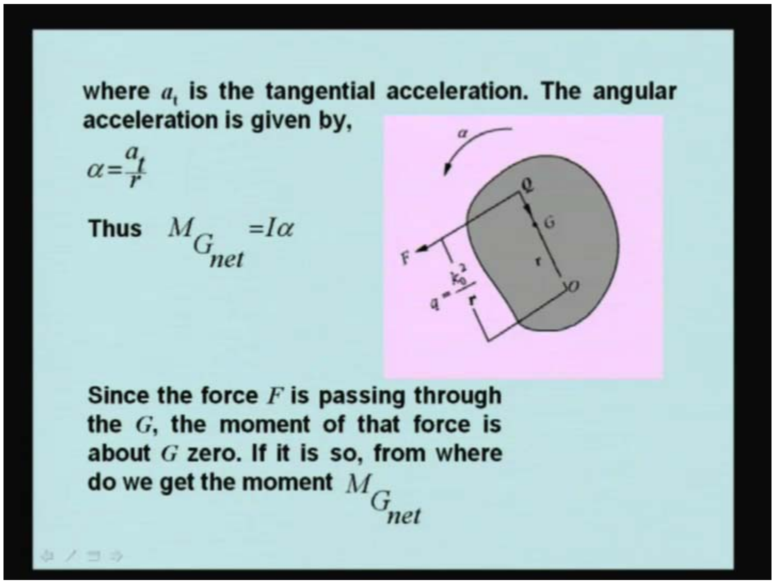

Where a_t is the tangential acceleration, the angular acceleration is given by alpha is equal to a_t times r. Thus, you will have M_{Gnet} is equal to I times alpha. Since the force F is passing through G, the moment of that force is about $G_{0.}$ Now, if it is from where we do M_G net then from where, because we have to balance this M_G net is equal to I alpha, obviously we get it from O, because there are some points at that O.

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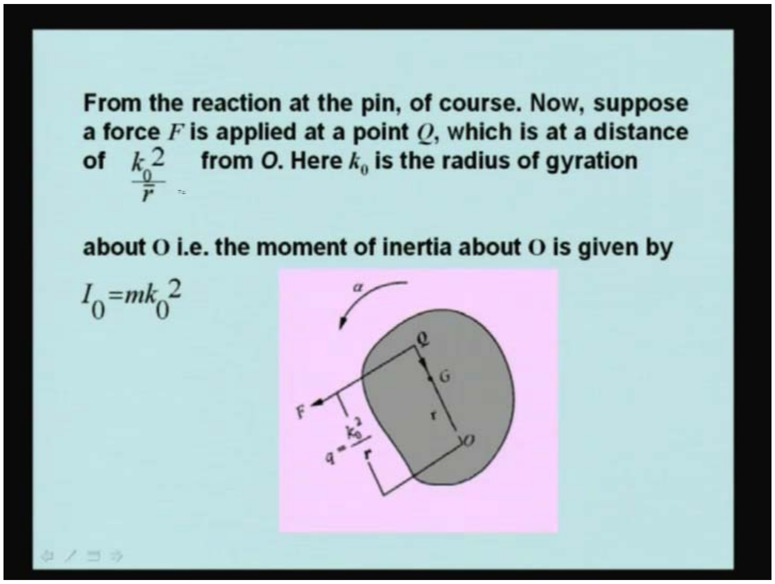

From the reaction at the pin you know that, because it is pinned at O we will get that. Suppose a force F is applied at a point Q, which is at a distance of Q is k_0 square by r from O. Instead of that you know that is passing through G, if now we apply to pass through Q, where k_0 is the radius of gyration about O; that is, the moment of inertia about O is given by I_0 is equal to mk_0 square, so this is given here.

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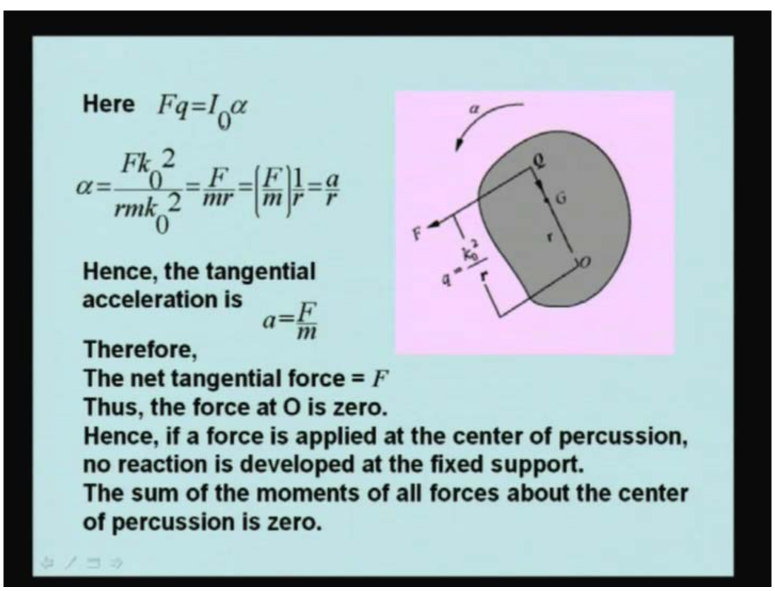

Then you take the moment about O. You will have that F_q is equal to I_0 times alpha, that is the basic equation about O you have applied. So, alpha is equal to a F F_q by I_0. We have an expression for Q. So, it is Fk_0 square divided by rmk_0 square is equal to F divided by mr and this is equal to F by m, a into1 by r; that means, this becomes equal to a by r. Hence, the tangential acceleration is, a is equal to F by m. Therefore, the net tangential force obviously must be F only. Thus, the force, the net tangential force is F and we have already applied the force F passing through this one Q. Therefore, this force applied by the pin must be 0. Therefore, we conclude that if a tangential force is applied at the center of percussion, then no tangential reaction is developed at the fixed support.

We can also conclude that the sum of the moments of all forces about the center of percussion will be 0, because if you take the moment about the center of percussion then that is 0. Therefore, if you want to get maximum advantage, sometimes you know batsman will hit the ball; in that case, if he hits the ball at the center of percussion, he will not feel any reaction at the hand. Therefore, that impact force will not be experienced by him and he can still provide the angular momentum to the bat.

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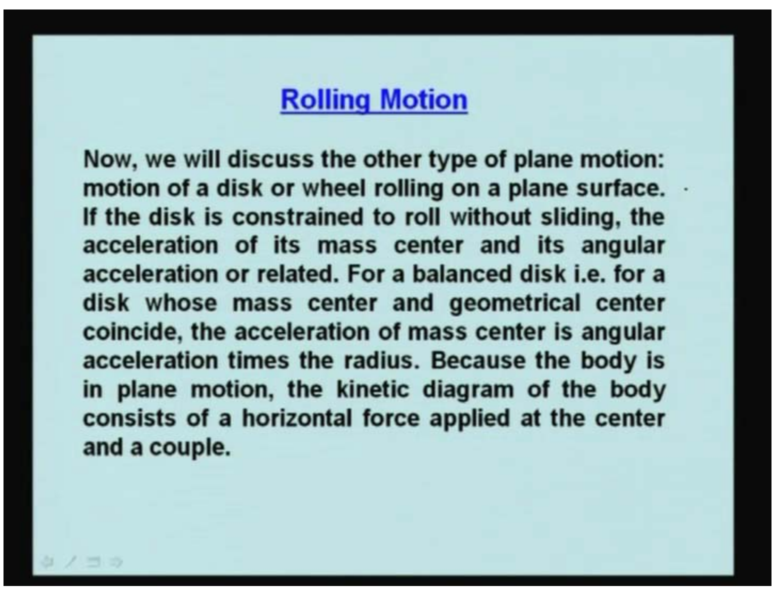

The center of percussion is very important in many situations to find out what is the center of percussion. Now we discuss the other type of motion that is called rolling motion. This is the motion of a disk or wheel rolling on a plane surface, the surface maybe horizontal or it maybe inclined to this one. If the disk is constrained to roll without sliding, the acceleration of its mass center and its angular acceleration are related; like, in the plane, in pure rolling motion the acceleration of the mass center and the angular acceleration will be related, but if there is some slippage taking place, then there is no relation. Now, you can consider the different cases, like one, you can have balanced disk. A balanced disk is that disk whose mass center and geometric center will coincide. If you have a cylinder, then if its mass center is located at the center of the cylinder, then it is called balanced. If mass center is different then you know that it is called unbalanced disk.

In that, the acceleration of mass center is angular acceleration times the radius, that is radius of this one, because the body is in plane motion. The kinetic diagram of the body consists of a horizontal force applied at the center and a couple. So, we can make the kinetic diagram because body is in the plane motion.

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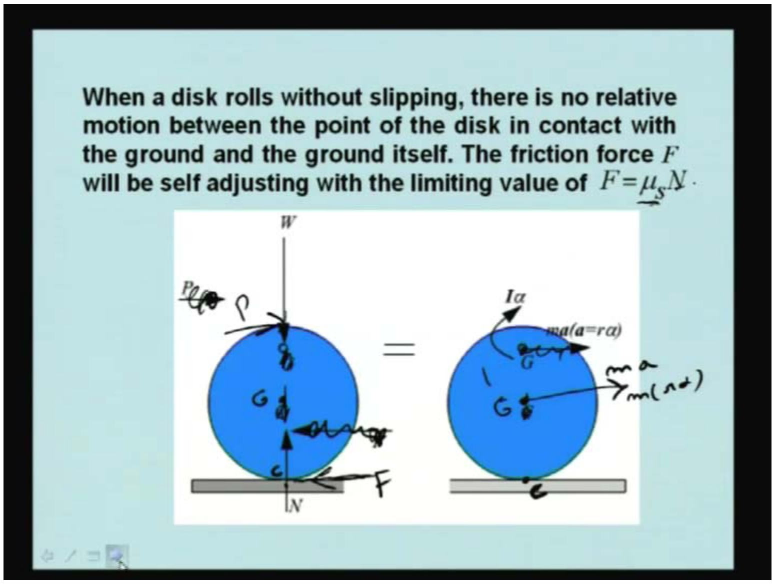

Let us consider this case. Here, this is the plane surface and the cylinder is rotating. This is the contact point C. From this, you are having a normal reaction N and then you are having the mass F. Then, there is a weight W. So, this is G, that mass center is G and this is W. You have some frictional force F. F actually will pass through this and then you may have some applied force acting here and that is P and its weight passes through its mass center G. This is equal to the equivalent kinetic diagram shown here. Here, the mass center is C and this is G that is passing through this. So, you have mass times acceleration and this will be mass times r alpha and this is I alpha.

When a disk rolls without slipping, there is no relative motion between the point of the disk in contact with the ground and the ground itself. The point C is the instantaneous center of rotation. You get a friction force F also here, but this force is self resisting; means, if you apply some force here P, the same amount of resistance is developed here with a limiting value of F is equal to mu_s times N. So, mu_s is the coefficient of friction into N.

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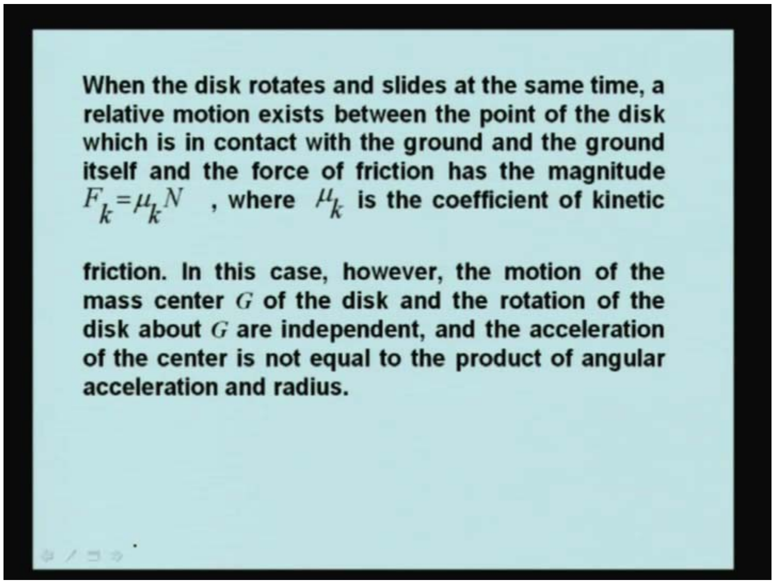

When the disk rotates and slides at the same time, a relative motion exists between the point of the disk which is in contact with the ground and the ground itself and the force of friction has the magnitude. In this case, F_k is equal to mu_k times N, where mu_k will be the coefficient of kinetic friction. Kinetic coefficient of kinetic friction is usually less than the coefficient of sliding friction. Sometimes, it maybe as less as that; that means, coefficient of kinetic friction maybe some 50% of the coefficient of sliding friction.

In other cases also, the difference may not be that much but there is some difference. So, you get F_k is equal to mu_k times N. In this case, however, the motion of the mass center G of the disk and the rotation of the disk about G are independent. You do not have the formula, like V is equal to omega r. These types of things cannot be done. Similarly, mass center acceleration cannot be said to be alpha times r; that will be less than or equal to mu_s times N.

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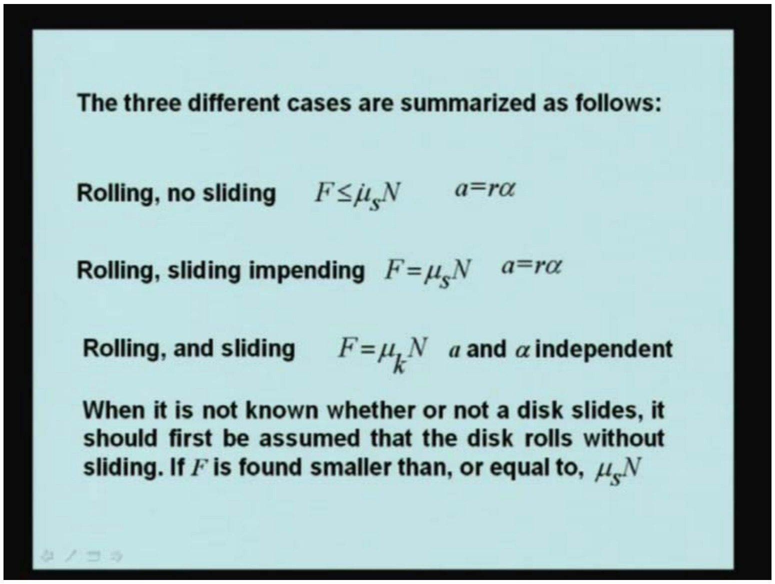

Either it will be less tangential friction force, either it will be less than mu_s times N or it maybe at most equal; because, mu_s times N is the limiting value and s is the coefficient of sliding friction. Then, you have one relation for geometry, that means a is equal to r times alpha. So, you have, a is equal to r times alpha, where r is the radius and alpha is the angular acceleration. Then you can have another situation that the pure rolling is taking place, but sliding is impending; that means, sliding is about to begin. In that case, F will be exactly equal to mu_s times N and you will have a is equal to r times alpha, where alpha is the angular acceleration and r is the radius of that disk.

Then, you can have the third condition in which there will be rolling but there will not be any sliding; both will go simultaneously. In that case, the equation one will be that F is equal to mu_k times N that normal force and tangential forces are related by kinetic coefficient of friction that is mu_k and a and alpha are independent in this case. We will not have any relation between a and alpha.

When it is not known whether a disk is sliding or rolling, then what should we do? We should first assume that the disk rolls without sliding. We assume that the disk is rolling without sliding and pure rolling motion is taking place. If F is found smaller than or equal to mu_s times N; that means, our assumption was correct.

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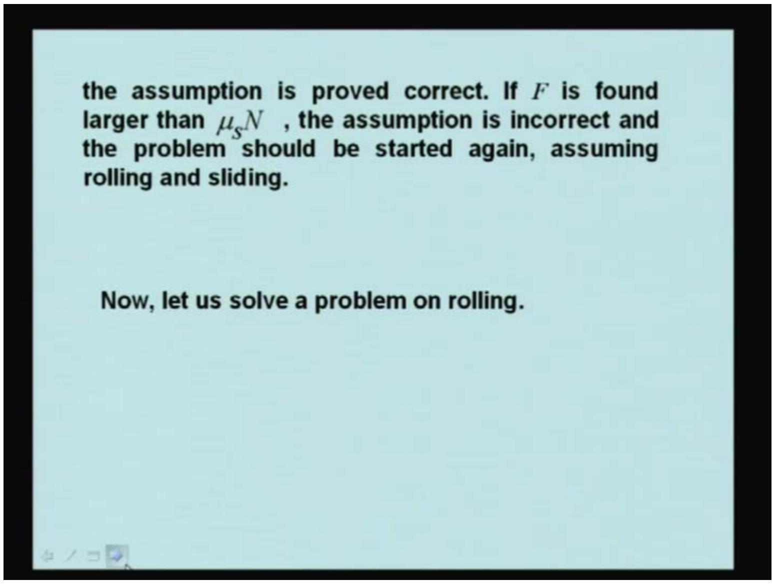

If F is found larger than mu_s times N then the assumption of pure rolling was incorrect. The problem should be started again. This time assuming that rolling and sliding. So, the equation and also the expression for force will change and F will now become equal to mu_k times N. Let us solve the problem on rolling.

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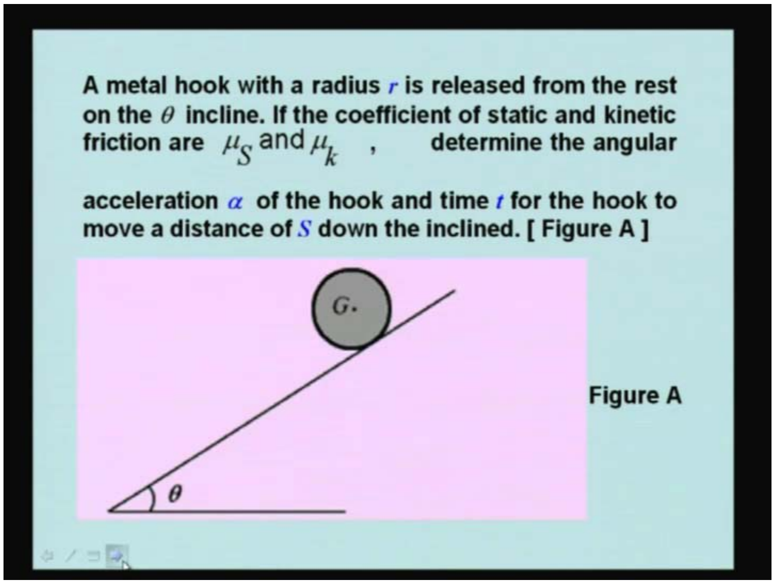

This will be like this. I have shown here an inclined plane; this is at angle theta and this is the metal hook G with a radius r that is released from the rest on the theta inclined. If the coefficient of static and kinetic friction is mu_s and mu_k then we have given that you determine the angular acceleration alpha of the hook and time t for the hook to move the distance of S down the inclined. So, this is shown here in the figure.

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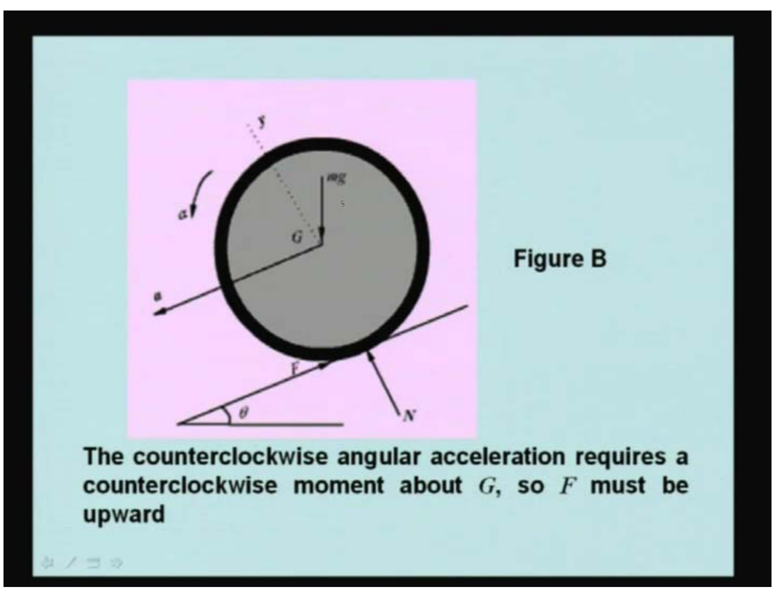

If it is a hook and it is a hollow that type of thing here then moment of inertia about G is given by mass times distance square radius square mra square. We may make use this relation.

If we make the free body diagram of this hook, you are having a force F, friction force which is upward and then you will have a normal force N and then you have got a mass. Its weight is passing through the center of gravity this is M_G and then this is a and this is alpha and this will be y. The counter clockwise angular acceleration requires, you have got counter clockwise acceleration because this part, mass is coming down here. So, acceleration is counter clockwise. So, you require a counter clockwise moment about G; that means F must be upward. So, whatever direction we have shown here is correct, that means it is counter clockwise.

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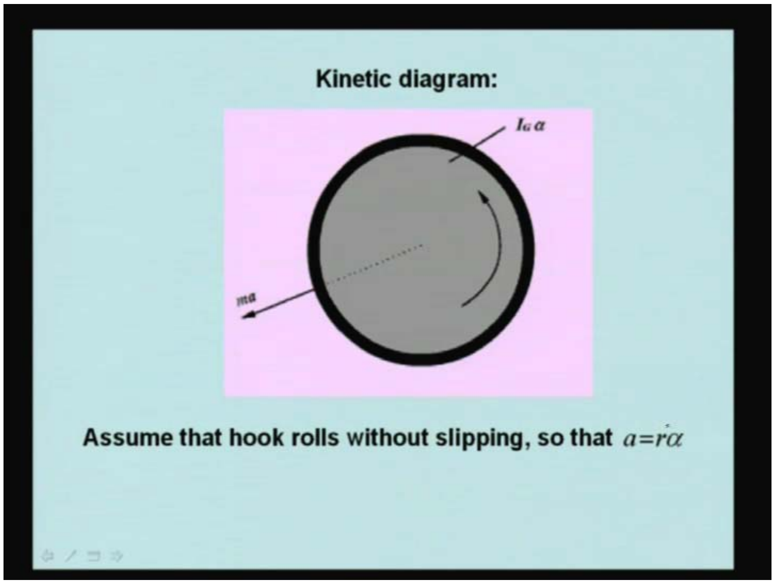

Then, we make the kinetic diagram. In that, we just show that mass times acceleration which is in the direction of inclined plane and that I_G times alpha. Now, assume that hook rolls without slipping, so that a is equal r times r alpha that equation is valid.

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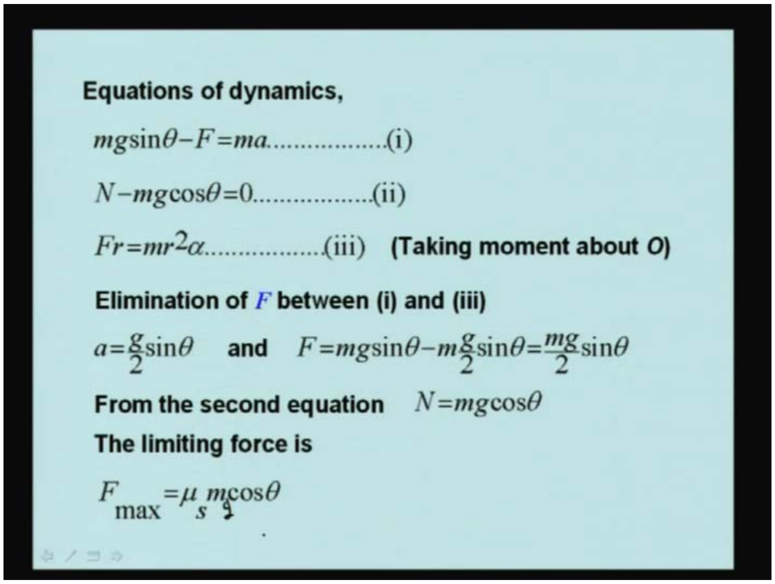

The three basic equations, equations of dynamics are written: one is that relating the tangential force. So, mg sin theta is the force due to gravity minus F is equal to mass times acceleration. Then you have got N minus mg cos theta is equal to 0; that is the normal direction. The third is that moment, mg is equal to I alpha. Therefore, what happens this is F into r is equal to mr square into alpha. That has been obtained by taking moment about the mass center m.

If we eliminate F between first and third, F is appearing. We eliminate F, because we do not know this F. What is that? So, eliminating that F, we get a is equal to g by 2 sin theta. This one we can put and of course N is equal mg cos theta. That thing we are not using right now. So, we find out that eliminating from this. We have eliminated this F and we put this. Then, we will be obtaining a is equal to g by 2 sin theta and F will be equal to mg sin theta minus mg by 2 sin theta, that means mg by 2 sin theta.

From the second equation, we get N is equal to mg cos theta. So, the limiting force is F_{max} is equal to mu_s times mg cos theta, this point is mg cos theta. We have to see that whatever force we have obtained that is mg by 2 sin theta, should be more than mu_s into mg cos theta.

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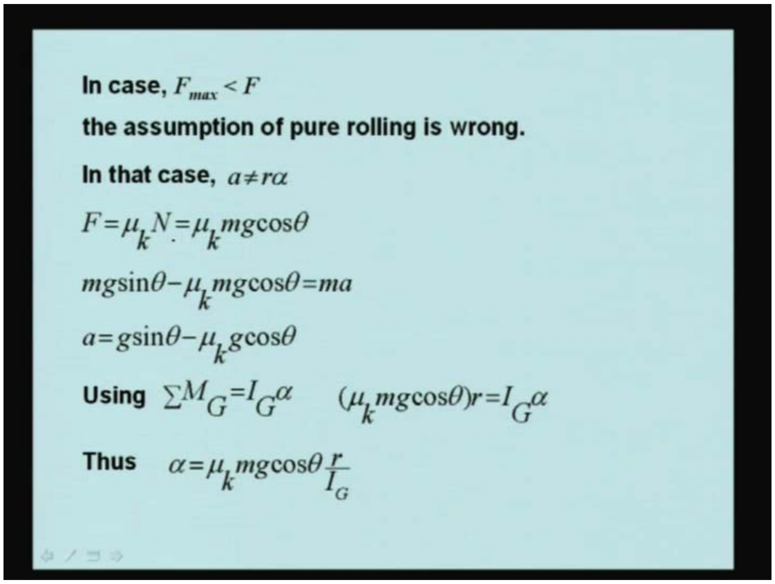

In case that you get F_{max} is smaller than F then the assumption of pure rolling is wrong. In this case, suppose you get that F_{max} by that thing, it is mu_s mg cos theta and if it comes out to be because this is the maximum possible value. In that case, a will not be equal to r alpha and you will be getting F is equal to mu_k times N. You still have the relation between the force and the normal reaction that is mu_k mg cos theta.

Now, you put this in the first equation, mg sin theta minus mu_k mg cos theta is equal to mass times acceleration. In this case, the acceleration will be g sin theta minus mu_k times g cos theta. You will get a different type of this one and we can of course use that sigma mg is equal to I_G alpha; that means, we can write mu_k times mg cos theta into r. That will be equal to I_G times alpha or alpha is equal to mu_k mg cos theta r divided by I_G. We can find out angular acceleration and linear acceleration, although they maybe unrelated.

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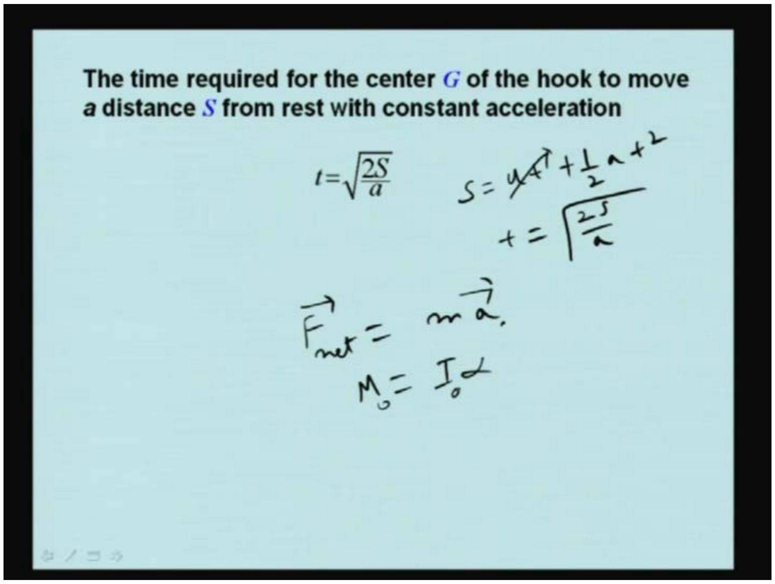

The time required for the center G of the hook to move a distance S from the rest with constant acceleration, obviously can be found by t is equal to under root 2 S by a, where 2 S is the distance because this is simple formula. S is equal to u_t plus half a_t square and this portion is being 0.

We know, t is equal to 2S by a under root. So, that means once we have found linear acceleration, be it in pure rolling motion or be it in rolling cum sliding motion, we have found. Therefore, we can always find out time and that is under root 2S by a and we can solve this problem. That way, by applying these basic equations that you know, F_{net} is equal to mass times the acceleration of the center that is the vector equation. Also, M is equal to I times alpha and M_0 is equal to I_0 times alpha. By applying these equations, we can solve any problem of kinetics in the plane motion.