Engineering Mechanics

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Kinetics-1

Module 11 Lecture 30
Direct and oblique impulse

Today, I am going to discuss about the impact of two bodies. We will discuss the direct and oblique impact. Although till now, we have studied the kinematics of particle, kinetics of particle and also we have discussed the kinematics of the bodies. Our assumption has been that bodies are perfectly rigid; that means they cannot deform. However, that when two bodies impact, there is some amount of deformation.

We will not study the deformation of the two bodies, but at least appreciate the concept that you know that two bodies when they collide, there is some amount of deformation and because of that both the bodies experience force. This point we will at least understand. Then, based on that you know that these bodies will affect each other. We will not really discuss those points that how much time it takes for collision of two bodies? We will assume that collision is instantaneous. As soon as the bodies impact, they separate out. If they do not separate out, in some cases, they remain distinct, you know that. Impact process finishes within no time that is our assumption.

We have to understand that this is idealization; because, in most of the cases this assumption may be justified. That is why we are going to do that. Although in later classes, you may study about the elastic and may be elastic context also. Another thing is that when two bodies are deforming, obviously, they will be of some finite size, but here we will not bring the size effect into the picture. We will assume that, as if there is an impact of two bodies of very small size. Therefore, they will be treated like a particle also.

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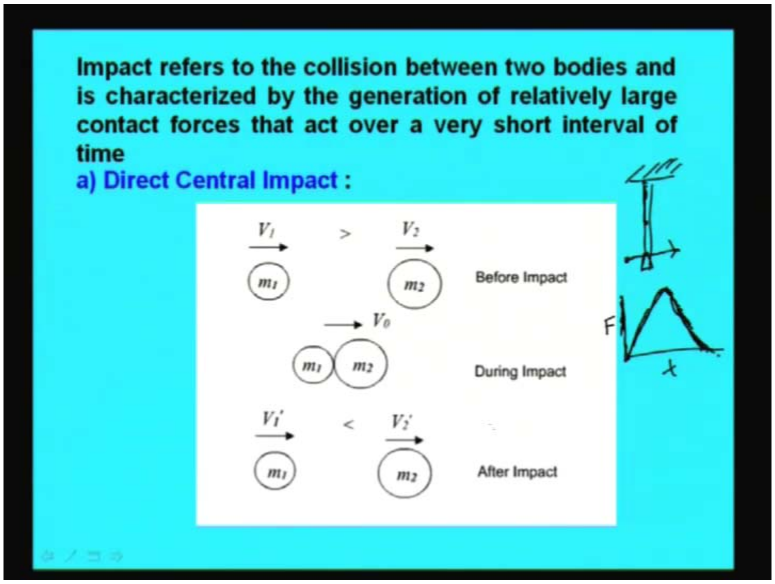

Let us see, what is the process of impact? Impact refers to the collision between two bodies and is characterized by the generation of relatively large contact forces that act over a very short interval of time. In collision, two bodies make contact and then large amount of contact force is generated. If body a, applies some force on body b, then body b also applies the same amount of force on body, a. There are mainly two types of impacts.

We will study direct central impact and we will also study oblique impact. What do we mean by direct impact? When the two bodies are making contact and we draw a normal line; that means if you draw a line normal to the contacting surface and the bodies are having velocity along that normal line which is common normal of the two bodies, then that will be called direct impact. In that case, the velocities basically are also in one line.

One dimensional equation can describe the motion and the central impact means that the mass centers are in the same line. If you make an impact, something like this, that there is a this one, that body and you are doing like this, that suppose this rod is hanging and one bar comes and hits at this one, this velocity of this rod is like this. The velocity of this point on the rod will also be in this direction. However, the mass center is different. Therefore, although this impact is direct, because common normal is this and the velocities are along the common normal, but its mass centers are not in the same line, this can be called direct non central impact. One can understand that also easily. But right now, we will not be discussing about that.

What happens if suppose two particles are making contact? Just see this figure that particle of mass m_1 is going to collide with the particle of mass m_2. We can say small balls instead of a particle. Velocity of V_1 is greater than V_2 that means V_1 is greater than V_2 velocity of per mass m_1 is more than velocity of mass m_2, before impact this is the situation. Since, this velocity is more than velocity of $V_{2,}$ body of mass m_1 catches body of mass m_2 and then there is an impact. When there is an impact, what happens is that, this mass m_1 is moving in that direction. It tries to move in that direction, the layers of that body m_2 there is a deformation.

We know that, Hooke's law of elasticity. You know, stress is proportional to strain; that when you apply the stress then there is a strain generated and it is like something similar to spring type of thing. Although here, what happens, the situations are very complicated because the contact area also keeps on changing. But you can appreciate that if we draw a force verses time diagram, this is force here and this is time. In the beginning, just when the body has touched another body, at that time, the force is 0; then after that, force tries to increase. It is increasing and after that this keeps on increasing and then more and more deformation starts taking place.

Then what happens, as more and more deformation is taking place that means a force is being applied on body $m_{2.}$ Therefore, its velocity is increasing; from V_2 it is increasing. At the same time, the body m_1 faces resistance, because that m_2 applies the force in the opposite direction. Therefore, its velocity keeps on decreasing and gradually, a point comes. At that point, the velocity of both the bodies become same and the force has reached maximum. That time the velocity of both is same that is $V_{0.}$

The condition also comes that both the bodies are moving with the same velocity, but then after that you know body m_1 cannot press body m_2 but if there is elasticity, because of elasticity there can be one situation that body m_1 has pressed it to maximum extent and both the bodies are moving at the same velocity. Therefore, it is not applying the force and keeps on moving with that velocity; but usually what will happen, after that there is some elasticity and this body m_2 will try to come back to the original position. Even body m_1 will have tendency to go to the original position or that means recover from deformation. Therefore, it will start applying the force in the opposite direction. Since, it will apply the force in the opposite direction that contact force will start decreasing like this and then finally, the contact force will become 0 at some point. This is the situation.

You know that contact force increases, reaches some maximum and then it comes down to 0. The exact profile may be different and it is not that necessary. This is just a schematic diagram showing that actually what may be going on. Once, in that process, a body m_2 was applying the force on m_1 in the opposite direction. Since they both were moving at the same velocity V_0, the velocity of m_1 will decrease and it will become V_1 prime. Simultaneously, the velocity of m_2 will increase, because if m_2 is applying a force on m_1, m_1 applies force on m_2. Therefore, during that process the velocity of this is increased. Therefore, velocity of particle m_2 will become V_2 prime. In this situation, after impact V_1 prime is certainly less than V_2 prime. In all the situations, you will see that when the two bodies collide and one body is moving with greater velocity, it contacts another body.

Then, after impact when they separate out, the velocity of the first body which was more will be necessarily less now and the another body's velocity will be more. That situation will be there after impact. This is the process which goes on inside this one and let me just describe it again.

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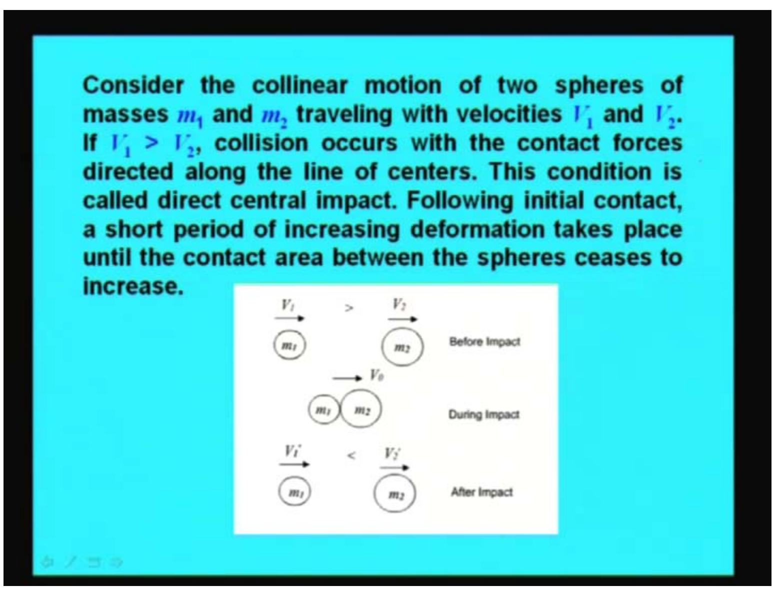

Consider the collinear motion of two spheres of masses m_1 and m_2 travelling with velocities V_1 and V_2. If V_1 is greater than V_2, then collision occurs with the contact forces directed along the line of centers. Contact forces are directed along the line of centers; that means along the normal. Therefore, this condition is called direct central impact, because the mass centers are in the same line and collision velocities are also taking along the common normal. So, this condition is called direct central impact.

Following initial contact, initial contact takes place then a short period of increasing deformation takes place. Deformation keeps on this one until the contact area between these spheres ceases to increase. Because after that velocity, both the velocities have become equal; naturally, that contact area will cease to increase. That is the situation.

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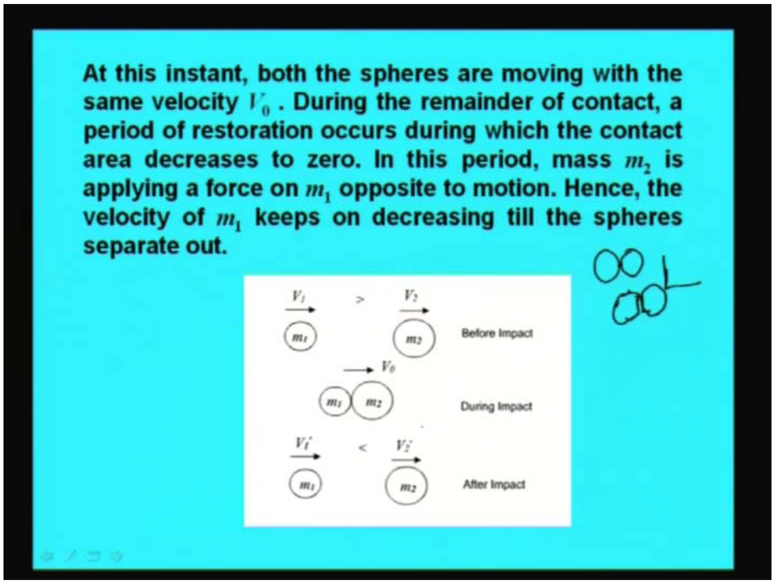

At this instant, both the spheres are moving with the same velocity V_0. When the contact area has ceased to increase; that means, contact area between the two spheres it is like this. In the beginning, there is like a point type of thing. That point is changing and contact area something like bodies have deformed. After deformation they become something like this. This is the type of thing. Contact area has increased. Like that, contact area keeps on increasing but direct contact area ceases. By that time, the force has reached the peak. After that the force cannot increase, because force is increasing because the body with greater velocity is pressing the body with a smaller velocity and contact area is keep on increasing. The force will be proportional to the contact area. That is also there.

Therefore, at some instant, both the bodies have reached velocity V_0. During the remainder of contact, a period of restoration takes place that is called restoration. Restoration occurs, during which the contact area decreases to 0. The contact area will slowly decrease to 0 in the case of elastic. But sometimes, the elasticity cannot be fully recovered. That is another thing. Then certain amount of plastic deformation will also be present. In many cases, there is sufficient amount of elasticity. In this period, mass m_2 is applying a force on m_1 opposite to motion. Hence, the velocity of mass m_1 keeps on decreasing till the spheres separate out. That is because, once the force has become 0, then they separate out because there is no force acting on that and the other body has picked up the velocity. Therefore, it will be separated out because other velocities are different.

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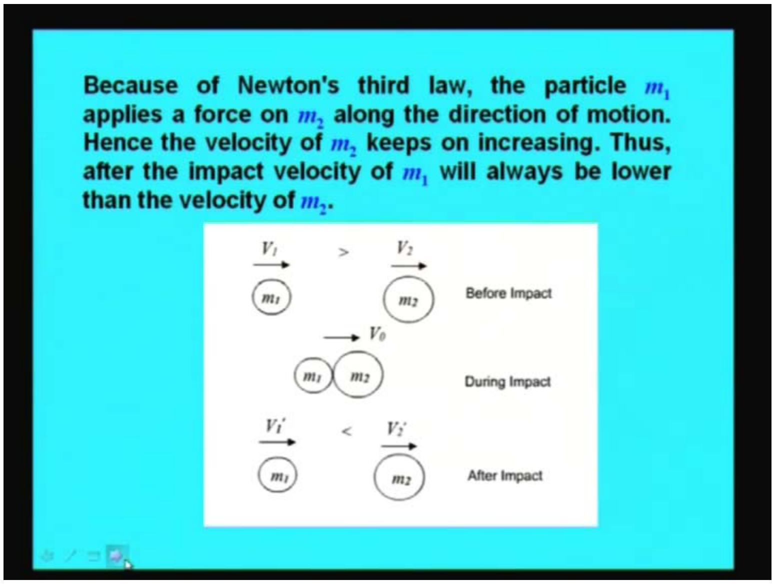

Because of Newton's law, the particle m_1 applies a force on m_2 along the direction of motion. Hence, the velocity of m_2 keeps on increasing. Thus, after the impact, velocity of m_1 will be always lower than the velocity of m_2. That is the point and it has to be noted out.

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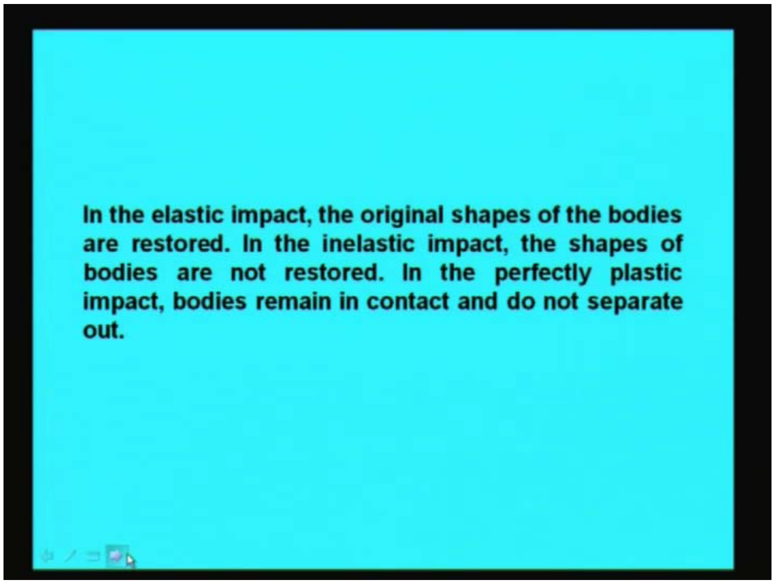

In the elastic impact, the original shapes of the bodies are restored. In the inelastic impact, the shapes of bodies are not restored. In the perfectly plastic impact, bodies remain in contact and do not separate out because there is nothing. No force is applied to decrease the area. Therefore, they keep moving with the common velocity and there is a plastic deformation. So, therefore they do not separate out. That is what happens in the perfectly plastic impact.

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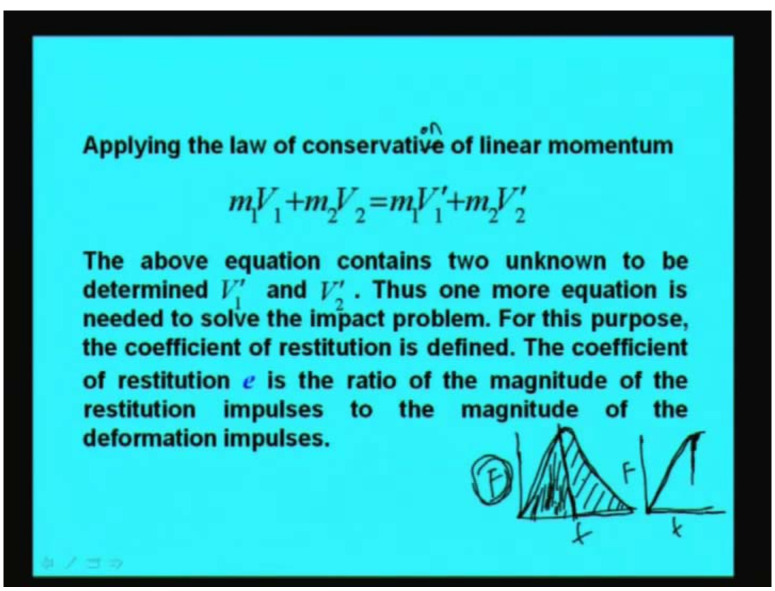

We apply the law of conservation of linear momentum. If we consider although there is a force on one particle and or body sphere, similarly, there is another force on another sphere or body. If we consider them together, there is no external force acting on these bodies; that force is internal because one body applies a force on the other body, other bodies also apply the equal amount of force. Net outside force is 0. We assume that friction is not playing any role. In fact, what happens, the contact duration is so small in most of the cases that tremendous amount of force is generated. Compared to that small amount of friction, force is often negligible. Therefore, we can apply the law of conservation of momentum to this one; that is law of conservation of linear momentum.

Linear momentum: Initially the velocity of one body is V_1 and the velocity of other body is V_2 and after that it becomes V_1 prime and V_2 prime respectively. The first body attains the velocity V_1 prime, second body is at V_2 prime; this is after the impact. We are not considering that small duration when really the bodies are touching each other or that they are moving with the common velocity; that we are not considering only that well before the impact and well after the impact.

We get the equation m_1 V_1 plus m_2 V_2 is equal to m_1V_1 prime plus m_2 V_2 prime. This equation contains two unknowns to be determined, V_1 prime and V_2 prime. Therefore, we must need two equations. Here, we are having only one equation. We cannot solve this thing. We need another one. So, one more equation is needed for this purpose and we define the coefficient of restitution. By that we can define that situation; that means, another equation that has to come from this is momentum equation, and another comes from the kinetic energy consideration. If the body is perfectly elastic then the kinetic energy must remain conserved.

We can write that equation also. We will have that one equation for the kinetic energy, initial kinetic energy. Then after that we will have another final kinetic energy. We will have for kinetic energy also. Therefore, by that we must know that how much fraction of kinetic energy has been lost. By that also, we can find out. Therefore, that is one way of doing that. But instead of doing the same thing, in a slightly different manner, we describe, that means loss in kinetic energy can be indicated by one quantity called the coefficient of restitution, e.

This coefficient of restitution e is the ratio of the magnitude of the restitution impulses to the magnitude of the deformation impulses. That is the thing. By this quantity, this is a very important quantity, suppose you say e is equal to one; that means, the kinetic energy is conserved. There is no loss of kinetic energy. If you say e is equal to 0, then there is a significant amount of loss of kinetic energy.

If you say e is equal to 0.5 then also there is some loss of kinetic energy. At e is equal to 1, there is no loss of kinetic energy. That is there, but we do not define that 1. We define the coefficient of restitution in a different way; ultimately it reads the same observation. We can relate these two kinetic energy also, but the thing is that here it becomes the coefficient of restitution. e is the ratio of the magnitude of the restitution impulses to the magnitude of the deformation impulses. What is the meaning of that restitution impulse and deformation?

See what impulse means force into time; product of force into time is called impulse. We integrate. That means, if we integrate F_{dt} or we draw that force versus time diagram. It is like, this is increasing and after that it is this one. Up till this point, there is a deformation process, because that body one is pressing the body two and contact area is increasing. After that it reaches a point; that means top point. Then the bodies try to restore their original shape. This is the restoration period or restore.

If we take the area up to this F that means F dt integrated this will be called deformation impulse. After that restoration period is there, if we take the area of this area will be called from here to here; that means, again this area, like this upto this will be called a restitution impulse. If we take the ratio of this area to that area then we get coefficient of restitution. If the contact is perfectly elastic, in that case, the deformation impulse will be equal to the restitution impulse. Whatever that impulse you have applied, same amount of impulse is getting recovered and therefore this ratio becomes 1. Otherwise ratio will be less than 1, if you do not allow bodies to restore.

That means, only it is rising, reaches the maximum contact area but then it is not separating out. In that case, if this is the situation, this is F and this is T and if it is increased but it is not coming down after that. Therefore, you have restitution impulse is 0; therefore, e will be 0 and one point has to be noted that you see what is this F? This F is the force on which body you can consider the force on both the bodies, because if body a is experiencing the force, the other body is also experiencing the same amount of the force. Net force, any way is 0 and this is what happens. Another case that if it keeps on applying the force and the restitution, one case I have shown like this and in another case, if the restitution does not take place, naturally that bodies will make contact, the force increases and after that this may become this one; that means, impulse may be 0 and it may just come down. Force may come down without making any change in the area.

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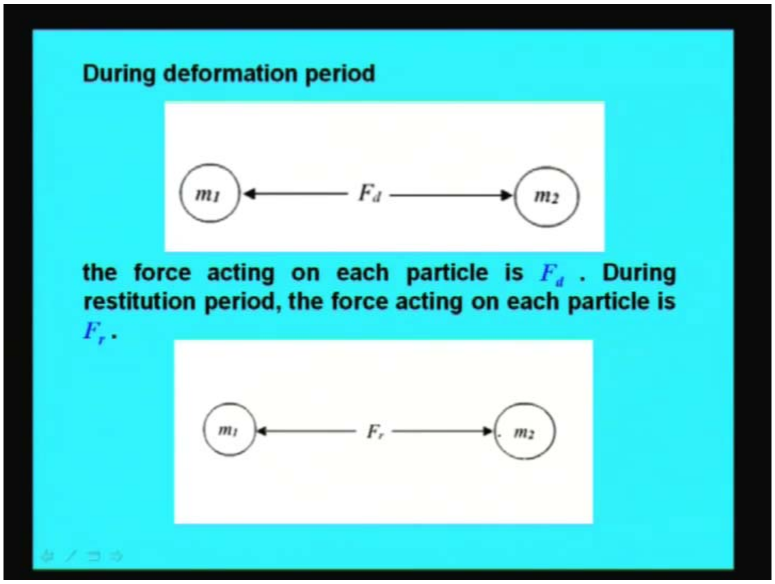

Here, let us see it carefully during the formation period m_1 and m_2 these are the two bodies, body m_1 is pressing body $m_{2,}$ therefore body m_2 is applying a force on body m_1 and that force is given by F_d, therefore that I am showing the direction of the force F_d on m_2 that is applied by body m_1 in turn body m_1 m_2 is applying a force on m_1 and that is in this direction, this is the situation the force acting on each particle is F_d.

During restitution period, the force acting on each particle is $F_{r.}$ Then also; the force is actually in the same direction, because this is $m_{2.}$ After that, you know that this is $m_{1.}$ So, during restitution period also bodies will experience the force that will be F_r.

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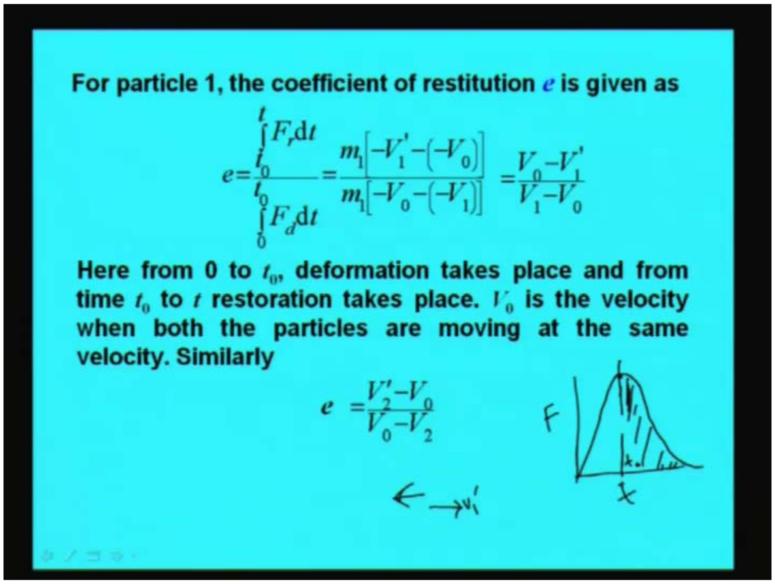

For particle1, if we consider just the particle one, we can consider any one particle. We consider particle 1, the coefficient of restitution is given as e is equal to t_0 to t. What is t_0? t_0 is that time at which bodies have started moving with the same velocity. The force has already reached maximum and this is like this. If bodies are deforming like this, this is time and this is F, this will be t_0.

From 0 to t_0 deformation takes place and we have e is equal to t_0 to t. Then, after that you have F_r into dt. Therefore, this becomes like this, t_0 to t F_r dt that force is keeping on increasing and it has become like this and divided by 0 to t_0. This t_0 to t means this period t_0 to t; that is restoration period. Another one will be, if the bodies will not be able to recover, then the time they will not be able to apply the force also. So, F_r will become 0. Therefore, this will become obviously that upper part will become 0 and then it is 0 to t_0 F_d dt.

At this stage, you can apply the impulse momentum equation; change in the momentum is equal to the impulse. During this period, that means t_0 to t, the velocity of the body one changes from V_0 to V_1 prime. We see that we have taken the direction of V_1 prime assuming like this, but the force is acting in this direction. So, they are in the opposite direction.

To be consistent, we will take this as minus V_1 prime, therefore what happens the final momentum is m_1 minus V_1 prime minus $V_{0.}$ This will be 0 to t_0 F_d dt that is m_1 minus $V_{0;}$ because, here also the final velocity is minus V_0 minus V_1. Therefore, we will be getting this and this will be equal to V_0 minus V_1 prime and divided by V_1 minus $V_{0.}$ We get one expression for e but although at this stage, we know that actually $V_{0,}$ therefore, we have just written the expression like this. Here, from 0 to t_0 deformation takes place and from time t_0 to t restoration takes place.

V_0 is the velocity when both the particles are moving at the same velocity. Similarly, we can have similar type of equation for the second particle also. For the second particle, we will be having e is equal to V_2 prime minus V_0 divided by V_0 minus $V_{2.}$ So, both the things are there, so both equations. In this, we can eliminate V_0 and we can get the expression for e like that.

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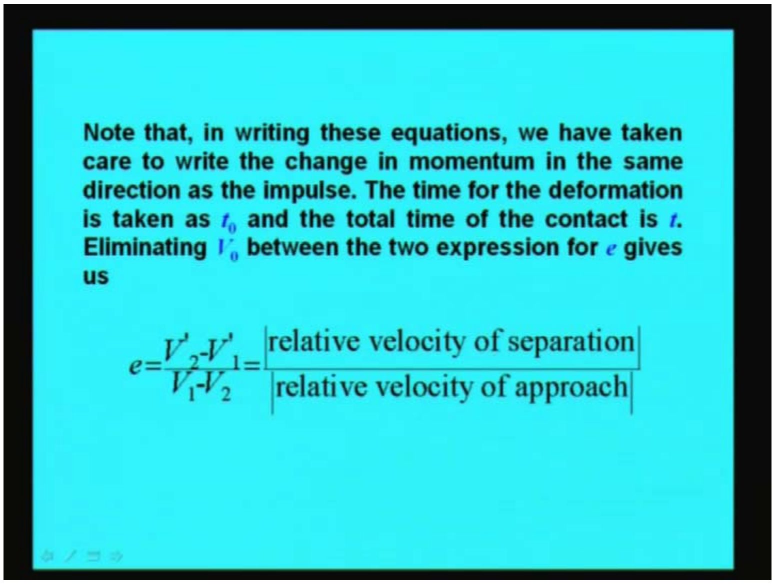

In writing these equations, we have taken care to write the change in momentum in the same direction as the impulse. The time for the deformation is as taken as t_0 and the total time of the contact is t. If we eliminate V_0 between the two expressions for e, then between the two expressions, the expression for e becomes e is equal to V_2 prime minus V_1 prime divided by V_1 minus $V_{2;}$ that means it is the relative velocity of separation V_2 prime minus V_1 prime, final velocity of body two and velocity of body one.

So, you see that both have started moving with a different velocity and they separate out with this relative velocity. This is called relative velocity of separation; other is called relative velocity of approach V_1 minus $V_{2.}$ So, this will be V_1 minus $V_{2.}$ So, the ratio of these two quantities is a positive quantity and that is called coefficient of restitution. That may range between 0 and 1.

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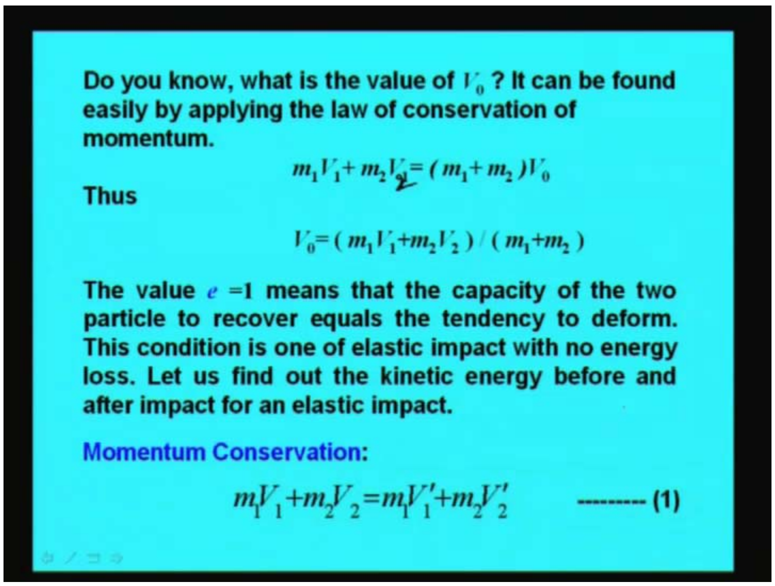

Do you know, what is the value of $V_{0?}$ Can we compute the value of V_0? It can be found easily by applying the law of conservation of momentum; that means, we can say m_1 V_1 plus m_2 V_2 is equal to m_1 plus m_2 into $V_{0.}$ So, we have m_1 V_1 plus m_2 V $_2$is equal to m_1 plus m_2 into $V_{2.}$ Thus we find out, V_0 is equal to m_1 V_1 plus m_2 V_2 divided by m_1 plus $m_{2.}$ That way we can find out anyway the intermediate velocity $V_{0,}$ which occurs for a short duration of time. Now, the value e is equal to 1 means that the capacity of the two particles to recover equals the tendency to deform. So, this condition is one of elastic impact with no energy loss. We have not talked about the kinetic energy, but now in this case, we will show that how there is no energy loss in this case.

Let us find out the kinetic energy before and after impact for an elastic impact. So, we have equations like we have equation for momentum conservation. We have m_1 V_1 plus m_2 V_2 is equal to m_1 V_1 prime plus m_2V_2 prime.

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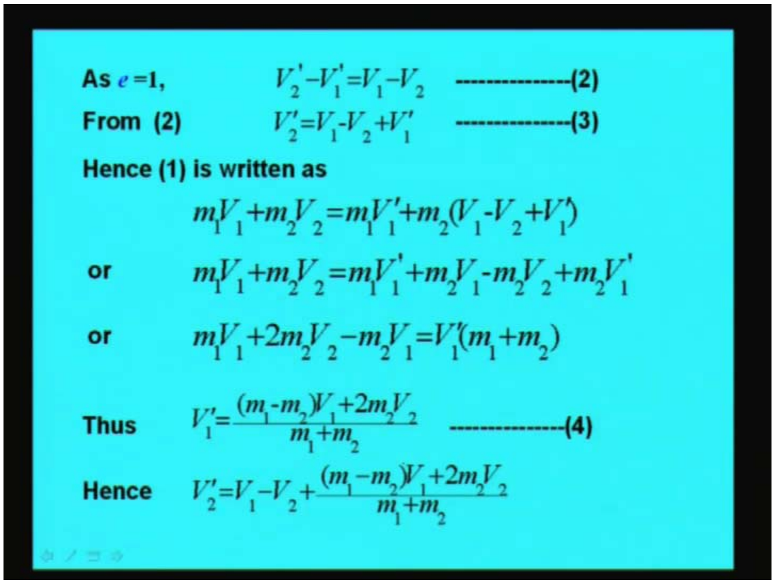

This is thing, as e is equal to one V_2 prime minus V_1 prime is equal to V_1minus $V_{2,}$ because V_2 prime minus V_1 prime is what? It is the relative velocity of separation. V_1 minus V_2 is what? It is the relative velocity of approach. This is V_2 prime minus V_1 prime is equal to V_1 minus V_2 and now from this we get V_2prime, is equal to V_1minus V_2 plus V_1 prime. Hence, we can write one as this momentum by this m_1 V_1plus m_2 V_2 is equal to m_1 V_1 prime plus m_2 V_1 minus V_2 plus V_1 prime.

We have substituted all these things or we will say, m_1 V_1 plus m_2 V_2 is equal to m_1V_1 prime plus m_2 V_1 minus m_2 V_2 plus m_2V_1 prime, or we take this portion, this side m_1 V_1 plus m_2 V_2 minus m_2 V_2 is equal to V_1 prime m_1 plus $m_{2.}$ Thus, we get V_1 prime is equal to m_1 minus m_2 into V_1 plus 2 m_2 V_2 divided by m_1 plus $m_{2.}$ So, we get this equation. Hence, V_2 prime because V_2 prime is V_1 minus V_2 plus V_1 prime. So V_2 prime is V_1 minus V_2 plus m_1 minus m_2 into V_1 plus 2 m_2 V_2 divided by m1 plus $m_{2.}$

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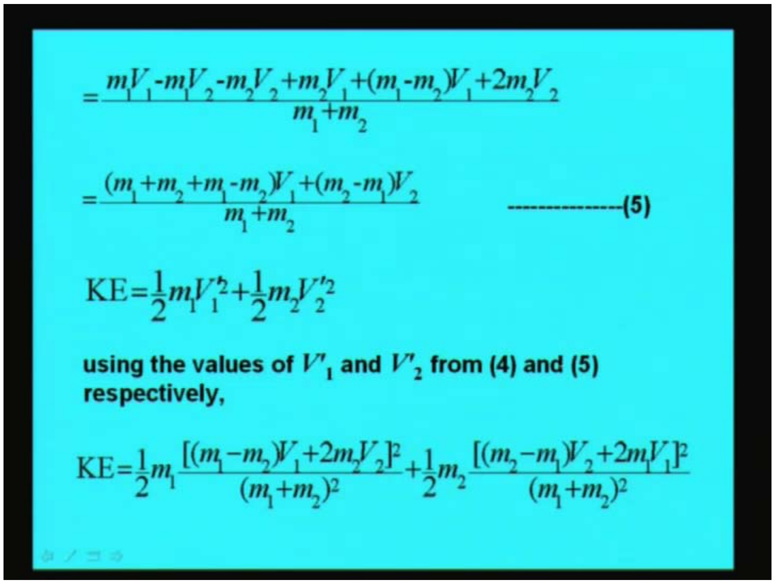

Then, after that we simplify this and we get something like this, m_1 V_1 minus m_1 V_2 minus m_2 V_2 plus m_2 V_1 plus m_1 minus m_2 V_1 plus 2 m_2 V_2 divided by m_1 plus m_2, or we write this like this; this is m_1 plus m_2 plus m_1 minus m_2 V_1 plus m_2 minus m_1 into V_2 divided by m1 plus m_2, now this is the expression for V_2 prime.

We have expressions for V_1 prime given by this equation 4 and then, we have expressions for V_2 prime given by this equation. Now, we write the kinetic energy impact; that is half m_1V_1 prime square plus half m_2 V_2 prime square. If we use the values of V_1 prime and V_2 prime from these equations 4 and 5 respectively, we get kinetic energy is equal to half m_1 and V_1 square; that means, m_1 minus m_2 multiplied by V_1 plus 2 m t_2 V_2 divided by m_1 plus m_2 whole square and this thing whole square on the top. Then plus half m_2 m_1 m_2 minus m_1 V_2 plus $2m_1V_1$ whole thing square divided by m_1 plus m_2 square. So, this is the thing and we have to do some algebra to simplify that.

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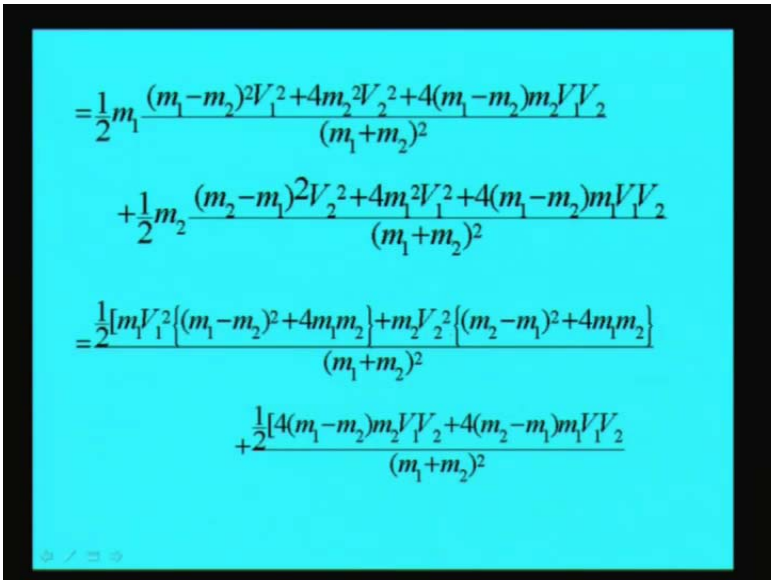

That is half m_1 and this will be m_1 minus m_2 square V_1 square plus 4 m_2 square V_2 square plus four m_1 minus m_2 m_2 V_1 V_2 divided by m_1 plus m_2 square plus half m_2 m_2 minus m_1 whole square V_2 square plus 4 m1 square V_1 square plus 4 m_1 minus m_2 into m_1 V_1 V_2 divided by m_1 plus m_2 whole square. So, that expression you are getting. Then, one has to adjust this. So, half can be taken as common. So, half m_1 V_1 square if you take, you will get m_1 minus m_2 square plus 4 m_1 m_2 plus you get a half is outside, totally outside of this, then m_2 V_2 square then m_2 minus m_1 whole square plus 4m_1 into m_2 divided by m_1 plus m_2 square and then you get, half here is equal to 4 m_1 minus m_2 into m_2 V_1 V_2 plus 4 m_2 minus m_1, m_1 into V_1 V_2 divided by m_1 plus m_2 whole square. So, this is this big expression you are getting.

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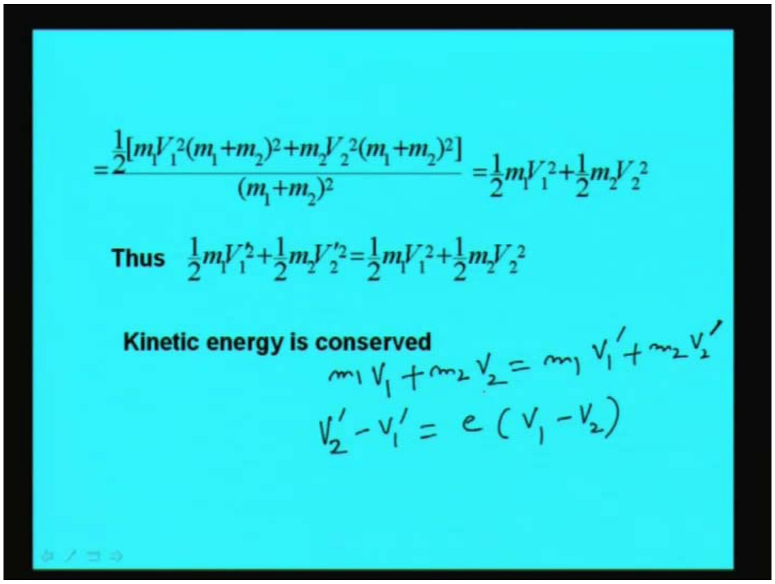

This is equal to half m_1 V_1 square m_1 plus m_2 square plus m_2 V_2 square bracket m_1 plus m_2 whole square divided by m_1 plus m_2 whole square. If you simplify, then this becomes half m_1 V_1 square plus half m_2 V_2 square. Thus, we get half m_1 V_2 prime square plus half m_2 V_2 prime square is equal to half m_1 V_1 square plus half m_2 V_2 square.

That means the final kinetic energy of the two particles is equal to the initial kinetic energy of the two particles. The kinetic energy of the particles or bodies change, but combined thing remains same, in which e is equal to 1. That we have demonstrated. Similarly, we can of course demonstrate that when the bodies are making perfectly elastic contact, then the kinetic energies that get changed.

After describing these things, it must be clear that it is easy to solve the problems of direct central impact. There you have to write one equation for the balance of linear momentum, the other equation will come from the coefficient of restitution or theory; that means, V_2 prime minus V_1 prime is equal to e times V_1 minus V_2. So, we will have two equations. So, there will not be difficulty in solving the problem of two unknowns. The two bodies are colliding, mass m_1 and with velocity V_1 it is moving. This another one is m_2 V_2 is equal to m_1 V_1 prime plus m_2 V_2 prime.

Your goal is just to find out V_1 prime and V_2 prime. So, here it is like this, where one equation is this and the other equation is given by this. V_2 prime minus V_1 prime is equal to e times V_1-V_2, is this is the thing. We got, velocity of approach is V_1-$V_{2;}$ velocity of separation is V_2 prime minus V_1 prime. So, therefore this is the thing.

This e, value of e must be known. It is a property factor of so many things. It depends on the material, it depends on the size of the balls, it depends on which velocities they are. You know that making impact or collision. So, like that this must be, but it will be given in a problem. Therefore, this is given. So, you have 2 equations and 2 unknowns; that is V_1 prime and V_2 prime. This can be easily solved.

We need not write anything about the kinetic energy in this case. So, first this is the thing. So, that way you can solve the problems of direct central impact. Now, we will be discussing about the oblique impact problem.

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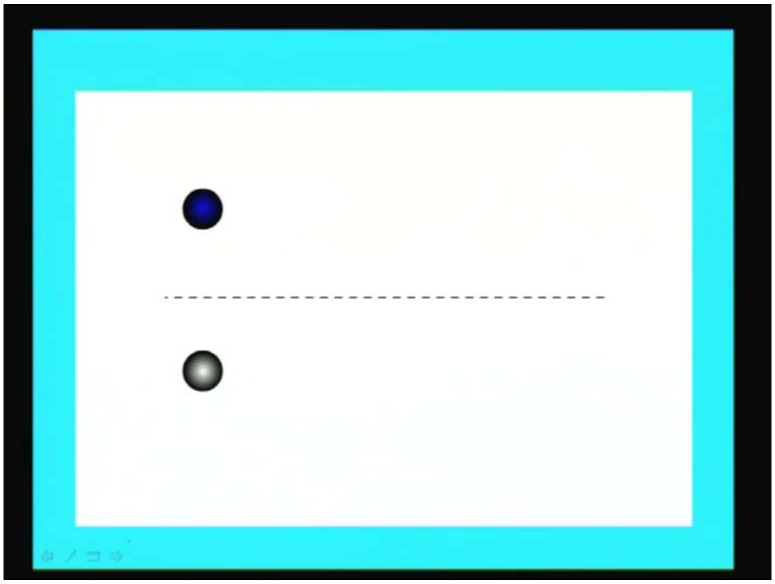

In this case, the particles collide at certain angle and if you know that it may make contact, then the common normal is in some different direction. So, in this case, here the two bodies like we have two spheres. They are going here and they may be moving with some inclination.

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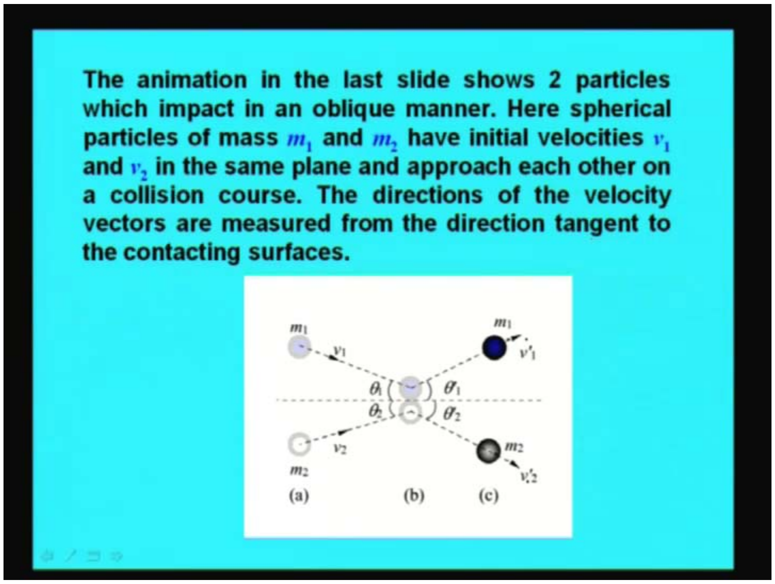

In this case, two bodies will make collision and the situation will be like this. There is a particle m_1 and there is a particle m_2; both are moving with the velocity V_1, this is moving with the velocity V_2, both may contact at this point, collide and after that they separate. So, you know that when they make contact, this is the normal direction and this direction is the tangential direction.

Now, this part, this is making that angle θ_1 from this direction t and this is making angle θ_2 from here. Similarly, that from the tangent direction this particle m_1 after the collision is making angle θ_1 prime and this is making an angle θ_2 prime. Therefore, this is V_1 prime and this is V_2 prime, this is m_1 and this is m_2. The direction of the velocity vectors are measured from the direction of tangent to the contacting surfaces. So, this is the case of the oblique contact, because they are making contact at angle. So, these are the three situations here, shown a b and c.

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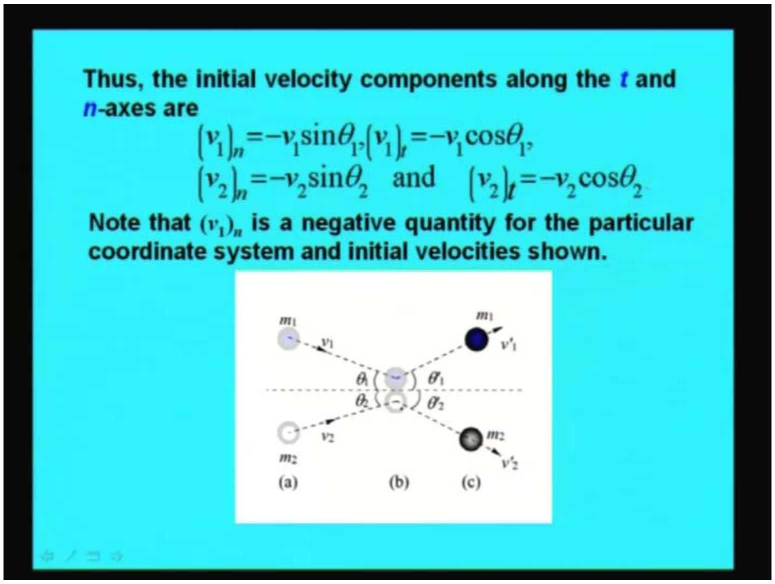

Thus, the initial velocity components along the t and n axes are, we can write that suppose, you have V_1 we can resolve. Then along the normal, V_1 n is equal to minus V_1 sin $theta_{1,}$ because it is coming downward. So, we have put a minus 1 and V_1 tangential. If we say that minus V_1 cos $theta_1$ in that direction, similarly, V_2 n is the velocity and that we have written minus V2 sin $theta_2$ and sin V_2 t is equal to minus V_2 cos $theta_{2.}$ V_1 depends on the coordinate system that whether you call that V_1 n positive or negative. If you take that, for this particular if you take upward direction as y, then it is like this.

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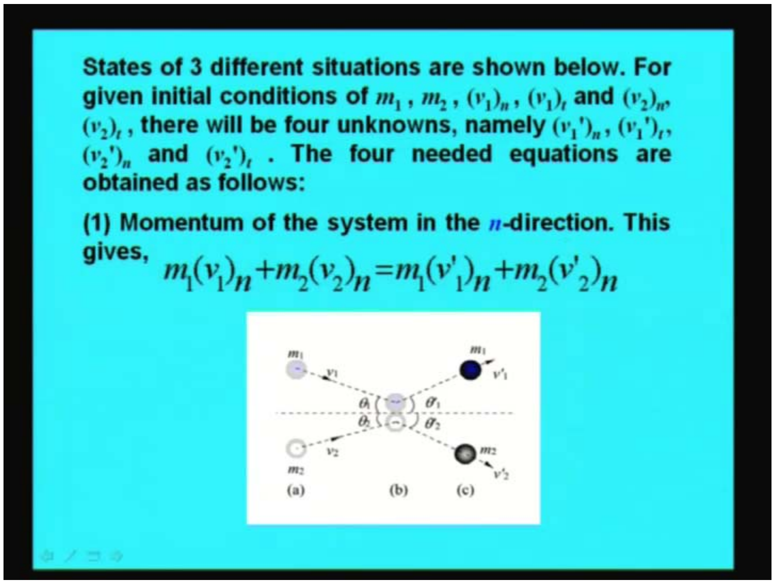

This situation is shown now. States of 3 different situations are shown below. For given initial conditions of m_1, m_2, V_1 n, V_2 n, there will be now four unknowns. We have got four unknowns namely, what are the four unknowns? V_1 prime n, we must know the normal component of that. You know both the velocity particles. What is this one? So, V_1 prime n. Then similarly, tangential components are also unknown. So, V_1 prime t and then V_2 prime n and V_2 prime t. Thus, you need four equations to solve that. From where will we get four equations?

What will you do to get four equations? There is no problem in solving this one; therefore, problem is solved. We just need four equations to find out unknowns. What are those unknowns normal after the impact, because before the impact we must know the velocities. Before the impact everything is known; masses are known, impacting velocities are known, at which angle they are going to make a contact is also known, because you know that you can join the line of contact and you can find out that angle also.

After the impact we must know the velocity. So, velocity has two components. Therefore, two particles. So, total four components, so four unknowns are there, or we can say V_1 prime V_2 prime and θ_1 prime and θ_2 prime are the unknown. So, we have to get four equations. Now, first equation we get, momentum of the system in the n direction gives m_1 V_1 n plus m_2 V_2 n is equal to m_1 V_1 prime n plus m_2 V_2 prime n. So, that means, in n direction the momentum conserved and that is what that you are getting. This is one equation, because there they are impacting and the forces are applied normal to this one. Although the forces are applied normal, their velocities in the normal direction will change. However, that a total net momentum is conserved and you get this equation. So, this is one equation. How will you get the other equations?

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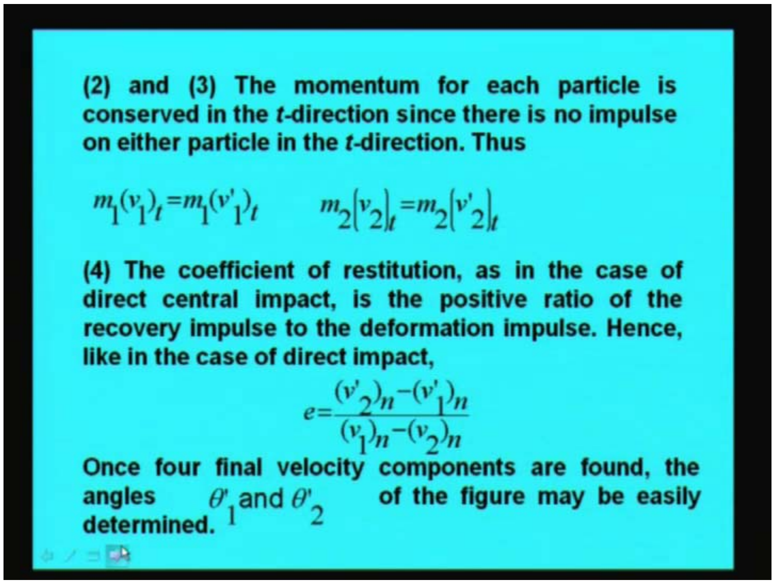

In the second and third equations we get from this consideration, if the there is a smooth contact then the momentum of each particle is conserved in the t direction. Otherwise also, you know that if the particles are making contact, then the momentum. When there is no impulse on either of the particles in the t direction, like smooth contact, then in that case m_1 V_1 t is equal to m_1 V_1 prime t; that means no tangential velocity change occurs for particle, a. Same thing is for particle b, means another particle also; that is m_2 V_2 t is equal to m_2 V_2 prime t. So, V_2 t is equal to V_2 prime, these are the conditions.

Otherwise, if friction is there, then the situation becomes difficult. Another condition, fourth condition is the coefficient of restitution as in the case of direct central impact. It is to be noted that the coefficient of restitution is defined as the ratio of the velocity of separation to velocity of approach. These velocities are always normal, not the tangential velocity. So, we define with respect to that normal velocity. So, the coefficient of restitution as in the case of direct central impact is the positive ratio of the recovery impulse to the deformation impulse.

Hence, like in the case of direct impact, we get here also e is equal to V_2 prime n minus V_1 prime n divided by V_1 n minus V_2 n. So, this is the thing normal. We have got now four equations, one is in terms of normal momentum it means conserved. Then for two particles, separately we have written that tangential this thing. If friction is known, then we have to know that friction force etc., and by that we will find out the change in velocity. But here, in the smooth contact you can easily find out and fourth equation is given by this one. So, four velocity components are found. Then, if you can find out four final velocity components, then we can determine angles theta_1 prime and theta_2 prime and our problem is solved. So, this method can be implied.

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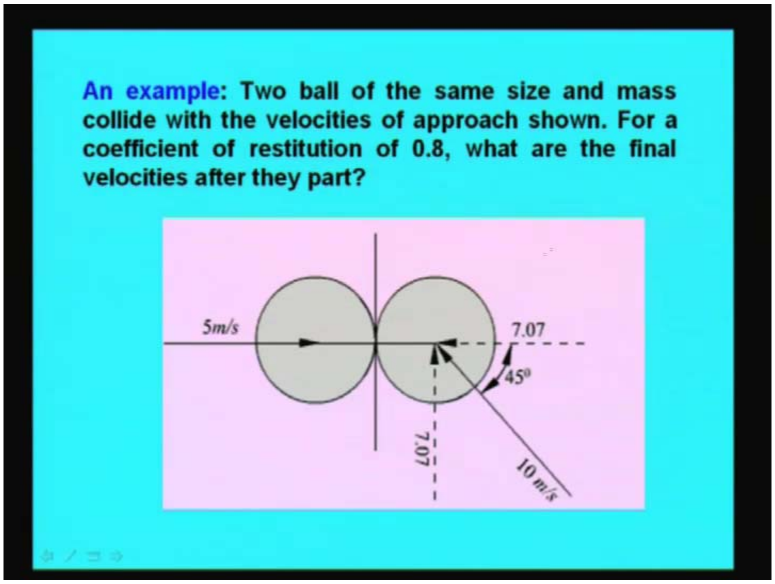

I give one example of oblique contact. Supposing, two balls of the same size and mass collide with the velocities of approach shown. Two balls of same size are colliding with the velocity shown for a coefficient of restitution of 0.8. What are the final velocities after they part? That question is there. So, what happens? That body, one body is going with 5 meter per second, other body is coming at 10 meter per second this is 10 meter per second but this velocity is at 45 degree to the common normal. In that case, resolve it into two components.

So, one component that normal component is 7.7, 7.07 and this is 1 meter per second, normal component is also 7.07 meter per second like that.

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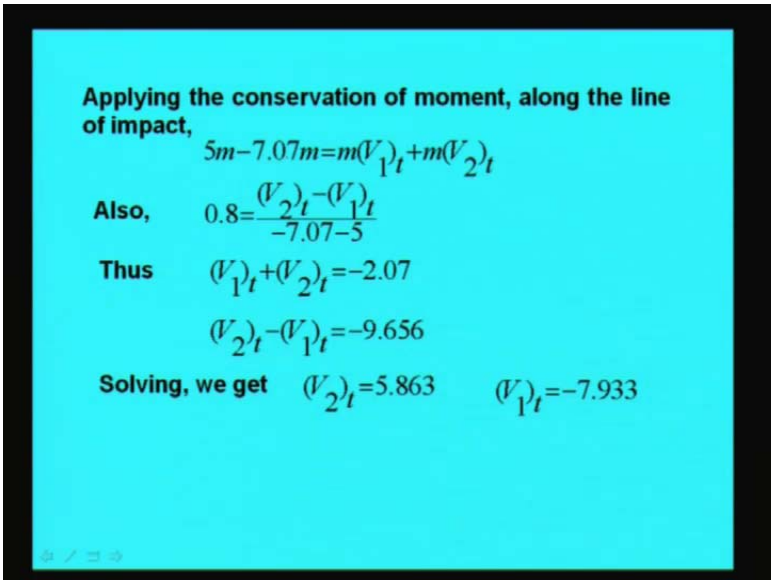

Then, you apply the conservation of momentum along the line of impact. So, 5 into mass, if mass is same. So, 5m minus 7.07m. In the beginning this velocity, this velocity is in opposite direction, so minus sign and is equal to m_1 V_1 t plus m_2 V_2 t normal, that is along the normal directions. Let us use the symbol n here. Also we have, coefficient of restitution is 0.8 given.

Therefore, 0.8 is equal to V_2 n minus V_1 n and divided by, in the beginning, minus 7.07 because this is minus 7.07 and the other is a minus 5; because these are minus 5, so velocity of approach. They are approaching towards each other with 7.07 plus 5, so it gets added. So therefore, we have this is V_2 V_1 n plus V_2 n is equal to, from the first moment conservation equation. This is actually momentum minus 2.07. Similarly, here you get V_2 n and V_1 n is equal to minus 9.656. Solving this, we get V_2 n is equal to 5.863 and V_1 n is equal to minus 7.933.

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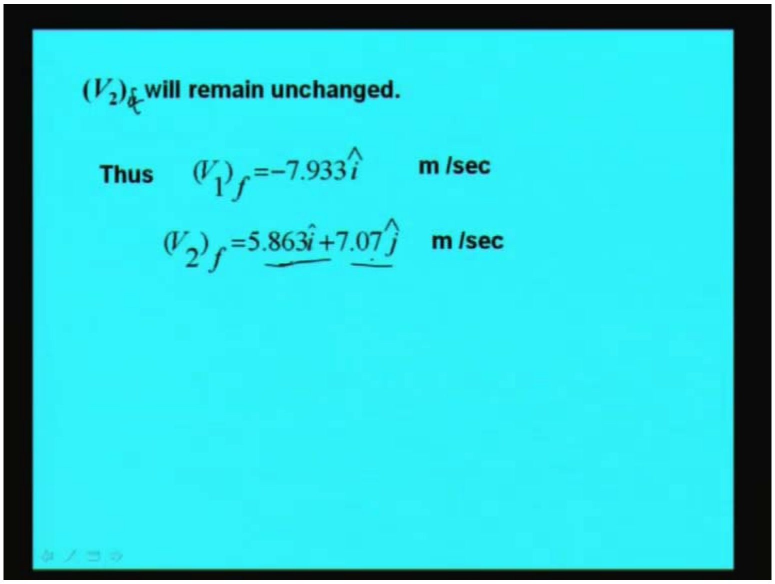

We have obtained like that. V_2 t, V_1 t n V_2 t remains same. Therefore, final velocity of that one particle can be written as minus 7.933 i in the i direction and the other one is actually 5.863 i plus 7.07 j in that direction, because you know that 7.07 components remains unchanged and only this part we have added in the i direction. So, this is what the velocity. By solving the four equations, we have been able to solve this problem. So, these are the basic concepts about the direct and central impact. Now, let me just tell you simple things and simple problems.

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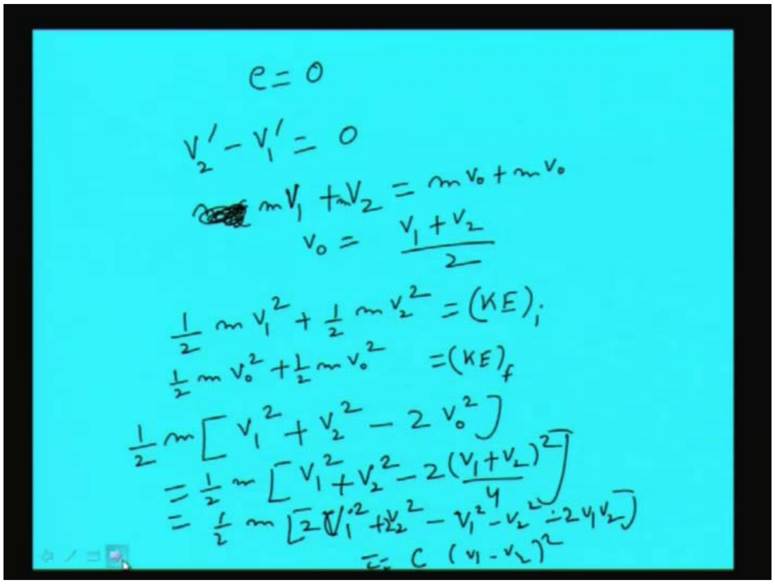

We discussed some problems, like suppose you want to show, in the case of the contact of two bodies with which a perfectly plastic impact e is equal to 0. In that case, V_2 prime minus V_1 prime will be 0; that means, both moves with the same velocity. Therefore, what happens in this case, you get mass into that, suppose the two particles of this one, two particles are of the same mass, we have to just simplify the situation. I am taking the two particles of the same mass. In that case, it will be initially, the velocity may be mV_1 plus $mV_{2.}$ This is m and finally the velocities become $V_{0,}$ same velocity.

This will become m V_0 plus m $V_{0.}$ All V_0 then will become basically V_1 plus V_2 by 2. In the beginning, you are having the kinetic energy half m_1 V_1 square plus half m_2 V_2 square, but finally he kinetic energy has. So, this is KE_i and KE_{final} is half m V_0 square plus half m V_0 square, that means $V_{0.}$ So, this is KE_{final}. So, what is the change in the kinetic energy is, that means half m is common. So, we can take half also common. So, basically we will get V $_1$ square plus V $_2$ square and then minus 2 V_0 square; that is, initial minus final. So, half m will remain common and this will become V_1 square plus V $_2$ square minus twice V $_1$ plus V $_2$ whole square by 4. So, this will become half m and this will become V_1 square. So, we can write that minus.

This will become V_1 by this one. This will become something like this; V_1 square plus V_2 square minus or 2 we can take common, then this becomes 2V $_1$ square 2 V_2 square minus V_1 square minus V_2 square minus twice V_1 V_2. So, the inside quantity can be written as something outside, because then V_1 minus V_2 whole square. Therefore, this energy is always is positive; that means, initial kinetic energy is always more than final kinetic energy.

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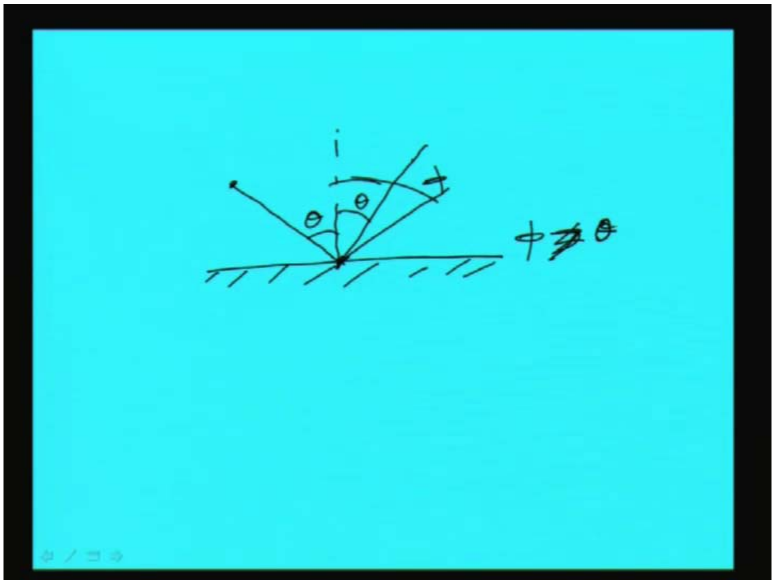

Similarly, I give one example of some oblique contact. Suppose there is a rigid floor and a ball is hitting here at phi theta angle from normal, it gets reflected. If there is a perfect elastic collision, then in that case, it will get reflected at the same theta. Why because in that case, the velocity of approach and velocity of this one, magnitude should be same, therefore theta. However, in the other case, if you know that e is not equal to this one, then the velocity of separation should reduce and therefore, you know it will move at something like this.

Therefore, this angle may be something phi, where phi will be less than theta, say more than theta. In this case phi will be more than theta. So, if a small ball hits that perfectly smooth rigid surface, in that case, just like a light gets reflected, in the same way it get reflected and otherwise this is the thing. So, this is one example of that oblique impact.