Engineering Mechanics

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Kinetics-1

Module 11 Lecture 29
Impulse and Momentum

In the last few lectures, we have studied the kinetics of a particle. We have first discussed the Newton’s second law of motion and how it can be used for solving the numerical problems of kinetics. Then, we discussed the D'Alembert's principle also, which is essentially the same as Newton’s second law. However, in the last lecture, we discussed another approach that is work energy theorem. Also, we discussed the conservation of energy principle. Using that we can solve the kinetics problem.

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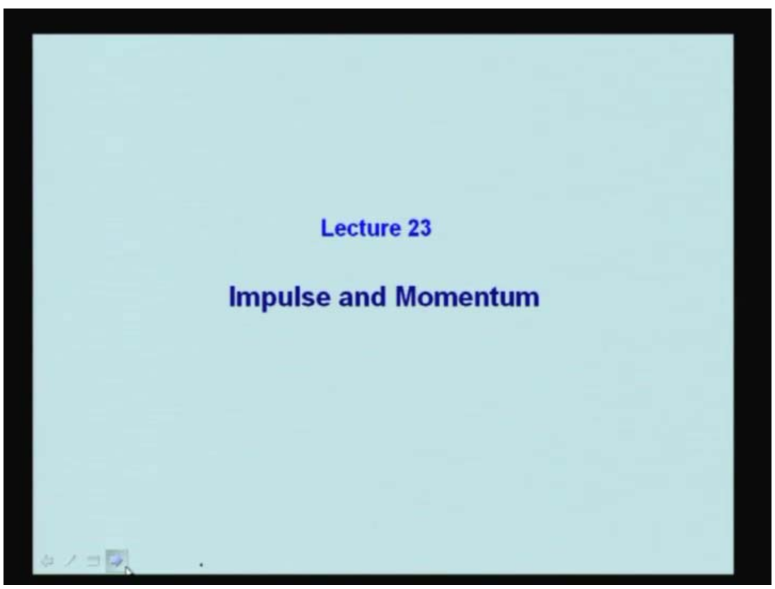

In this lecture, we will be discussing about another approach of solving the problems of kinetics of a particle; that is, impulse and momentum method. You know that mass and velocity of a particle are the two things which influence motion and forces. Suppose, you hit a wall with 100 gram steel bar and another man hits with a 1 kg steel bar, which one will provide more force to the wall? Which will do greater damage? You know that it depends on the mass. Mass, no doubt, has some influence but it also depends on the velocity.

If 100 gram bar and 1kg bar both are moving at the same velocity then naturally the 1kg bar will do more damage. However, if the 100 gram is bar is moving with a tremendously high velocity and 1kg bar is moving slowly then 100gram bar will do more damage to the wall. Therefore, the mass and velocity are two important things influencing the force, which can be applied by the moving body.

We coined a function, F is equal to that is function of mass and velocity that is half mv square; that is called kinetic energy. In this function, mass is having a power 1, but velocity is having a power 2. Why cannot we think of many other types of functions?

One simple function is that if we say, that function is mass times velocity and we get some quantity instead of telling that mass into velocity square, we can have mass into velocity which is simpler. One can have, of course that mass into velocity cube but we are not familiar with such type of physical quantity, which is popular.

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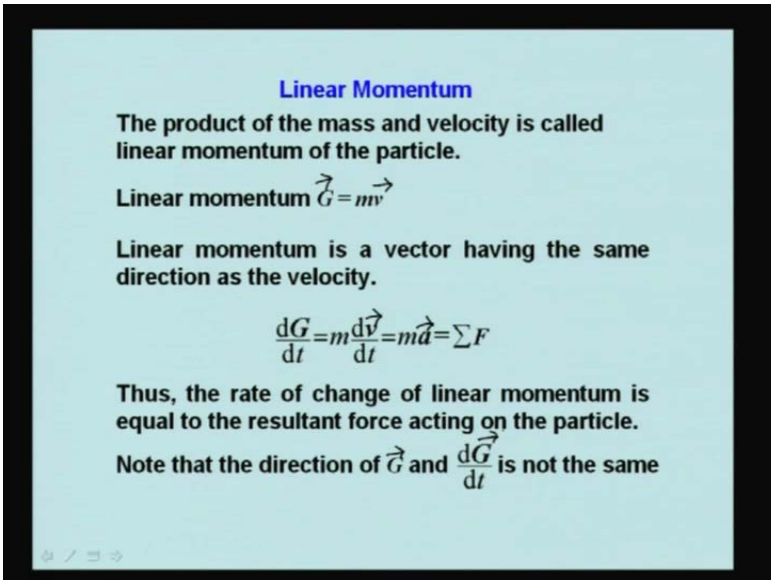

If we just take mass into velocity; that means, we define a function of mass and velocity in which that function G is mv. The product of the mass and velocity is called the linear momentum of the particle. Liner momentum G is equal to mv. Now linear momentum is a vector having the same direction as the velocity. Kinetic energy is a scalar; however, the linear momentum is a vector. It has the same direction as the velocity. It is proportional to mass and velocity. If you differentiate the linear momentum with time, you get mass into dv by dt. If the mass is constant, this becomes equal to mass times acceleration which is equal to sigma F, by Newton's second law. Thus, the rate of change of linear momentum is equal to the resultant force acting on the particle.

Therefore, Newton's second law can be expressed in this form also; that is, the rate of change of linear momentum is equal to the resultant force acting on the particle. Note that, the direction of G and dG by dt is not the same. If you differentiate a vector, you do not get a vector or you do not always get a vector in the same direction. V and dv by dt are having, may have different directions. Therefore, in this case G and dG by dt are having different directions.

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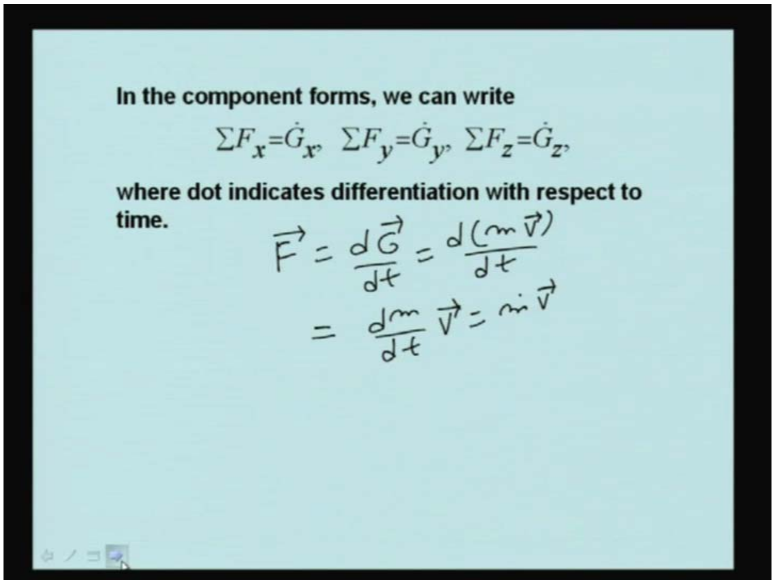

In the component forms, we can write, sigma F_x is equal to G dot $_x$, sigma F_y is equal to G dot $_y$, sigma F_z is equal to G dot $_z$, where dot indicates differentiation with respect to time. We can also write, F is equal to dG by dt, even when the mass is changing and you can have them in this case d mv by dt.

If you have a situation, in which mass is changing like in fluid mechanics problems, in a control volume, but if velocity remains the same, then you may have, dm by dt into V which is m dot V. This is the thing but in this course, mainly we will be discussing rigid body behavior.

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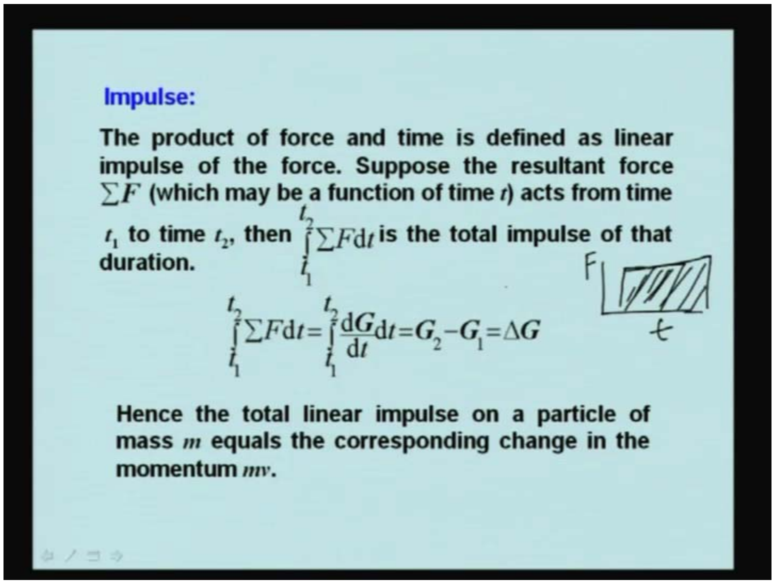

We define the other term that is impulse. You know that when we apply a force on an object, it causes acceleration. If we remove the force, the acceleration vanishes. Therefore, it is important that how much time that force has been acting. The force as well as time both provides the some action. Therefore, the product of force and time is defined as linear impulse of the force. Suppose, the resultant force sigma F, which may be a function of time t acts from time t_1 to time t_2, then t_1 to t_2 integrated sigma F dt is the total impulse of that duration.

Total impulse is the area under F_t curve. If you plot force on this axis in one dimensional case and this is t, then if the force is acting between, then this area is sigma F dt; that is the total impulse of that duration.

Now $t_{1\ to}$ t_2 sigma F dt can be written as t_1 to t_2 dG by dt by Newton’s second law into dt which gives you dG; that means G_2 minus G_1or delta G. Hence, the total linear impulse on a particle of mass m equals the corresponding change in the momentum m; that is mv. By this, we can find out change in momentum without that is, or in other words, change in velocity, without really finding out acceleration.

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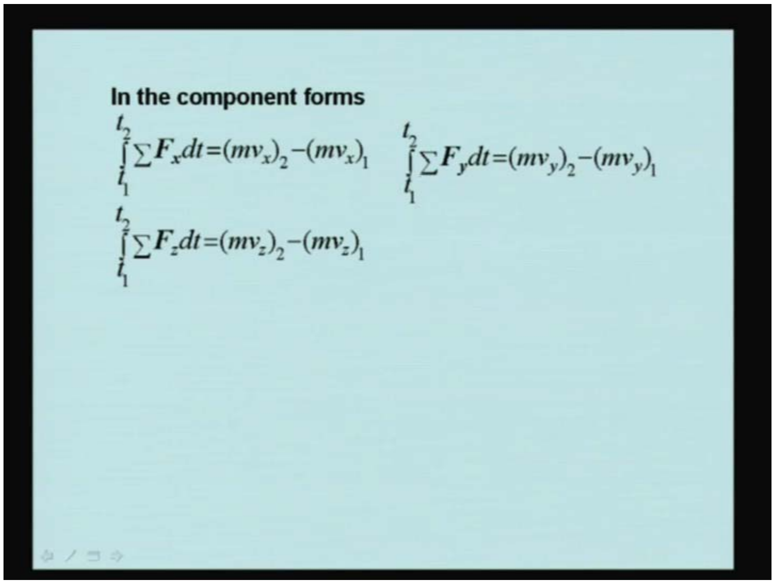

In the component form these equations are like this t_1 to t_2 sigma Fx dt is equal to m vx_2 minus m vx_1, t_1 to t_2 sigma Fy dt is equal to m vy_2, where vy is the velocity component in y direction minus m vy_1, t_1 to t_2 sigma Fz dt is equal to m vz_2 minus m vz_1, where vz is the velocity component in the z direction.

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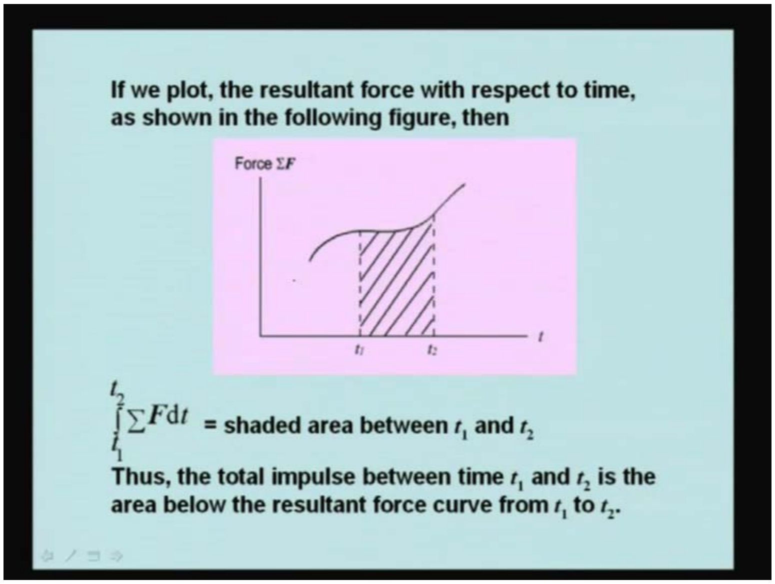

If we plot the resultant force with respect to time, then t_1 to t_2 sigma F dt is the shaded area between t_1 and t_2.

If we have the three components F_x, F_y, F_z, then we can plot F_x component with time, F_y component with time, F_z component with time and we can find out the area that is sigma F dt is equal to this thing. Like that, we can calculate the final momentum. Thus, the total impulse between time t_1 and t_2 is the area below the resultant force curve from t_1 to t_2. Total impulse is a vector quantity.

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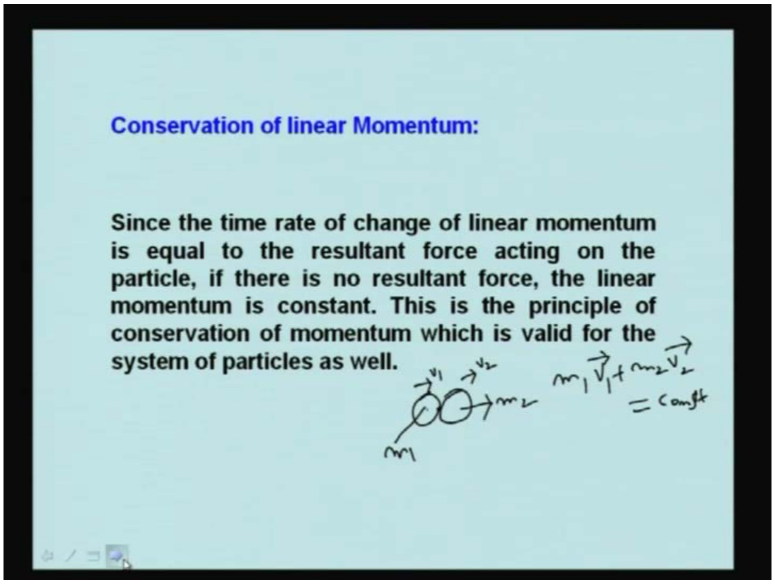

We have that very important principle that is called conservation of linear momentum. Since the time rate of change of linear momentum is equal to the resultant force acting on the particle, if there is no resultant force, the linear momentum is constant. This is the principle of conservation of momentum which is valid for the system of particles as well.

If two particles are colliding, one particle is exerting a force on the other particle. Similarly, the other particle is also exerting the force on this particle, but there is no resultant force on the system of particles. Therefore, the linear momentum will remain conserved. These particles may have different velocities; however, if this particle's mass is $m_1$and this particle's mass is m_2, this velocity is V_1 and this is V_2, m_1 V_1 plus m_2 V_2 will remain constant, because no force has been applied from outside. This is called conservation of linear momentum.

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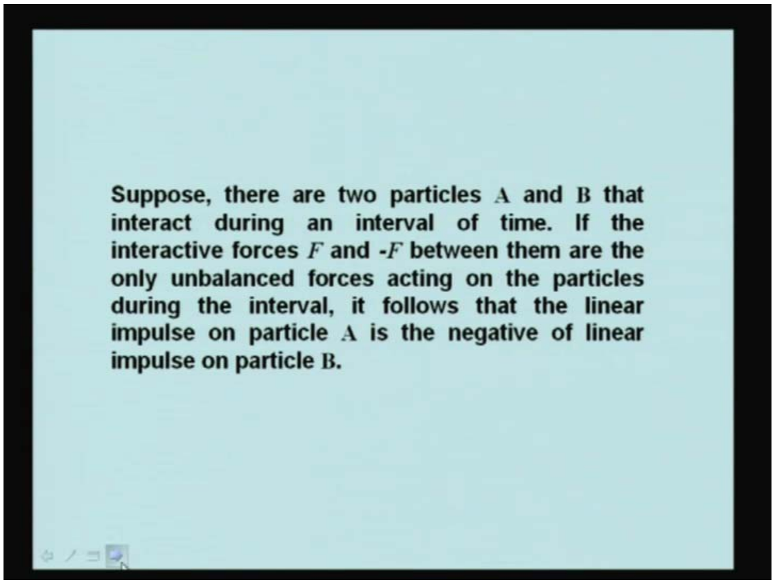

Suppose there are two particles A and B that interact during an interval of time. If the interactive force is F and minus F between them are only unbalanced forces acting on the particles during the interval, it follows that the linear impulse on particle A is the negative of linear impulse on particle B.

Here, I am using the word interaction. The particles need not physically collide. They may apply only the forces and affect the motion, like the gravitational field on charge particles applying the forces on each other. It is not necessary that there should be impact of the particles.

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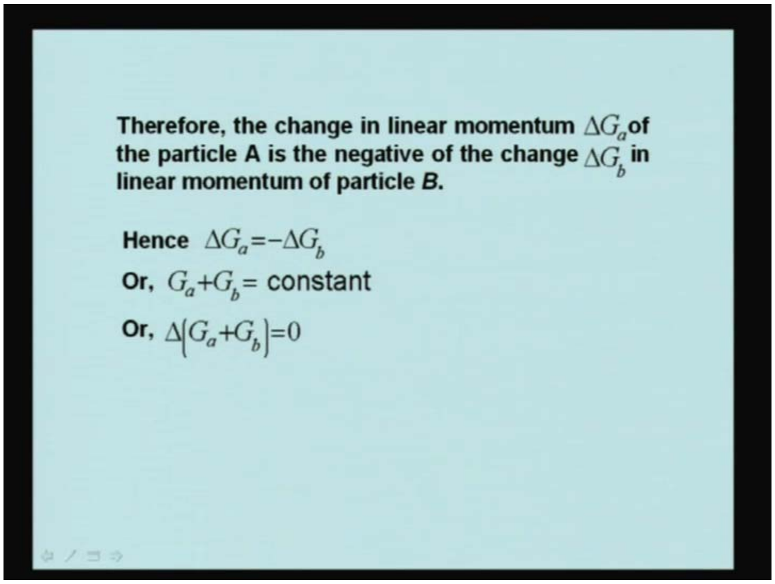

Change in linear momentum delta G_a of the particle A is the negative of the change delta G_b in linear momentum of particle B; that is delta G_a is equal to minus delta G_b or G_a plus G_b is equal to constant or delta G_a plus G_b is equal to 0.

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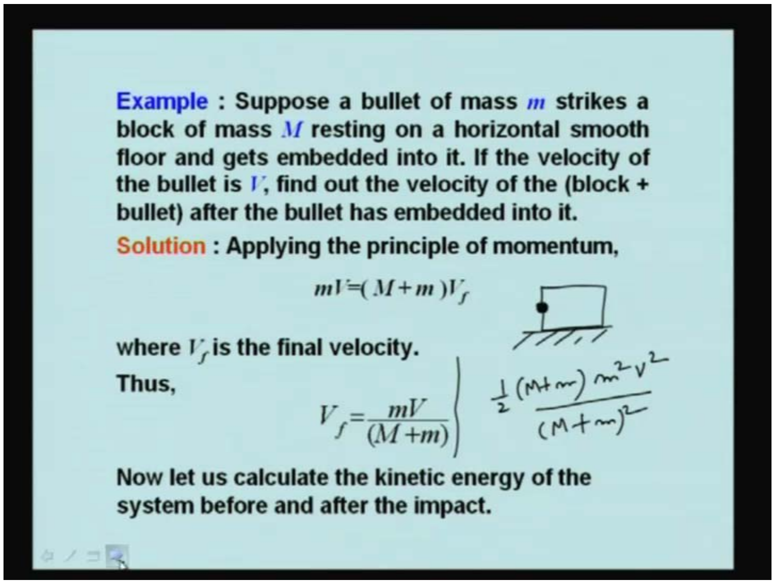

We have been using this principle. For example, a bullet of mass m strikes a block of mass M resting on a horizontal smooth floor and gets embedded into it. If the velocity of bullet is V, find out the velocity of the block plus bullet after the bullet has embedded into it.

Applying the principle of momentum, since there is no external force acting, so that mV is equal to M plus m V_f where V_f is the final velocity. Thus, V_f is equal to mV divided by M plus m. This is the final velocity of the bullet block.

If the block is resting on a smooth floor and a bullet of mass m moving with velocity capital V has embedded in it, then the final velocity of the block will be m capital V divided by M plus m. If the block is kept on a rough floor instead of the smooth floor then what happens?

In that case, if the bullet is striking, there is a frictional force also acting; but the magnitude of that force is, if we assume that the impact duration is very small and then the force coming due to impact of the bullet is so high that the friction can be neglected. Therefore, in the beginning V_f may still be can considered as mV divided by M plus m. However, after that, its velocity will keep on reducing because of the friction.

Let us calculate the kinetic energy of the system before and after the impact.

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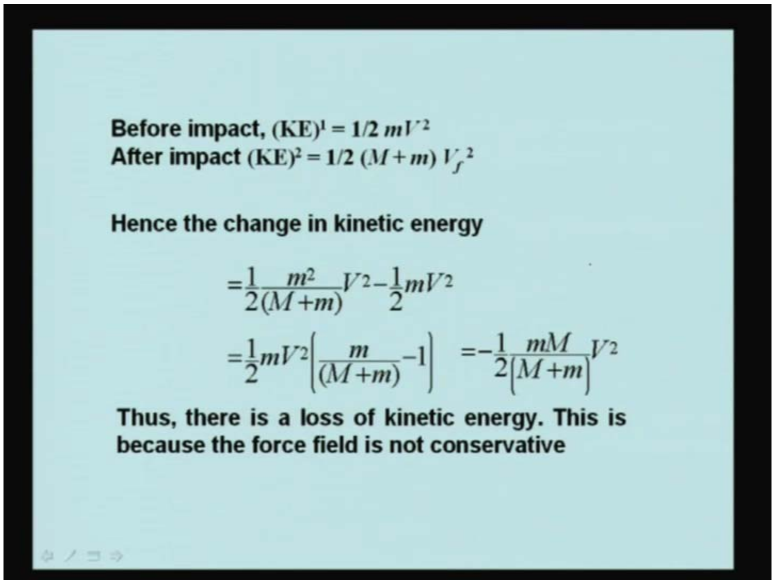

Before impact, kinetic energy is equal to half mV square; after impact, kinetic energy has become half M plus m V_f square. Hence, the change in kinetic energy is equal to half mV square divided by M plus m V square minus half mV square; because, the final velocity is mV M plus m. Therefore, kinetic energy before, in the beginning, the kinetic energy was only half mV square. After the impact, block is moving with velocity mV square. So, therefore half M plus m m square V square divided by M plus m whole square. This gives you half mV square M plus m V square minus half mV square.

One can take half mV square common which gives you, m divided by M plus m minus 1 that is equal to minus half mM divided by M plus m V square. Thus, there is a loss of kinetic energy. This is because the force field is not conservative; in this case the velocity. The loss of this one has occurred; minus half m M divided by M plus m divided by V square.

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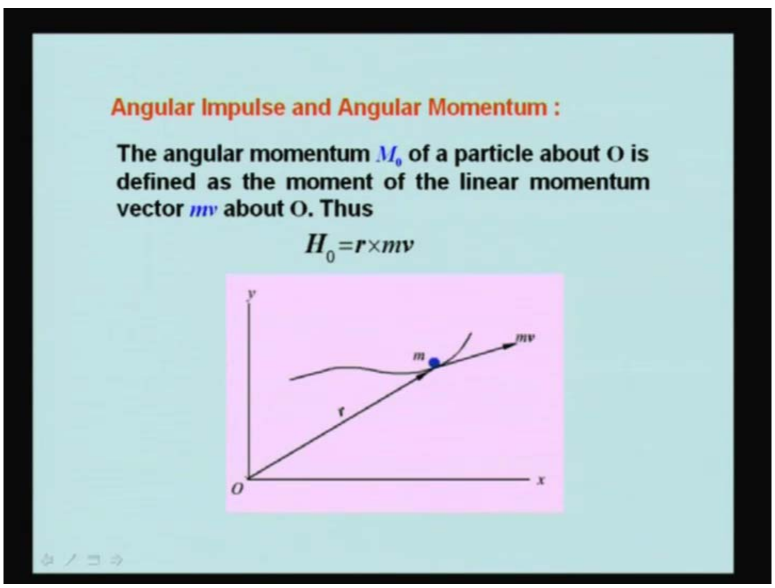

If we define another term just like we defined linear impulse and linear momentum, we define now angular impulse and angular momentum. The angular momentum M_0 of a particle about O is defined as the moment of the linear momentum vector mv about O. Thus, H_0 will be r cross mv. This is a particle and it is having a linear momentum mv. Direction of the linear momentum is same as the direction of the velocity. It is a position vector with respect to one fixed point O and is given as r. Then, we are defining the linear momentum about this point as r cross product mv.

Now, the cross product of two vectors is also a vector. Therefore, the angular momentum is a vector; that is, angular momentum is also called moment of the linear momentum.

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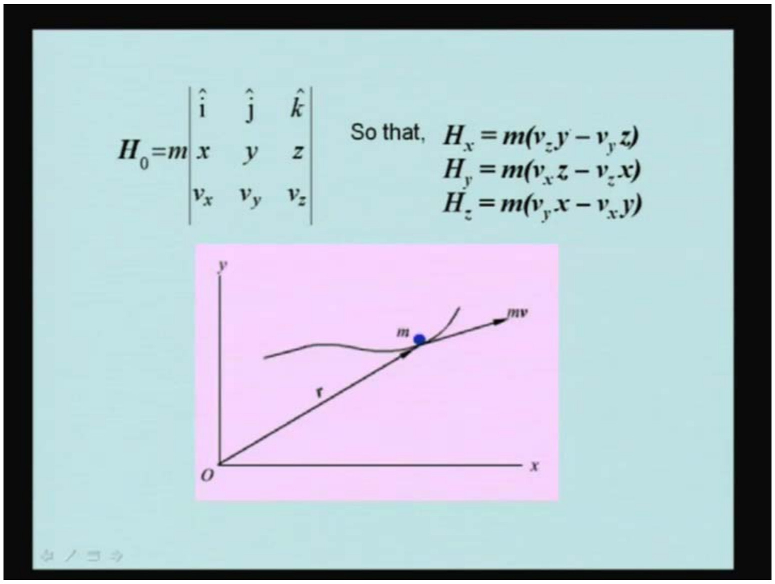

In the component form, this can be written like this. Cross product of r cross mv gives us, H_0 is equal to m determinant i, j, k, x, y, z, v_x, v_y, v_z that is H_x is equal to m v_z y minus v_y z. H_y is equal to m v_x z minus v_z x. H_z is equal to m v_y x minus v_x y. These are the components of the angular momentum.

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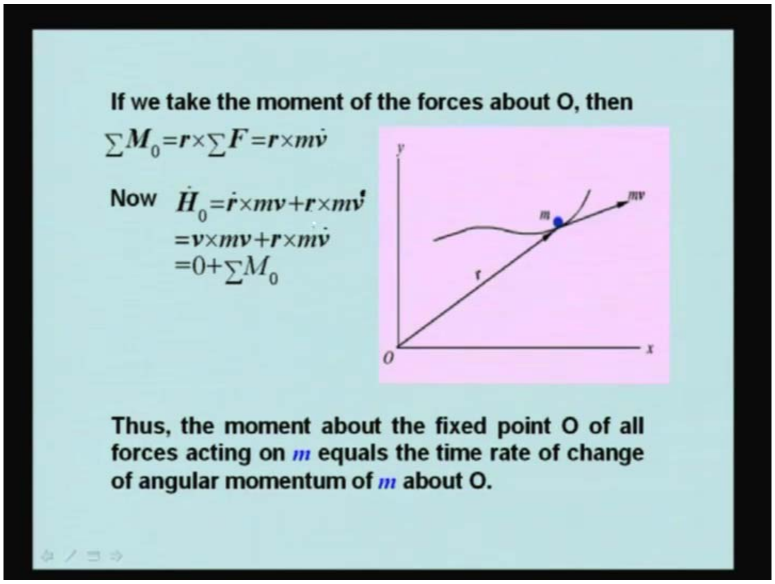

If we take the moment of the forces about O, forces which act on the particle then sigma M_0 will be r cross sigma F; that is, r cross mv dot because sigma F is equal to mv dot.

Since H_0 is defined as r cross mv, we can take the time derivative of a_0 and this H dot 0 is equal to r dot cross mv plus r cross mv dot. r dot is nothing but V velocity, so v cross mv plus r cross mv dot r cross mv dot is equal to that. This can be written has H_0 is equal to r cross mv.

It has come v cross mv. Cross product of a vector with itself is 0. m is a scalar, it can any way be taken outside. So, this term has become 0. Therefore, you have got r cross mv dot which is nothing but sigma M_0 as shown here. Therefore, H dot O is equal to sigma $M_{0.}$ Thus, the moment about the fixed point O of all forces acting on m equals the time rate of change of angular momentum of m about O.

We find out the moment of the forces about the fixed point O. Then, it should be time rate of change of angular momentum about this one.

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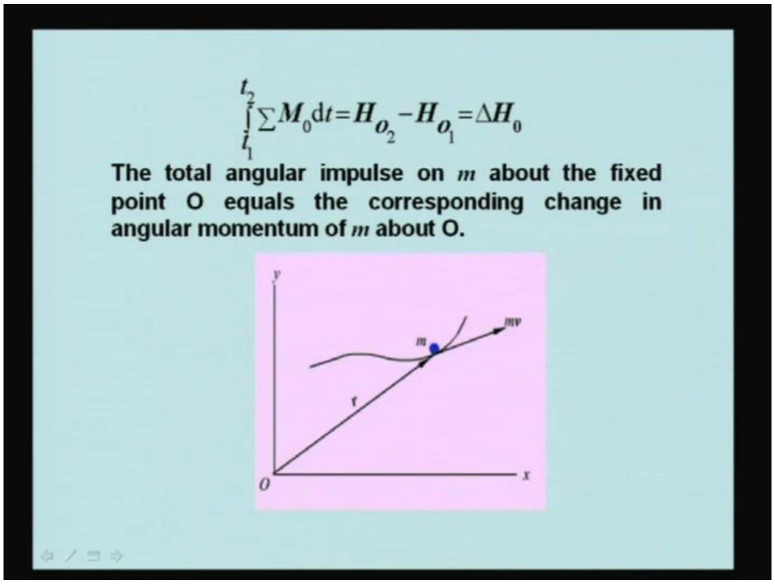

This equation is very useful. Or you can write t_1 to t_2 sigma M_0 dt is equal to H_{02} minus H_{01} or H_{02} minus H_{01} that is equal to delta H_0. Thus, the total angular impulse on m about the fixed point O equals the corresponding change in angular momentum of m about O. This is called t_1 to t_2 sigma M_0 dt is called angular impulse.

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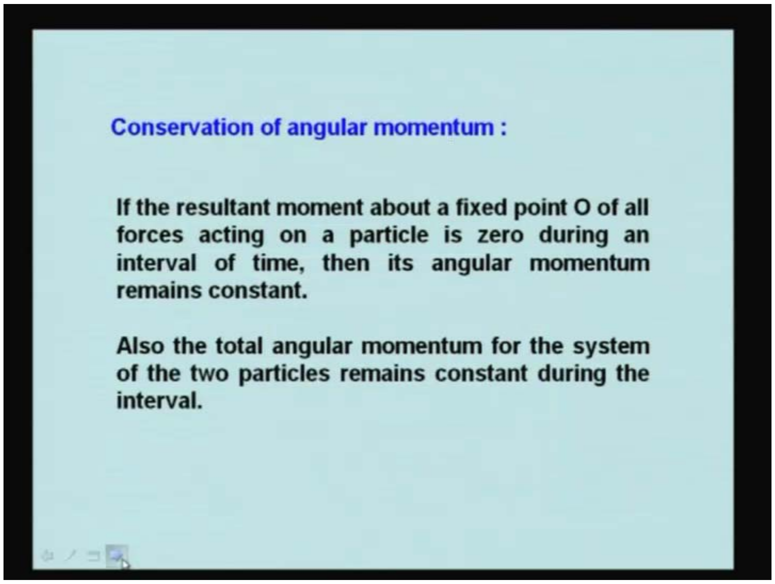

Like we have the principle of conservation of linear momentum, we also have principle of conservation of angular momentum. We see the equation H dot$_0$ is equal to sigma M_0. If sigma M_0 that is the moment of the forces about O is 0, then what happens, then H dot O will become 0 and in that case, this will be H dot 0 is 0; that means, H_0 is constant. Thus, if the resultant moment about a fixed point O of all forces acting on a particle is 0 during an interval of time then, its angular momentum remains constant. Also, the total angular momentum for the system of the two particles remains constant during the interval. This means that this is valid also for the system of particles.

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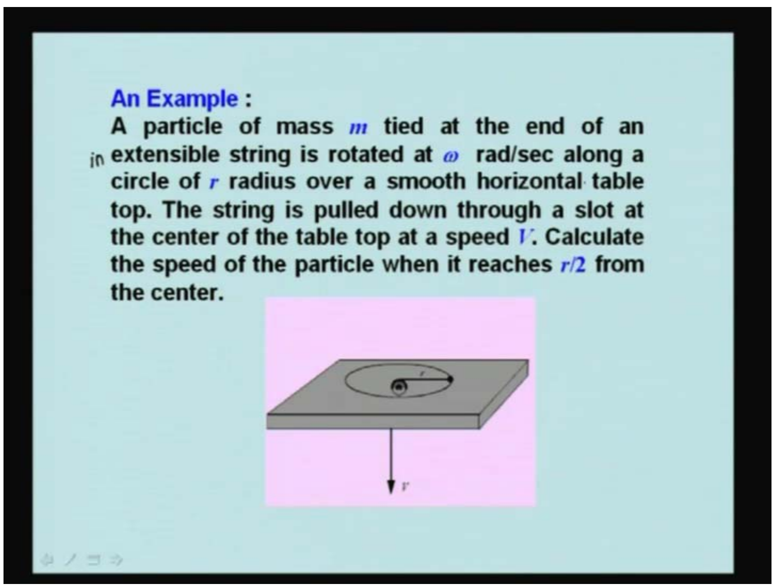

We present, one example based on the conservation of angular momentum. A particle of mass m tied at the end of an inextensible string is rotated at omega radian per second along a circle of r radius over a smooth horizontal table top. The string is pulled down through a slot at the center of the table top at a speed V. Calculate the speed of the particles when it reaches r by 2 from the center.

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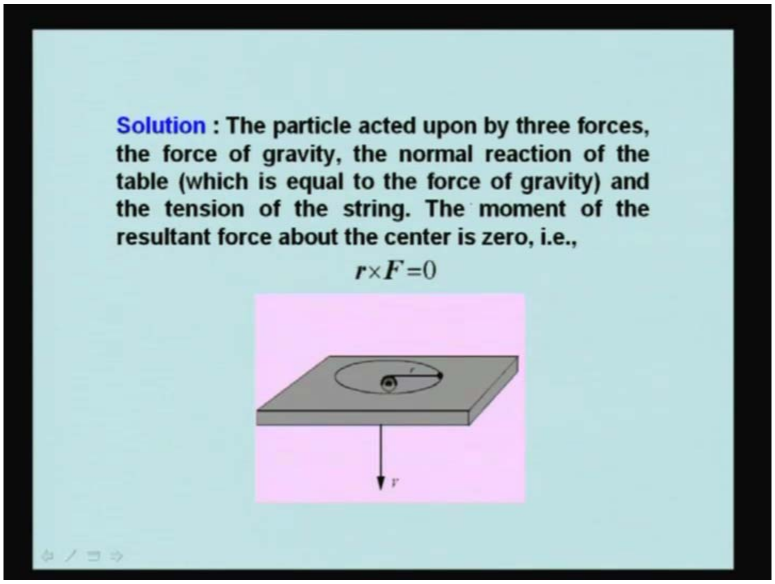

Now, the particle is acted by three forces: The force of gravity, the normal reaction of the table which is equal to the force of gravity and the tension of the string. The moment of the resultant force about the center is 0; that is r cross F is equal to 0.

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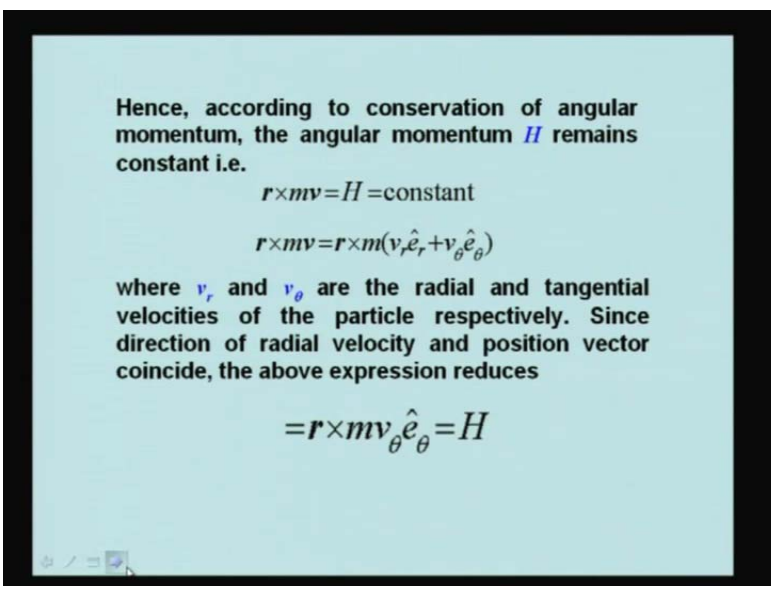

Then, hence according to conservation of angular momentum, the angular momentum H remains constant; that is r cross mv is equal to H remains constant. r cross mv can be written as r cross m

v_r e_r plus v_{theta} e_{theta}, because the string is rotating and it has got v and also it is been pulled. So, it has got two components that is e v_r and v_{theta}. Thus, r cross m v_r e_r plus v_{theta} e_{theta} where v_r and v_{theta} are the radial and tangential velocities of the particle respectively. Since direction of radial velocity and position vector coincide, this expression reduces to r cross mv_{theta} e_{theta} that is H constant.

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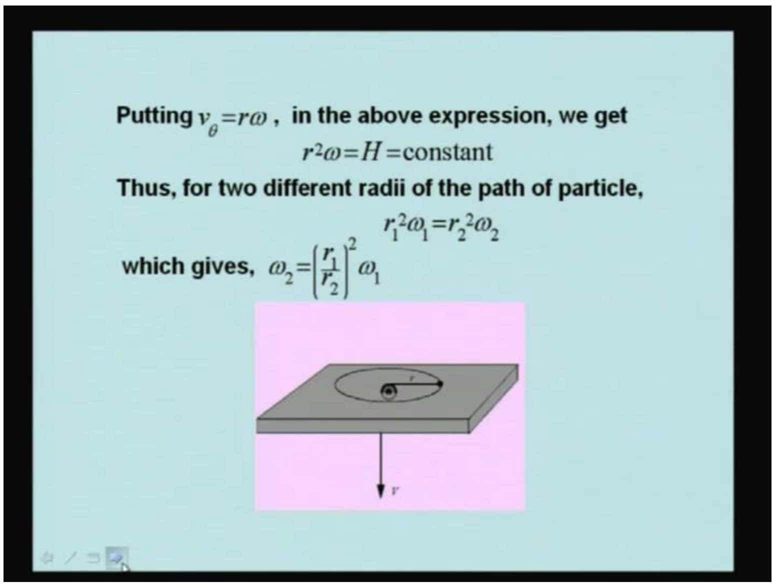

Now, putting v_{theta} is equal to r omega in this expression, we get r square omega is equal to H, that is constant. We got v_{theta}; you put r omega for v_{theta}, so this is omega square r, that is constant. Thus, for two different radii of the path of particle r_1 square $omega_1$ is equal to r_2 square $omega_2$, which gives $omega_2$ is equal to r_1 by r_2 whole square $omega_1$.

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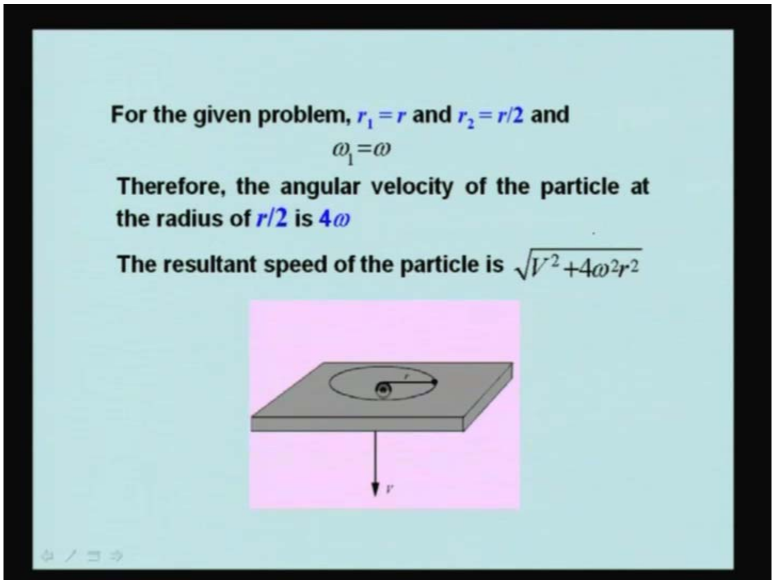

From the given problem, r_1 is equal to r and r_2 is equal to r by 2 and $omega_1$ is equal to omega. Therefore, the angular velocity of the particle at the radius is 4 omega. The resultant speed of the particle is under root V square plus four omega square r square; that is, the velocity of the particle. This is the application of conservation of angular momentum. We will be doing and discussing some problems based on the impulse and momentum and also the work and energy theorem.

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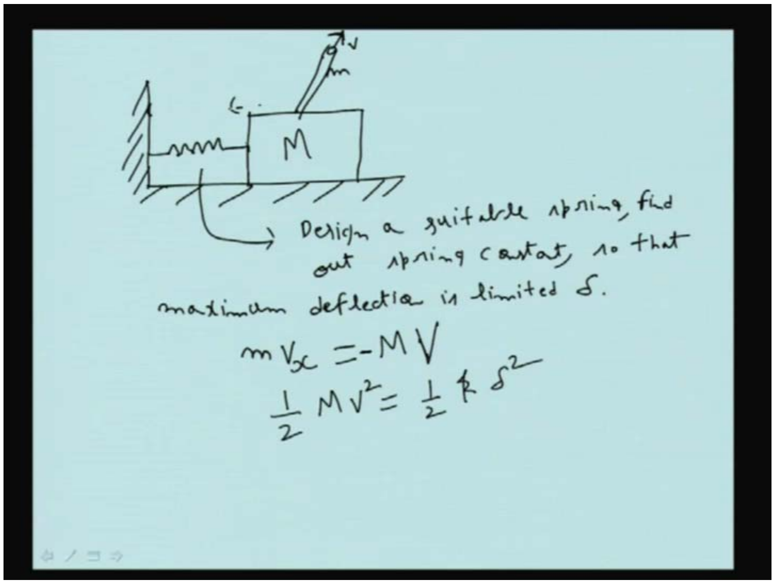

One problem can be of this type.

If there is a block acted by this, which is held by spring from here, you fire a bullet thorough a gun. Once it is fired, then some velocity V and the mass of the bullet is m. This mass may be M. Then you have to find out that if you have designed a spring whose stiffness should not whose displacement that is.

The question may be, design problem may be that design a suitable spring which means find out a spring constant so that maximum displacement deflection is limited to delta. In this case, you can apply the conservation of momentum principle to find out the horizontal velocity of mass m. Initially the momentum remains conserved. Therefore, mass times V_x where V_x is the horizontal component of the velocity should give you mass times the velocity of the particle block V.

Once we have find out the velocity, this velocity will be in the negative direction; that means when the bullet is going this way, it will go in this way. This block will now possess the potential energy kinetic energy. That kinetic energy will be stored as potential energy in spring. That means, half mv square is equal to half k delta square delta, because that is limiting deflection. So, you can find out the value of k, that is this thing.

If friction is also present then you have to find out energy lost in friction. Then half mv square will not be equal to half k delta square. In that case, you can manage with little less stiffer spring. Some energy is also observed by the friction, so this is this type of problem.

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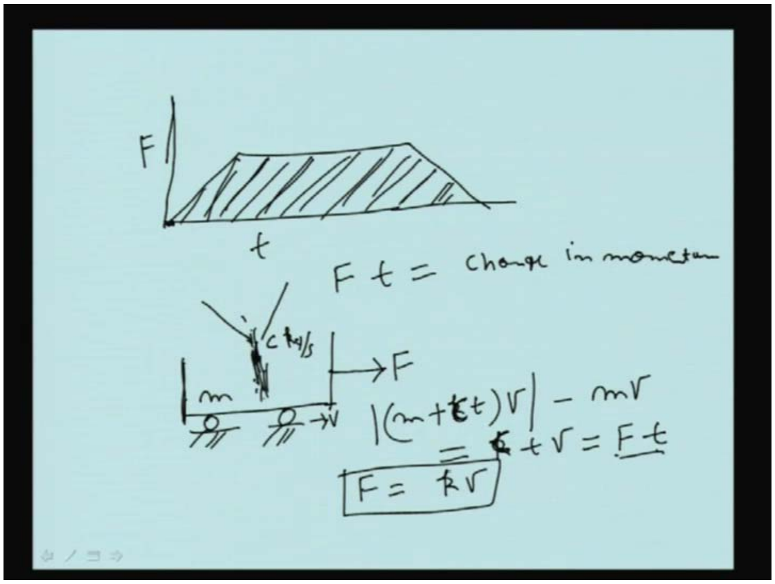

Another problem can be of this type. When the force profile is given to you and that if force versus time like this and you are required to find out the velocity of a particle which starts at rest from 0 and reaches this, then what is its velocity after time t? In that case, you can apply the impulse momentum principle.

Change in momentum will be equal to the total impulse which is nothing but this area. You can thus find out the final velocity of the particle without doing much effort. Last lecture, we did a problem that if the truck heavy truck and a light car both are moving with the same kinetic energy, if equal amount of breaking force is applied which one will stop by covering less distance? The answer was that both will cover the same distance without stopping.

We put a question that if a truck and car are moving with the same linear momentum and if the same amount of breaking force is applied which will stop first? In this case, both will stop first because the change in momentum is same in both the cases. F_t is equal to change in momentum. If the force is also same, then the time will be same. This problem also has been done.

We may have the problems of that kind, where the mass is changing. Suppose, you have a cart in which you are putting some sand through harper, at some rate that is c kilogram per second. If the mass of the cart is m, then, in order to maintain the constant velocity, what force F needs to be applied to neglect the friction? In this case, at certain time t, change in momentum is equal to m plus kt because the mass after time t is m plus c t. This is, c kg multiplied by V that is at certain time. Then, minus mV that is in the beginning when nothing was there, then the total change in momentum is equal to k t V which is equal to force times Ft. Thus, we get F is equal to k times V.

We need to apply a force which will be equal to velocity which is proportional to the velocity of the cart and this is c t, rate of filling the materials, F is equal to cV, c kg per second is being filled. So, this is what that you have learnt.

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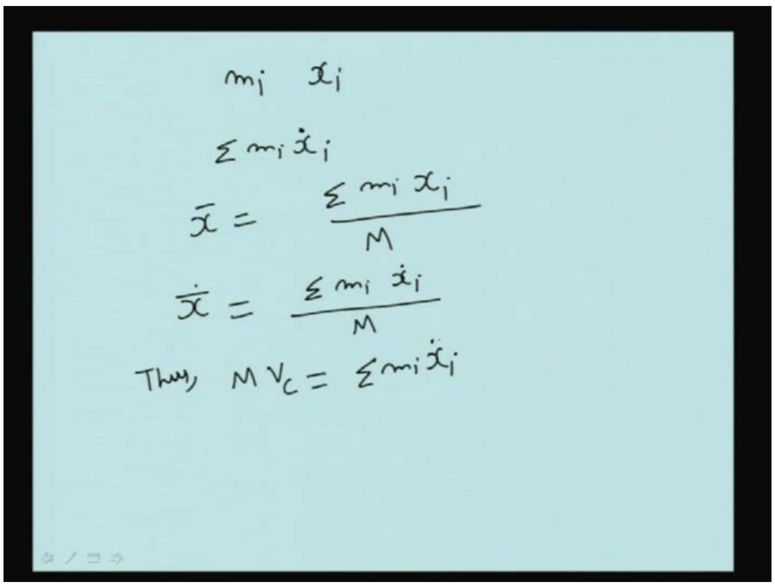

It can be shown that momentum of a system of the particles will be equal to the momentum of the mass center of the particle. Momentum of the mass center of the particle is defined as the total mass into the velocity of mass center.

We know, if we have system of particles and let us say i th particle is of mass m_i and its position is x_i, then sigma m_i x_i dot is the total momentum of the system.

Also, we have, mass center x is equal to sigma m_i, x_i dot. The x_i is the position, so x_i dot is by velocity, sigma m_i x_i divided by mass M. I can differentiate the both sides with respect to time and get a relation; that is x dot is equal to sigma m_i x_i dot divided by M. Thus, M into V_c, where V_c is x bar dot that is the velocity of mass center that is equal to sigma m_i x_i dot. So, velocity of the mass center of a system of particle is, that is momentum of the mass center of the particles is sigma m_i x dot $_i$. Thus, we can use this equation for system of particles.

Let us do one interesting problem based on the angular momentum principle.

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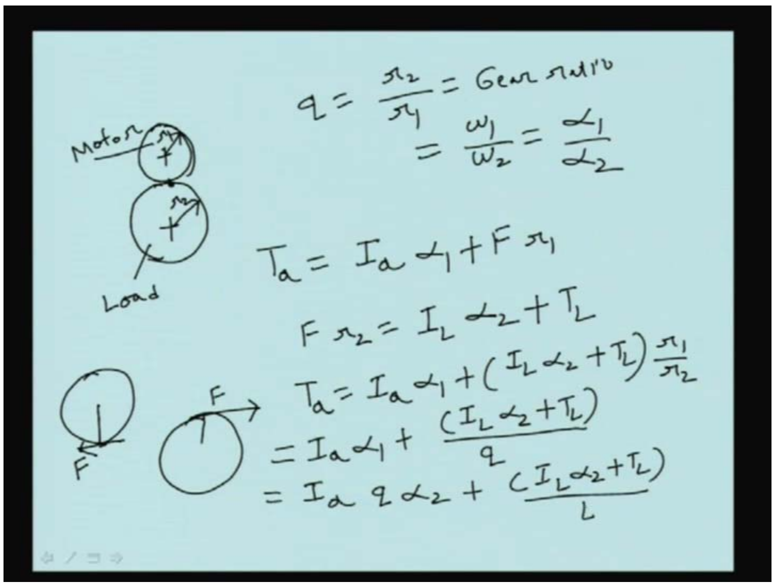

We have got a pinion that is a small gear, connected with that is gear and this is attached with the motor. This is motor gear and this is attached towards the load side. If this radius is r_1 this is r_2 then q is equal to r_2 by r_1 is called gear ratio. There are two gears; so, it is gear ratio which is equal to omega$_1$ by omega$_2$ which is equal to alpha$_1$ by alpha$_2$. The linear velocity is same actually for both the gears. So, omega$_1$ into r_1 is equal to omega$_2$ into r_2.

If we indicate that torque by the motor is T_a that is basically I_a times, where I_a is the inertia of the armature plus this gear I times alpha$_1$, alpha$_1$ is the angular acceleration plus if we make a free body diagram of this, you see that it is having I_a alpha plus it is having a resisting force F present here due to the other gear. That is how it transmits the torque, because it is rotating this gear.

Suppose, it rotates in clockwise direction then opposing force has to come here like this. So, that thing is given by F into r_1; that is you have to overcome that also. So, we have to apply Fr_1.

Similarly, for the other gear, you have F is the force applied by the pinion. So, F r_2 is equal to I_L times $alpha_2$ plus T_L, where T_L is the loaded torque. Therefore, I can eliminate it from here and put T_a is equal to I_a time's $alpha_1$ e plus I_L times $alpha_2$ plus T_L. r_1 by r_2, which can be written as I_a times $alpha_1$ plus I_L $alpha_2$ plus T_L divided by q r_1 by r_2, that is this r_1 by r_2 has been replaced by 1 by q, so this is not there.

We have to find out the gear ratio for minimum torque. For that you do, d T_a by dq is equal to I_a times; let us first represent T_a in that form, because I may say that $alpha_2$ will be prescribed; that is, load has to accelerate with certain angular accelerations $alpha_2$, that means pick up of the load.

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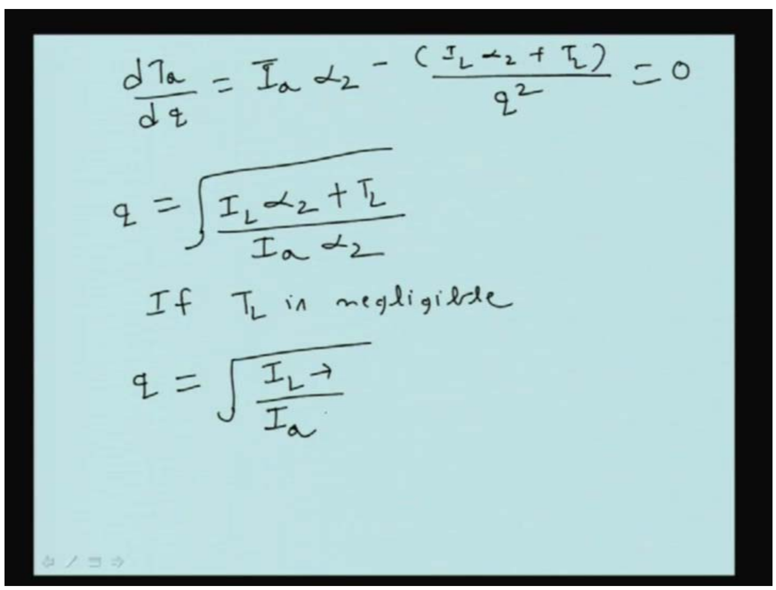

Therefore, better to put it I_a as q times $alpha_2$ plus I_L $alpha_2$ plus T_L divided by q or d T_a by now d T_a by dq is equal to I_a time's $alpha_2$ minus I_L $alpha_2$ plus T_L divided by q square. This is equal to 0 for minimizing the torque. Therefore, q is equal to under root I_L $alpha_2$ plus T_L divided by I_a $alpha_2$. This is the optimum gear ratio. If T_L is negligible, then q is equal to under root I_L divided by I_a. This will give this.

Therefore, in this case, the gear ratio is not dependent on $alpha_{2.}$ It is the ratio between the moment of inertias of the load; momentum of inertia of the load and moment of inertia of the armature. This is what that answer has come and we have discussed about. Now, I am giving one example of finding out the friction torque of a disc experimentally.

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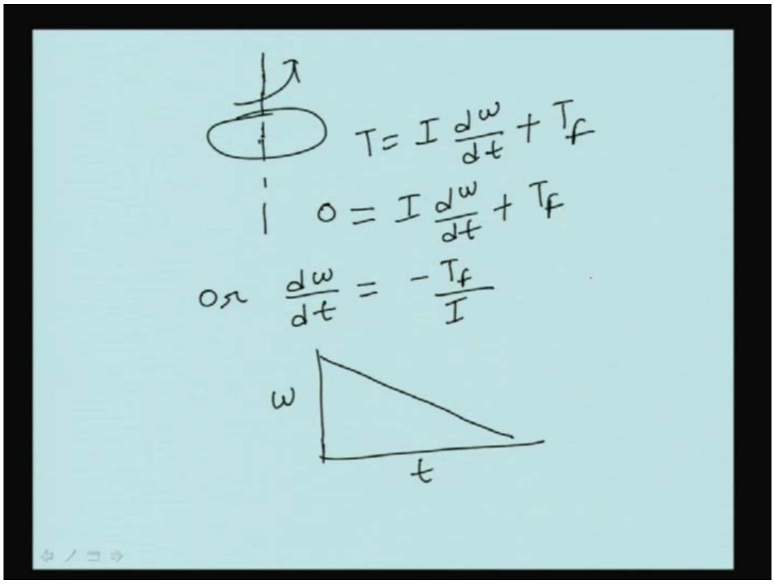

Suppose you take a disc. This disc is rotating on the bearing. In this case, T is equal to I d omega by dt pus $T_{f,}$ where T_f is the friction torque. If you rotate the disc and after that, remove the torque then the disc stops after some time. Therefore for that, equation of motion is 0 plus I d omega by dt plus T_f or d omega by dt is equal to minus If T_f by e minus T_f by e of i.

That means, if you plot omega versus time curve and you can take the slope of that. By that, you can find out the friction torque. Friction torque T_f may be I times d omega dt, this is the thing about that.

In this lecture, we have discussed about the linear momentum equation and also angular momentum equation. What is the relation between the rate of change of linear momentum with the resultant force and the relation of rate of change of angular momentum with the applied moments?

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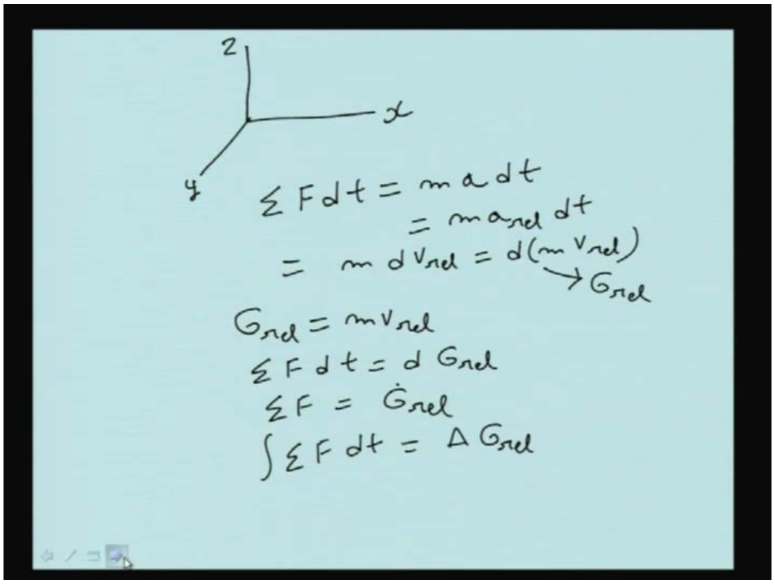

If we have taken for calculating the angular momentum, we have taken the fixed point, if we have a negative frame of reference, then we can have like this. Relative to x y z some frame of reference sigma and if it is moving with a constant velocity then sigma F dt will be m absolute acceleration times dt. Since, it is moving with a constant velocity, therefore this will be m a_{rel} dt, which will be equal to m d V_{rel} or dm $V_{rel.}$ Therefore, sigma F dt; that means, this I can write as $G_{rel.}$ That means, linear momentum with reference to another reference frame x y z which is moving with a constant velocity.

G_{rel} is equal to m V_{rel} or sigma F dt is equal to d G_{rel} or sigma F is equal to G dot $_{rel.}$ The equation of the impulse momentum is valid; that means, we can also write sigma F dt is equal to delta G_{rel} in the relative frame of reference which is moving with a constant velocity with respect to the system.

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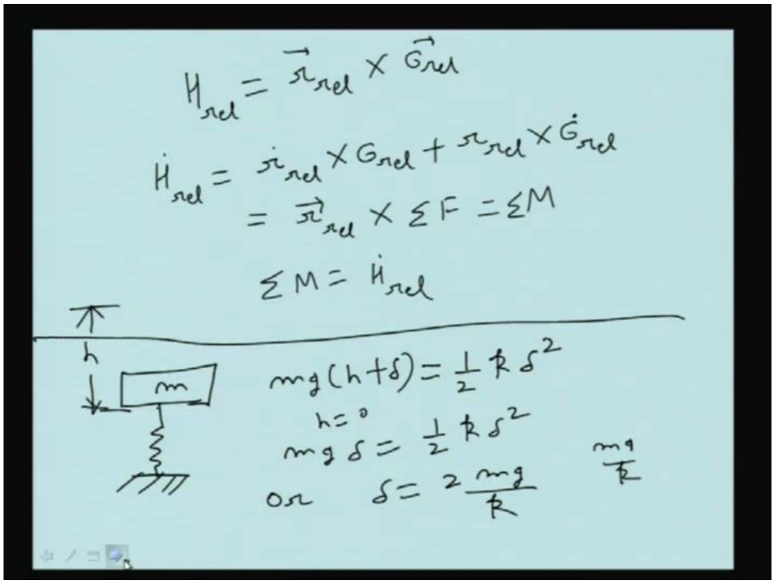

In fact, we can also see that if we have the angular momentum about this thing. So, this is H_{rel} is equal to r_{rel} plus $G_{rel.}$ If you take the time derivative, H dot_{rel} is equal to r dot_{rel} plus G_{rel} plus r_{rel} cross G dot_{rel}.

Now r dot_{rel} and G cross $_{rel}$ are in the same direction, therefore this is 0. This becomes r $_{rel}$ cross sigma F which is equal to sigma M, therefore we have this equation as well that sigma M is equal to H $dot_{rel.}$ Thus, relative to that a reference frame which is moving with a constant velocity, we have all the equations. Not only these, Newton's second law is valid, but we also have the valid impulse momentum equations.

We have already shown that even work kinetic energy theorem is valid provided you find out the work in that relative frame of reference and change in kinetic energy in that reference frame.

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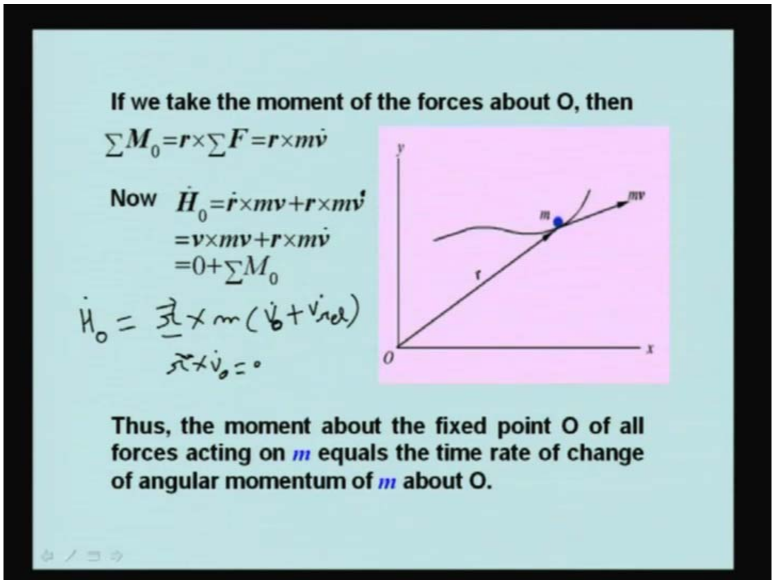

If we pay attention to equation for angular impulse angular momentum, here we have got the angular momentum. We see that we have derived it like this; H dot_o is equal to r plus mv dot. If we have a system moving with a constant velocity, then in that term its effect is not experienced; means, I can have, if it is valid for mv dot, it is also valid for m v plus c dot. Therefore, naturally this relation holds good for point O which is moving with a constant velocity, but there r is a fixed point or this may not, if that point is having some velocity, which is acceleration which is in the direction of r then also this holds good. Because, suppose you have a point O, H dot O is equal to r cross m. This is V of that particle; that means, I indicate by V_o plus V_{rel} and I say that this is basically dot.

If you can somehow make r cross V dot O equal to 0, then this equation can be written as H dot O is equal to r cross m V_{rel}; that means; I can have the relative velocities only. Therefore, the third condition is that V dot O is in the same direction as r; that means, point that particle is accelerating towards r away from the particle. In these situations, this can be used.

Let us do a problem of this. Whenever you are provided a problem, you have to think that which principle will be more convenient to apply.

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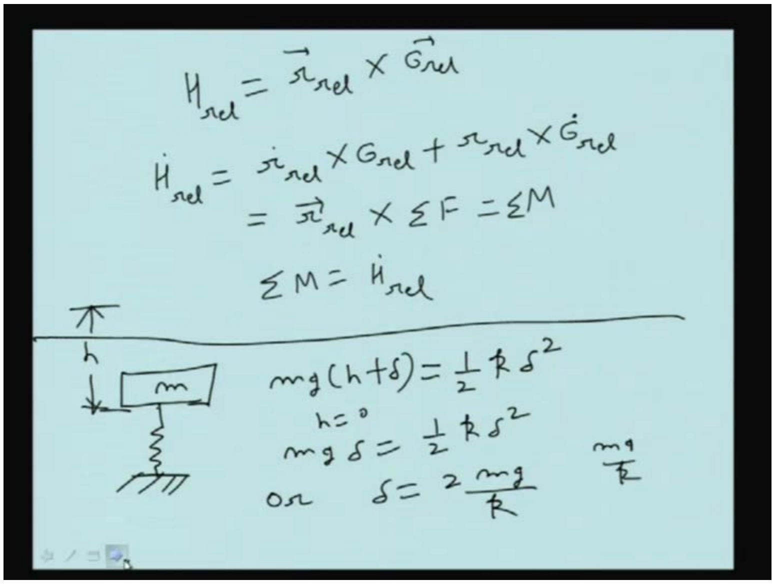

For example, if the problem is that there is a spring and you put a mass m suddenly on it, what is the deflection of the spring? In this case, mass has been put suddenly. In this case, the application of conservation of energy seems quite appropriate.

If we drop a mass m from a distance height h from the reference unstretched position of the spring, then the spring gets compressed by delta mg h plus delta is the change in the potential energy. From the conservation of energy principle, this should be same as the potential energy stored in the spring during its maximum deflection. At that time, the kinetic energy is 0. Therefore, this gives us this relation.

This gives a quadratic equation, which can be solved to find out delta if the particle is dropped from a height h. In case, h is equal to 0 then this becomes mg delta; that means you just have put the particle suddenly on the spring that is half k delta square or delta is equal to 2 mg by k. 2 mg by k and in that case the deflection will be two times the static deflection static deflection is mg by k.

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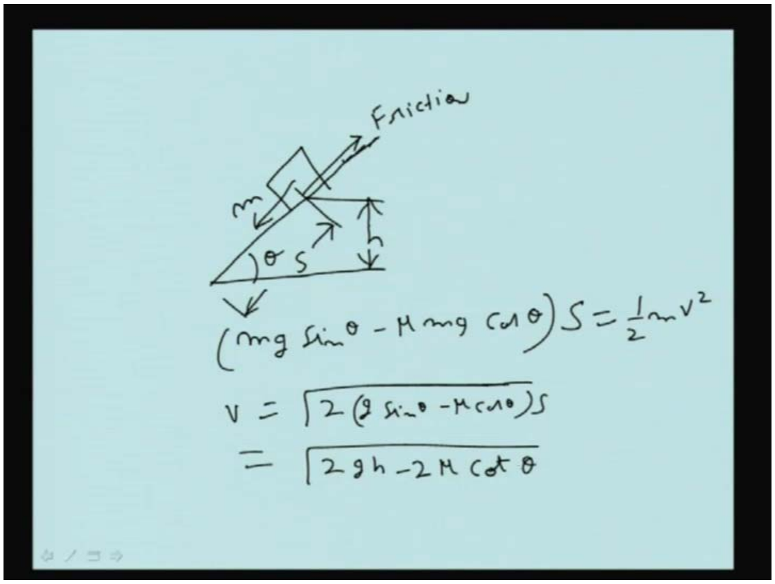

Similarly, if you have been provided a problem where particle of mass theta is particle at inclined plane which is inclined at theta this particle is sliding down. This friction force is also acting here. In this case, what will be the velocity when the particle has travelled distance s along the inclined surface?

In this case, also the principle of the work energy that is conservation of energy can be applied, provided, you take into account the frictional forces also. That is, we can see, it is just work energy theorem. mg sin theta minus mu mg cos theta are the forces and S is the distance covered; that is, half mv square. Therefore, V is equal to under root 2 g sin theta minus mu cos theta into S, which can be written as 2 g h minus h is this height minus 2 mu cot theta. Due to friction, the velocity will decrease.

We see that understanding of all the principles is important; that is Newton’s second law and D'Alembert's principle which are basically one, and then impulse momentum and energy, work energy methods. Whenever that is appropriate you should apply this one in order to get the answer.

However, it is true that by following any method you should be able to get the answers. Only thing is that in some cases, a particular method may take more time.