Engineering Mechanics

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Kinetics-1

Module 11 Lecture 27
Kinetics of a Particle

In this lecture, we will discuss the kinetics of a particle.

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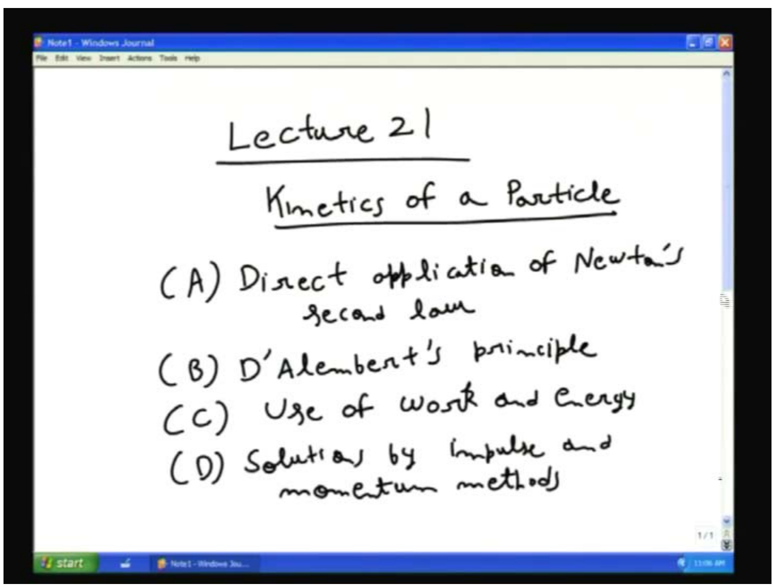

We have already studied kinematics of a particle in which we obtained the velocity of a particle by differentiating its displacement vector, acceleration by differentiating the velocity vector. In this lecture, we are going to discuss the effect of forces on the motion of the particle. In kinematics, we did not consider the effect of the forces. In kinetics, we will study the effect of the forces on the motion of a particle.

The particle, due to influence of the forces will change its velocity. There are four approaches to the solution of kinetics problem: (A) Direct application of Newton's law, direct application of

Newton’s second law. (B) D'Alembert's principle (C) Use of work and energy (D) solutions by impulse and momentum methods.

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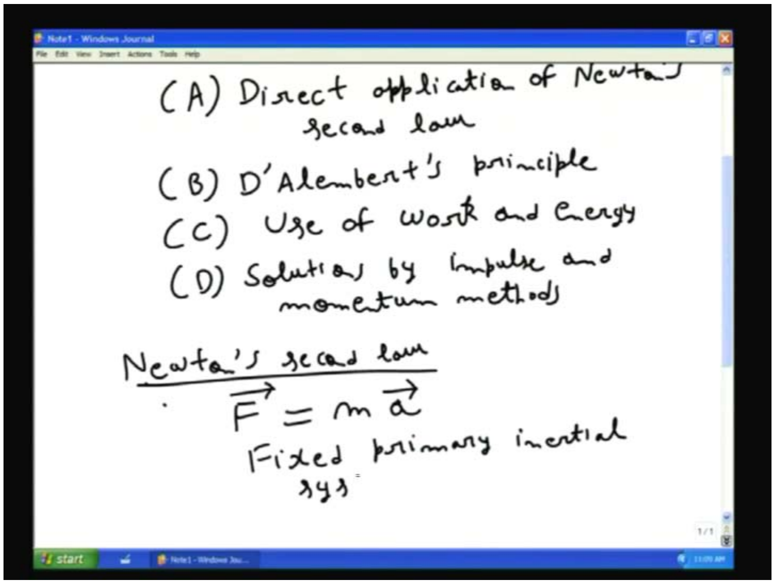

Now, we discuss these one by one. First is the direct application of Newton’s second law. It is the second law which is widely used in solving the kinematics problem. Newton’s second law states, the acceleration of a particle is proportional to resultant force acting on it and is in the direction of this force. This is a vector equation; that is, F is equal to m, where m is the mass of the particle and it is scalar, but F is the acceleration vector. So, F is equal to m into a. We keep on using this law widely; however, the relation can be verified only experimentally. There is no theoretical proof. That is why, F must be equal to mass times acceleration. Moreover, this law is not valid in all frames of reference. For this to be valid, we have to assume the existence of a fixed primary inertial system. If we can assume a fixed primary inertial system, this is an imaginary set of reference axis that is assumed to have no translation or rotation in space.

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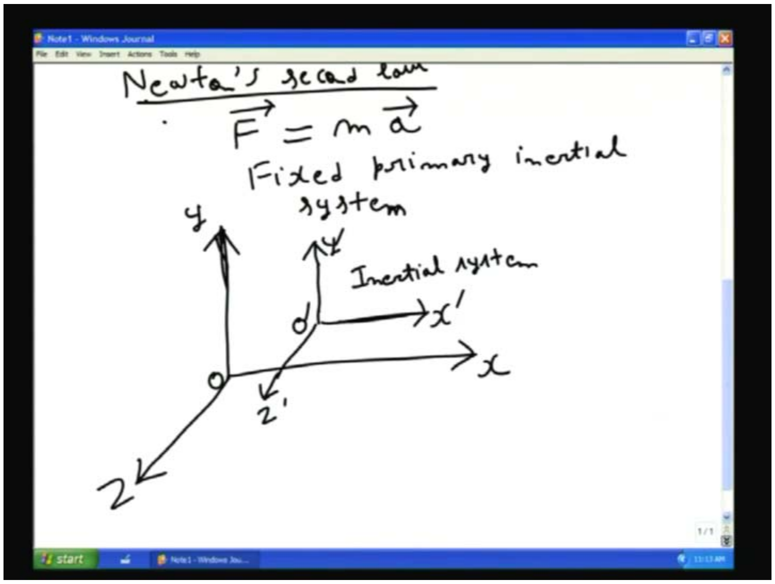

Assume that O is a point here; this is x, this is y and this is z. This axis system is totally fixed. Its central point O does not move at all. x y and z axis does not rotate at all. If we can find out such reference system, this is called Primary inertial reference system. This system mostly has to be called imaginary, because we cannot find out for sure if a reference system is not moving. If we are sitting in a room, we can draw the axis xyz. We will see that these axes are not rotating.

However, if we observe outside the earth in the space, we see the earth is rotating; therefore, the axes are also really rotating. Therefore, it is difficult to find out this thing, but for all practical purposes in solving the problems of machines to be assumed that earth can be considered as the primary inertial system. This F is equal to ma is valid only in the primary inertial system. Newton's second law is valid in primary inertial system as well as with respect to any non rotating reference system that translates with respect to the primary system with a constant velocity. Such a system is called an inertial system.

If this is a primary inertial system, this is also valid here. This is x dash, y dash, z dash, o dash is another reference system that is moving at a constant velocity with respect to this system. This system will be called inertial system. If this x dash, y dash, z dash system is rotating with a constant even at a constant angular velocity with respect to xyz, this will not be called inertial system. So for inertial system the reference system x dash, y dash, z dash has to move with a constant velocity or be stationary with respect to xyz.

Therefore, the Newton's second law is valid in the inertial reference system, only that type of thing. Newton has provided three laws in his book Philosophiae Naturalis Principia Mathematica in 1687. These three laws can be in very crude manner. They can be stated like that: first law; an object in motion will remain in motion unless acted upon by another force; second is force equals mass times acceleration; third is that, for every action there is an equal but opposite reaction. You have been using the first law and third law in solving the problems of statics also, because the first law will require that if the body has to be at rest then the net forces acting on that will be zero. The second law was not used in solving the statics is problem.

Third law says that there has to be equal and opposite reaction. You have used that in solving the first problem. If first member apply a force at another member, the equal and opposite force is applied on it by the second member. These laws can be stated in a more sophisticated manner. As the first law says, if no external force acts on a particle, then it is possible to select a set of reference frames called inertial reference frames, observed from which the particle moves without any change in velocity.

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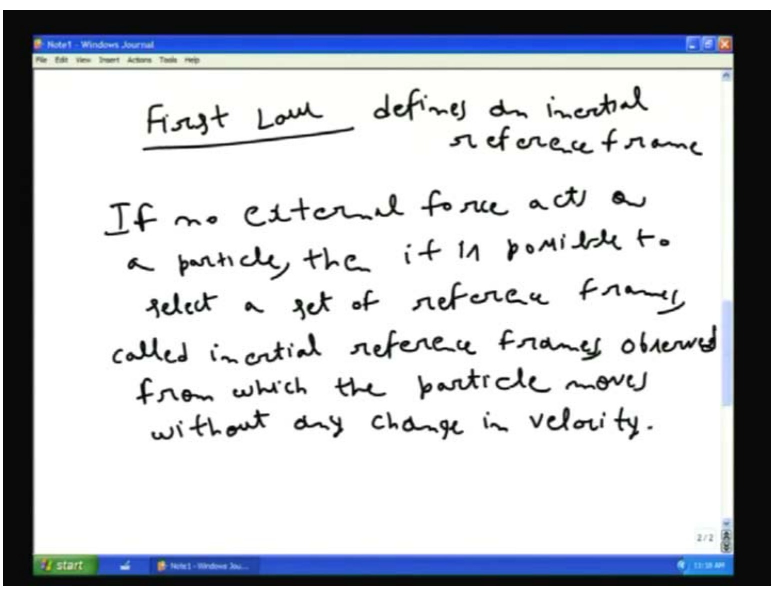

The first law defines an inertial reference frame. If the first law says, if no external force acts on a particle, then it is possible to select a set of reference frames called inertial reference frames observed from which from which the particle moves without any change in velocity. With the statement that it is possible to select a set of reference frames called inertial reference frame, observed from which the particle moves without any change in velocity indicates that there are other frames, reference frames about which the particle may move without the application of the external force.

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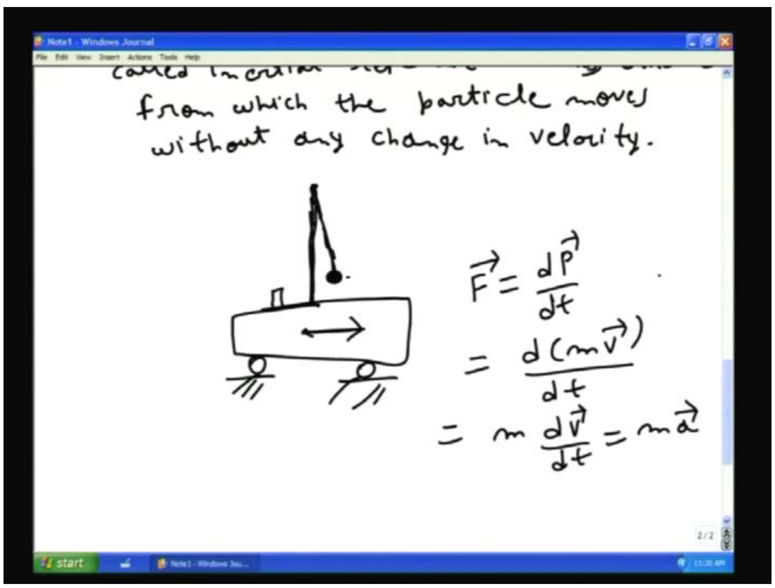

For example, you take a cart. On this cart, put a pillar and suspend it with a thread of small ball. The ball keeps on deflecting depending on the acceleration of the cart. Although no horizontal force is acting, as it is, a person sitting on it is observing that there is no horizontal force acting on the ball. Therefore, the first law is valid only in an inertial frame. So, by this you can have this one; definition of inertial frame. The second law says that observed from an inertial reference frame, the net force on a particle is proportional to time rate of change of its linear momentum; that means in the second law, F is equal to dp by dt, where P is the linear momentum. In classical mechanics, P is equal to mv dt and if the mass is kept constant, then this becomes m dv by dt which is equal to mass times acceleration.

This law is often stated as F is equal to ma. This is valid only in an inertial reference frame. An inertial reference frame is that frame in which, if no external forces act on a particle then the particle moves without any change in velocity. Therefore, the definition of inertial frame comes from the first law. The second law shows that force is equal to mass into acceleration. The third law states that whenever a exerts force on b, simultaneously, b exerts force on a with the same magnitude in the opposite direction.

The third law brings into picture, the concept of a force. A force can be applied by only some other body; It is not an independent quantity. A particle can have velocity irrespective of any other body; what is happening on the other body is of not concern. A particle can have mass, but the particle cannot have force acting on it unless there is another body. On that body also, a force will be applied.

Therefore, the third law may provide a concept of the force. Force has to be applied by the agent and it is not an independent quantity. Strong form of the third law says that two forces act along the same line. However, there are cases in which the forces of same magnitude opposite direction but they need not be in the same line. Particularly in the two current carrying conductors, when the magnetic forces act on them, their line of action is different.

In Stating of these laws, there have been certain changes compared to the original laws; that means these laws have been put in a refined language for a period of time. Now, this first law was generally stated something like this; an object at rest will remain at rest unless acted upon by an external and unbalanced force. An object in motion will remain in motion unless acted upon by an external and unbalanced force. This law is also called the law of inertia. It is our common observation that an object at rest remains at rest unless acted upon by an external and unbalanced force.

However, we have to see in a certain manner that an object in motion will remain in motion unless acted upon by an external and unbalanced force. It is not possible to avoid these forces on a moving body. When a body moves, there is friction; there are wind resistances. So, really, any experiment has not been done, which shows that a body keeps on moving unless acted upon by this thing. Then, second law was stated as the rate of change of momentum of a body is proportional to the resultant force acting on the body and is in the same direction. The exact original translation was like this; the alteration of motion is a proportional to the motive force impressed and is made in the direction of the right line, in which that force is impressed.

If a force generates motion, a double force will generate double the motion and a triple force, triple the motion. Whether the force b is impressed altogether and at once, or gradually and successively, this motion is being always directed the same way with the generating force. If the body moved before is added to or subtracted from the formal motion accordingly as they directly correspond with or directly contrary to each other are obliquely joint when they are oblique, so as to produce a new motion from bounded from the determination of both. Here, one has to see that Newton has used the word motion instead of momentum. Here, these are the some changes in the languages of the salon.

We have to understand, that the first law and second law require the presence of the inertial reference frame. They are not valid in non-inertial systems. Many times, mistakes are caused due to the application of these laws in non-inertial systems. Therefore, one has to be careful in applying Newton’s law to only an inertial frame of reference. We will illustrate the application of Newton’s law by one example.

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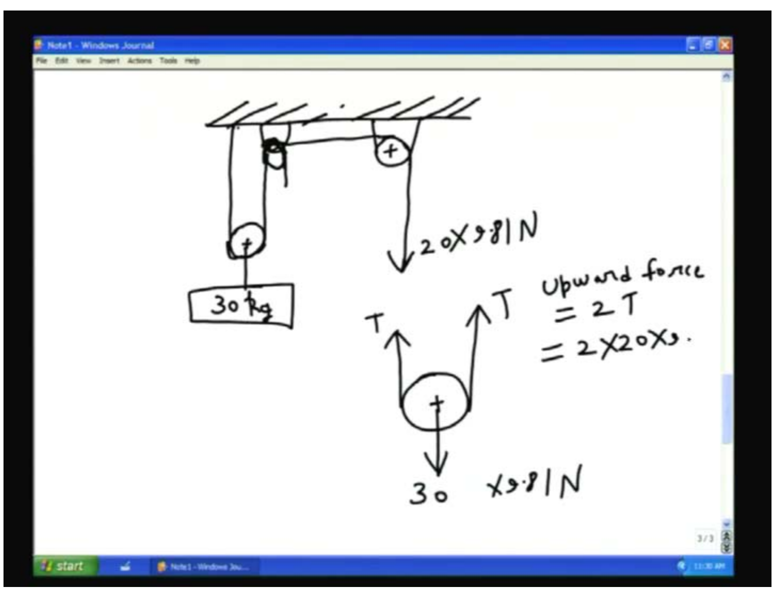

Consider pulley mass system. This is a pulley, where this string is passing. Then another pulley then this string goes like this and this is going here. If a force of 20 into 9.81 Newton is applied, a mass of 30 kg is suspended from here. We have to find out the acceleration of 30 kg mass, when the cart is being pulled by 20 into 9.81 Newton force. We neglect friction and the mass of the pulley and then make the free body diagram of 30 kg mass as well as pulley attached to it.

Tension in the cart will be everywhere equal to 20 into 9.81 Newton; because, there is no friction present. Making the free body diagram we get of the pulley, we see a 30 kg mass is acting here. Force is acting here. So, 30 kg into 9.81 will be the force. So, 30 into 9.81 Newton. T is like this; this is T, upward force. In this case, it is 2 T which is 2 into 20 into 9.81 downward forces is 30 into 9.81.

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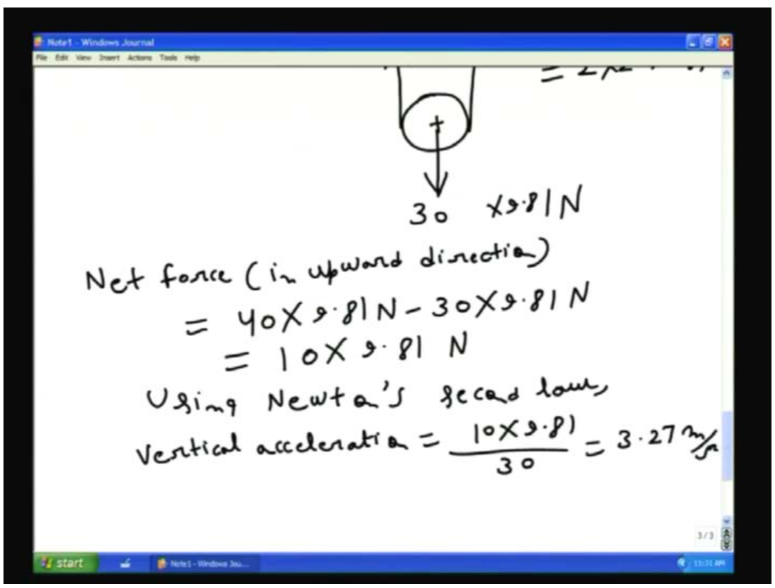

Therefore, net force in upward direction is equal to 40 into 9.81 Newton minus 30 into 9.8 Newton is equal to ten into load 9.81 Newton. Now, using Newton's second law vertical acceleration is equal to 10 into 9.81 divided by 30, which is equal to 3.27 meter per Second Square. If instead of applying 20 into 9.8 Newton force like this, here if we hang a 20 kg weight from the free end of the string, will the acceleration be same as before? Intuitive reason ensures that it will not be. Because, the extra mass has been [Refer Slide Time: 28:45] in which you have to make some force to accelerate that mass as well. Let us try to solve this problem.

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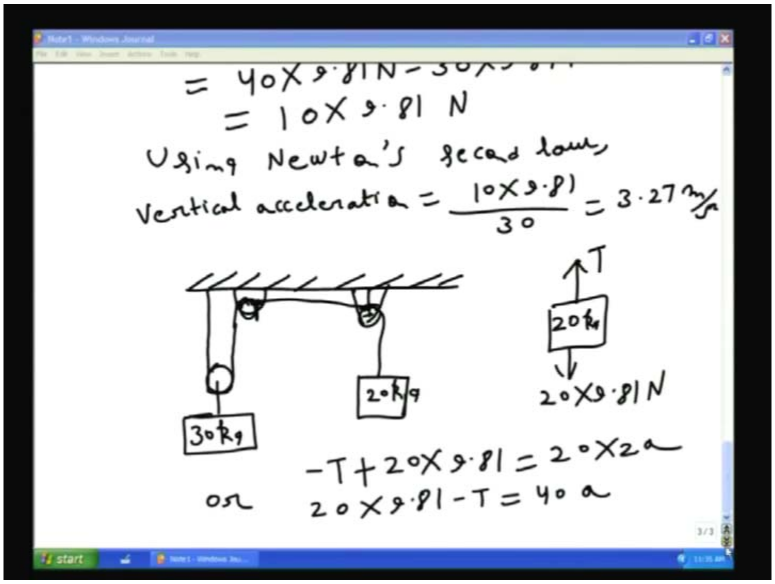

This one is and this is pulley here. Another pulley or which it holds, then this is 20 kg; this is 30kg.

Now, making the free body diagram of 20 kg mass, we see that 20 kg mass is subjected to force 20 into 9.81 Newton due to gravity and a force T upward. Now, minus T plus 20 into 9.81 will be equal to 20 into acceleration of the mass of 20 kg. What is the acceleration of the mass of 20 kg? If the acceleration of 30 kg mass is a, the acceleration will be 2a; because, if 30 kg mass goes up by a distance of x, the 20 kg mass will move down by a distance of 2x, because there are 2 strings. Even it has moved at a distance of x, this string had been shortened by using the x.

This also has been shortened. The same goes here, so 2x. Therefore, this is 20 into 2a or 20 into 9.81 minus T is equal to 40a. We may make the free body diagram of 30 kg mass which is like this.

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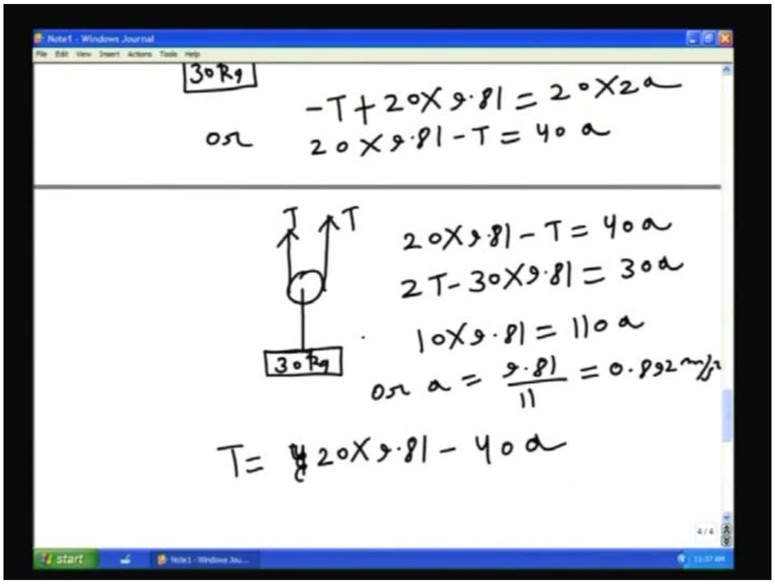

That is 2T minus 30 into 9.81 is equal to 30a. Solving this equation that is one equation, 20 into 9.81 minus T is equal to 40 a; other, 2T minus 30 into 9.81 is equal to 30 a, we get 10 into 9.81 is equal to 110 a, or a is equal to 9.81 by 11 is equal to 0.892 meter per second square.

We see that the acceleration has got quite reduced. Similarly, we also will see that from the previous slides that in this case, T will be equal to 20 t into 9.81 minus 40a. So, this will create reduction in tension in a way. Then, the particle 30 kg mass will experience tension because of the presence of this thing. So, this is like that. There can be various examples of applying Newton’s law.

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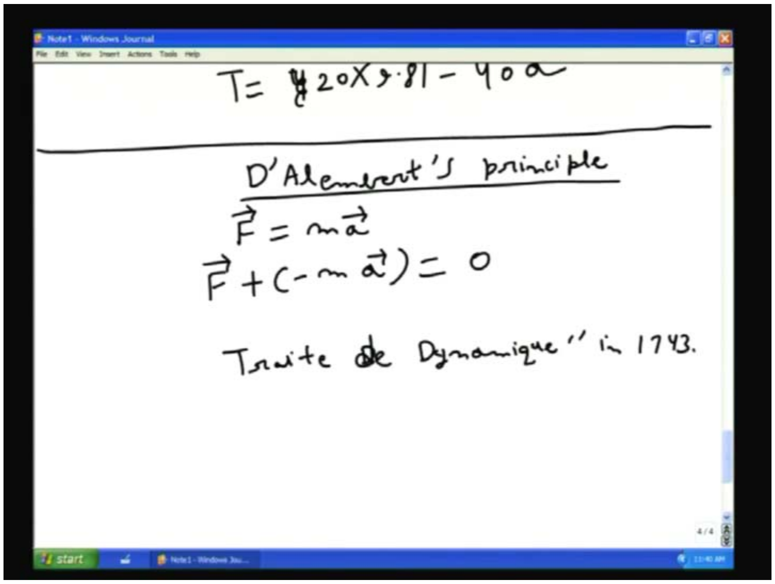

We discuss another principle that is called D'Alembert's principle. Newton's second law is F is equal to ma. We can write it as F plus minus ma is equal to 0; F is the resultant of external forces applied on the particle. We can treat minus ma also as a force. Then, we can say that the body is in equilibrium under the action of external forces, resultant external forces and force minus ma. This fictitious force is known as inertial force and the artificial state of equilibrium created is known as dynamic equilibrium.

The transformation of a problem in dynamics, to one in static has been known as D'Alembert's principle. D'Alembert's published his work in his traite de dynamic in 1743 after Newton. This is nothing but the Newton's law is stated in another way so that the problem of dynamics can be solved as the problem of static. This minus ma can be called as inertia force. Inertia force is a fictitious force; it is not a real force.

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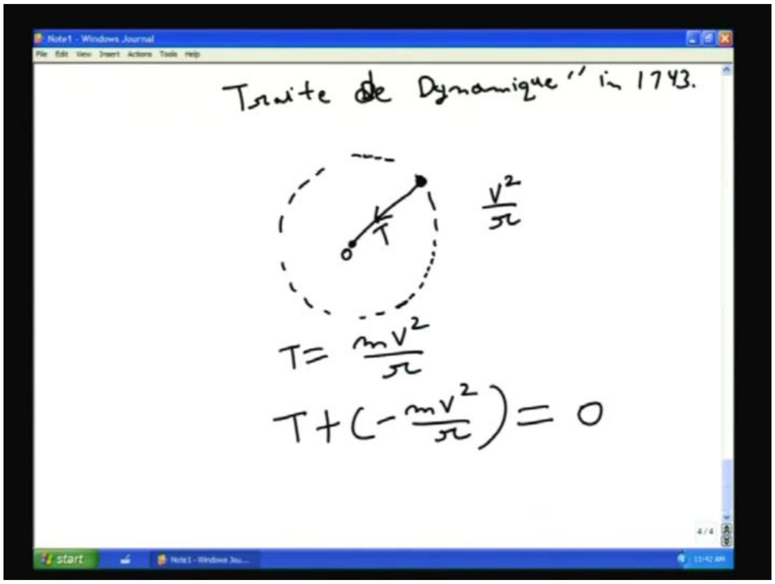

Assume that a particle is rotated in a horizontal plane by means of a string. For an external observer, the particle is moving and it has a centripetal acceleration V square by r. There is a tension T which pulls the particle towards center. For an external observer, the particle is moving with a centripetal acceleration of V square by r. Newton's law can be applied and we get T is equal to m V square by r.

Now, suppose the observer is sitting in the particle itself, for him the particle is not moving, but he is seeing that particle is being pulled by a force T. Thus, he will feel that there is an outward force that is balancing this force. This fictitious outward force is called inertia force. Therefore, particle is balanced due to presence of T and minus mV square by r is equal to 0. You can either apply Newton's second law or D'Alembert's principle. However, we must keep in mind that Newton's second law is valid in the inertial reference frame. In the same way, so is the case for D'Alembert's principle. We have to be careful that always the acceleration should be measured from the inertial reference frame.

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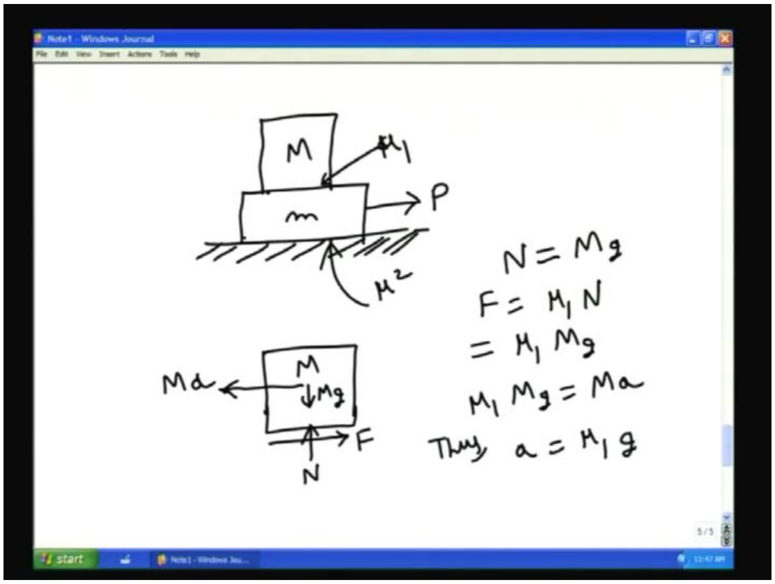

Let us do one problem. Suppose you have a cart of mass m, small m, on which there is crate of mass M. It is being pulled by force P. The coefficient of friction between the crate and cart is $mu_{1;}$ that means coefficient of friction here is $mu_{1.}$ Coefficient of friction between the cart and the road is $mu_{2.}$ Find out the maximum value of force P such that crate do not shift. Determine the maximum allowable magnitude of P and the corresponding acceleration of the cart. We make the free body diagram of the mass, big mass M.

If we make the free body diagram of mass M, we see that it is subjected to force due to gravity that is Mg. Then, the vertical reaction which is put by the car on it is N, then there is no force acting in the horizontal direction. There is only a friction force which maybe in this direction, that is F. Then, there is a force and it is accelerating. Therefore, I can put inertia force as Ma opposite to the acceleration, because if the acceleration is in this direction, inertia force is F is equal to Ma.

We can apply D'Alembert's principle and we consider the static balance of this. We have to understand that here the acceleration a has to be measured with respect to the ground which can act as inertial reference frame not with respect to the particle m or cart M. With respect to cart M, the acceleration is 0. So, applying where vertical force by this gives N is equal to $M_{g,}$ friction force F is equal to m_{u1} times N. Applying the Coulomb’s law which is equal to m_{u1} times $M_{g.}$

If you balance these two forces, F is equal to M_a that means mu_1 M_g is equal to $M_{a.}$ Thus a, is equal to mu_1g. So, this is the maximum possible acceleration of the cart which can take the mass, big mass M without slipping.

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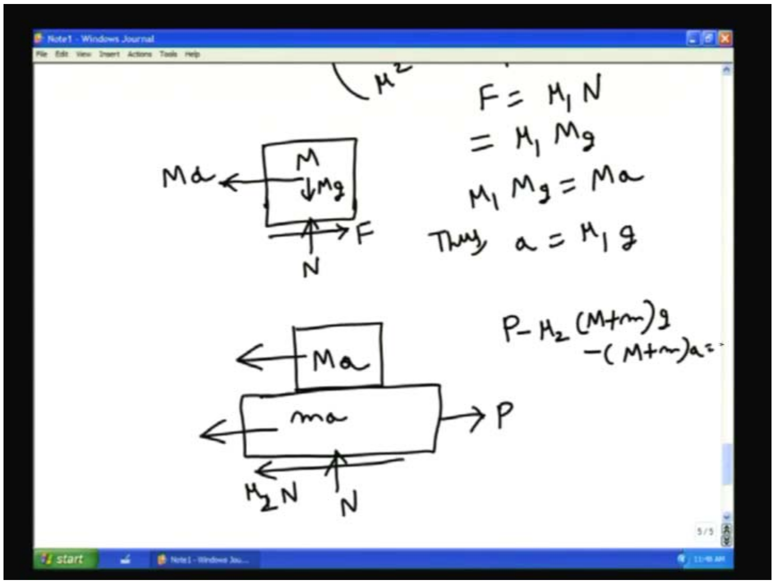

Now, from the free body diagram of the crate plus cart in this inverse m, this is the cart M, this is force N, mu_2 N. P this is P minus mu_2 M plus M_g minus M plus M into a equal to 0.

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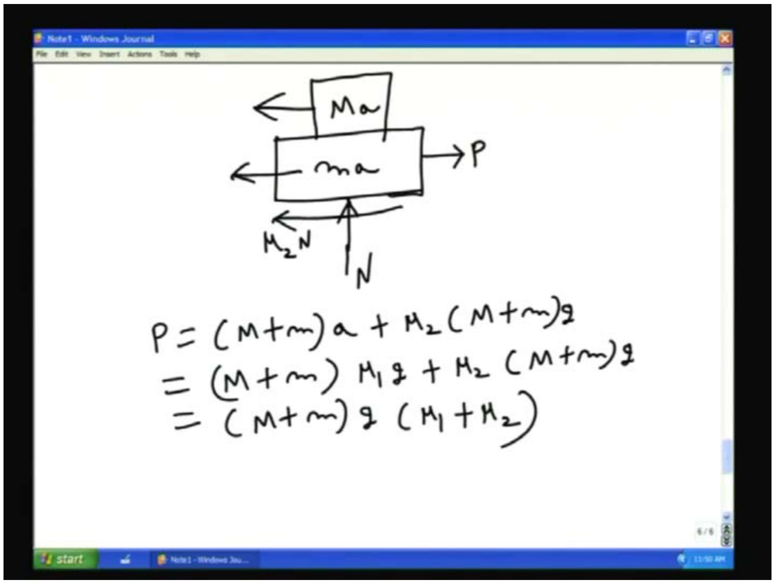

This is Ma, ma this is N mu$_2$N P. So, P is equal to M plus m into a plus mu$_2$ times M plus m into g, or this is M plus m mu$_1$ g plus m$_{u2}$ M plus m into g, or M plus mg mu$_1$ plus mu$_{2.}$ So, this is the value of the maximum possible force P. So, this has been done. We will discuss about the work and energy approach and solutions are by impulse and momentum methods in the subsequent lectures. Before that, let us solve some problems which include the Newton's second law.

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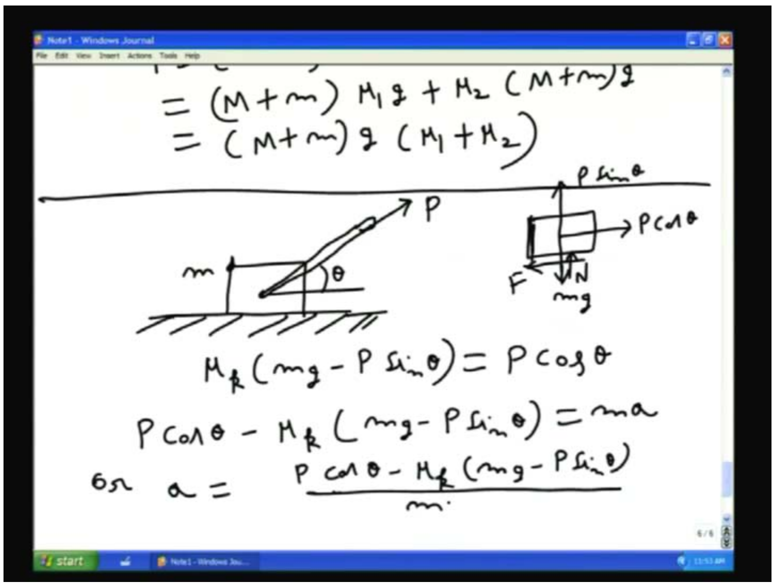

A particle, suppose a block is resting on this one, mass m. A force is applied by pulling it through a rod which is making an angle theta. If the force magnitude is P, find out the angle theta, optimum value of that angle theta which will provide maximum acceleration to this mass. Coefficient of static friction is mu$_s$ and coefficient of kinetic friction is mu$_{k.}$ During the motion of the block, only the kinetic force will be responsible. mu$_k$ m$_g$ minus P sin theta, because if we make the free body of this one then there is a force m$_{g,}$ there is a friction force F, then P sin theta and this N. I am showing N here and this is P cos theta.

The vertical force by this gives N is equal to mg minus P sin theta. So, mg minus P sin theta into mu$_k$ is equal to P cos theta mu$_k$ is F is equal to mu$_k$ times m$_g$ minus P sin theta is equal to P cos theta, if there is no acceleration. If there is acceleration, then P$_k$ cos theta, net force is P$_k$ cos theta minus mu$_k$ $_{mg}$ minus P sin theta is equal to m$_{a,}$ or a is equal to P cos theta minus mu$_k$ m$_g$ minus P sin theta divided by m.

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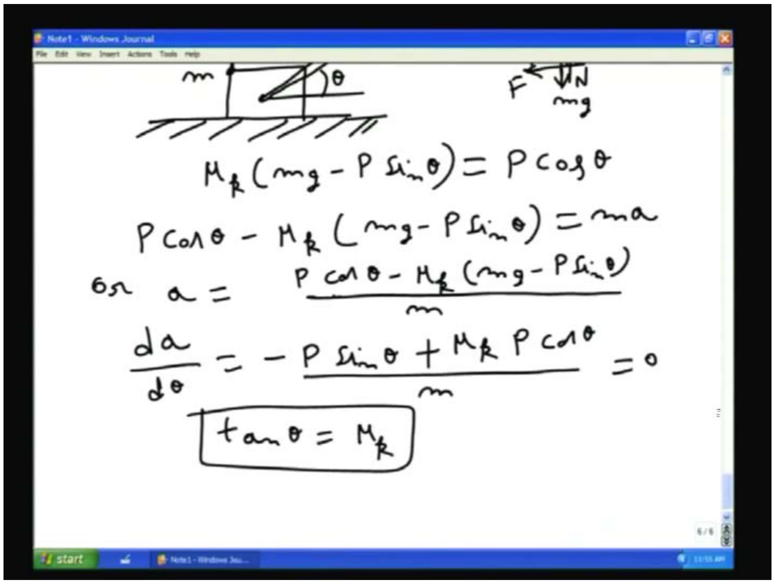

Therefore, the acceleration is dependent on theta; acceleration is a function of theta in this case. If we take for which value of theta the acceleration will be maximum, we have to put da by d theta is equal to minus P sin theta plus mu$_k$ P cos theta divided by m equal to 0; which gives tan theta is equal to mu$_k$. When tan theta is equal to mu$_k$, at that point the acceleration is actually at a maximum. We can differentiate da by d theta once more and confirm that tan theta is equal to mu$_k$ provides minimum.

However, from physical consideration, we can see that minimum acceleration may be provided when theta is equal to 90 degree. That time, the acceleration or if you know P, you apply in the opposite direction then the block will start moving in the opposite direction. So, between 0 and 90 degree may be that maxima will be at the corner; that means theta equal to 90 degree. Now, let us see another problem.

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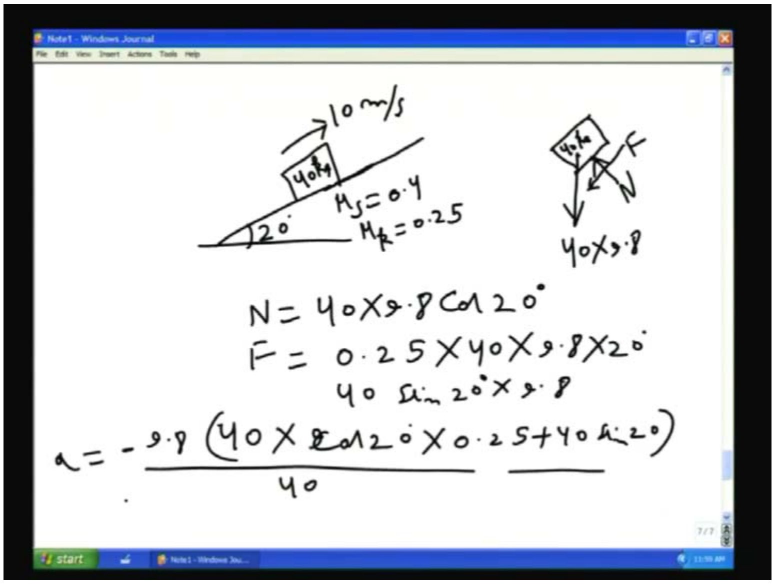

If there is a block 20 degree, you are having a block 40 kg and it is moving at 10 meter per second, then you have to find out the acceleration of the block, if mu_s is equal to 0.4 and mu_k is equal to 0.25. mu_s is the static coefficient of friction which decides when the particle is in equilibrium; mu_k is the kinetic coefficient of friction. So, it should be used when the particle is moving. Now, in this case the net force is on this direction. We can find out how.

If we make a free body diagram of 40 kg bar, we will get normal reaction of the plane as N. Then its weight is acting vertically downward that is 40 into 9.8. Now, there is a friction force which is opposing its motion, that is F. There is no motion perpendicular to the plane. Therefore, N is equal to 40 into 9.8 cos 20 degree. Therefore, the friction force will be equal mu_k that is 0.25 into 40 into 9.8 into 20 degree. Component of the force rate along the inclined plane is 40 sin 20 degree into 9.8.

Therefore, net force in Newton is 40 into 9.81 a 40 into cos 20 into 0.25 plus 40 sin 20 this is 9.8 out. Therefore, acceleration will be found. That is, if it is moving at ten meter per second at this moment, therefore opposite to the velocity, the acceleration is negative, this is minus 40. Therefore, if we want to find out that when the particle will stop, we can find out T is equal to V divided by a, which comes out to be 1.767 second. We can also find out the distance to the particle before it is stops, by d is equal to V square by 2a.

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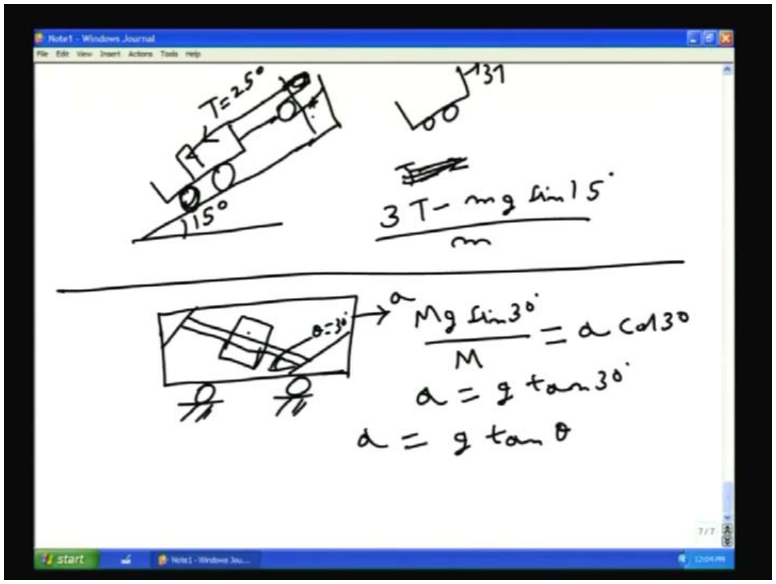

See another example. This is a cart and it is being pulled by a string. A man may be sitting here and he is pulling the string like this. In this case, if he is pulling with a force 250, if we make the free body diagram of the cart, we see there are these three. If we cut it here, if we cut it here then we see 3T shown, if a force there will be 3T on that system. Therefore, if T is equal to 200, 3 T minus m_g sin 15 divided by m will be the acceleration.

If we take one cart like this and a block on this slide, if this angle is theta degree and the block which is moving is of mass Mg, we will see the force due to gravity along this direction is M_g sin 30 divided by M is equal to the net acceleration, that is a cos 30; because, if this cart is having an acceleration a, its component along this is a cos 30. Therefore we get, a is equal to theta is equal to 30 degree, a is equal to g time 30, or if this angle was theta, a would have been g times theta.

Therefore, you can make a device to measure the acceleration of a moving bus by means of this type of arrangement. Take that a small gadget, in which there is a rod. Rod's angle can be adjusted and there is a block which moves on a frictionless surface; there it means reduce the friction. Then, when this block gets balanced the cart's acceleration is given by, a is equal to g time theta. So, by this you can find out the acceleration of this thing. We have discussed few problems on the application of Newton's law or D'Alembert's principle. Next, we will discuss other approaches to find out problems.