Engineering Mechanics

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Kinematics

Module 10 - Lecture 24

Kinematics of a particle moving on a curve

Today, I am going to discuss about kinematics of a particle moving on a curve. As we have discussed in the previous lectures that a particle's position in a space can be specified by a vector. When the particle goes from that position to another position then there is a displacement; that displacement is change in the position vectors at two instance of time. If you differentiate that with respect to time you get the velocity.

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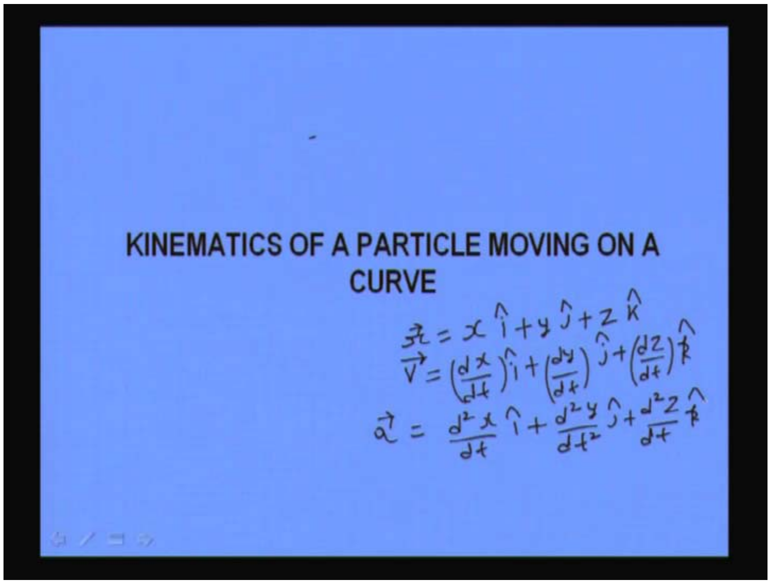

If the position vector can be expressed in terms of components along x-axis, y-axis and z-axis that is r is equal to xi plus yj plus zk, then it is very easy to find out its velocity by differentiating this with respect to time. Therefore, velocity in this case will be dx by dt i plus dy by dt j plus dz by dt k. Here, x, y and z are known as a function of time. Similarly, acceleration can be found out by d square x by dt i plus d square y by dt square j plus d square z by dt square k. However, in many situations, one may not know the explicit expressions for x, y and z as a function of time. Instead, it may be known that the particle is moving on a curve.

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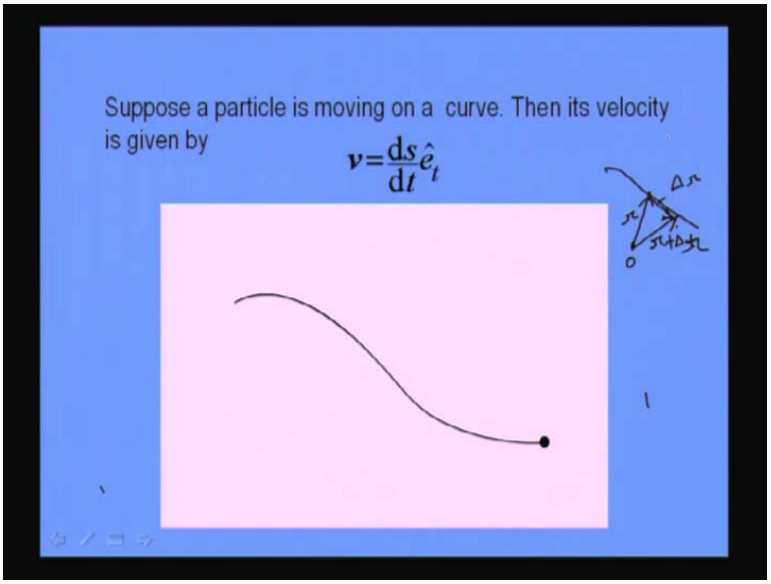

In this case, if the particle is moving on a curve then its velocity can be given by v is equal to ds by dt $e_{t.}$ This is very easy to see here that suppose this is the origin; now origin is somewhere. This is the curve and this is origin O. At one instant of time, the position is r; after that the position may become r plus this is next position, so this is one position. Then you have got another position that is after time t r plus delta r. So, this difference is indicated by delta r.

Now, if these two points are very close by, then delta r will be approximately equal to ds that is the arc length. Therefore, the magnitude is known as ds by dt e. Then the direction is tangential to the path because as delta r will approach the tangent, as the distance between two points is very small, the velocity of the particle is given by v is equal to ds by dt e_t where et is the unit tangent vector.

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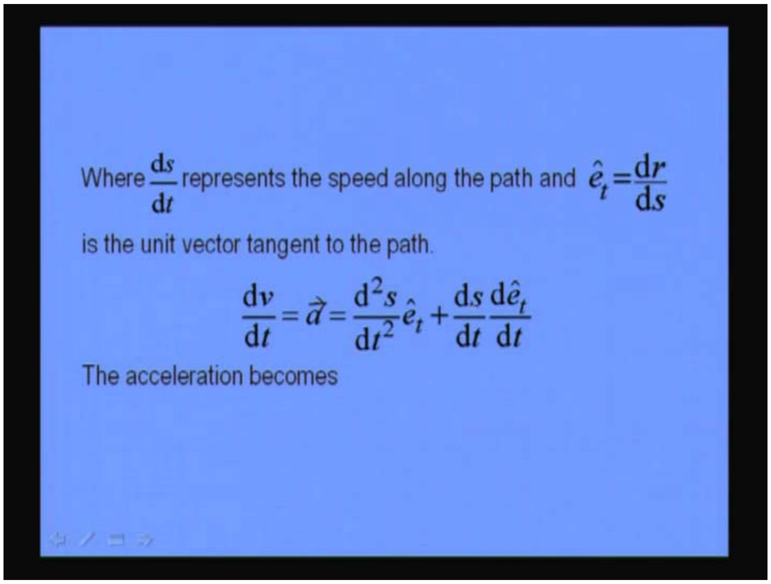

Now ds by dt basically represents the speed along the path. It is a scalar quantity that indicates s is the distance travelled by the particles. So, ds by dt is the speed and e_t is tangential to the path. Therefore, et is given by dr by ds that is unit vector tangent to the path. The acceleration becomes dv by dt. You differentiate velocity vector with respect to time. Again, you get a vector quantity that is a and this is equal to d square s by dt square et because you have had expression for velocity is ds by dt e_t.

Both are functions of time. If the particle was moving in a straight line then et was constant; it was not a function of time. Therefore, its derivative will be 0, but in this case, ds by dt is a function of time. Also, e_t is a function of time. Therefore, if you differentiate then dv by dt is equal to a, which is equal to de square s by dt square e_t ; that is, we applied the product rule for differentiation, first differentiated with this one, then second differentiated e_t with respect to time. So, you get another term ds by dt d e_t by dt. Therefore, this is the expression for acceleration.

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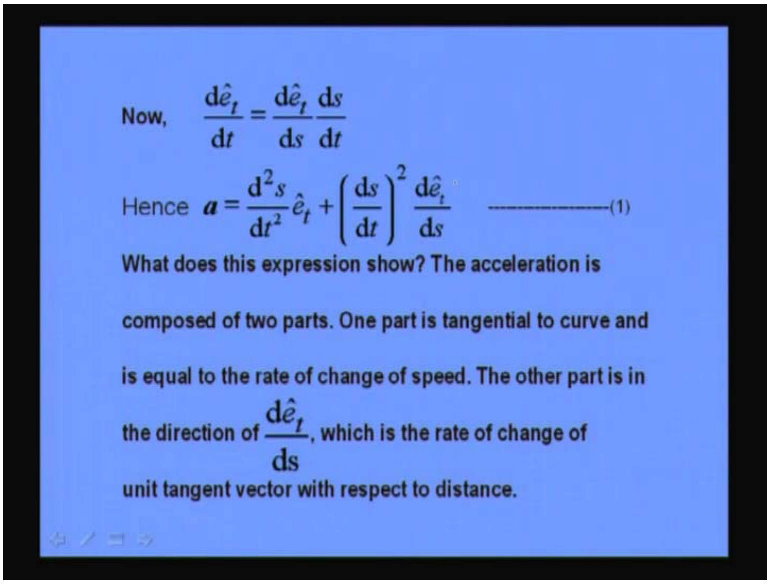

The acceleration has got two components; that is one is tangential, that is d square s by dt square it is nothing but the derivative of the speed; but the second component is very important this is ds by dt de$_t$ by dt. Let us see, how we can simplify. Now, de$_t$ by dt is basically de$_t$ by ds into ds by dt. It can be written like this. Therefore, acceleration becomes d square s by dt square e$_t$ plus ds by dt square de$_t$ by ds.

What does this expression show? It shows that the acceleration is composed of two parts. One part is tangential to curve and is equal to the rate of change of speed. The other part is in the direction of de$_t$ by ds, which is the rate of change of unit tangent vector with respect to distance. Then, there is a square of speed term also and ds by dt is the speed.

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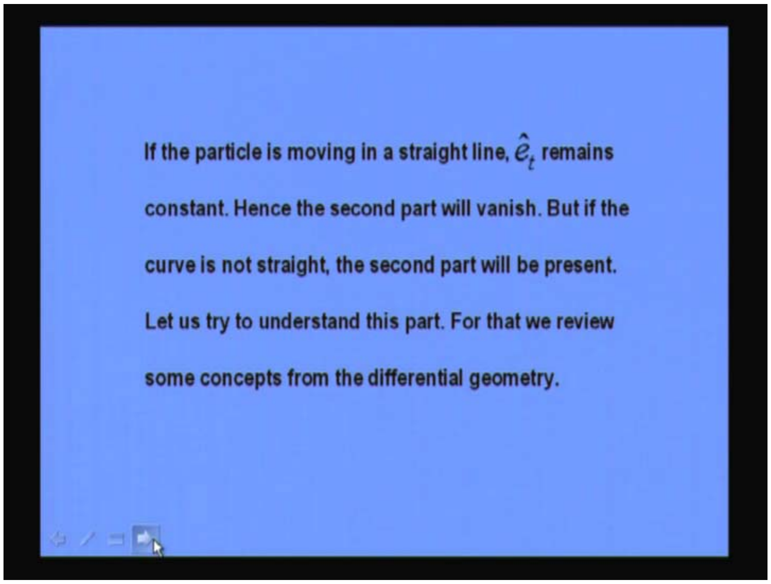

If the particle is moving in a straight line, then e_t remains constant. Hence, the second part will vanish. When the particle is moving in the straight line this part vanishes because e_t is constant and it is not changing with respect to s but first part remains. Therefore, it is very easy to find out the acceleration there. If the curve is not straight, the second part will be present. Let us try to understand this part. Before we start discussion about that let us view some concepts from the differential geometry.

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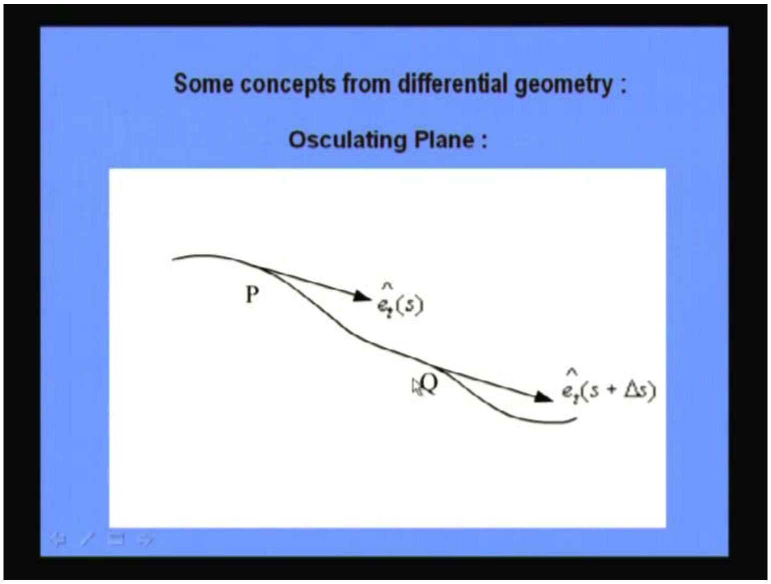

Taking some concepts from differential geometry, usually we have been studying planar curves; the curves which are contained in this. In general, a curve can be made in a space also. For example, you see that a curve has been shown here. Two points have been indicated; those are P and Q. They are on the curve and this curve is in a space which is a spatial curve; it is not a planar curve. In that case, also, one can try tangent on the curve at position s that is e_t(s) and the next one is e_t(s plus delta s). So, these are the two tangent vectors drawn in this one.

A curve if we can find out, plane which encompasses these two lines that is e_t(s) and this one. If we keep the distance between two planes e_t(s) e_t(s plus delta s), the distance between two points is kept very small. In that case, it is possible to find out the plane which will have these two lines lying in it that plane and it will be called osculating plane.

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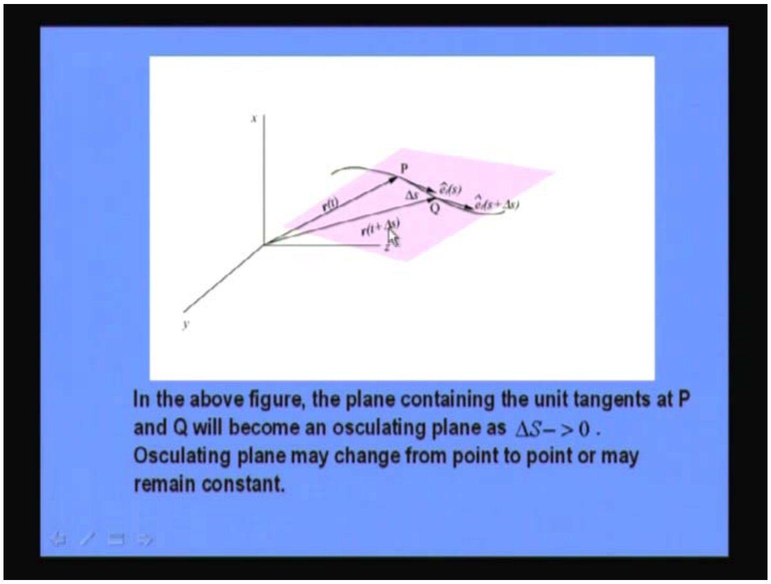

In this figure, at point P a tangent has been drawn that is called ets. This is the situation at time rt. At time t plus delta t, another tangent has been drawn; this is et s plus delta s. Length PQ is half length delta s, if the plane containing the unit tangents P and Q will become an osculating plane as delta s tends to 0. So, osculating plane may change from point to point or may remain constant. If the motion is taking place only in a plane, then that plane itself is an osculating plane. Osculating means kissing. Osculating plane just kisses the path; that is why it is called osculating plane.

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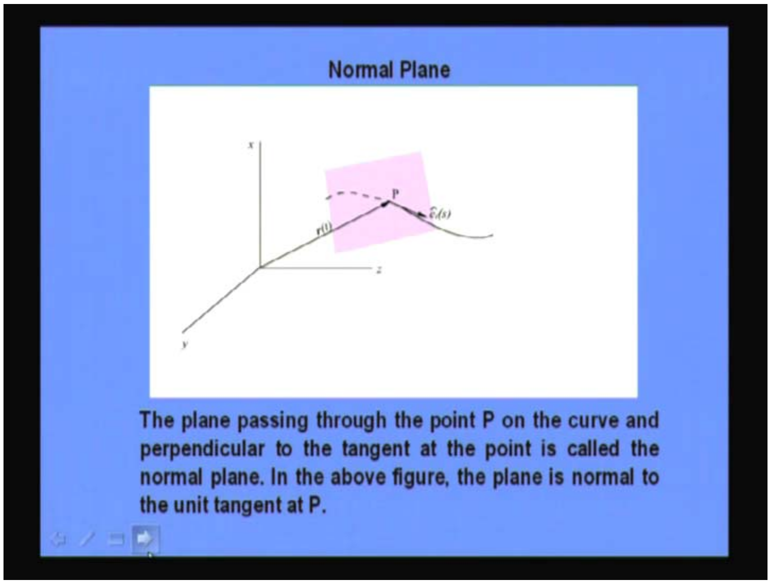

Now, we define another plane that is called normal plane. The plane passing through the point P on the curve and perpendicular to the tangent at the point is called normal plane. You see here, at this point P, its position vector has been indicated by rt. Draw a tangent at this point, that is, ets and this plane is perpendicular to the tangent line. Therefore, this curve is coming out this side and that side it has been shown by dotted. This plane is normal to the unit tangent at P and is called normal plane.

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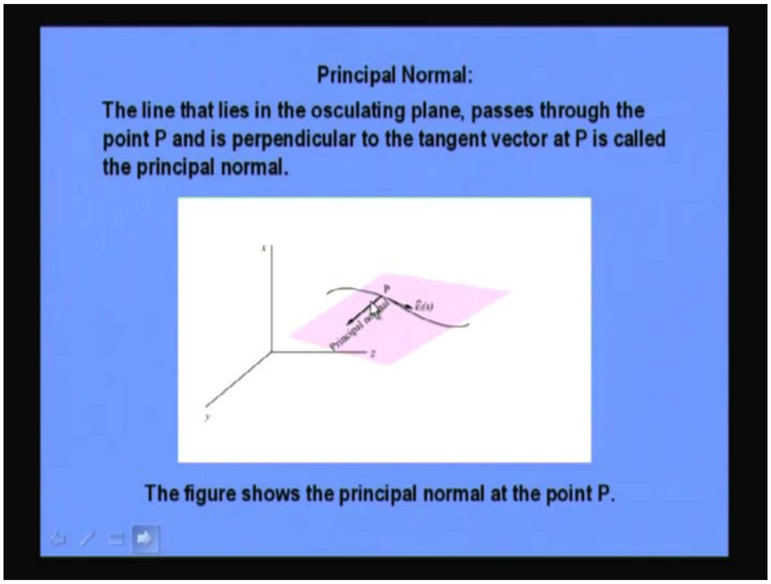

We define principal normal. The line that lies in the osculating plane passes through point P and is perpendicular to the tangent vector at P is called the principal normal. In this case, this is a unit tangent vector then this is a principal normal.

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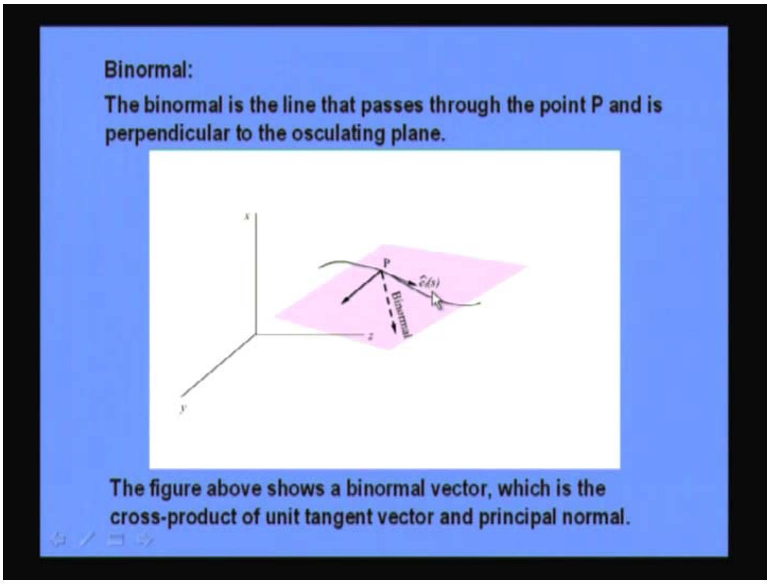

Another plane is vector is defined, that is binormal. The binormal is the line that passes through the point P which is perpendicular to the osculating plane. So, binormal like if this is an osculating plane, then binormal is downward perpendicular to the plane; this is shown here. A binormal is basically the cross product of unit tangent vector ets and principal normal. It is a cross product which will be perpendicular to both these vectors.

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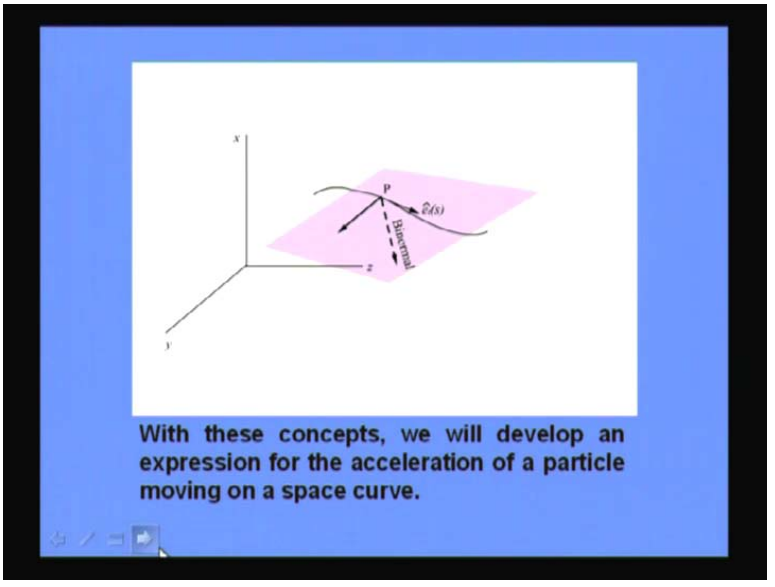

With these concepts, we will develop an expression for the acceleration of an article moving on a space curve.

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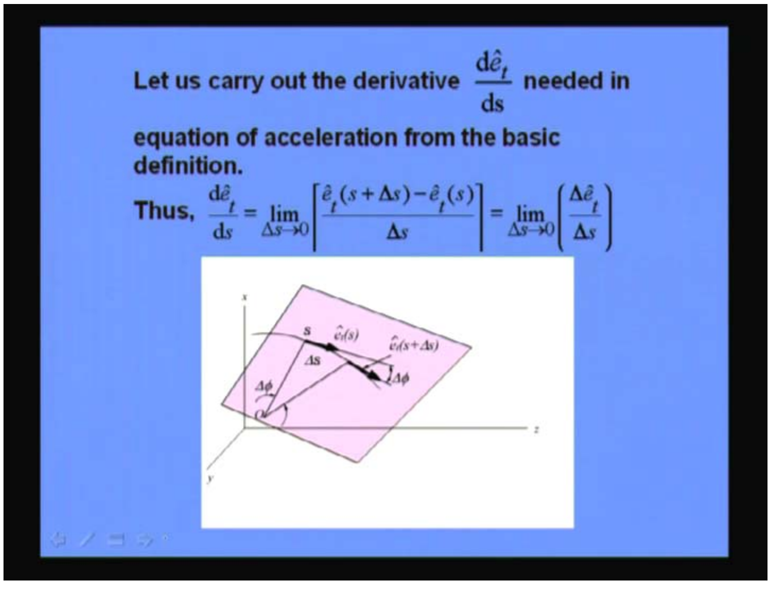

We need to find out the derivative de_t by ds e_t is the unit tangent vector it is needed in equation of acceleration. From the basic definition de_t by ds is a limit delta s tends to 0 e_t s plus delta s minus $e_t(s)$ divided by delta s which can be written as limit delta s tends to 0 delta e_t by delta s. Here, it has to be noted that difference between these two vectors will be a vector which has direction as well as magnitude. Therefore, the derivative of unit tangent vector is having the direction as well as magnitude. So, it is a vector quantity.

This shows an osculating plane. If we show a unit tangent vector at point s by $e_t(s)$ and after the particle has travelled a distance of s plus delta s, this becomes e_t(s plus delta s). The angle between these two tangent vectors is delta phi, if you draw a normal at s. Similarly, you draw a normal at this point; both the normals intersect at point O and this angle will also be delta phi. So, $e_t(s)$ plus delta s minus $e_t(s)$ is the vector difference of these two unit tangents.

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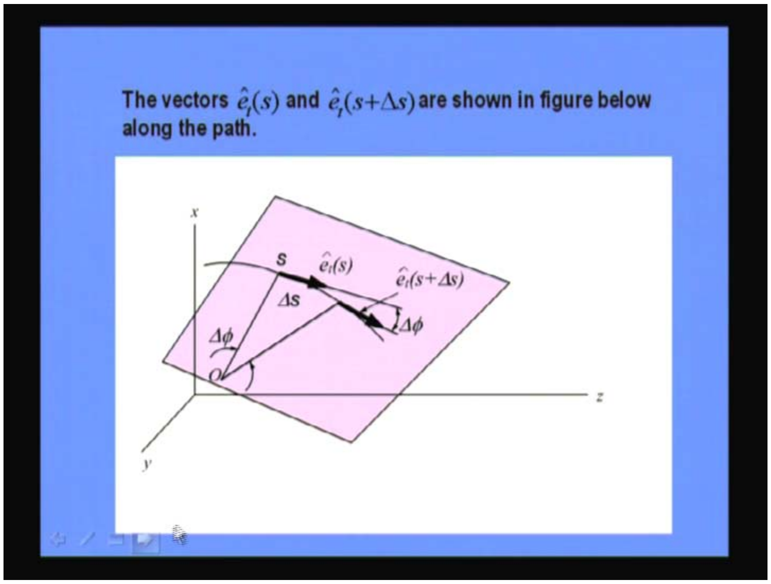

Now, these vectors have been shown along the path. This is delta s.

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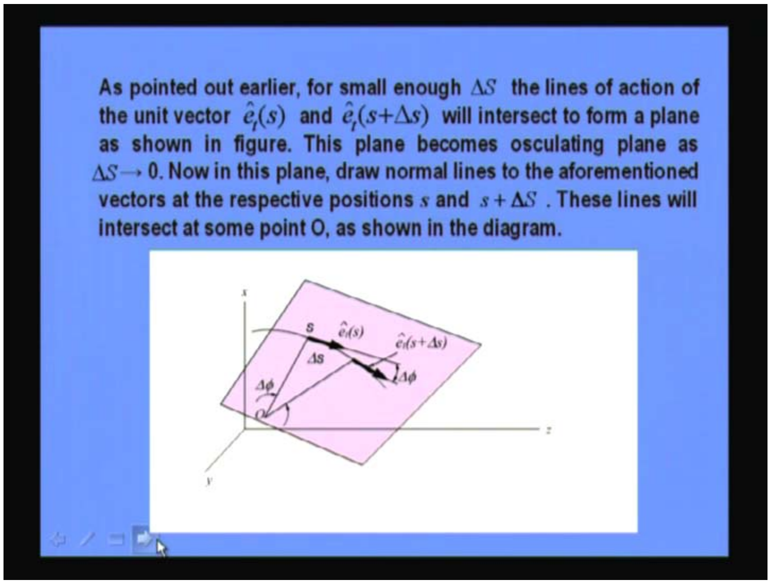

As pointed out earlier, for a small enough delta S the line of action of the unit vector e_t(s) and e_t(s) plus delta s will intersect to form a plane as shown in the figure. This plane is called osculating plane when delta S approaches 0. We have learned normal lines to these vectors and they intersect at point O - which has been explained earlier.

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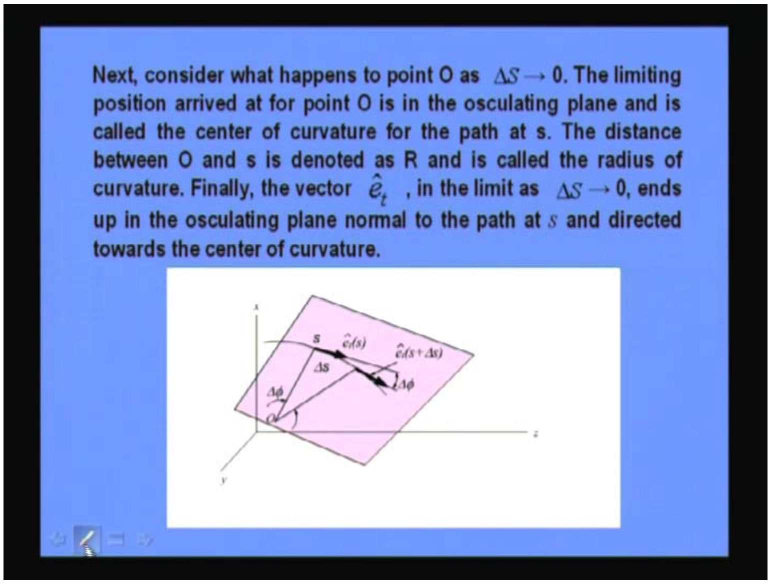

Let us consider, to see that what happens to point O as delta S tends to 0, the limiting positions arrived at point O is in the osculating plane and is called the center of curvature for the path at s this is by definition. The distance between O and S is denoted as R and is called the radius of the curvature. Finally, the vector e_t in the limit as delta s tends to 0, ends up in the osculating plane normal to the path at s and directed towards the vector. The vector delta e_t in the limit as delta S tends to 0 ends up in the osculating plane, normal to the path at S and it is directed towards the center of curvature.

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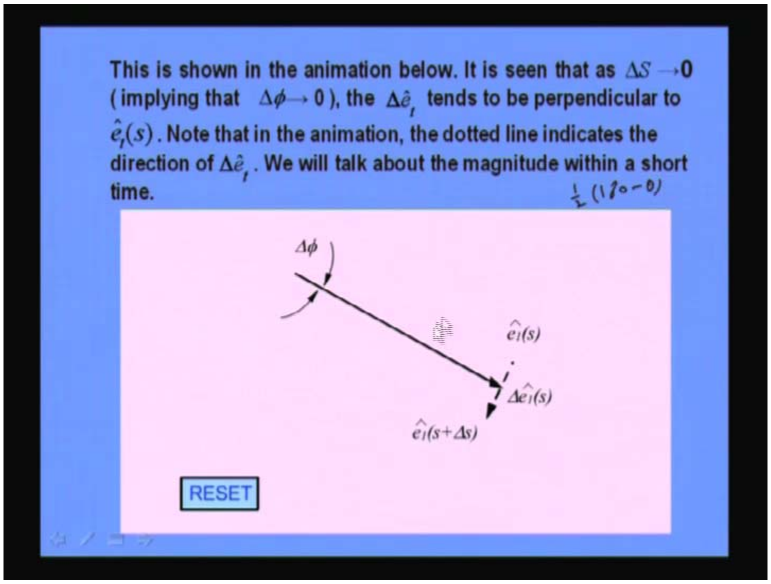

This is explained here. It is seen that as this animation makes it clear, it is seen that as these are the two vectors $e_t(s)$ e_t(s plus delta S) and this is the difference between them. This is indicated by delta $e_t(s)$, as delta phi tends to 0; delta e_t tends to be perpendicular to $e_t(s)$. Because delta phi is tending to 0, these two angles are equal and both are unit vectors. So, they are basically 180 minus theta divided by half. Why theta tends to 0? These angles become 0 in a triangle. So this is what that happens. Now, we will now talk about the magnitude also. Let us see it again, see this is what that is shown.

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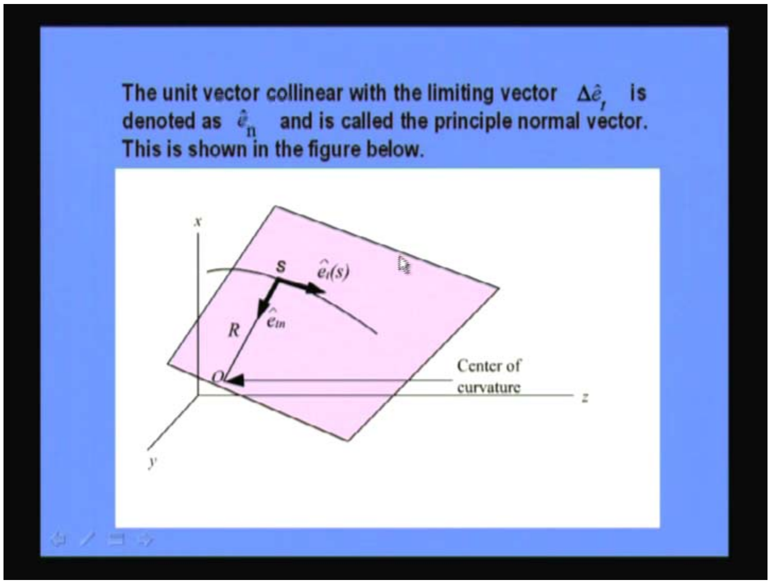

The unit vector collinear with limiting vector delta e, e_t is denoted as e_n and is called the principle normal vector - this is shown in the figure below. So here, this is ets and another vector which we will go towards center of curvature is indicated by en. This is called the principle normal vector.

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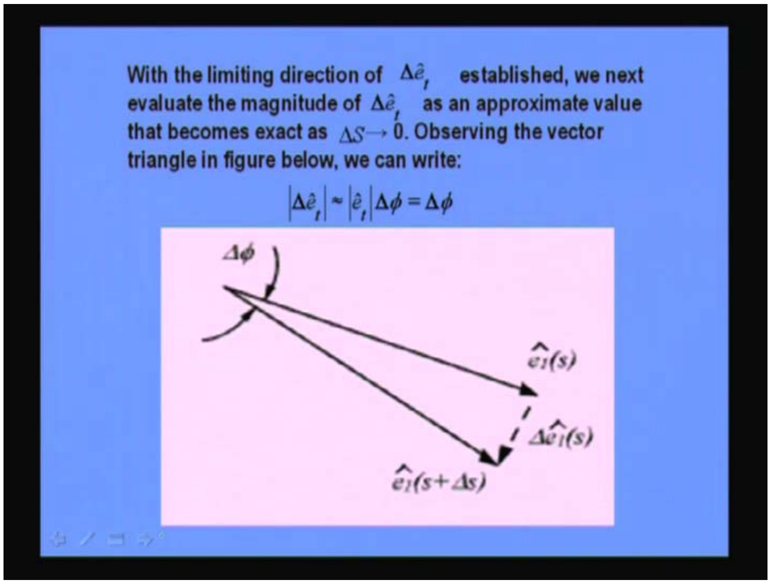

When the limiting direction of delta e_t with the limiting direction of delta e_t established, we evaluate the magnitude of delta e_t as an approximate value that becomes exact as delta S tends to 0. We observe the vector triangle in the figure. In this we can write, delta e_t is equal to approximately e_t times delta phi. This angle is delta phi; therefore, this can be written as e_t times delta phi magnitude of e_t times delta phi. Magnitude of e_t is one because it is unit vector. So, this becomes equal to delta phi.

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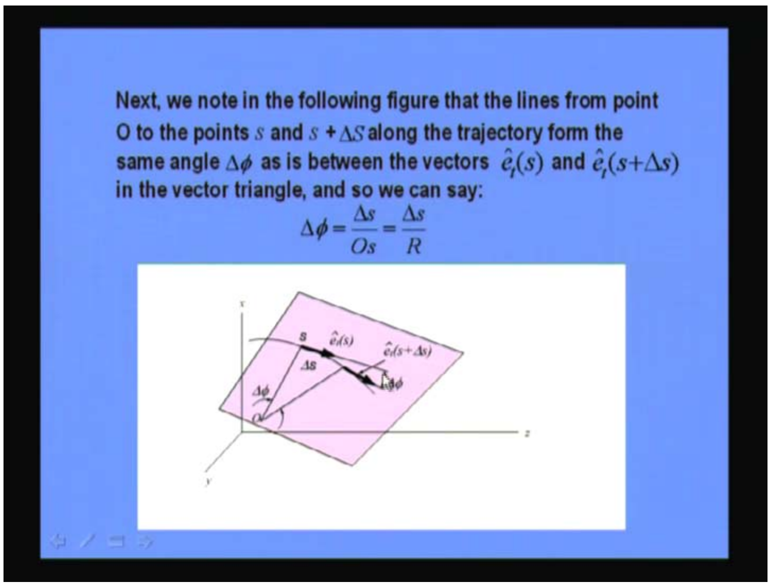

Next, we note in this figure, that the lines from point O to point s and s plus delta S along the trajectory form the same angle delta phi as is between the vectors e_t s and e_t s plus delta s, because angle between $e_t(s)$ and e_t(s plus delta S) is delta phi. Therefore, angle between the normal to these two vectors will also be delta phi. Therefore, this angle is delta phi and delta phi is equal to delta s divided by Os which is equal to delta s divided by R, where R is the radius of curvature.

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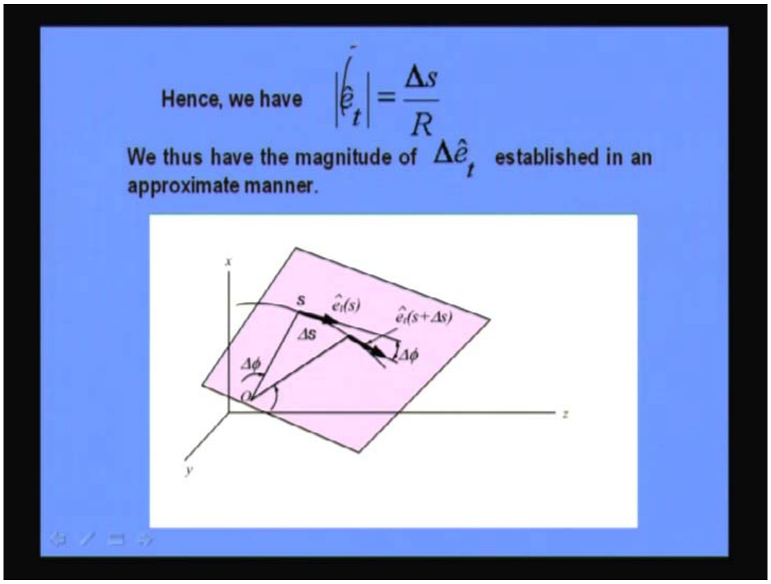

Hence, we have e_t is equal to delta S by R. We thus have the magnitude of delta e_t established in an approximate manner, that is delta et is equal to delta S by R. Therefore, we have established the magnitude of delta e_t in an approximate manner. In an approximate manner, delta e_t is nothing but delta S divided by R.

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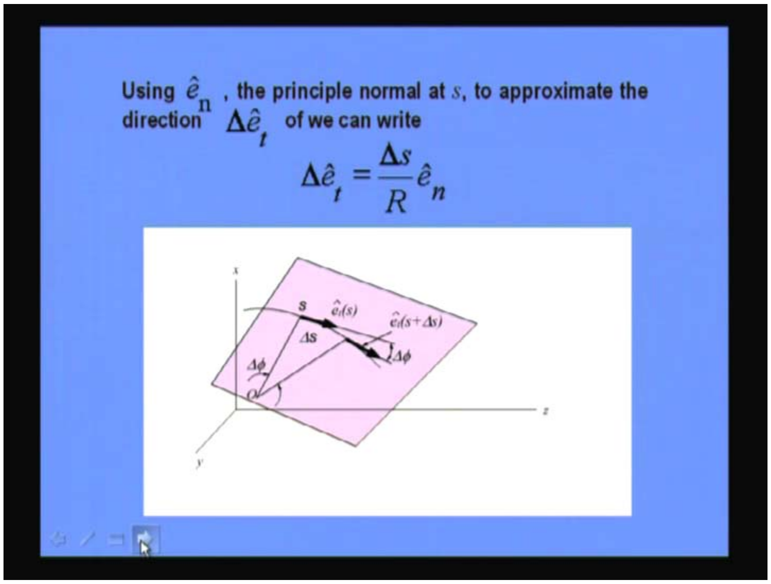

Using e_n, the principle normal at s, to approximate the direction delta e_t because we can write delta e_t is equal to delta s divided by R into e_n, where e_n is the unit vector normal to the tangent.

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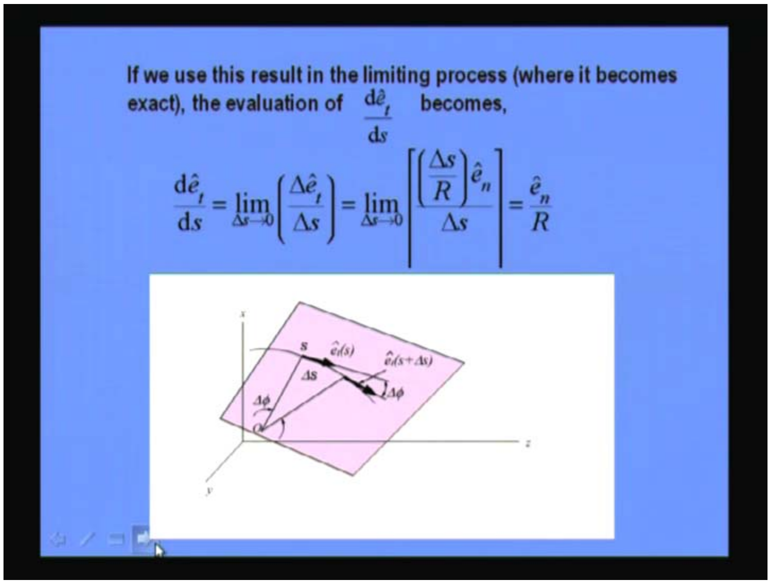

If we use this result in the limiting process where it becomes exact, the evaluation of d e_t by ds becomes d e_t by ds is equal to limit delta s tends to 0 delta e_t by delta s is equal to limit delta s tends to 0 delta s by R e_n divided by delta s which is equal to e_n by R. Therefore, we can say that d e_t by ds will be equal to e_n by R; that means if you differentiate the unit tangent vector with respect to s, if the magnitude of that differentiation is 1 by R where R is the radius of curvature and it is in the direction of principal normal that is e_n.

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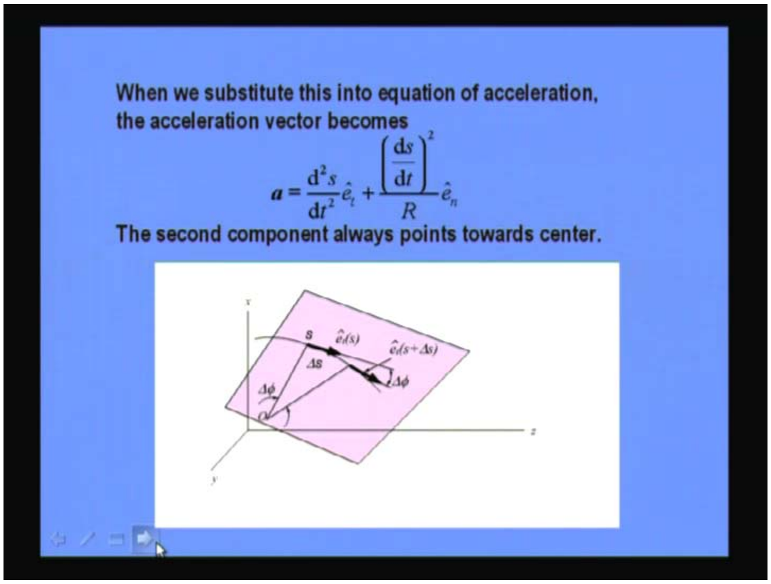

Therefore, when we substitute this into equation of acceleration, the acceleration vector becomes a is equal to d square s by dt square e_t plus ds by dt square divided by R into e_n. The second component always points towards center. It is to be noted that d square s by dt square can be a positive quantity or negative quantity. Therefore the tangential component of the acceleration can be positive or negative. However, the second expression shows that ds by dt square is always positive. So, the normal component is always directed towards the center of curvature.

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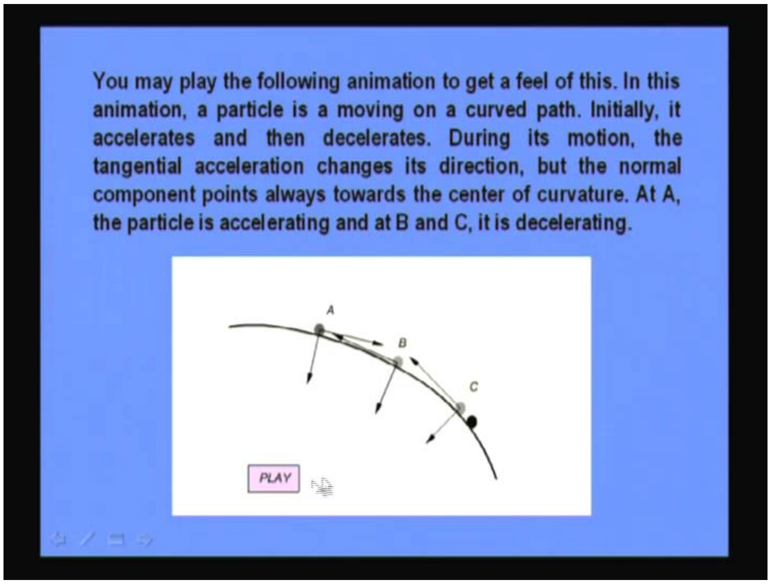

To show this point, this animation has been shown, displayed. Here, in this animation, the ball A moves on a curved path reaches point B then it reaches point C. In this case, here at this point A the particle A is accelerating; that means its speed is increasing. Therefore, the acceleration vector has been shown towards this side; that means, the normal vector is towards the center of curvature. At point B the particle decelerates; that means its speed starts decreasing. Therefore, the tangential component is along this direction. However, the normal component is still towards the center of curvature. At point C also the normal component is towards center of curvature. However, the tangential component is along opposite direction.

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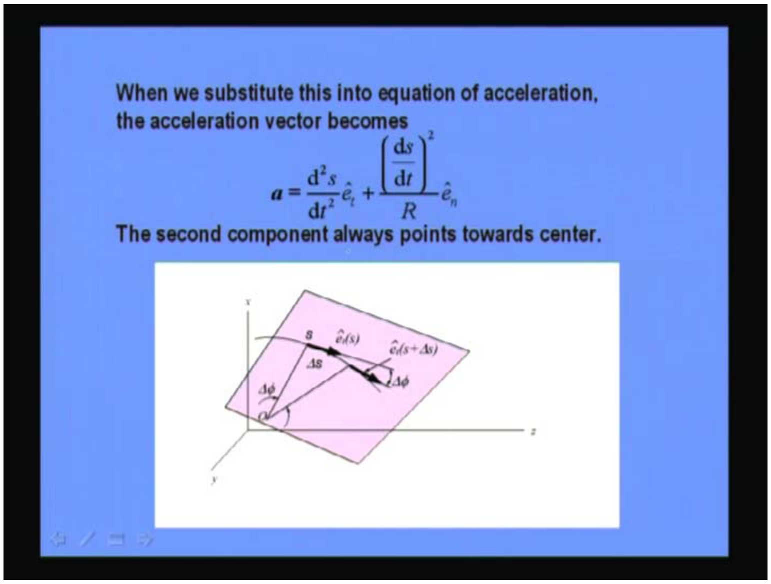

This expression is very important; that is a is equal to d square s by dt square e_t plus ds ds by dt square by R into e_n. We will be making use of this expression quite often.

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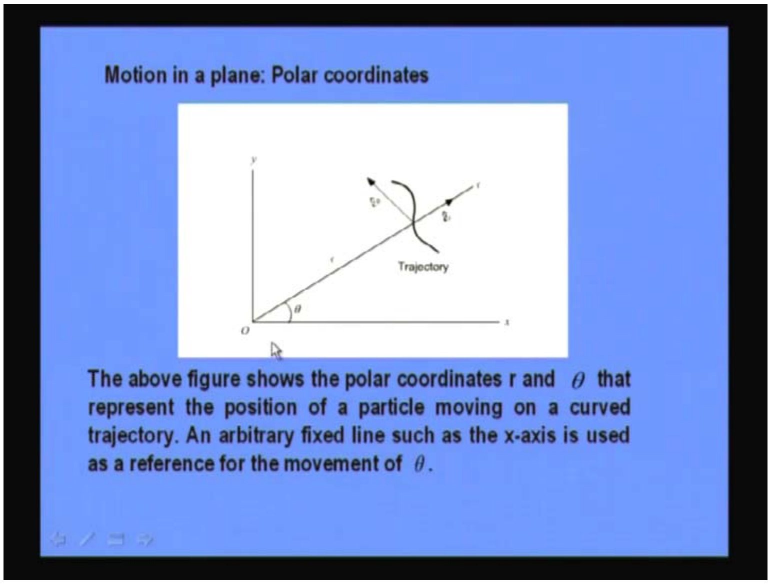

We discuss motion in a plane and use the polar coordinates. This is a curve that is shown here; that is the trajectory of the particle and its position can be indicated by xy coordinates or by r and theta; r is the radial distance of the particle from the center O and theta is the angle whose position vector makes with respect to fixed datum which has been chosen here as a x-axis. The polar coordinates r theta can be transformed in the form of x and y functions of x and y.

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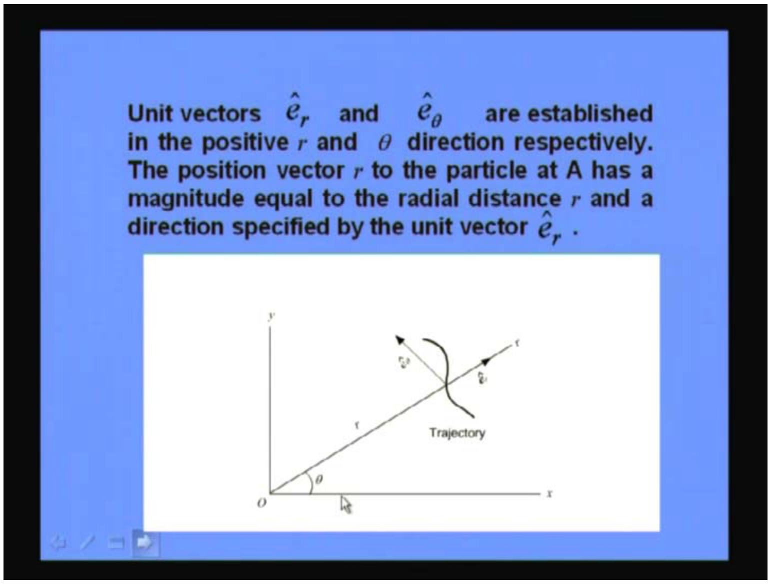

We can define unit vectors e_r and e_{theta} established in the positive r and theta direction respectively. So, unit vector e_r is along r direction and unit vector e theta is x_n. The position vector r to the particle a has a magnitude equal to the radial distance r and a direction is specified by the unit vector e_r, So, although at this point we have two unit vectors e_r into e_{theta}, when we say that position vector of the particle, it is expressed only in terms of this position vector is equal to r times $e_{r.}$ Theta terms do not come here at this stage. It is very easy to sometimes specify its position by just one quantity. Otherwise, we have to specify x, y both here. This is shown here.

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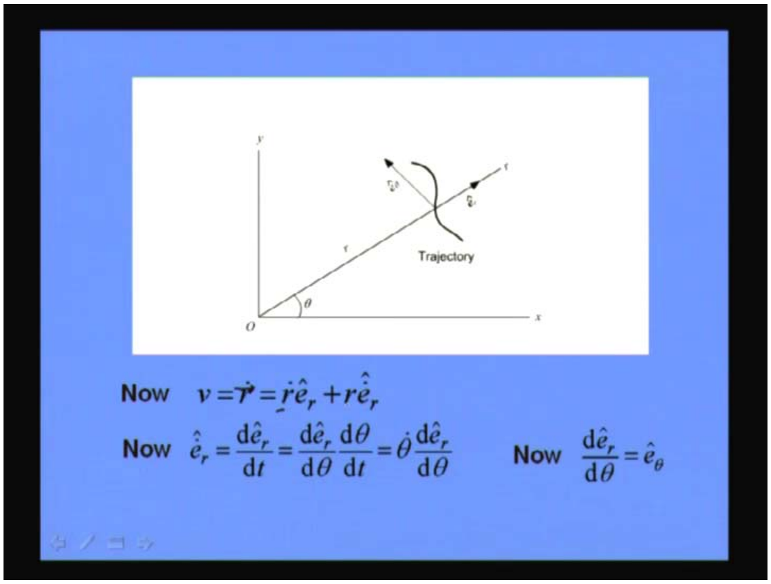

Once we know the position vector, we can differentiate position vector and obtain the velocity v is equal to r dot. Here, dot indicates the derivative with respect to time that is equal to this r is a vector that is r dot; that is, this is a scalar r into e_r plus r which is a scalar quantity. This is differentiation of the unit vector that is e_r t. Now, e dot r is basically dr by dt. Because the particle is moving, its radial position keeps changing. Therefore, dr by dt will not be 0. dr by dt can be written as d e_r by d theta into d theta by dt that is equal to theta dot dr by d theta. Now dr by d theta can be written as e_{theta}.

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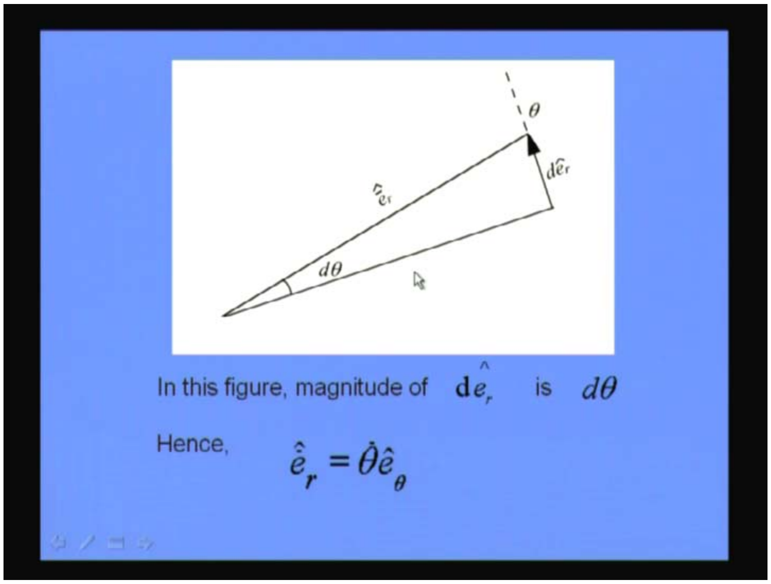

This point can be explained here. Now, supposing you have a unit vector e_r here and after a rotation of delta theta, unit vector goes to the next position. Difference is given by delta e_r in the unit; if delta theta is very small this can be written as d e_r only. So, in this figure magnitude of de_r is nothing but d theta because these are the unit vectors. So, unit vector times unit length into d theta. Hence, if we find out the derivatives, if we take the derivative of d e_r by dt it will be d theta by dt that is theta dot and the direction is along the theta direction. This can be explained very clearly that as d theta tends to 0, direction of dr will be towards perpendicular to e_r. Therefore, e dot r is equal to theta dot e_{theta}.

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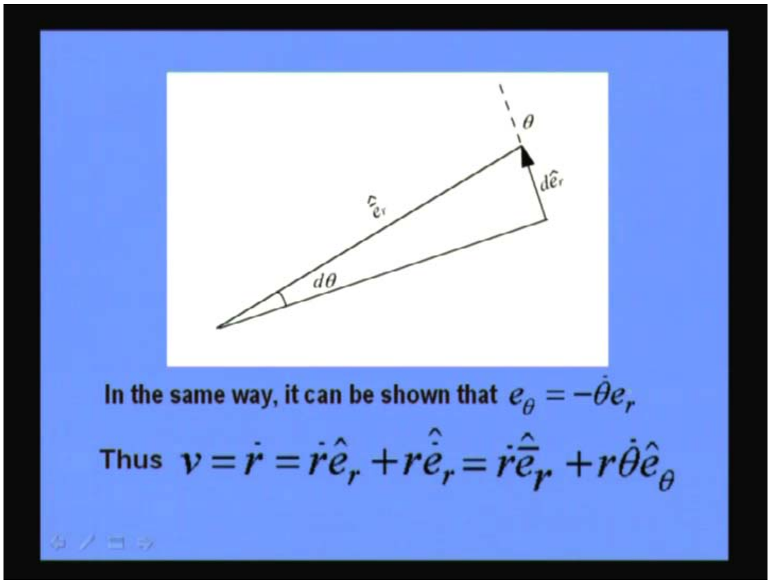

In the same way, it can be shown that e_{theta} is equal to minus theta dot $e_{r.}$ Here, minus term is equal to theta dot is the can be written as omega. Thus, velocity of the particle in polar coordinate is written as r dot which is r dot e_r plus r e dot r is equal to r dot e_r plus r theta dot e_{theta}. Therefore, velocity has two components; one component is along the radial direction and that is nothing but dr by dt change in the radial radius plus r, another component along the theta direction and that component r times theta dot.

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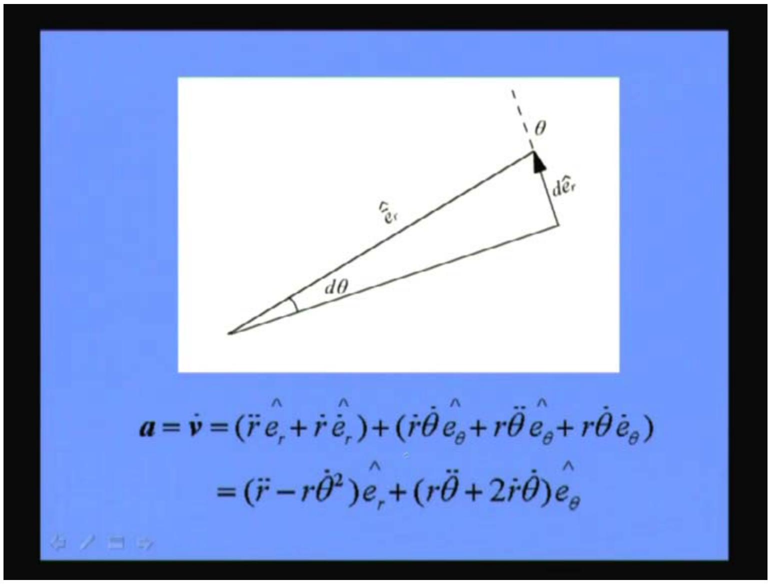

By differentiating this expression v with respect to time, we can obtain the expression for acceleration. Acceleration is given by a is equal to v dot which is r double dot e_r plus r dot e_r dot r plus r dot theta dot e_{theta} plus r theta double dot plus e_{theta} plus r theta dot e dot theta. Now e dot r can be written as theta dot e_{theta}. Similarly, we know e dot theta also can written as minus theta dot e_r. Therefore, this expression simplifies to r double dot minus r theta dot square e_r plus r theta double dot plus 2 r dot theta dot e_{theta}.

Unit vector e_r and unit vector e_{theta} both are orthogonal to each other. In this case, acceleration has two components; one component is r double dot minus r theta dot square, another component is r theta double dot plus two r dot theta dot e_{theta}. Here we can see that if the particle is moving on a circular path where its radius remains constant then r double dot will be 0. Even then, there will be a component of acceleration along the radial direction and that is equal minus r theta dot square. Similarly, there will be tangential components if the particle is moving with a constant angular speed then r theta double dot will be 0; but still if the particle is moving simultaneously in the radial direction, then there will be another component that is two r dot theta e_{theta}.

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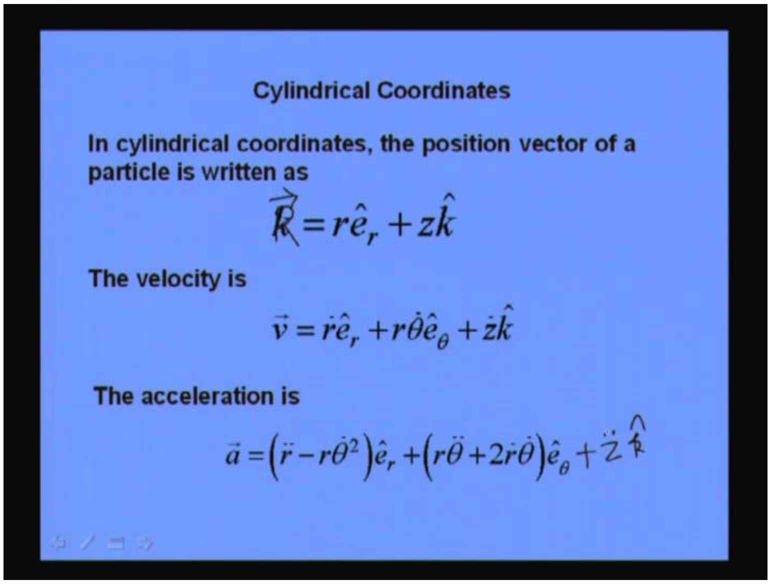

In the cylindrical co-ordinates the position vector of a particle is written as r times er plus zk. This R is equal to r times e_r plus zk. Therefore, the velocity is v is equal to r dot e_r plus r theta dot e_{theta} plus z dot k. The acceleration is given by r dot minus r theta dot square e_r plus r theta double dot plus two r dot theta dot e_{theta} plus z double dot k.

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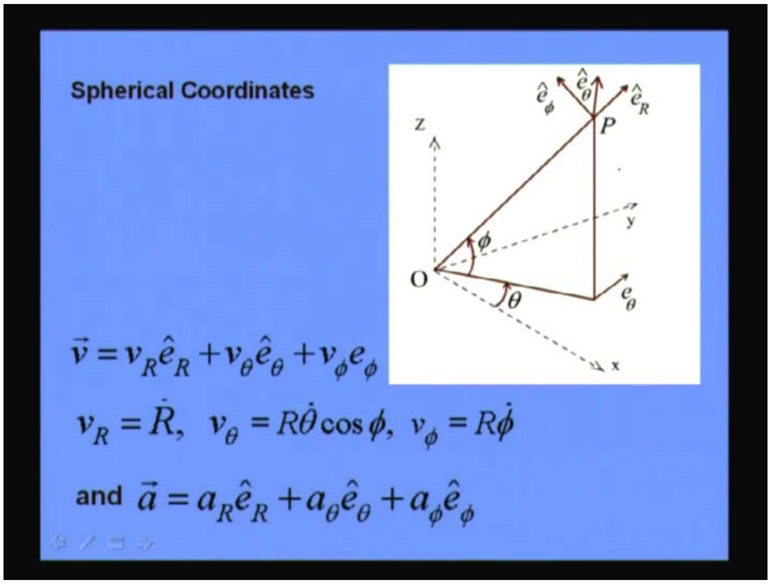

In spherical coordinates, we can imply the spherical coordinate also and get the position of a particle P. Here, three coordinate systems have been shown. This is Cartesian coordinate system and indicated by x y and z. Then we have cylindrical coordinates and we have spherical coordinates. In this, particle is at position P. In the cylindrical coordinates, its position is indicated by R that is z and theta; theta is the angle which the projection of the position vector of the points makes with the x-axis in the xy plane.

We have the position vector OP. Take its projection in xy plane. This is making angle theta here, that is this one. So, in the cylindrical coordinates, we have r theta z. Similarly, in the spherical coordinate one coordinate is OP; that is the radial distance of the point P from O. Then we have got theta, then we have got phi. In this case, in a spherical coordinate its velocity is indicated by v_R into e_R plus v_{theta} e_{theta} plus v_{phi} into e_{phi}, where e_R e_{theta} and e_{phi} are the unit vectors along R theta and phi directions. v_R is equal to R dot, v_{theta} is equal to R theta dot cos phi, v_{phi} is equal to R phi dot.

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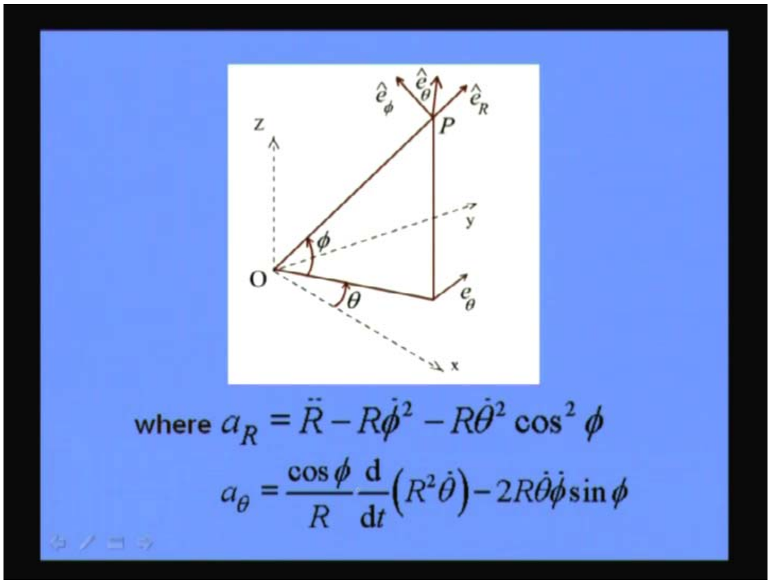

Similarly for acceleration, we have the expression acceleration a is equal to a_R into e_R plus a_{theta} into e_{theta} plus a_{phi} into e_{phi} where a_R is equal R double dot minus R phi dot square minus R theta dot square cos square phi and a_{theta} is equal to cos phi by R d by dt R square theta dot minus 2 R theta dot phi dot sin phi.

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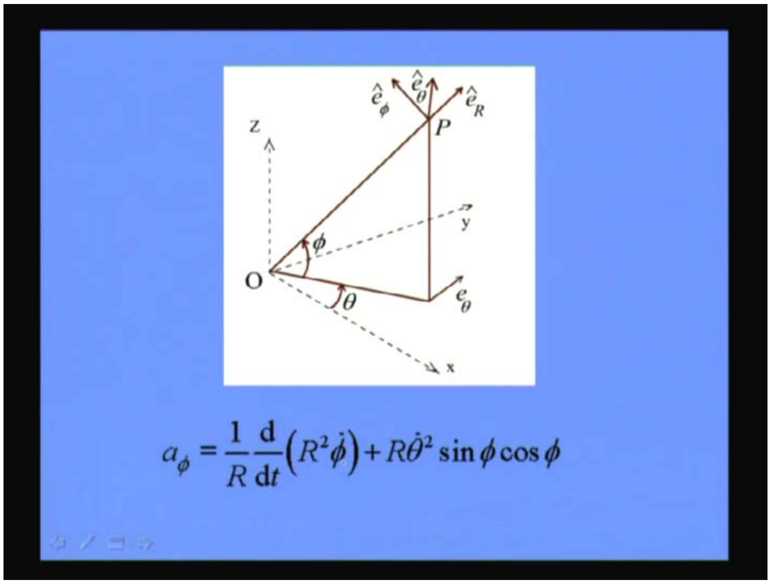

a_{phi} is given by 1 by R d by dt bracket R square phi dot plus R theta dot square sin phi cos phi. These expressions are convenient when the particle is moving in a spherical path and you are asked to find out the velocity and acceleration components. Now we will be doing one simple problem based on this lecture.

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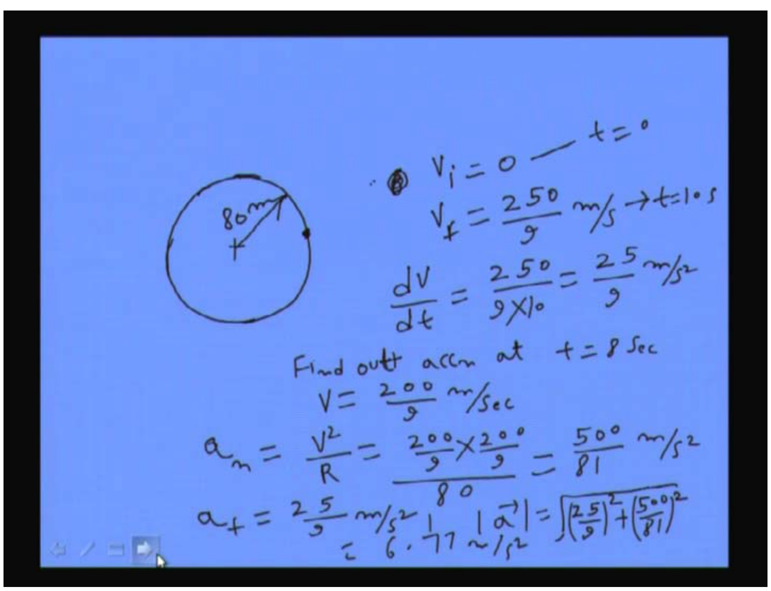

Let us consider the case of a particle moving on a circular path. Now, this particle is going from this point around the circle. If it has been provided to you that in the beginning its speed is 0, we initial it to 0 and its final speed is 250 by 9 meter per second. Then, it has been mentioned that this particle attains a speed of 250 by 9 meter per second in 10 seconds. Therefore, we know that dv by dt is equal to 250 by 9 by 10 that is 25 by 9 meter per second, provided the speed has been attained in an uniform manner. Total time from this V_i is equal to 0 and t is equal to 0; this is at t is equal to 10 seconds. If it is desired to find out the acceleration at t is equal to 8 seconds; that means find out acceleration at t is equal to 8 seconds. At 8, the velocity is attaining the value of 250 by 9 in 10 seconds in a uniform manner. Therefore, the velocity after 8 seconds will be equal to 25 into 8 by 9 that is 200 by 9.

V at that point will be 200 by 9 meter per second. Therefore, the normal component of the acceleration a_m will be equal to V square by R where R is the radius of the circle. Supposing, the radius of the circle has been given as 80 meter, then you have got the component that is 200 by 9 into 200 by 9 divided by 80 which comes out to be 500 by 81 meter per second square.

Then, there will be tangential component of this one. Tangential component 80, its magnitude will be nothing but dv by dt that is 25 by 9 meter per second square, this is per second square. Therefore, the magnitude of the acceleration a is equal to under root 80 square that is 25 by 9 square plus 500 by 81 square which after simplification, will come out to be 6.77 meter per second square. Now, let us discuss one or two more cases which will imply the concepts developed in this lecture.

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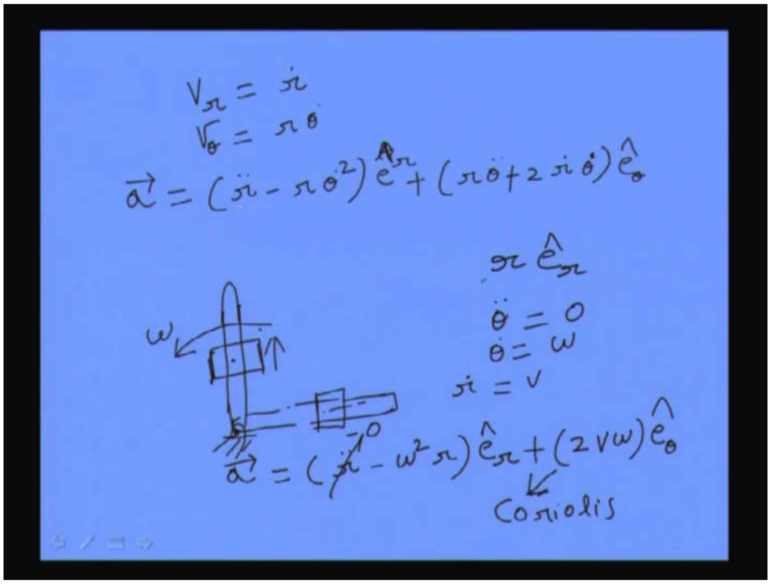

It was mentioned that the velocity component in polar coordinate has two components; V_r is equal to r dot and V_{theta} is equal to r theta dot. Similarly, the acceleration a can be written as r dot dot minus r theta dot square into e_R plus r theta dot dot plus 2 r dot theta dot. This is e theta, this is $e_{R.}$ Now, we will make use of this equation. Let us consider a problem in which there is a ring which is rotating about this thing in a counter clockwise direction with constant speed omega. On this ring, there is a block which slides. This block is sliding on the ring; that means it is going up and ring is rotating in the counter clockwise direction. You may assume that in the beginning the ring was somewhere here and the block may be at some position r.

After that block moves on the ring at that surface and at the same time its angular position is same as the angular position of the ring. Therefore, the block's position is indicated by r times that is e_R that you have. This is at any location you have radial; therefore, it is very convenient to indicate the position of the block just by cylindrical coordinates. This is unit vector e_r but unit vector keeps on rotating. So, it keeps on changing. In this case, just using the expression for acceleration we can see. If it is assumed that ring is rotating with a uniform angular velocity, then theta double dot will be 0, theta dot will be omega.

Then, the acceleration of the block which is moving will be written as a is equal to r dot dot minus omega square r e_r plus r theta double dot that is 0. Therefore, 2 and r dot can be written as

V, which is the relative velocity of the block on the ring. So it is two V times omega e_{theta}. Now that we are familiar with these components, we see that acceleration has got these two components; one is along the radial direction and another is along the theta direction. Along the radial direction, this is r dot dot but as we already know that r dot is equal to V, therefore r dot dot will be dv by dt.

If the blocks slide on the ring with a constant speed then r dot dot this will also go to 0 and you will be left with minus omega square r $e_{r;}$ that is there will be one acceleration whose magnitude will be omega square r and it will be directed towards negative r direction; that means towards center of O. This component is known as the centripetal acceleration particle. Apart from the centripetal acceleration, you also get another component in the theta direction that is 2V times omega. This component is called Cariole's component. When a block slides on a rotating ring, we get another component of the acceleration that is called Carioli's component and its magnitude is given by 2V times omega, where omega is the angular speed of the ring. This comes due to sliding.

If V is equal to 0, then this component will not be present. In general, there are four components here r double dot that is basically dv by dt; then you have centripetal portion r theta dot dot which comes due to angular acceleration. Then, there is a Carioli's component of acceleration.

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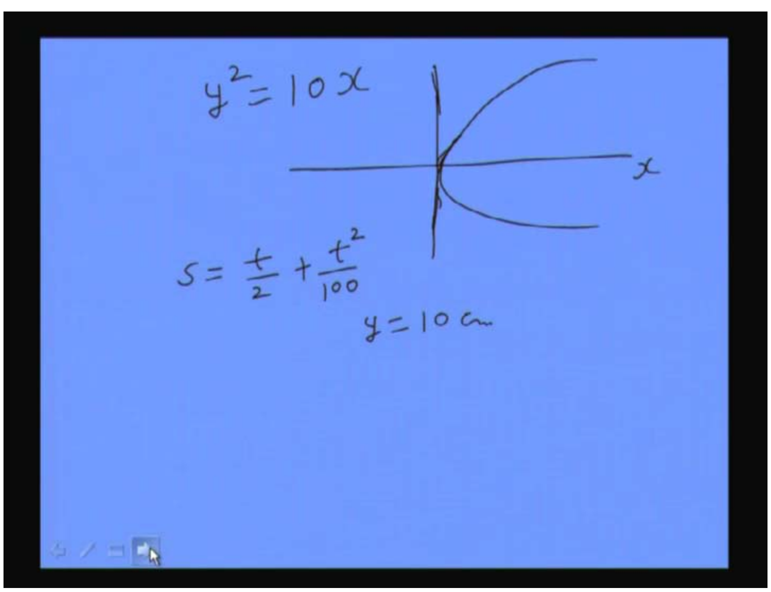

Let us discuss one more problem. It is given that y square is equal to 10x is certain path. This is the equation of parabola. x can be only positive, it cannot be negative because y square cannot be negative for real numbers; this is shown here like this. If it has been told that s is equal to t by 2 plus t square by 100 and it is desired to find out its acceleration when y is equal to 10, y is equal to 10 unit or maybe 10 centimeter; at that point how do you find out the acceleration? You have a known relation between s and t. s is the distance travelled by the particle. To solve this problem, we can find out the acceleration; one component of the acceleration, that is tangential component. We have developed these equations in the beginning. Let us go back to the previous that slides.

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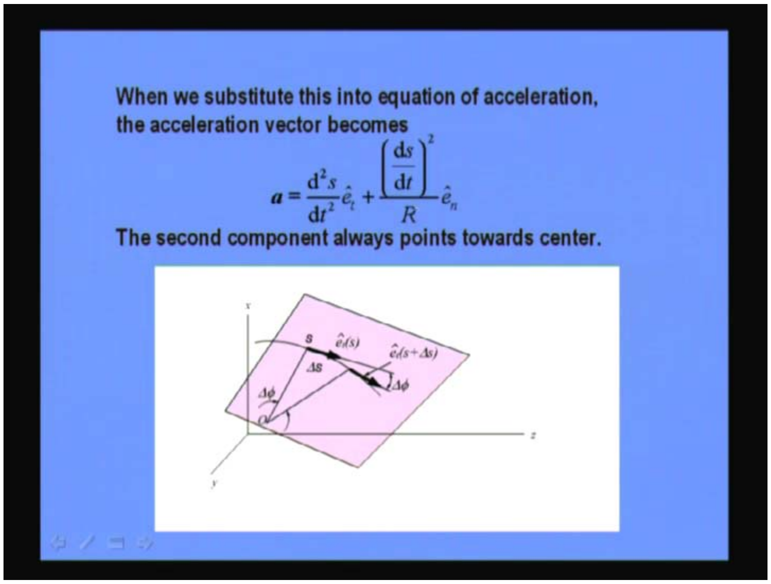

Here, we have finally established this equation d square s by dt square $e_{t.}$ If you know s as a function of time, it is very easy to differentiate this and tell that this is the tangential component. Similarly, this is ds by dt whole square. If you know the radius of curvature you can find out the other component. The point is that you have not been given time. Although you can take the derivative, you do not find out the value of the derivative. You need to know the time.

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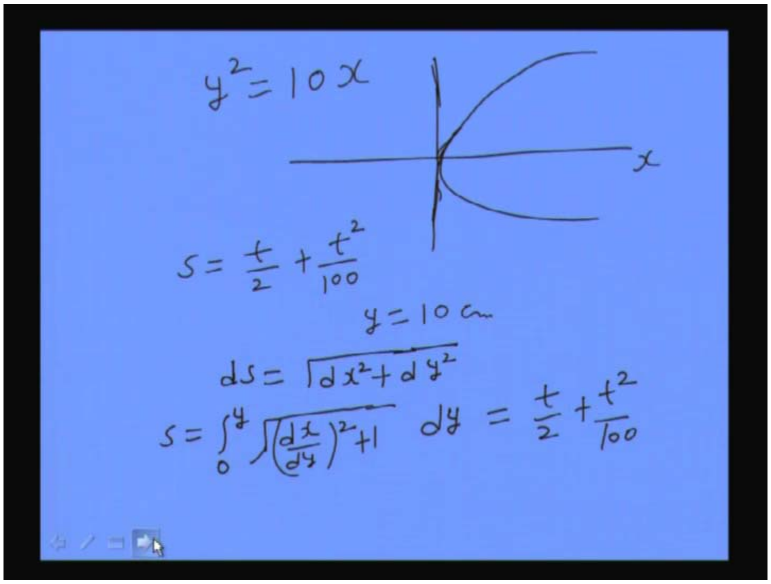

But it has been mentioned that y is equal to 10 centimeters at that point. So, how do we take at this problem? ds can be written as under root dx square plus dy square. Now, we can write s is equal to 0 to y under root dx by dy square plus 1 into dy, that is the expression for s.

dx by dy can be found out from this; because you know x, x is equal to x square y square by 10. You can find out dx by dy square plus 1 and then you can integrate and this can give you this thing. So, you get expression for s at that point. Then you know that this s is equal to t by 2 plus t square by 100.

Solving this equation, you can find out the value of t. As you have found the value of t, you can find out the component of acceleration. How do we find out the radius of curvature?

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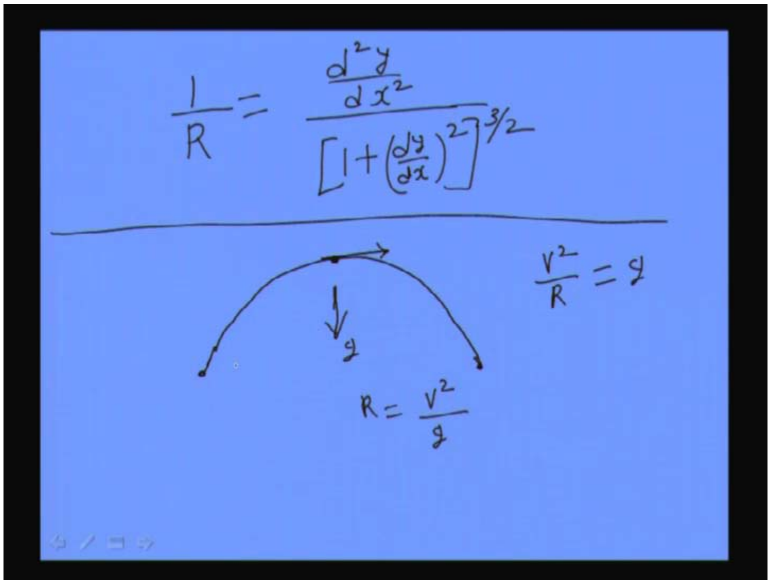

Radius of curvature for a planar curve is given by 1 by R which is called curvature, that is equal to d square y by dx square divided by 1 plus dy by dx whole square to the power 3 by 2. If you know the equation of y, we can always find out these things. So, this is like this. Now, we will discuss one more problem. You know in projectile motion, if a particle is thrown, if somebody asks you to find out the radius of curvature when the particle has moved to top position. For finding out that you know the velocity of the projectile at that horizontal velocity remains same. So, with whatever velocity you projected, the velocity will remain same. Now, V square by R is the normal component of acceleration, which is nothing but z at this point. So, this is equal to z.

Therefore, R is equal to V square by z where V is the horizontal component of the velocity of the projectile. Using that you can find out the radius of curvature. Although the radius of curvature could have been found by the equation of the parametric equation of this parabola.